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Estimation and assessment of measures of the natural rate of interest: Evidence from Latin American economies with inflation targeting

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Estimation and assessment of measures of the natural rate of interest: Evidence from Latin American economies with inflation targeting. *

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Abstract

We estimate and assess two measures of the natural rate of interest (NRI) for two Latin American economies, Chile and Perú, where monetary policy is conducted under inflation targeting. The first NRI measure is obtained through the estimation of a time varying parameter vector autoregression model with stochastic volatility (TVP-VAR-SV) as in Lubik y Matthes (2015), which we denote TVP-NRI. The second NRI measure is based on a recent methodology proposed by Benati (2020) and Benati (2023), which exploits the relationship between the interest rate and money velocity (Benati-NRI). In order to assess these two measures of NRI, we propose a new and simple criterion based on the idea that NRI is not expected to react to shocks that have no long-run effect on real interest rate (i.e. transitory shocks). The results for Chile and Peru indicate that TVP-NRI measures are relatively superior.

Key words : Natural rate of interest, monetary policy, TVP-VAR-SV, money velocity, permanent and transitory shocks, LATAM-5.

JEL classification : C11, C32, E52

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1 Introduction

The period of low levels of interest rates experienced by many countries since the global financial crisis has been a key driver of recent research on the so-called natural rate of interest (NRI), a concept introduced by Wicksell in 1898 (Woodford, 2003). Most studies have provided evidence that the NRI has displayed a downward trend (Zhang et al. , 2021; Wang y Kwan , 2021; Holston et al. , 2017). (Holston et al., 2023) finds that the NRI has remain low even during Covid-19 and the recent period of high and persistent inflation that pushed back to higher interest rates.

The NRI, a key concept in monetary policy under the New Keynesian framework, is defined as the interest rate at which the economy achieves both price and economic stability (Laubach y Williams , 2003), or the interest rate that stabilizes both inflation and the output gap (Woodford, 2003). The comparison between the short-term real interest rate and the NRI is very useful to determine the stance of monetary policy: if the real short-term interest rate is below (above) the NRI, then monetary policy is expansive (contractive). Also, the NRI can be used in order to assess potential threatens to financial stability; for instance, excessive risk-taking can arise if real interest rates are below the NRI. Research on the NRI has increased in recent years motivated by the downward trend of real interest rates observed in many countries since the global financial crisis, as well as by its rebound observed during the current high inflation period.

A main practical drawback of the NRI is that it is a non-observable, theoretical variable and therefore needs to be estimated. The literature provides several alternatives to estimate the NRI. A first approach is based on the use of univariate filters (such as the Hodrick-Prescott filter) or multivariate filters (such as the Kalman filter) which can be combined with theoretical relationships (Laubach y Williams , 2003; Holston et al. , 2017, among others). A second approach relies on the estimation of econometric models, such as vector autoregressive models with time-varying parameters and stochastic volatility (Lubik y Matthes , 2015; Wang y Kwan , 2021, among others) and cointegration (Benati, 2020, 2023). A third alternative is the estimation and/or calibration of theoretical models (Doojav and Gantumur , 2020;

Zhang et al. , 2021). Additionally, Borio (2021) suggests the use of indicators that capture the expectations of market participants that is beyond the direct control of the central bank, such as a real forward rate.

In this paper, we estimate and assess two measures of the NRI for two Latin American economies that conduct monetary policy under inflation targeting (LATAM-5): Chile and Peru. The first NRI measure is obtained through the estimation of a time varying parameter vector autoregression model with stochastic volatility (TVP-VAR-SV) as in Lubik y Matthes (2015), which we denote TVP-NRI. Under this framework, a TVP-VAR-SV is estimated using three variables: the rate of growth of GDP, inflation and the ex ante real interest rate. The model is estimated using standard Bayesian techniques as in Primiceri (2005). From this TVP-VAR-SV model, the NRI is obtained as the long-run (5-year) forecast of the observed ex ante real interest rate.

The second NRI measure is based on a recent methodology proposed by Benati (2020) and Benati (2023), which exploits the relationship between the interest rate and money velocity (B-NRI). Benati (2023) defines the NRI as the permanent component of the ex-post real monetary policy interest rate. The estimation of the NRI is based on the idea that the speed of circulation of M1, defined as the ratio between nominal GDP and nominal M1, is an approximation of the permanent component of the nominal interest rate (R_t^P). Then, M1 velocity can be considered as a reliable estimate of the nominal NRI. Benati (2023) proposes a two-step procedure to estimate the real NRI: (i) project the monetary policy rate on M1 velocity in order to obtain an estimate of the nominal NRI, and (ii) calculate the real TNI as the difference between the nominal TNI and either the inflation target or the average inflation for the estimation sample. Econmetrically, this procedure requires that the monetary policy rate on M1 velocity are cointegrated.

In order to assess the adequacy of these NRI measures, we propose a new and simple criterion: the RI is not expected to react to shocks that have no long-run effect on real interest rate, i.e. NRI does not react to transitory shocks. To test whether a given NRI, r_t^N, meets the proposed criterion, we estimate a two-variable VAR for the real interest rate, r_t, and the NRI, r_t^N, and identify a transitory and a permanent shock imposing the restriction that a transitory shock has no effect on the ex ante real interest rate in the long-run, following the long-run identification strategy proposed by Blanchard and Quah (1989). Then, r_t^N is a good NRI measure if (i) the response of r_t^N to a transitory shock is close to zero, and (ii) the contribution of the transitory shock to the variance of r_t^N is very small.

We employ quarterly data obtained from the corresponding central banks. The sample covers the period 1995Q4-2023Q2 for Peru and 1997Q1-2023Q2 for Chile. The main results indicate that TVP-NRI measures are relatively superior.

The rest of the paper is organized as follows. Section 2 contains the description of the methodology employed to estimate and assess alternative NRI measures. Section 3 presents the main results, and section 4 concludes.

2 Methodology

We estimate and assess two measures of NRI. The measures of NRI are estimated using two recent approaches: (i) a time varying parameter vector autoregression model with stochastic volatility (TVP-VAR-SV) as in Lubik y Matthes (2015), and (ii) the projection of the interest rate on M1 velocity, as proposed by Benati (2023). The assessment of these measures is performed using a novel criterion based on the following idea: the NRI is not expected to react to shocks that have no long-run effect on real interest rate, i.e. NRI does not react to transitory shocks.

2.1 TVP-SV-VAR

The first NRI measure is obtained through the estimation of a time varying parameter vector autoregression model with stochastic volatility (TVP-VAR-SV) as in Lubik y Matthes (2015). We denote this measure as TVP-NRI.

The specification of the VAR includes three variables: the rate of growth of real GDP,

g_t, inflation, π_t, and ex ante real interest rate, r_t. If we define $y_t = (g_t, \pi_t, r_t)'$, then a TVP-VAR-SV with 2 lags (a standard specification in the literature) can be written as:

$$y_t = c_t + B_{1,t}y_{t-1} + B_{2,t}y_{t-2} + u_t \qquad t = 1, \dots, T. \tag{1}$$

where u_t is a 3×1 vector of heterocedastic shocks normally distributed, with mean zero and a 3×3 variance covariance matrix Ω_t, i.e. $u_t \sim N(0, \Omega_t)$; c_t is a 3×1 vector of time varying coefficients that represent the intercepts for every equation, $B_{1,t}$, $B_{2,t}$, and Ω_t are matrices that contain time varying coefficients. The variance covariance matrix Ω_t is descomposed as:

$$\Omega_t = A_t^{-1}\Sigma_t\Sigma_t'A_t^{-1'}$$

where A_t and Σ_t are given by:

$$A_t = \begin{pmatrix} 1 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 \end{pmatrix} \qquad , \qquad \Sigma_t = \begin{pmatrix} \sigma_{1,t} & 0 & 0 \\ 0 & \sigma_{2,t} & 0 \\ 0 & 0 & \sigma_{2,t} \end{pmatrix}$$

Thus, the TVP-VAR-SV can be written as:

$$y_t = c_t + B_{1,t}y_{t-1} + B_{2,t}y_{t-2} + A_t^{-1}\Sigma_t\varepsilon_t \qquad , t = 1, \dots, T.$$

where $Var(\varepsilon_t) = I_n$. If we use the Kronecker product $\otimes$ and define the following block matrix:

$$X_t' = I_3 \otimes [1, y_{t-1}, y_{t-2}]$$

then we can re-write the model as:

$$y_t = X_t'B_t + A_t^{-1}\Sigma_t\varepsilon_t \tag{2}$$

In order to get a time varying structural VAR is key that matrix A is time varying, so that any shock has a time varying effect on each variable.

Given the equivalence between equations 3 and 4, the analysis of the model will be based on the parameters specified in 4. First, let's define a vector α_t that contains all the parameters of matrix A different from 0 and 1:

$$\alpha_t = [\alpha_{21,t}, \alpha_{31,t}, \alpha_{32,t}]$$

and a vector σ_t that contains all the main diagonal parameters of matrix Σ:

$$\sigma_t = [\sigma_{1,t}, \sigma_{2,t}, \sigma_{3,t}]$$

The time varying parameters of the model are assumed to follow a random walk process as follows:

$$\beta_t = \beta_{t-1} + v_{\beta,t}$$
$$\alpha_t = \alpha_{t-1} + v_{\alpha,t}$$
$$\log \sigma_t = \log \sigma_{t-1} + v_{\sigma,t}$$

Notice that $\log \sigma_t$ being a random walk implies that every σ_t evolves as a geometric random walk and thus represent a stochastic volatility model (the variances generated by $\log \sigma_t$ are not observable) in contrast to an ARCH model.

The model assumes that $\epsilon_t, v_{\beta,t}, v_{\alpha^i,t}$, and $v_{\sigma,t}$ are jointly normally distributed and mutually independent, with:

$$Var(v_{\beta,t}) = Q$$
$$Var(v_{\alpha,t}) = S$$
$$Var(v_{\sigma,t}) = W$$

where Q, S, and W are positive definite matrices. We denote $V = Var(\varepsilon_t, v_{\alpha,t}, v_{\sigma,t})$.

The estimation of the time-varying parameters of the structural VAR model is done using

Bayesian methods as in Primiceri (2005) and Del Negro y Primiceri (2015).[1] The posterior distributions of the parameters of interest contained in B, A, and Σ and the hyperparameters of V are evaluated using a bayesian approach because the difference between parameters and shocks is not straight. The posterior numerical evaluation of the parameters of interest is done using Gibbs sampling (a particular variant of Markov chain Monte Carlo methods or MCMC), which consists of drawing from lower dimensional conditional posteriors as opposed to the high dimensional joint posterior of the whole parameter set (Primiceri , 2005).

Following Primiceri (2005), we assume: (1) the initial states for the coefficients, the covariances, log volatilities and hyperparmeters are independent of each other, (2) the priors for the hyperparameters Q, W and the blocks of S are distributed as independent inverse-Wishart, and thus (3) normal priors on the entire sequences of $B's$, $\alpha's$, and $\log \sigma's$ (conditional on Q, W, and S). As discussed in Primiceri (2005), the order of the variables is important. The estimation of the model is done simulating the distribution of the parameters of interest given the data. The MCMC algorithm employed to generate a sample from the joint posterior of (B^T, A^T, Σ^T, V) is described in Del Negro y Primiceri (2015).

We use the first 6 years to calibrate the priors (24 quarters). Following Wang y Kwan (2021), we draw 10,000 samples from the posterior distribution and discard the first 2,000 samples (the chain should converge). Following Lubik y Matthes (2015), the NRI is the conditional 5-year (long run) forecast of the observed ex ante real interest rate:

$$r_t^* = (0, 0, 1) * E_t(y_{t+20}) = (0, 0, 1) * X_t' \beta_t^{20}$$

so that we can obtain an estimate of the NRI for every quarter.

2.2 Projection of interest rate on M1 velocity

The second NRI measure is based on a recent methodology proposed by Benati (2020) and Benati (2023), which exploits the relationship between the interest rate and money velocity (B-NRI). This approach follows the idea of Cochrane (1994) to estimate the

[1] The order of the Gibbs sampling is corrected as Del Negro y Primiceri (2015).

permanent component of the gross national product by exploiting the informational content of consumption.

Benati (2023) defines the NRI as the permanent component of the ex-post real monetary policy interest rate. The estimation of the NRI is based on the idea that the speed of circulation of M1, defined as the ratio between nominal GDP and nominal M1, is an approximation of the permanent component of the nominal interest rate (R_t^P). Then, M1 velocity can be considered as a reliable estimate of the nominal NRI. The permanent component of the nominal NRI, R_t^N, is determined by permanent shocks to inflation, π_t^P, and to the natural rate of real interest rate r_t^N:

$$R_t^P = \pi_t^P + r_t^N \tag{3}$$

If the monetary policy regime is such that inflation can be considered stationary (e.g. an inflation targeting regime), then permanent changes in the velocity of M1 reflect permanent fluctuations in the NIR, and thus the following long-run relationship holds:

$$V_t = \beta_1 + \beta_2 r_t^N + v_t \tag{4}$$

Based on this, Benati (2023) proposes a two-step procedure to estimate the real NRI: (i) project the monetary policy rate on M1 velocity in order to obtain an estimate of the nominal NRI, and (ii) calculate the real TNI as the difference between the nominal TNI and the inflation target of the average inflation for the estimation sample. Econometrically, The first step can be achieved in two ways: using a cointegrated VAR for interest rate and money velocity as in Cochrane (1994) or projecting interest rate onto money velocity using an OLS regression.

2.3 Assessment of NRI estimates

In order to assess the adequacy of these NRI measures, we propose a new and simple criterion: the RI is not expected to react to shocks that have no long-run effect on real interest rate, i.e. NRI does not react to transitory shocks. To test whether a given NRI, r_t^N, meets the proposed criterion, we estimate a two-variable VAR for the ex-ante real interest rate r_t and r_t^N, and identify a transitory and a permanent shock imposing the restriction that a transitory shock has no effect on the ex ante real interest rate in the long-run. Then, r_t^N is a good NRI measure if (i) the response of r_t^N to a transitory shock is close to zero, and (ii) the contribution of the transitory shock to the variance of r_t^N is very small.

Formally, let r_t denote the real ex ante real interest rate, r_t^N the natural rate of interest (NRI), and Δr_t and Δr_t^N the corresponding first differences of both series. Under the assumption that both Δr_t and Δr_t^N are stationary time series, and that the levels are non-stationary, the vector moving average (VMA) representation for Δr_t and Δr_t^N in terms of fundamental innovations can be written as:

$$\begin{bmatrix} \Delta r_t \\ \Delta r_t^N \end{bmatrix} = \begin{bmatrix} \overline{r} \\ \overline{r}^N \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} \phi_{11}(i) & \phi_{12}(i) \\ \phi_{21}(i) & \phi_{22}(i) \end{bmatrix} \begin{bmatrix} \varepsilon_{t-i}^P \\ \varepsilon_{t-i}^T \end{bmatrix} \tag{5}$$

where ε_t^T and ε_t^P represent two disturbances affecting the ex ante real interest rate and the NRI measure. Given that the first difference of both series are assumed to be stationary, then none of these disturbances will have long-run effects on them. However, these disturbances might have long-run effects on the level of the rate of interest.

We identify a transitory and a permanent shock in equation 5 using a long-run restriction as in Blanchard and Quah (1989): an innovation to ε_t^T has no long-run effect on the ex ante real interest rate, i.e. $\sum_{i=0}^{\infty} \phi_{12}(i) = 0$. Under this identification strategy, we can use the impulse response functions (IRFs) and variance decomposition (VD) analysis to assess whether r_t^N reacts to transitory shocks. Specifically, r_t^N is a good NRI measure if: (i) its cumulative response to ε_t^T shocks, $\sum_{i=0}^{\infty} \phi_{22}(i)$, is close to zero, and (ii) the contribution of ε_t^T shocks to fluctuations in r_t^N is close to zero.

3 Results

In this section we present the results for Chile and Peru.[2] The data is obtained from the corresponding central banks. The sample covers the period 1995Q4-2023Q2 for Peru and 1997Q1-2023Q2 for Chile.

The estimation of the TVP-SV-VAR is based on quarterly data for three variables: real GDP growth, the ex-ante real interest rate, and inflation. Real GDP growth is computed as percentage changes on year-over-year basis. Inflation is defined as the annual growth rate of CPI excluding food and energy. The ex-ante real interest rate is calculated as the difference between the policy interest rate (for the case of Chile) or the short-term interest rate (interbank rate for Peru)[3] and inflation expectations, which is calculated as a four-quarter moving average of past inflation. Given that we use 24 quarters as the presample to calibrate the priors and include 2 lags in the VAR, the estimated NRI starts in 2002Q2 for Peru and 2003Q3 for Chile. The estimation of TVP-VAR-SV was performed in R using the package "bvarsv" Krueger (2022).

For the estimation based on Benati (2023), the real interest rate as the difference between nominal interest rate and the inflation target. The velocity of M1 is defined as the ratio between nominal GDP and M1. We estimate two versions of Benati NRI: (i) a simple projection using OLS (Benati-OLS), and (ii) a cointegrated VAR with long-run restrictions (Benati-VECM).

Figures 1 and 2 display the estimated TVP-NRI for Peru and Chile, respectively. For comparison purposes, we use the restricted sample associated to the TVP-NRI estimates for each country. In both cases, there is evidence of a downward trend in the real NRI. Also, in both cases the Benati-NRI estimates display a relatively greater downward slope. For the case of Peru, the TVP-NRI is higher than the two Benati-NRI estimates.

[2]We are still working on results for Brazil, Colombia, and Mexico.
[3]The policy rate started in September 2003.

Figure 1. *Estimated NRI for Peru*

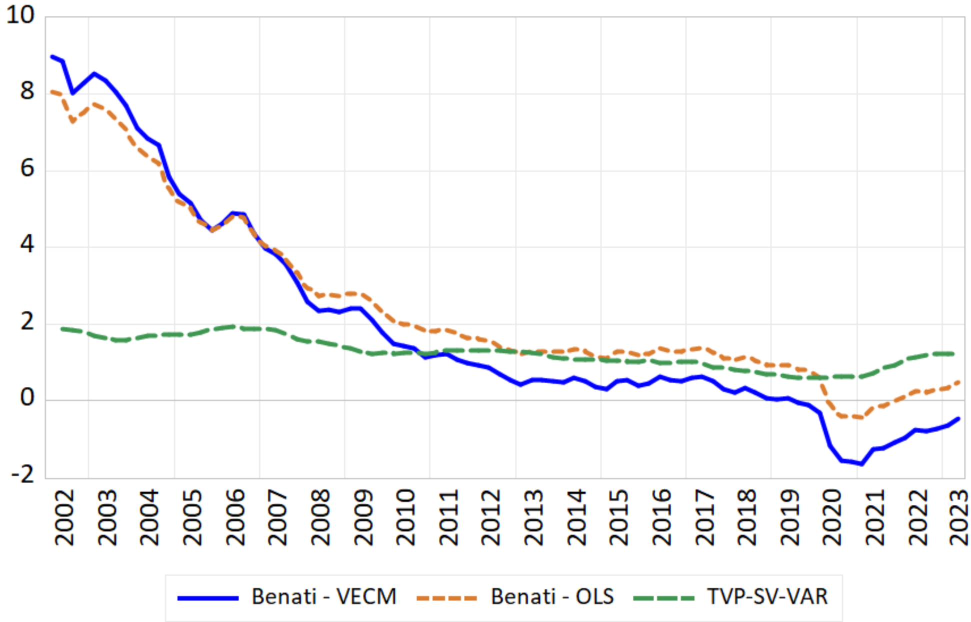
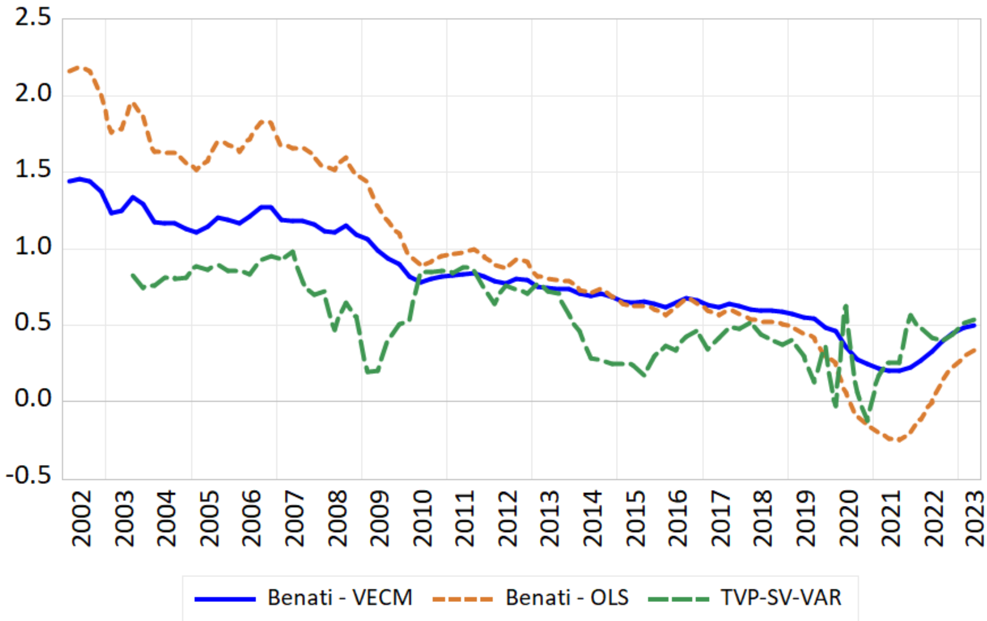


Figure 2. *Estimated NRI for Chile*



In order to compare the alternative NRI estimates, we apply the methodology proposed in subsection 2.3. The first criterion is based on the comparison of the cumulative IRF of the NRI measure after a structural temporary shock: a good measure of NRI should display a cumulative response close to 0. Table 1 shows the results for Peru. All NRI measures display a similar response to a temporary structural shock: a small reaction and only in the first period. Therefore, according to this criterion, all three NRI measures are equally useful.

Table 1. *NRI estimates for Peru Cumulative impulse-response function*

Quarter	Benati-VECM	Benati-OLS	TVP-VAR-SV
1	-0.04*	-0.03*	0.01*
2	0.00	0.00	0.01
3	0.04	0.04	0.01
4	0.09	0.07	0.01
8	-0.01	-0.01	0.00
12	0.01	0.01	0.00
16	0.00	0.00	0.00
20	0.00	0.00	0.00

Table 2 shows the results for Chile. Again, all NRI measures display a similar response to a temporary structural shock: a small reaction and only in the first two periods. Therefore, as in the case of Peru, all three NRI measures are equally useful according to this criterion.

Table 2. *NRI estimates for Chile: Cumulative impulse-response function*

Quarter	Benati-VECM	Benati-OLS	TVP-VAR-SV
1	-0.02*	-0.04*	0.04*
2	-0.02*	-0.05*	0.04*
3	-0.01	-0.01	0.01
4	0.00	0.01	0.02
8	0.01	0.01	0.00
12	0.00	0.00	0.00
16	0.00	0.00	0.00
20	0.00	0.00	0.00

The second criterion is based on the forecast error variance decomposition of the NRI measure: the contribution of a temporary structural shock to the fluctuations in the NRI should be close to zero. Table 3 shows the results for Peru, which indicate that in the long run (after 20 quarters or 5 years), the contribution of the shock to the fluctuations of the

NRI is smaller for the TVP-NRI, and thus it is a better NRI measure.

Table 3. *NRI estimates for Peru: Variance decomposition*

Quarter	Benati-VECM	Benati-OLS	TVP-VAR-SV
1	2.98	2.98	16.99
2	4.49	4.49	10.78
3	7.37	7.37	10.10
4	9.96	9.96	9.69
8	17.43	17.43	10.38
12	17.75	17.75	10.43
16	17.85	17.85	10.44
20	17.86	17.86	10.44

Table 4 shows the results for Chile. In this case, the contribution of the shock to the fluctuations of the NRI is smaller for the TVP NRI at all horizons, and thus it is a better NRI measure.

Table 4. *NRI estimates for Chile: Variance decomposition*

Quarter	Benati-VECM	Benati-OLS	TVP-VAR-SV
1	42.66	42.66	5.74
2	36.89	36.89	5.12
3	48.59	48.59	8.38
4	49.31	49.31	8.32
8	48.68	48.68	8.67
12	49.39	49.39	8.68
16	49.52	49.52	8.69
20	49.56	49.56	8.69

Overall, the analysis suggests that TVP-NRI measures can be considered relatively superior given that their fluctuations response less to structural temporary shocks.

4 Conclusions

We estimated and assessed two measures of the natural rate of interest (NRI) for Chile and Perú: (i) TVP-NRI based on the estimation of a time varying parameter vector autoregression model with stochastic volatility (TVP-VAR-SV) as in Lubik y Matthes (2015), and (ii) a Benati-NRI measure based on a recent methodology proposed by Benati (2023). We assessed the alternative NRI measures using a novel criterion proposed in this paper: the RI is not expected to react to shocks that have no long-run effect on real interest rate, i.e. NRI does not react to transitory structural shocks. The results for Chile and Peru indicate that TVP-NRI measures are relatively superior.

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