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Information Heterogeneity and the Role of Foreign Exchange Interventions

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Abstract

In this paper we introduce information heterogeneity in the FX market in line with Bacchetta and Wincoop (2006) in attempt to extend their results to a DSGE framework. We show that the introduction of information heterogeneity, in the shape of both dispersed information about fundamentals and non-fundamental based shocks, generates a magnification effect that can help obtaining a better approximation to the empirical moments. Also, FX intervention can improve the connection between the exchange rate and its fundamentals. Finally, in the methodological front, we extend the procedure proposed by Townsend (1983) to solve dynamic stochastic general equilibrium (DSGE) models with heterogeneous expectations.

Key words: Foreign exchange microestructure, Exchange rate dynamics, Exchange Rate Intervention, Monetary policy.

JEL Classification: E4, E5, F3, G15.

The microstructure approach to exchange rates literature has given economists key insights to understand the behaviour of exchange rates. The limited explanatory power that observed macro fundamentals have on the exchange rate, coined 'the exchange rate determination puzzle', has found answers in the microstructure literature. As Evans and Lyons (2002) found, exchange rate movements can be explained largely by order flows. Orders received by dealers and the exchange rate have a direct connection through *portfolio balance* effects. Nonetheless, this channel is not the sole one through which order flows affect exchange rate prices. As Lyons (2006) explains, order flows convey private information. The author identifies at least three different channels at work. The first one, is related to information about transitory risk premia, as dealers possess better information about their own inventories and the inventories of other dealers. Using this information dealers possess an informative advantage over the general public. The second channel is related to the aggregate position of dealers, reflected in the portfolio balance, which in the eyes of the dealers, are changes in the aggregate position which are undiversifiable across themselves. As Vitale (2011) explains, heterogeneous information might emerge in relation to this channel as certain dealers can have superior information regarding the aggregate position of the market, such as the case when Central Banks intervene through a subset of dealers. The third one is related to asset payoffs. Dealers could have as well private information regarding future interest rate differentials or, perhaps closer to reality, dealers could either interpret this information in a different way or have access to different information regarding other dealers' expectations about the future differentials.

As Bacchetta and van Wincoop (2011) explain, typically in macroeconomic models foreign exchange (FX) market participants are assumed to: i) have identical information; ii)perfectly know the model; iii) use the available information at all times. Assumptions that are quite inconsistent with the way FX markets operate. The authors show that relaxing these assumptions allows explaining various exchange rate puzzles, such as the disconnection between exchange rates and fundamentals and the forward premium puzzle. In this line, we extend the model introduced Montoro and Ortiz (2016) by relaxing the first assumption, acknowledging that FX dealers can have access to different sources of information and can have different expectations about future macroeconomic variables. As shown by Bacchetta and Wincoop (2006) in a more tractable model, these characteristics magnify the response of the exchange rate to unobserved variables and generate a disconnection in the short run between the exchange rate and observed fundamentals. In a related work, Vitale (2011) extends Bacchetta and Wincoop (2006) model in a partial equilibrium framework to analyse the impact of FX intervention on FX markets. The resulting model is useful to analyse how FX intervention influences exchange rates via both a portfolio-balance and an information related channel.

The goal of the present paper is twofold. First, to introduce information heterogeneity into a Neo-Keynesian general equilibrium model and verify the role it plays in the determination of exchange rates and the *disconnection puzzle*. The second objective is to understand the role of FX interventions in this setup. Different from previous research, we treat information heterogeneity in a model where the interest rate is endogenous and reacts to the exchange rate through the effects the latter has on inflation.

In this way, there is an explicit channel through which the FX market microstructure, FX interventions and monetary policy interact; a channel we consider worthwhile studying in more detail.[1]

On the technical side, the presence of heterogeneous information poses a challenge in terms of the solution method. Now, the variance of exchange rate changes will not only affect the risk premium charged by dealers for holding foreign currency assets in their portfolio, but will be a key element in the information extraction exercise that dealers perform. For this reason we follow an approach in line with Townsend (1983) and Bacchetta and Wincoop (2006). We solve a signal extraction problem of the investors to calculate the average expected depreciation rate in the modified uncovered interest parity (UIP) condition with an endogenous risk premium, which feeds from the rational expectations solution of the model.

We are not the first ones to treat the *exchange rate disconnection puzzle* from a general equilibrium perspective. Wang (2007) studies the role that the home-bias effect in consumption has in the ratio of volatilities between the exchange rate and the macroeconomic variables, though the results are driven by ad-hoc UIP shocks. Evans and Lyons (2007) work a two-country general equilibrium model with initially not publicly observed information that is assimilated by the exchange rate at a slow pace. In this model dealers form heterogeneous expectations about central bank reactions to changes in the economy and revise their expectations using the information contained in the order flow. Gabaix and Maggiori (2015), Itskhoki and Mukhin (2017) and Cavallino (2019) present models in which exchange rate fluctuations are driven by capital flows, while Fanelli and Straub (2016) depict a model in which the Central Bank portfolio can affect the interest rate differential in a small open economy with incomplete capital markets and impact the behaviour of carry-traders. A model with collateral constraints is introduced by Chang (2018) in which FX intervention is effective only when these constraints bind. Alla et al. (2017) present a model in which FX intervention can reduce the volatility of the economy in presents of risk appetite shocks. Finally, Adler et al. (2016) work in a small open economy setup in which agents learn if the central bank follows a Taylor rule or a 'fear of floating' type of behaviour.

This paper is connected to some other strands of the literature such as the models of noisy rational expectations (see Brunnermeier (2001) for a survey) and imperfect information (see Woodford (2001), Mankiw and Reis (2007) and Sims (2003)).

In the next section, the model in Montoro and Ortiz (2016) is extended to take into account

[1]In countries with a dollarized financial system and agents with dollarized liabilities an additional channel is present. In this case exchange rate fluctuations generate balance sheet effects on households and firms, with consequences for interest rates in the banking sector. For a discussion of this channel, see Céspedes et al. (2004).

information heterogeneity in the dealers' market. Section 3 discusses the solution method. In section 4 we present the results of the model. The last section concludes.

1 The model

The model describes a small open economy with nominal rigidities, in line with the contributions from Obstfeld and Rogoff (1995), Chari et al. (2002), Galí and Monacelli (2005), Christiano et al. (2005) and Devereux et al. (2006), with the key difference that the exchange rate is determined in a market of risk adverse dealers. Different from the model seen in Montoro and Ortiz (2016), now dealers in the FX market will receive heterogeneous information, as in Bacchetta and Wincoop (2006), raising an information extraction problem that will affect exchange rate dynamics.

1.1 Dealers

As in the baseline model, there is a continuum of dealers in the interval $[0, 1]$ operating in the domestic economy. Each dealer ι receives ϖ_t^ι and $\varpi_t^{\iota,cb}$ in domestic bond sale and purchase orders from households and the central bank, and $\varpi_t^{\iota*}$ and $\varpi_t^{\iota*,cb}$ in foreign bond sale orders from foreign investors and the central bank, respectively. These orders are exchanged among dealers, that is $\varpi_t^\iota + \varpi_t^{\iota,cb} + S_t\left(\varpi_t^{\iota*} + \varpi_t^{\iota*,cb}\right) = B_t^\iota + S_t B_t^{\iota*}$, where B_t^ι and $B_t^{\iota*}$ are the ex-post holdings of domestic and foreign bonds by dealer ι, respectively. The exchange rate S_t is defined as the price of foreign currency in terms of domestic currency, such that a decrease (increase) of S_t corresponds to an appreciation (depreciation) of the domestic currency. At the end of the period, any profits -either positive or negative- are transferred to the households.[2]

Dealers are risk-averse and short-sighted. They select an optimal portfolio allocation in order to maximise the expected utility of their end-of-period returns, where their utility is given by a CARA utility function. The one-period dealer's horizon gives tractability and captures the feature that FX dealers tend to unwind their FX exposure at the end of any trading period, as explained by Vitale (2011). The problem of dealer ι is

$$\max_{B_t^{\iota*}} -E_t^\iota e^{-\gamma\Omega_{t+1}^\iota}$$

subject to:

$$\varpi_t^\iota - \varpi_t^{\iota,cb} + S_t\left(\varpi_t^{\iota*} + \varpi_t^{\iota*,cb}\right) = B_t^\iota + S_t B_t^{\iota*} \tag{1}$$

where γ is the coefficient of absolute risk aversion and E_t^ι is the rational expectations operator conditional on the information that dealer ι possesses at time t, $\mathcal{I}_t^\iota$, thus:

$$E_t^\iota[\cdot] \equiv E[\cdot \mid I_t^\iota]$$

[2]Under the present formulation FX transactions carried out for commercial purposes will only affect the exchange rate through their impact in the domestic interest rate though not through variations in the order flow faced by dealers.

Ω_{t+1}^{ι} represents total investment after returns, given by:

$$\begin{aligned}\Omega_{t+1}^{\iota} &= (1+i_t)\, B_t^{\iota} + (1+i_t^*)\, S_{t+1} B_t^{\iota *} \\ &\approx (1+i_t)\left[\varpi_t^{\iota} - \varpi_t^{\iota,cb} + S_t\left(\varpi_t^{\iota *} + \varpi_t^{\iota *,cb}\right)\right] + (i_t^* - i_t + s_{t+1} - s_t)\, B_t^{\iota *}\end{aligned}$$

where we have made use of the resource constraint of dealers. We have log-linearised the excess of return on investing in foreign bonds and $s_t = \ln S_t$. Since the only non-predetermined variable is s_{t+1}, assuming it is normal distributed with time-invariant variance, the first order condition for the dealers is:

$$0 = -\gamma\left(i_t^* - i_t + E_t^{\iota} s_{t+1} - s_t\right) + \gamma^2 B_t^{\iota *} \sigma^2$$

where $\sigma^2 = var_t\left(\Delta s_{t+1}\right)$ is the conditional variance of the depreciation rate. Then, the demand for foreign bonds by dealer ι is given by the following portfolio condition:

$$B_t^{\iota *} = \frac{i_t^* - i_t + E_t^{\iota} s_{t+1} - s_t}{\gamma \sigma^2} \tag{2}$$

1.2 FX market equilibrium

Foreign bonds equilibrium in the domestic market should sum FX market orders from foreign investors (capital inflows) and central bank FX intervention, that is:

$$\int_0^1 B_t^{\iota *} d\iota = \int_0^1 \left(\varpi_t^{\iota *} + \varpi_t^{\iota *,cb}\right) d\iota = \varpi_t^* + \varpi_t^{*,cb}.$$

Replacing the FX market equilibrium condition in the aggregate demand for foreign bonds yields the following arbitrage condition:

$$\bar{E}_t s_{t+1} - s_t = i_t - i_t^* + \gamma\sigma^2(\varpi_t^* + \varpi_t^{*,cb}) \tag{3}$$

where $\bar{E}_t s_{t+1}$ is the average rational expectation of the next period exchange rate across all dealers. Given that dealers have access to different sets of information, expected exchange rate depreciation would differ among them as well. Condition (3) determines the exchange rate, and adds three new elements to the traditional uncovered interest rate parity condition.[3] On the right-hand side, we note the presence of central bank market orders, reflecting the *portfolio balance effect* of FX interventions. The second novel element is the presence of the exchange rate volatility, which scales the impact of interventions and portfolio capital flows shocks in the exchange rate. We call this the *volatility channel.*

Finally, on the left-hand side we find the average rational expectation of the next period exchange rate, reflecting the presence of heterogeneous information. In our model, dealers will form both conditional moments present in condition (3) through a signal extraction problem. As we discuss, the way in which the central bank intervenes could affect both in the manner in which information is processed and in the information available to agents.

[3] See Obstfeld and Rogoff (1995) for an example of the standard UIP condition.

1.3 Information structure

Two sources of information heterogeneity among dealers are considered: first, we assume dealers face idiosyncratic shocks in the amount of customer orders from foreign investors and, second, they will also receive noisy signals about some future shocks. The later assumption seems reasonable, since regularly dealers form their own forecasts from different models or own experiences, generating heterogeneity in spite of having access to the same data.

In particular, we assume the foreign investor exposure for each dealer is equal to the average plus an idiosyncratic term:

$$\varpi_t^{\iota *} = \varpi_t^* + \varepsilon_t^{\iota} \tag{4}$$

where ε_t^{ι} has an infinite support, so that knowing one‘s own foreign investor exposure provides no information about the average exposure as in Bacchetta and Wincoop (2006). ϖ_t^* is unobservable and follows an AR(1) process:

$$\varpi_t^* = \rho_{\varpi^*} \varpi_{t-1}^* + \varepsilon_t^{\varpi^*} \tag{5}$$

where $\varepsilon_t^{\varpi^*} \sim N\left(0, \sigma_{\varpi^*}^2\right)$. We consider the case in which this autoregressive process is known by all agents.

We assume that dealers observe past and current fundamental shocks, while they also receive private signals about some future shocks. More precisely, we assume dealers receive one signal each period about the foreign interest rate one period ahead.[4] That is, at time t dealer ι receives a signal

$$v_t^{\iota} = i_{t+1}^* + \varepsilon_t^{v\iota},\ \varepsilon_t^{v\iota} \sim N\left(0, \sigma_{v\iota}^2\right) \tag{6}$$

where $\varepsilon_t^{v\iota}$ is independent from i_{t+1}^* and other agents' signals. This idiosyncratic signal can be reconciliated with the fact that dealers have different models to forecast future fundamentals, so each can imperfectly observe future variables with an idiosyncratic noise. We also assume that the average signal received by investors is i_{t+1}^*, that is $\int_0^1 v_t^{\iota} d\iota = i_{t+1}^*$. The foreign interest rate follows an AR(1) process known by dealers:

$$i_t^* = \rho_{i^*} i_{t-1}^* + \varepsilon_t^{i^*} \tag{7}$$

where $\varepsilon_t^{i^*} \sim N\left(0, \sigma_{i^*}^2\right)$. Dealers solve a signal extraction problem for the unknown innovations $\left(\varepsilon_t^{\varpi^*}, \varepsilon_{t+1}^{i^*}\right)$, given the observed depreciation rate and signal $\left(\Delta s_t, v_t^d\right)$.[5]

As Bacchetta and Wincoop (2006), we consider a *common knowledge* (CK) benchmark. In this case, the signal about future interest rates becomes public, but remains noisy. Agents only

[4]This assumption can be extended to the case where dealers receive each period a vector of signal of a set of fundamental variables.

[5]As explained by Bacchetta and Wincoop (2006), if the $\mathcal{B}(L)$ polynomial in equation (15) is invertible, knowledge of the depreciation rate at times $t-1$ and earlier and of the interest rate shocks at time t and earlier, reveals the shocks ε^{ϖ^*} at times $t-1$ and earlier. That is, $\varepsilon_{t-s}^{\varpi^*}$ becomes observable at time t for $s \geq 1$.

extract information from this signal, since the equilibrium exchange rate stops being informative. For a detailed description see Section 2.B.3 in the appendix.

1.3.1 FX intervention

We describe two different FX intervention strategies for the central bank, aside of the no intervention scenario. First, the central bank can perform a rule based intervention taking into account the changes in the exchange rate. We call this strategy "the Δs rule".

$$\varpi_t^{*cb} = \phi_{\Delta s}\Delta s_t + \varepsilon_t^{cb,1} \tag{8}$$

According to this rule, when there are depreciation (appreciation) pressures on the domestic currency, the central bank sells (purchases) foreign bonds to prevent the exchange rate from fluctuating. $\phi_{\Delta s}$ captures the intensity of the response of the FX intervention to pressures in the FX market. Second, the monetary authority can take into account misalignments of the real exchange rate as a benchmark for FX intervention. We call this strategy "the RER rule".

$$\varpi_t^{*cb} = \phi_{rer} rer_t + \varepsilon_t^{cb,2} \tag{9}$$

The rest of the model describes the behaviour of households, firms, the external sector and a monetary policy authority, which participates actively in the FX market through discretionary or rule-based interventions. We refer the reader to Montoro and Ortiz (2016), for a complete description of the model and a thorough explanation of the differences among these three FX intervention strategies.

2 Computational strategy

The computational strategy consists of dividing the system of log-linearized equations into two blocks. In the first block we take into account all the equations but the risk-premium adjusted UIP, which is included in the second block. Then, we solve for the rational expectations equilibrium of the first block taking the depreciation rate as an exogenous variable. This solution feeds into the second block to solve for the policy function of the depreciation rate. Note that with this computational strategy we are also eliminating any informational spillovers between dealers and other economic agents, such as households and firms. However, the segmented information in the FX market seems a reasonable assumption, since it takes into account that dealers have access to private information, which is not known by other economic agents.

Accordingly, in the first block the depreciation rate only appears in the real exchange rate equation:

$$rer_t = rer_{t-1} + \Delta s_t + \pi_t^* - \pi_t \tag{10}$$

This system of equations can be written as:

$$A_0 \begin{bmatrix} X_t \\ E_t Y_{t+1} \end{bmatrix} = A_1 \begin{bmatrix} X_{t-1} \\ Y_t \end{bmatrix} + A_2 \Delta s_t + B_0 \epsilon_t \tag{11}$$

where $X_t = [rer_t, i_t, \pi_t^*, w_t^*, i_t^*, ...]'$ is a size n_1 vector of backward looking variables, $Y_t = [\pi_t, ...]'$ is a size n_2 vector of forward looking variables, such as $n_T = n_1 + n_2 + 1$ is the number of endogenous variables. ϵ_t is the vector of observable shocks in the model. $A_2 = [1, 0..0]'$ is a $(n_1 + n_2) \times 1$ matrix.[6]

The second block corresponds to the risk-premium adjusted UIP condition:

$$\overline{E}_t \Delta s_{t+1} = i_t - i_t^* + \gamma\sigma^2 \left(\varpi_t^* + \varpi_t^{*,cb}\right) \tag{12}$$

In the first stage we find the rational expectations solution of the system in (11) using the perturbation method, taking as exogenous Δs_t.[7] That is, we find the policy functions:

$$Y_t = M_1 X_{t-1} \tag{13}$$

$$X_t = M_2 X_{t-1} + M_3 \Delta s_t + M_4 \epsilon_t \tag{14}$$

In the second stage we use the previous solution to find the policy function of Δs_t using Townsend (1983) method. More precisely, we conjecture a solution for Δs_t as a function of infinite lag polynomials of the shocks in the model.

$$\Delta s_t = \mathcal{A}(L)\varepsilon_{t+1}^{i^*} + \mathcal{B}(L)\varepsilon_t^{\varpi^*} + \mathcal{D}(L)\zeta_t \tag{15}$$

where $\varepsilon_{t+1}^{i^*}$ is an innovation to the future foreign interest rate (i_{t+1}^*), the fundamentals over which agents receive a signal, and $\varepsilon_t^{\varpi^*}$ is the shock to the unobservable capital flow (ϖ_t^*), which can be inferred with a lag. $\mathcal{A}(L)$ and $\mathcal{B}(L)$ are infinite lag polynomials, while $\mathcal{D}(L)$ is a vector of infinite lag polynomials operating ζ_t, the vector of remaining shocks. [8]

In the second stage we solve for the signal extraction problem of the dealers for the unobserved innovations $\left(\varepsilon_t^{\varpi^*}, \varepsilon_{t+1}^{i^*}\right)$, using both the depreciation rate and their private signal $\left(\Delta s_t, v_t^d\right)$, which serves to calculate the average expectation of the future depreciation rate and its conditional variance in equation (12) as functions of shocks.[9]

The next step involves relating the coefficients of (12) to those on the conjectured solution (15). This yields a system of non-linear equations on the unknown coefficients of $\mathcal{A}(L)$, $\mathcal{B}(L)$ and $\mathcal{D}(L)$. Although this is an infinite-order set of equations, we can exploit the recursive pattern present among the coefficients. We are able to solve the system through a numerical approach that limits the number of lags affecting the solution, effectively imposing zeros after a certain lag. This lag is determined numerically, through an iterative process. See appendix B for details on the computational strategy.

[6] Since information heterogeneity only enters the model through the exchange rate, the unobservable shocks are excluded from the first step.

[7] We use Dynare to solve for the rational expectations of the first block. More information see: Villemont (2011) and Adjemian et al. (2012).

[8] Notice that ϵ_t and ζ_t are not exactly the same, since the latter can also include FX intervention shocks.

[9] In turn, given the solution of the first block, we can express the endogenous variables in (12) as function of shocks as well.

3 Model Dynamics

Our interest lies in understanding first and foremost, how information heterogeneity affects the connection between the exchange rates and the "traditional" fundamentals. [10] These are the variables that affect the exchange rate determination in traditional monetary models (i.a., interest rate differentials). We follow Bacchetta and Wincoop (2006) by solving the model for different values for the parameters that govern the inference problem that dealers face.

3.1 Calibration

With respect to the baseline model studied in Montoro and Ortiz (2016), this extension presents an additional parameter which affects the precision of the private signal (σ_ν). This value is set at 0.08 for the baseline calibration, the same standard deviation assumed by Bacchetta and Wincoop (2006). There are two key parameters for the signal extraction problem: the standard deviation of noise in the signal (σ_ν), the standard deviation of the capital flows shock (σ_{ω^*}). We study the properties of the simulated series under different values for these parameters. For a discussion on the calibrated values for the rest of parameters in the model see Montoro and Ortiz (2016).

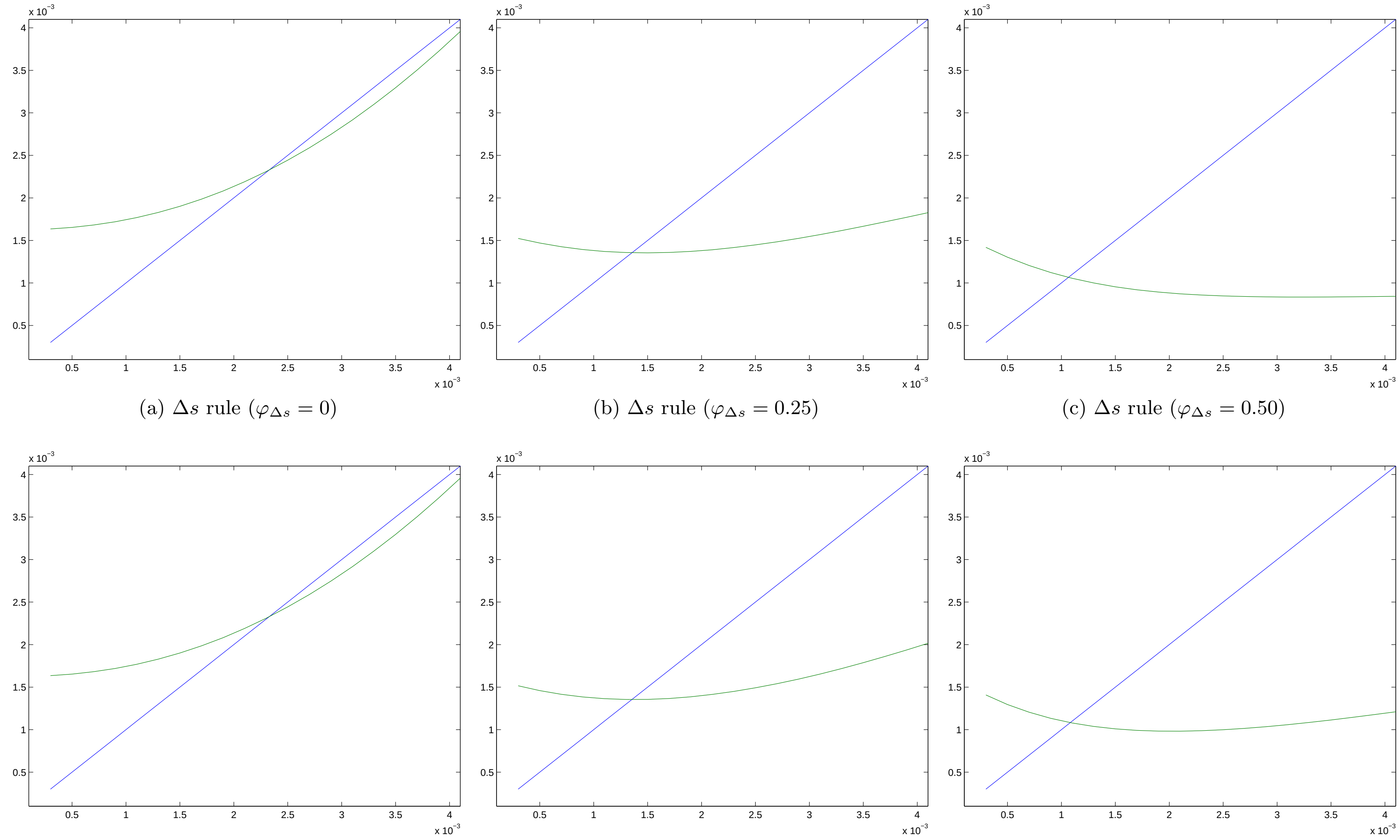
[10] Bacchetta and Wincoop (2006) treat portfolio flow shocks as "non-fundamental" variables. Vitale (2011) considers that, given the importance of order flows for the determination of exchange rates, these should be considered fundamentals too.

Table 1: **Baseline Calibration**

Parameter	*Value*	*Description*
β	0.9975	Consumers time-preference parameter.
χ	0.5	Labour supply elasticity.
γ_c	1	Risk aversion parameter.
ε	0.75	Elasticity of substitution btw. home and foreign goods.
ε_X	0.75	Elasticity of substitution btw. exports and foreign goods.
ψ	0.6	Share of domestic tradables in domestic consumption.
θ_H	0.75	Domestic goods price rigidity.
θ_M	0.5	Imported goods price rigidity.
θ_X	0.5	Exported goods price rigidity.
ψ_b	0.01	Portfolio adjustment costs.
φ_π	1.5	Taylor rule reaction to inflation deviations.
γ	500	Absolute risk aversion parameter (dealers)
ϕ_ϖ	0.5	Net asset position over GDP ratio
ϕ_C	0.68	Consumption over GDP ratio
σ_x	0.01	S.D. of all shocks x
ρ_x	0.5	AR(1) coefficient for all exogenous processes
σ_ν	0.08	S.D. of noise in signal.

3.2 Variance fixed-point problem

The risk premium-adjusted uncovered interest parity condition (equation 3) is a function of the variation of the exchange rate. In turn, this variable depends on the RE equilibrium of the model. Different from the full information case, the solution now involves a search for the undetermined coefficients in the lag polynomials $\mathcal{A}(L)$, $\mathcal{B}(L)$ and $\mathcal{D}(L)$, defined in (15). We conjecture a variance and solve for the unknown coefficients. In Figure 1 we plot the mappings of the conjectured and the implied conditional variance of the depreciation rate for different parameterisations of the FX-intervention reaction function. Intersections with the 45-degree straight line correspond to fixed points for the conditional variance of the depreciation rate. The results found under full information carry over to the heterogeneous information case, as under both rules of FX intervention there is only a unique and stable equilibrium. Also, the intensity of FX intervention reduces the RE equilibrium variance of the exchange rate change. The value of the variances is similar as well, although this model presents an extra shock given by the noise in the public signal.



(a) Δs rule ($\varphi_{\Delta s} = 0$) (b) Δs rule ($\varphi_{\Delta s} = 0.25$) (c) Δs rule ($\varphi_{\Delta s} = 0.50$)

(d) RER rule ($\varphi_{rer} = 0$) (e) RER rule ($\varphi_{rer} = 0.15$) (f) RER rule ($\varphi_{rer} = 0.30$)

Figure 1: Existence of equilibria under FX intervention rules (HI)

Note: Simulations involved 61 values for the conjectured variances of the change of the exchange rate. When the intervention parameter under both rules is zero, we replicate the values for the pure discretional intervention case.

3.3 The effects of heterogeneous information

Bacchetta and Wincoop (2006) proved that by adding heterogeneous information in an exchange rate determination model it is possible to account for the short-run disconnection between the exchange rate and observed fundamentals. Instead, the exchange rate becomes closely associated to order flow, which the author associates to the private information component of total market orders. The mechanism at work is a magnification effect of unobserved fundamentals, such as portfolio capital flows in our model, on the exchange rate. Under heterogeneous information, there is rational confusion since when the exchange rate changes dealers do not know whether this is driven by unobserved fundamentals (e.g.: portfolio capital flows) or by information about future macroeconomic fundamentals held by other dealers (e.g.: foreign interest rates).

The rational confusion magnifies the impact of the unobserved capital flows on the exchange rate, an effect Bacchetta and Wincoop (2006) called *the magnification effect.* As we have explained, agents will have now two different signals. The first is the private information about the fundamental. The second is the equilibrium exchange rate - more precisely the unknown component of this rate. As unobservable fundamental capital flows impact the exchange rate, agents will confuse them with changes in observable fundamentals and will react to them, amplifying the effect of capital flows. This magnification effect depends on the precision of the public signal (the exchange rate) relative to the precision of the private signal (v_t^d). Figure 2 shows the difference in the contemporaneous response to capital flow shocks between the HI and CK cases. The magnification effect increases with σ_v and decreases with σ_{ω^*}. This is in line with our previous observations. As the private signal becomes noisier, dealers will rely more on the equilibrium exchange rate as a source of information. Thus, liquidity based capital inflows and outflows effects in the exchange rate will be amplified. By contrast, an increase in σ_{ω^*} will reduce the magnification effect. In this case, the exchange rate loses power as a signal, since its dynamics will be more affected by capital flows instead of traditional macro fundamentals.

To show the role of heterogeneous information and the magnification effect in explaining the disconnection effect in our setup we perform simulations of the model and the calculate the R2 of the regression between the exchange rate and the observable variables in the model and contrast it with the imperfect common knowledge case. Under imperfect common knowledge agents will fully observe the aggregate capital flows and follow the same signal, however, this signal will be a noisy one. Hence, their forecast error will have an effect on the equilibrium exchange rate. [11]

[11] For a detailed explanation of how the model works under imperfect common knowledge see section 2.B.3 in the appendix.

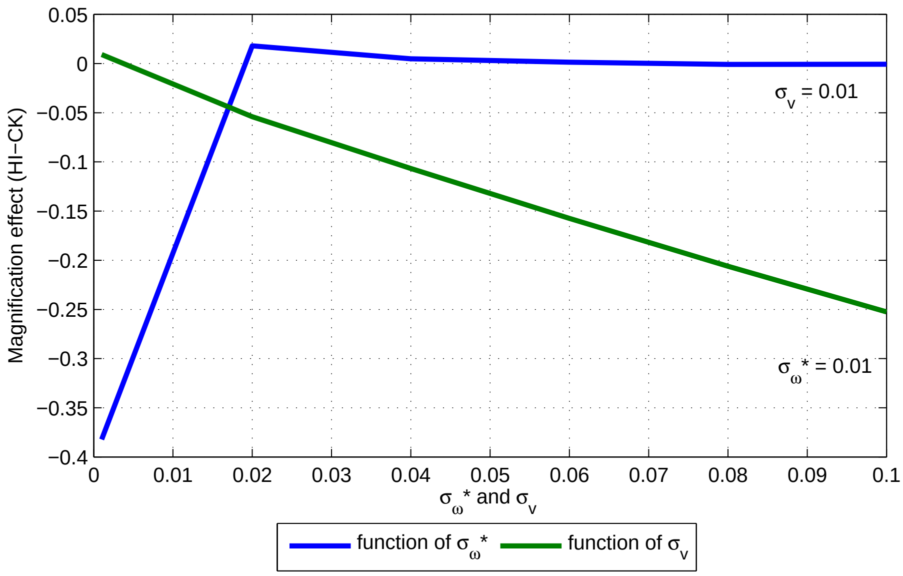


Figure 2: Magnification effect for different values of σ_{ω^*} and σ_ν

Note: Figure shows the difference in the contemporaneous response of the variation in the exchange rate s to a one standard deviation capital flows shock ($\varepsilon_t^{\omega^*}$) under heterogeneous information (HI) and common knowledge (CK), for different values of σ_{ω^*} and σ_ν.

3.4 FX intervention under heterogeneous information

FX intervention can affect the magnification effect and the connection of the exchange rate with observed fundamentals. We show in figure 3 the response on impact of the exchange rate change to future foreign interest rate shocks (i^*_{t+1}) and unobserved capital flows shocks (ϖ^*_t) , that is coefficients a_1 and b_1 respectively. We show in the first column the responses in a model with common knowledge, defined as one in which all dealers have access to the same information, and in the second column the responses in a model with heterogeneous information.[12] In the last column we present the differences between the responses in heterogeneous information and the common knowledge models. These responses are plotted for different values of the standard deviation of unobserved capital flow shocks $\left(\sigma_{\varpi^*_t}\right)$, for three degrees of FX intervention intensity under the Δs rule (no intervention, $\phi_{\Delta s} = 0.25$, and $\phi_{\Delta s} = 0.25$).

The following things are important to notice: i) In both the imperfect common knowledge and heterogeneous information cases, FX intervention dampens the impact of both unobserved capital flow shocks and future foreign interest rate shocks. ii) the standard deviation of unobserved capital flow shocks $\left(\sigma_{\varpi^*_t}\right)$ affects the responses under heterogeneous information, but not under common knowledge. This is because the response of the exchange rate depends on the precision of the signals only in the former model. iii) There is evidence of a magnification effect.

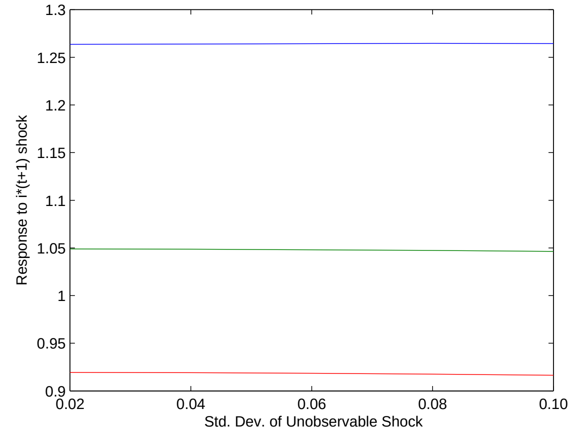
[12]Therefore, in a common knowledge model capital flows become an observable variable and all dealers observe signal shock (ε_t^{vd})

That is, the response to unobservable capital flow shocks is much stronger in the heterogeneous information than in common knowledge model. The opposite is true for the response to future foreign interest rate shocks. iv) The magnification effect is larger when the intensity of FX intervention is stronger. The main mechanism for this result is that, when FX intervention reduces the exchange rate volatility it also increases the precision of the public signal, which amplify the magnification effect[13] [14]

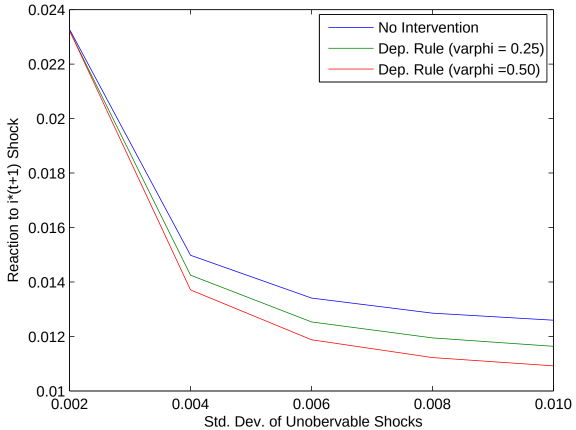
These results shed light of an additional effect that intervention can have in the FX market, that is the magnified response of the exchange rate to unobservable shocks, such as capital flows. However, the magnification effect is not strong enough to increase the disconnection between the exchange rate and observed fundamentals. Figure 4 reports the R^2 of regressions of Δs_t on unobserved capital flows shocks (ϖ_t^*) and future interest rates (i_{t+1}^*). As shown, FX intervention reduces the contribution of unobserved capital flow shocks to exchange rate changes, and as a counterpart increases the connection between observed fundamentals and the exchange rate.

[13] On the other hand, as shown in figure 3f, the magnification effect is larger when the unobservable capital flows are more volatile, because that increases the exchange rate volatility.

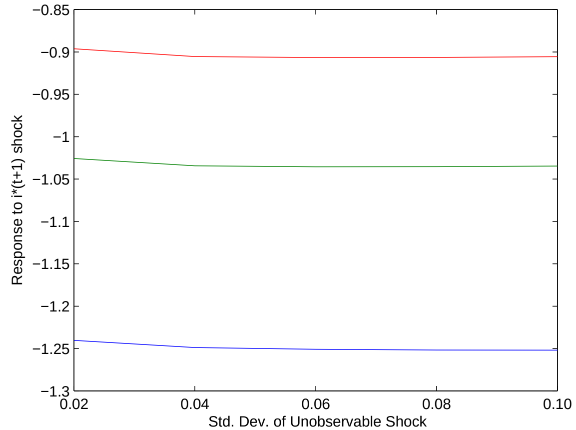
[14] However, this result could change if intervention can bring additional information about future fundamentals to the FX market, as analysed by Vitale (2011).



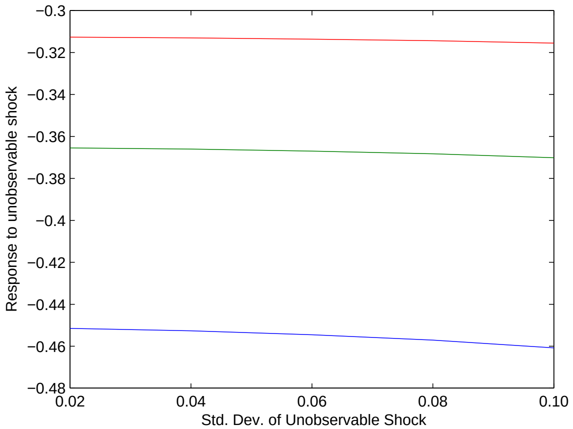
(a) Reaction to a i^*_{t+1} - CK



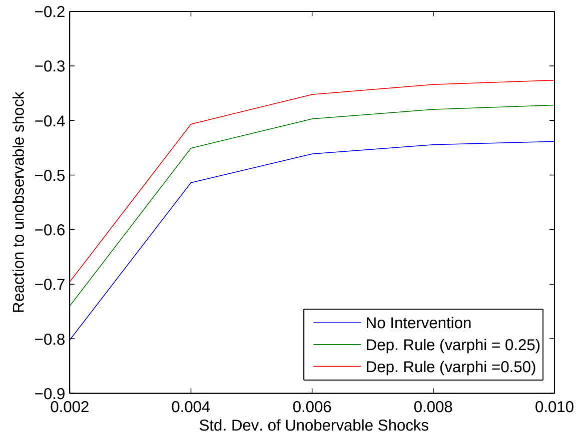
(b) Reaction to a i^*_{t+1} - HI



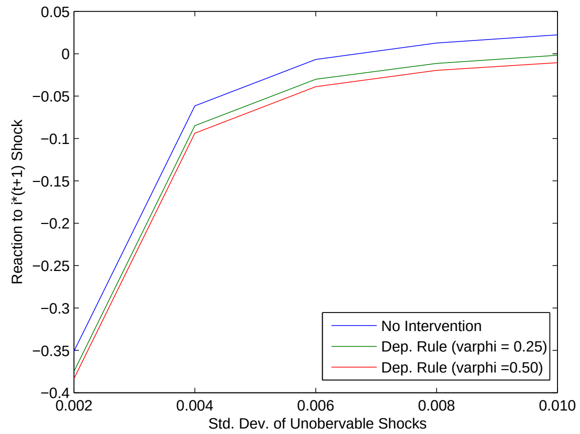
(c) Difference (HI - CK)



(d) Reaction to a ω^*_t - CK

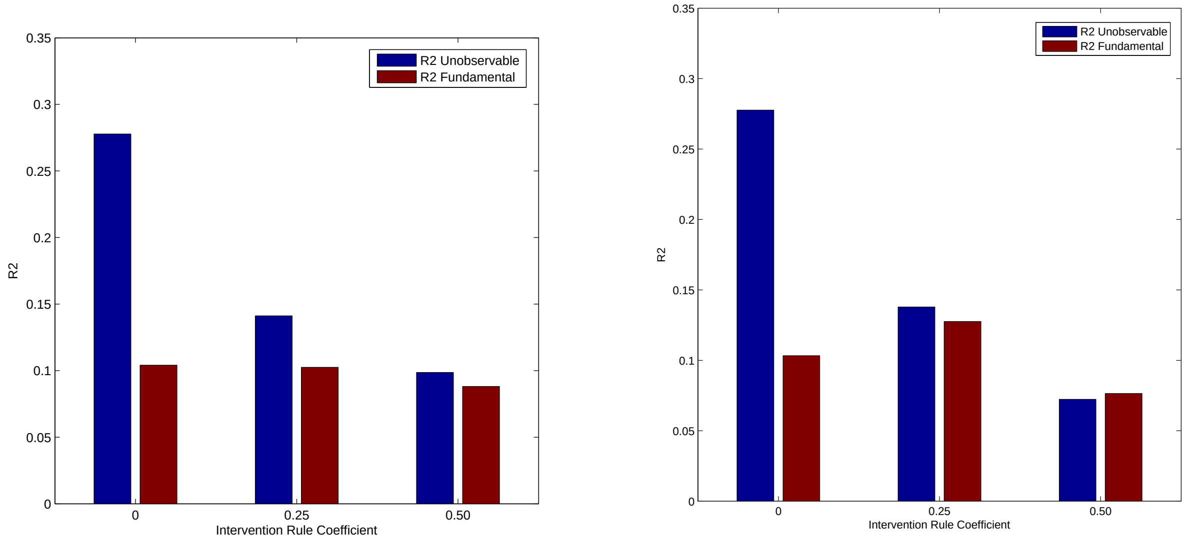


(e) Reaction to a ω^*_t - HI



(f) Magnification Effect (HI - CK)

Figure 3: Reaction to unobservable and fundamental shocks under Heterogeneous Information and Common Knowledge



(a) Reaction to a i^*_{t+1} - CK

(b) Reaction to a i^*_{t+1} - HI

Figure 4: Regression of Δs on unobservable and fundamental shocks - Δs rule

Note: Figure shows the R2 statistic of the regressions of Δs_t on $\varepsilon_t^{\omega^*}$ and on $\varepsilon_{t+1}^{i^*}$. Average of 20 simulations reported. Each simulation involve a sample of 1000 observations for the model. Regression includes an intercept.

4 Conclusions

In this paper we introduce heterogeneous information across dealers in the FX markets in line with Bacchetta and Wincoop (2006) into a DSGE model for a small open economy. We confirm that the *magnification effect*, which amplifies the contemporaneous impact of capital flow shocks, is still present in our framework. This effect is generated by the rational confusion emerging as dealers are unable to identify the source of shocks. The presence of the endogenous response of interest rates to changes in the exchange rate generates a channel between monetary policy and the information extraction problem of agents. Moreover, this framework allows us to study the interaction between exchange rate interventions by the central bank and the magnification effect observed under heterogeneous information. We find that FX interventions can reduce the contribution of unobserved capital flows shocks to the exchange rate, also increasing its connection with observed fundamentals. Despite these findings, the relationship between the degree of FX intervention and the connection to fundamentals is not monotonic. Finally, on the technical side, we propose an extension of Townsend (1983) that can be useful to solve DSGE models with heterogeneous information.

Further research should introduce richer dynamics in the information setup, such as central banks operating in a hidden way as in Vitale (2011), increasing the information dispersion through FX interventions, or central banks that reveal public signals through interventions. We consider that the setup presented here is capable of handling these problems. We leave these extensions for future research.

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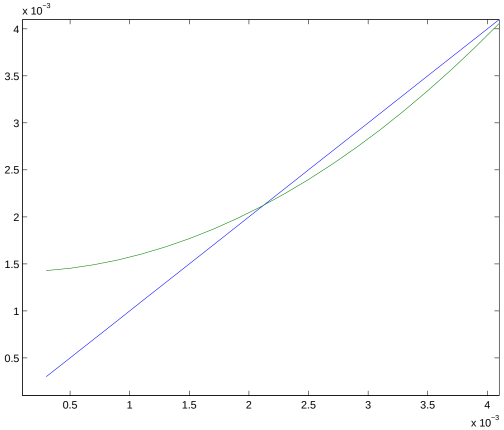
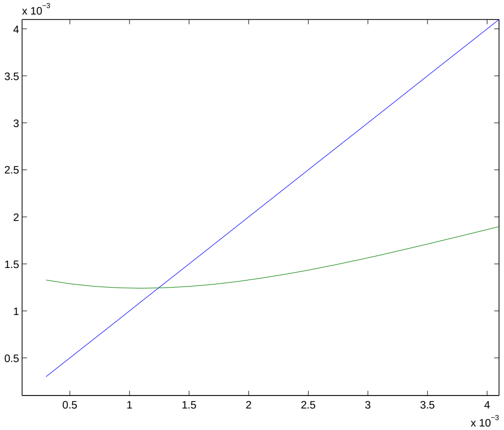
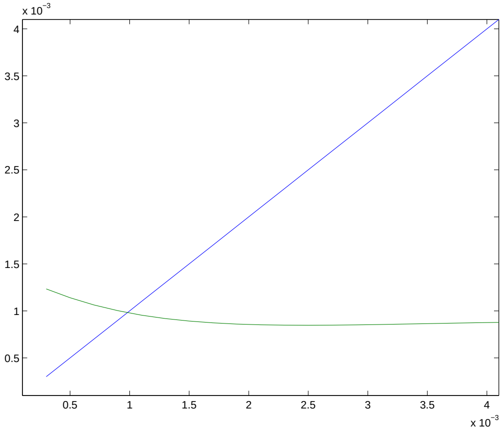
2.A Figures



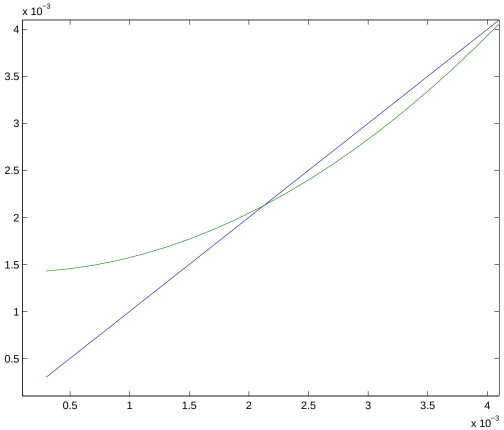
(a) Δs rule ($\varphi_{\Delta s} = 0$)



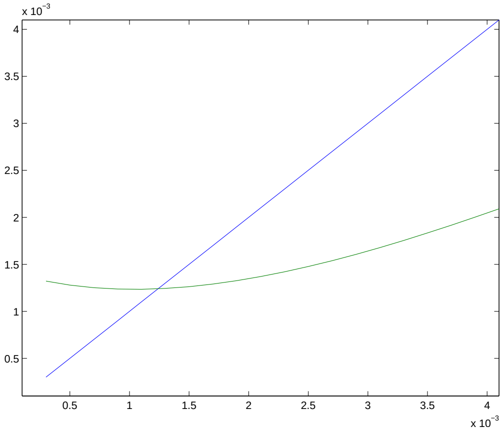
(b) Δs rule ($\varphi_{\Delta s} = 0.25$)



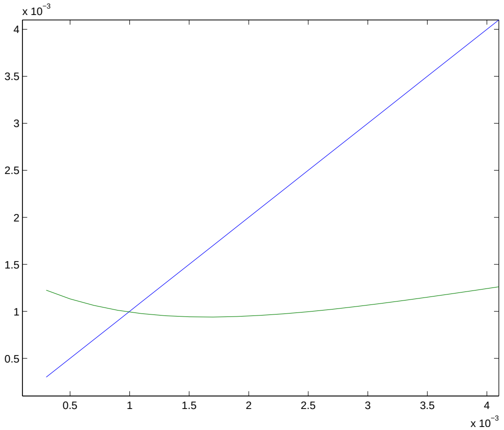
(c) Δs rule ($\varphi_{\Delta s} = 0.50$)



(d) RER rule ($\varphi_{rer} = 0$)



(e) RER rule ($\varphi_{rer} = 0.15$)



(f) RER rule ($\varphi_{rer} = 0.30$)

Figure 5: Existence of equilibria under FX intervention rules (CK)

Note: Simulations involved 61 values for the conjectured variances of the change of the exchange rate. When the intervention parameter under both rules is zero, we replicate the values for the pure discretional intervention case.

2.B Details of the computational strategy

The log-linearised system of equations of the model can be written as:

$$A_0 \begin{bmatrix} X_t \\ E_t Y_{t+1} \end{bmatrix} = A_1 \begin{bmatrix} X_{t-1} \\ Y_t \end{bmatrix} + A_2 \Delta s_t + B_0 \epsilon_t \tag{16}$$

and

$$\overline{E}_t \Delta s_{t+1} = i_t - i_t^* + \gamma \sigma^2 \left(\varpi_t^* + \varpi_t^{*,cb} \right) \tag{17}$$

where $A_2 = [1, 0..0]'$ is a $(n_1 + n_2) \times 1$ matrix and the definitions of the other matrices and vectors are in section 2. This is the state space form of the model.

2.B.1 Solving the first block

As an illustration, we will solve the system in (16) under some simplifying assumptions. For a more general solution, see Villemot (2011). The system in (16) can be written as:[15]

$$\begin{bmatrix} X_t \\ E_t Y_{t+1} \end{bmatrix} = A_0^{-1} A_1 \begin{bmatrix} X_{t-1} \\ Y_t \end{bmatrix} + A_0^{-1} A_2 \Delta s_t + A_0^{-1} B_0 \epsilon_t$$

or

$$\begin{bmatrix} X_t \\ E_t Y_{t+1} \end{bmatrix} = A \begin{bmatrix} X_{t-1} \\ Y_t \end{bmatrix} + \begin{bmatrix} a_{11} \Delta s_t \\ 0_{(n_1+n_2-1)\times 1} \end{bmatrix} + B \epsilon_t$$

after making $A = A_0^{-1} A_1$, $B = A_0^{-1} B_0$ and a_{11} the $(1,1)$ element of A_0^{-1}. Using the Jordan decomposition of $A = P \Lambda P^{-1}$, it becomes:

$$P^{-1} \begin{bmatrix} X_t \\ E_t Y_{t+1} \end{bmatrix} = \Lambda P^{-1} A \begin{bmatrix} X_{t-1} \\ Y_t \end{bmatrix} + \begin{bmatrix} p_{11} a_{11} \Delta s_t \\ 0_{(n_1+n_2-1)\times 1} \end{bmatrix} P^{-1} B \Delta s_t + P^{-1} C \epsilon_t$$

Making $R = P^{-1}B, \Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix}, P^{-1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}, R = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$ and p_{11} the $(1,1)$ element of P^{-1}. Λ_1 (Λ_2) is the diagonal matrix of stable (unstable) eigenvalues of size n_1 (n_2). The system of equations become:

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} X_t \\ E_t Y_{t+1} \end{bmatrix}$$
$$= \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} X_{t-1} \\ Y_t \end{bmatrix} + \begin{bmatrix} p_{11} a_{11} \Delta s_t \\ 0_{(n_1+n_2-1)\times 1} \end{bmatrix} + \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \epsilon_t.$$

[15] Assuming A_0 is invertible, otherwise we can generalise this for the case of non-invertible matrix.

Making $\widetilde{X}_{t-1} = P_{11}X_{t-1} + P_{12}Y_t$, $\widetilde{Y}_t = P_{21}X_{t-1} + P_{22}Y_t$, the system becomes:

$$\begin{bmatrix} \widetilde{X}_t \\ E_t\widetilde{Y}_{t+1} \end{bmatrix} = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \begin{bmatrix} \widetilde{X}_{t-1} \\ \widetilde{Y}_t \end{bmatrix} + \begin{bmatrix} p_{11}a_{11}\Delta s_t \\ 0_{(n_1+n_2-1)\times 1} \end{bmatrix} + \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \epsilon_t$$

According to Blanchard & Kahn, given that Λ_2 is the diagonal of unstable eigenvalues, the only stable solution is given by: $\widetilde{Y}_t = 0 = P_{21}X_{t-1} + P_{22}Y_t$.

Then, the solution for the forward looking variables is:

$$Y_t = (P_{22})^{-1} P_{21}X_{t-1}. \tag{18}$$

The solution for the system of stable (backward looking) equations is:

$$\widetilde{X}_t = \Lambda_1\widetilde{X}_{t-1} + \begin{bmatrix} p_{11}a_{11}\Delta s_t \\ 0_{(n_1-1)\times 1} \end{bmatrix} + R_1\epsilon_t \tag{19}$$

2.B.2 Solving the second block

The $MA(\infty)$ representation of the first block

Now we change the classification of endogenous variables in the block 1 to focus in the ones which are part of the minimum state variables (MSV) set. We call these variables Z_t, while the rest of endogenous variables is referred as Z_t^-. In our case the Z_t is formed by *12* variables as defined in appendix B.

The transition and policy functions can be written as:

$$\begin{bmatrix} Z_t \\ Z_t^- \end{bmatrix} = \begin{bmatrix} W \\ W^- \end{bmatrix} Z_{t-1} + \begin{bmatrix} V \\ V^- \end{bmatrix} \epsilon_t^* \tag{20}$$

where $\epsilon_t^{*\prime} = [\epsilon_t', \Delta s_t]$ appends the depreciation rate in the vector of shocks. Evaluating the transition function in $t-1$ and replacing it in (20), we have:

$$\begin{bmatrix} Z_t \\ Z_t^- \end{bmatrix} = \begin{bmatrix} W \\ W^- \end{bmatrix} \left(WZ_{t-2} + V\epsilon_{t-1}^*\right) + \begin{bmatrix} V \\ V^- \end{bmatrix} \epsilon_t^*$$

Repeating this process many times, we get:

$$\begin{bmatrix} Z_t \\ Z_t^- \end{bmatrix} = \begin{bmatrix} W \\ W^- \end{bmatrix} \left[(W)^n Z_{t-n-1} + (W)^{n-1} V\epsilon_{t-n}^* + ... + WV\epsilon_{t-2}^* + V\epsilon_{t-1}^*\right] + \begin{bmatrix} V \\ V^- \end{bmatrix} \epsilon_t^*$$

Which allows us to write the solution as a $MA(\infty)$:

$$\begin{bmatrix} Z_t \\ Z_t^- \end{bmatrix} = \begin{bmatrix} W \\ W^- \end{bmatrix} \sum_{i=1}^{\infty} (W)^{i-1} V\epsilon_{t-i}^* + \begin{bmatrix} V \\ V^- \end{bmatrix} \epsilon_t^* \tag{21}$$

Given the form of matrix W, the impact of shocks diminish over time, allowing us to approximate the solution using a fixed number of lags. We focus in the solution for i_t in this step and replace it back into (17). In our setup i_t^* follows an exogenous process which is easy to express as a function of shocks. Finally, the last term, $\gamma\sigma^2\left(\varpi_t^* + \varpi_t^{*,cb}\right)$ is a combination of the conditional volatility term σ^2, the first order autoregressive process of ϖ_t^* and other endogenous variables in the policy rule for $\varpi_t^{*,cb}$, that can also be expressed as function of shocks.

Conditional moments and solution method

In order to calculate the the conditional volatility of the depreciation rate, we need to make use of the strategy proposed by Bacchetta and van Wincoop (2006), based on Townsend (1983).

First we conjecture a solution for the depreciation of exchange rate of the form:

$$\Delta s_t = \mathcal{A}(L)\varepsilon_{t+1}^{i^*} + \mathcal{B}(L)\varepsilon_t^{\varpi^*} + \mathcal{D}(L)'\zeta_t \tag{22}$$

where $\mathcal{A}(L)$ and $\mathcal{B}(L)$ are infinite order lag polynomials, while $\mathcal{D}(L)$ is an infinite order lag polynomials vector operating ζ_t, the vector all other shocks in the model. Writing $\mathcal{A}(L) = a_1 + a_2L + a_3L^2 + ...$ (and a similar definition for $\mathcal{B}(L)$ and $\mathcal{D}(L)$), we evaluate forward the conjecture (22) to obtain the value in $t+1$.

$$\Delta s_{t+1} = a_1\varepsilon_{t+2}^{i^*} + b_1\varepsilon_{t+1}^{\varpi^*} + d_1'\zeta_{t+1} + \vartheta'\xi_t + \mathcal{A}^*(L)\varepsilon_t^{i^*} + \mathcal{B}^*(L)\varepsilon_{t-1}^{\varpi^*} + \mathcal{D}^*(L)'\zeta_t \tag{23}$$

where $\xi_t = \left(\varepsilon_{t+1}^{i^*}, \varepsilon_t^{\varpi^*}\right)'$ contains the unobservable innovations, $\vartheta' = (a_2, b_2)$ stands for the parameters associated to these shocks, $\mathcal{A}^*(L) = a_3 + a_4L + ...$ (similar definition for $\mathcal{B}^*(L)$) and $\mathcal{D}^*(L) = d_2 + d_3L + ...$ The last three terms in 23, $\mathcal{A}^*(L)\varepsilon_t^{i^*} + B^*(L)\varepsilon_{t-1}^{\omega^*} + \mathcal{D}^*(L)'\zeta_t$ represent all observable and past known shocks. Taking expectations for dealer ι over the previous equation yields:

$$E_t^\iota(\Delta s_{t+1}) = \vartheta' E_t^\iota(\xi_t) + \mathcal{A}^*(L)\varepsilon_t^{i^*} + \mathcal{B}^*(L)\varepsilon_{t-1}^{\varpi^*} + \mathcal{D}^*(L)\zeta_t \tag{24}$$

while the conditional variance as a function of unobservable innovation is given by:

$$var_t(\Delta s_{t+1}) = a_1^2 var_t(\varepsilon_{t+2}^{i^*}) + b_1^2 var_t(\varepsilon_{t+1}^{\varpi^*}) + (d_1)' var_t(\zeta_{t+1}) d_1 + \vartheta' var_t(\xi_t)\vartheta. \tag{25}$$

Here $\sigma^2 \equiv var_t(\Delta s_{t+1})$ is constant given that $var_t(\xi_t)$ is also constant. In order to obtain the conditional moments we need to obtain the conditional expectation and variance of the unobservable component ξ_t. The computation of the conditional moments is then obtained following Townsend (1983) and Bacchetta and Wincoop (2006).

FX traders extract information from the observed variation of the exchange rate Δs_t and the signal v_t^ι. To focus on the informational content of observable variables, we subtract the known components from these observables and define these new variables as Δs_t^* and $v_t^{\iota*}$. We follow the notation in Bacchetta and Wincoop (2006). The measurement equation on this part of the problem is given by:

$$Y_t^\iota = H'\xi_t + w_t^\iota \tag{26}$$

where $Y_t^\iota = (\Delta s_t^*, v_t^{\iota *})'$, $w_t^\iota = (0,\, \varepsilon_t^{v\iota})'$, and

$$H' = \begin{bmatrix} a_1 & b_1 \\ 1 & 0 \end{bmatrix}$$

The unconditional means of ξ_t and w_t^ι are zero, while we define their unconditional variances as $\widetilde{P}$ and R respectively. Following Townsend (1983), we can write:

$$E_t^\iota(\xi_t) = MY_t^\iota \tag{27}$$

where:

$$M = \widetilde{P}H\left(H'\widetilde{P}H + R\right)^{-1}.$$

For the conditional variance of the unobservable component we have, $P \equiv var_t(\xi_t)$, where

$$P = \widetilde{P} - MH'\widetilde{P}. \tag{28}$$

Substituting (26) and (27) in (24) and averaging over dealers gives the average conditional expectation of the variation of the exchange rate in terms of the shocks:

$$\overline{E}_t \Delta s_{t+1} = \vartheta' MH'\xi_t + \mathcal{A}^*(L)\varepsilon_t^{i^*} + \mathcal{B}^*(L)\varepsilon_{t-1}^{\varpi^*} + \mathcal{D}^*(L)\zeta_t. \tag{29}$$

Replacing the FX intervention policy strategy in equation (17), the $MA\,(\infty)$ representation of the endogenous variables (21), and the definition of σ^2 from (25), we obtain:

$$\overline{E}_t \Delta s_{t+1} = \hat{\imath}_t - \hat{\imath}_t^* + \gamma\sigma^2\left(\varpi_t^* + \varphi_{\Delta s}\Delta s_t + \varphi_{rer} rer_t + \varepsilon_t^{cb}\right) \tag{30}$$

$$\begin{aligned} \overline{E}_t \Delta s_{t+1} = {} & \mathcal{F}_i(L)\epsilon_t^* - \mathcal{G}(L)\varepsilon_t^{i^*} + ... \\ & + \gamma\left[(a_1^2 var_t(\varepsilon_t^{i^*}) + b_1^2 var_t(\varepsilon_t^{\varpi^*}) + (d_1)' var_t(\zeta_t) d_1 + \vartheta' P\vartheta\right] \\ & \times\left[\mathcal{J}(L)\varepsilon_t^{\omega^*} + \varphi_{\Delta s}\Delta s_t + \varphi_{rer}\mathcal{F}_{rer}(L)\epsilon_t^* + \varepsilon_t^{cb}\right] \end{aligned} \tag{31}$$

where $\mathcal{F}_z(L)\epsilon_t^*$ stands for $z_t = \{i_t, rer_t\}$, $\mathcal{G}(L)\varepsilon_t^{i^*}$ for i_t^*, and $\mathcal{J}(L)\varepsilon_t^{\omega^*}$ for ω_t^*. This is the "fundamental equation" $MA(\infty)$ representation.

To solve for the parameters of $\mathcal{A}(L)$, $\mathcal{B}(L)$ and $\mathcal{D}(L)$ we need to match the coefficients from equations (29) and (31).

Solution of parameters

Now we go through the algebra. Define $z_j^{y,x} \equiv \frac{dy_t}{dx_{t-j+1}}$ as the linear impulse response in the first step of the endogenous variable y_t with respect to the exogenous variable x_{t-j+1}. With this auxiliary variable we identify the parameters multiplying each shock. For this, we use the method of undetermined coefficients comparing equations (29) and (31).

Solution without rule-based FX intervention For simplicity, we solve first for the parameters assuming first there is no rule-based FX intervention, that is: $\varphi_{\Delta s} = \varphi_{rer} = 0$.

We start taking derivatives to the right hand side of equations (29) and (31) with respect to $\varepsilon_t^{i^*}, \varepsilon_{t-1}^{i^*}, ..., \varepsilon_{t-s+3}^{i^*}$, respectively:

$$
\begin{aligned}
a_3 &= \frac{di_t}{d\varepsilon_t^{i^*}} + \left(\frac{di_t}{d\Delta s_t} \frac{d\Delta s_t}{d\varepsilon_t^{i^*}} + \frac{di_t}{d\Delta s_{t-1}} \frac{d\Delta s_{t-1}}{d\varepsilon_t^{i^*}} \right) - \frac{di_t^*}{d\varepsilon_t^{i^*}} \\
a_4 &= \frac{di_t}{d\varepsilon_{t-1}^{i^*}} + \left(\frac{di_t}{d\Delta s_t} \frac{d\Delta s_t}{d\varepsilon_{t-1}^{i^*}} + \frac{di_t}{d\Delta s_{t-1}} \frac{d\Delta s_{t-1}}{d\varepsilon_{t-1}^{i^*}} + \frac{di_t}{d\Delta s_{t-2}} \frac{d\Delta s_{t-2}}{d\varepsilon_{t-1}^{i^*}} \right) - \frac{di_t^*}{d\varepsilon_{t-1}^{i^*}} \\
&\vdots \\
a_s &= \frac{di_t}{d\varepsilon_{t-s+3}^{i^*}} + \sum_{j=1}^{s-1} \left(\frac{di_t}{d\Delta s_{t+1-j}} \frac{d\Delta s_{t+1-j}}{d\varepsilon_{t-s+3}^{i^*}} \right) - \frac{di_t}{d\varepsilon_{t-s}^{i^*}}
\end{aligned}
$$

In this case the direct effect is zero, because i_t^* only appears in the risk-premium adjusted UIP condition, that is $\frac{di_t}{d\varepsilon_{t-s+3}^{i^*}} = 0$. Then the solution for a_3, a_4, ...is given by:

$$
a_s = \sum_{j=1}^{s-1} z_{s-j}^{i,\Delta s} a_j - \rho_{i^*}^{s-3} \qquad \text{for } s \geqslant 3 \tag{32}
$$

Similarly, taking derivatives with respect to $\varepsilon_{t-1}^{\varpi^*}, \varepsilon_{t-2}^{\varpi^*}, ..., \varepsilon_{t-s+2}^{\varpi^*}$, yields:

$$
\begin{aligned}
b_3 &= \frac{di_t}{d\varepsilon_{t-1}^{\varpi^*}} + \left(\frac{di_t}{d\Delta s_t} \frac{d\Delta s_t}{d\varepsilon_{t-1}^{\varpi^*}} + \frac{di_t}{d\Delta s_{t-1}} \frac{d\Delta s_{t-1}}{d\varepsilon_{t-1}^{\varpi^*}} \right) + \gamma\sigma^2 \frac{d\varpi_t^*}{d\varepsilon_{t-1}^{\varpi^*}} \\
&\vdots \\
b_s &= \frac{di_t}{d\varepsilon_{t-s+2}^{\omega^*}} + \sum_{j=1}^{s-1} \left(\frac{di_t}{d\Delta s_{t+1-j}} \frac{d\Delta s_{t+1-j}}{d\varepsilon_{t-s+2}^{i^*}} \right) + \gamma\sigma^2 \frac{d\varpi_t^*}{d\varepsilon_{t-s+2}^{i^*}}
\end{aligned}
$$

Similarly to the previous case, the direct effect is zero here, that is $\frac{di_t}{d\varepsilon_{t-s+2}^{\varpi^*}} = 0$. Then the solution for b_3, b_4, ... is given by:

$$
b_s = \sum_{j=1}^{s-1} z_{s-j}^{i,\Delta s} b_j + \gamma\sigma^2 \rho_{\omega^*}^{s-2} \qquad \text{for } s \geqslant 3 \tag{33}
$$

Using the same approach, we take derivatives with respect to $\varepsilon_t, \varepsilon_{t-1}, ..., \varepsilon_{t-s}$ for $\varepsilon \in \zeta$:

$$
\begin{aligned}
d_2^\varepsilon &= \frac{di_t}{d\varepsilon_t} + \left(\frac{di_t}{d\Delta s_t} \frac{d\Delta s_t}{d\varepsilon_t} \right) + \gamma\sigma^2 \left(\mathcal{I}_{\varepsilon=\varepsilon^{\varpi^*,cb}} \right) \\
d_3^\varepsilon &= \frac{di_t}{d\varepsilon_{t-1}} + \left(\frac{di_t}{d\Delta s_t} \frac{d\Delta s_t}{d\varepsilon_{t-1}} + \frac{di_t}{d\Delta s_{t-1}} \frac{d\Delta s_{t-1}}{d\varepsilon_{t-1}} \right) + \gamma\sigma^2 \left(\rho_\varepsilon \mathcal{I}_{\varepsilon=\varepsilon^{\varpi^*,cb}} \right) \\
&\vdots \\
d_s^\varepsilon &= \frac{di_t}{d\varepsilon_{t-s+2}} + \sum_{j=1}^{s-1} \left(\frac{di_t}{d\Delta s_{t+1-j}} \frac{d\Delta s_{t+1-j}}{d\varepsilon_{t-s+2}} \right) + \gamma\sigma^2 \left(\rho_\varepsilon^{s-2} \mathcal{I}_{\varepsilon=\varepsilon^{\varpi^*,cb}} \right)
\end{aligned}
$$

where $\mathcal{I}_{\varepsilon=\varepsilon^{\varpi*,cb}}$ is an indicator value of 1 when the shock ε equals $\varepsilon^{\omega^*,cb}$. This system is summarised by:

$$d_s^{\varepsilon} = z_{s-1}^{i,\varepsilon} + \sum_{j=1}^{s-1} z_{s-j}^{i,\Delta s} d_j + \gamma\sigma^2 \left(\rho_{\varepsilon}^{s-2} \mathcal{I}_{\varepsilon=\varepsilon^{\varpi*,cb}}\right) \tag{34}$$

which is valid for $s \geqslant 2$. Note also that $\frac{di_t}{d\varepsilon_{t-s+2}} = 0$ when $\varepsilon = \varepsilon^{\omega^*,cb}$.

This set of equations (32), (33) and (34) allows us to express the whole system as a function of parameters a_1, a_2, b_1, b_2 and the vector of parameters d_1.

Taking derivatives with respect to the two unobservable shocks $\left\{\varepsilon_{t+1}^{i^*}, \varepsilon_t^{\varpi^*}\right\}$ we get:

$$(\vartheta' M H')_1 = z_1^{i,\Delta s} a_1, \tag{35}$$

$$(\vartheta' M H')_2 = z_1^{i,\Delta s} b_1 + \gamma\sigma^2. \tag{36}$$

By substituting back the values for the matrices, we obtain a non-linear system of equations on the unknowns:

$$[a_2 \; b_2]\, M \begin{bmatrix} a_1 \\ 1 \end{bmatrix} = z_1^{i,\Delta s} a_1 \tag{37}$$

$$[a_2 \; b_2]\, M \begin{bmatrix} b_1 \\ 0 \end{bmatrix} = z_1^{i,\Delta s} b_1 + \gamma\sigma^2 \tag{38}$$

Note that considering (37) and (38) we have two equations and four unknowns, which impedes us to solve for the system. Bacchetta and Wincoop (2006) overcome this problem by proving that the coefficients in the lag polynomials follow a recursive pattern. Assuming non-explosive coefficients, they are able to obtain additional restrictions on the values of the coefficient in the lag-polynomial. In our case, the interest rate is endogenous, meaning a feedback is present from the effect of unobservable shocks into the exchange rate and from there into the interest rate. This feedback effect makes the relationship across the coefficients in the lag polynomials a function of the solution in the first block and of the assumed FX intervention strategy. For this reason we follow instead a numerical approach that limits the number of lags affecting the solution. We set up the non-linear system of equations on the first elements of both infinite lag polynomials and search for a numerical solution using the trust-region-dogleg method implemented by MATLAB. The extra restrictions in our case are given by selecting a limit to the lags and setting the parameters associated with this lag at zero.[16] Since these are functions of the first parameters (the unknowns), we can solve the system and obtain the solution. We change sequentially this limit and derive new solutions in each step. The algorithm stops when

[16]Note that Bacchetta and Wincoop (2006) guess a solution for the *level* of the exchange rate, while we solve for its *first difference*. Our method implicitly assumes the first difference of the exchange rate is stationary. We consider that in our setup our assumption is less restrictive.

a fixed point is achieved, revealing that the inclusion of additional lags has a negligible effect on the result.[17]

The system of equations: We can represent the system of equations using some auxiliary matrices.

The A system The set of equations in (32) can be written as:

$$\begin{bmatrix} a_3 \\ a_4 \\ \vdots \\ a_{n+1} \\ a_{n+2} \end{bmatrix} = \begin{bmatrix} z_1^{i,\Delta s} & 0 & \dots & 0 & 0 \\ z_2^{i,\Delta s} & z_1^{i,\Delta s} & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ z_{n-1}^{i,\Delta s} & z_{n-2}^{i,\Delta s} & \dots & z_1^{i,\Delta s} & 0 \\ z_n^{i,\Delta s} & z_{n-1}^{i,\Delta s} & \dots & z_2^{i,\Delta s} & z_1^{i,\Delta s} \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \\ \vdots \\ a_n \\ a_{n+1} \end{bmatrix} - \begin{bmatrix} 1 \\ \rho_{i^*} \\ \vdots \\ (\rho_{i^*})^n \\ (\rho_{i^*})^{n-1} \end{bmatrix} + + a_1 \begin{bmatrix} z_2^{i,\Delta s} \\ z_3^{i,\Delta s} \\ \vdots \\ z_n^{i,\Delta s} \\ z_{n+1}^{i,\Delta s} \end{bmatrix} \tag{39}$$

These equations can be written in the matrix form, after assuming that the value of $a_{n+2} \longrightarrow 0$:

$$Z_1 A = Z_2^i A - X_{i^*} + a_1 Z_3^{i,\Delta s} \tag{40}$$

where $Z_1 = \begin{bmatrix} 0_{(n-1)\times 1} & I_{n-1} \\ 0 & 0_{1\times(n-1)} \end{bmatrix}$, $A = [a_2, ..., a_{n+1}]'$ is a $n \times 1$ vector, Z_2^i is the lower triangular matrix that pre-multiplies A, $X_{i^*} = \left[1, \rho_{i^*}, ..., (\rho_{i^*})^{n-1}\right]'$, and $Z_3^{i,\Delta s} = \left[z_2^{i,\Delta s}, z_3^{i,\Delta s}, ..., z_{n+1}^{i,\Delta s}\right]'$

The B system:

Similarly, equations (33) can be written as:

$$Z_1 B = Z_2^i B + \gamma\sigma^2 \rho_{\varpi^*} X_{\varpi^*} + b_1 Z_3^{i,\Delta s} \tag{41}$$

where $B = [b_2, b_3, .., b_{n+1}]'$ and $X_{\varpi^*} = \left[1, \rho_{\varpi^*}, ..., (\rho_{\varpi^*})^{n-1}\right]'$.

The D system

In the same vein, the system for $D^\varepsilon = [d_1^\varepsilon, d_2^\varepsilon, .., d_n^\varepsilon]'$ is the following

$$\begin{aligned} Z_1 D^\varepsilon &= Z_2^i D^\varepsilon + Z_3^{i,\varepsilon} \quad \text{when } \varepsilon \neq \varepsilon^{\varpi^*,cb} \\ Z_1 D^{\varpi^*,cb} &= Z_2^i D^{\varpi^*,cb} + \gamma\sigma^2 X_{\varpi^*,cb} \quad \text{otherwise} \end{aligned}$$

[17] We set the fixed-point algorithm convergence criterion over the maximum difference in the values of the coefficients associated with the unobservable shocks.

where $Z_3^{i,\varepsilon} = \left[z_1^{i,\varepsilon}, z_2^{i,\varepsilon}, ..., z_n^{i,\varepsilon}\right]'$ and $X_{\varpi^*,cb} = \left[1, \rho_{\varpi^*,cb}, ..., (\rho_{\varpi^*,cb})^{n-1}\right]'$.

The complete system of equations.

Then, after making use of $Z = Z_1 - Z_2^i$, the total system of non-linear equations becomes:

$$
\begin{aligned}
[a_2\ b_2]\, M \begin{bmatrix} a_1 \\ 1 \end{bmatrix} &= z_1^{i,\Delta s} a_1 \\
[a_2\ b_2]\, M \begin{bmatrix} b_1 \\ 0 \end{bmatrix} &= z_1^{i,\Delta s} b_1 + \gamma\sigma^2 \\
A &= -Z^{-1}\left(X_{i^*} - a_1 Z_3^{i,\Delta s}\right) \\
B &= Z^{-1}\left(\gamma\sigma^2 \rho_{\varpi^*} X_{\varpi^*} + b_1 Z_3^{i,\Delta s}\right) \\
D^{\varepsilon} &= Z^{-1} Z_3^{i,\varepsilon} \\
D^{\varpi^*,cb} &= \left(\gamma\sigma^2\right) Z^{-1} X_{\varpi^*,cb} \\
\sigma^2 &= a_1^2 var_t(\varepsilon_{t+2}^{i^*}) + b_1^2 var_t(\varepsilon_{t+1}^{\varpi^*}) + (d_1)' var_t(\zeta_{t+1}) d_1 + \vartheta' var_t(\xi_t)\vartheta
\end{aligned} \tag{42}
$$

Note the system has $n \times \#$ of shocks $+3$ equations and unknowns, which only $n \times 2 + 3$ are non-linear equations (those corresponding to the B and $D^{\varpi^*,cb}$ system and the equations for a_1, b_1 and σ^2).

4.0.1 Solution with FX intervention rules

When we allow for FX intervention, the equations (32), (33), (34), (37) and (38) are replaced by:

$$a_s = \sum_{j=1}^{s-1} z_{s-j}^{i,\Delta s} a_j - \rho_{i^*}^{s-3} + \gamma\sigma^2 \left[\varphi_{\Delta s} a_{s-1} + \varphi_{rer} \sum_{j=1}^{s-1} z_{s-j}^{rer,\Delta s} a_j\right] \tag{43a}$$

$$b_s = \sum_{j=1}^{s-1} z_{s-j}^{i,\Delta s} b_j + \gamma\sigma^2 \left[\rho_{\omega^*}^{s-2} + \varphi_{\Delta s} b_{s-1} + \varphi_{rer} \sum_{j=1}^{s-1} z_{s-j}^{rer,\Delta s} b_j\right] \tag{43b}$$

$$d_s^{\varepsilon} = z_{s-1}^{i,\varepsilon} + \sum_{j=1}^{s-1} z_{s-j}^{i,\Delta s} d_j + \gamma\sigma^2 \left[\begin{array}{c} \rho_{\varpi^*,cb}^{s-2} \mathcal{I}_{\varepsilon=\varepsilon^{\varpi^*,cb}} + \\ \varphi_{\Delta s} d_{s-1} + \varphi_{rer} \sum_{j=1}^{s-1} z_{s-j}^{rer,\Delta s} d_j^{\varepsilon} \end{array}\right] \tag{43c}$$

$$[a_2\ b_2]\, M \begin{bmatrix} a_1 \\ 1 \end{bmatrix} = z_1^{i,\Delta s} a_1 + \gamma\sigma^2 \left(\varphi_{\Delta s} a_1 + \varphi_{rer} z_1^{rer,\Delta s} a_1\right) \tag{43d}$$

$$[a_2\ b_2]\, M \begin{bmatrix} b_1 \\ 0 \end{bmatrix} = z_1^{i,\Delta s} b_1 + \gamma\sigma^2 \left(1 + \varphi_{\Delta s} b_1 + \varphi_{rer} z_1^{rer,\Delta s} b_1\right) \tag{43e}$$

We can also express this with linear algebra. For example, the A system can be written as:

$$Z_1 A = Z_2^i A + \gamma\sigma^2 \left(\varphi_{\Delta s} I_n + \varphi_{rer} Z_2^{rer}\right) A - X_{i^*} + a_1 \left(Z_3^{i,\Delta s} + \gamma\sigma^2 \varphi_{rer} Z_3^{rer,\Delta s}\right)$$

Then, after making use of $Z^{FX} = Z_1 - Z_2^i - \gamma\sigma^2 \left(\varphi_{\Delta s} I_n + \varphi_{rer} Z_2^{rer}\right)$, where Z_2^{rer} is a lower triangular matrix, analogous to Z_2^i, with $z_1^{rer,\Delta s}$ as elements of its main diagonal, the total system of non-linear equations becomes:

$$\begin{aligned}
[a_2\ b_2]\, M \begin{bmatrix} a_1 \\ 1 \end{bmatrix} &= z_1^{i,\Delta s} a_1 + \gamma\sigma^2 \left(\varphi_{\Delta s} + \varphi_{rer} z_1^{rer,\Delta s}\right) a_1 \\
[a_2\ b_2]\, M \begin{bmatrix} b_1 \\ 0 \end{bmatrix} &= z_1^{i,\Delta s} b_1 + \gamma\sigma^2 \left(1 + \varphi_{\Delta s} b_1 + \varphi_{rer} z_1^{rer,\Delta s} b_1\right) \\
A &= -\left(Z^{FX}\right)^{-1} \left[X_{i^*} - a_1 \left(Z_3^{i,\Delta s} + \gamma\sigma^2 \varphi_{rer} Z_3^{rer,\Delta s}\right)\right] \\
B &= \left(Z^{FX}\right)^{-1} \left(\gamma\sigma^2 \rho_{\varpi^*} X_{\varpi^*} + b_1 \left(Z_3^{i,\Delta s} + \gamma\sigma^2 \varphi_{rer} Z_3^{rer,\Delta s}\right)\right) \\
D^{\varepsilon} &= \left(Z^{FX}\right)^{-1} Z_3^{i,\varepsilon} \\
D^{\varpi^*,cb} &= \left(\gamma\sigma^2\right) \left(Z^{FX}\right)^{-1} X_{\varpi^*,cb} \\
\sigma^2 &= a_1^2 var_t(\varepsilon_{t+2}^{i^*}) + b_1^2 var_t(\varepsilon_{t+1}^{\varpi^*}) + (d_1)' var_t(\zeta_{t+1}) d_1 + \vartheta' var_t(\xi_t) \vartheta \qquad (44)
\end{aligned}$$

where $M = \frac{1}{(a_1)^2 \sigma_{i^*}^2 \sigma_v^2 + (b_1)^2 \sigma_{\omega^*}^2 \left(\sigma_{i^*}^2 + \sigma_v^2\right)} \begin{bmatrix} a_1 \sigma_{i^*}^2 \sigma_v^2 & (b_1)^2 \sigma_{\omega^*}^2 \sigma_{i^*}^2 \\ b_1 \sigma_{\omega^*}^2 \left(\sigma_{i^*}^2 + \sigma_v^2\right) & -a_1 b_1 \sigma_{i^*}^2 \sigma_{\omega^*}^2 \end{bmatrix}$

and $P = var_t(\xi_t) = \frac{\sigma_{i^*}^2 \sigma_v^2 \sigma_{\omega^*}^2}{(a_1)^2 \sigma_{i^*}^2 \sigma_v^2 + (b_1)^2 \sigma_{\omega^*}^2 \left(\sigma_{i^*}^2 + \sigma_v^2\right)} \begin{bmatrix} (b_1)^2 & -a_1 b_1 \\ -a_1 b_1 & (a_1)^2 \end{bmatrix}$

2.B.3 The problem with common knowledge (CK)

In the common knowledge benchmark, investors share the same signal about the future fundamental values. Information is common but incomplete. All investors receive:

$$v_t = i_{t+1}^* + \varepsilon_t^v, \qquad \varepsilon_t^v \sim N(0, \sigma_v^2)$$

Under common knowledge ϖ_t^* becomes observable, because we get rid of the idiosyncratic shocks. Thus, capital flows shocks will only affect the economy through the portfolio balance channel. In the signal extraction problem dealers have to infer information only for $\xi_t^{CK} = \varepsilon_{t+1}^{i^*}$. We must assume now that the equilibrium exchange rate depends directly on the ε_t^v shock, the noise of the signal common to all agents. We guess a solution of the type:

$$\Delta s_t = \mathcal{A}(L)\varepsilon_{t+1}^{i^*} + \mathcal{B}(L)\varepsilon_t^{\omega^*} + \mathcal{D}(L)\zeta_t + \Psi(L)\varepsilon_t^v \qquad (45)$$

Notice that now the $\varepsilon_t^{\omega^*}$ shock is observable and we have a new term in the solution for the 'now relevant' signal noise.

The only relevant signal for the problem under common knowledge is given by υ_t.[18] Following Townsend (1983), we obtain:

$$E_t(\varepsilon_{t+1}^{i^*}) = \hat{M}\upsilon_t^*,$$

where

$$\hat{M} = \frac{\sigma_{i^*}^2}{\sigma_{i^*}^2 + \sigma_\upsilon^2}$$

and

$$\upsilon_t^* = \varepsilon_{t+1}^{i^*} + \varepsilon_t^\upsilon \tag{46}$$

is the unknown component of the signal υ_t at time t. We first obtain an expression for s_{t+1}, using (45):

$$\begin{aligned} \Delta s_{t+1} = a_1\varepsilon_{t+2}^{i^*} + b_1\varepsilon_{t+1}^{\omega^*} + \psi_1\varepsilon_{t+1}^\upsilon + d_1\zeta_{t+1}' + \vartheta^{CK}\xi_t^{CK} + ... \\ ... + \mathcal{A}^*(L)\varepsilon_t^{i^*} + \mathcal{B}^*(L)\varepsilon_t^{\omega^*} + \mathcal{D}^*(L)\zeta_t + \Psi^*(L)\varepsilon_{t-1}^\upsilon \end{aligned} \tag{47}$$

where $\vartheta^{CK} = [a_2]$ and we have grouped the shocks known at t in the lag polynomials denoted with $(^*)$. Now, taking expectations over (47):

$$E(\Delta s_{t+1}) = a_2 E(\varepsilon_{t+1}^{i^*}) + A^*(L)\varepsilon_t^{i^*} + B^*(L)\varepsilon_t^{\omega^*} + D^*(L)\zeta_t + \Psi^*(L)\varepsilon_{t-1}^\upsilon \tag{48}$$

where we have used the fact that $E_t(\varepsilon_t^\upsilon) = 0$. Now we take the second moment:

$$var_t(\Delta s_{t+1}) = a_1^2\sigma_{i^*}^2 + b_1^2\sigma_{\omega^*}^2 + \psi_1^2\sigma_\upsilon^2 + (d_1)' var_t(\zeta_{t+1})d_1 + \vartheta_{CK}' P^{CK}\vartheta_{CK}$$

Note that:

$$\begin{aligned} E(\varepsilon_{t+1}^{i^*}) &= E_t(\xi_t^{CK}) = \hat{M}Y_t \\ &= \hat{M}\left(\varepsilon_{t+1}^{i^*} + \varepsilon_t^\upsilon\right) \end{aligned}$$

Then, the equation (48) becomes:

$$E(\Delta s_{t+1}) = a_2\frac{\sigma_{i^*}^2}{\sigma_{i^*}^2 + \sigma_\upsilon^2}\left(\varepsilon_{t+1}^{i^*} + \varepsilon_t^\upsilon\right) + A^*(L)\varepsilon_t^{i^*} + B^*(L)\varepsilon_t^{\omega^*} + D^*(L)\zeta_t + \Psi^*(L)\varepsilon_{t-1}^\upsilon$$

this equation is equivalent to (29) in the heterogeneous information case. We compare the coefficients with respect to (31).

Equations (43d) and (43e) now become:

$$a_2^{CK}\frac{\sigma_{i^*}^2}{\sigma_{i^*}^2 + \sigma_\upsilon^2} = z_1^{i,\Delta s}a_1^{CK} + \gamma\sigma_{CK}^2\left(\varphi_{\Delta s}a_1^{CK} + \varphi_{rer}z_1^{i,\Delta s}a_1^{CK}\right) \tag{49}$$

$$a_2^{CK}\frac{\sigma_{i^*}^2}{\sigma_{i^*}^2 + \sigma_\upsilon^2} = z_1^{i,\Delta s}\psi_1^{CK} + \gamma\sigma_{CK}^2\left(\varphi_{\Delta s}\psi_1^{CK} + \varphi_{rer}z_1^{i,\Delta s}\psi_1^{CK}\right) \tag{50}$$

[18] It is straightforward to verify this. The unknown part of the equilibrium variation of the exchange rate is given by $\Delta s_t^* = a_1\varepsilon_{t+1}^{i^*} + \psi_1\varepsilon_t^\upsilon$. Since a_1 would be equal to ψ_1, it is clear the equilibrium exchange rate brings no additional information.

from this equations we obtain that $a_1^{CK} = \psi_1^{CK}$. Since agents only observe the sum of both the fundamental an noise shock, it stands to reason that the contemporaneous reaction to both shocks must be the same.

Additionally, we have a set of equations for $\Psi^*(L)$:

$$\psi_s = \sum_{j=1}^{s-1} z_{s-j}^{i,\Delta s}\psi_j + \gamma\sigma^2 \left[\varphi_{\Delta s}\psi_{s-1} + \varphi_{rer}\sum_{j=1}^{s-1} z_{s-j}^{rer,\Delta s}\psi_j\right] \tag{51}$$

The system of equations then becomes:

$$\begin{aligned}
A_{CK} &= -\left(Z^{FX}\right)^{-1}\left[X_{i^*} - a_1^{CK}\left(Z_3^{i,\Delta s} + \gamma\sigma^2\varphi_{rer}Z_3^{rer,\Delta s}\right)\right] \\
B_{CK} &= \left(Z^{FX}\right)^{-1}\left[\gamma\sigma^2\rho_{\varpi^*}X_{\varpi^*} + b_1^{CK}\left(Z_3^{i,\Delta s} + \gamma\sigma^2\varphi_{rer}Z_3^{rer,\Delta s}\right)\right] \\
\Psi_{CK} &= \left(Z^{FX}\right)^{-1}\left[\psi_1^{CK}\left(Z_3^{i,\Delta s} + \gamma\sigma^2\varphi_{rer}Z_3^{rer,\Delta s}\right)\right] \\
D_{CK}^{\varepsilon} &= \left(Z^{FX}\right)^{-1} Z_3^{i,\varepsilon} \\
D_{CK}^{\varpi^*,cb} &= \left(\gamma\sigma^2\right)\left(Z^{FX}\right)^{-1} X_{\varpi^*,cb} \\
a_2^{CK}\frac{\sigma_{i^*}^2}{\sigma_{i^*}^2 + \sigma_v^2} &= z_1^{i,\Delta s}a_1^{CK} + \gamma\sigma_{CK}^2\left(\varphi_{\Delta s}a_1^{CK} + \varphi_{rer}z_1^{i,\Delta s}a_1^{CK}\right) \\
\psi_1^{CK} &= a_1^{CK} \\
b_2^{CK} &= z_1^{i,\Delta s}b_1^{CK} + \gamma\sigma^2\left(1 + \varphi_{\Delta s}b_1^{CK} + \varphi_{rer}Z_1^{rer}b_1^{CK}\right) \\
var_t(\Delta s_{t+1}) &= a_1^2\sigma_{i^*}^2 + b_1^2\sigma_{\omega^*}^2 + \psi_1^2\sigma_v^2 + (d_1)'var_t(\zeta_{t+1})d_1 + \vartheta_{CK}'P^{CK}\vartheta_{CK}
\end{aligned}$$

Once again, the remaining restrictions come from imposing zeros at a given lag for the whole model, since the rest of the elements in the lag polynomials can be expressed as a function of ones associated with the unknowns.