# Piecewise linear trends and cycles in primary commodity prices

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**Abstract**

We extend the methodology put forward in Yamada and Yoon (2014, *Journal of International Money and Finance*, 42(C), 193-207) to analyze the trend and cyclical behavior of relative primary commodity prices. These authors propose the use of the so-called $\ell_1$-filter that renders piecewise linear trends of the underlying data. Our focus in on the calibration of such filter and its implications for the empirical analysis of primary commodity prices, especially the interpretation given to the resulting trend. We also illustrate how suitably calibrated filters may be used to compute piecewise linear (super) cycles, whose turning points are easy to identify.

**JEL Classification** : C22, E30, O13, Q02.
**Keywords** : Commodity prices, piecewise linear trends, Hodrick-Prescott filter, super cycles.

## 1 Introduction

The Prebisch-Singer hypothesis (PSH henceforth) plays a central role in the study of the evolution of primary commodity prices. It claims that the relative prices of primary commodities in terms of manufactures are driven by a secular downward trend. Since such declining terms of trade for primary exporters have profound policy implications, testing the validity of the PSH has always been a subject of great empirical interest. Since the early contributions of Cuddington and Urzua (1989) and Cuddington (1992), the bulk of the literature examines the PSH through the lens of unit root tests (see Ghoshray, 2011, for a recent account). It is decided whether a particular relative price should be modeled as a trend stationary or as difference stationary process, possibly subject to structural breaks. Then, a negative estimated time slope or drift is taken as supportive evidence on the PSH. See Cuddington et al. (2008) for a comprehensive survey.

Recently, Yamada and Yoon (2014) suggest an alternative, fresh view to assess the PSH. They propose a direct attack to the problem by estimating the trend component of relative prices using the $\ell_1$-filter advanced in Kim et. al (2009). This filter resembles in many ways the celebrated Hodrick and Prescott (1997) filter, but has the peculiarity that the resulting trend is piecewise linear. Thus, the changes in slope of the estimated trend, which are very easy to detect, reflect structural breaks in the underlying time series. This feature of the $\ell_1$-filter makes it very suitable for the empirical evaluation of the PSH, since the segments of the piecewise trend that are negatively sloped are to be associated with periods when the PSH prevails. With this technique, Yamada and Yoon conclude that the PSH holds “sometimes”.

Like the Hodrick and Prescott filter, the characteristics of the $\ell_1$-trend depend on a smoothing parameter $\lambda$. However, in clear contrast to the case of the Hodrick and Prescott filter, the literature on the $\ell_1$-filter is still incipient and a remaining open question, of considerable practical importance, is how to suitably calibrate $\lambda$. In their empirical exploration, Yamada and Yoon estimate $\ell_1$-trends for several relative commodity prices using the same value of $\lambda$. This gives the impression that a uniform filter is applied to different time series. Nevertheless, it is shown below that this is not the case, rendering the various estimated trends incomparable.

The first goal of this paper is to reexamine the evidence put forward by Yamada and Yoon, with special focus on the calibration of the $\ell_1$-filter and the interpretation of their estimated trends. To this end, we further exploit the analogies between the $\ell_1$-filter and the readily interpretable Hodrick and Prescott filter as a low-pass filter. In particular, the smoothing parameter of Hodrick and Prescott filter is tightly related to a cutoff period $T$, such that the estimated trend accounts for the movements of the time series of $T$ years or more. Such a direct interpretation is not possible for the $\ell_1$-filter because, we argue, a meaningful calibration of its smoothing parameter needs to be data-dependent and, hence, case-specific. It is argued, however, that the $\ell_1$-trend can be made close enough to a $T$-year Hodrick and Prescott trend, so that it may be interpreted as a $T$-year piecewise linear trend. Besides the robustness check to the results of Yamada and Yoon, we also apply the $\ell_1$-filter methodology to the extended database of relative commodity prices of Harvey et al. (2010).

The second goal is explore the notion of a super cycle *vis-á-vis* long-run trends in primary commodity prices, recently brought (back) to attention by Cuddington and Jerrett (2008) and Erten and Ocampo (2013), using the $\ell_1$-filter. It is well-known that a band-pass filter, which isolates the cyclical component of a series, can be constructed as the differences between two low-pass filters. Thus, using suitably calibrated $\ell_1$-filters we are able to obtain estimates of the cycles in primary commodities prices. Importantly, the derived cycles are also piecewise linear, a fact that facilitates enormously the identification of their turning points. It is worth mentioning that the slope of this analysis is limited to the characterization of the cycles. Important questions related to their causes and commonalities are left open for future research.

The remaining of this paper is organized as follows. Section 2 discusses methodological issues. Section 3 presents empirical results. Even though the details on individual relative prices may differ considerably from those reported in Yamada and Yoon, the main conclusion that the PSH holds sometimes remain. The same holds true for the Harvey et al. data. In both cases, short-run trends differ significantly from long-run trends, which provides evidence for super cycles in relative commodity prices. Their properties and turning points are also documented. Section 4 gives closing remarks.

## 2 Methodological issues

Let us introduce some notation. Let $\mathbf{z} \in \mathbb{R}^n$ be a $n$-vector whose $i$-th element is denoted by $z_i$. The length of $\mathbf{z}$ may be measured by alternative norms: namely, the Euclidean $\ell_2$-norm $\| \mathbf{z} \|_2^2 = \sum_i (z_i)^2$ or the $\ell_1$-norm $\| \mathbf{z} \|_1 = \sum_i |z_i|$. On the other hand, consider a $(n-2) \times n$ matrix $\mathbf{D}$ such that $\mathbf{Dz} \in \mathbb{R}^{n-2}$ is a $(n-2)$-vector whose entries correspond to the second differences of the entries of $\mathbf{z}$; the $i$-th element of $\mathbf{Dz}$ is $\Delta^2 z_i = z_{i+2} - 2z_{i+1} + z_i$ for $i = 1, 2, \ldots, n-2$.

### 2.1 The Hodrick-Prescott filter and its interpretation

By *trend* we refer to the smoothed version of a time series that isolates its medium- to long-run movements from short-run fluctuations (cycles, seasonality, noise, among others). The Hodrick and Prescott filter, henceforth $\ell_2$-filter, is admittedly the most popular univariate method in economics for trend estimation, and its properties have been thoroughly documented (see, inter alia, King and Rebelo, 1993).

Given an $n$-vector of data $\mathbf{y}$, the $\ell_2$-trend is obtained by minimizing

$$Q_2(\mathbf{x}, \lambda_2 \mid \mathbf{y}) = \| \mathbf{y} - \mathbf{x} \|_2^2 + \lambda_2 \| \mathbf{Dx} \|_2^2 \,, \tag{1}$$

with respecto to $\mathbf{x}$. Determining the $\ell_2$-trend, $\mathbf{x}_2$, can be regarded as a fitting problem, where the error $\| \mathbf{y} - \mathbf{x} \|_2^2$ is minimized, subject to smoothing constraints captured by the penalty term $\| \mathbf{Dx} \|_2^2$. The constant $\lambda_2$ controls the trade-off between goodness of fit and smoothness. The larger $\lambda_2$, the smoother the resulting $\ell_2$-trend. It is well-known that if $\lambda_2 \to 0$, then the $\ell_2$-trend converges to the original series, $\mathbf{x}_2 \to \mathbf{y}$, whereas at the other extreme, if $\lambda_2 \to \infty$, then $\mathbf{x}_2$ approaches the linear trend that fits the data best.

Since (1) is a quadratic function of $\mathbf{x}$, its minimizer is a linear function of $\mathbf{y}$, i.e. it can be expressed as $\mathbf{x}_2 = \mathbf{A}(\lambda_2)\mathbf{y}$, where $\mathbf{A}(\lambda_2)$ depends on $\lambda_2$ but does not depend on $\mathbf{y}$. This linearity, well understood by practitioners, is manifested when the data is subject to an affine transformation. More precisely, given $\lambda_2$, if $\mathbf{x}_2$ is the $\ell_2$-trend of $\mathbf{y}$, then $a\mathbf{x}_2 + c$ is the $\ell_2$-trend of $a\mathbf{y} + c$, where $a$ and $c$ are arbitrary scalars. In fact, it is simple to verify that $Q_2(a\mathbf{x} + c, \lambda_2 \mid a\mathbf{y} + c) = a^2 Q_2(\mathbf{x}, \lambda_2 \mid \mathbf{y})$, which is proportional to $Q_2(\mathbf{x}, \lambda_2 \mid \mathbf{y})$. Thus, the objective functions of the original and transformed problems lead to the same filtering operation $\mathbf{A}(\lambda_2)$.

Furthermore, the linearity of the $\ell_2$-filter allows us to study its properties in the frequency domain (see, inter alia, Kaiser and Maravall, 1999; Gómez, 2001). The $\ell_2$-filter is a low-pass filter, designed to preserve variation in the data associated to low frequencies (in the time domain, the medium-to-long term) and to attenuate or to remove high-frequency (short-term) variation. This aspect of the filter is quite useful to calibrate the value of $\lambda_2$ and to give a precise interpretation to the estimated trend. For data sampled $s$ times per year ($s = 1$, annual; $s = 4$, quarterly; $s = 12$, monthly) and a *cutoff period* of $T$ years, setting

$$\lambda_2 = \frac{1}{16} \sin\left(\frac{\pi}{Ts}\right)^{-4} \tag{2}$$

produces a trend that can be interpreted as the *component of the series that isolates the series fluctuations of T years or more*, whereas the residual series $\mathbf{y} - \mathbf{x}_2$ captures variation of less than $T$ years (see Gómez, 2001). In the business cycles literature, where the $\ell_2$-filter is extensively used, the popular choice of $\lambda_2 = 1600$ for quarterly data ($s = 4$) is associated with a cutoff period of $Ts = 39.7$ quarters or $T = 9.9$ years, which reflects the consensus that fluctuations in economic data beyond approximately 10 years are to be attributed to trend developments (see King and Rebelo, 1993; Kaiser and Maravall, 1999). The same cutoff period of 9.9 years corresponds to $\lambda_2 = 6.6$ for annual data ($s = 1$) and $\lambda_2 \simeq 129000$ for monthly data ($s = 12$). Since $\lambda_2$ is an increasing function of $T$, then the $\ell_2$-trend is unsurprisingly smoother for larger values of $T$.

### 2.2 Data dependency of the $\ell_1$-filter

Kim et. al (2009) propose an interesting variation to the Hodrick and Prescott filter. In particular, the $\ell_2$-norm in the penalty term in (1) is replaced by an $\ell_1$-norm, so the $\ell_1$-trend is obtained by minimizing

$$Q_1(\mathbf{x},\lambda_1 \mid \mathbf{y}) = \| \mathbf{y} - \mathbf{x} \|_2^2 + \lambda_1 \| \mathbf{D}\mathbf{x} \|_1 . \tag{3}$$

The constant $\lambda_1$ is also a smoothing parameter, and Kim et. al (2009) show that like the $\ell_2$-filter, the $\ell_1$-trend converges to the original data vector, $\mathbf{x}_1 \to \mathbf{y}$, when $\lambda_1 \to 0$, whereas it converges to the least squares linear trend as $\lambda_1 \to \infty$. However, unlike the $\ell_2$-trend which is a continuous function of time, the $\ell_1$-trend is a piecewise function of time that connects $k+1$ linear segments, where $k$ is the number of "structural breaks" in the series. Note that $k$ is expected to decrease with $\lambda_1$. See Yamada and Jin (2013) and Yamada and Yoon (2014) for further discussion on the workings of this filter.

A remaining open question is how to suitably calibrate $\lambda_1$. This task is complicated by the fact that, unlike the $\ell_2$-trend, the $\ell_1$-trend is a *nonlinear* function of $\mathbf{y}$, schematically $\mathbf{x}_1 = \mathbf{B}(\lambda_1,\mathbf{y})\mathbf{y}$ where $\mathbf{B}(\lambda_1,\mathbf{y})$ depends on $\lambda_1$ *and* $\mathbf{y}$, which prevents us to obtain clear-cut expressions like (2) for this filter. The main difficulty seems to be that an adequate calibration of $\lambda_1$ *needs to be data-dependent*.

This can be illustrated in several ways. For instance, Kim et. al (2009) show that the $\ell_1$-trend becomes invariant to $\lambda_1$, and equal to the least squares linear trend, for all $\lambda_1 \geq \lambda_{\max} = \| (\mathbf{D}'\mathbf{D})^{-1}\mathbf{D}'\mathbf{y} \|_\infty$, where $\| \mathbf{z} \|_\infty = \max_i |z_i|$ is the $\ell_\infty$-norm. That $\lambda_{\max}$ depends on $\mathbf{y}$ points out, of course, to the data-driven nature of the choice of $\lambda_1$. On the other hand and more formally, it is not difficult to verify that if $\mathbf{x}_1$ is the $\ell_1$-trend of $\mathbf{y}$, then $a\mathbf{x}_1 + c$ is *not* the $\ell_1$-trend of $a\mathbf{y} + c$ *unless* $\lambda_1$ changes to reflect the transformation of the data. Indeed, now we have $Q_1(a\mathbf{x}+c,\lambda_1 \mid a\mathbf{y}+c) = a^2 Q_1(\mathbf{x},\lambda_1/a \mid \mathbf{y})$, which is not proportional to $Q_1(\mathbf{x},\lambda_1 \mid \mathbf{y})$. Thus, given $\lambda_1$, the objective functions for the original and transformed problems imply *different* filtering operations, respectively $\mathbf{B}(\lambda_1,\mathbf{y})$ and $\mathbf{B}(\lambda_1/a, a\mathbf{y}+c) \neq \mathbf{B}(\lambda_1, a\mathbf{y}+c)$.

### 2.3 Calibration of the $\ell_1$-filter

When $\lambda_1$ is set in an arbitrary fashion, the nonlinearity of the $\ell_1$-filter hinders the interpretability of the estimated trend. Given the analogies with the $\ell_2$-filter that originally motivated the $\ell_1$-filter, a sensible course of action is to choose $\lambda_1$ such that the resulting $\ell_1$-trend, whose properties are obscure, resembles an $\ell_2$-trend whose properties as a low-pass filter are well understood. Put it differently, since the $\ell_2$-trend with parameter $\lambda_2$ amounts to a $T$-year trend, where $T$ is implicitly defined in (2), then the "closest" $\ell_1$-trend might well be interpreted as a $T$-year piecewise linear trend. This is the approach followed in the empirical section below.

To elaborate, let $\mathbf{x}_2(\lambda_2)$ denote the $\ell_2$-trend that uses $\lambda_2$ as a smoothing parameter, and $\mathbf{x}_1(\lambda_1)$ denote the $\ell_1$-trend that uses $\lambda_1$ as a smoothing parameter (to avoid clutter, we leave the dependency on $\mathbf{y}$ implicit). In the literature, a *same fitting error* criterion is used. Define

$$E(\lambda_1,\lambda_2) = \| \mathbf{y} - \mathbf{x}_2(\lambda_2) \|_2^2 - \| \mathbf{y} - \mathbf{x}_1(\lambda_1) \|_2^2 . \tag{4}$$

Kim et. al (2009) determines the value of $\lambda_2$ that makes $E(\cdot,\cdot) = 0$ for a given $\lambda_1$. It can be shown that $E(\lambda_1,\lambda_2)$ is a continuous function of $\lambda_2$, with $E(\lambda_1,0) < 0$ and $\lim_{\lambda_2\to\infty} E(\lambda_1,\lambda_2) > 0$ for $\lambda_1 \in (0,\lambda_{\max})$. Thus, a value $\lambda_2^*$ such that $E(\lambda_1,\lambda_2^*) = 0$ can always be found.

On the other hand, Yamada and Jin (2013) conduct the converse exercise and seek to determine the value of $\lambda_1$ that makes (4) equal to zero for a given $\lambda_2$. Here, we have that for a positive and finite $\lambda_2$, $E(0,\lambda_2) > 0$ and $E(\lambda_1,\lambda_2) < 0$ for any $\lambda_1 \geq \lambda_{\max}$. However, given that the number of linear segments in $\mathbf{x}_1(\lambda_1)$ may change discreetly with a marginal change in $\lambda_1$, $E(\lambda_1,\lambda_2)$ may not be continuous in $\lambda_1$. Hence, a suitable value of $\lambda_1$ in this case may be, for instance, a minimizer of $|E(\lambda_1,\lambda_2)|$, which brings (4) as close to zero as possible. Such minima always exist and satisfy $\lambda_1^* \in (0,\lambda_{\max})$.[1]

To sum up, the $\ell_1$-trend that minimizes the above criterion *given* $\lambda_2$ may be interpreted as a $T$-year piecewise

[1] An alternative criterion would be to minimize the discrepancies between both trends directly $D(\lambda_1,\lambda_2) = \| \mathbf{x}_2(\lambda_2) - \mathbf{x}_1(\lambda_1) \|_2^2$. The function $D(\cdot,\cdot)$ is also continuous in $\lambda_2$, with a positive finite global minimum, and it may also be discontinuous in $\lambda_1$ with local minima $0 < \lambda_1 < \lambda_{\max}$. In practice, the results found by minimizing either (4) or $D$ are likely to be similar.

linear trend. Conversely, *for a given* $\lambda_1$, the value of $\lambda_2$ found by minimizing the above criterion can be used to determine the cutoff period $T$ that can be associated to the piecewise linear trend.

### 2.4 Cycles

Following Cuddington and Jerrett (2008) and Erten and Ocampo (2013), it is of interest to explore whether primary commodity relative prices exhibit long-lasting cycles. It is well-known that a band-pass filter, which isolates series variation within a frequency window (i.e., the cycle), can be constructed as the difference between two low-pass filters (see inter alia Gómez, 2001). Thus, the $\ell_1$ cyclical component of $\mathbf{y}$ can be estimated simply as

$$\mathbf{c} = \mathbf{x}_1(\lambda_1^{\text{ST}}) - \mathbf{x}_1(\lambda_1^{\text{LT}}), \tag{5}$$

where $\mathbf{x}_1(\lambda_1^{\text{ST}})$ and $\mathbf{x}_1(\lambda_1^{\text{LT}})$ are, respectively, estimated "short-term" and "long-term" $\ell_1$-trends: as discussed previously, $\lambda_1^{\text{ST}}$ is the smoothing parameter associated to a $T^{\text{ST}}$-year trend, whereas $\lambda_1^{\text{LT}}$ corresponds to a $T^{\text{LT}}$-year trend. Cuddington and Jerrett (2008) suggest that super cycles are associated to a series component lasting from twenty to seventy years, whereas long-run trends should be defined as a component whose variation lasts at least seventy years. Thus, we set $T^{\text{LT}} = 20$ and $T^{\text{LT}} = 70$.

A particularly appealing feature of the estimated cyclical component $\mathbf{c}$ is that, just like its constituent $\ell_1$-trends, it is piecewise linear. Therefore, the identification of turning points and the measurement of cycle durations are straightforward. The leading algorithm for dating cycles is that of Bry and Boschan (1971). Most of the algorithm deals with the noisiness of the data, and incorporates a number of smoothing operations that are redundant for an already smooth estimate such as $\mathbf{c}$. Thus, our approach to identify turning points in $\mathbf{c}$ takes only a few steps from the original algorithm. In particular, if $c_i$ denotes the $i$-th element of $\mathbf{c}$, then...

... a peak occurs in period $i$ if $c_i \geq c_{i-h}$ and $c_i \geq c_{i+h}$ for $h = 1, 2, \ldots, H_P$, and $c_i > K_P$,

... a trough occurs in period $i$ if $c_i \leq c_{i-h}$ and $c_i \leq c_{i+h}$ for $h = 1, 2, \ldots, H_T$, and $c_i < -K_T$,

... peaks and troughs alternate.

A peak is defined as a local maximum within a window of width $2H_P$. Also, the peak should be significant by exceeding a predetermined threshold $K_P$. A trough follows an analogous definition. Finally, by convention, alternation is enforced: a peak is to be followed by a trough and vice versa. In our empirical exploration, we set $H_P = H_T = 20$ years, and $K_P = K_T = 10\%$ of the standard deviation of the cycle-plus-noise component $\mathbf{y} - \mathbf{x}_1(\lambda_1^{\text{LT}})$. Surely, different setups of the algorithm would render different set of turning points; yet, these choices lead to very sensible dates that can even be visually detected.

## 3 Results

In this section we use the $\ell_1$-filter to illustrate various issues on its calibration, to assess the PSH and to estimate super cycles. We use two datasets well-known in the literature, accounting to a total of 49 series of relative primary commodity prices (many of them may refer to the same relative commodity price but measured differently). The output of our estimations is abundant. We present relevant statistics in tables for all time series, but to save space in the figures we display results only for selected prices. The complete output is available as an online supplement to this paper.

### 3.1 Extended Grilli and Yang (EGY) data

We first present some interpretations and robustness checks to Yamada and Yoon (2014). We use the same annual data, over the period from 1900 to 2010 ($n = 111$ observations), for 24 primary commodities (11 foodstuff, 7 nonfood soft commodities and 6 metals). The dataset is the major extension of the popular Grilli and Yang (1988) dataset documented in Pfaffenzeller et. al (2007).[2] The series of interest are 100 times the logarithm of the ratio of the prices of each commodity to a manufacturing unit value index.

### Trends and the PSH

Table 1 presents the results, where the commodity prices are sorted as in Yamada and Yoon. Figure 1 shows the evolution of selected relative prices (two food, two nonfood and two metals) and their $\ell_1$-trends under various calibrations.

The first block of Table 1 replicates the results in Yamada and Yoon, which are obtained by applying the $\ell_1$-filter to each of the 24 series after setting $\lambda_1 = 1000$.[3] As discussed above, even though this may look like uniform filtering of the series, it is not. For each series, the value of $\lambda_2$ that made the discrepancy (4) equal to zero was computed (column "$\lambda_2$") and so the corresponding cutoff period (column "$T$"). It can be seen that $\lambda_2$ and $T$ vary widely, with $T$ ranging from below 40 years (for lamb, rubber, tin and silver) to more than 70 years (for cotton and zinc).

On the other hand, the figures in the column labeled "$k$" represents the number of slope changes in the estimated piecewise linear trends, as reported in Yamada and Yoon. It is common to find episodes where a single underlying change in the data produces various shifts in the piecewise linear trends. Most of them are surely transitional (or not very significant in magnitude) and hence we also report, in braces, a cleansed number that restricts the duration of the linear segment to be more than 5 years. Thus, $k + 1$ can be regarded as the number of regimes in the sample, as estimated by the $\ell_1$-filter.

Finally, the column "PSH" reports the proportion of years where the $\ell_1$-trend is negatively sloped, as a simple measure of the prevalence of the PSH. These proportions also vary considerably, averaging 0.55 across commodities.

The second block of the table shows the results of an exercise closer to uniform filtering. Yamada and Yoon's trends have an average cutoff period of 53 years. Thus, the 50-year $\ell_2$-trend was computed for each series (using $\lambda_2 = 4021$) and the corresponding $\ell_1$-trend was obtained by minimizing $E$. The column "$\lambda_1$" presents the resulting smoothing constants. Their variation with respect to the benchmark $\lambda_1 = 1000$ mirrors the differences between the values of $\lambda_2$ implicit in Yamada and Yoon's calculations and $\lambda_2 = 4021$. As expected, the number of structural breaks in the piecewise linear trends, $k$, is smaller [larger] for commodities whose $\lambda_1$ are greater [less] than 1000. Curiously, on average $k$ is the same as in Yamada and Yoon's baseline results. Lacking of a better explanation, we conjecture that this equivalence is driven by the fact that both sets of results have, on average, the same cutoff period.

The conclusions regarding the incidence of the PSH are robust. Out of 24 commodity prices, the proportions of negatively slope trends are significantly different, between the baseline and the 50-year $\ell_1$-trend, in only 5 instances: higher in the case of cocoa, jute and zinc, and lower in the case of maize and cotton. These proportions average 0.57 across commodities.

Following the definitions for short-term and long-term trends suggested by Cuddington and Jerrett (2008), the last two blocks of Table 1 display the results for the 20 and 70-year trends. Note that since the PSH is concerned to long-term trends, attention should be paid to the latter. In this point, the inconvenience of the data-dependent nature of the calibration of $\lambda_1$ arises again. The only conclusion we can safely reach about this smoothing parameter is that, compared to the 50-year trend, it should be smaller for the 20-year trend and larger for the 70-year trend. The $E$ criterion answers by how much.

Figure 2 presents the differences graphically. It shows the periods when the slopes of the piecewise linear

[2] Publicly available at `www.stephan-pfaffenzeller.com`.

[3] **Important:** There are some minor differences in implementation with respect to Yamada and Yoon (2014). First, the data in our case is 100 times Yamada and Yoon's. Second, we compute the $\ell_1$-filter by minimizing (3), using the freely available codes provided by Kim et. al (2009) at `www.stanford.edu/~boyd/l1_tf`. In our notation, Yamada and Yoon would minimize $Q_{YY} = \| \mathbf{y} - \mathbf{x} \|_2^2 + 50\lambda_{YY} \| \mathbf{D}\mathbf{x} \|_1$ instead. Thus, the value of $\lambda_{YY} = 20$ used in their paper corresponds to $\lambda_1 = 1000$. Despite these marginal differences, we were able to replicate Yamada and Yoon's results exactly.

trends are found to be negative for each commodity. The inspection of the 70-year trends provides support for the PSH in all the sample period for 5 commodities (rice, sugar, palmoil, rubber and aluminium), and for 3 commodities (wheat, maize and wool) in most of the sample. On the other hand, the PSH does not hold at all for 3 commodities (beef, lamb and timber), and barely holds in 4 cases (coffee, tin, lead and zinc). For the remaining 9 commodities (cocoa, tea, banana, cotton, jute, hide, tobacco, copper and silver), the PSH holds "sometimes", in about half the sample. Relative to the results in Yamada and Yoon, the PSH statistic is almost identical for 10 commodities, lower for 5 and higher for 9. Thus, our results would be slightly more supportive of the PSH.

#### Super cycles

The notion of a super cycle can be informally tested by comparing the properties of the 20 and 70-year trends in Table 1. In particular, it is found that the PSH statistic is significantly different in most cases (17 out of 24) which points out to an important long-lasting cyclical component in these series. These discrepancies are apparent in Figure 2. To explore this issue more deeply, Table 2 reports the turning points associated to each relative commodity price, identified using the algorithm described in section 2.4, whereas Figure 4 shows the evolution of the cyclical component of selected relative prices.

Our findings are in agreement with those previously documented in the literature (e.g., Cuddington and Jerrett, 2008; Erten and Ocampo, 2013). It is argued that the presence of super cycles in primary commodity prices is a demand-driven phenomenon fueled by the surge of large industrial economies. In recorded modern history, prolonged expansive phases in commodity prices have been associated to the American industrialization (late 19th century), the post-war reconstruction of Europe and Japan (mid 20th century), and the Chinese vigorous economic expansion (late 20th century). The median number of peaks and troughs identified by the $\ell_1$-filter is 2, which indeed suggests the presence of two cyclical waves during the sample period (the 20th century). The median durations of a contractionary phase, i.e. the transition from peak to trough, and of an expansionary phase, i.e. the transition from through to peak, are both about 20 years, suggesting a cycle duration of 40 years. Moreover, most prices produce a trough surrounding 1920 (the end of the American industrialization momentum), a peak during the 1950s (the reconstruction of Europe), and a trough by the late 1980s (the beginning of the Chinese boost).

Surely, aggregation hides certain distinctive patterns among group of commodities. Food prices tend to have shorter cycles (a median durantion of 30 years) than metals (50 years), with other soft commodities being an intermediate case (25 years). Furthermore, the contractionary phase is much longer in food prices (a median of 17 years against a median of 13 years of the expansionary phase), whereas the expansionary phase lasts more in metals (a median duration of 25 years against 22 years of the peak to trough transition).

Figure 4 shows the proportion of commodities in each group that, in a given period, are experiencing an expansionary phase. The cyclical behavior is apparent and so are the commonalities across commodity groups. Erten and Ocampo (2013) conclude that the World GDP cycle is a good predictor of commodity prices cycles, in both cases agricultural goods and metals, which explains the high correlation in these cycles. More interestingly, however, a glance at the Figure suggests that cycles in food prices tend to lead the cycles in metal prices. To the best of my knowledge, such phenomenon has been overlooked by the literature and seems to be an interesting open question for future reseach.

### 3.2 Harvey, Kellard, Madsen and Wohar (HKMW) data

Next, we repeat the analysis but employing the much longer dataset of 25 relative commodity prices constructed by Harvey et al. (2010).[4] Again, the series of interest are 100 times the logarithm of the commodity prices deflated with a manufacturing value-added price index. The dataset ends in 2005. For eight commodity prices (beef, coal, gold, lamb, lead, sugar, wheat and cotton) it begins in 1650; it begins in 1670 for cotton, 1673 for tea, 1687 for rice and silver, 1709 for coffee, 1741 for tobacco, 1782 for pig iron,

[4] The data are publicly available at the *Review of Economics and Statistics Dataverse* (`http://hdl.handle.net/1902.1/15039`).

1800 for cocoa, copper and hide, 1808 for tin, 1840 for nickel, 1850 for aluminium, 1853 for zinc, 1859 for oil, and 1900 for banana and jute. Thus, depending on data availability, the number of observations ranges between $n = 106$ and $n = 356$.

Relative to the EGY dataset, it excludes 4 commodities (maize, palmoil, rubber and timber) but includes 5 different commodities (coal, gold, pig iron, nickel and oil). Whenever both datasets are comparable (basically, the 20th century, from 1900 to 2005), the main differences lie on the way the deflator (a manufacture price index) is computed, with the index in the EGY data having a lower growth rate. Thus, on *a priori* grounds the Grilli and Yang data is expected to provide less supporting evidence of the PSH.

### Trends and the PSH

Table 3 and Figure 6 present the results, where the commodity prices are sorted according to data availability. Figure 5 shows the evolution of selected relative prices (two food, two nonfood and two metals) with information for the complete 1650-2005 period, and their 20-year and 70-year $\ell_1$-trends.

For the sake of comparison to the results obtained with the EGY dataset, Table 3 presents computations for the PSH statistic using all available information, and for the subsample that corresponds to the 20th century (in the column labeled "PSHXX"). Focusing on the 70-year trend to assess the PSH, the aggregate conclusions reached with the 20th century data are qualitative the same as before. The average PSH statistic is 0.57 across commodities, and the evidence for the PSH is local and not global. The inspection of the 70-year trends provide support for the PSH for 4 commodities (wheat, wool, pig iron and aluminium) in all the subsample period, and for 2 commodities (lamb and copper) in most of the subsample. The PSH does not hold at all only for oil, and it barely holds for 5 commodities (beef, coal, gold, tin and jute). For the remaining 13 commodities, the PSH holds "sometimes", in about half the sample period.

Out of the 20 common commodities in the EGY and HKMW databases, the EGY results are more supportive of the PSH only for 2 commodities (rice and sugar), whereas the HKMW data provides more empirical support for the PSH in 8 cases (coffee, wheat, beef, lamb, wool, copper, lead and zinc). Most of these discrepancies are explained by differences in the sources of information and measurement methods, as well as the aforementioned differences in the manufacture price index used as a deflator. Yet, in the case of wheat, lead and copper a further and important source of discrepancy is that EGY dataset includes the period 2006-2010, where these particular relative commodity prices increased substantially, thereby affecting the slope of the $\ell_1$-trends near the end of the sample. On the other hand, in the case of sugar the (negative) slope of its $\ell_1$-trend with the HKMW data is much steeper, given the influence of higher prices during the end of the 19th century. This makes the change in slope around the 1940s to be more pronounced than with the EGY data, to the extend that it becomes positive for about 3 decades.

The longer span of the HKMW allows us to evaluate whether the conclusions for the 20th century also hold in a broader historical perspective. The corresponding average PSH statistic in Table 3 is 0.55, an average that nets out the fact that, compared to all available information, the 20th century is more supportive of the PSH in 7 cases, and less supportive in 6 cases. The first group includes a moderate but sustained decrease in the relative prices of coal, rice and coffee from the mid 18th century to the late 19th century, and a sharp fall in the relative prices of sugar, nickel, zinc and oil from the mid to the late 19th century. The second group includes a sustained increase during the 19th century in the relative price of lamb, wool, pig iron and copper, and since the mid 18th century to the late 19th century in the relative price of wheat and cotton. All these comparisons are apparent in Figure 6.

### Super cycles

The differences between the 20 and 70-year trends reported in Table 3 point out to the present of a cyclical component in these series. Table 4 reports the turning points associated to each relative commodity price, and Figure 7 shows the evolution of the cyclical component of selected relative prices.

As before, for the sake of comparison, we first focus on the events of the 20th century. The number of coincidences between the turning points identified with the EGY and HKMW datasets is remarkable. The dates of most of these turning points are in agreement between both datasets, sometimes displaying minor differences of few years. There are, of course, some important exceptions. Namely, a peak in the price of lead in 1952 identified in the HKMW dataset, but dubbed as insignificant in the EGY data; a trough in the price of silver in 1943, which is identified in 1960 in the EGY data; a peak in the price of tobacco in 1960, identified much later, in 1984, with the EGY data; a trough in the price of aluminum in 1993 in the EGY which is not identified in the HKMW data; and a trough in the price of banana in 1999 in the EGY data, dubbed as insignificant in the HKMW data. Despite the differences, the similarity of the results suggest that the $\ell_1$-filter is able to produce piecewise lineal cycles that are robust to measurement errors.

As stressed by Harvey et al. (2010), the mere existence of super cycles implies that a century long data may be too short to adequately assess the properties of such cycles. The availability of various centuries of observations in the HKMW dataset, thus, gives us the opportunity to perform such task. When compared the full sample figures in Table 4 with those corresponding to the 20th century a clear pattern emerges: with very few exceptions (the cases of tin and to a lesser extent copper and wool), the duration of the cycles has decreased significantly in the 20th century. Put it differently, the cyclical behavior in commodity markets has accentuated relatively recently in history. These dynamics coincide with the more frequent surge of industrial powers, since the mid 19th century. Figure 8 illustrates, through the proportion of commodity prices experiencing an expansionary phase, how super cycles have become faster by commodity group. Unlike Figure 4, the longer dataset allows us to depict the super cycle derived from the American industrialization in the 19th century.

## 4 Concluding remarks

We have extended the methodology put forward in Yamada and Yoon (2014) to analyze the behavior of relative primary commodity prices. Our focus has been the calibration of the $\ell_1$-filter, that renders piecewise linear trends of the underlying data. In particular, it has been argued that the evaluation of the PSH should be made on a suitably calibrated long-term trend (in our case, associated to a cutoff period of 70 years). Even though we did find important differences for individual prices, the aggregate qualitative conclusion of Yamada and Yoon, that the PSH holds locally rather than globally, remains robust.

Furthermore, we have extended the analysis by estimating and characterizing super cycles in relative primary commodity prices, using the difference of suitably calibrated $\ell_1$-filters to approximate the workings of a band-pass filter. An attractive feature of this method is that the resulting cycle is also piecewise linear, which simplifies considerably the identification of turning points. With this approach, we find clear evidence of super cycles in almost all series analyzed, which, in agreement with Cuddington and Jerrett (2008), Harvey et al. (2010) and Erten and Ocampo (2013), highlights the importance of these unobserved component in primary commodity prices. We reckon an interesting avenue for future research is to further enquire the drivers and cross-correlations amongst these cycles.

On the other hand, a revealing finding in Yamada and Yoon (2014) that is also present in our estimations, is that if the underlying trend of relative primary commodity prices is to be modeled as a broken linear trend, then the number of slope changes to be considered should be, at least, moderate (see Tables 1 and 3). This has important implications for the literature on unit root tests applied to primary commodity prices. It is well-known that such tests have low power (i.e., the propensity of favoring the presence of unit roots in the data) against unmodeled structural breaks in the alternative hypothesis. Since most of the literature focus on unit root tests allow for, at most, 2 structural breaks under the alternative hypothesis (see inter alia Ghoshray, 2011), their conclusions may be misleading. Furthermore, the evidence of super cycles in these series may also affect the power of unit root tests, which would tend to confound the inherent persistence of a long-lasting, but stationary, cyclical phenomenon with unit root nonstationarity. Thus, it seems promising to explore more powerful unit root procedures that may take into account the possibility of several slope changes and super cycles, at least as applied to primary commodity prices.

Finally, it is worth mentioning that the piecewise linearity of the $\ell_1$-trend makes it an interesting tool for future research in various empirical economic branches, beyond the PSH. For instance, Kim et. al (2009) compute the trend of the S&P 500 index to identify bearish and bullish episodes in the stock market, whereas Yamada and Jin (2013) use the filter to determine the output gap in Japan. We hope our discussion on calibration to be a useful guidance for future applications.

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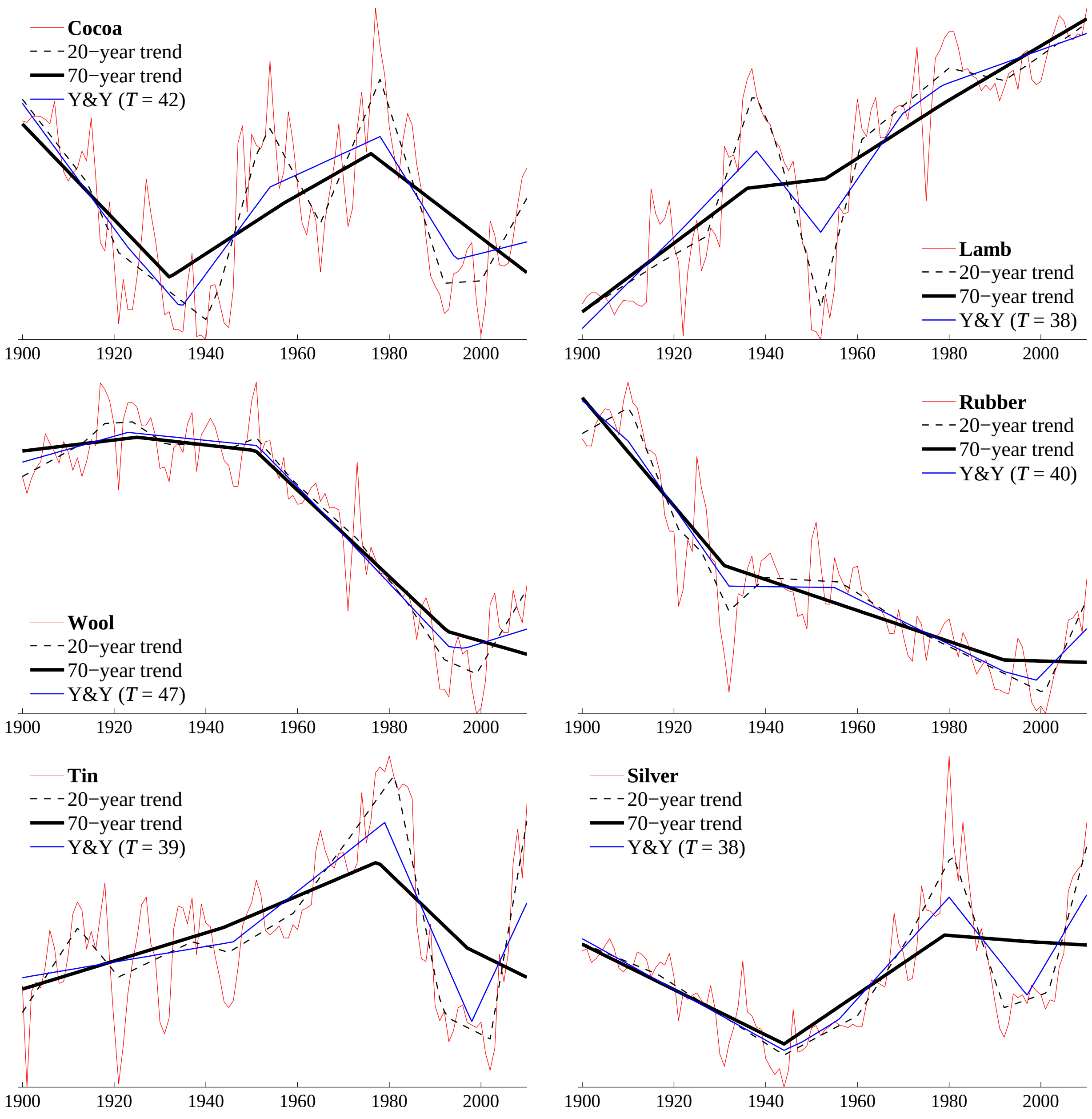
**Table 1.** *Estimation results (EGY dataset)*

| | | Yamada and Yoon ($\lambda_1$ = 1000) | | | 50-year trend ($\lambda_2$ = 4021) | | | 20-year trend ($\lambda_2$ = 104) | | | 70-year trend ($\lambda_2$ = 15426) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $T$ | $\lambda_2$ | $k$ | PSH | $\lambda_1$ | $k$ | PSH | $\lambda_1$ | $k$ | PSH | $\lambda_1$ | $k$ | PSH |
| Coffee | 50 | 3980 | 6 [5] | 0.64 | 1004 | 6 [5] | 0.64 | 156 | 19 [11] | 0.52‡ | 1719 | 4 [3] | 0.30‡* |
| Cocoa | 42 | 1959 | 7 [5] | 0.47 | 1511 | 5 [5] | 0.61† | 168 | 13 [9] | 0.59 | 2793 | 4 [3] | 0.60 |
| Tea | 54 | 5635 | 4 [3] | 0.64 | 848 | 4 [3] | 0.64 | 108 | 15 [8] | 0.67 | 1451 | 4 [3] | 0.47‡* |
| Rice | 52 | 4843 | 4 [4] | 0.91 | 926 | 4 [4] | 0.91 | 158 | 12 [9] | 0.51‡ | 1457 | 4 [2] | 1.00‡* |
| Wheat | 65 | 11152 | 5 [3] | 0.74 | 670 | 7 [5] | 0.74 | 119 | 11 [9] | 0.57‡ | 1089 | 4 [4] | 0.73 * |
| Maize | 59 | 7915 | 5 [4] | 0.85 | 769 | 7 [5] | 0.75† | 115 | 12 [7] | 0.50‡ | 1180 | 5 [4] | 0.85‡* |
| Sugar | 50 | 3937 | 6 [5] | 0.63 | 1008 | 6 [5] | 0.63 | 314 | 10 [7] | 0.55 | 1363 | 3 [3] | 1.00‡* |
| Beef | 43 | 2175 | 4 [3] | 0.35 | 1751 | 3 [2] | 0.35 | 181 | 10 [8] | 0.42 | 3069 | 3 [3] | 0.00‡* |
| Lamb | 38 | 1342 | 7 [5] | 0.13 | 1746 | 3 [3] | 0.14 | 195 | 12 [8] | 0.25‡ | 2407 | 3 [3] | 0.00‡* |
| Banana | 66 | 12287 | 5 [3] | 0.53 | 536 | 7 [5] | 0.55 | 84 | 12 [9] | 0.50 | 1108 | 5 [2] | 0.52 |
| Palmoil | 59 | 7688 | 4 [3] | 0.69 | 747 | 7 [5] | 0.69 | 132 | 12 [6] | 0.49‡ | 1432 | 5 [3] | 1.00‡* |
| Cotton | 73 | 18216 | 2 [2] | 0.84 | 649 | 5 [4] | 0.68† | 100 | 16 [9] | 0.60 | 929 | 4 [3] | 0.66 |
| Jute | 58 | 7260 | 6 [4] | 0.32 | 809 | 8 [4] | 0.51† | 100 | 17 [9] | 0.47 | 1445 | 3 [2] | 0.44 |
| Wool | 47 | 3194 | 7 [4] | 0.66 | 1109 | 6 [3] | 0.65 | 78 | 13 [10] | 0.64 | 2168 | 5 [3] | 0.77‡* |
| Hide | 56 | 6186 | 4 [3] | 0.63 | 789 | 5 [3] | 0.65 | 140 | 9 [8] | 0.66 | 1347 | 4 [3] | 0.62 |
| Tobacco | 54 | 5522 | 5 [3] | 0.40 | 745 | 5 [3] | 0.39 | 75 | 12 [9] | 0.35 | 1688 | 4 [3] | 0.40 |
| Rubber | 40 | 1596 | 5 [5] | 0.90 | 1351 | 5 [4] | 0.89 | 235 | 11 [7] | 0.75‡ | 2509 | 2 [2] | 1.00‡* |
| Timber | 64 | 10785 | 3 [3] | 0.00 | 615 | 4 [4] | 0.00 | 123 | 11 [8] | 0.22‡ | 1169 | 3 [3] | 0.00 * |
| Copper | 42 | 1938 | 4 [4] | 0.58 | 1340 | 5 [4] | 0.57 | 152 | 11 [8] | 0.69‡ | 2033 | 3 [3] | 0.58 * |
| Aluminum | 60 | 8595 | 2 [2] | 1.00 | 644 | 5 [4] | 1.00 | 134 | 12 [8] | 0.54‡ | 3021 | 1 [1] | 1.00 * |
| Tin | 39 | 1503 | 4 [3] | 0.17 | 1641 | 4 [3] | 0.18 | 137 | 15 [9] | 0.35‡ | 2224 | 4 [3] | 0.30‡ |
| Silver | 38 | 1290 | 8 [4] | 0.55 | 1613 | 5 [3] | 0.56 | 240 | 10 [7] | 0.50 | 2167 | 3 [3] | 0.68‡* |
| Lead | 41 | 1833 | 6 [4] | 0.40 | 1421 | 4 [3] | 0.38 | 131 | 12 [9] | 0.76‡ | 2373 | 4 [3] | 0.35 * |
| Zinc | 76 | 21455 | 2 [2] | 0.28 | 608 | 6 [5] | 0.51† | 161 | 7 [7] | 0.50 | 895 | 4 [3] | 0.28‡* |
| Average | 53 | | 5 [4] | 0.56 | | 5 [4] | 0.57 | | 12 [8] | 0.53* | | 4 [3] | 0.57 |

**Notes:** The number of observations is $n = 111$. "$k$" is the number of slope changes of the $\ell_1$-trend, and in braces the number of significant changes (each linear segment lasts at least 5 years). "PSH" is the proportion of years where the $\ell_1$-trend is negatively sloped. In the first block, "$\lambda_2$" is the value of $\lambda_2$ that makes $E(1000, \lambda_2) = 0$, whereas "$T$" is the corresponding cutoff period, using equation (2). In the remaining blocks, "$\lambda_1$" is the value of $\lambda_1$ that minimizes $|E(\lambda_1, \lambda_2)|$.

$z$-scores were computed to test $H_0 : p = \pi$, $z = (p - \pi)/\sqrt{\pi(1-\pi)/n}$. "†" denotes rejection of $H_0$ ($|z| > 2$) for $p$ = PSH in the 50-year trend and $\pi$ = PSH in Yamada and Yoon's calibration; "‡" denotes rejection of $H_0$ for $p$ = PSH in the 20 or 70-year trend and $\pi$ = PSH in the 50-year trend; "$*$" denotes rejection of $H_0$ for $p$ = PSH in the 70-year trend and $\pi$ = PSH in the 20-year trend.

**Figure 1.** *Selected relative primary commodity prices and piecewise linear trends (EGY dataset)*




**Notes:** The (red) thin lines are 100 times the logarithm of the relative commodity price. The remaining lines are the piecewise linear trends computed for the values of $\lambda_1$ given in Table 1.

**Figure 2.** *Periods of negatively sloped piecewise linear trends (EGY dataset)*

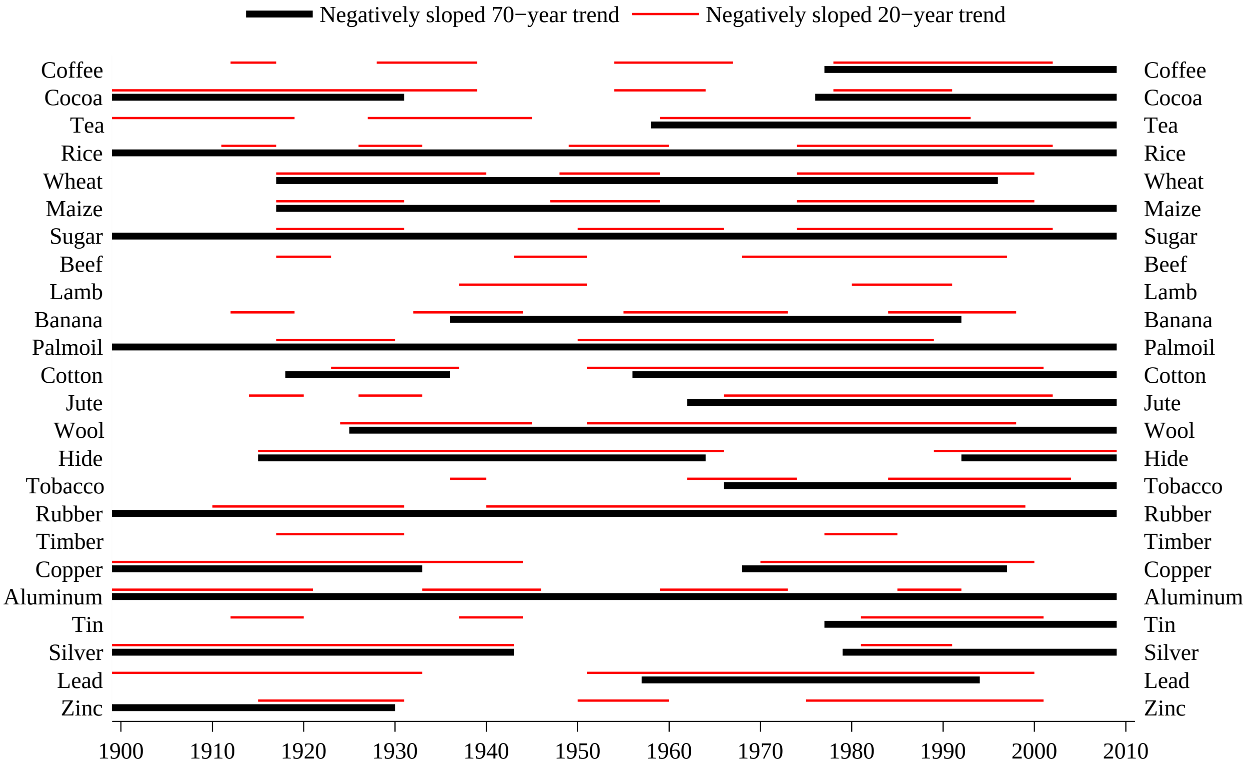
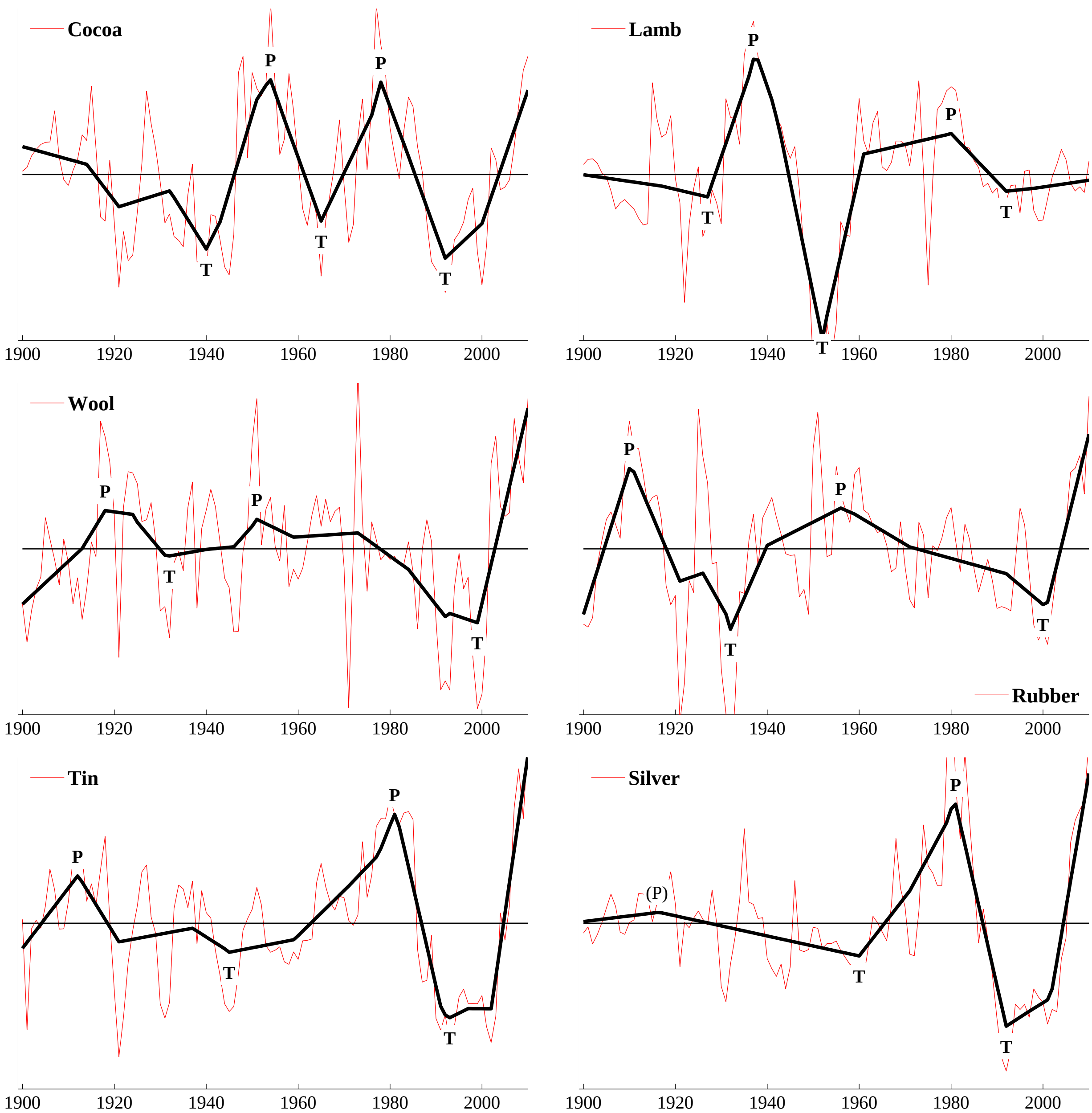


**Table 2.** *Turning points and duration of super cycles (EGY dataset)*

| | | Peaks ($P$) | | Troughs ($T$) | Mean duration in years | | | |
|---|---|---|---|---|---|---|---|---|
| | $N_P$ | Dates | $N_T$ | Dates | $P \to P$ | $T \to T$ | $T \to P$ | $P \to T$ |
| Coffee | 3 | 1928, 1954, 1978 | 3 | 1940, 1968, 1991 | 25 | 26 | 12 | 13 |
| Cocoa | 2 | 1954, 1978 | 3 | 1940, 1965, 1992 | 24 | 26 | 14 | 13 |
| Tea | 2 | 1927, 1960 | 3 | 1920, 1946, 1993 | 33 | 37 | 11 | 26 |
| Rice | 2 | 1911, 1974 | 2 | 1934, 1987 | 63 | 53 | 40 | 18 |
| Wheat | 3 | 1917, 1948, 1974 | 3 | 1941, 1960, 1991 | 29 | 25 | 11 | 18 |
| Maize | 3 | 1917, 1947, 1974 | 3 | 1932, 1960, 1990 | 29 | 29 | 15 | 15 |
| Sugar | 3 | 1920, 1950, 1974 | 3 | 1932, 1967, 1986 | 27 | 27 | 13 | 14 |
| Beef | 2 | (1917), 1942, 1968 | 3 | 1924, 1952, 1998 | 26 | 37 | 17 | 20 |
| Lamb | 2 | 1937, 1980 | 3 | 1927, 1952, 1992 | 43 | 33 | 19 | 14 |
| Banana | 3 | 1911, 1932, 1955 | 3 | 1920, 1944, 1999 | 22 | 40 | 12 | 22 |
| Palmoil | 2 | 1917, (1950), 1979 | 2 | 1931, (1961), 1990 | 62 | 59 | 48 | 13 |
| Cotton | 2 | 1923, 1951, (1977) | 2 | 1938, (1969), 2002 | 28 | 64 | 13 | 33 |
| Jute | 2 | 1914, 1966 | 2 | 1934, 2003 | 52 | 69 | 32 | 29 |
| Wool | 2 | 1918, 1951 | 2 | 1932, 1999 | 33 | 67 | 19 | 31 |
| Hide | 3 | 1915, 1949, 1989 | 2 | 1923, 1967 | 37 | 44 | 24 | 13 |
| Tobacco | 2 | 1921, 1984 | 3 | 1913, 1941, 2005 | 63 | 46 | 26 | 21 |
| Rubber | 2 | 1910, 1956 | 2 | 1932, 2000 | 46 | 68 | 24 | 33 |
| Timber | 2 | 1917, 1942, (1977) | 2 | 1909, 1932, (1967), (2001) | 25 | 23 | 9 | 15 |
| Copper | 2 | 1916, 1970 | 2 | 1945, 2001 | 54 | 56 | 25 | 30 |
| Aluminum | 2 | 1933, 1985 | 3 | 1922, 1947, 1993 | 52 | 36 | 25 | 11 |
| Tin | 2 | 1912, 1981 | 2 | 1945, 1993 | 69 | 48 | 36 | 23 |
| Silver | 1 | (1916), 1981 | 2 | 1960, 1992 | | 32 | 21 | 11 |
| Lead | 1 | (1951), 1979 | 2 | 1934, (1961), 2001 | | 67 | 45 | 22 |
| Zinc | 2 | 1915, 1950, (1975) | 2 | 1933, (1961), 2002 | 35 | 69 | 17 | 35 |

**Notes:** The number of observations is $n = 111$. $N_P$ is the number of peaks and $N_T$ is the number of troughs, identified as explained in section 2.4. The dates in parenthesis are turning points eliminated due to insignificance or alternation.

**Figure 3.** *Piecewise cyclical components of selected relative primary commodity prices (EGY dataset)*




**Notes:** The (red) thin lines are deviations of the actual data from its 70-year trend. The (black) thick lines depict deviations of the 20-year trend from the 70-year trend, our measure of cycle. Peaks are labeled with “P” and troughs with “T”. (P) or (T) indicate turning points that were eliminated due to insignificance or alternation.

**Figure 4.** *Proportion of relative primary commodity prices in expansionary phases (EGY dataset)*

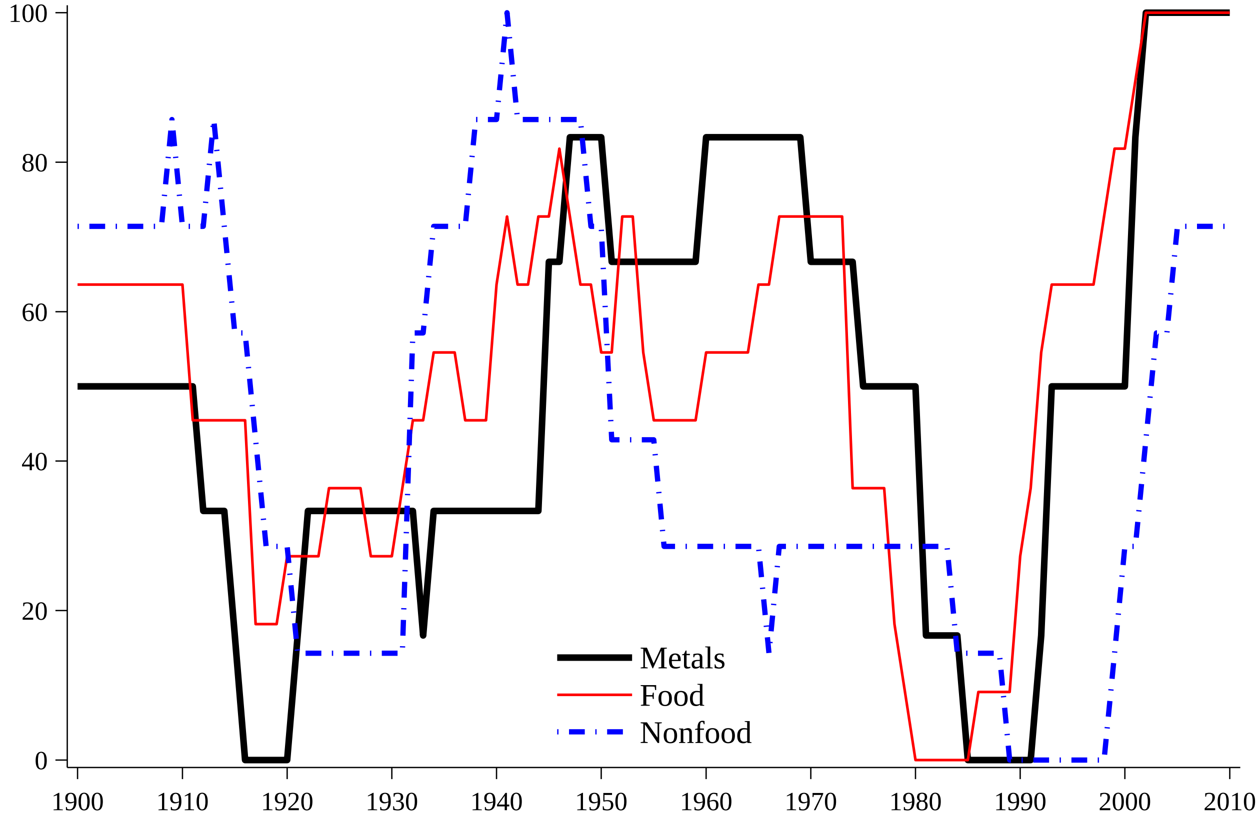


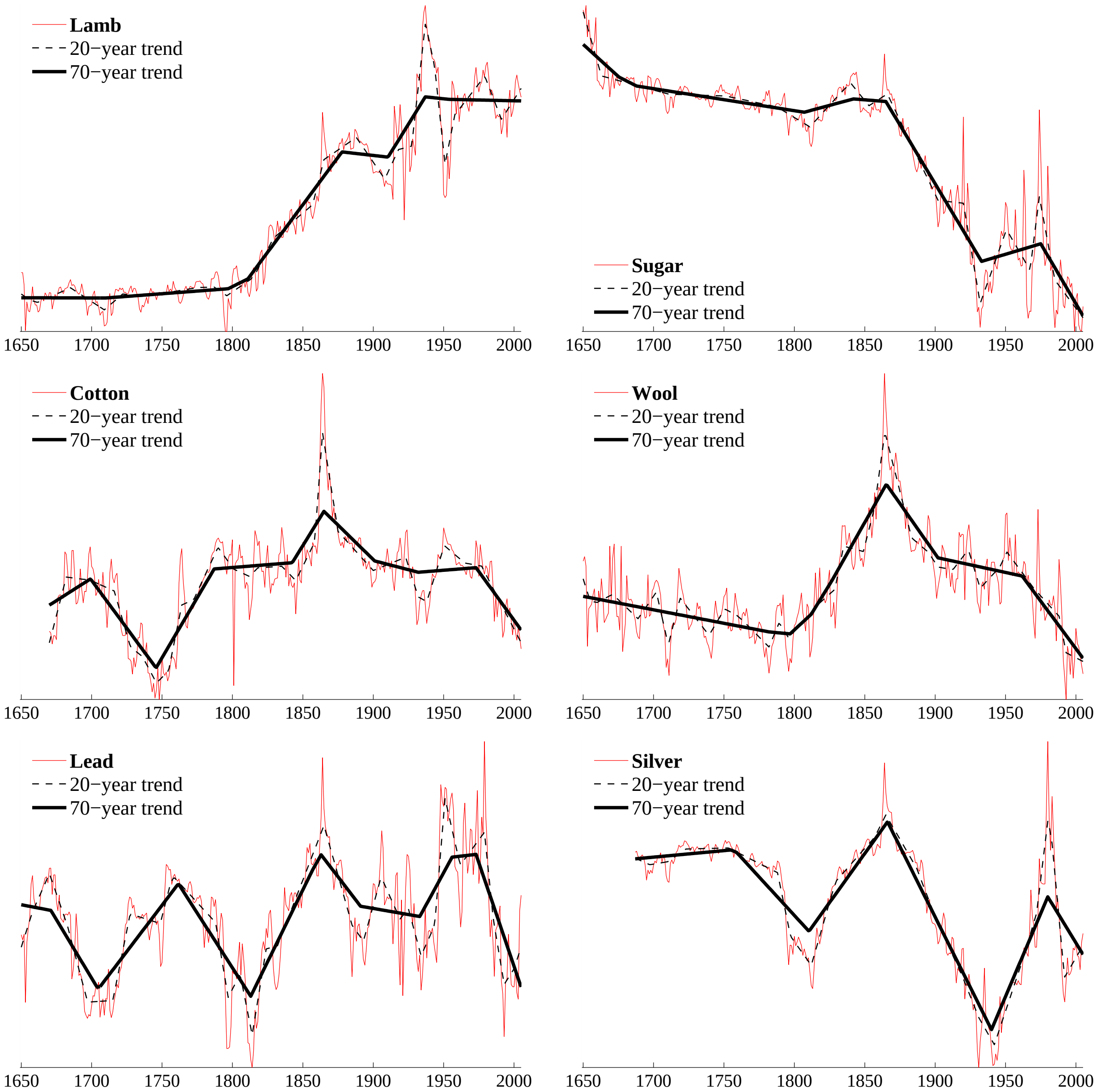
**Table 3.** *Estimation results (HKMW dataset)*

| | | 20-year trend ($\lambda_2$ = 104) | | | | | 70-year trend ($\lambda_2$ = 15426) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $n$ | $\lambda_1$ | $k$ | $D$ | PSH | PSHXX | $\lambda_1$ | $k$ | $D$ | PSH | PSHXX |
| Beef | 356 | 116 | 35 [26] | 13 | 0.34 | 0.48 | 1765 | 7 [6] | 51 | 0.09* | 0.30* |
| Coal | 356 | 76 | 47 [28] | 12 | 0.45 | 0.31 | 2010 | 8 [7] | 45 | 0.50 | 0.24* |
| Gold | 356 | 127 | 36 [21] | 16 | 0.52 | 0.62 | 2368 | 13 [9] | 36 | 0.38* | 0.22* |
| Lamb | 356 | 137 | 33 [22] | 15 | 0.30 | 0.32 | 1676 | 9 [7] | 45 | 0.45* | 0.74* |
| Lead | 356 | 112 | 39 [28] | 12 | 0.48 | 0.45 | 1633 | 14 [10] | 32 | 0.58* | 0.62* |
| Sugar | 356 | 240 | 22 [17] | 20 | 0.81 | 0.75 | 2694 | 8 [7] | 45 | 0.78 | 0.60* |
| Wheat | 356 | 108 | 38 [27] | 13 | 0.48 | 0.68 | 1041 | 11 [10] | 32 | 0.65* | 1.00* |
| Wool | 356 | 119 | 44 [27] | 13 | 0.62 | 0.71 | 2370 | 9 [7] | 45 | 0.81* | 1.00* |
| Cotton | 336 | 203 | 32 [25] | 13 | 0.60 | 0.66 | 3088 | 11 [8] | 37 | 0.44* | 0.61 |
| Tea | 333 | 112 | 40 [27] | 12 | 0.71 | 0.62 | 2242 | 8 [7] | 42 | 0.69 | 0.60 |
| Rice | 319 | 147 | 35 [21] | 15 | 0.53 | 0.44 | 1443 | 10 [8] | 35 | 0.81* | 0.69* |
| Silver | 319 | 156 | 26 [19] | 16 | 0.50 | 0.51 | 3729 | 8 [5] | 53 | 0.48 | 0.62* |
| Coffee | 297 | 163 | 40 [24] | 12 | 0.69 | 0.59 | 3040 | 9 [6] | 42 | 0.86* | 0.62 |
| Tobacco | 265 | 111 | 34 [21] | 12 | 0.44 | 0.54 | 1248 | 8 [6] | 38 | 0.25* | 0.42* |
| PigIron | 224 | 93 | 26 [18] | 12 | 0.71 | 0.67 | 1085 | 8 [6] | 32 | 0.71 | 1.00* |
| Cocoa | 206 | 193 | 25 [16] | 12 | 0.52 | 0.62 | 3327 | 11 [7] | 26 | 0.55 | 0.58 |
| Copper | 206 | 105 | 22 [13] | 15 | 0.51 | 0.54 | 2666 | 8 [5] | 34 | 0.53 | 0.71* |
| Hide | 206 | 116 | 19 [15] | 13 | 0.46 | 0.49 | 1837 | 8 [6] | 29 | 0.46 | 0.60* |
| Tin | 198 | 115 | 23 [15] | 12 | 0.22 | 0.21 | 3284 | 5 [4] | 40 | 0.14* | 0.26 |
| Nickel | 166 | 151 | 24 [13] | 12 | 0.62 | 0.60 | 2171 | 5 [4] | 33 | 0.63 | 0.42* |
| Aluminum | 156 | 257 | 14 [9] | 16 | 0.92 | 0.89 | 5868 | 5 [4] | 31 | 1.00* | 1.00* |
| Zinc | 153 | 126 | 15 [10] | 14 | 0.59 | 0.56 | 1315 | 4 [4] | 31 | 0.72* | 0.60 |
| Oil | 147 | 286 | 15 [9] | 15 | 0.70 | 0.58 | 2848 | 4 [3] | 37 | 0.22* | 0.00* |
| Banana | 106 | 78 | 13 [8] | 12 | 0.48 | 0.48 | 1527 | 2 [1] | 53 | 0.49 | 0.49 |
| Jute | 106 | 111 | 13 [9] | 11 | 0.55 | 0.55 | 2769 | 4 [2] | 35 | 0.36* | 0.36* |
| Average | | | 31 [21] | 14 | 0.54 | 0.55 | | 9 [7] | 39 | 0.55 | 0.57 |

**Notes:** The number of observations for each relative prices is "$n$". "$k$" is the number of slope changes of the $\ell_1$-trend, and in braces the number of significant changes (each linear segment lasts at least 5 years). "$D$" is the average duration of each linear segment, $D = n/(k+1)$. "PSH" ["PSHXX"] is the proportion of years where the $\ell_1$-trend is negatively sloped [since 1900]. "$\lambda_1$" is the value of $\lambda_1$ that minimizes $|E(\lambda_1, \lambda_2)|$.

"$*$" denotes rejection of $H_0 : p = \pi$ (an absolute $z$-score) for $p$ = PSH in the 70-year trend and $\pi$ = PSH in the 20-year trend; in this case, rejection is taken as evidence of a super cycle in the relative price.

**Figure 5.** *Selected relative primary commodity prices and piecewise linear trends (HKMW dataset)*




**Notes:** The (red) thin lines are 100 times the logarithm of the relative commodity price. The remaining lines are the piecewise linear trends computed for the values of $\lambda_1$ given in Table 3.

**Figure 6.** *Periods of negatively sloped piecewise linear trends (HKMW dataset)*

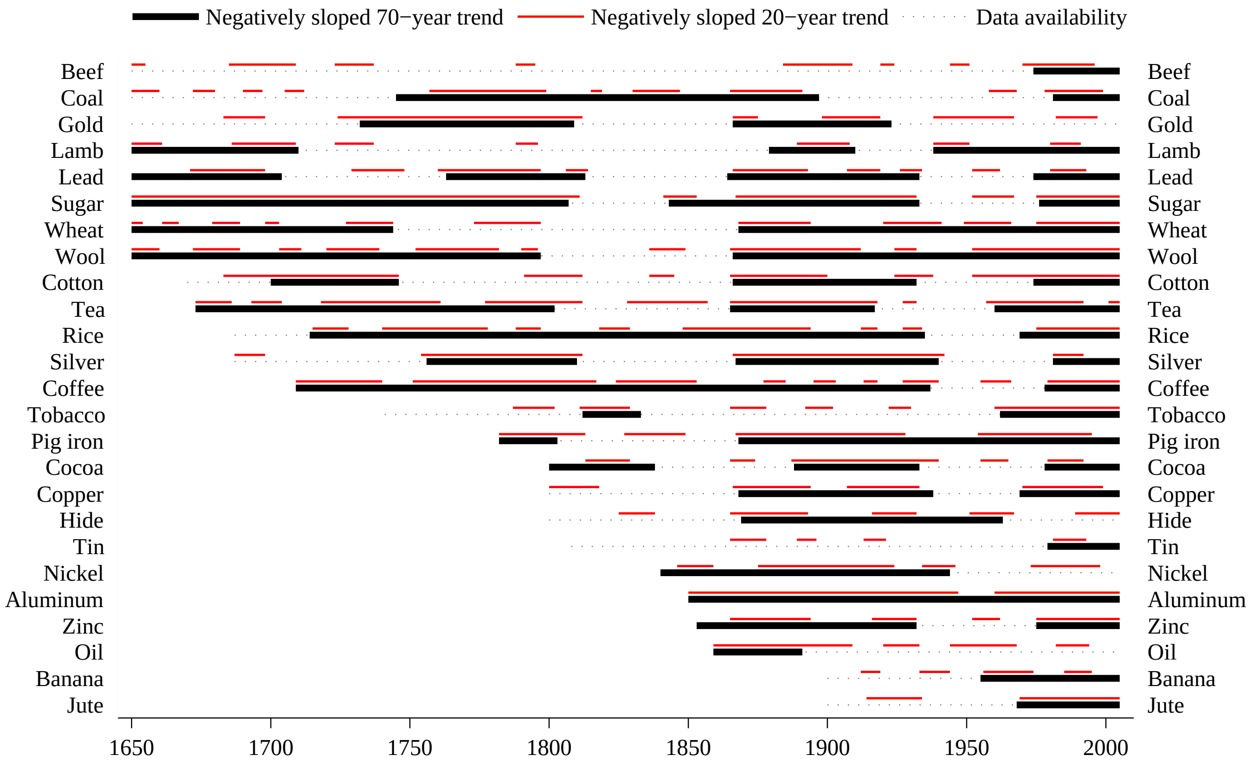
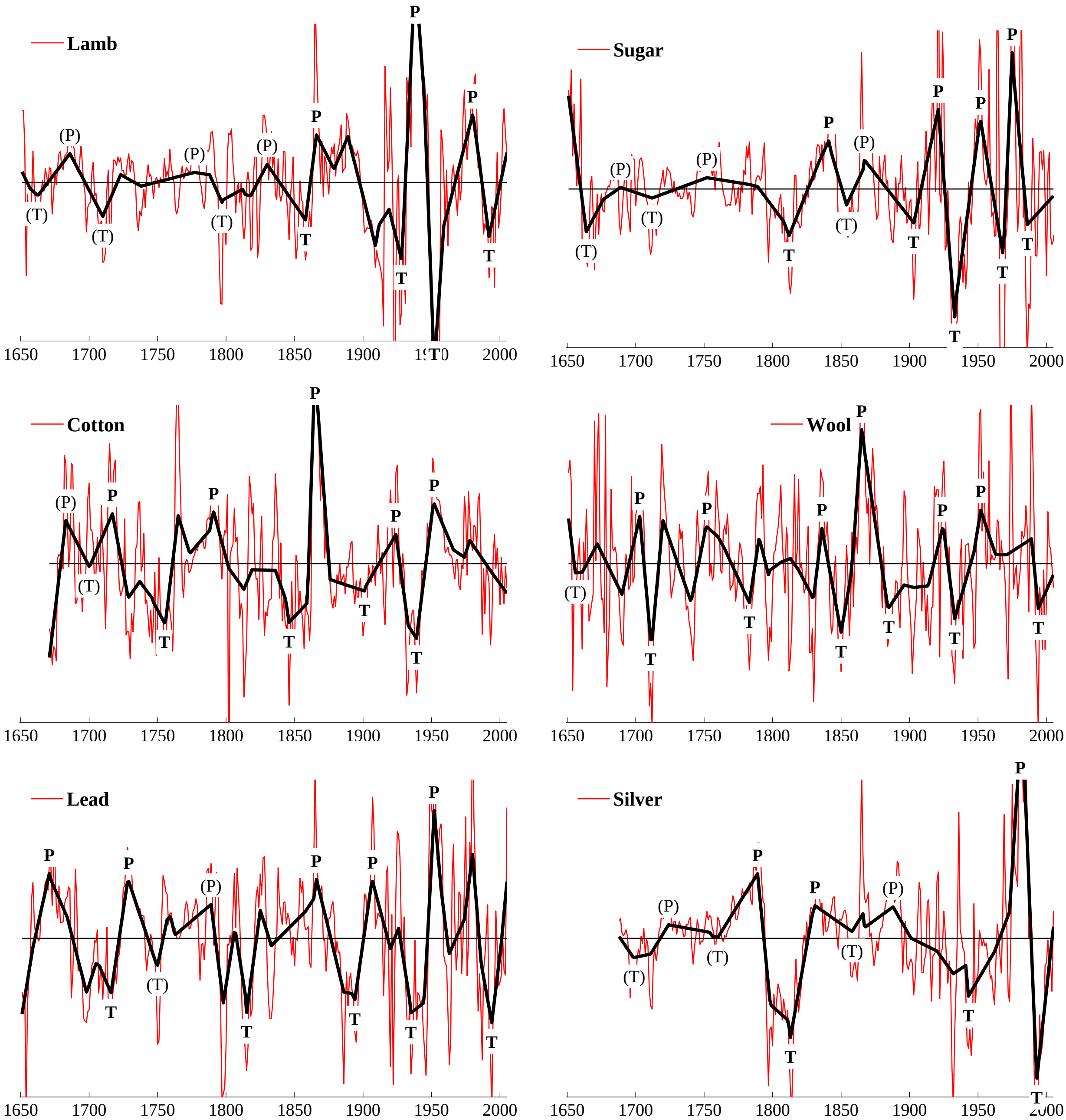


**Table 4.** *Turning points and duration of super cycles (HKMW dataset)*

| | | Peaks ($P$) | | Troughs ($T$) | Mean duration in years | | | |
|---|---|---|---|---|---|---|---|---|
| | $N_P$ | Dates | $N_T$ | Dates | $P \to P$ | $T \to T$ | $T \to P$ | $P \to T$ |
| Beef | 3 {2} | 1866, 1944, 1970 | 4 {3} | 1851, 1925, 1953, 1997 | 52 {26} | 49 {36} | 17 {18} | 32 {18} |
| Coal | 3 {1} | 1690, 1865, 1978 | 4 {2} | 1681, 1800, 1969, 1995 | 144 | 105 {26} | 28 { 9} | 77 {17} |
| Gold | 4 {3} | 1789, 1907, 1938, 1982 | 4 {3} | 1799, 1920, 1968, 1999 | 64 {38} | 67 {40} | 47 {16} | 18 {20} |
| Lamb | 3 {2} | 1866, 1938, 1980 | 4 {3} | 1858, 1928, 1952, 1992 | 57 {42} | 45 {32} | 15 {19} | 29 {13} |
| Lead | 5 {2} | 1671, 1729, 1866, 1907, 1952 | 5 {2} | 1716, 1815, 1894, 1935, 1994 | 70 {45} | 70 {59} | 24 {17} | 46 {35} |
| Sugar | 4 {3} | 1841, 1921, 1952, 1975 | 5 {4} | 1812, 1903, 1933, 1968, 1986 | 45 {27} | 44 {28} | 18 {15} | 25 {13} |
| Wheat | 7 {3} | 1698, 1727, 1773, 1868, 1920, 1949, 1975 | 8 {3} | 1655, 1704, 1745, 1798, 1895, 1942, 1967, 1992 | 46 {28} | 48 {25} | 29 { 8} | 19 {19} |
| Wool | 6 {2} | 1703, 1752, 1836, 1865, 1924, 1952 | 6 {2} | 1711, 1783, 1850, 1885, 1933, 1994 | 50 {28} | 57 {61} | 33 {19} | 21 {26} |
| Cotton | 5 {2} | 1717, 1791, 1865, 1924, 1952 | 4 {2} | 1755, 1846, 1901, 1939 | 59 {28} | 61 {38} | 23 {18} | 36 {15} |
| Tea | 6 {2} | 1693, 1718, 1777, 1828, 1927, 1957 | 6 {3} | 1705, 1761, 1797, 1920, 1947, 1992 | 53 {30} | 57 {36} | 15 { 9} | 37 {28} |
| Rice | 5 {2} | 1715, 1788, 1818, 1912, 1975 | 5 {2} | 1729, 1798, 1830, 1935, 1989 | 65 {63} | 65 {54} | 50 {40} | 15 {19} |
| Silver | 3 {1} | 1789, 1831, 1981 | 3 {2} | 1813, 1943, 1993 | 96 | 90 {50} | 28 {38} | 49 {12} |
| Coffee | 6 {3} | 1751, 1824, 1895, 1928, 1955, 1985 | 7 {4} | 1741, 1818, 1854, 1904, 1941, 1969, 1992 | 47 {29} | 42 {29} | 19 {18} | 23 {11} |
| Tobacco | 6 {2} | 1783, 1811, 1865, 1892, 1922, 1960 | 6 {2} | 1770, 1803, 1830, 1879, 1916, 1942 | 35 {38} | 34 {26} | 16 {12} | 19 {20} |
| PigIron | 3 {1} | 1827, 1867, 1954 | 4 {2} | 1800, 1850, 1929, 1965 | 64 | 55 {36} | 23 {25} | 32 {11} |
| Cocoa | 4 {2} | 1813, 1865, 1955, 1979 | 4 {3} | 1830, 1941, 1966, 1993 | 55 {24} | 54 {26} | 21 {14} | 30 {13} |
| Copper | 3 {2} | 1865, 1907, 1970 | 4 {2} | 1819, 1894, 1946, 2000 | 53 {63} | 60 {54} | 28 {24} | 33 {35} |
| Hide | 4 {3} | 1865, 1916, 1951, 1989 | 4 {2} | 1839, 1894, 1933, 1968 | 41 {37} | 43 {35} | 22 {20} | 21 {17} |
| Tin | 3 {2} | 1865, 1913, 1982 | 3 {2} | 1897, 1946, 1994 | 59 {69} | 49 {48} | 26 {36} | 26 {23} |
| Nickel | 2 {1} | 1875, 1973 | 2 {2} | 1947, 1999 | 98 | 52 {52} | 26 {26} | 49 {26} |
| Aluminum | 3 {2} | 1889, 1934, 1980 | 2 {1} | 1899, 1948 | 46 {46} | 49 | 34 {32} | 12 {14} |
| Zinc | 3 {2} | 1865, 1916, 1952 | 2 {1} | 1884, 1933 | 44 {36} | 49 | 26 {19} | 18 {17} |
| Oil | 3 {3} | 1920, 1944, 1982 | 4 {3} | 1883, 1934, 1970, 1995 | 31 {31} | 37 {31} | 20 {11} | 18 {18} |
| Banana | 3 {3} | 1909, 1933, 1956 | 3 {3} | 1920, 1945, 1975 | 24 {24} | 28 {28} | 12 {12} | 14 {14} |
| Jute | 2 {2} | 1914, 1969 | 2 {2} | 1935, 1994 | 55 {55} | 59 {59} | 34 {34} | 23 {23} |

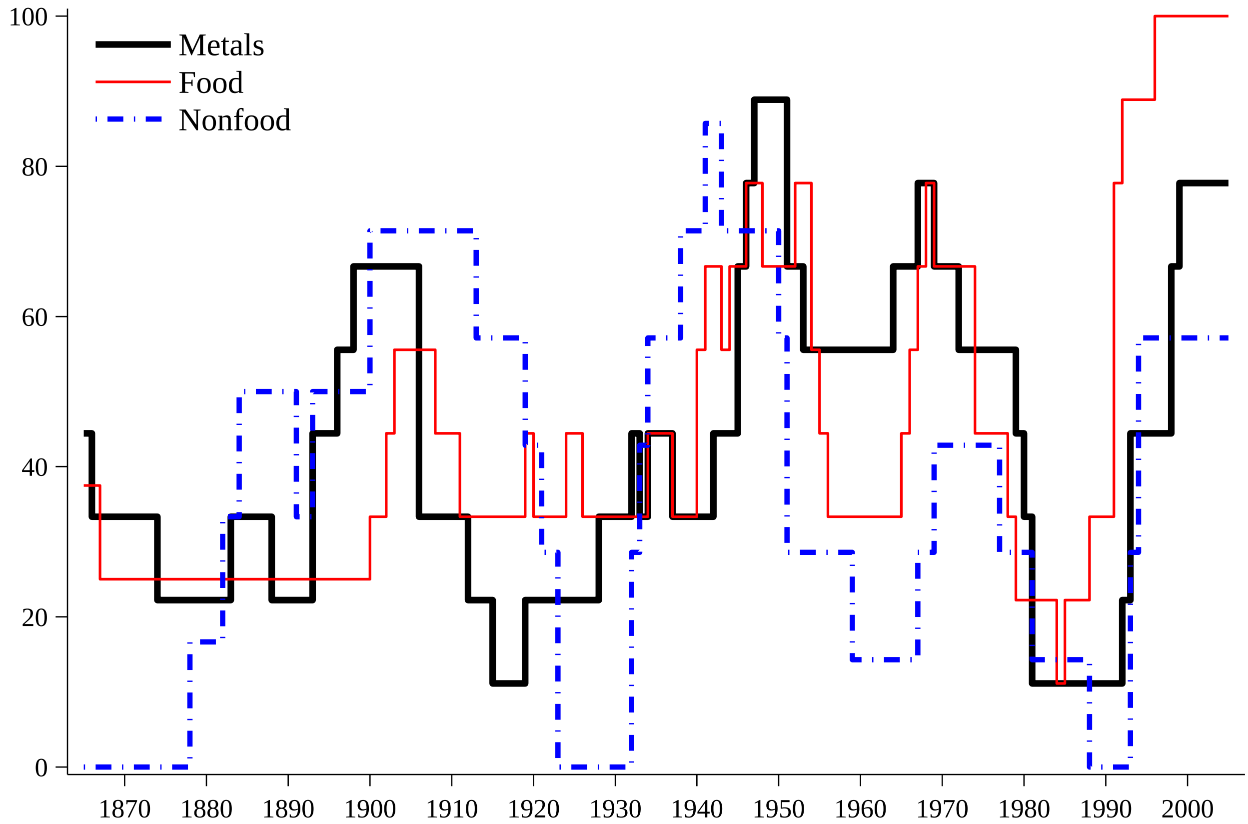
**Notes:** $N_P$ is the number of peaks and $N_T$ is the number of troughs, identified as explained in section 2.4. To save space, only significant turning points are shown. The complete set of turning points (including those eliminated due to insignificance or alternation) are shown in the online supplement to this paper.

**Figure 7.** *Piecewise cyclical components of selected relative primary commodity prices (HKMW dataset)*




**Notes:** The (red) thin lines are deviations of the actual data from its 70-year trend. The (black) thick lines depict deviations of the 20-year trend from the 70-year trend, our measure of cycle. Peaks are labeled with “P” and troughs with “T”. (P) or (T) indicate turning points that were eliminated due to insignificance or alternation.

**Figure 8.** *Proportion of relative primary commodity prices in expansionary phases (HKMW dataset)*



## Supplementary material (not intended for publication): EGY dataset

**Figure.** *Relative commodity prices, piecewise linear trends and super cycles (EGY dataset)*

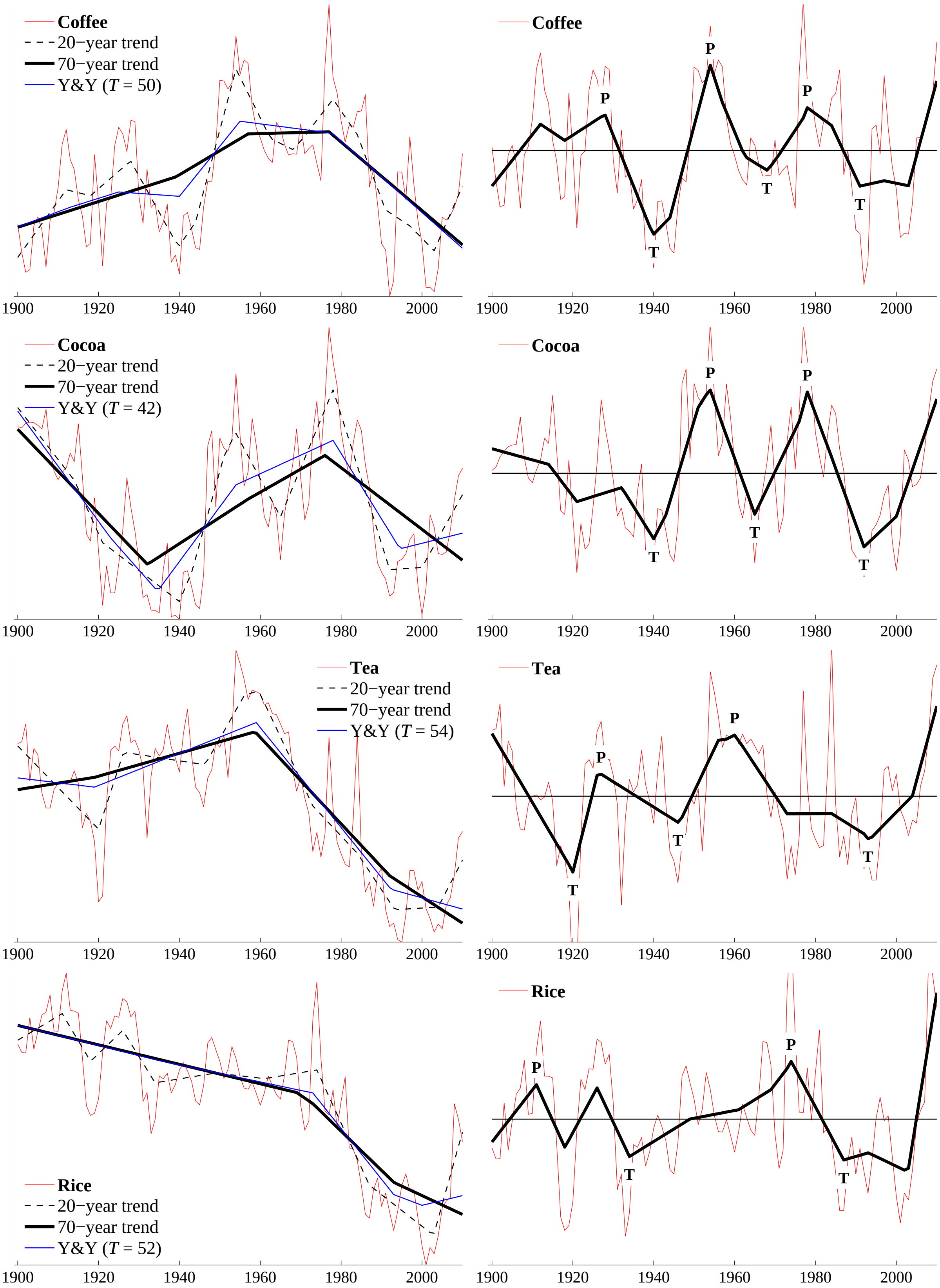



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**Figure (cont').** *Relative commodity prices, piecewise linear trends and super cycles (EGY dataset)*

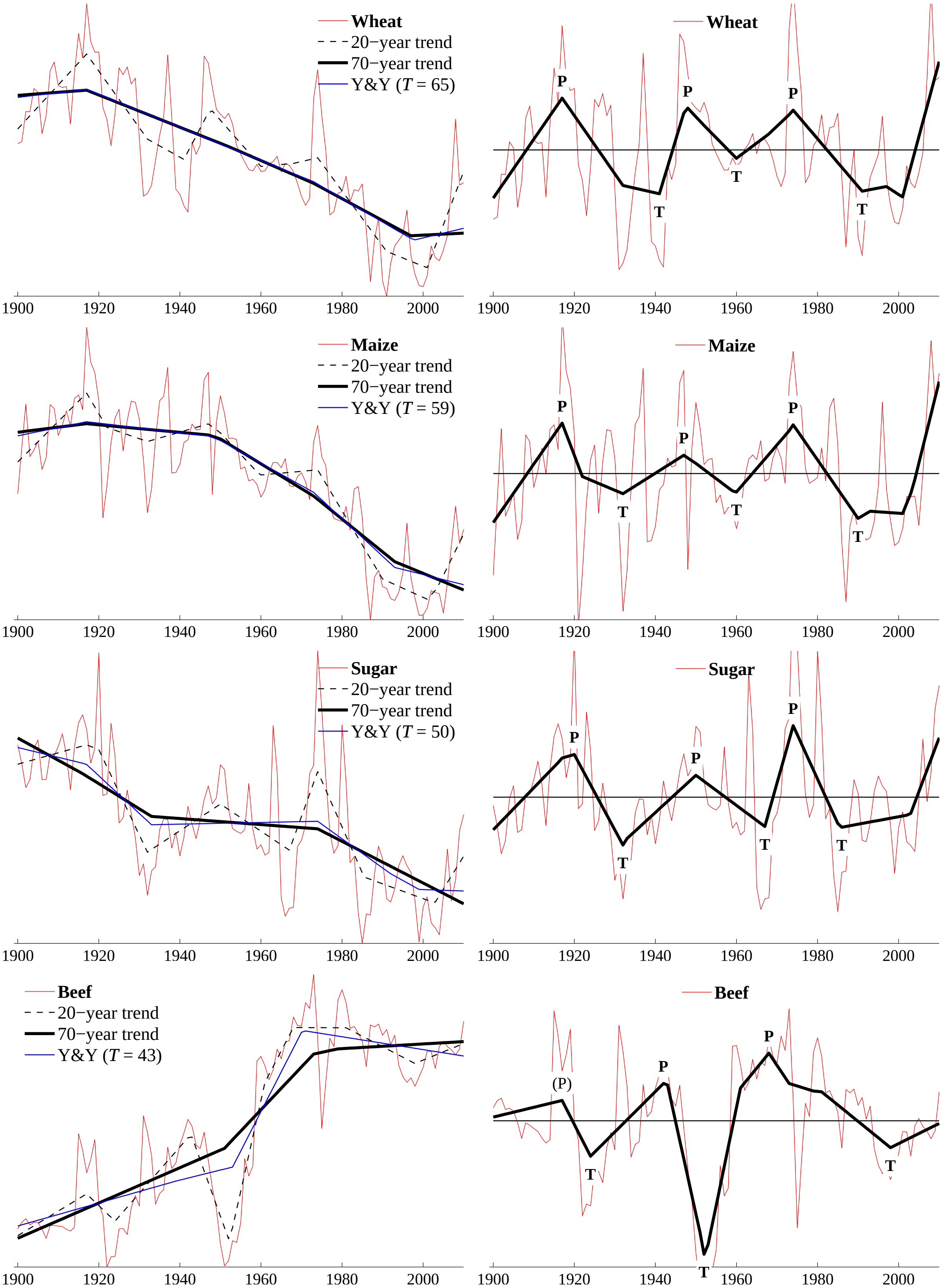


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**Figure (cont').** *Relative commodity prices, piecewise linear trends and super cycles (EGY dataset)*

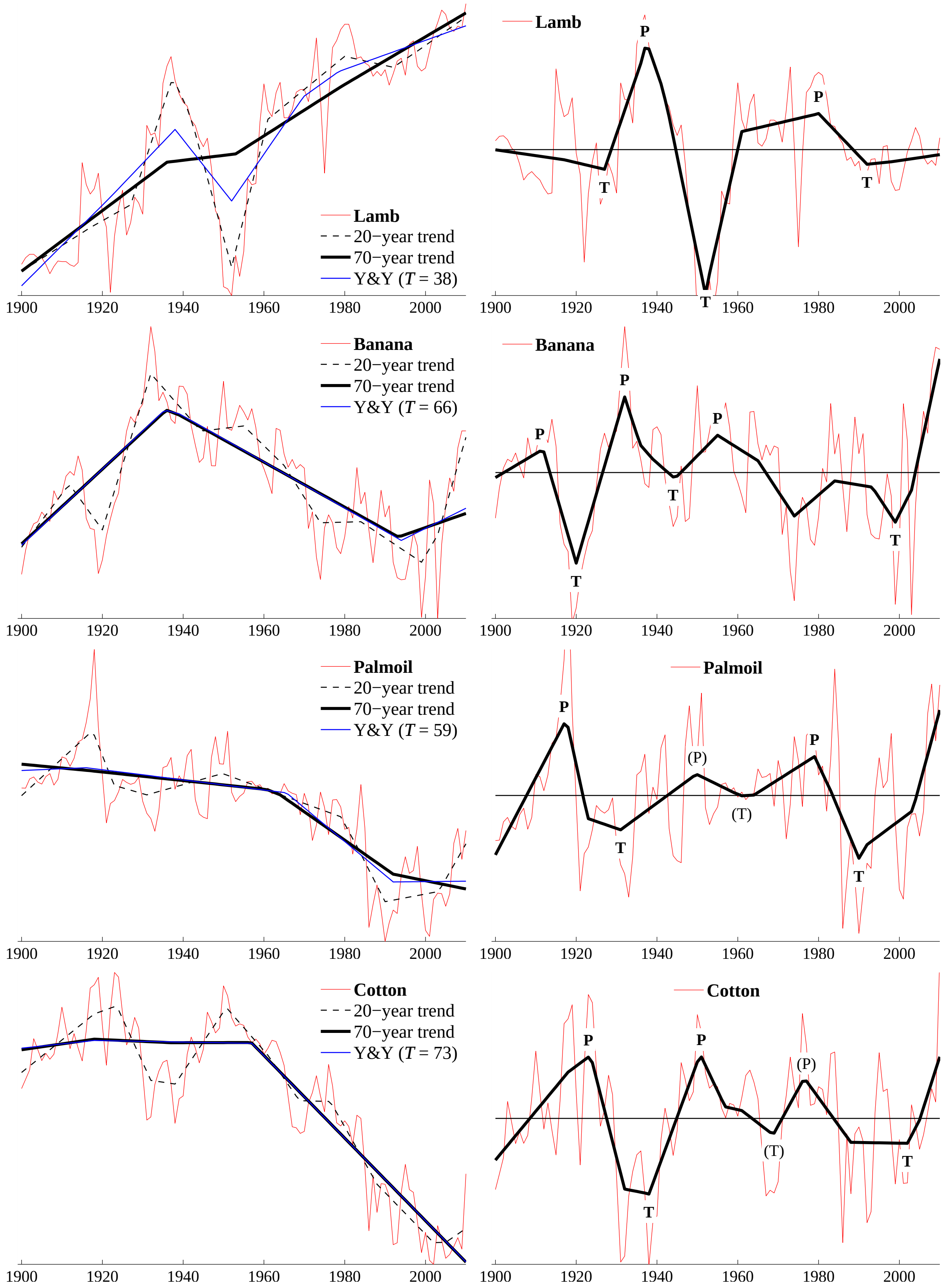


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**Figure (cont').** *Relative commodity prices, piecewise linear trends and super cycles (EGY dataset)*

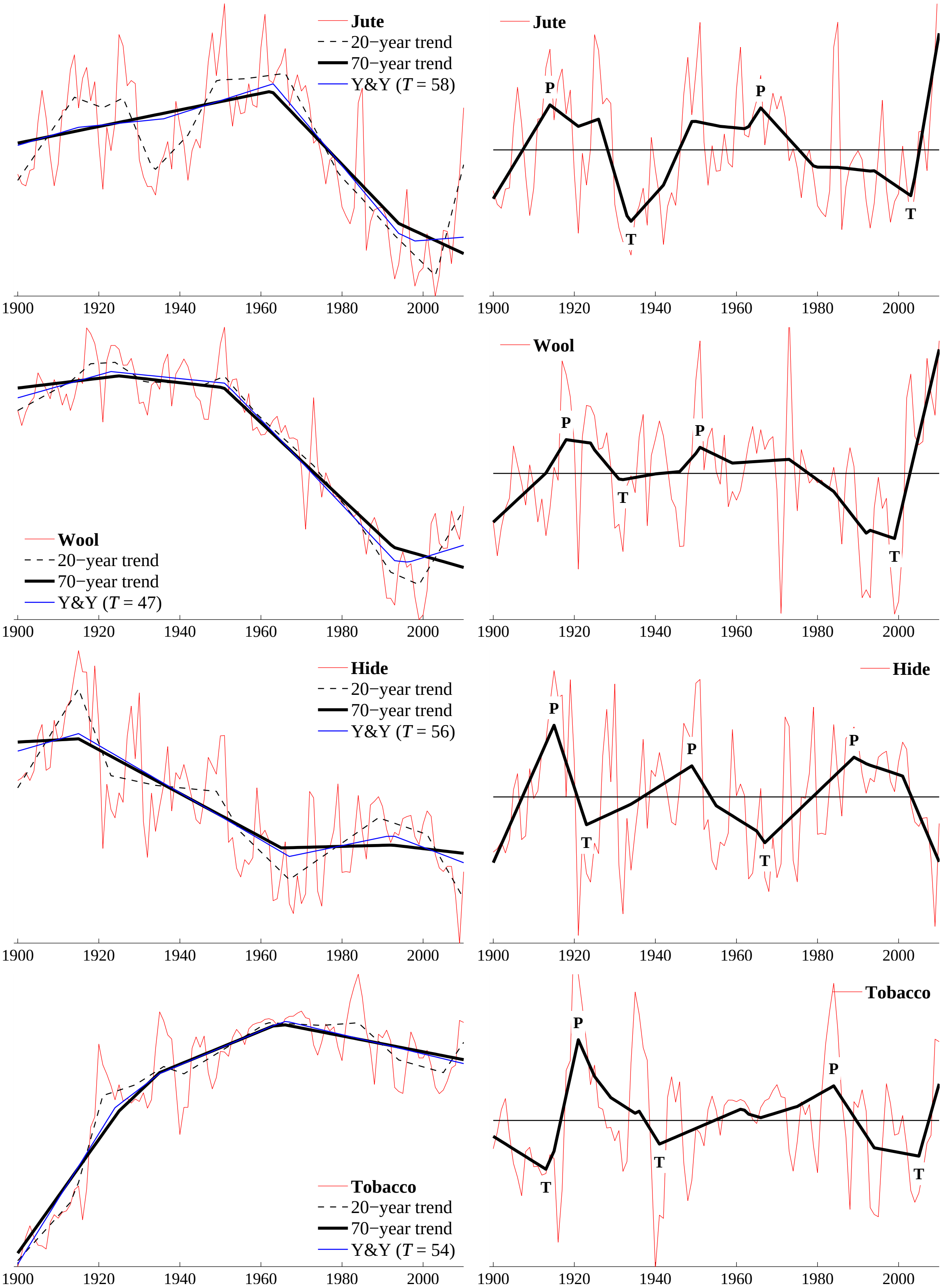


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**Figure (cont').** *Relative commodity prices, piecewise linear trends and super cycles (EGY dataset)*

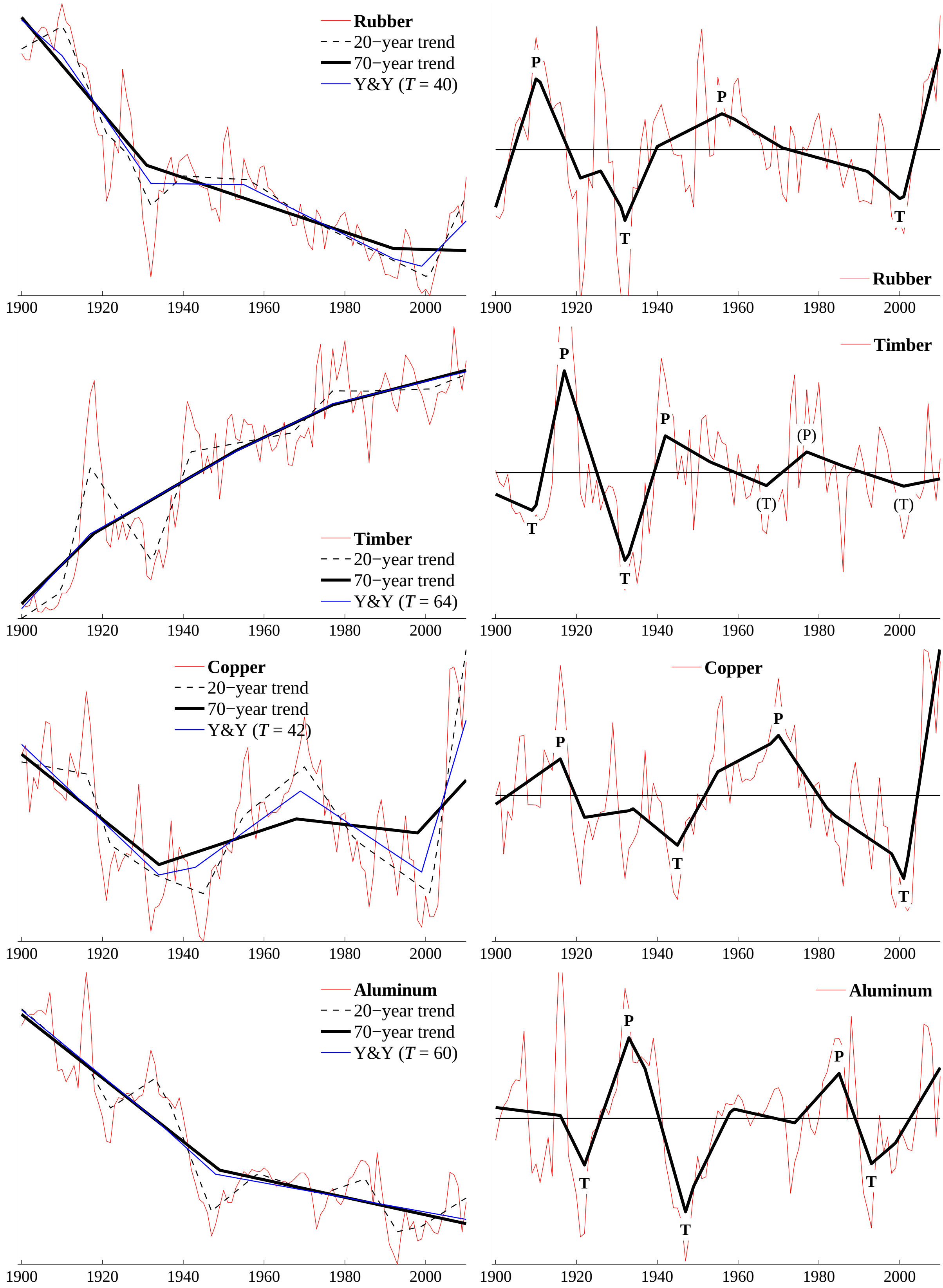


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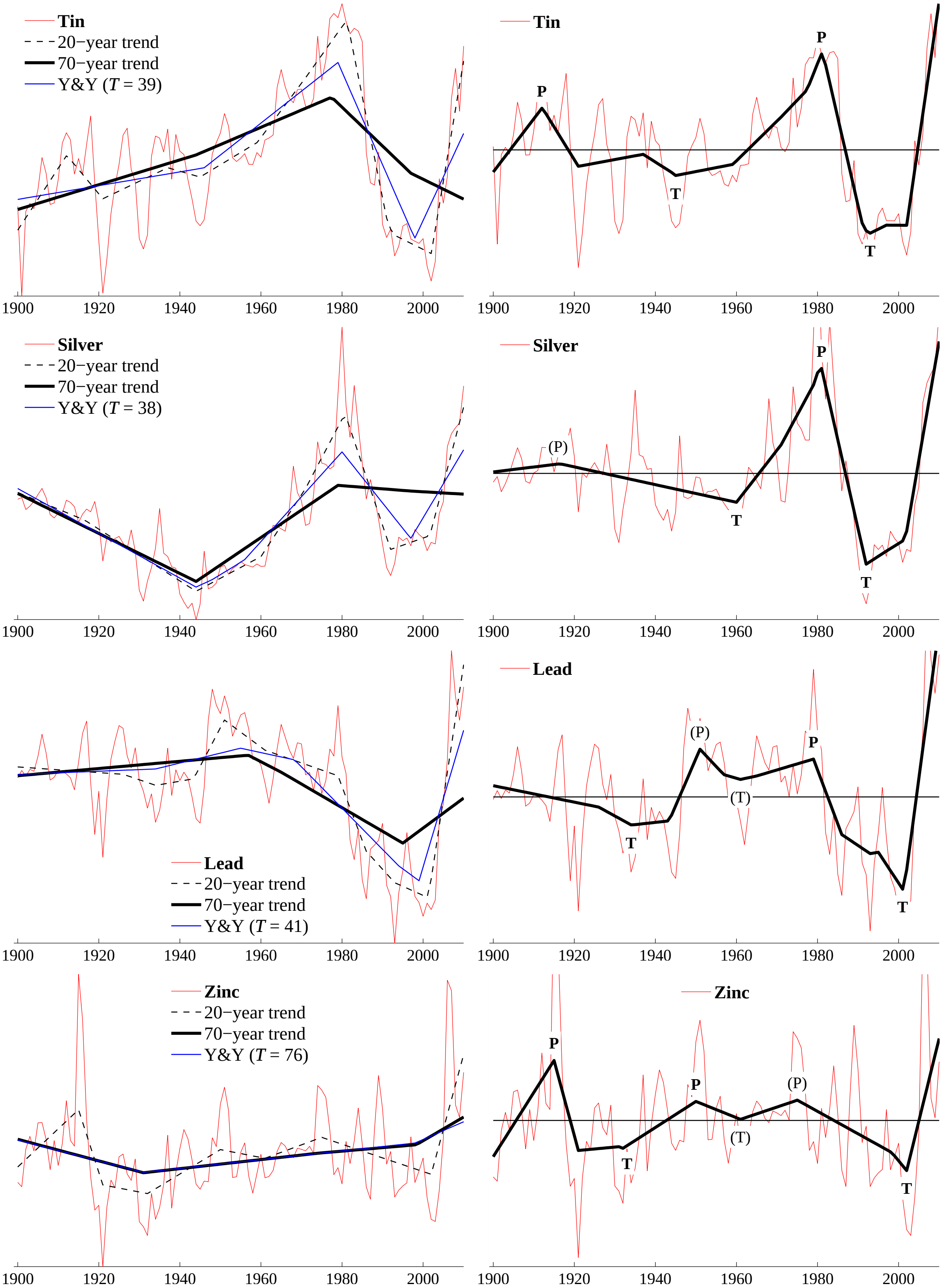
**Figure (cont').** *Relative commodity prices, piecewise linear trends and super cycles (EGY dataset)*




**Notes:** The panels on the first column show 100 times the logarithm of the relative commodity price (as the red, thin line), together with various estimated $\ell_1$-trends, using the calibrated the values of $\lambda_1$ given in Table 1. In the panels on the second column, the thin and thick lines are, respectively, deviations of the actual data and its 20-year trend from the 70-year trend. Peaks are labeled with "P" and troughs with "T", with (P) or (T) indicating turning points that were eliminated due to insignificance or alternation.

## Supplementary material (not intended for publication): HKMW dataset

**Figure.** *Relative commodity prices, piecewise linear trends and super cycles (HKMW dataset)*

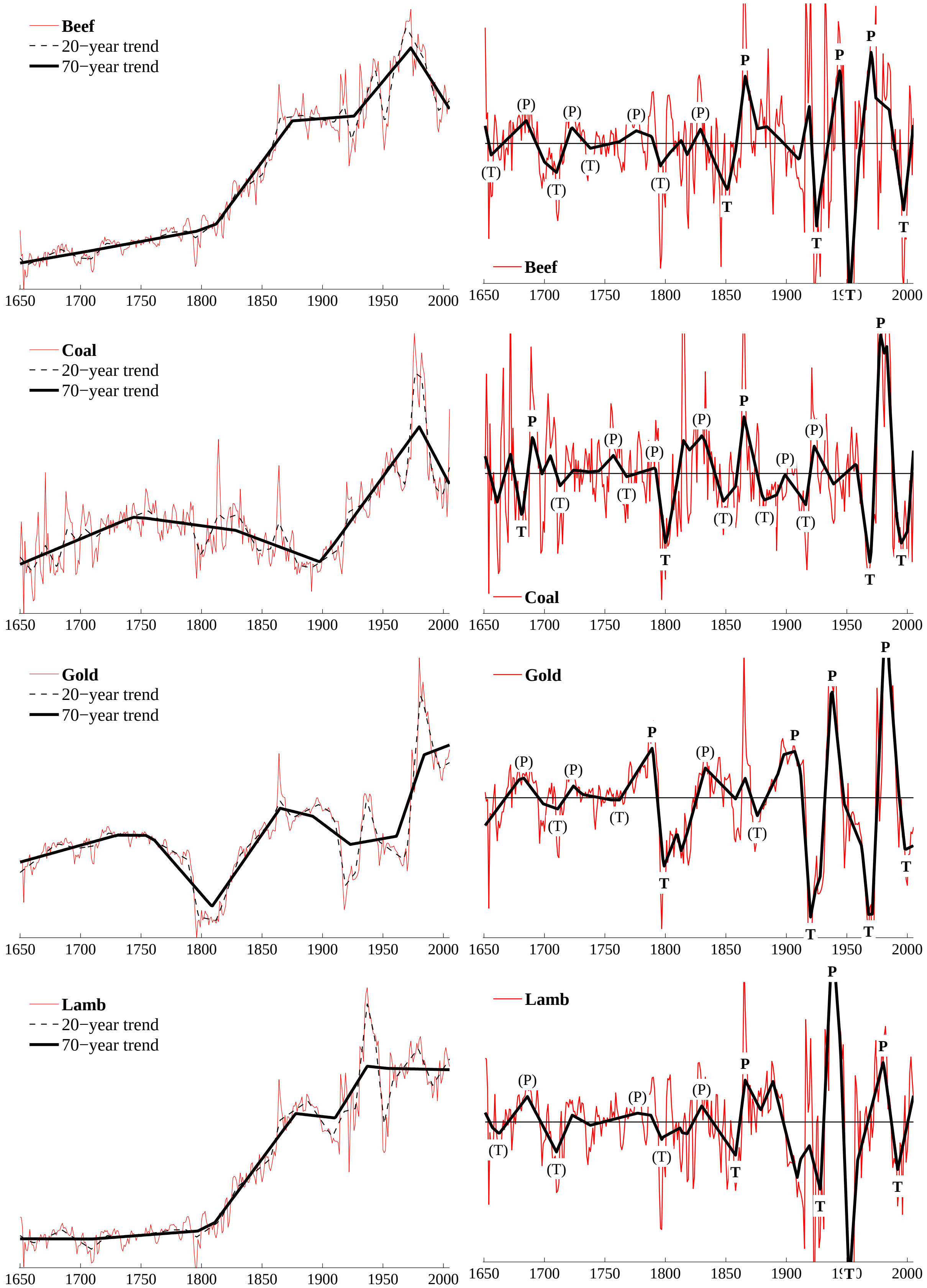



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**Figure (cont').** *Relative commodity prices, piecewise linear trends and super cycles (HKMW dataset)*

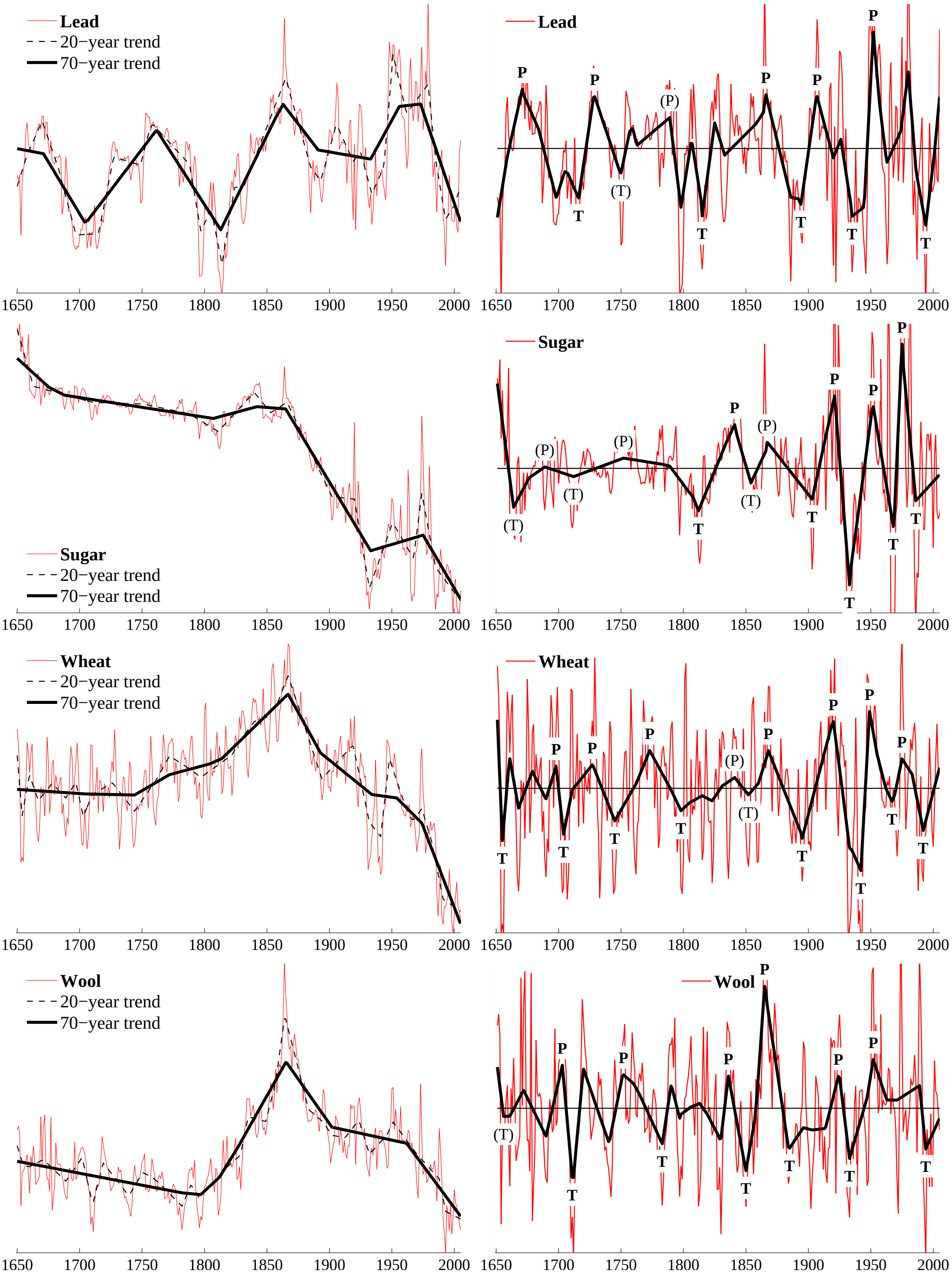


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**Figure (cont').** *Relative commodity prices, piecewise linear trends and super cycles (HKMW dataset)*

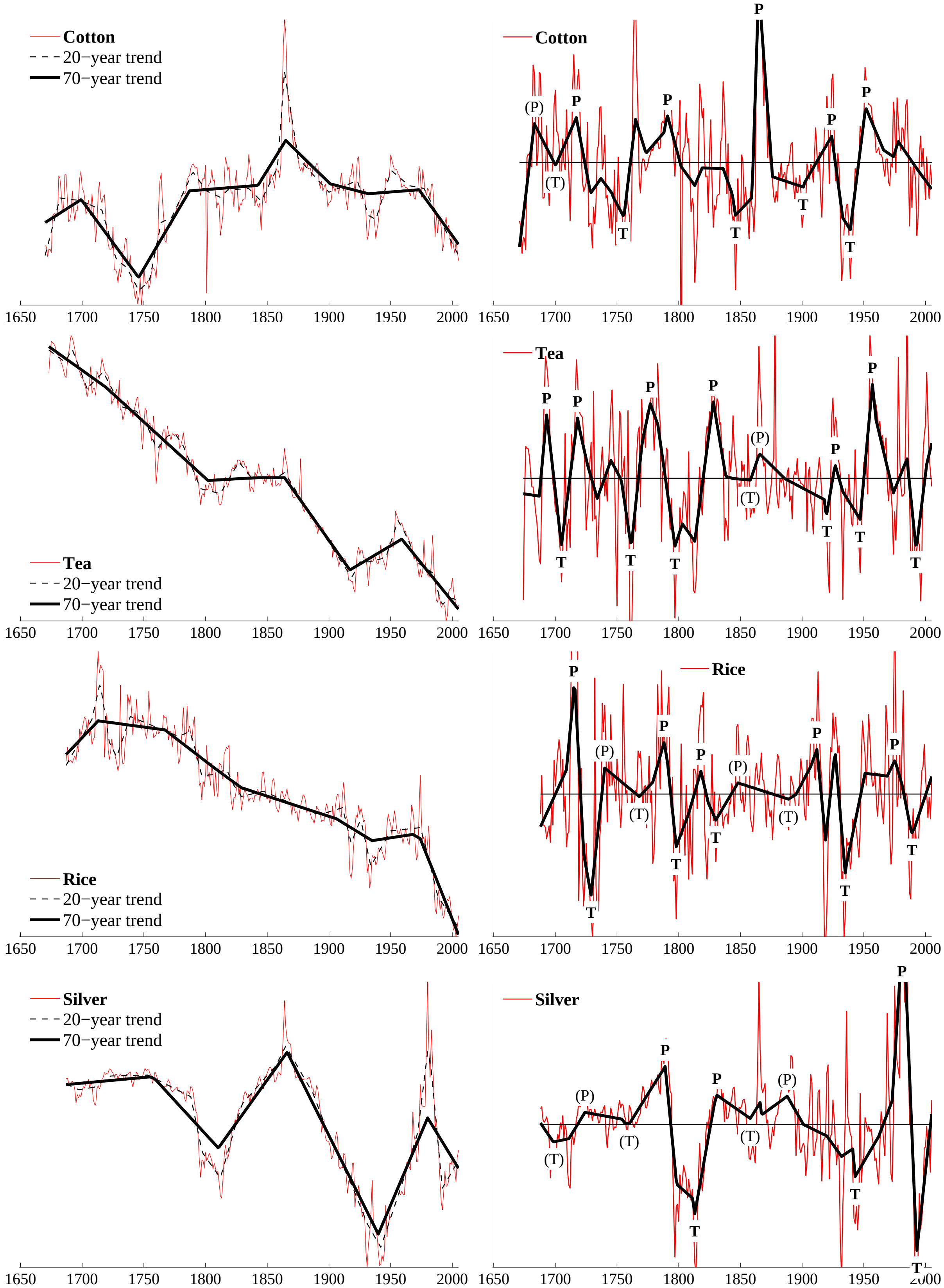


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**Figure (cont').** *Relative commodity prices, piecewise linear trends and super cycles (HKMW dataset)*

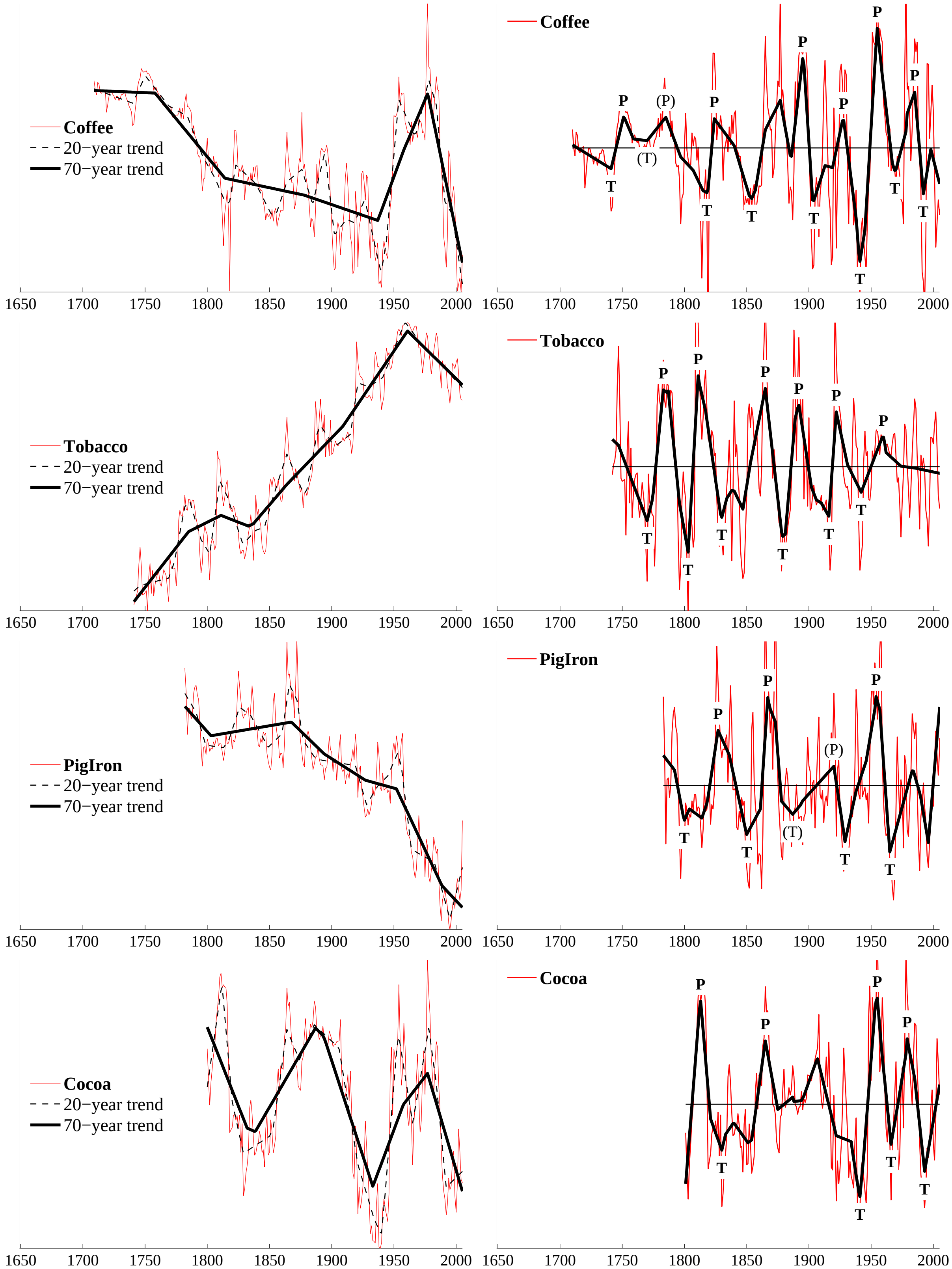


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**Figure (cont').** *Relative commodity prices, piecewise linear trends and super cycles (HKMW dataset)*

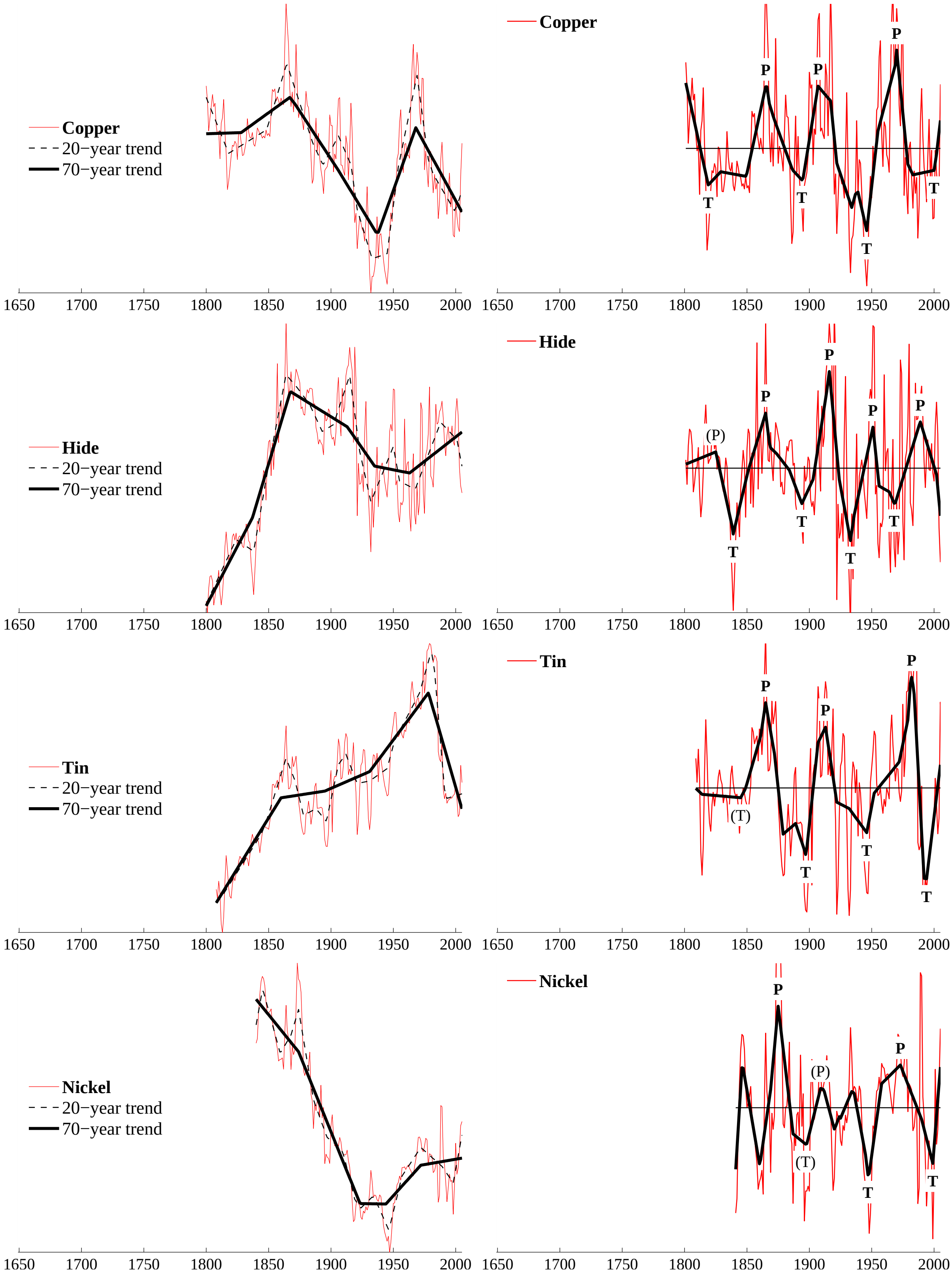


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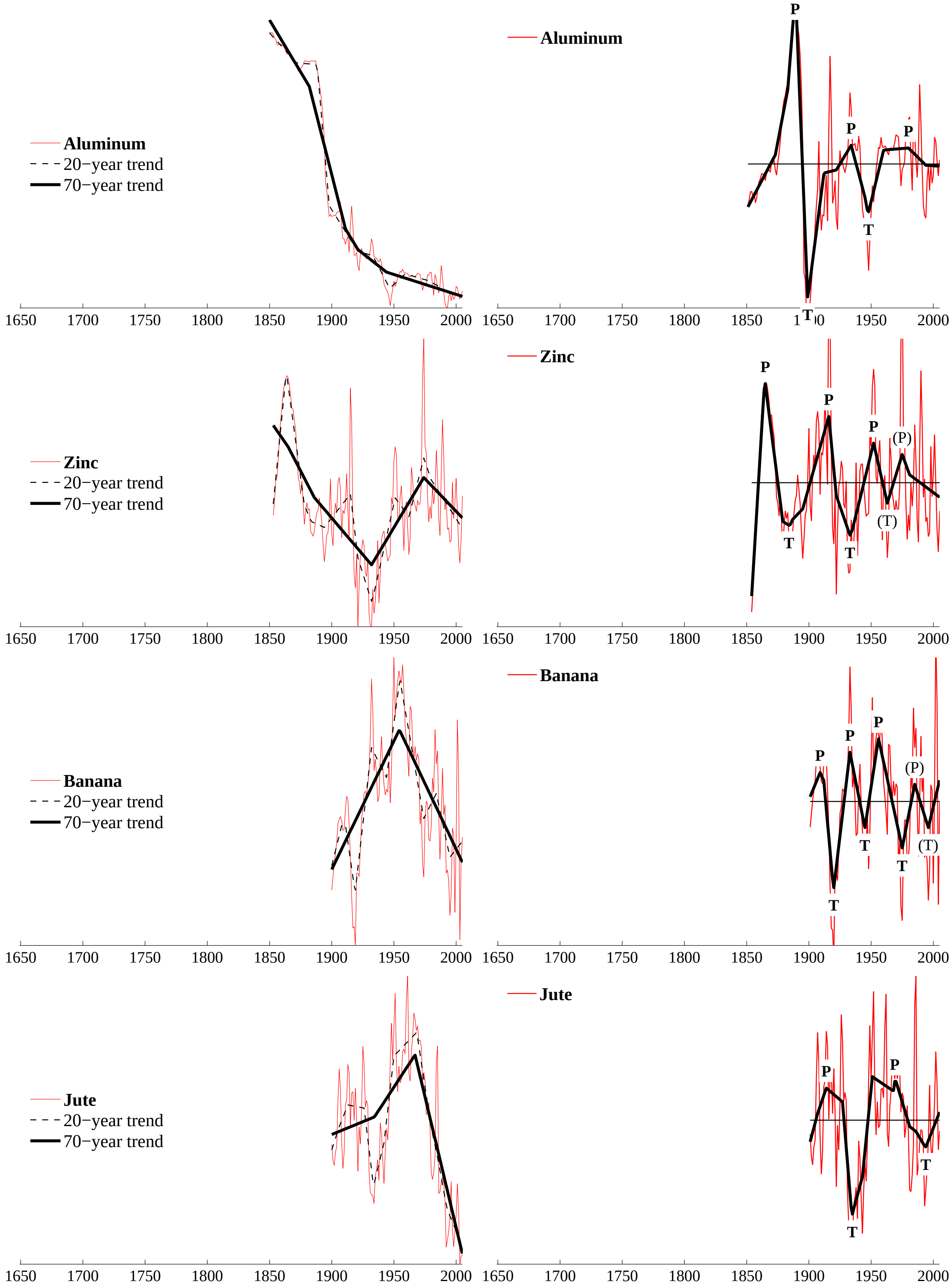
**Figure (cont').** *Relative commodity prices, piecewise linear trends and super cycles (HKMW dataset)*




**Notes:** The panels on the first column show 100 times the logarithm of the relative commodity price (as the red, thin line), together with various estimated $\ell_1$-trends, using the calibrated the values of $\lambda_1$ given in Table 3. In the panels on the second column, the thin and thick lines are, respectively, deviations of the actual data and its 20-year trend from the 70-year trend. Peaks are labeled with "P" and troughs with "T", with (P) or (T) indicating turning points that were eliminated due to insignificance or alternation.