

# Capital Flows and Bank Risk-Taking Jorge Pozo\*

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# Capital Flows and Bank Risk-taking<sup>\*</sup>

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#### Abstract

I build up a framework to study the dynamics of the default probability of banks and the excess bank risk-taking in an emerging economy. I calibrate the model for the 1998 Peruvian economy. The novelty result is that an infinity-period model creates an intertemporal channel that amplifies banks' incentives to take excessive risk. I simulate the sudden stop that hit Peru in 1998 as a negative shock on the foreign borrowing limit of banks. The model accurately predicts the substantial short-term rise in the morosity rate through the rise of the excess bank risk-taking after the sudden stop.

Keywords: Sudden stop, bank risk-taking, prudential policy. JEL Classification: E44, F41, G01, G21, G28.

## 1 Introduction

In 1998, the Peruvian economy experienced a sudden stop in capital flows. After 1998Q3, there was a reduction of short-term capital flows (see, figure 1), which was interestingly accompanied by an almost immediate increase of the morosity rate in the banking system (see, figure 2), suggesting a higher risk to the banks' loans. The morosity rate jumped from 6.4 during 1998Q3 to 10.3 in 1999Q2. According to Dancourt (2015), 11 of 26 banks were bankrupt or were bailed out, including the second- and fifth-largest banks (by amount of deposits), during the 1998-2000 banking crisis in Peru.

This motivates the building of a framework to examine the dynamics of the default probability of banks and the excess bank risk-taking after a sudden stop in an emerging economy, such as Peru's, which allows us to understand the behavior of the morosity rate in the sudden stop that hit Peru in 1998. Hence, I calibrate the model for the Peruvian

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economy to simulate the 1998 sudden stop. The sudden stop is modeled as a reduction of the foreign borrowing limit faced by banks. I start the economy from its stochastic steady state, and then I simulate a gradual adjustment of the foreign borrowing limit, where the initial fall is not anticipated, afterward, agents correctly anticipate the path of the foreign borrowing limit.

To do this, I develop an infinite-period model. In this model banks that receive fully insured domestic and foreign deposits face limited liability and a binding foreign borrowing limit. From the literature we know that in a two-period model the interaction of limited liability and deposit insurance results in an inefficiently high level of loans and hence excess bank risk-taking, see e.g. Collard et al. (2017). In this paper the bank risk-taking involves the volume of credit and not the type of credit as in Collard et al. (2017). In this paper, in order to have an infinite-period version, I assume an exogenous law of motion for the banks' net worth.

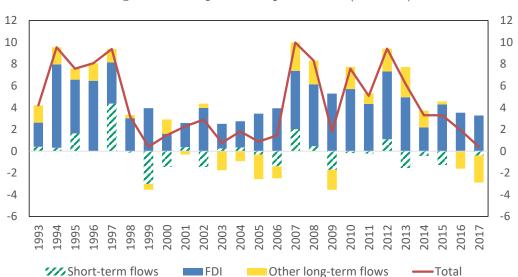


Figure 1: Net private capital flows (% GDP)

Updated Graph 2 from Gondo et al. (2009). Source: Central Reserve Bank of Peru (CRBP).

In contrast to the two-period model, the infinite-period model creates an intertemporal channel, represented by a shadow value of banks' net worth, that is sensitive to the state of the economy. The novelty result, based on a quantitative analysis, is that this intertemporal channel substantially increases excess bank risk-taking in the long-term, and thus generates a stronger short-term positive response of excess bank risk-taking after the unanticipated initial fall of the foreign borrowing limit in the sudden stop simulation.

The intuition behind this result is as follows: the fact that banks have limited liability and deposit insurance, not only in the present but also in the future, creates incentives to overestimate (from the banks' perspective) by even more the expected marginal benefits of the banks' loans. As a result, the intertemporal channel amplifies the inefficiency,

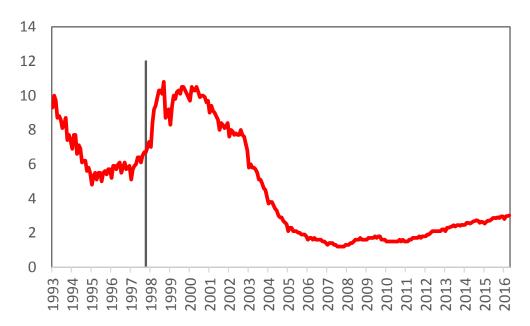


Figure 2: Morosity rate of the Peruvian banking system (%)

Source: The Superintendency of Banking, Insurance and Private Pension Fund Administrators (SBS). Following the SBS criteria, the morosity rate is defined as the share of credits overdue or in judicial collection. According to the SBS, loans to big firms are overdue if their scheduled payments (principal and interests) have more than 15 days of delay. Loans to small firms are overdue if their scheduled payments have more than 30 days of delay. And mortgage and personal loans are overdue if their scheduled payments have more than 90 days of delay.

i.e., this channel amplifies the difference between the level of loans with its the socially efficient level, which leads to a higher excess bank risk-taking in the long-term.<sup>1</sup> Consequently, if banks know ex-ante that there is going to be a gradual reduction of the foreign borrowing limit of banks, this intertemporal channel creates a stronger short-term positive response of the banks' excess risk-taking behavior because they internalize the higher future marginal benefits of loans from increasing the current level of loans.

I conduct a qualitative analysis of the model calibrated to Peru data. In particular, I calibrate the model to mimic the data of 1998Q3. In the long-term equilibrium (stochastic steady state) bank loans are 5.79% inefficiently high, and the (annualized) default probability of banks, that by construction is 3%, is eight times the probability under a regulated economy (the decentralized competitive equilibrium under an optimal policy intervention). However, when abstracting from the intertemporal channel, loans are only 0.15% inefficiently high and the default probability is almost the same as in the regulated economy.

Peru's 1998 sudden stop saw a reduction of the short-term foreign liability of financial system to GDP ratio (excluding Central Reserve Bank of Peru, CRBP), from 7.5% in

<sup>&</sup>lt;sup>1</sup>The long-term values involve the stochastic steady state values. Later, I explain why I focus on the stochastic and not in the deterministic steady state.

1998Q3 to an average of 1% during 2003. In order to simulate the same reduction in the model, I assume a gradual 87% reduction of the foreign borrowing limit. The sudden stop in the model produces a long-term increase of the (quarterly) default probability of banks, from 0.74% to 4.02%, which represents an increase of 328 basis points (bps); while, when abstracting from the intertemporal channel, the default probability increases from 0.1% to 0.45%, which represents only an increment of 35 bps. Furthermore, two quarters after the initial reduction of the foreign borrowing limit, the banks' default probability, the excess marginal benefits of loans and relative excess loans increases, respectively, by 55, 17 and 222 bps. In contrast, when abstracting from the intertemporal channel, the intertemporal channel, these variables increase, respectively, by 7, 1 and 10 bps. Hence, the intertemporal channel amplifies the response of the excess bank risk-taking.

The model helps us understand the substantial (60%) short-term increment in the Peruvian morosity rate after the sudden stop from 6.4 during 1998Q3 to 10.3 in 1999Q2, which is captured in the model by the short-term rise in the default probability, which becomes 1.6 its initial value. Hence, according to the model, the substantial short-term increment of the morosity rate was the result of banks significantly updating their risk-taking incentives in the short-term. As a result, an ex-ante regulation intended to reduce these excess risk-taking incentives would have reduced this large short-term response in the morosity rate and hence would have reduced the severity of the 1998-2000 banking crisis. Finally, the sudden stop simulation mimics very well the reduction in bank foreign debt to bank credit ratio from 25% to 5% over the 1998Q3-2006Q4 period.

This paper is organized as follows. Section 2 discusses the related literature. Section 3 presents the competitive equilibrium. Section 4 describes the problem of the domestic social planner. Section 5 describes the long-term equilibrium. Section 6 compares the competitive equilibrium with the social planner equilibrium. The quantitative analysis is presented in section 7. Section 8 provides conclusions.

## 2 Literature review

This work is related to the literature that explores the role of financial intermediaries in DSGE models, Gertler and Karadi (2011), Gertler and Kiyotaki (2011) and Gertler, Kiyotaki and Queralto (2012). The financial friction in these models is presented in the form of a moral hazard problem. These models mainly highlight the amplification channel of the financial friction and several policies that reduce financial instability and improve domestic welfare. By contrast, this work focuses on a small open economy where the limited liability and deposit insurance creates inefficiencies within the banking sector. In contrast to Bernanke et al. (1999), I am not assuming heterogeneous banks facing idiosyncratic shocks and so the probability of default does not represent the fraction of firms that default. In this paper, identical banks facing the same shocks defaults in bad states of the economy.

As in Gertler et al. (2012) I am interested in the stochastic steady states (also known as risky steady states) rather than in the deterministic steady states since I focus on measuring the level of risk taken by the banking sector. However, in contrast to Gertler et al. (2012), I endogenously model the default probability of banks. The concept of the stochastic (or risky) steady state is also implemented in Hills et al. (2016). They develop a stylized New Keynesian model with a lower bound for the interest rate. The result is that the inflation and policy rate are lower, and output is higher in the risky steady state than in the deterministic steady state. This is because, in the risky steady state, agents account for the uncertainty associated with future shocks and hence with a non-negative probability that the interest rate lower bound binds in the future.<sup>2</sup> De Groot (2014) solves a banks' portfolio problem as a part of a DSGE model around a risky steady state and studies the effects of domestic monetary policy on banking sector risk-taking.<sup>3</sup>

This work is also related to the literature that studies emerging economies and macroprudential policies, Bianchi (2011), Bianchi and Mendoza (2015), and Mendoza (2010). Mendoza (2010) presents a business cycle model for an open economy. The occasionally binding international borrowing constraint produces an amplification of the shocks that results from a Fisherian deflation. In contrast, this paper focuses on the inefficiencies caused by banks' decisions about risk-taking.

Regarding the studies of the capital requirements, as a macroprudential policy, the available papers can be divided into two groups. The first group is related to marketbased capital requirement rules, and the second group is related to exogenous capital requirement rules. The second group of research, Darracq, Kok and Rodriguez-Palenzuela (2011), Rubi and Carrasco (2014), Benes, Kumhof and Laxton (2014), and Angeloni and Faia (2013), is more closely related to this document. They develop a DSGE model for closed economies that conclude that countercyclical capital requirements, as in Basel III, are better than Basel II and Basel I. My contribution to this literature is, again, to open the economy.

The exercise of the sudden stop simulation presented here is similar in some sense to the one presented in Fornaro (2018). To study the deleveraging process by peripheral euro area countries, Fornaro (2018) develops a model of a continuum of small open economies and assesses the effects of a permanent tightening of the foreign borrowing limit. As in

<sup>&</sup>lt;sup>2</sup>Similar results are obtained regarding the simulated means of inflation, interest rate and output in Guerrieri and Iacoviello (2015) that also develops an New Keynesian model with zero lower bound, Table 4.

<sup>&</sup>lt;sup>3</sup>De Groot (2014) presents a quantitative New Keynesian business cycle model, with the purpose to assess how altering the monetary policy rule affects banks' risk-taking incentives at the steady state, which could not be achieved, as is well known, by assuming deterministic steady states. This motivates solving the model around the risky steady state.

this paper, Fornaro (2018) assumes a gradual reduction of the foreign borrowing limit and that the initial reduction is not anticipated.

# 3 Competitive equilibrium

This study uses an infinite-period open economy model. Agents are composed of an identical and large number of domestic households, domestic banks and foreign investors. Each period, domestic households decide how much to consume and save. They can save only through deposits in domestic banks. Banks face limited liability and receive short-term deposits from domestic households and foreign investors, and make short-term risky investments (loans). All deposits are fully insured by the domestic government. Domestic banks are owned by domestic households. All agents are risk-neutral.

Banks also face a foreign borrowing limit. This limit aims to capture two frictions, which I abstract from modeling explicitly: those between the domestic banks and foreign investors and those between domestic government and foreign investors. In good or bad states of the economy, domestic banks might have incentives to divert the funds that have to be paid to foreign investors. Similarly, the government might prefer not to pay to foreign investors when banks do not fully honor their debt (i.e., when banks default). Hence, in order to eliminate these incentives, foreign investors limit the amount of lending to domestic banks. Furthermore, since domestic depositors have more ability than foreign investors to enforce domestic banks and government to honor their debts, they do not limit the amount of lending to banks.

The model assumes that the cost of risk-free foreign deposits is lower than the cost of risk-free domestic deposits. This is to characterize a small open economy that is exposed to capital flows. This assumption is based on having a Fed rate smaller than the Peruvian monetary policy rate. Note that since the foreign interest rate is smaller than the domestic interest rate, the foreign borrowing limit is always binding. This is because even when there is not more space to invest in risky loans, banks benefit from investing in domestic risk-free bonds. For simplicity, there are not firms and banks intermediate capital. Households cannot borrow from foreign investors and cannot invest directly in risky assets. They invest indirectly through domestic bank deposits. Another assumption in this dynamic model is that I assume an exogenous rule for the law of motion of banks' net worth. This results in an exogenous banks' dividend policy.

The utility of the representative domestic household at time t is given by,

$$W_t = \mathbb{E}_t \left\{ \sum_{i=0}^{\infty} \beta^i C_{t+i} \right\},\tag{1}$$

where  $\mathbb{E}_t(.)$  refers to the expectation conditional to the available information at time t,

 $\beta$  is the constant discount factor of the domestic household, and  $C_t$  is the consumption level at period t. The budget constraint of the household at time t is,

$$C_t + D_t = \omega^H + \bar{R}_{t-1}^D D_{t-1} + \Pi_t - T_t, \qquad (2)$$

where  $\omega^H$  is a fixed exogenous income and aims to capture the resources from sectors in the economy that do not require, or that do not have access to, banks' credit,<sup>4</sup>  $D_t$  is the level of one-period deposits held in the bank by domestic households (also referred to as domestic deposits),  $\Pi_t$  are the banks' dividends,  $T_t$  are lump-sum government taxes and  $\bar{R}_t^D$  is the gross return agreed upon at time t for the domestic deposits held from t to t+1 in the domestic bank. Since the model assumes deposit insurance, the government ensures depositors always receive the agreed-upon gross return for their deposits. As a result, each time the bank defaults, the government collects lump-sum taxes from domestic households to pay depositors. Hence, as suggested by (2), domestic depositors always receive the agreed-upon payment  $\bar{R}_{t-1}^D D_{t-1}$ .

The representative domestic household seeks to maximize (1), subject to the budget constraint, (2). The first order condition for  $D_t$  requires,

$$1 = \beta E_t \{ \bar{R}_t^D \}. \tag{3}$$

Hence, the supply curve of domestic deposits is perfectly elastic at  $\bar{R}_t^D = \frac{1}{\beta} \forall t.^5$ 

The model assumes the opportunity cost of foreign investors is  $R_t^F$  which is assumed to be an exogenous process. As in the case of the domestic deposits, under limited liability and full deposit insurance, foreign depositors receive the agreed gross return for the foreign deposits held from t to t+1,  $\bar{R}_t^F$ . Since foreign investors are risk-neutral, and foreign deposits are fully insured by the government, it must hold in equilibrium that,

$$\bar{R}_t^F = R_t^F. \tag{4}$$

Hence, the supply of foreign deposits is perfectly elastic at  $R_t^F$  and up to  $\phi_t$ , due to the foreign borrowing limit.<sup>6</sup>

Since all banks are identical and the shock in the economy is aggregate, we can speak of a representative bank. That is, we can think of the banks described here as the aggregate banking sector. The role of banks is to receive short-term deposits from domestic households and foreign investors and intermediate funds between savers and

<sup>&</sup>lt;sup>4</sup>The goal of  $\omega^H$  as explained in the calibration section is to match the target level of consumption to GDP ratio.

<sup>&</sup>lt;sup>5</sup>Note that equation (3) also holds in a competitive equilibrium under unlimited liability. <sup>6</sup>Note that equation (4) also holds in a competitive equilibrium under unlimited liability.

productive assets. The balance sheet equation of banks is,

$$K_t = D_t + D_t^F + N_t, (5)$$

where  $K_t$  is banks' capital (loans),  $N_t$  is banks' net worth, or equity, and  $D_t^F$  are the shortterm deposits held by foreign investors (also referred to as foreign deposits) in domestic banks. Hence, banks use their initial net worth and domestic and foreign deposits to finance their assets or capital holdings,  $K_t$ . I will use the terms "capital", "loans" and "credit" interchangeably.

When banks intermediate  $K_t$  of capital in period t, there is a payoff of  $Z_{t+1}K_t^{\alpha}$  in period t+1, plus leftover capital  $(1-\delta)K_t$ ,  $\delta$  is the capital depreciation rate, and  $Z_{t+1}$ determines the capital productivity for banks. Capital productivity follows a log-normal AR(1) process,  $Z_{t+1}=exp(\mu_z(1-\rho_z)+\rho_z log(Z_t)+e_{t+1}^z)$ , where  $\mu_z$  is the unconditional mean of  $log(Z_t)$ ,  $\rho_z$  is the persistence of the productivity, and  $e_t^z$  is the independent and identically distributed productivity shock,  $e_t^z \sim N(0, \sigma_{e^z}^2)$ . I define f and F as the probability density function (pdf) and the cumulative density function (cdf), respectively, of  $e_t^z$ . Banks face a limit on short-term foreign borrowing,

$$D_t^F \le \phi_t. \tag{6}$$

where  $\phi_t$  is exogenously given.

By definition, the net operating income of banks at t + 1 is given by,

$$NOI_{t+1} = (1 - \delta)K_t + Z_{t+1}K_t^{\alpha} - \bar{R}_t^D D_t - \bar{R}_t^F D_t^F - N_t,$$
(7)

which is the difference at t+1 between the total revenues of the bank,  $(1-\delta)K_t + Z_{t+1}K_t^{\alpha}$ , and its total obligations,  $\bar{R}_t^D D_t + \bar{R}_t^F D_t^F$ , minus banks' net worth at time t. The timing is as follows: at the beginning of period t,  $Z_t$  and  $\phi_t$  are known, and banks pay dividends,  $d_t$ , to the banks' owners, and hence the initial banks' net worth for the following period,  $N_t$ , is known. Finally, banks choose the optimal level of loans,  $K_t$ , and hence of domestic deposits,  $D_t = K_t - \phi_t - N_t$ .

The value at time t of the bank is given by the net present value of future dividends:

$$V_t = \mathbb{E}_t \left\{ \sum_{i=0}^{\infty} \beta^i d_{t+i} \right\}.$$
(8)

Since banks are owned by domestic households, I discount the dividends using the domestic households' discount factor. Since both domestic and foreign deposits are risk-free and the interest rate in domestic deposits is higher than foreign deposits, banks prefer to borrow as much as they can from foreign investors in equilibrium. I assume that in, equilibrium, the foreign borrowing limit is binding  $\forall t$  and hence  $D_t^F = \phi_t \forall t$ . Later, I show how the calibration looks so that, in equilibrium,  $\phi_t$  is low enough and thus the foreign borrowing limit is always binding.

Since banks have limited liability, the banks' owners cannot put up their own wealth to pay for the banks' obligations. Hence, under limited liability banks default at t + 1, if the banks' available income is not enough to cover the agreed obligations, i.e., banks default if,

$$(1-\delta)K_t + Z_{t+1}K_t^{\alpha} < \bar{R}_t^D D_t + \bar{R}_t^F D_t^F.$$
(9)

The latter statement is equivalent to saying that each time the banks' net operating income is lower than  $-N_t$ , banks default, since banks' owners are not going to honor the unpaid banks' debt, i.e., banks default if,

$$NOI_t + N_t < 0. (10)$$

For simplicity, I assume that there are no default costs, and that banks can continue operating during the next period after defaulting. When the banks default at t+1, I further assume that banks' owners are not going to inject new net worth into banks at t+1. In that case, when banks default, dividends and net worth are both zero,

$$d_{t+1} = 0, \qquad N_{t+1} = 0. \tag{11}$$

When banks do not default, i.e., when (9) does not hold, I assume banks allocate a fraction  $0 < \gamma < 1$  of the gross profits,  $NOI_{t+1} + N_t$ , to shareholders as dividends, i.e.,

$$d_{t+1} = \gamma \left( NOI_{t+1} + N_t \right), \tag{12}$$

where, for simplicity,  $\gamma$  is exogenous and constant across time. Following the spirit of Jermann and Quadrini (2012), this is an extreme case in which firms cannot adjust the dividends to gross profits ratio.<sup>7</sup> As a result, the law of motion of banks' net worth is given by,

$$N_{t+1} = (1 - \gamma) (NOI_{t+1} + N_t).$$

This assumption rules out an explosive accumulation of bank's net worth. In general, banks' dividends and net worth can be written, respectively as,<sup>8</sup>

$$d_{t+1} = \gamma \left[ NOI_{t+1} + N_t \right]^+,$$
(13)

<sup>&</sup>lt;sup>7</sup>Indeed, Jermann and Quadrini (2012) assumes quadratic costs on the adjustment of the level of dividends and not on the dividends to gross profits ratio. They model these costs in order to formalize the rigidities affecting the substitution between debt and equity. Another way of thinking about the adjustment costs, as commented by Jermann and Quadrini (2012), is that it captures the preferences of managers for dividends smoothing.

<sup>&</sup>lt;sup>8</sup>The expression  $[.]^+ = max\{., 0\}.$ 

$$N_{t+1} = (1 - \gamma) \left[ NOI_{t+1} + N_t \right]^+, \tag{14}$$

where,

$$NOI_{t+1} + N_t = (1 - \delta)K_t + Z_{t+1}K_t^{\alpha} - \bar{R}_t^D D_t - \bar{R}_t^F D_t^F.$$

I define a productivity shock  $e_{t+1}^{z,*}$ , so that for a given set of  $K_t$ ,  $D_t$ ,  $N_t$  and  $D_t^F$  if at time t+1, the shock is low enough that  $e_{t+1}^z < e_{t+1}^{z,*}$ , banks default. As a result, the probability at time t that banks default at t+1 is given by,

$$p_t = F(e_{t+1}^{z,*}). \tag{15}$$

In particular,  $e_{t+1}^{z,*}$  is solved in,

$$max\{Z_{t+1}^*, 0\} = exp(\mu_z(1-\rho_z) + \rho_z ln(Z_t) + e_{t+1}^{z,*}),$$

where  $Z_{t+1}^*$  makes banks' revenues equal to their obligations each , i.e.,

$$(1-\delta)K_t + Z_{t+1}^* K_t^{\alpha} = \bar{R}_t^D D_t + \bar{R}_t^F D_t^F,$$
(16)

From (13-14), banks' dividends can be rewritten as a function of banks' net worth,

$$d_t = \frac{\gamma}{1 - \gamma} N_t. \tag{17}$$

Banks optimally choose  $\{D_t, D_t^F, N_t\}$ , in order to maximize the net present value of future dividends, (8), subject to the law of motion of banks' net worth, (14), where banks' dividends and capital are given by equations (17) and (5), respectively. Earlier I stated that the foreign borrowing limit is always binding, i.e.,  $D_t^F = \phi_t \forall t$ . Hence, banks optimally choose only  $\{D_t, N_t\}$ . The Lagrangian is,

$$L_t = \mathbb{E}_t \left\{ \sum_{i=t}^{\infty} \beta^{i-t} \left( \frac{\gamma N_i}{1-\gamma} + \lambda_i \left[ (1-\gamma) \left[ NOI_i + N_{i-1} \right]^+ - N_i \right] \right) \right\},\tag{18}$$

where  $\lambda_i$  is the Lagrange multiplier associated with the law of motion of banks' net worth, equation (14). This is the shadow value (or marginal value) of banks' net worth. If at time *i*, I exogenously give the economy  $1/(1 - \gamma)$  units of banks' gross profits, which allows for one unit more of banks' net worth accumulation, banks' welfare increases by  $\lambda_i$ .

According to (17), banks' net worth affects dividends and it, which in turn affect banks' utility, according to the right-hand side of equation (14),  $N_t$  also affect banks' capacity to accumulate net worth. The latter is because  $N_t$  increases  $K_t$ , which in turn affects  $NOI_{t+1}$ . Similarly,  $D_t$  affects, according to (14), banks' capacity to accumulate net worth. For convenience, I rewrite the Lagrangian as,

$$L_{t} = \frac{\gamma}{1+\gamma} N_{t} + \lambda_{t} \left[ (1-\gamma) \left[ NOI_{t} + N_{t-1} \right]^{+} - N_{t} \right] + \beta \mathbb{E}_{t} \left\{ \frac{\gamma}{1+\gamma} N_{t+1} + \lambda_{t+1} \left[ (1-\gamma) \left[ NOI_{t+1} + N_{t} \right]^{+} - N_{t+1} \right] \right\} + \beta^{2} \mathbb{E}_{t} L_{t+2}.$$

And I replace the expectation for integrals when it is convenient,

$$L_{t} = \frac{\gamma}{1-\gamma} N_{t} + \lambda_{t} \left[ [NOI_{t} + N_{t-1}]^{+} (1-\gamma) - N_{t} \right] + \frac{\gamma\beta}{1-\gamma} \mathbb{E}_{t} N_{t+1} + \beta \int_{e_{t+1}^{z,*}}^{+\infty} \lambda_{t+1} \left[ NOI_{t+1} + N_{t} \right] (1-\gamma) dF(e_{t+1}^{z}) - \beta \mathbb{E}_{t} \{ \lambda_{t+1} N_{t+1} \} + \beta^{2} \mathbb{E}_{t} L_{t+2}.$$
(19)

The first order condition for domestic deposits,  $D_t$ , yields,

$$0 = \beta \int_{e_{t+1}^{z,*}}^{+\infty} \lambda_{t+1} \left( 1 - \delta + Z_{t+1} \alpha K_t^{\alpha - 1} - \bar{R}_t^D \right) (1 - \gamma) dF(e_{t+1}^z) + \beta \lambda_{t+1} \left( (1 - \delta) K_t + Z_{t+1}^* K_t^{\alpha} - \bar{R}_t^D D_t - \bar{R}_t^F \phi_t \right) (1 - \gamma) f(e_{t+1}^{z,*}) \frac{\partial e_{t+1}^{z,*}}{\partial D_t}.$$

Since  $(1-\delta)K_t + Z_{t+1}^*K_t^{\alpha} - \bar{R}_t^D D_t - \bar{R}_t^F D_t^F = 0$ , the first order condition for  $D_t$  becomes,

$$\beta \int_{e_{t+1}^{z,*}}^{+\infty} \lambda_{t+1} \left( 1 - \delta + Z_{t+1} \alpha K_t^{\alpha - 1} - \bar{R}_t^D \right) (1 - \gamma) dF(e_{t+1}^z) = 0.$$
<sup>(20)</sup>

Observing the Lagrangian, equation (19), we see that one-unit increase of domestic deposits,  $D_t$ , is not going to directly increase banks' future dividends, i.e.,  $\frac{\gamma}{1-\gamma}N_{t+1}$ , but rather banks' capacity to accumulate banks' net worth and to pay dividends, and thus the partial derivatives of future banks' net worth with respect to  $D_t$  is zero. Hence, the unit increment of  $D_t$  produces an increase in banks' gross profits by  $1 - \delta + Z_{t+1}\alpha K_t^{\alpha-1} - \bar{R}_t^D$ , which in turn raises their capacity of net worth accumulation. This is, the unit increase of  $D_t$  raises the right-hand side of equation (14), by  $(1 - \delta + Z_{t+1}\alpha K_t^{\alpha-1} - \bar{R}_t^D)(1 - \gamma)$ each time banks do not default. Hence, the marginal effect on banks' utility is computed by multiplying this marginal increase with the shadow value of future banks' net worth,  $\lambda_{t+1}$ . In equilibrium, the expected discounted value of this marginal effect on banks' utility has to be zero, as suggested by (20).

As in the two-period model version, the limited liability and deposit insurance make banks not internalize the states of nature when banks have negative gross profits, i.e.,  $NOI_{t+1} + N_t < 0$ . However, at this stage it is not possible to conclude that banks might overestimate the marginal benefits of capital, since the effect of the shadow value of banks' net worth on banks' decisions it is not clear. In section 5, after presenting the social planner allocation, I compare both allocations and conclude, supported by numerical results, that banks' credit is inefficiently high in the long-term.

The first order condition for banks' net worth,  $N_t$ , yields,

$$\frac{\gamma}{1-\gamma} - \lambda_t + \beta \int_{e_{t+1}^{z,*}}^{+\infty} \lambda_{t+1} \left( 1 - \delta + Z_{t+1} \alpha K_t^{\alpha - 1} \right) (1-\gamma) dF(e_{t+1}^z) = 0, \qquad (21)$$

which is the so-called Euler condition. From equation (19), a unit increase in banks' net worth produces the following effects on banks' utility: an increase by  $\frac{\gamma}{1-\gamma}$ , since dividends depend on net worth, according to (17); a reduction by  $\lambda_t$ , which captures the reduction in banks' utility, due to an excess of one unit of bank's net worth; and a discounted expected marginal increase of  $\lambda_{t+1} \left(1 - \delta + Z_{t+1}\alpha K_t^{\alpha-1}\right) (1-\gamma)$  each time banks do not default at t + 1. This latter captures the gains in banks' utility due to the marginal increases in the banks' capacity to accumulate net worth, adjusted by the shadow value of net worth, conditional to a non-default event. This is because the unit increase of  $N_t$  increases  $K_t$  and then increases banks' revenues, which in turn increases banks' gross profits and thus their capacity to accumulate net worth (right-hand side of equation 14).

As equation (20) suggests, compared to a two-period model, the novelty of this dynamic version is given by the presence of the shadow value of banks' net worth,  $\lambda_{t+1}$ , that multiplies the marginal net benefits of banks' credit,  $1-\delta+Z_{t+1}\alpha K_t^{\alpha-1}-\bar{R}_t^D$ . This  $\lambda_{t+1}$ captures the intertemporal effects when choosing the optimal level of domestic deposits and, hence, the optimal level of banks' credit. In other words, in this multi-period model, banks have to account for the fact that the banks' decisions on current credit affect not only banks' profits in the next period but also profits in the following periods, through banks' net worth.<sup>9</sup> As a result, the intertemporal channel in the model is represented by the presence of  $\lambda_t$ , the shadow value of banks' net worth, which dynamics is determined by combining the banks' first order conditions, equations (20-21) and since in equilibrium  $\beta \bar{R}_t^D = 1$ ,

$$\lambda_t = \frac{\gamma}{1 - \gamma} + \int_{e_{t+1}^{z,*}}^{+\infty} \lambda_{t+1} dF(e_{t+1}^z)(1 - \gamma).$$
(22)

Equation (22) equalizes the marginal cost (left-hand side) and the marginal benefits (right-hand side) of an exogenous one-unit increase of banks' net worth at time t. The marginal cost is given by the shadow value (or marginal value) of banks' net worth at time t. The marginal benefits are given by the marginal increase of the dividends of  $1/(1-\gamma)$ at time t, which in turn increases banks' utility by the same magnitude, according to (8). In addition, the unit increase of  $N_t$  increases future banks' revenues and their capacity to accumulate net worth, the right-hand side of (14), which in turn increases banks' utility each time banks do not default. This latter marginal utility is represented by the second

<sup>&</sup>lt;sup>9</sup>Appendix B shows that  $\lambda_{t+1}$  is not independent of  $e_{t+1}$  and hence the intertemporal channel is validated; otherwise, we are back to the two-period version

term of the right-hand expression of (22).

The clearing condition for the goods market is,<sup>10</sup>

$$C_t = \omega^H + (1 - \delta)K_{t-1} + Z_t K_{t-1}^{\alpha} - K_t + \phi_t - \bar{R}_{t-1}^F \phi_{t-1}.$$
(23)

By equation (23), the domestic consumption and hence the welfare of the domestic economy is independent of the  $\gamma$ . It means that aggregate consumption is not affected by the liability structure and the leverage ratio of domestic banks. Expression (23) can be rearranged to obtain the expression for the gross domestic product,

$$GDP_t = C_t + I_t + G_t + XN_t = G_t + Y_t,$$
(24)

where  $G_t$  are the government expenses,  $XN_t = Y_t - C_t - I_t$  are the net exports,  $Y_t = \omega^H + Z_t K_{t-1}^{\alpha}$ is output, and  $I_t = K_t - (1-\delta)K_{t-1}$  are the investments. The net foreign asset position is,

$$NFA_t = -\phi_t,\tag{25}$$

The law of motion for the stock of net foreign assets, i.e., the current account, is,

$$NFA_t - NFA_{t-1} = CA_t = Y_t - C_t - I_t - \phi_{t-1}(\bar{R}_{t-1}^F - 1),$$
(26)

where the current account,  $CA_t$ , is given by the sum of net exports,  $Y_t - C_t - I_t$ , and the net interest payment on the stock of net foreign assets owned by the domestic economy at the start of the period,  $\phi_{t-1}(1 - \bar{R}_{t-1}^F)$ .

Equilibrium Definition: Equations (3), (4), (5), (7), (13), (14), (16), (20), (21), (23) determine the eight endogenous variables  $(K_t, D_t, C_t, N_t, d_t, NOI_t \bar{R}_t^D, \bar{R}_t^F, Z_{t+1}^*, \lambda_t)$  as a function of the state variables  $(N_{t-1}, K_{t-1}, D_{t-1})$  together with the exogenous stochastic process  $Z_t$ , the exogenous deterministic processes  $\phi_t, G_t, \omega^H$ .

For simplicity, I assume,  $G_t=G$ ,  $\phi_t=\phi$  and  $R_t^F=R^F \forall t$ . As I stated before since I am interested in modeling a small open economy facing capital inflows and sudden stops, I would like to have in equilibrium that  $R_t^F < R_t^D$ . For this, I assume  $R^F < 1/\beta$ , or equivalently that the risk-free foreign interest rate is lower than the domestic risk-free interest rate.

# 4 Domestic social planner equilibrium

The domestic social planner aims to maximize the welfare of the domestic economy. Since domestic households are the owners of domestic banks, the utility of domestic

 $<sup>^{10}\</sup>mathrm{The}\ \mathrm{proof}\ \mathrm{in}\ \mathrm{Appendix}\ \mathrm{A}.$ 

households,  $W_t$ , is going to be a fair measure of the welfare of the domestic economy.

Hence, the domestic social planner optimally chooses  $\{K_t, D_t^F\}$  to maximize the domestic households' utility, (1), subject to the market clearing condition of goods, (23), and the foreign borrowing limit, (6).<sup>11</sup> Since I am assuming the foreign borrowing limit is always binding, the domestic social planner only optimally chooses  $K_t$ . The first order condition for  $K_t$  is,

$$\beta \mathbb{E}_t \left\{ 1 - \delta + Z_{t+1} \alpha K_t^{\alpha - 1} \right\} - 1 = 0.$$
(27)

Hence, the socially efficient level of capital is,

$$K_t = \left(\frac{\mathbb{E}_t\{Z_{t+1}\}\alpha}{1/\beta - (1-\delta)}\right)^{\frac{1}{1-\alpha}},\tag{28}$$

where,

$$\mathbb{E}_t\{Z_{t+1}\} = exp(\mu_z(1-\rho_z) + \rho_z log(Z_t) + 0.5\sigma_{e^z}^2).$$

Socially efficient equilibrium is definition as follows: equations (28), (4) and (23) determine the three endogenous variables  $(K_t, C_t, \bar{R}_t^F)$  as a function of the state variable  $K_{t-1}$ , together with the exogenous stochastic process  $Z_t$ , the exogenous processes  $\phi_t$  and  $R_t^F$ , and the constant variable  $\omega^H$ .

Appendix C solves for the competitive equilibrium, but under unlimited liability. In other words, it solves the maximization problem of the bank under unlimited liability. It proves that the equilibrium level of capital under unlimited liability is the same as in the social planner equilibrium. Hence, the allocation under unlimited liability is also efficient.

# 5 The long-term equilibrium: the stochastic steady state

In contrast to the standard approach, I am interested in the stochastic steady state of the model and not in the deterministic steady state. This is because I want to consider a limit behavior of the economy where agents are aware of the existence of future shocks hitting the economy, i.e., where agents take into account the likelihood of a future very low productivity shock so that the bank might default in the future. This allows me to compute the stochastic steady state value of the default probability of banks. If I use the deterministic steady state instead, the probability of default is going to be zero, since under the deterministic steady state, agents do not consider the existence of future shocks, and, thus, the steady state of the model in the competitive equilibrium is going

<sup>&</sup>lt;sup>11</sup>Since the SP is also subject to the foreign borrowing limit, this is commonly known in the literature as the constrained social planner.

to be the same as the steady state of the domestic social planner equilibrium.

Coeurdacier et al. (2011) develop a new way of approximating the dynamics of stochastic macroeconomic models, by jointly solving for the linear dynamics of the state variables and the stochastic steady state, which is called risky steady state. An application of this is found in Gertler et al. (2012). However, in this paper, I aim to numerically compute the exact value of the stochastic steady state and the exact dynamics of the variables. Similar to the risky steady state literature (see, e.g., Coeurdacier et al., 2011, and De Groot, 2013), the stochastic steady state is defined as the equilibrium at which the variables stay constant in the presence of expected future shocks, but when the values of these shocks turn out to be zero.

Next, I present the long-term equilibrium or the stochastic steady state of the model in the competitive equilibrium and in the social planner equilibrium.

#### 5.0.1 Competitive equilibrium

Note that I calibrate the model so that in the stochastic steady state banks do not default, i.e., it holds that,

$$(1-\delta)K_{sss} + Z_{sss}K^{\alpha}_{sss} > \bar{R}^D_{sss}D_{sss} + \bar{R}^F_{sss}\phi, \qquad (29)$$

but there is positive default probability that in the next period banks default, i.e., it holds that,

$$(1-\delta)K_{sss} < \bar{R}^D_{sss}D_{sss} + \bar{R}^F_{sss}\phi, \tag{30}$$

where the subscript sss refers to the stochastic steady state value of the variable. Equation (30) says that if in the next period the worst state of nature is realized, banks are not able to fully payback depositors.

Following the definition of the stochastic steady state, I can write the system of equations that determines the stochastic steady state values of  $\bar{R}_{sss}^D$ ,  $\bar{R}_{sss}^F$ ,  $K_{sss}$ ,  $N_{sss}$ ,  $D_{sss}$  and  $\lambda_{sss}$  as follows:

$$\bar{R}^{D}_{sss} = \frac{1}{\beta}, \qquad \bar{R}^{F}_{sss} = R^{F}, \tag{31}$$

$$\int_{e^{z,*}}^{\infty} \lambda (1 - \delta + \alpha Z K_{sss}^{\alpha - 1} - \bar{R}_{sss}^D) f(e^z) d(e^z) = 0, \qquad (32)$$

$$\lambda = \frac{\gamma}{1-\gamma} + \int_{e^{z,*}}^{+\infty} \lambda'(1-\gamma)f(e^z)de^z = 0, \tag{33}$$

$$N_{sss} = (1 - \gamma) \left[ (1 - \delta) K_{sss} + Z_{sss} K^{\alpha}_{sss} - \bar{R}^{D}_{sss} D_{sss} - \bar{R}^{F}_{sss} \phi \right],$$
(34)

$$K_{sss} = D_{sss} + \phi + N_{sss},\tag{35}$$

where,  $e^z \sim N(0, \sigma_{e^z})$ ,  $Z_{sss} = exp(\mu_z)$ , and,

$$Z = exp(\mu_{z}(1 - \rho_{z}) + \rho_{z}log(Z_{sss}) + e^{z}),$$
$$Z_{sss}^{*} = (\bar{R}_{sss}^{D}D_{sss} + \bar{R}_{sss}^{F}\phi - (1 - \delta)K_{sss})/K_{sss}^{\alpha},$$
$$e_{sss}^{z,*} = ln(Z_{sss}^{*}) - \mu_{z}(1 - \rho_{z}) - \rho_{z}log(Z_{sss}).$$

Since the interest rates are constant in equilibrium, the stochastic steady state values of these are the same constants, equation (31). Equation (32) is found from equation (20) by making  $K_t = K_{sss}$  and by being aware of the existence of future shocks. Similarly, equation (33) is found from (22). In the long-term  $\lambda_t(Z_t, N_t)$  converges to a function  $\lambda$ that depends on Z, and  $\lambda'$  is its next-period function. More formally, since the shadow value of net worth is a function of equity and productivity level, i.e.,  $\lambda_t = \lambda_t(Z_t, N_t)$ , then  $\lambda = \lambda(Z, N(Z, K_{sss}, N_{sss}))$  and  $\lambda' = \lambda'(Z', N(Z', \hat{K}(Z, N), N)))$ , where apostrophe means next-period,  $\hat{K}(Z, N)$  is the privately optimal credit for given Z and N,

$$Z' = exp(\mu_z(1-\rho_z) + \rho_z \log(Z) + e^z),$$
$$N(x, y, z) = (1-\gamma) \left[ (1-\delta)y + xy^{\alpha} - \bar{R}^D_{sss}(y-z-\phi) - \bar{R}^F_{sss}\phi \right]$$

Also, equation (34) is obtained from (14) by making  $N_t = N_{sss}$ ,  $K_t = K_{sss}$ ,  $D_t = D_{sss}$  and assuming that the current shock is zero, i.e.,  $Z_{t+1} = exp(\mu_z(1 - \rho_z) + ln(Z_{sss}) + 0)$ , which leads to  $Z_{t+1} = Z_{sss}$  since  $Z_{sss} = exp(\mu_z)$ . This latter is because following the definition of the stochastic steady state  $Z_{sss}$  is found in  $Z_{sss} = exp(\mu_z(1 - \rho_z) + \rho_z ln(Z_{sss}) + 0)$ . Finally, Appendix E shows the stochastic steady state values of the other variables. I will use "stochastic steady state" or "long-term" interchangeably.

From (34-35) I can write the stochastic steady state of the bank's net wort as,

$$N_{sss} = \frac{(1-\gamma)}{1-(1-\gamma)(1/\beta)} \left[ (1-\delta - 1/\beta)K_{sss} + Z_{sss}K_{sss}^{\alpha} + (1/\beta - R^F)\phi \right].$$
 (36)

By (36), a higher  $\gamma$  leads to a lower  $N_{sss}$  for the given  $K_{sss}$ . A higher  $\gamma$  implies a lower fraction kept as net worth. This leads to higher domestic deposits for a given level of loans. This situation increases the size of the liabilities and reduces the net worth even further. In order to have a positive net worth at the stochastic steady state, since  $(1 - \delta - 1/\beta)K_{sss} + Z_{sss}K_{sss}^{\alpha} + (1/\beta - R^F) > 0$ , as suggested by Appendix G, I calibrate the model so that,

$$\gamma > 1 - \beta.$$

#### 5.0.2 Social planner equilibrium

Since I am assuming  $\phi_t = \phi$  and  $R_t^F = R^F$ , the stochastic steady state value of domestic credit (capital) is given by,

$$K_{sss} = \left(\frac{\mathbb{E}_{sss}\{Z\}\alpha}{1/\beta - (1-\delta)}\right)^{\frac{1}{1-\alpha}},\tag{37}$$

where,  $\mathbb{E}_{sss}\{.\}$  denotes the expectation condition to being in the stochastic steady state. Hence,

$$\mathbb{E}_{sss}\{Z\} = exp\left(\mu_z(1-\rho_z) + \rho_z ln(Z_{sss}) + 0.5\sigma_{e^z}^2\right).$$

Note that in the social planner equilibrium, there exists a closed-form solution for the level of loans, which does not depend on characteristics of the banking sector. In fact, the social planner's optimal decision does not tell us anything regarding the banks' leverage ratio or the banks' liability composition.

# 6 Comparison of the competitive equilibrium and social planner equilibrium

Here, I compare the social planner equilibrium (the efficient allocation) with the competitive equilibrium. In other words, I asses the effects of limited liability and deposit insurance and the effects of the intertemporal channel on bank credit's decisions. Hence, I aim to answer the following question: Is banks' credit (capital) in the competitive equilibrium inefficiently high, as in the two-period version? Recall that in the social planner equilibrium, capital is given by equation (28),

$$K_t^{SP} = \left(\frac{\mathbb{E}_t\{Z_{t+1}\}\alpha}{1/\beta - (1-\delta)}\right)^{\frac{1}{1-\alpha}},\tag{38}$$

where the superscript SP refers to the social planner allocation. In the competitive equilibrium, equation (20) can be rewritten as,

$$K_t^{CE} = \left(\frac{\frac{\mathbb{E}_t\{Z_{t+1}\lambda_{t+1}|e_{t+1}^z \ge e_{t+1}^{z,*}\}}{\mathbb{E}_t\{\lambda_{t+1}|e_{t+1}^z \ge e_{t+1}^{z,*}\}}\alpha}{1/\beta - (1-\delta)}\right)^{\frac{1}{1-\alpha}},$$

where the superscript CE refers to the allocation in the competitive equilibrium. For convenience, I rewrite  $K_t^{CE}$  as,

$$K_t^{CE} = \left(\frac{\left[\mathbb{E}_t\{Z_{t+1}|e_{t+1}^z \ge e_{t+1}^{z,*}\} + \frac{Cov_t\{Z_{t+1}\lambda_{t+1}|e_{t+1}^z \ge e_{t+1}^{z,*}\}}{\mathbb{E}_t\{\lambda_{t+1}|e_{t+1}^z \ge e_{t+1}^{z,*}\}}\right]\alpha}{1/\beta - (1-\delta)}\right)^{\frac{1}{1-\alpha}},$$
(39)

where  $Cov_t\{.\}$  refers to the covariance function conditional to information up to time t. By definition  $\mathbb{E}_t\{Z_{t+1}|e_{t+1}^z \ge e_{t+1}^{z,*}\}\ge \mathbb{E}_t\{Z_{t+1}\}$ . Clearly, if banks do not internalize the effect of their decisions of credit on banks' dividends of more than one-period ahead, i.e., if

 $Cov_t\{Z_{t+1}\lambda_{t+1}|e_{t+1}^z \ge e_{t+1}^{z,*}\}=0$ , then  $K_t^{CE} > K_t^{SP}$  and, as in the two-period model, the only source of the inefficiency is banks not internalizing the next period's negative losses and, hence, overestimating the next period's marginal benefits of loans.

To examine the sign of  $Cov_t \{Z_{t+1}\lambda_{t+1} | e_{t+1}^z \ge e_{t+1}^{z,*}\}$  I investigate how  $\lambda_t$  depends on  $e_t$ from equation (22). Appendix B shows that  $\lambda_{t+1}$  is not independent of  $e_{t+1}^z$ . In fact,  $\lambda_{t+1}$ is a function of the state variables  $N_{t+1}$  and  $Z_{t+1}$ . Hence, it is not possible to drop  $\lambda_{t+1}$ from the first order condition of  $D_t$ , equation (20), and thus it is not possible to take it out of the integral in equation (22). Due to the complexity of the expression for  $\lambda_t$ , reported in Appendix B, it is not possible to formally prove that this is positively (or negatively) correlated with  $e_t$ .

The dependence of  $\lambda_t$  on  $e_t^z$  determines the effects of the intertemporal channel on bank credit and excess risk-taking. Visually, from (22)  $\lambda_t$  depends on  $e_{t+1}^{z,*}$ . In particular, the higher the  $e_{t+1}^{z,*}$ , the lower the  $\lambda_t$ . The intuition is that the higher the likelihood that banks default in the future, the lower the probability that an exogenous unit of banks' net worth increases banks' capacity to accumulate net worth and hence increases bank dividends in the future, which reduces the shadow value of banks' net worth,  $\lambda_t$ . In order to assess how  $\lambda_t$  depends on  $e_t^z$ , I examine how  $e_t$  affects  $e_{t+1}^{z,*}$ . Appendix F shows that,

$$\frac{\partial e_{t+1}^{z,*}}{\partial e_t^z} = \omega_{K_t} \rho_z Z_t^{\rho_z} K_{Z_t^{\rho_z}} + \left(\omega_{K_t} K_{N_t} - \omega_{N_t}\right) \frac{\partial N_t}{\partial e_t^z} - \rho_z,\tag{40}$$

where  $\omega_{K_t}$ ,  $\omega_{N_t} > 0$ ,  $K_{Z_t^{\rho_z}} > 0$  and  $K_{N_t} < 0$  are the partial derivatives of  $K_t$  with respect to  $Z_t^{\rho_z}$  and  $N_t$ , respectively, and then  $(\omega_{K_t}K_{N_t} - \omega_{N_t}) < 0$ ; and

$$\frac{\partial N_t}{\partial e_t^z} = (1 - \gamma) Z_t K_{t-1}^{\alpha} > 0, \tag{41}$$

In the partial derivative of interest, equation (40), three forces explain the relationship between  $e_{t+1}^z$  and  $e_t^z$ :

• The first force (first term),  $\omega_{K_t} \rho_z Z_t^{\rho_z} K_{Z_t^{\rho_z}}$ : since the productivity shock is persistent

(i.e.,  $\rho_z > 0$ ), a high  $e_t^z$  leads to a high demand for credit. Then, the higher the credit, the higher the default probability and the higher  $e_{t+1}^{z,*}$ . Therefore, this forces creates a negative relationship between  $\lambda_t$  and  $e_t$  and hence decreases the excess bank risktaking. Intuition: when the economy receives a positive productivity shock, which is expected to last, and banks live more than one-period, banks are less motivated to take excessive risk since they want to default a reduced number of times so they collect higher positive profits (associated with the good economic prospective) an increased number of times. In other words, in a good economic prospective banks want to ensure their high profits and hence take less excessive risk.

- The second force (second term),  $(\omega_{K_t}K_{N_t}-\omega_{N_t})\frac{\partial N_t}{\partial e_t^z}$ : a high productivity shock leads to a high net worth (i.e.,  $\frac{\partial N_t}{\partial e_t^z} > 0$ ) reducing the default probability. This reduces  $e_{t+1}^{z,*}$ . Therefore, this forces creates a positive relationship between  $\lambda_t$  and  $e_t$  and hence increases the excess bank risk-taking.
- The third force (third term),  $\rho_z$ : This is the direct effect of  $e_t^z$  on  $e_{t+1}^{z,*}$ . A higher  $e_t^z$  or a higher  $Z_t$  reduces the required size of productivity shock such that banks do not default; consequently, a high  $e_t^z$  reduces  $e_{t+1}^{z,*}$  and creates a positive relationship between  $\lambda_t$  and  $e_t$ . Hence, this force also increases excess bank risk-taking.

The intuition behind the second and third force is as follows: in a good economic prospective, banks default less and hence they benefit more from dividends. This increases banks' incentives to issue excess loans and hence take more excess risk in order to increase profits and hence dividends. In other words, due to the limited liability and deposit insurance each period banks overestimate the marginal benefits of credit; and since they live forever, they optimally choose current credit internalizing its effect on the excess marginal benefits of future credit through the law of motion of banks' net worth. Hence, these two forces, compared to the two-period model, make that banks overestimate even more the marginal benefits of credit.

Note that if the first force dominates the other two, the inter-temporal channel reduces the excess bank risk-taking that exists in the two-period model. Interestingly, if this negative effect of the intertemporal channel on excess bank risk-taking is strong enough, bank credit could be inefficiently low.

As  $\gamma$  converges to 1 (i.e., there is less net worth accumulation), the second force diminishes, while the first and third forces are not affected. As a result, as  $\gamma$  converges to one, it is expected to observe a less positive (or more negative) relationship between  $\lambda_t$  and  $e_t$ . As a result, the weaker the net worth accumulation, the less important the intertemporal channel and hence the smaller the amplification of the excess bank risktaking. In other words, a smaller accumulation of banks' net worth reduces the impact of current banks' decisions about credit on dividends of more than one-period ahead. In the extreme case, where  $\gamma = 1$ , dividends  $d_{t+2}$ ,  $d_{t+3}$ ,  $d_{t+4}$ , ... are independent of banks' optimal decisions about  $K_t$ , and thus the intertemporal channel disappears, and the excess bank risk-taking is similar to a two-period model.

A low persistence of the productivity shock; on the on hand, reduces the first force, which leads to a more positive (or less negative) relationship between  $\lambda_t$  and  $e_t$ . On the other hand, reduces the third force leading to a more negative (or less positive) relationship between  $\lambda_t$  and  $e_t$ . As a result, the final effect of the intertemporal channel on excess bank risk-taking is not clear. Numerical results, presented later, shows that as  $\rho_z$  converges to zero, it is expected to observe a more positive (or less negative) relationship between  $\lambda_t$  and  $e_t$ . This increases excess bank risk-taking. In particular, if the shock is not persistent (i.e.,  $\rho_z=0$ ), the intertemporal channel amplifies the excess bank risk-taking since only the second force is present.

In the benchmark calibration ( $\rho_z=0.65$ ), numerical results suggest that in the longterm (the stochastic steady state) the correlation between  $\lambda_t$  and  $e_t^z$  is mainly positive (see figure 3).<sup>12</sup> Therefore, in the long-term since  $\mathbb{E}_t\{Z_{t+1}|e_{t+1}^z \ge e_{t+1}^{z,*}\} \ge \mathbb{E}_t\{Z_{t+1}\}$  and  $Cov_t\{Z_{t+1}\lambda_{t+1}|e_{t+1}^z \ge e_{t+1}^{z,*}\} > 0$ , bank credit is inefficiently high, i.e.,

$$K_t^{CE} > K_t^{SP}.$$
(42)

In addition, comparing (39) with its equivalent expression for a two-period model,

$$K_t^{CE} = \left(\frac{\mathbb{E}_t\{Z_{t+1}|e_{t+1}^z \ge e_{t+1}^{z,*}\}\alpha}{1/\beta - (1-\delta)}\right)^{\frac{1}{1-\alpha}},$$

since in the long-term  $Cov_t\{Z_{t+1}\lambda_{t+1}|e_{t+1}^z \ge e_{t+1}^{z,*}\}>0$ , the intertemporal channel in this multi-period model amplifies the distortions on bank credit in the long-term. As a result, in the long-term the excess bank risk-taking is amplified due to the presence of the shadow price (or marginal value) of banks' net worth,  $\lambda_t$ , which captures the intertemporal channel of banks' optimal decisions about their credit levels. Recall that the bank risk-taking involves the volume of bank credit and not the type of credit.

### 6.1 Domestic welfare losses

In this section, I present the welfare losses of the domestic economy. As I said before, a fair measure of the domestic welfare is households' utility since they are the owners of domestic banks. Hence, the welfare losses  $(WL_t)$  are defined as the difference in the households' utility under the social planner equilibrium,  $W_t^{SP}$ , and their utility under the

<sup>&</sup>lt;sup>12</sup>In the long term:  $Z_{t-1} = Z_{sss}$  and  $N_t = N_{sss}$ .

competitive equilibrium,  $W_t^{CE}$ ,

$$WL_t = W_t^{SP} - W_t^{CE} = \mathbb{E}_t \left\{ \sum_{i=t}^{\infty} \beta^{i-t} (C_i^{SP} - C_i^{CE}) \right\}.$$

Using the clearing condition of the goods market,  $WL_t$  can be rewritten as,

$$WL_{t} = \mathbb{E}_{t} \left\{ \sum_{i=t}^{\infty} \beta^{i-t} \left[ (K_{i}^{CE} - K_{i}^{SP}) + (1-\delta)(K_{i-1}^{SP} - K_{i-1}^{CE}) + Z_{i} \left( (K_{i-1}^{SP})^{\alpha} - (K_{i-1}^{CE})^{\alpha} \right) \right] \right\}.$$

Welfare losses are explained by the differences in banks' credit under the competitive equilibrium and the social planner equilibrium. Therefore, I define  $\nu_t = (K_t^{CE} - K_t^{SP})/K_t^{SP}$ .100 in order to capture the size of the inefficiency in the domestic economy. Hence,  $\nu_t$ , which I call the relative excess loans, directly measures the excess bank risk-taking. Also, I define  $\theta_t$  as the excess marginal benefit of capital or bank loans:

$$(1 - \delta) + \alpha \mathbb{E}_t \{ Z_{t+1} \} (K_t^{CE})^{\alpha - 1} + \theta_t = R_t^B.$$
(43)

So,  $\theta_t$  represents another measure of the inefficiency in the competitive equilibrium. If bank credit is efficient,  $\theta_t=0$ . Since in the stochastic steady state capital is inefficiently high,  $\theta_{sss}>0$ . In addition, since the default probability is positively related to banks' incentive to take excess risk, I also focus on its behavior to predict the behavior of the excess bank risk-taking.<sup>13</sup> In fact, it is more natural to link the concept of the default probability with the concept risk-taking.<sup>14</sup>

As a result, a policy that restores the efficient level of capital is enough to obtain zero welfare losses and is going to be an optimal macroprudential policy.

## 7 Quantitative analysis

## 7.1 Calibration

In the model, each period represents a quarter. Table 1 reports the values of the parameters and the sources or targets to calibrate those. Capital's share in output is set to  $\alpha=0.33$ , which is a standard value in the literature. The gross risk-free foreign interest rate is set to  $R^F=1.0124$ , which is the average of the real Fed rate over the 1998M1-2004M12 period.<sup>15</sup> The mean of  $log(Z_t)$ ,  $\mu_z$ , is normalized to zero without loss

<sup>&</sup>lt;sup>13</sup>Since under limited liability and deposit insurance banks do not internalize their losses each time they default, the higher the default probability the higher excess bank risk-taking.

<sup>&</sup>lt;sup>14</sup>Note that the optimal regulation seeks for  $\nu_t$  and  $\theta_t$  to converge to zero; meanwhile, the default probability converge to its level under a regulated economy, which is not necessarily zero.

<sup>&</sup>lt;sup>15</sup>I choose this period since it contains the sudden stop episode matter of the study.

of generality. Note that I already assumed  $\mu_{e^z}=0$ . The persistence of the productivity shock is set to  $\rho_z=0.65$  as in Céspedes et al. (2016).

The remaining parameters are chosen using Peruvian economy data. The calibration strategy thus consists of choosing values for the parameters so that the stochastic steady state of the model matches some key aspects of the Peruvian economy. The discount factor is set to  $\beta$ =0.9855, in order to match an annualized real gross interest rate of 1.06, which is the average of the interbank real gross interest rate in the Peruvian economy for the period 1998M1-2004M12. This period captures the 1998 sudden stop and the deleveraging episode. This results in a real interest rate spread of 4.76%. The parameter  $\gamma$  affects the leverage ratio, as suggested by the Appendix I, but since  $\gamma$  interacts with the effects of the intertemporal channel as suggested by (22), this is calibrated to match a short-term response of the default probability in the sudden stop simulation, similar to the morosity rate's response observed in the data (see figure 2): two quarters after the reduction of the initial reduction of the foreign liability (FL) to GDP ratio, the morosity rate rose 60%.<sup>16</sup> As a result, I set  $\gamma$ =0.50 in the model so that the default probability two quarters after the initial foreign borrowing reduction becomes 1.6 times its initial value.

The depreciation rate,  $\delta$ , is set to 0.115, so that, in the long-term equilibrium, the limited liability is binding and the leverage ratio is 9.5, which is the value of the riskweighted assets to net worth ratio in the Peruvian banking system in the third quarter of 1998.<sup>17</sup> This is meant to capture the size of the leverage in the quarter preceding the sudden stop. This is higher than what is suggested by the literature.<sup>18</sup> The exogenous domestic households' income is set to  $\omega^H=4.7$ , so that the stochastic steady state value of the annualized private consumption to GDP ratio in the model equals 72.1%, which is the value of the ratio in the third quarter of 1998 for the Peruvian economy. The exogenous government expenses are set to G=1.2, in order to match an annualized credit to GDP ratio in the model of 28.4%, which is the value of the private credit by banks to GDP ratio in the third quarter of 1998 for the Peruvian economy. The exogenous borrowing limit,  $\phi$ , is set to 2.39, in order to match a foreign liability to GDP ratio of 7.5%, which is the value of the short-term foreign liability of the financial system (excluding CRBP) to

<sup>&</sup>lt;sup>16</sup>The morosity rate of only loans to firms, which is the best counterpart in the data, is only available since 2001. Before to 2001, there is only available the morosity rate of total loans, which is shown in figure 2. In 1998Q3 the share of bank loans to firms was 86%. This number has declined slowly so that in 2002Q4 the share was 80%. This means that in the period of study the morosity rate of total loans is a very good approximation of the morosity rate of loans to firms. Recently, in 2017Q4 the share of bank loans to firms was 60%.

 $<sup>^{17}</sup>$  Note that over the period 1998M1-2003M12 the private credit of the banking system accounts on average for 80% of the total private credit of the financial system.

<sup>&</sup>lt;sup>18</sup>Appendix G shows how the leverage ratio and foreign debt to credit ratio define a lower bound for  $\delta$ . In addition, it suggests that a high  $\delta$  is associated with a low leverage ratio. In addition, Appendix J shows that using  $\delta$  to math the desired credit to GDP ratio leads to unrealistic levels of  $\delta$ .

GDP ratio for the Peruvian economy in the third quarter of 1998.<sup>19</sup> Finally, the standard deviation of the productivity shock  $\sigma_{e^z}$  is set to 0.998, in order to match an annualized default probability of 3%, which is small, as suggested by the empirical and theoretical literature.

The competitive equilibrium cannot be solved analytically and thus I solve it using numerical simulations. I employ a global solution method in order to deal with the nonlinearities. Appendix K describes the numerical solution method.

Table 2 shows the stochastic steady state, or long-term values of key indicators in the social planner equilibrium (regulated economy) in column 1, the competitive equilibrium in column 2, the competitive equilibrium abstracting from the intertemporal channel, i.e., when assuming  $\lambda_t$  is independent of the productivity shock,  $e_{t+1}^z$ , and hence  $Cov_t\{Z_{t+1}\lambda_{t+1}|e_{t+1}^z\geq e_{t+1}^{z,*}\}=0 \forall t$  in column 3 and the competitive equilibrium under unlimited liability (regulated economy) in column 4, which leads to the same credit allocation as in the social planner equilibrium. Note that the default probability under unlimited liability, by definition, is zero. However, the value 0.09% reported in the table is the long-term monthly probability that the net operating income plus the net worth becomes negative in the next period. Hence, this can be interpreted as the default probability of banks in a regulated competitive equilibrium, i.e., the default probability that exists in the competitive equilibrium under optimal policy intervention.

In the competitive equilibrium, long-term loans are 5.79% inefficiently higher than in the social planner equilibrium. The long-term default probability of banks is 0.74%, which is eight times that of the regulated economy. When abstracting from the intertemporal channel, the stochastic steady state value of the default probability is only 0.10%, which is close to that of the regulated economy, and loans are only 0.15% inefficiently high. In addition, in the competitive equilibrium, the excess marginal benefit is 0.48%, while when abstracting from the intertemporal channel this is 0.01%. Hence, the intertemporal channel amplifies the inefficiency and hence the excess bank risk-taking in the long-term equilibrium.

Figure 3 shows the long-term equilibrium of the shadow value of banks' net worth as a function of  $e^z$ , i.e.,  $\lambda$  that appears in equations (32-33), in the competitive equilibrium (dashed line) and in the unlimited liability equilibrium (solid black line).<sup>20</sup> The figure

<sup>&</sup>lt;sup>19</sup>In the stochastic steady state  $\omega^H$  accounts for 82% of the consumption, and  $\omega^H + G$  for the 74% of the GDP. Later, in the sudden stop simulation section I show that this calibration also matches the observed banks' foreign debt to banks' credit ratio observed in September 1998 and its dynamics after the sudden stop. Hence, if I abstract from  $\omega^H$  and G, i.e., if I avoid matching the levels of credit to GDP and consumption to GDP ratio, and I only focus on matching the foreign debt to credit ratio, i.e., in the banking sector, the rest of the parameter values are going to be very similar. Hence, the quantitative results are going to be almost identical.

 $<sup>^{20}\</sup>lambda$  is solved using the value function iteration method. To do this I use equation 22 (where  $\lambda_t$  is expressed in a recursive way) and the policy functions (i.e., the privately optimal credit level as a function of the state variables).

Description		Value	Source / Target
Discount factor	β	0.986	Gross domestic rate = $1.060$ (annual)
Gross foreign interest rate	$R^F$	1.003	Gross foreign rate = $1.0124$ (annual)
Capital's shares in output	$\alpha$	0.330	Standard value
Capital depreciation ratio	$\delta$	0.115	Bank Leverage ratio
Dividend policy	$\gamma$	0.500	Short-term dynamics of $p_t$
Foreign borrowing limit	$\phi$	2.390	FL to GDP ratio
Government Expenses	G	1.166	Bank Credit to GDP ratio
Households' exogenous income	$\omega^H$	4.665	Consumption to GDP ratio
Mean of log $Z_1$	$\mu_z$	0.000	Normalized
Std. Dev. of the productivity shock	$\sigma_{e^z}$	0.998	Default Probability = $3\%$ (annual)
Persistence of the shock	$\rho_z$	0.650	Céspedes et al. $(2016)$

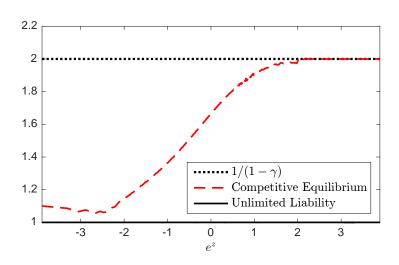
 Table 1: Parameters

Table 2: Stochastic steady states

Description	Variables	(1) SP	(2) CE	(3) CE <sup>†</sup>	(4) CE-ULL
Bank leverage ratio FL to GDP ratio (%) Bank credit to GDP ratio (%) Consumption to GDP ratio (%) FL to credit ratio (%) Bank default probability (%)	$\begin{array}{c} K_{sss}/N_{sss} \\ \phi/(4.GDP_{sss}) \ (\%) \\ K_{sss}/(4.GDP_{sss}).100 \\ C_{sss}/GDP_{sss}.100 \\ \phi/K_{sss}.100 \\ p_{sss}.100 \end{array}$	7.61 26.96 72.65 28.21	9.53 7.54 28.39 72.07 26.67 0.74	$\begin{array}{c} 8.78 \\ 7.61 \\ 27.00 \\ 72.63 \\ 28.17 \\ 0.10 \end{array}$	$\begin{array}{c} 8.76 \\ 7.61 \\ 26.96 \\ 72.65 \\ 28.21 \\ 0.09 \end{array}$
Excess marginal benefits (%) Relative excess loans (%)	$ heta_{sss}.100 \ (K_{sss}^{CE}/K_{sss}^{SP}$ -1).100	-	$0.48 \\ 5.79$	$\begin{array}{c} 0.01 \\ 0.15 \end{array}$	-

 $CE^{\dagger}$ : Competitive equilibrium abstracting from the intertemporal channel, i.e., assuming  $\lambda_t$  is independent of  $e_t^z$ . CE-ULL: Competitive equilibrium under unlimited liability. FL= Foreign liabilities =  $\phi$ .  $GPD_{sss}=G+Y_{sss}$ .  $Y_{sss}=\omega^H+Z_{sss}K_{sss}^{\alpha}$ 

shows how in the stochastic steady state  $\lambda$  is first for small values of  $e^z$  decreasing on it and then increasing on it. Hence, in the context of the discussion presented in section 6 the first force dominates the other forces, then this force is dominated. Furthermore, the shadow value of net worth is always higher than its value in the unlimited liability equilibrium, for any  $e^z$ . The intuition is as follows: in contrast to the unlimited liability equilibrium, under limited liability, when total gross revenues are lower than total obligations, equity holders (banks' owners) do not have to pledge their own money to fully pay back depositors. This clearly leads to a higher marginal benefit of an exogenous unit increase of equity (net worth) and hence to a higher shadow value of net worth.



## Figure 3: Lambda $\lambda$

## 7.2 Simulation: Peru 1998 sudden stop

I calibrate the sudden stop simulation in the model by observing the dynamics of the short-term foreign liability of the financial system to GDP ratio presented in figure 4. The figure shows that this ratio (solid blue line) gradually decreases after the third quarter of 1998, from 7.5% to an average of 1% during 2003.

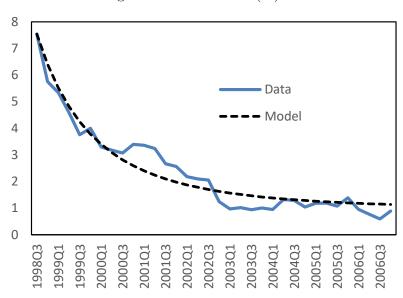


Figure 4: FL to GDP (%)

Data: FL=the short-term foreign obligations of the financial system from 1998Q3 to 2006Q4. Model: FL=foreign borrowing limit. Source: CRBP.

I consider that the economy starts from its stochastic steady state at time t=0. The sudden stop simulation consists of a shock to the foreign borrowing limit. In particular,

I decrease  $\phi$  from 2.39 to 0.32 (i.e.,  $\phi^{old}=2.39$  and  $\phi^{new}=0.32$ ). This implies a reduction of 87% in the exogenous foreign borrowing limit. This is in order to observe in the model a reduction of the stochastic steady state value of foreign liability to GDP ratio of 6.5%, which is the same as that observed in the Peruvian data in the 1998 sudden stop (from 7.5% in 1998Q3 to an average of 1% during 2003).

In real life (see figure 4), the adjustment of the borrowing limit is gradual. So I assume it follows the log-linear path,

$$log(\phi_t) = \rho_\phi log(\phi_{t-1}) + (1 - \rho_\phi) log(\phi^{new}),$$

for  $t \ge 1$ . The initial fall in the foreign borrowing limit happening in t=1 is not anticipated by agents, while from the period 1 onward, agents correctly anticipate the path of  $\phi_t$ . I set  $\rho_{\phi} = 0.92$  in order to match the dynamics of the reduction of the short-term foreign liability to GDP ratio in the Peruvian economy over the period 1998Q3-2003Q4. Indeed, figure 4 shows that the dynamic behavior of the FL to GDP ratio in the model (dashed black line) matches very well the dynamics observed in the Peruvian data (solid blue line). Appendix M describes the solution method of the dynamics of the sudden stop simulation.

Figure 5 shows the dynamics for key variables of the sudden stop simulation from t=1 to t=30 in the social planner equilibrium (solid blue line), the competitive equilibrium (dashed red line) and the competitive equilibrium abstracting from the intertemporal effect (dotted red line). Note that the sudden stop simulation, i.e., the gradual reduction in the foreign limit, does not affect the capital level in the social planner equilibrium, but increases the long-term default probability by 28 bps (from 0.09% to 0.37%), which indeed corresponds to the regulated economy. In the competitive equilibrium, in the long-term the simulated sudden stop produces an increase of the default probability of 328 bps (from 0.74% to 4.02%). Banks' loans increase by 8.3% and hence the relative excess loans increase by 880 bps (from 5.79% to 14.59%). When abstracting from the intertemporal channel, in the long-term the default probability increases by 35 bps (from 0.10% to 0.45%), loans increase by only 0.5%, and the relative excess loans increase by 51 bps (from 0.15% to 0.66%).

As calibrated in the competitive equilibrium two quarters after the initial foreign borrowing reduction, the default probability of banks becomes 1.6 times its initial value. Furthermore, in the short-term after the unanticipated initial foreign borrowing reduction, i.e., at t=3, the default probability, the excess marginal benefits and the relative excess loans increase, respectively, by 55, 17 and 222 bps. However, when abstracting from the intertemporal channel, the same variables increases by 7, 1 and 10 bps, respectively.

Hence, figure 5 shows that, in the competitive equilibrium, there are substantial

responses of in the banks' default probability, the excess marginal benefits of loans, and the relative excess loans. However, smaller reactions are observed when abstracting from the intertemporal effect. Hence, the intertemporal channel significantly increases the response of the excess bank risk-taking. This is because banks internalize the effects of their current risk-taking positions on future dividends.

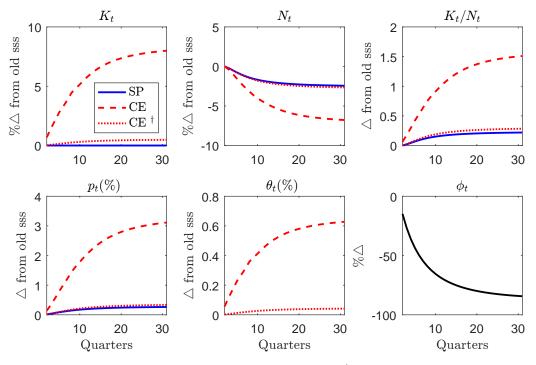


Figure 5: Sudden stop simulation

SP: Social planner equilibrium. CE: Competitive equilibrium. CE<sup>†</sup>: Competitive equilibrium abstracting from the intertemporal channel.

The simulation helps us understand the significant increase in the morosity rate after the initial foreign debt reduction. Ceteris paribus, the gradual reduction of cheap borrowing leads to higher future default probabilities, which in turn increases banks' incentives to take excessive risk. In this multi-period model, it is possible to observe how banks internalize the excess marginal benefits of issuing more loans in the future, once they are aware of the gradual foreign borrowing reduction, which results in a significant increase in current loans and thus an increase in banks' current excess risk-taking. With optimal regulation, the increase in the morosity rate and bank risk-taking would not have been as big as it was, and the banking system would have been more resilient to the sudden stop faced by the Peruvian economy in 1998. This is because the goal of the optimal policy would have been to reduce banks' incentives to take excessive risk.

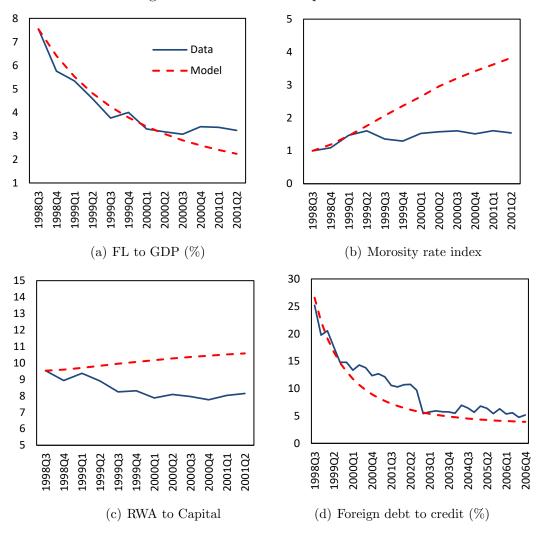


Figure 6: 1998 sudden stop: model vs. data

Source: CRBP, SBS. I assume the counterpart of the morosity rate in the model is a lineal function of the bank default probability. Hence, in order to compare the model counterpart of the morosity rate with the data, I scale the morosity rate in the model and the bank default probability in the model to 1 in 1998Q3. RWA to Capital = Risk weighted assets to bank net worth. In the model this is  $K_t/N_t$ . Foreign Debt = Foreign debt and obligations of the banking system. Credit = Banking system credit to the private sector. In the model, foreign debt to credit is  $\phi_t/K_t$ 

Figure 6 compare the dynamics of the model with the data after the sudden stop in 1998Q3. As stated before, the model matches very well the dynamics of the FL to GDP ratio. In addition, according to figure 6.b it matches very well the significant shortterm increase of the morosity rate, through the increase of the bank default probability. However, from figure 6.c the model does not match the reduction of the bank leverage ratio observed in the data. This is because the model does not capture the reduction in credit growth, from 29.5% in September 1998 to very negative growth rates in 2000. I leave for future work an extension of the model that captures this reduction of the credit growth in order to study its effect on the main results of the model. Figure 2 shows that the morosity rate decreases from 10.0 in 2001Q2 to an average of 1.3 in 2008, which is not captured by the model. This reversal in the morosity rate was due to several policy measures implemented since 2001, including the inflation target system and the accumulation of sufficient foreign-exchange reserves, which promoted economic and financial stability (see, Dancourt, 2015). Hence, it seems that these policies did not allow the morosity rate to converge in the long-term to higher levels; instead, they reduced the excess risk-taking by banks, which was also accompanied by a recovery in credit growth.

Interestingly, figure 6.d shows that the model and the sudden stop simulation match fairly well the level and dynamics of the foreign debt to credit ratio in the banking system that decreases from 25% to 5% over the 1998Q3-2006Q4 period. Hence, instead of focusing on the calibration and sudden stop simulation in the short-term foreign liabilities of the financial system to GDP ratio, I could focus on the ratio of foreign debt to credit in the banking system. For this latter approach, I am not necessarily interested in matching the consumption to GDP ratio and the FL to GDP ratio, and thus I can assume  $\omega^{H}=0$ and G=0, without affecting the main results of the model.

#### 7.2.1 Robustness check

I now examine the sensitivity of the long-term equilibrium (stochastic steady state) and the sudden stop simulation to changes in two key parameters: the one that describes the law of motion of banks' net worth,  $\gamma$ , and the persistence of the productivity shock,  $\rho_z$ . In particular, I solve the model for  $\gamma=0.54$ , which is higher than its baseline calibration ( $\gamma=0.50$ ); and for  $\rho_z=0.78$  and  $\rho_z=0.00$ , which are, respectively, higher and lower than its baseline calibration ( $\rho_z=0.65$ ). Table 3 summarizes the results of the robustness analysis and compares those with the baseline calibration. For comparison reasons each time I change any parameter, I recalibrate  $\sigma_{e^z}$ ,  $\delta$ ,  $\phi$ , G and  $\omega_H$ , so the stochastic steady state values of the default probability, leverage ratio, FL to GDP ratio, credit to GDP ratio, and consumption to GDP ratio are not affected. Next, I compare first the long-term equilibriums and then the sudden stop simulations.

In the long-term, by construction, leverage ratio and default probability do not change. When  $\gamma$  is higher (row 2), as commented before, the amplification effect, caused by the intertemporal channel, on excess bank risk-taking is smaller (see columns 2-4). For instance, the default probability is only 2.3 times its value than when abstracting from the intertemporal channel, while in the baseline calibration it is 7.9 times. In other words, a lower accumulation of net worth diminishes the amplification effect on the long-term excess bank risk-taking. When the productivity shock is not persistent (i.e.,  $\rho_z=0$ , row 3), the amplification effect, caused by the intertemporal channel, is more significant (see columns 3-5). In this case, the default probability is 103 times than when abstracting from the intertemporal channel, while in the baseline calibration it is only 7.9 times. Interestingly, in the case of a more persistent shock (i.e.,  $\rho_z=0.78$ , row 4) the intertemporal channel reduces the excess bank risk-taking such that in the long-term credit is 0.9% inefficiently low. This is because the first force dominates the other two forces described in section 6.

	Ir	itial	999.	+-0	Sudden stop simulation											
	Initial SSS: $t=0$				t=3					t=sss						
	$_{U}CE p_{0}^{CE} p_{0}^{CE} \nu_{0}^{CE}$			$\frac{\boxed{\bigtriangleup p_t}{CE \ CE^{\dagger}} \frac{\bigtriangleup \nu_t}{(5)/(6)} \frac{\bigtriangleup \nu_t}{CE \ CE^{\dagger}}}_{(8)/(9)}$				$\triangle p_t$			$\Delta \nu_t$					
	$\nu_0$	$p_0^{SP}$	$p_0^{CE^{\dagger}}$	$\nu_0^{CE^{\dagger}}$	CE	$CE^{\dagger}$	(5)/(6)	CE	$CE^{\dagger}$	(8)/(9)	CE	$CE^{\dagger}$	(11)/(12)	CE	$CE^{\dagger}$	(14)/(15)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(1) Baseline	5.8	7.9	7.2	38.6	0.55	0.07	7.9	2.22	0.10	22.2	3.28	0.35	9.4	8.81	0.51	17.3
(2) $\gamma = 0.54$	4.1	2.6	2.30	8.77	0.31	0.11	2.8	1.20	0.15	8.0	1.27	0.43	3.0	3.76	0.61	6.2
(3) $\rho_z = 0.00$	9.4	104	103	887	0.28	0.01	28.0	0.75	0.02	37.5	1.25	0.10	12.5	2.61	0.14	18.6
(4) $\rho_z = 0.78$	-0.9	0.8	0.52	-0.45	0.86	0.37	2.3	3.53	0.54	6.5	50.3	1.76	28.6	147	2.58	57.0

Table 3: Robustness check

 $\nu_t^x = (K_t^x - K_t^{SP})/K_t^{SP}$ .100. t=0 refers to the initial (old) stochastic steady state. t=sss refers to the new stochastic steady state. The initial foreign borrowing limit reduction is at t=1. CE: Competitive equilibrium. SP: Social planner (or regulated competitive equilibrium). CE<sup>†</sup>: competitive equilibrium abstracting from the intertemporal channel. SSS: stochastic steady state. For comparison reasons in all robustness exercises the sudden stop simulation  $\phi^{new} = (1-87\%)\phi^{old}$  in order to observe a 6.5% reduction of the SSS value of the FL to GDP ratio as in the baseline calibration. Baseline: Baseline calibration, Table 1.

Regarding the sudden stop simulation, the smaller the net worth accumulation (i.e., higher  $\gamma$ ), the lower the absolute response of default probability in the competitive equilibrium with respect to the absolute response when abstracting from the intertemporal channel. This holds in the short-term (column 7) and in the long-term (column 13), and also when observing the relative excess loans (columns 10 and 16). For instance, the relative absolute response of the relative excess loans shrinks from 7.9 in the baseline calibration to 2.8. As a result, a higher  $\gamma$  leads to a smaller effect of the intertemporal channel on the sudden stop simulation. When the shock is not persistent (i.e.,  $\rho_z=0$ ), the relative responses of these variables increase. Finally, when the economy starts from an inefficiently low credit (i.e.,  $\rho_z=0.78$ ), in the short term the relative responses of these variables decrease, but in the long-term increase.

## 7.3 Productivity shock

Here, I aim to assess how the model behaves after a productivity shock. To construct the impulse response I state that: (i) at t=0 the economy starts from its stochastic steady state, (ii) the positive and negative shocks at t=1 are  $+0.25\sigma_{e^z}$  and  $-0.25\sigma_{e^z}$ , respectively, and (iii) ex-post the following shocks are zero, i.e.,  $e_t^z=0$ ,  $\forall t > 1$ . So the economy converges to the stochastic steady state after the shocks.<sup>21</sup>

Figures 12 and 13, in Appendix O, show the impulse response functions after a negative and positive productivity shock, respectively, in the social planner equilibrium, the competitive equilibrium and the competitive equilibrium abstracting from the intertemporal channel. In what follows I provide a detailed explanation.

In the social planner equilibrium after a negative productivity shock, since the shock is persistent, it produces a persistent reduction of credit, and in the regulated economy a persistent reduction of net worth. Since the net worth reduction is larger than credit reduction, leverage ratio increases. The net worth reduction pushes the default probability up, while the credit reduction pushes the default probability down. In equilibrium, the former dominates and default probability increases.

Similarly, in the competitive equilibrium, after the same shock, the net worth reduction pushes the default probability up. However, the credit reduction (driven by the persistent negative productivity shock) pushes the default probability down. In equilibrium, the former effect dominates, and so the default probability increases. The higher default probability increases banks' incentives to take excess risk, which pushes credit up. As a result, after a negative productivity shock, the excess bank risk-taking increases (i.e.,  $\theta_t$  increases) and thus there is a smaller reduction of bank credit in the competitive equilibrium (dashed line) than in the social planner equilibrium (solid line). Furthermore, in the competitive equilibrium, default probability increment is larger than in the social planner equilibrium. This is because of the larger reduction of net worth, which is caused by the smaller reduction of banks' credit. The larger reduction of net worth and the smaller reduction of credit explain the larger increase of leverage ratio in the competitive equilibrium. Comparing the competitive equilibrium with and without the intertemporal channel (i.e., the dashed line with the dotted line), the intertemporal channel amplifies the excess bank risk-taking. Therefore, the intertemporal channel amplifies fluctuations of leverage ratio, default probability and excess bank risk-taking, while reduces credit fluctuation.

Similarly, after a positive productivity shock, in the competitive equilibrium the default probability, excess bank risk-taking and leverage ratio fluctuate more than in the social planner equilibrium, while capital fluctuates less. And the intertemporal channel amplifies the fluctuations of the leverage ratio, default probability and excess bank risk-taking, while diminishes credit fluctuation.

Next, I examine how these results change for different calibrations:  $\gamma=0.54$ ,  $\rho_z=0$  and  $\rho_z=0.78$ . In general, the intertemporal channel amplifies the fluctuations. Figures 14-19, in Appendix O, report the results.

 $<sup>^{21}</sup>$ To construct the impulse responses functions I use the policy function from solving banks' problem in its recursive as it was done in Appendix K.

When the accumulation of banks' net worth is smaller (i.e., when  $\gamma$  is higher), the amplification effect of the intertemporal channel on the fluctuations of default probability, excess bank risk-taking and leverage ratio decreases (see figures 14-15). In the case of a non-persistent productivity shock (i.e.,  $\rho_z=0$ ), credit fluctuations in the competitive equilibrium are also larger than in the social planner equilibrium. This is because the only force affecting credit are the incentives to take excess risk (i.e., the increased default probability) and so credit in the social planner does not change. Therefore, the intertemporal channel also amplifies credit fluctuations (see figures 16-17). When  $\rho_z$  is higher (for instance,  $\rho_z=0.78$ ), the opposite holds (see figures 18-19). In other words, in the competitive equilibrium credit fluctuates much less than under the social planner equilibrium.

# 8 Conclusions

I build a framework to understand the dynamics of the default probability and the excess bank risk-taking after a sudden stop in an emerging economy, such as in Peru. To do this, I develop a multiperiod model by assuming an exogenous law of motion for banks' net worth. The novelty result is that, compared to the two-period model, the infinite-period model creates an intertemporal channel, represented by the shadow value of bank equity that is sensitive to the state of nature, which amplifies the short-term effect of a sudden stop on the excess bank risk-taking. I calibrate the model for the Peruvian economy and a reduction of the foreign borrowing to simulate the 1998 sudden stop. The model accurately predicts the substantial short-term rise in the morosity rate, that occurs after the sudden stop, through the short-term rise in bank default probability and hence in excess bank risk-taking.

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## Appendix A Competitive equilibrium: Market clearing condition

Here, I prove how I obtain (23). Recall that budget constraint of the domestic household is,

$$C_t + D_t = \omega^H + \bar{R}_{t-1}^D D_{t-1} + \Pi_t - T_t.$$

Recall  $\Pi_t$  are banks' dividends and  $T_t$  are the lump-sum government taxes. Hence,

$$\Pi_t = d_t = \gamma \left[ NOI_t + N_{t-1} \right]^+$$

Since taxes are given by the amount of banks need in order to ensure depositors receive the full payment each time the bank default,

$$T_t = [-NOI_t - N_{t-1}]^+$$

Inserting the expressions of  $\Pi_t$  and  $T_t$  into the budget constraint and since  $D_t = K_t - D_t^F - N_t$ ,

$$C_t = \omega^H - K_t - D_t^F + [NOI_t + N_{t-1}]^+ - [-NOI_t - N_{t-1}]^+.$$

Since  $[NOI_t + N_{t-1}]^+ - [-NOI_t - N_{t-1}]^+ = NOI_t + N_{t-1}$  and in equilibrium  $D_t^F = \phi_t \ \forall t$ ,

$$C_t = \omega^H + (1 - \delta)K_{t-1} + Z_t K_{t-1}^{\alpha} - K_t + \phi_t - \bar{R}_{t-1}^F \phi_{t-1}.$$

#### Appendix B Competitive equilibrium: $\lambda_t$

Recall by (22) that,

$$\frac{\gamma}{1-\gamma} - \lambda_t + \int_{e_{t+1}^{z,*}}^{+\infty} \lambda_{t+1} dF(e_{t+1}^z)(1-\gamma) = 0,$$

where  $dF(e_{t+1}^z) = f(e_{t+1}^z)d(e_{t+1}^z)$ . Rearranging,

$$\lambda_t = \int_{e_{t+1}^{z,*}}^{+\infty} \lambda_{t+1} dF(e_{t+1}^z)(1-\gamma) + \frac{\gamma}{1-\gamma}.$$
(44)

I follow the repeated forward substitution method in order to solve it. I first shift (44) one period ahead:

$$\lambda_{t+1} = \int_{e_{t+2}^{z,*}}^{+\infty} \lambda_{t+2} dF(e_{t+2}^z)(1-\gamma) + \frac{\gamma}{1-\gamma}.$$
(45)

Inserting (45) into (44),

$$\lambda_t = \int_{e_{t+1}^{z,*}}^{+\infty} \left\{ \int_{e_{t+2}^{z,*}}^{+\infty} \lambda_{t+2} dF(e_{t+2}^z)(1-\gamma) + \frac{\gamma}{1-\gamma} \right\} dF(e_{t+1}^z)(1-\gamma) + \frac{\gamma}{1-\gamma}.$$
(46)

Solving,

$$\lambda_t = \int_{e_{t+1}^{z,*}}^{+\infty} \int_{e_{t+2}^{z,*}}^{+\infty} \lambda_{t+2} dF(e_{t+2}^z) dF(e_{t+1}^z) (1-\gamma)^2 + \frac{\gamma}{1-\gamma} (1-F(e_{t+1}^{z,*}))(1-\gamma) + \frac{\gamma}{1-\gamma}.$$
 (47)

In general, for n=1, 2, 3, ...,

$$\lambda_{t} = \int_{e_{t+1}^{+\infty}}^{+\infty} \int_{e_{t+2}^{z,*}}^{+\infty} \dots \int_{e_{t+n}^{z,*}}^{+\infty} \lambda_{t+2} dF(e_{t+n}^{z}) \dots dF(e_{t+2}^{z}) dF(e_{t+1}^{z}) (1-\gamma)^{n} + \frac{\gamma}{1-\gamma} + \frac{\gamma}{1-\gamma} (1-F(e_{t+1}^{z,*})) (1-\gamma) + \frac{\gamma}{1-\gamma} \int_{e_{t+1}^{z,*}}^{+\infty} (1-F(e_{t+2}^{z,*})) dF(e_{t+2}^{z}) dF(e_{t+1}^{z}) (1-\gamma)^{2} + \frac{\gamma}{1-\gamma} \int_{e_{t+1}^{z,*}}^{+\infty} \int_{e_{t+2}^{z,*}}^{+\infty} (1-F(e_{t+3}^{z,*})) dF(e_{t+2}^{z}) dF(e_{t+1}^{z}) (1-\gamma)^{3} + \dots + \frac{\gamma}{1-\gamma} \int_{e_{t+1}^{z,*}}^{+\infty} \dots \int_{e_{t+n-2}^{z,*}}^{+\infty} (1-F(e_{t+n-1}^{z,*})) dF(e_{t+n-2}^{z}) \dots dF(e_{t+1}^{z}) (1-\gamma)^{n-1}, \quad (48)$$

where for m=1, 2, 3, ...,

$$e_{t+m}^{z,*} = \log\left(\left((R^B - (1-\delta))K_{t+m-1} - (R^B - R^F)\phi - R^B N_{t+m-1}\right) / K_{t+m-1}^{\alpha}\right) - \mu_z + \rho_z^{m+1}\mu_z - \rho_z^{m+1}\log(Z_{t-1}) - \sum_{i=1}^m \rho_z^i e_{t+m-i}.$$

For  $n=\infty$ , the first term of the right-hand side of equation (48) converges to zero. Then, (48) becomes,

$$\lambda_{t} = \frac{\gamma}{1-\gamma} + \frac{\gamma}{1-\gamma} (1 - F(e_{t+1}^{z,*}))(1-\gamma) + \frac{\gamma}{1-\gamma} \int_{e_{t+1}^{z,*}}^{+\infty} (1 - F(e_{t+2}^{z,*})) dF(e_{t+1}^{z})(1-\gamma)^{2} + \frac{\gamma}{1-\gamma} \int_{e_{t+1}^{z,*}}^{+\infty} \int_{e_{t+2}^{z,*}}^{+\infty} (1 - F(e_{t+3}^{z,*})) dF(e_{t+2}^{z}) dF(e_{t+1}^{z})(1-\gamma)^{3} + \dots + \frac{\gamma}{1-\gamma} \int_{e_{t+1}^{z,*}}^{+\infty} \dots \int_{e_{t+n-2}^{z,*}}^{+\infty} (1 - F(e_{t+n-1}^{z,*})) dF(e_{t+n-2}^{z}) \dots dF(e_{t+1}^{z})(1-\gamma)^{n-1}.$$
(49)

Next, I prove that  $\lambda_t < \frac{1}{1-\gamma}$ . To do this I evaluate each term of the right-hand side from equation (49): It is easy to notice that,

$$\frac{\gamma}{1-\gamma} (1 - F(e_{t+1}^{z,*}))(1-\gamma) < \gamma,$$
$$\frac{\gamma}{1-\gamma} \int_{e_{t+1}^{z,*}}^{+\infty} \left(1 - F(e_{t+2}^{z,*})\right) dF(e_{t+1}^z)(1-\gamma)^2 < \gamma(1-\gamma),$$

and so on. In general,  $\forall \ n \geq 3$ ,

$$\frac{\gamma}{1-\gamma} \int_{e_{t+1}^{z,*}}^{+\infty} \dots \int_{e_{t+n-2}^{z,*}}^{+\infty} \left(1 - F(e_{t+n-1}^{z,*})\right) dF(e_{t+n-2}^{z}) \dots dF(e_{t+1}^{z}) (1-\gamma)^{n-1} < \gamma (1-\gamma)^{n-2}.$$
(50)

Finally, by (50), it is easy to see that,

$$\lambda_t < \frac{\gamma}{1 - \gamma} \left( 1 + (1 - \gamma) + (1 - \gamma)^2 + \dots \right) = \frac{1}{1 - \gamma}.$$
 (51)

## Appendix C Competitive equilibrium: Unlimited liability

Is the allocation under unlimited liability the same to the allocation of the domestic social planner equilibrium, in which the domestic social planner aims to maximize domestic households' utility subject to the foreign borrowing limit? Yes, it is the same. Hence, the allocation under unlimited liability is going to be efficient. Next, I show the maximization problem of banks under unlimited liability and the corresponding equilibrium conditions, and compare these with the social planner equilibrium. By definition the dynamics of banks' net worth is given by,

$$N_{t+1} = N_t + NOI_{t+1} - d_{t+1}.$$
(52)

Under unlimited liability banks always honor their obligations. When the net operating income plus the banks' net worth at the beginning of the period t+1 is negative or, equivalently, when the total banks' obligations are higher than the total available income of the banks, i.e.,

$$NOI_{t+1} + N_t < 0, \qquad or \qquad (1 - \delta)K_t + Z_{t+1}K_t^{\alpha} < \bar{R}_t^D D_t + \bar{R}_t^F D_t^F, \tag{53}$$

banks' owners have to honor the unpaid deposits,  $\bar{R}_t^D D_t + \bar{R}_t^F D_t^F - (1-\delta)K_t - Z_{t+1}K_t^{\alpha}$ , so that domestic and foreign depositors always get the agreed gross return for their deposits. In other words, when (53) holds, banks transfer negative dividends to owners by the amount of,

$$d_{t+1} = NOI_{t+1} + N_t = (1 - \delta)K_t + Z_{t+1}K_t^{\alpha} - \bar{R}_t^D D_t - \bar{R}_t^F D_t^F < 0,$$
(54)

and hence from (52) this yields to zero net worth at the end of the period, i.e.,  $N_{t+1}=0$ . When the banks' total revenues are higher than total obligations, i.e., when (53) does not hold, I assume that the dividends paid to banks owners (domestic households) follow the same law of motion than under limited liability and deposit insurance, i.e.,

$$d_{t+1} = \gamma \left( NOI_{t+1} + N_t \right). \tag{55}$$

Inserting (55) into (52) I obtain the law of motion of the bank's net worth,

$$N_{t+1} = (1 - \gamma) \left( NOI_{t+1} + N_t \right).$$
(56)

In general, the dividends and net worth are given respectively by,

$$d_{t} = g_{1,t}(Z_{t}, D_{t-1}, D_{t-1}^{F}, N_{t-1}) = \begin{cases} \gamma (NOI_{t} + N_{t-1}) & if (53) \ doesn't \ hold, \\ NOI_{t} + N_{t-1} & if (53) \ holds, \end{cases}$$
(57)

$$N_{t} = g_{2,t}(Z_{t}, D_{t-1}, D_{t-1}^{F}, N_{t-1}) = \begin{cases} (1-\gamma) (NOI_{t} + N_{t-1}) & if (53) \ doesn't \ hold, \\ 0 & if (53) \ holds. \end{cases}$$
(58)

Banks seek to maximize (8) subject to the dynamics of bank's net worth, (58), where bank's credit (capital) and bank's divided are defined by the equations (5) and (57) respectively. I continue assuming that the foreign borrowing limit, (6), binds in equilibrium.

The Lagrangian is,

$$L_{t} = \mathbb{E}_{t} \left\{ \sum_{i=t}^{\infty} \beta^{i-t} \left[ g_{1,i} + \lambda_{i} \left( g_{2,i}(Z_{i}, D_{i-1}, D_{i-1}^{F}, N_{i-1}) - N_{i} \right) \right] \right\},\$$

where  $\lambda_t$  is the Lagrange multiplier associated with the dynamics of bank's net worth, equation (58). For convenience, I rewrite the Lagrangian as,

$$L_t = g_{1,t} + \lambda_t (g_{2,t} - N_t) + \beta \mathbb{E}_t \{ g_{1,t+1} + \lambda_{t+1} (g_{2,t+1} - N_{t+1}) \} + \beta^2 \mathbb{E}_t \{ L_{t+2} \},$$

where,

$$\mathbb{E}_{t} \left\{ g_{1,t+1} + \lambda_{t+1} (g_{2,t+1} - N_{t+1}) \right\} = \int_{e_{t+1}^{z,*}}^{+\infty} \gamma(NOI_{t+1} + N_{t}) dF + \int_{-\infty}^{e_{t+1}^{z,*}} (NOI_{t+1} + N_{t}) dF + \int_{e_{t+1}^{z,*}}^{+\infty} \lambda_{t+1} (1 - \gamma) (NOI_{t+1} + N_{t}) dF - \mathbb{E}_{t} \{\lambda_{t+1}N_{t+1}\}.$$

The first order condition for  $D_t$  yields,

$$0 = \beta \int_{e_{t+1}^{z,*}}^{+\infty} \gamma (1 - \delta + \alpha Z_{t+1} K_t^{\alpha - 1} - \bar{R}_t^D) dF + \beta \int_{-\infty}^{e_{t+1}^{z,*}} (1 - \delta + \alpha Z_{t+1} K_t^{\alpha - 1} - \bar{R}_t^D) dF + \beta \int_{e_{t+1}^{z,*}}^{+\infty} \lambda_{t+1} (1 - \gamma) (1 - \delta + \alpha Z_{t+1} K_t^{\alpha - 1} - \bar{R}_t^D) dF.$$
(59)

The first order condition for  $N_t$  yields,

$$0 = -\lambda_{t} + \beta \int_{e_{t+1}^{z,*}}^{+\infty} \gamma (1 - \delta + \alpha Z_{t+1} K_{t}^{\alpha - 1}) dF + \beta \int_{-\infty}^{e_{t+1}^{z,*}} (1 - \delta + \alpha Z_{t+1} K_{t}^{\alpha - 1}) dF + \beta \int_{e_{t+1}^{z,*}}^{+\infty} \lambda_{t+1} (1 - \gamma) (1 - \delta + \alpha Z_{t+1} K_{t}^{\alpha - 1}) dF.$$
(60)

Since there is unlimited liability, the interest rates of domestic and foreign deposits are free of risk. Hence, in equilibrium it must hold that,

$$\bar{R}_t^D = 1/\beta, \qquad \bar{R}_t^F = R^F.$$
(61)

Since in equilibrium  $\bar{R}_t^D = 1/\beta$ , from (59) and (60),

$$0 = -\lambda_t + \int_{e_{t+1}^{z,*}}^{+\infty} \gamma dF + \int_{-\infty}^{e_{t+1}^{z,*}} dF + \int_{e_{t+1}^{z,*}}^{+\infty} \lambda_{t+1} (1-\gamma) dF.$$
(62)

Interestingly, Appendix D shows that in equilibrium  $\lambda_t=1$  and hence it does not depend

on the state of nature at time t. Thus, the first order condition of  $D_t$  becomes,

$$0 = \beta \left( 1 - \delta + \alpha \mathbb{E}_t \{ Z_{t+1} \} K_t^{\alpha - 1} - 1/\beta \right).$$

Comparing with equation (27) banks' credit (capital) under unlimited liability in this multiperiod model is socially efficient. Under unlimited liability, banks' owners receive only a fraction of  $NOI_t + N_t$  when the latter is positive; however, they receive the full  $NOI_t + N_t$  when this is negative. This means that banks are fully internalizing the losses in the bad states of nature and also fully internalizing the profits in the good state of natures. Note also that the allocation of capital is independent of  $\gamma$ . In order words, a change of  $\gamma$  does not produce any distortion.

In this framework the stochastic steady state values of bank's credit (capital), domestic deposits and bank's net worth are described by the following system of equations,

$$K_{sss} = \left(\alpha \frac{\mathbb{E}_{sss}\{Z\}}{(1/\beta) - (1 - \delta)}\right)^{\frac{1}{1 - \alpha}},$$

$$N_{sss} = (1 - \gamma) \left[(1 - \delta)K_{sss} + Z_{sss}K_{sss}^{\alpha} - (1/\beta)D_{sss} - R^{F}\phi\right],$$

$$K_{sss} = D_{sss} + \phi + N_{sss},$$
(63)

where,

$$E_{sss}\{Z\} = exp\left(\mu_z(1-\rho_z) + \rho_z ln(Z_{sss}) + 0.5\sigma_{e^z}^2\right).$$

Note that  $\gamma$  pins down the bank's liability composition and therefore the bank's leverage ratio. The SSS values for the other variables are shown in Appendix E.

#### Appendix D Unlimited liability: $\lambda_t$

Recall (62),

$$0 = -\lambda_t + \int_{e_{t+1}^{z,*}}^{+\infty} \gamma dF + \int_{-\infty}^{e_{t+1}^{z,*}} dF + \int_{e_{t+1}^{z,*}}^{+\infty} \lambda_{t+1} (1-\gamma) dF.$$

For convenience, I rewrite it as,

$$\lambda_t = 1 - (1 - \gamma) + (1 - \gamma)F(e_{t+1}^{z,*}) + \int_{e_{t+1}^{z,*}}^{+\infty} \lambda_{t+1}(1 - \gamma)dF.$$
(64)

I follow the repeated forward substitution method in order to solve it. I, first, shift (64) one period ahead:

$$\lambda_{t+1} = \gamma + (1-\gamma)F(e_{t+2}^{z,*}) + \int_{e_{t+2}^{z,*}}^{+\infty} \lambda_{t+2}(1-\gamma)dF.$$
(65)

Inserting (65) into (64),

$$\lambda_{t} = 1 - (1 - \gamma)^{2} + (1 - \gamma)^{2} F(e_{t+1}^{z,*}) + (1 - \gamma)^{2} \int_{e_{t+1}^{z,*}}^{+\infty} F(e_{t+2}^{z,*}) dF(e_{t+1}^{z,*}) + (1 - \gamma)^{2} \int_{e_{t+1}^{z,*}}^{+\infty} \int_{e_{t+2}^{z,*}}^{+\infty} \lambda_{t+2} dF(e_{t+2}^{z}) dF(e_{t+1}^{z}).$$
(66)

The procedure is repeated by forwarding (64) two periods ahead, then inserting the expression for  $\lambda_{t+2}$  into (66), we get,

$$\begin{aligned} \lambda_t &= 1 - (1 - \gamma)^3 + (1 - \gamma)^3 F(e_{t+1}^{z,*}) + (1 - \gamma)^3 \int_{e_{t+1}^{z,*}}^{+\infty} F(e_{t+2}^{z,*}) dF(e_{t+1}^z) \\ &+ (1 - \gamma)^3 \int_{e_{t+1}^{z,*}}^{+\infty} \int_{e_{t+2}^{z,*}}^{+\infty} F(e_{t+3}^{z,*}) dF(e_{t+2}^z) dF(e_{t+1}^z) \\ &+ (1 - \gamma)^3 \int_{e_{t+1}^{z,*}}^{+\infty} \int_{e_{t+2}^{z,*}}^{+\infty} \int_{e_{t+3}^{z,*}}^{+\infty} \lambda_{t+3} dF(e_{t+3}^z) dF(e_{t+2}^z) dF(e_{t+1}^z). \end{aligned}$$

I continue in this way and the general form (for n=1,2,3,...) becomes,

$$\lambda_{t} = 1 - (1 - \gamma)^{n} + (1 - \gamma)^{n} F(e_{t+1}^{z,*}) + \sum_{i=1}^{n-1} \left\{ (1 - \gamma)^{n} \int_{e_{t+1}^{z,*}}^{+\infty} \dots \int_{e_{t+i}^{z,*}}^{+\infty} F(e_{t+i+1}^{z,*}) dF(e_{t+i}^{z}) \dots dF(e_{t+1}^{z}) \right\} + (1 - \gamma)^{n} \int_{e_{t+1}^{z,*}}^{+\infty} \int_{e_{t+2}^{z,*}}^{+\infty} \dots \int_{e_{t+n}^{z,*}}^{+\infty} \lambda_{t+n} dF(e_{t+n}^{z}) \dots dF(e_{t+2}^{z}) dF(e_{t+1}^{z}).$$
(67)

Letting  $n \rightarrow +\infty$ , the marginal value of bank's net worth becomes,

$$\lambda_t = 1. \tag{68}$$

#### Appendix E Stochastic steady state

Once it is known the values  $K_{sss}$ ,  $N_{sss}$  and  $D_{sss}$ , we can obtain the values for the rest of variables,

$$I_{sss} = \delta K_{sss},$$

$$C_{sss} = \omega^H - I_{sss} + Z_{sss} K^{\alpha}_{sss} - (R^F - 1)\phi,$$
(69)

$$GDP_{sss} = G + Y_{sss},$$
$$Y_{ssss} = \omega^{H} + Z_{sss}K^{\alpha}_{sss},$$
$$CA_{sss} = 0,$$
$$NFA_{sss} = -\phi.$$
$$d_{sss} = NOI_{sss} = \frac{\gamma}{1 + \gamma}N_{sss}.$$

Note that in the stochastic steady state the bank's dividends are the same to the net operating income in order to not increase or decrease the level of bank's net worth.

# **Appendix F** Proof of the expression for $\frac{\partial e_{t+1}^{z,*}}{\partial e_t^z}$

Recall that  $e_{t+1}^{z,*}$  is solved in:

$$max\{Z_{t+1}^*, 0\} = exp(\mu_z(1-\rho_z) + \rho_z ln(Z_t) + e_{t+1}^{z,*}),$$

where  $Z_{t+1}^*$  satisfies,

$$(1-\delta)K_t + Z_{t+1}^* K_t^{\alpha} = \bar{R}_t^D D_t + \bar{R}_t^F D_t^F,$$
(70)

If  $Z_{t+1}^* \leq 0$ , then,

$$\frac{\partial e_{t+1}^{z,*}}{\partial e_t^z} = 0,$$

otherwise, if  $Z_{t+1}^* \ge 0$ , which is the interesting case, then,

$$\frac{\partial e_{t+1}^{z,*}}{\partial e_t^z} = \frac{\partial ln(Z_{t+1}^*)}{\partial e_t^z} - \rho_z \frac{\partial ln(Z_t)}{\partial e_t^z},\tag{71}$$

From (70),

$$\frac{\partial ln(Z_{t+1}^*)}{\partial e_t^z} = \left(\frac{\bar{R}_t^D - (1-\delta)}{(\bar{R}_t^D - (1-\delta))K_t - \phi(\bar{R}_t^D - \bar{R}_t^F) - \bar{R}_t^D N_t} - \frac{\alpha}{K_t}\right) \frac{\partial K_t}{\partial e_t^z} - \frac{\bar{R}_t^D}{(\bar{R}_t^D - (1-\delta))K_t - \phi(\bar{R}_t^D - \bar{R}_t^F) - \bar{R}_t^D N_t} \frac{\partial N_t}{\partial e_t^z}.$$
(72)

And, since  $ln(Z_t) = \mu_z(1 - \rho_z) + \rho_z log(Z_{t-1}) + e_t^z$ ,

$$\frac{\partial ln(Z_t)}{\partial e_t^z} = 1. \tag{73}$$

Inserting (72) and (73) into (71),

$$\frac{\partial e_{t+1}^{z,*}}{\partial e_t^z} = \omega_{K_t} \frac{\partial K_t}{\partial e_t^z} - \omega_{N_t} \frac{\partial N_t}{\partial e_t^z} - \rho_z, \tag{74}$$

where

$$\omega_{K_t} = \left(\frac{R_t^D - (1 - \delta)}{(\bar{R}_t^D - (1 - \delta))K_t - \phi(\bar{R}_t^D - \bar{R}_t^F) - \bar{R}_t^D N_t} - \frac{\alpha}{K_t}\right) > 0,$$
$$\omega_{N_t} = \frac{\bar{R}_t^D}{(\bar{R}_t^D - (1 - \delta))K_t - \phi(\bar{R}_t^D - \bar{R}_t^F) - \bar{R}_t^D N_t} > 0.$$

Furthermore, taking the patial derivative to  $N_t$  with respect to  $e_t^z$ ,

$$\frac{\partial N_t(Z_t, K_{t-1}, N_{t-1})}{\partial e_t^z} = (1-\gamma) \frac{\partial Z_t}{\partial e_t^z} K_{t-1}^\alpha = (1-\gamma) Z_t K_{t-1}^\alpha, \tag{75}$$

And, taking the partial derivative to  $K_t$  with respect to  $e_t^z$ ,

$$\frac{\partial K_t(Z_t, N_t)}{\partial e_t^z} = \frac{\partial Z_t^{\rho_z}}{\partial Z_t} \frac{\partial K_t}{\partial Z_t^{\rho_z}} + K_{N_t} \frac{\partial N_t}{\partial e_t^z},$$
$$\frac{\partial K_t(Z_t, N_t)}{\partial e_t^z} = \rho_z Z_t^{\rho_z} K_{Z_t^{\rho_z}} + K_{N_t} \frac{\partial N_t}{\partial e_t^z},$$
(76)

where  $K_{Z_t^{\rho_z}} > 0$  and  $K_{N_t} < 0$  are the partial derivatives of  $K_t$  with respect to  $Z_t$  and  $N_t$ , respectively. These means that it is expected that a higher initial equity reduces the default probability of banks, which in turn reduces the excess risk-taking incentives and credit; and also that a higher productivity ceteris paribus increases the marginal productivity of capital and hence pushing credit up. Inserting equations (75-76) into equation (74),

$$\frac{\partial e_{t+1}^{z,*}}{\partial e_t^z} = \omega_{K_t} \left( \rho_z Z_t^{\rho_z} K_{Z_t^{\rho_z}} + K_{N_t} \frac{\partial N_t}{\partial e_t^z} \right) - \omega_{N_t} \frac{\partial N_t}{\partial e_t^z} - \rho_z$$

Solving,

$$\frac{\partial e_{t+1}^{z,*}}{\partial e_t^z} = \omega_{K_t} \rho_z Z_t^{\rho_z} K_{Z_t^{\rho_z}} + \left(\omega_{K_t} K_{N_t} - \omega_{N_t}\right) \frac{\partial N_t}{\partial e_t^z} - \rho_z,$$

where  $\omega_{K_t} K_{N_t} - \omega_{N_t} < 0.$ 

### Appendix G Calibration of $\delta$

Recall that due to the limited liability banks default at t + 1 if,

$$(1-\delta)K_t + Z_{t+1}K_t^{\alpha} \le R^B D_t + R^F \phi_t$$

Also, recall that in order to have a binding limited liability, which is of the interest of this paper, I need to calibrate the model so that at t in equilibrium there is a positive default probability of banks in t+1. This means that I calibrate the model so that in the worst state of nature, i.e.,  $Z_{t+1}=0$ , banks' revenues are lower than banks' obligations, i.e.,

$$(1-\delta)K_t \le R^B D_t + R^F \phi_t. \tag{77}$$

I rewrite (77) as,

$$\delta \ge 1 - R^B + (R^B - R^F) \frac{\phi}{K_t} + R^B \frac{N_t}{K_t}.$$
(78)

Hence, the capital depreciation rate is lower bounded. This lower bound depends on the banks' leverage ratio,  $K_{sss}/N_{sss}$ , and foreign debt to credit ratio,  $\phi/K_{sss}$ . It implies that if we have as targets the leverage ratio,  $K_t/N_t$ , and the foreign debt ratio,  $\frac{\phi}{K_t-N_t}$ , then we are affecting the possible values that  $\delta$  could take in order to have a binding limited liability. In particular, the lower the desired leverage ratio, the higher the lower bound for  $\delta$ .

From equation (36),

$$N_{sss} = \frac{(1-\gamma)}{1-(1-\gamma)(1/\beta)} \left[ (1-\delta - 1/\beta)K_{sss} + Z_{sss}K_{sss}^{\alpha} + (1/\beta - R^F)\phi \right].$$
(79)

In general, in equilibrium capital can be written as,

$$K_{sss} = \left(\frac{\alpha Z_{sss}^+}{1/\beta - (1-\delta)}\right)^{\frac{1}{1-\alpha}},\tag{80}$$

where,

$$Z_{sss}^{+} = \frac{\int_{e^{z,*}}^{\infty} \lambda Z f(e^{z}) d(e^{z})}{\int_{e^{z,*}}^{\infty} \lambda f(e^{z}) d(e^{z})}$$

Inserting (80) into (79),

$$\frac{N_{sss}}{K_{sss}} = \frac{(1-\gamma)}{1-(1-\gamma)(1/\beta)} \left[ (1/\beta - 1 + \delta) \left( \frac{1}{\alpha} \frac{Z_{sss}}{Z_{sss}^+} - 1 \right) + (1/\beta - R^F) \frac{\phi}{K_{sss}} \right], \quad (81)$$

Since I am assuming a small default probability,  $\frac{1}{\alpha} \frac{Z_{sss}}{Z_{sss}^+} > 1$ . Hence, ceteris paribus, a higher  $\delta$  reduces the leverage ratio,  $K_{sss}/N_{sss}$ . Also note that since  $\frac{1}{\alpha} \frac{Z_{sss}}{Z_{sss}^+} > 1$ ,  $(1 - \delta - 1/\beta)K_{sss} + Z_{sss}K_{sss}^{\alpha} + (1/\beta - R^F)\phi > 0$ 

From (16) with variables at their stochastic steady state values,

$$(1 - \delta)K_{sss} + Z^*_{sss}(K_{sss})^{\alpha} = (1/\beta)D_{sss} + R^F\phi.$$
 (82)

Inserting (80) into (82) it can be obtained,

$$\frac{1}{\alpha} \frac{Z_{sss}^*}{Z_{sss}^+} = \left(1 - \frac{1/\beta - R^F}{1/\beta - (1 - \delta)} \frac{\phi}{K_{sss}} - \frac{N_{sss}}{K_{sss}} \frac{1/\beta}{1/\beta - (1 - \delta)}\right).$$
(83)

Also, ceteris paribus a higher  $\delta$  leads to smaller leverage ratio,  $K_{sss}/N_{sss}$ .

#### Appendix H Calibration of $\sigma_{e^z}$

Recall from equation (83),

$$\frac{1}{\alpha} \frac{Z_{sss}^*}{Z_{sss}^+} = \left(1 - \frac{1/\beta - R^F}{1/\beta - (1 - \delta)} \frac{\phi}{K_{sss}} - \frac{N_{sss}}{K_{sss}} \frac{1/\beta}{1/\beta - (1 - \delta)}\right).$$

Rewriting  $\frac{1}{\alpha} \frac{Z_{sss}^*}{Z_{sss}^+}$  as  $\frac{Z_{sss}^*}{Z_{sss}} \frac{1}{\alpha} \frac{Z_{sss}}{Z_{sss}^+}$  and taking the logarithm,

$$ln(Z_{sss}^{*}) = \mu_{z} + ln\left(\frac{\alpha Z_{sss}^{+}}{Z_{sss}}\right) + ln\left(\Sigma\right)$$

where,

$$\Sigma = 1 - \frac{1/\beta - R^F}{1/\beta - (1 - \delta)} \frac{\phi}{K_{sss}} - \frac{N_{sss}}{K_{sss}} \frac{1/\beta}{1/\beta - (1 - \delta)},$$

and from (81) solving for  $Z_{sss}^+/(\alpha Z_{sss})$ ,

$$\frac{1}{\alpha} \frac{Z_{sss}}{Z_{sss}^+} = \frac{1}{(1/\beta - 1 + \delta)} \left( \frac{N_{sss}}{K_{sss}} \frac{1 - (1 - \gamma)(1/\beta)}{(1 - \gamma)} - (1/\beta - R^F) \frac{\phi}{K_{sss}} \right) + 1.$$

Then,

$$e_{sss}^{z,*} = ln(Z_{sss}^*) - \mu_z(1-\rho_z) - \rho_z ln(Z_{sss}) = \left[ln\left(\frac{\alpha Z_{sss}^+}{Z_{sss}}\right) + ln\left(\Sigma\right)\right].$$

Finally,

$$\frac{e_{sss}^{z,*}}{\sigma_{e^{z}}} = \frac{1}{\sigma_{e^{z}}} \left[ ln \left( \frac{\alpha Z_{sss}^{+}}{Z_{sss}} \right) + ln \left( \Sigma \right) \right].$$

Hence, the default probability of banks,  $\Phi(\frac{\ln(Z_{sss}^*)}{\sigma_{e^z}})$ , where  $\Phi$  is the cdf of the standard normal, is defined as a function of the parameters  $\{\beta, R^F, \delta, \gamma, \sigma_{e^z}\}$  and the leverage ratio,  $K_{sss}/N_{sss}$  and the foreign debt participation on credit,  $\phi/K_{sss}$ . As a result, for a given set of parameters  $\{\beta, R^F, \delta, \gamma, \sigma_{e^z}, \rho_z\}$ , and for given targets of the leverage ratio and foreign debt to credit ratio, I calibrate the volatility of the shock,  $\sigma_{e^z}$ , to match the target default probability.

#### **Appendix I** Calibration of $\gamma$

Recall equation (36),

$$\frac{N_{sss}}{K_{sss}} = \frac{(1-\gamma)}{1-(1-\gamma)(1/\beta)} \left[ (1/\beta - 1 + \delta) \left( \frac{1}{\alpha} \frac{Z_{sss}}{Z_{sss}^+} - 1 \right) + (1/\beta - R^F) \frac{\phi}{K_{sss}} \right].$$
(84)

Since the default probability of bank is very small,

$$Z_{sss} \simeq Z_{sss}^+$$

Inserting it into (84),

$$\frac{1}{1-\gamma} \simeq (1-\delta)\frac{K_{sss}}{N_{sss}} + \frac{1}{\alpha}\frac{K_{sss}}{N_{sss}}(1/\beta - (1-\delta)) - (1/\beta)\frac{D_{sss}}{N_{sss}} - R^F \frac{\phi}{N_{sss}}$$

Since  $D_{sss} = K_{sss} - N_{sss} - \phi$ , the expression becomes,

$$\frac{1}{1-\gamma} \simeq (1/\beta - R^F) \frac{\phi}{N_{sss}} + (1/\beta) + \frac{K_{sss}}{N_{sss}} \left(\frac{1}{\alpha} - 1\right) (1/\beta - 1 + \delta).$$

Solving for  $\gamma$ ,

$$\gamma \simeq 1 - \frac{1}{\frac{K_{sss}}{N_{sss}} \left( (1/\beta - R^F) \frac{\phi}{K_{sss}} + (1/\alpha - 1)(1/\beta - 1 + \delta) \right) + (1/\beta)}.$$

This means that for given targets of the leverage ratio,  $K_{sss}/N_{sss}$ , and the foreign debt to credit ratio,  $\phi/K_{sss}$ , we can approximately calibrate  $\gamma$ . For instance, for a given level of foreign debt to credit ratio and taking as given the rest of the parameters a higher  $\gamma$ is expected to lead to higher leverage.

## Appendix J Why is it necessary to assume $\omega^H > 0$ and/or G > 0 to match the desired credit to GDP ratio?

Assuming  $\omega^{H}=0$  and G=0, the stochastic steady state of the credit to GDP ratio is given by  $\frac{K_{sss}}{Z_{sss}K_{sss}^{\alpha}}$ . Using (80) it can be rewritten as,

$$\frac{K_{sss}}{Z_{sss}K_{sss}^{\alpha}} = \frac{\alpha}{1/\beta - 1 + \delta} \frac{Z_{sss}^+}{Z_{sss}}$$

Since the default probability is small,  $\delta$  governs this ratio. Next, I show that in this case (when  $\omega^H=0$  and G=0) it is not feasible to calibrate  $\delta$  in order to match the target credit

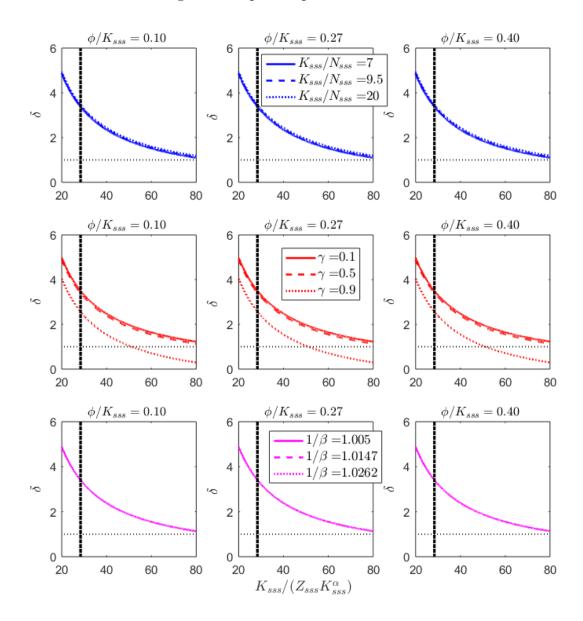
to GDP ratio of 29%. Dividing by  $K_{sss}$  equation (79)

$$\frac{N_{sss}}{K_{sss}} = \frac{(1-\gamma)}{1-(1-\gamma)(1/\beta)} \left[ (1-\delta-1/\beta) + \frac{Z_{sss}K_{sss}^{\alpha}}{K_{sss}} + (1/\beta - R^F)\frac{\phi}{K_{sss}} \right].$$
 (85)

Equation (85) shows the relationship of the leverage ratio,  $K_{sss}/N_{sss}$ , the credit to GDP ratio,  $K_{sss}/(Z_{sss}K_{sss}^{\alpha})$ , and the foreign debt to credit ratio of the model in the stochastic steady sate.

Figure 7 plots  $\delta$  founds in equation (85) as a function of the credit to GDP ratio,  $K_{sss}/(Z_{sss}K_{sss}^{\alpha})$ . This relationship is shown for different values of the foreign debt to credit ratio, leverage ratio,  $K_{sss}/N_{sss}$ ,  $\gamma$  and  $\beta$ . Recall in the baseline calibration,  $\phi/K_{sss} = 27\%$ ,  $K_{sss}/N_{sss} = 9.5$ ,  $\gamma = 0.50$  and  $1/\beta = 1.047$ . The vertical dotted black lines indicate the target of the credit to GDP ratio, 28.39%.

The figure concludes that in order to match this desired ratio it is required an unrealistic high level of the capital depreciation rate (i.e., higher than 1). Hence, in order to match the credit to GDP ratio of the data it is necessary to include exogenous components in the GDP, i.e. to assume  $\omega^H > 0$  and/or G > 0.



#### Figure 7: Capital depreciation rate: $\delta$

#### Appendix K Recursive Competitive Equilibrium

The model is solved using the value function iteration method. To this end, first, I rewrite banks' problem in its recursive form with two state variables, the initial level of bank' net worth, N, and the current level of productivity level, Z. Bankers need to choose the level of credit, K. The maximization problem of the banker can then be rewritten as,

$$V(Z, N) = \max_{K'} \beta \mathbb{E}_{Z'|Z} \{ \gamma / (1 - \gamma) N'(K, Z', N) + V(Z', N') \},\$$

where,

$$N' = (1 - \gamma) \left[ (1 - \delta)K + Z'(K)^{\alpha} - 1/\beta (K - \phi - N) - R^F \phi \right]^+,$$

subject to foreign borrowing limit, where V is the present value of futures dividends. The optimal decision rule of credit, denoted as  $\hat{K}(Z, N)$ , solves the recursive optimization problem of the households.

Second, I derive the optimal policy function  $\hat{K}(Z, N)$ . To compute this I discretize the endogenous state variable N using a grid with 92 non-equidistant points and K using a grid with 4253 non-equidistant points. In particular, I accumulate more points where it is expected to observe a kink or a curvature in the policy functions. The productivity process is approximated using the quadrature procedure following Tauchen (1998) and Tauchen & Hussey (1991) for 251 points. To improve accuracy by using spline interpolations on the N grid.

Finally, to compute the stochastic steady states of bank loans and bank net worth I look for the long-term values of these obtained from the policy function,  $\hat{K}(Z, N)$ , and the law of motion of bank net worth. To do so I start with some arbitrary initial values of credit and net worth and assume that the realized productivity shocks are zero, i.e. Z' = Z. I use the spline interpolation method to evaluate the policy function at points that are not in the grid.

#### Appendix L Policy Functions

Figure 8 plots the policy functions of bank credit,  $\hat{K}(Z_{sss}, N)$ , and default probability,  $\hat{p}(Z_{sss}, N)$ , in the social planner equilibrium, competitive equilibrium and the competitive equilibrium abstracting from the intertemporal channel, for  $Z=Z_{sss}$ . The figure reports the solutions under the baseline calibration and other calibrations discussed in the robustness section.

In the social planner equilibrium, from equation (28) the level of credit is independent of the initial level of net worth. As a result, for a given Z, the policy function (the dashed black line) is fully horizontal. However, the probability that gross profits  $NOI_{t+1}+N_t$ become negative (or the default probability in a regulated economy) increases with a lower net worth.

In contrast, from equations (20-21) in the competitive equilibrium, bank credit is not independent of the initial equity. This is because banks' net worth affects the default probability of banks, which in turn alters bank credit and risk-taking decisions. In particular, the lower the initial net worth, the lower the capacity of the bank to absorb losses in the next period and hence the larger the probability that in the next period banks default (see right plots in figure 8), and hence the higher the banks' incentive to take excessive risk and then the higher the level of credit (see left plots in figure 8).

Note that in the baseline calibration (figure 8.a) abstracting from the intertemporal

effect leads to smaller default probability, i.e., the solid red line is above the dashed blue line. This is due to the amplification effects of the distortions due to the intertemporal effect. Consequently, bank credit is higher when considering the intertemporal effect. Also, notice that the when net worth is high enough so that next period default probability is zero, the credit level is still inefficiently high (i.e., in the left plot the solid red line is above the black dashed line). This is because the default probabilities in n periods ahead for n > 1 are not necessarily zero. The higher the N, the smaller the difference of the credit in the CE and the SP equilibrium.

These results are robust, for instance, when net worth accumulation is slower, i.e.,  $\gamma=0.54$  (see figure 8.b) or when the productivity shock is not persistent, i.e.,  $\rho_z=0$ , (see figure 8.c). However, figure 8.d shows that for a high enough persistence of the shock,  $\rho_z=0.78$ , when  $Z=Z_{sss}$  and for high enough values of N, bank credit is inefficiently low, i.e., the solid red line is below the dotted black line, and when N increases, which leads to smaller default probability, bank credit converges to the efficient level.

#### Appendix M Recursive Equilibrium: Simulation

The dynamics of the sudden stop simulation is solved using the value function iteration method. To this end, I rewrite banks' problem in its recursive form with three state variables, the initial bank net worth, N, the current productivity level, Z, and the current size of foreign deposits,  $\phi$ . Recall that since the foreign borrowing limit is always binding foreign deposits equal the foreign borrowing limit,  $\phi$ . Bankers need to choose the future level of credit, K. The maximization problem of the banker can then be rewritten as,

$$V(Z, N, \phi) = \max_{K'} \beta \mathbb{E}_{Z'|Z} \{ \gamma / (1 - \gamma) N'(K, Z', N, \phi) + V(Z', N', \phi') \},\$$

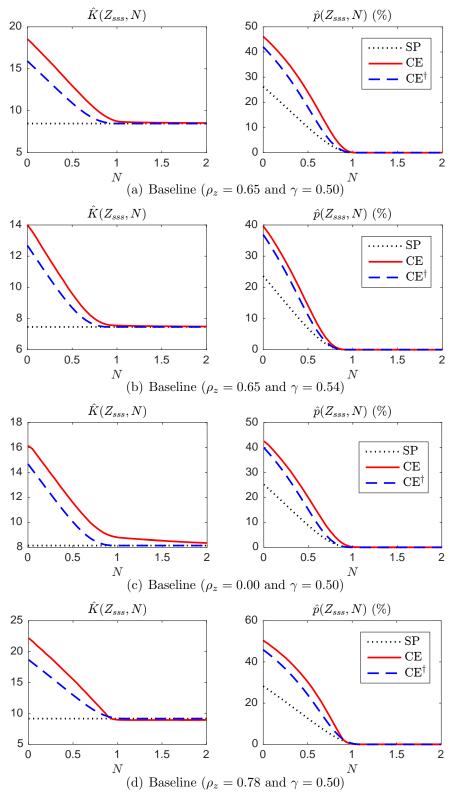
where,

$$N' = (1 - \gamma) \left[ (1 - \delta)K + Z'(K)^{\alpha} - 1/\beta (K - \phi - N) - R^{F} \phi \right]^{+},$$
$$ln(\phi') = \rho_{\phi} \phi + (1 - \rho_{\phi}) \phi^{new},$$

subject to foreign borrowing limit. The optimal decision rule of credit, denoted as  $\hat{K}(Z, N, \phi)$ , solves the recursive optimization problem of households.

Second, I derive the optimal policy function  $\hat{K}(Z, N, \phi)$ . To compute this I discretize the endogenous state variable N using a grid with 85 non-equidistant points, K using a grid with 1069 non-equidistant points and  $\phi$  using 4 equidistant points. The productivity process is approximated using the quadrature procedure following Tauchen (1998) and Tauchen & Hussey (1991) for 251 points. To improve accuracy I use spline interpolations on the N grid and on  $\phi$  grid. Finally, in order to find the dynamics:





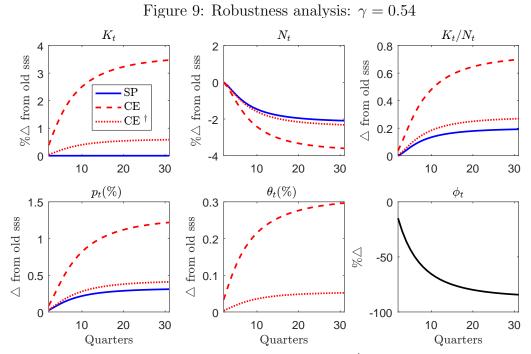
SP: Social planner equilibrium. CE: Competitive equilibrium. CE<sup> $\dagger$ </sup>: Competitive equilibrium abstracting from the intertemporal channel.

- The economy starts at t=0 at its stochastic steady state. In other words, I set  $N_0=N_{sss}, Z_0=Z_{sss}$  and  $\phi_0=\phi^{old}$ . And I assume the realized productivity shocks,  $e_t^z$ , are zero.
- Since the foreign borrowing limit reduction that occurs at t=1 is not anticipated at  $t=0, K_0=K_{sss}$ .
- For t>0, I evaluate the policy function  $K_t(Z_t, N_t, \phi_t)$ , where the dynamics of the state variables are:

$$Z_{t} = exp(\mu_{z}(1-\rho_{z})+\rho_{z}ln(Z_{t-1})),$$
$$N_{t} = (1-\gamma)\left[(1-\delta)K_{t-1} + Z_{t}K_{t-1}^{\alpha} - 1/\beta(K_{t-1}-\phi_{t-1}-N_{t-1}) - R^{F}\phi_{t-1}\right]^{+},$$
$$\phi_{t} = exp(\rho_{\phi}\phi_{t-1} + (1-\rho_{\phi})\phi^{new}),$$

• Finally, I use the spline interpolation method to evaluate the policy function at points out of the grids.

#### Appendix N Sudden Stop Simulation - Robustness



SP: Social planner equilibrium. CE: Competitive equilibrium.  $CE^{\dagger}$ : Competitive equilibrium abstracting from the intertemporal channel.

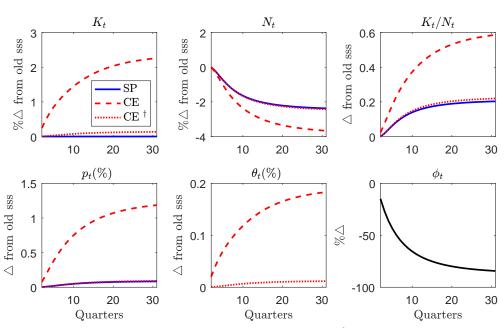


Figure 10: Robustness analysis:  $\rho_z=0.00$ 

SP: Social planner equilibrium. CE: Competitive equilibrium. CE<sup> $\dagger$ </sup>: Competitive equilibrium abstracting from the intertemporal channel.

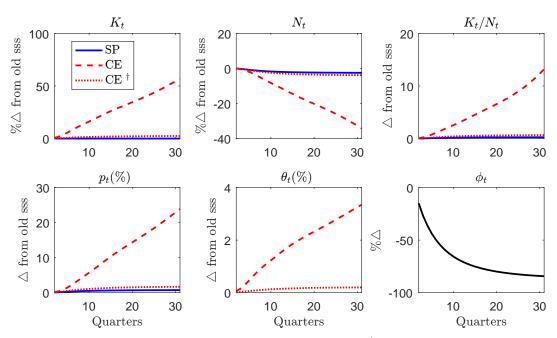


Figure 11: Robustness analysis:  $\rho_z=0.78$ 

SP: Social planner equilibrium. CE: Competitive equilibrium.  $CE^{\dagger}$ : Competitive equilibrium abstracting from the intertemporal channel.

#### Appendix O Productivity shock - Robustness

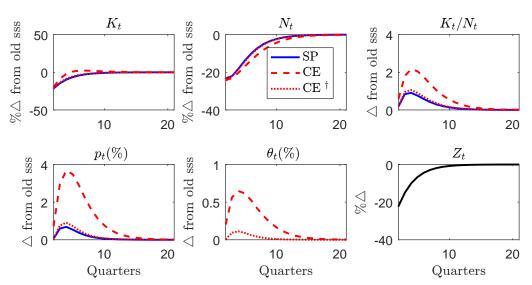
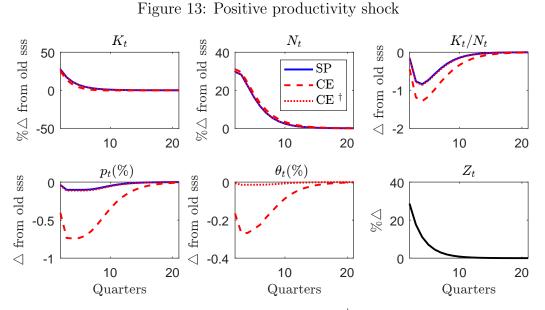


Figure 12: Negative productivity shock

SP: Social planner equilibrium. CE: Competitive equilibrium. CE<sup> $\dagger$ </sup>: Competitive equilibrium abstracting from the intertemporal channel.



SP: Social planner equilibrium. CE: Competitive equilibrium. CE<sup> $\dagger$ </sup>: Competitive equilibrium abstracting from the intertemporal channel.

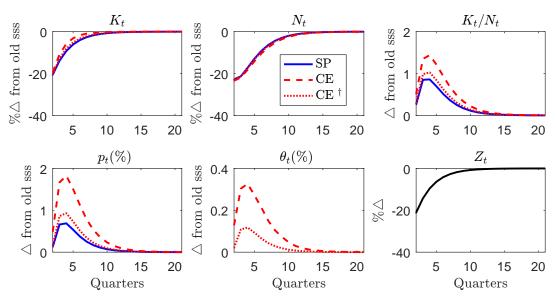
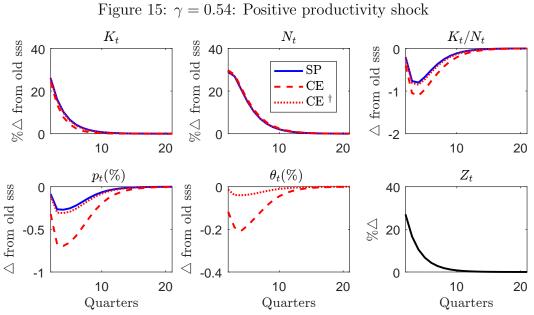


Figure 14:  $\gamma = 0.54$ : Negative productivity shock

SP: Social planner equilibrium. CE: Competitive equilibrium. CE<sup> $\dagger$ </sup>: Competitive equilibrium abstracting from the intertemporal channel.



SP: Social planner equilibrium. CE: Competitive equilibrium.  $CE^{\dagger}$ : Competitive equilibrium abstracting from the intertemporal channel.

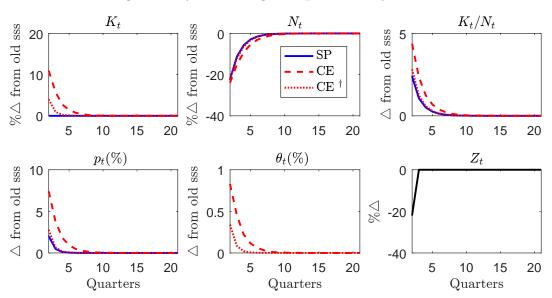


Figure 16:  $\rho_z = 0$ : Negative productivity shock

SP: Social planner equilibrium. CE: Competitive equilibrium. CE<sup> $\dagger$ </sup>: Competitive equilibrium abstracting from the intertemporal channel.

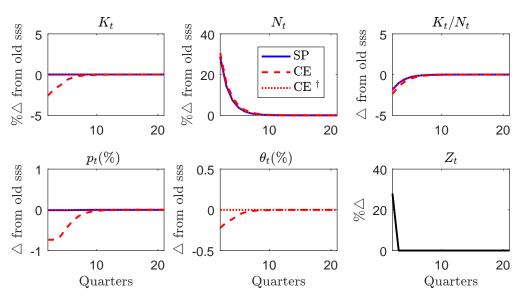
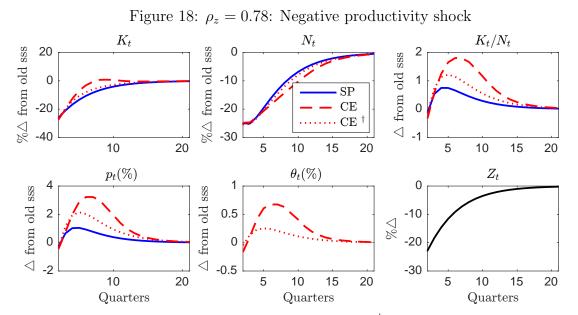
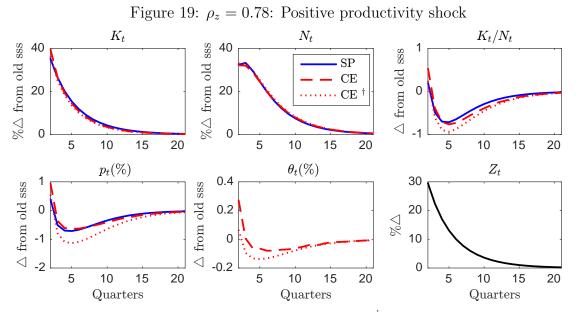


Figure 17:  $\rho_z = 0$ : Positive productivity shock

SP: Social planner equilibrium. CE: Competitive equilibrium.  $CE^{\dagger}$ : Competitive equilibrium abstracting from the intertemporal channel.



SP: Social planner equilibrium. CE: Competitive equilibrium.  $CE^{\dagger}$ : Competitive equilibrium abstracting from the intertemporal channel.



SP: Social planner equilibrium. CE: Competitive equilibrium.  $CE^{\dagger}$ : Competitive equilibrium abstracting from the intertemporal channel.