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Abstract

The goal of this paper is to explain a recent regularity observed in economies in which central banks have moved from using a money aggregate as the instrument for the conduction of monetary policy towards a short-term interest rate (for example Peru in 2002). In particular, in those economies we observe that, after the change in the policy instrument, there is a decrease in the macroeconomic volatility accompanied by a reduction in the average level of both inflation and interest rates vis-à-vis an increase in the average level of money aggregates (an increase in the money demand).

In order to explain the previous stylized fact, a second order solution of a general equilibrium model for a small open economy is evaluated. By analyzing the second order solution we relax the assumption of certainty equivalence which permits consider the role of uncertainty (risk) in the equilibrium solution of the economy. The previous solution takes into account the reduction of macroeconomic uncertainty (risk) as a consequence of changing the instrument (from money aggregates to interest rate rules), helping to explain the stylized fact.

Our findings show that the use of the interest rate as the instrument for the conduction of monetary policy induces a reduction of macroeconomic risks. In turn, the previous reduction has driven a decrease in the average level of interest rates and inflation which is consistent with the increase in the demand for money observed in Peru in the 2000s. Hence, the recent increase in the growth rate of money aggregates should not be linked, whatsoever, to higher inflation rates.

JEL Classification: E52, E42, E12, C63

Keywords: Small Open Economy Model, Incomplete Markets, Second Order Solution

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1 Motivation

In many central banks has become a common practice to set their monetary policy by targeting a short term interest rate. Instead, the use of money aggregates as the policy instrument has been greatly downgraded in the implementation of monetary policy. Thus, financial innovations, difficulties in determining the relevant money aggregate and unobservable money demand shocks, among others, have caused money aggregates to have a minor role in central banking policy implementation. Moreover, from the theoretical point of view, as Clarida, Galí and Gertler (1999) pointed out, the advantage of using interest rate rules with respect to a money rule depends upon whether or not a money demand shock is observable. In fact, in a world where money demand is perfectly observable, then it does not matter whether a central bank sets the short term interest rate or the money supply as the instrument, because both instruments can give the same macroeconomic outcome. Instead, when the money demand is not perfectly observable by the policy maker, then a money demand shock can induce a volatile behavior of the interest rate, which feeds out into volatility of output and other macro-variables. This is why many central banks, in practice, have moved from money aggregates toward interest rate rules. In table 1 we report some countries that in implementing the IT regime have changed the instrument from money aggregates to interest rate rules.

Country	Adoption years				
	Other instruments	Money aggregates Instrument	Transition to interest rate instrument	Interest rate instrument	Inflation Target
Australia	1945 - Dec. 1983 (fixed exchange rate)	Dec. 1983 - Ene. 1985	May. 1985 - Dec. 1989	from Ene. 1990	1994
Canada	1935 - 1950 and 1962 - 1970 (fixed exchange rate)	----	----	1950 - 1962 and from 1970	1991
Hungary	----	1987 - 1991	1992 - 1994	from 1995	2001
New Zealand	----	Mid 1980s - Ene. 1999	----	from Feb. 1999	1991
Mexico	before 1995 (fixed exchange rate)	desde 1995	From 2002	----	2002
Peru	----	antes del 2001	2001 - 2002	from 2003	2002
Thailand	1945 - Jun. 1997 (fixed exchange rate)	Jul. 1997 - May. 2000	----	from May. 2000	2000

Table 1: Change of policy instrument

The change in the policy instrument has brought about changes in the dynamic properties of the main macroeconomic series. In Table 2 we report the change in both mean and standard deviations for Perú, Thailandia and México for CPI inflation, short term interest rates and money growth[1]. The previous countries have moved from using money aggregates as the

[1] The data for the standard deviation of the short term interest was detrended using the Hodrick Prescott filter and the data for the standard deviation of the growth rate real demand for M3 is seasonaly adjusted .

target to interest rates targets. Notoriously, the change in the instrument was accompanied by a reduction in the volatility of the macroeconomic variables reported in table 2. The reduction in volatility was followed by a significant decrease in the means of both inflation and the short term interest rate vis-a-vis an important increase in the mean of the money aggregates[2].

Table 2: change in moments after change in policy instrument

	Peru		Thailand		Mexico	
	Means	S.D.	Means	S.D.	Means	S.D.
CPI inflation	-5.4	-3.2	-1.5	-33.2	-23.3	-72.4
Shor term interest rate	-11.2	-70.5	-7.1	-91.3	-10.0	-50.0
Money demand growth	3.2	-26.4	1.9	-31.2	24.2	81

Moreover, there have been observed other key features of the data after the adoption of an interest rate rule, such as an increase in money growth higher than the rate of inflation, and increase in the mean in output and a reduction in the correlation between them (not shown in the table). Thus, as an example, during the last decade the Peruvian economy has experienced two well defined episodes of aggregate volatility, with the period 1994-2001 displaying higher volatility of inflation, interest rate and output than the period 2001-2005. These episodes coincide with two different ways in which monetary policy conduct its actions, the first one characterized by a money rule target and the second one by an interest rate rule. Table 3 shows that the standard deviations of inflation, and the nominal interest rate were materially higher before the switch of the instrument. For the period spanning from 1994 until 2001, the sharp fall in CPI inflation was linked with a persistent decrease in the monetary base growth rate. In this period the correlation between the growth rate of M3 and CPI inflation was close to 1. Interestingly, this relationship breaks after 2001 where both, an upward trend in base money growth and a steady level of inflation are observed. Because of this, the correlation between the growth rate of M3 and CPI inflation becomes close to 0. The previous result suggests, somehow, a less relevant role for money to explain inflation dynamics after the change in the instrument.

The above empirical evidence suggests that, to the extent that there is a connection between decrease in volatilities and change in the means of key macroeconomic variables following the change in the instrument, it seems that the cycles must be studied in a non-conventional way in order to capture the aforementioned effects of volatilities over the means.

Then, the objective of this paper is to develop as a first pass a quantitative model capable of explaining the connection between volatilities and means within the cycle as a consequence of the change in the monetary policy instrument. In particular, we try to capture the following empirical regularity: reduction in volatilities trigger a decrease in mean in both inflation, and interest rate along with an increase in the mean of money aggregates. In doing so, we evaluate

[2] De Gregorio (2003) reported different episodes in which countries with low inflation coincided with rapid money growth: Chile and other OECD countries (Australia, Canada, Germany, Japan, Korea, Norway, South Africa, Sweden, Switzerland y United States). He argues that in the context of a monetary policy based on inflation targeting, where the policy instrument is the interest rate, it is possible for money to fluctuate widely without jeopardizing the inflation target.

Table 3: Perú, main moments - nominal variables
(In percentages)

	Sample		
	1994.1-2001.4	2002.1-2005.4	1994.1-2005.4
Mean			
CPI Inflation	8,4	1,9	6,4
Short term interest rate	13,4	3,0	9,5
Δ% M0	15,2	15,2	15,2
Δ% M1	16,4	16,9	16,6
Δ% M3	19,9	5,4	15,3
Standard deviation			
CPI Inflation	6,3	1,4	6,0
Short term interest rate	4,6	0,6	6,3
Δ% M0	12,2	6,8	10,7
Δ% M1	16,2	6,6	13,8
Δ% M3	13,5	4,1	13,2
Cross correlation with CPI Inflation (*)			
Short term interest rate	17,9	-6,5	58,4
Δ% M0	69,0	5,2	51,5
Δ% M1	85,0	13,5	66,6
Δ% M3	91,4	-3,5	91,0

(*) Adjusted by Forbes and Rigobon

the second order solution of stochastic small open economy model in the line of the "New Open Economy Macroeconomics" literature.

Typically, researchers analyze the properties of stochastic general-equilibrium monetary models, relying on a certainty equivalence assumption by approximating exact equilibrium relationships with log-lineralizations around the steady state. However, in the aforementioned solutions it is no possible to analyze higher moments effects of the equilibrium, nor the relationship among different moments. Thus, the second order solution allow us to relax the certainty equivalence assumption and to take into account the second moments effects over means. Due to the second order solution, in our model uncertainty matters and therefore it has an impact not only in the international prices but also on the price setting-decisions, output determination, money demand and so on.

In our analisys we are isolating one of the three effects that the adoption of an inflation target regime have. An IT regime changes the way expectations are formed, anchoring the expected inflation to the target. It also changes the policy rule of the policymaker, since the adoption of the IT regime has the explicit rule of control of the inflation. And finally, the IT regime must be accompained with the standard operative procedure of using the interest rate as the policy instrument. In our analysis we have disentangled the last effect, since we consider policy rules are both consistent in their objectives of inflation and output gap.

In order to capture only the change in the policy instrument we consider two policy rules: 1) an interest rate feedback rule and 2) a money rule. Moreover, in order to isolate the effects of the election of the policy instrument from the reaction that different rules can have to inflation, we make both rules equivalent under the absence of a money demand shock. Therefore, both rules respond the same way to any shock, except for money demand shocks. A money demand shock enhances volatility to the money aggregate regime which is transmitted to the mean of the variables. In this regard, once we introduce an unobservable demand shock we are able to calibrate our model to allow for realistic changes in volatilities once we change the instrument rule.

Our quantitative exercises show that the model does a reasonably good job at characterizing the key features of an economy in which the central bank changes its instrument from money aggregates toward interest rate rules. In particular, the model captures the decrease in the mean in inflation and the nominal interest rate vis-a-vis the increase in the mean in the money demand as a consequence of the reduction of the nominal volatility following the policy change. Moreover, we can identify three sources of risk premium which interact among them in order to get the overall change in means such as: the effect of the volatility of interest rate in the money demand portfolio decision, the risk premium components of the UIP on the levels of interest rate and nominal depreciation, and the effect of price uncertainty on inflation in the Phillips curve.

Finally other authors have introduced the second order approach in closed and open economies, however, most of the work have mainly focused on normative issues. Thus, Benigno and Woodford (2004) implement the second order solution to evaluate optimal monetary and fiscal policy in a closed economy. Ferrero (2005) extends Beningo and Woodford (2003) to the open economy counterpart. Benigno and Benigno (2004) used the second order approach to evaluate the optimal policy in a two-country model with complete markets. Similarly, De-Paoli (2004, 2005) evaluates optimal monetary policy for a small open economy under different financial markets structures. The closer work to ours is the one by Obstfeld and Rogoff (1998) in which they develop an explicit stochastic NOEM model relaxing the assumption of certainty equivalence. Based on simplified assumptions, they obtain analytical solutions for the level exchange rate premium. Different from Obstfeld Rogoff (1998) we perform a quantitative evaluation of the second order solution. We also differ from them by enriching the asset market structure allowing for stationary net foreign assets[3].

In section 2 we present the model in which we consider money demand and foreign interest rate shocks. In this section we show how the second order solution might affect the equilibrium conditions. In the first part of section 3 we present the results on the second and first moments of the economic variables and we do some sensibility analysis to the calibration of the parameters. Section 4 concludes.

2 The model

We consider a two country open economy model with imperfect competition and price rigidities á la Calvo´ in the line of Obstfeld and Rogoff (1995), Clarida, Gali and Gertler (2002), Benigno and Benigno (2002) and others. We allow for tradable goods and home bias to generate deviations from purchasing power parity. Home bias depends on the degree of openeness and

[3]Obstfeld and Rogoff (1998) employs a key simplifying feature which implies that the current account is always zero in equilibrium.

the relative size of the economy. We characterize the small open economy by taking the limit to the home size to zero as in Sutherland (2003)[4].

2.1 Preferences

There are two countries, H(Home) and F (Foreign). Population in the home country belongs to the interval $[0, n]$, while in the foreign economy it is in the segment $(n., 1]$. Similarly, firms at home produce goods on the interval $[0, n]$ and are indexed by h. Foreign firms do so on the interval $(n, 1]$ and are indexed by f . The utility function of a representative household h in country H is given by:

$$E_t \sum_{i=0}^{\infty} \beta^i U \left[C_t, \frac{M_t}{P_t}, N_t(h) \right] \tag{1}$$

$$U_t = \left[\frac{1}{1-\rho} (C_t)^{1-\rho} + \zeta \frac{\exp(\varepsilon_t)}{1-\epsilon} \left(\frac{M_t}{P_t} \right)^{1-\epsilon} - \frac{1}{n} \int_0^n \frac{(N_t(h))^{1+\eta}}{1+\eta} dh \right] \tag{2}$$

E_t denotes the expectation conditional on the information set at date t, and β is the intertemporal discount factor, with $0 < \beta < 1$. ρ and $\eta > 0$ represent the coefficient of risk aversion and the inverse of the elasticity of the labor supply, respectively; ε_t is a money demand shock which is not observed by the policy maker[5].

Agents get utility from consumption C and from holding domestic real money balances, $\frac{M}{P}$. They get desutility from supplying working hours, N. C is a Dixit-Stiglitz aggregator of home and foreign goods, defined as:

$$C \equiv \left[v^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{3}$$

where $\theta > 0$ is the intratemporal elasticity of substitution between home and foreign tradable goods and C_H and C_F are the consumption sub-indices that refer to the consumption of home-produced and foreign-produced goods. Following, Sutherland (2002) $(1-v)$ is a function of.the relative size of the foreing economy $1-n$ and the degree of opennes,$\gamma : (1-v) = (1-n)\gamma$

Similarly, the corresponding consumption index for foreign households is given by

$$C^* \equiv \left[v^{*\frac{1}{\theta}} C_H^{*\frac{\theta-1}{\theta}} + (1-v^*)^{\frac{1}{\theta}} C_F^{*\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{4}$$

with $v^* = n\gamma$. The super-index $*$ indicates a variable of the foreign economy.

[4] De Paoli (2004) analyze welfare by using a second order solution in a small open economy. By departing from a two-country model, she treats the small open economy as a limiting case.

[5] We assume that the set of information that the policymaker has does not include ε_t . However, ε_t is revealed when the money market clears, therefore it will affect the determination of either the money supply or the nominal interest rate, depending on the policy instrument used.

The optimal allocation of consumption across goods gives the following indexes for consumption of home-produced and foreign produced goods $C_H(C_H^*)$ and $C_F(C_F^*)$:

$$
\begin{array}{ll}
C_H \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c(z)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}} & C_F \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 c(z)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}} \\
C_H^* \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c^*(z)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}} & C_F^* \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 c^*(z)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}}
\end{array} \tag{5}
$$

where $\sigma > 1$ is the elasticity of substitution across the differentiated products[6]. The aggregate consumption price indexes for the home and foreign economy are:

$$
P \equiv \left[vP_H^{1-\theta} + (1-v)\, P_F^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{6}
$$

$$
P^* \equiv \left[v^* P_H^{*1-\theta} + (1-v^*)\, P_F^{*1-\theta}\right]^{\frac{1}{1-\theta}} \tag{7}
$$

The aggregate demands for each good come from aggregating over all the individuals:

$$
\begin{aligned}
y^d(h) &= \left(\frac{p(h)}{P_H}\right)^{-\sigma} \left(\frac{P_H}{P}\right)^{-\theta} \left[vC + \frac{v^*(1-n)}{n} Q^\theta C^*\right] \\
y^d(f) &= \left(\frac{p(f)}{P_F}\right)^{-\sigma} \left(\frac{P_F}{P}\right)^{-\theta} \left[\frac{(1-v)\, n}{1-n} C + (1-v^*)\, Q^\theta C^*\right]
\end{aligned} \tag{8}
$$

where we define the real exchange rate as the ratio of relative consumption price indexes: $Q = SP^*/P$.

In the case of a small open economy we take limit to the economy size to $n \to 0$, and the aggregate demands become:

$$
\begin{aligned}
y^d(h) &= \left(\frac{p(h)}{P_H}\right)^{-\sigma} \left(\frac{P_H}{P}\right)^{-\theta} \left[(1-\gamma)\, C + \gamma Q^\theta C^*\right] \\
y^d(f) &= \left(\frac{p^*(f)}{P_F^*}\right)^{-\sigma} \left(\frac{P_F^*}{P^*}\right)^{-\theta} C^*
\end{aligned} \tag{9}
$$

notice that in the case of a small open economy the demand for home-produced goods depends on both domestic and foreign consumption. However, because of the small relative size of domestic economy, the demand for foreign-produced goods is affected uniquely by foreign consumption[7].

[6] In the same, way we can obtain the price indexes for home-produced and foreign produced goods $P_H(P_H^*)$ and $P_F(P_F^*)$:

$$
\begin{array}{ll}
P_H \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n p(z)^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}} & P_F \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 P(z)^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}} \\
P_H^* \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n p^*(z)^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}} & P_F^* \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 P^*(z)^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}}
\end{array}
$$

[7] We define the terms of trade as the relative price of foreign-produced over home-produced goods: $T = P_F/SP_H^* = P_F/P_H$. In the case of a small open economy, as $n \to 0$, we can express the real exchange rate as a function of the terms of trade of the following form: $Q_t = \left[\frac{T_t^{1-\theta}}{(1-\gamma)+\gamma T_t^{1-\theta}}\right]^{\frac{1}{1-\theta}}$

The money demand is given by the intratemporal first order condition:

$$\zeta \exp\left(\varepsilon_t\right) m_t^{-\epsilon} = \frac{i_t}{1+i_t} C_t^{-\rho} \tag{10}$$

where $m = M/P$ are real money balances and ε are the shocks to the money demand that are not observed by the policy maker. If these shocks were not present, then a policy maker could follow a rule based on money aggregates consistent with a rule based on interest rate. The fact that there is an unexpected component in the money demand makes that such policy rules have different effects in the economy. Under an interest rate feedback rule the policy maker fix the interest rate and the money supply is endogenous determined and the money demand shock will affect only this variable. On the other hand, under money aggregates rule, the policy maker fix the money supply and the interest rate is determined in the money market equilibrium, and the shocks to the money demand will affect the volatility of the interest rate and the rest of the variables on the economy.

The previous mechanism is neatly captured under the assumption of separability in the utility function and/or the abcense of transactions cost that depend on C and m[8]. If we relax any of these two assumptions, we will have that money has an additional role by directly affecting the aggregate demand and any policy instrument will have an additional channel to affect the solution of the model. We abstract from that additional channel to gain intuition, but we also include a more general form of utility function allowing for non-separabilities in the last section.

2.2 Price setting mechanism

The firm´s price setting behavior is modelled through a Calvo-type mechanism. We assume that prices are subject to changes at random intervals. In each period a seller faces a fixed probability $(1-\alpha)$ of adjusting the price, irrespective on how long it has been since the last change had occured. In this model suppliers behave as monopolists in selling their products. The optimal choice of producers that can set their price .$\widetilde{p}_t(z)$ at time t is given by:

$$E_t\left\{\sum_{T=0}^{\infty} (\alpha\beta)^T U_c\left(C_T\right)\left(\frac{\widetilde{p}_t(z)}{P_{H,T}}\right)^{-\sigma} Y_{H,T}\left[\frac{\widetilde{p}_t(z)}{P_T} - \mu rmc_t\right]\right\} = 0 \tag{11}$$

where $\mu = \frac{\sigma}{\sigma-1}$ is the mark up that the monopolist charge over the marginal costs.

Given the Calvo-type setup, the the price index evolves according the following law of motion:

$$P_{H,t}^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha)\left(p_t(h)\right)^{1-\sigma} \tag{12}$$

Following Benigno and Woodford (2004), we write the optimal price and the inflation dynamics

[8]See McCallum (2001)

recursively, introducing K_t and F_t as auxiliary variables:

$$\begin{aligned}
\alpha\left(\pi_{H,t+1}\right)^{\sigma-1} &= 1-(1-\alpha)\left(\frac{K_t}{K_t}\right)^{\frac{1-\sigma}{1+\eta\sigma}} \\
K_t &= \mu Y_{H,t}C_t^{-\rho}rmc_t+\alpha\beta E_t\left(\pi_{H,t+1}\right)^{\sigma(1+\eta)}K_{t+1} \\
F_t &= \left(\frac{P_{H,t}}{P_t}\right)Y_{H,t}C_t^{-\rho}+\alpha\beta E_t\left(\pi_{H,t+1}\right)^{\sigma-1}F_{t+1}
\end{aligned} \tag{13}$$

2.3 The asset market structure

As we mention in the introduction, incomplete markets amplifies second moments effects due to uncertainty on consumption[9]. Thus, the incomple markets assumption permit us to introduce the effects of both the exchange rate risk and monetary policy uncertainty to the levels of the nominal exchange rate, inflation and the nominal interest rate. By taking the second order approximation we relax the certainty equivalence assumption which is more relevant under the incomplete asset market structure[10].

We have chosen to model incomplete markets in which two risk-free one-period nominal bonds denominated in domestic and foreign currency are traded, and a cost of bond holdings is introduced to achieve stationarity[11]. One bond is denominated in domestic currency and the other one in foreign currency. Then, the real budget constraint of the domestic household h will be given by

$$\frac{B_{H,t}^h}{P_t\left(1+i_t\right)}+\frac{S_tB_{F,t}^h}{P_t\left(1+i_t^*\right)\phi\left(\frac{S_tB_{F,t}}{P_t}\right)}\leq\frac{B_{H,t-1}^h+S_tB_{F,t-1}^h}{P_t}+\frac{W_t^hN_t^h}{P_t}-C_t^h-\frac{\Pi_t^h}{P_t} \tag{14}$$

where W_t^h is the nominal wage. Π_t^h are nominal profits for home consumer. We assume that each consumer holds one firm in each sector (domestic firms are located in the interval $[0,n]$ and the size of the home population is normalized to n) and there is no trade in firms' shares. $B_{H,t}$ is household h's holding of the risk free nominal bond, in Home currency. $B_{F,t}$ is household h's holding of the risk-free nominal bond in Foreign currency. The function $\phi(.)$ depends on the real holdings of the foreign assets in the entire economy, and therefore is taken as given by the domestic household[12]. $\phi(.)$ will allow us to obtain a well-defined steady

[9] Under complete markets the real exchange rate at the equilibrium equalize the ration of marginal utilities at every period. Therefore, the second order solution does not have any effect over the previous equilibrium condition.

[10] Relaxing the certainty equivalence assumption will affect also the price setting decision of firms by the presence of nominal rigidities. Obstfeld and Rogoff (1998) and Rankin (1998) focus on the effects of monetary uncertainty through the aggregate supply and welfare.

[11] We follow Benigno (2001). Schmitt-Grohe and Uribe (2001) and Kollmann (2002) develop small open-economy models introducing the same cost to achieve stationarity. See Benigno and Thoenissen (2003) and Selaive and Tuesta (2003) for applications in two country DSGE models.

[12] As Benigno, P.(2001) points it out, some restrictions on $\phi(.)$ are necessary: $\phi(0)=1$; assumes the value 1 only if $B_{F,t}=0$; differentiable; and decreasing in the neighborhood of zero.

state, and to capture the costs of undertaking positions in the international asset market[13]. We assume $\phi\left(\frac{S_t B_{F,t}}{P_t}\right) = \phi\left(\frac{S_t B_{F,t}}{P_t}\right) = 1 + \delta\left(\exp\left(-\frac{\frac{S_t B_{F,t}}{P_t} - \bar{b}}{\bar{C}}\right) - 1\right)$, as in Schmitt-Grohe and Uribe (2001), which is decreasing and convex on $\frac{S_t B_{F,t}}{P_t}$. We further assume that the initial level of wealth is the same across all households belonging to the same country. This assumption combined with the fact that all households within a country work for all firms sharing the profits in equal proportion, implies that within a country all the households face the same budget constraint. In their consumption decisions, they will choose the same path of consumption. We can then drop the index h and consider a representative household for each country.

The conditions characterizing the allocations of domestic and foreign consumption, and holding of nominal bonds are:

$$U_C(C_t) = (1+i_t)\beta E_t\left\{U_C(C_{t+1})\frac{P_t}{P_{t+1}}\right\} \tag{15}$$

$$U_C^*(C_t^*) = (1+i_t^*)\beta E_t\left\{U_C^*\left(C_{t+1}^*\right)\frac{P_t^*}{P_{t+1}^*}\right\} \tag{16}$$

$$U_C(C_t) = (1+i_t^*)\phi\left(\frac{B_{F,t}S_t}{P_t}\right)\beta E_t\left\{U_C(C_{t+1})\frac{P_t S_{t+1}}{P_{t+1}S_t}\right\} \tag{17}$$

$$\frac{S_t B_{F,t}}{P_t(1+i_t^*)\phi\left(\frac{B_{F,t}S_t}{P_t}\right)} = \frac{S_t B_{F,t-1}}{P_t} + \frac{P_{H,t}Y_{H,t}}{P_t} - C_t \tag{18}$$

In addition, combining the conditions characterizing the allocations of domestic and foreing consumption, and holding of nominal bonds, we get the non-linear version of the UIP :

$$\frac{(1+i_t)}{(1+i_t^*)} = \frac{\phi\left(\frac{S_t B_{F,t}^h}{P_t}\right)E_t U_C(C_{t+1})\frac{P_t S_{t+1}}{P_{t+1}S_t}}{E_t U_C(C_{t+1})\frac{P_t}{P_{t+1}}} \tag{19}$$

2.4 Alternative policy rules

From the empirical evidence it is clear that moving from money aggregates toward interest rules has generated a decrease in macroeconomics volatility and the consequent reduction in the mean. In order to capture only the effects of moving from money aggregates we need to define a money rule comparable to an interest rate rule in absence of a money demand shock. For simplicity we assume an interest rate feedback-rule that only reacts to movements in current inflation

$$\widehat{i}_t = \gamma_\pi \widehat{\pi}_t \tag{20}$$

The linearized demand for real balances is given by

$$\widehat{M}_t - \widehat{P}_t = \frac{\rho}{\epsilon}\widehat{C}_t - \frac{1}{\epsilon}\frac{\beta}{1-\beta}\widehat{i}_t + \frac{1}{\epsilon}\varepsilon_t \tag{21}$$

[13] Another way to describe this cost is to assume the existence of intermediaries in the foreign asset market (which are owned by the foreign households) who can borrow and lend to households of country F at a rate $(1+i^*)$, but can borrow from and lend to households of country H at a rate $(1+i^*)\phi(.)$.

It is well known that if ε_t is perfectly observable, there is no difference between using $\widehat{M_t}$ and $\widehat{i_t}$ as a policy instrument. In particular, given the interest rate feedback-rule defined in equation (20), by using the aggregate demand equation (21) , it is possible to back out a time path for $\widehat{M_t}$ consistent with the taylor rule. Thus, by combining the aggregate demand with the taylor rule under the assumption that ε_t is perfectly observable we get

$$\widehat{M_t} - \widehat{M_{t-1}} = \frac{\rho}{\epsilon}\left(\widehat{C}_t - \widehat{C}_{t-1}\right) + \left(1 - \frac{\gamma_\pi}{\epsilon}\frac{\beta}{1-\beta}\right)\widehat{\pi}_t + \frac{\gamma_\pi}{\epsilon}\frac{\beta}{1-\beta}\widehat{\pi}_{t-1} \tag{22}$$

Notice however that if we allow for a money demand shock, it is not possible to get a money rule consistent with a taylor rule. An unexpected money demand shock will enhance an interest rate volatility which will then amplifies both consumption and output volatilities via the euler equation. Instead, when the interest rate is the target the monetary authority lets the money supply to adjust to the money demand shock. Therefore, there is not impact on output or inflation since the central bank perfectly accommodates.[14].

3 Results

3.1 The mechanism

We have identified three main sources of uncertainty from which volatility feeds into the means of inflation, nominal interest rate and money demand, they are the portfolio decision, which affects the level of money demand, the UIP which affects the level of the interest rate, and finally the Phillips curve which affects the level of inflation.

3.1.1 Portfolio decision:

The money demand is given by:

$$\zeta \exp\left(\varepsilon_t\right) m_t^{-\epsilon} = \frac{i_t}{1+i_t} C_t^{-\rho} \tag{23}$$

that have a order expansion of the following form:

$$\widehat{m}_t = \frac{\rho}{\epsilon}\widehat{C}_t - \frac{1}{\epsilon\bar{i}}\widehat{i}_t + \frac{1}{2}\frac{1+\bar{i}}{\epsilon\bar{i}^2}\left(\widehat{i}_t\right)^2 + \frac{1}{\epsilon}\varepsilon_t + \mathcal{O}\left(\|\xi\|^3\right)$$

where $\bar{i} = \beta^{-1} - 1$ is the steady state nominal (real) interest rate.

Through this mechanism, the higher the volatility of the interest rate, the higher the level money demand. Since the nominal interest rate of domestic bonds is the oportunity cost of

[14]See Clarida, Galí and Gertler (1999) for a brief discussion why is preferable to use an interest rate targets under the optimal policy instead of a money target rule. Woodford (2003) also pointed out that since in practice central banks can not estimate the random disturbance in the money demand, adoption of money-rules necessarily results in an interest rate variations that can be eliminated by using an interest rate rule.

money, the higher the interest rate volatility the lower the demand for domestic bonds and the higher de demand for money. However, there are other first order effects that affect the level of money demand, such as the consumption level and the interest rate level. In equilibrium all these factors interact to affect the level of the money demand.

3.1.2 Uncovered Interest Rate Parity

From the first order conditions for the domestic consumer for domestic and foreign bonds we can obtain the non-linear version of the uncvered interest rate parity:

$$\frac{(1+i_t)}{(1+i_t^*)} = \frac{\phi\left(\frac{S_t B_{F,t}^h}{P_t}\right) E_t \frac{U_C(C_{t+1})}{U_C(C_t)} \frac{P_t S_{t+1}}{P_{t+1} S_t}}{E_t \frac{U_C(C_{t+1})}{U_C(C_t)} \frac{P_t}{P_{t+1}}} \tag{24}$$

Which has the following second order expansion:

$$\widehat{i}_t - \widehat{i}_t^* = E_t \Delta \widehat{S}_{t+1} - \delta \widehat{b}_t + RP_t + \mathcal{O}\left(\|\xi\|^3\right)$$

Where: $RP_t = -\delta \widehat{b}_t + \left[\frac{1}{2}\left[Var\left(\widehat{r}_t^*\right) - Var\left(\widehat{r}_t\right)\right]\right] - \rho cov\left(\Delta \widehat{C}_{t+1}, \Delta \widehat{S}_{t+1}\right)$
where $r_t = E_t\left(1+i_t\right)\frac{P_t}{P_{t+1}}$ and $r_t^* = E_t\left(1+i_t^*\right)\frac{P_t S_{t+1}}{P_{t+1} S_t}$

The uncovered interest rate parity condition is an arbitrage condition between the domestic and foreign bonds for the individuals of the domestic country. The higher the net foreign asset position, the lower the cost of financing abroad and the lower the real return of domestic bonds. Similarly, the higher the variance of the real return of the domestic bond, the lower the demand for that bond and the lower the return.

The third component is related to the consumption CAPM. Under that approach, the return of an asset is positively related to its covariance between the marginal rate of consumption. Because individuals look forward to smooth consumption, when the marginal utility of consumption is low, they look to save to for periods when the marginal utility is high. If an asset has higher returns in those episodes, then it will demanded at a lower return than an asset with positive covariance.

The higher the volatility of the real exchange rate with respect to the inflation, the higher the demand for domestic bonds and the higher its interest rate. The higher the covariance between the marginal utility of consumption and the nominal exchange rate, the higher the gap between the domestic and the foreign interest rates. (Positive correlation between exchange rate and consumption, foreing bonds demand a lower interest rate).

3.1.3 The Phillips Curve:

The non-linear form of the Phillips curve is given by:

$$\alpha\left(\pi_{H,t}\right)^{\sigma-1} = 1 - (1-\alpha)\left(\frac{K_t}{F_t}\right)^{\frac{1-\sigma}{1+\sigma\eta}}$$

where K_t and F_t are the auxiliary variables defined recursively in the previous section. The second order expansion of the Phillips curve is:

$$\widehat{\pi}_{H,t} = \kappa\widehat{rmc}_t + \beta E_t \widehat{\pi}_{H,t+1} - \frac{1}{2}\frac{\sigma - 1}{1-\alpha}\left[\kappa\widehat{rmc}_t + \beta E_t \widehat{\pi}_{H,t+1}\right]^2 + \frac{1}{2}\alpha\beta G\left(\widehat{rmc}_t, E_t\widehat{\pi}_{H,t+1}\right) + \mathcal{O}\left(\|\xi\|^3\right)$$

where $\kappa = \frac{1-\alpha}{\alpha}\frac{1-\alpha\beta}{1+\eta\sigma}$, $\widehat{rmc}_t$ is the average real marginal costs defined in the appendix and $G > 0$ is a function that depends on the second moments of the marginal costs and next period inflation.

The effect of the second moments on the level of inflation is positive, higher macroeconomic volatility implies higher risk for the firms on the price level they put, and this implies higher inflation. Moreover, when prices are more sticky, α higher, this risk premium is higher. Moreover, higher α implies a flatter Phillips curve, which implies higher volatility on real variables but lower volatility on nominal variables. In overall, the effect of the degree of price stickyness (α) in the mean of inflation would depend on which effect is higher.

3.2 Parameterization

We set a quarterly discount factor, β, equal to 0.99 which implies an annualized rate of interest of 4%. The share of foreign goods in consumption $\gamma = 0.4$. For the coefficient of risk aversion parameter, ρ, we choose a value of 5. Regarding this parameter, Eichenbaum et.al (1988) fiand a range between 0.5 and 3. On the other hand, Hall (1988) suggest a value greater than 5. The inverse of the elasticity of labour supply,η, is calibrated equal to 0.5, similar to those used in the RBC literature.We choose a degree of monopolistic competition, σ, equal to 7.66. This implies a mark-up of 15%. The elasticity of substitution between home and foreign goods, θ, is 1.5 as assumed in Chari, Kehoe and McGrattan (2002).

For the monetary rule, we have a *sui-generis* rule in terms of cpi inflation and consumption, in order to have a policy rule comparable in both regimes. We follow Taylor (1993) and set the coefficient $\phi_\pi = 1.5$. We fix the probability of not adjustin prices, $\alpha = 2/3$. The process for the interest rate $\rho_v = 0.96$ and $var\left(v\right) = (0.009)^2$. We calibrate the process of money demand shocks in order to explain 50% of the variance of the interest rate in the case of a money rule.

Note that the monetary policy regime changes the impulse responce to money demand shocks. However, the response to other kind of shock, such us to international interest rate shocks, is the same in both regimes as it is shown in figures 1 and 2.

3.3 Results

In this section we analyze the effect of a change in volatilities over the means. In our analysis, the traditional method based on the log-linearization around the steady state permits to analyze the second moments of the endogenous variables, such us the variance and covariance. However, in order to analyze the first moments, the mean, we need a higher order solution. This is why we use a second order approach, which is based on the perturbation method and consist on solving the policy function of the endogenous variables as a second order polynomial on the exogenous variables.

3.3.1 First order solution: effects on the variances and covariaces

We use the first order solution of the model in order to calculate the unconditional second moments under both type of monetary instruments, money aggregates vs. interest rates. As we can see in the following table, the variances and covariances of the endogenous variables differ under both regimes.

Table 4: second moments effects.

	$sd(i)$	$sd(\pi)$	$sd(M^d/P)$	$sd(\Delta s)$	$sd(y_H)$
Money rule	0.64%	0.76%	11.18%	5.87%	2.14%
Interest rate rule	0.43%	0.38%	11.25%	5.70%	1.03%

From this results we can see that a change in the policy instrument from money aggregates to interest rates decreases the macroeconomic volatility.

The second moments change because of the shocks to the money demand. The mechanism is that under a money aggregates rule the interest rate is residual and any shock in the money demand will affect the interest rate, which will have effects on all the real variables. The higher the volatility of money demand, the higher the overall macroeconomic volatility. On the other hand, under interest rate rules, the money supply is residual and absorves all the money demand shocks and decreases macroeconomic volatility.

3.3.2 Second Order Solution: Effects on the mean

We use the perturbation method, developed originally by Judd (1998), Collard and Julliard (2001) and Uribe and Schmitt-Grohe (2004), to find a second order approximation of the solution of the model. This method consists in obtaining the coefficients of a taylor expansion of the solution of the model near to the steady state using a system of equations that come from the differentiation of the equilibriu conditions fo the model. For instance, if there are only international interest rate shocks (v)and money demand shocks(ε), the solution for an endogenous variable x is given by:

$$x = b_\sigma \sigma^2 + b_v v + b_\varepsilon \varepsilon + b_{v\varepsilon} v\varepsilon + \frac{1}{2} b_{vv} v^2 + \frac{1}{2} b_{\varepsilon\varepsilon} \varepsilon^2$$

The coefficients of the first order terms, b_v and ε, are equal to those of the log-linearised solution of the model. The second order solution only adds additional terms to the log-linearised solution, $b_{v\varepsilon}$,b_{vv} and $b_{\varepsilon\varepsilon}$, preserving the existing terms. Additionaly, b_σ is a constant that depend on the variance of the shocks, as it is shown in Uribe and Schmitt-Grohe (2004).

The first moments of the endogenous variables can be calculated using this second order solution, for example $E(x) = b_\sigma \sigma^2 + \frac{1}{2} b_{\varepsilon\varepsilon} Var(\varepsilon) + \frac{1}{2} b_{vv} Var(v) + \mathcal{O}\left(\|\xi\|^3\right)$. i

In the next table we show the change in the unconditional mean of the nominal interest rate, inflation and money demand due to a change on the policy instrument.

Table 5: first moments effects

	Ei	$E\pi$	EM^d/P
Money instrument	1.83%	0.84%	38.3%
Interest rate instrument	1.00%	0.00%	45.4%
Difference	-0.83%	-0.84%	7.1%

From this results we can see that under the baseline calibration the change of the monetary policy instrument, from a money aggregate to interest rates, reduces the unconditional mean of the nominal interest rate an inflation by .83%, which increases the unconditional mean of money demand by 7%.

As mentioned before, we have three main mechanisms through wich macroeconomic volatility feeds into the level of inflation, interest rates and money demand as the form of risk premiums. The risk premium of the Phillips curve feeds into the mean of inflation, that feeds into the mean of the nominal interest rate and the money demand. There are also the mechanism of the risk premium of the UIP that feeds into the level of the interest rate and that of the portfolio decision, that feeds from volatility of the nominal interest rate into the level of the money demand.

In the appendix we show some sensitivity analysis on the changes of the standard deviation and the mean of the nominal interest rate due to a change of regime for the risk aversion (ρ), the inverse of the labor supply elasticity (η) and the degree of price stickyness (α).

4 Conclusions

In this paper we have accounted for several features observed in economies that have recently moved from a monetary policy in which the instrument was a money aggregate toward an interest rate target. In paticular, in those economies, changes in volatilities as a consequence of the instrument change have significant affected the means of the main nominal variables such as interest rate, inflation and money aggregates. The aforementioned features can not be capture with standard log-linear model. Thus, in this paper we contribute to the discussion by evaluting a second order solution of a typical small open economy in order to account for effect of volatilities over means. The model allows us to compute different forms of risk-premia on inflation, interest rate and money aggregates as it is observed in the data Furthermore, we identify three sources of risk premium that affect the levels of money demand, interest rate and inflation. The first of them is related to the effect of the volatility of interest rate in the money demand portfolio decision. The second source of risk premium comes from the UIP and affects the levels of interest rate and nominal depreciation, and the last source corresponds to the effect of price uncertainty on inflation through the Phillips curve. These three sources of risk premium provide a link between macroeconomic volatility and the unconditional means of economic variables. In this sense, a reduction in the volatility of the nominal interest rate and inflation implies a reduction in the risk premium of holding domestic currency, which in equilibrium generates a fall in expected interest rate and inflation, which causes an increase in money demand.

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A Sensibility analysis

In this section we analyze numerically the effects of changing some parameters on the changes of the standard deviation and the mean of the nominal interest rate due to a change of regime.

Changing risk aversion (ρ)		
	Δ s.d.	Δ Mean
$\boldsymbol{\rho = 5}$	$-\mathbf{33}\%$	$-\mathbf{0.83}\%$
$\rho = 1$	-30%	-0.98%
$\rho = 0.5$	-27%	-1.10%

Changing inverse labor supply (η)		
	Δ s.d.	Δ Mean
η= **0.5**	$-\mathbf{33}\%$	$-\mathbf{0.83}\%$
$\eta = 1$	-22%	-0.73%
$\eta = 5$	-61%	-0.28%

Changing price stickyness (α)		
	Δ s.d.	Δ Mean
$\alpha = 0.75$	-23%	-0.61%
$\boldsymbol{\alpha} = \mathbf{0.66}$	$-\mathbf{33}\%$	$-\mathbf{0.83}\%$
$\alpha = 0.50$	-53%	-1.25%

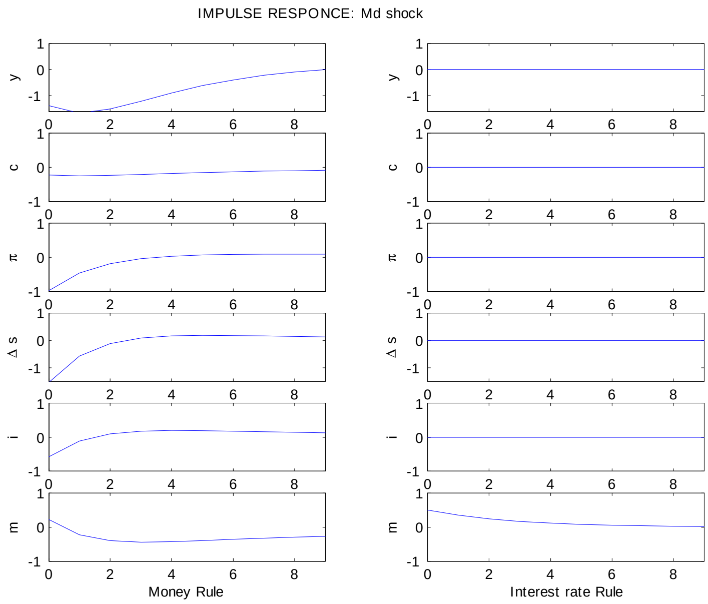

Figure 1: Impulse Responses - Money demand shock.

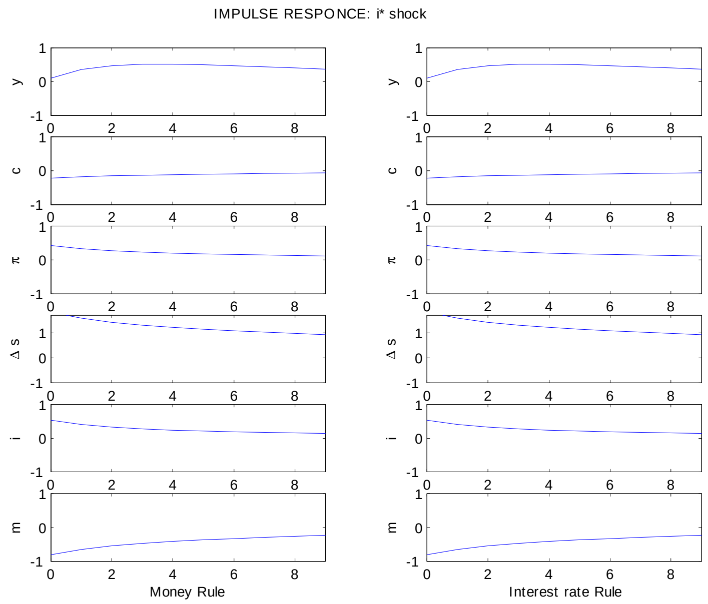

Figure 2: Impulse Responses - Foreign Interest Rate Shock.