# Measuring the Stance of Monetary Policy in a Time-Varying World 

Fernando Pérez Forero*

* Banco Central de Reserva del Perú

DT. ${ }^{\circ}$. 2018-004<br>Serie de Documentos de Trabajo<br>Working Paper series<br>Abril 2018

Los puntos de vista expresados en este documento de trabajo corresponden a los de los autores y no reflejan necesariamente la posición del Banco Central de Reserva del Perú.

# Measuring the Stance of Monetary Policy in a Time-Varying World * 

Fernando J. Pérez Forero ${ }^{\dagger}$

5th April 2018


#### Abstract

Knowing the stance of monetary policy is a general theme of interest for academics, policy makers and private sector agents. The mentioned stance is not necessarily observable, since the Fed have used different monetary instruments at different points in time. This paper provides a measure of this stance for the last forty five years, which is a weighted average of a pool of instruments. We extend Bernanke and Mihov (1998)'s Interbank Market model by allowing structural parameters and shock variances to change over time. In particular, we follow the recent work of Canova and Pérez Forero (2015) for estimating non-recursive TVCVARs with Bayesian Methods. The estimated stance measure describes how tight/loose was monetary policy over time and takes into account the uncertainty related with posterior estimates of time varying parameters. Finally, we present how has monetary transmission mechanism changed over time, focusing our attention in the period after the Great Recession.


C11, E51, E52, E58
SVARs, Interbank Market, Operating Procedures, Monetary Policy Stance, Time-varying parameters, Bayesian Methods, Multi-move Metropolis within Gibbs Sampling

[^0]
## 1 Introduction

Knowing the stance of monetary policy is a general theme of interest for academics, policy makers and private sector agents. It provides an important piece of information to understand the current state of the economy and contributes to the expectations formation of future states. Despite its importance, it has been difficult to have an exact measure of this stance in the past, given the lack of consensus on which were the instruments of monetary policy and operating procedures at each point in time. In the recent years this task has turned even more difficult because of the introduction of the so-called Unconventional Monetary Policies (UMP) and because the Federal Funds Rate (FFR) hit the Zero-Lower-Bound (ZLB), given that the latter used to be considered as the core instrument at least for the last two decades. The purpose of this paper is to provide a measure of the policy stance that takes into account changes over time of the operating procedures of the Fed.

In practice, Monetary Policy is implemented through the intervention of the Fed in the interbank market of reserves. In this market, each participant has to meet a reserve requirement set by the Fed in advance. To do that, the banks use the interbank loans market. Therefore, the banks that have a deficit in reserves can borrow funds from the ones that have excessive reserves. These loans are granted only if the borrowers have an amount of collateral equivalent to the amount of credit that is going to be lent from another bank. The equilibrium price of this market is the interbank interest rate, i.e. the FFR. In this context, the Fed performs open market operations (OMO) in order to set the supply of reserves and a thereby affect the equilibrium outcome of this market. In particular, if a bank can not meet its reserve requirement, it will need to borrow reserves from the Fed at the Discount Window (DW), and the obtained funds will be called Borrowed Reserves (BR). Given the Total Reserves (TR) stock that market participants have at the end of the period, the difference is called Non-Borrowed Reserves $(\mathrm{NBR}=\mathrm{TR}-\mathrm{BR})$, i.e. Reserves that are obtained through open market operations. Finally, the way these operations (OMO and DW) are implemented is what we call the Fed's operating procedures. In particular, if we take a look to the recent monetary history of the United States, we can find evidence of different episodes with different operating procedures.

The implementation in each case depends on what was the Fed's target at each point in time, e. g. borrowed reserves, non-borrowed reserves, total reserves or the federal funds rate (see, Cosimano and Jansen (1988), Cosimano and Sheehan (1994), Bernanke and Mihov (1998), among others).

The empirical approach in this paper is based on Structural Vector Autorregressions (SVARs). These time series models have been popular for identifying and assessing the dynamic effects of monetary policy shocks. When using these models, the FFR has been considered as the core instrument since the seminal work of Bernanke and Blinder (1992) ${ }^{12}$. On the other hand, Christiano and Eichenbaum (1992) and Strongin (1995) used reserves of banks as a monetary policy instrument. Bernanke and Mihov (1998) reconcile these two strands of the literature with an eclectic approach that identifies monetary policy shocks as a linear combination of innovations in different instruments and, subsequently Christiano et al. (1999) summarizes this empirical literature and takes stock of what we learned about SVARs and monetary policy shocks during the 1990's decade. In the previous decade (2001-2010) the SVARs have been extended to solve some problems related with the identification procedures. First, Bernanke et al. (2005) suggest that a large amount of information needs to be used to solve the price-puzzle. In second place, the instability of parameters has been taken into account and also regime changes were explicitly discussed (see Cogley and Sargent (2005), Primiceri (2005) and Sims and Zha (2006)). Although useful, all these extensions still considered the FFR as the core instrument for monetary policy. Clearly, the approaches that used the FFR as policy instrument are unsuitable to discuss the role of the UMPs and the interest rate at the ZLB. In this regard, recent attempts to identify monetary policy with the interest rate at the ZLB and the active use of the UMPs can be found in Baumeister and Benati (2013) and also Peersman (2011). In particular, these authors use sign restrictions as in Canova and De Nicoló (2002) and Uhlig (2005), and they identify the UMP shocks that are different from standard FFR innovations. UMP has different dimensions, and an innovation in each of them must be associated with monetary policy actions. The problem is that it is difficult to interpret the role of the shocks for the sample portion before

[^1]the financial crisis. As a result, considering a different policy shock for each UMP dimension (as in Baumeister and Benati (2013) and Peersman (2011)) is not necessarily a good strategy. We believe that the strategy of identifying monetary policy shocks as a linear combination of innovations in different instruments, as in Bernanke and Mihov (1998), should be reconsidered, given the various dimensions of UMPs (e.g. to influence Financial Markets conditions through Large Scaled Asset Purchases, Forward Guidance, Direct Financial Intermediation, Quantitative Easing) together with the $\mathrm{FFR}^{3}$. Thus, in this paper we use Bernanke and Mihov (1998)'s approach to take into account the multi-dimensionality of monetary policy and the possibility that different instruments matter at different points in time.

The building blocks of our approach are as follows. Bernanke and Mihov (1998) characterize Federal Reserve's operating procedures and provide a measure of the stance of monetary policy for the period 1965-1996. Essentially, their model has an interbank market of reserves where monetary policy can be implemented, depending on the parameter values of the model, by targeting either interest rates (price of reserves) or the supply of reserves. What makes this model useful is its capability to identify monetary policy shocks for different contexts and instruments (see eq. (12) in the mentioned reference). These authors make explicit their concern about stability of parameters along their sample of analysis because the operating procedures might have changed over time (e.g. Volcker's experiment in early 1980s or the recent UMPs in our case). Instead, when measuring policy stance one should take into account that the weight of each instrument is likely to be time varying. These weights are nonlinear functions of the estimated structural parameters. Thus, in order to study the posterior distribution of the path of the monetary policy stance and the weights taken by each component, we follow Canova and Pérez Forero (2015), who extend the setup of Time-Varying Coefficients (TVC) VARs with Stochastic Volatility (Primiceri, 2005) to deal with non-recursive and potentially over-identified SVAR models. In particular, we extend the framework used by Bernanke and Mihov (1998) to account for the monetary policy (UMPs) practices. The model can be used to study the role of Quantitative Easing (QE), since it identifies demand and supply shocks to

[^2]reserves and the discount window operations shocks. However, the model needs to be slightly modified in order to capture Large Scaled Asset Purchases (LSAPs) and Forward Guidance (the announcement of future path for interest rates), actions aimed to affect medium and long-term interest rates. Thus, we include the shadow interest rate as a proxy of these measures (Wu and Xia, 2015). It is important to remark that this is different from including a specific equation for UMP shocks ${ }^{4}$. One potential limitation of our approach is that we do not make explicit the role of communication (i.e. FOMC meeting Releases, Minutes, Speeches, etc.) and the recently introduced interest paid for holding reserves. Our strategy to characterize the Monetary Policy Stance is robust in terms of specification, since we are allowing structural parameters in both policy and non-policy blocks to vary over time. On the other hand, we believe that part of the effect of FOMC communication is captured through changes in shadow interest rate, which is included in our SVAR.

We find that the stance of monetary policy has varied quite a lot over the last 45 years. It was loose for the first half of the 1970s and roughly neutral for the second half, it becomes tighter at the beginning of Volcker's period, i.e. the so-called Volcker's disinflation experiment (19801982) and then becomes loose again. Volcker's period ends with a relatively tight stance but showing more uncertainty than before. Greenspan's first ten years (1987-1996) exhibit a tight stance with a short period of loose policy in 1989. A long episode of loose stance (1996-2001) is observed with a subsequent neutral stage (2002-2003). Last Greenspan's years (2003-2005) display a relatively tight stance but shows an upward trend starting in late 2004. The stance turns to be loose when Bernanke's period starts until the outbreak of the Great Recession in 2007:Q4, when the stance turns to be tight again since 2008:Q4. We finally observe a reversal of this pattern after the implementation of UMPs, when the stance turns to be relatively loose in 2011-2013. This result is also in line with Beckworth (2011), who claims that the Monetary Policy Stance was relatively tight in 2008. After the tapering talk of may 2013, the stance turned to be less loose until our days, a result that is in line with the recent events observed, i.e. the FFR was raised in December 2015, and it is expected to be hiked again in December

[^3]2016. The relative weights of these instruments are time-varying, where the most important result is the weight of zero for the FFR at the end of the sample, consistent with the binding ZLB. What matters here is the fact that the model is capable of capturing significant changes in operating procedures.

Model estimates allow us to explore time variations in the transmission of policy shocks. Overall, the transmission of monetary policy shocks is stable for a large portion of our sample, but it exhibits significant changes after the outbreak of the Great Financial Crisis and the achievement of the ZLB. We find that the effect of expansionary policy shocks on the spreads is positive before 2007, but turns to be negative afterward. The latter is consistent with the purpose of UMPs, i.e. since the FFR is constant, the objective is to cut medium and long term interest rates. We also find evidence of a vanishing liquidity effect over time and that monetary policy shocks became relatively more volatile during Volcker's episode and during the Great Recession.

In sum, the approach this paper presents is capable of capturing changes in monetary policy implementation across different episodes. We present a monetary policy stance index that might be be useful for both policy makers and researchers. However, more work is needed for exploring the explicit role of communication in UMPs, the announcement of future paths of interest rates and credibility. We believe that these type of issues should be explored in a richer setup and therefore we leave it for future agenda. In this regard, some structural models that incorporate different dimensions of UMPs can be found in Gertler and Karadi (2011), Cúrdia and Woodford (2011) and Chen et al. (2012).

The document is organized as follows: section 2 presents the Structural VAR model used for the analysis, section 3 describes the estimation procedure, section 4 presents an estimate of the monetary policy stance, sections 5 and 6 explore the transmission mechanism and the volatility of monetary policy shocks, respectively and section 7 concludes.

## 2 The Model

### 2.1 A Structural Dynamic System

We are interested in characterize a dynamic setting in order to identify monetary policy shocks and for that purpose we closely follow the method proposed by Bernanke and Blinder (1992) and Bernanke and Mihov (1998). That is, assume that the structure of the economy is linear and given by

$$
\begin{aligned}
Y_{t} & =c_{t}^{n p} D_{t}+\sum_{i=0}^{p} R_{i, t} Y_{t-i}+\sum_{i=0}^{p} T_{i, t} P_{t-i}+C_{t}^{n p} v_{t}^{n p} \\
P_{t} & =c_{t}^{p} D_{t}+\sum_{i=0}^{p} S_{i, t} Y_{t-i}+\sum_{i=0}^{p} G_{i, t} P_{t-i}+C_{t}^{p} v_{t}^{p}
\end{aligned}
$$

where $Y_{t}$ is a vector of macroeconomic variables, $P_{t}$ is a vector of monetary policy instruments, $c_{t}^{n p}$ and $c_{t}^{p}$ are matrices of coefficients on a vector of deterministic variables $D_{t}$ and $v_{t}^{n p}$ and $v_{t}^{p}$ are vectors of structural shocks that can hit the economy at any point in time $t=1, \ldots, T$ with

$$
v_{t}^{k} \sim N\left(\mathbf{0}, \Sigma_{t}^{k} \Sigma_{t}^{k \prime}\right) ; \quad k=\{n p, p\}
$$

where $\Sigma_{t}^{k} \Sigma_{t}^{k \prime}$ is a diagonal positive definite matrix and $\operatorname{Cov}\left(v_{t}^{n p}, v_{t}^{p}\right)=\mathbf{0}$.
Notice that here we allow for potential time variation in matrix coefficients and variances and therefore we include the index $t$ for each of them. As in the mentioned references, we assume that macroeconomic variables $Y_{t}$ do not react within the next period to innovations in policy instruments, i.e. $T_{0, t}=\mathbf{0} \forall t$, so that

$$
\begin{align*}
& Y_{t}=c_{t}^{n p} D_{t}+\sum_{i=0}^{p} R_{i, t} Y_{t-i}+\sum_{i=1}^{p} T_{i, t} P_{t-i}+C_{t}^{n p} v_{t}^{n p}  \tag{1}\\
& P_{t}=c_{t}^{p} D_{t}+\sum_{i=0}^{p} S_{i, t} Y_{t-i}+\sum_{i=0}^{p} G_{i, t} P_{t-i}+C_{t}^{p} v_{t}^{p}
\end{align*}
$$

where we assume that a period $t$ is a quarter. For now we can say that the structure of the economy (1) takes the form of a system of Vector Autorregressions (VAR) of order $p$. In order
to be in line with a textbook representation of this type of system we present a more compact setup. Denote the vector of variables $\mathbf{y}_{t} \equiv\left[Y_{t}^{\prime}, P_{t}^{\prime}\right]^{\prime}$, the vector of intercepts $c_{t} \equiv\left[c_{t}^{n p \prime}, c_{t}^{p \prime}\right]^{\prime}$ and the matrices

$$
\begin{align*}
A_{t} & \equiv\left[\begin{array}{ll}
A_{11, t} & A_{12, t} \\
A_{21, t} & A_{22, t}
\end{array}\right]=\left[\begin{array}{cc}
I-R_{0, t} & \mathbf{0} \\
-S_{0, t} & I-G_{0, t}
\end{array}\right]  \tag{2}\\
A_{i, t} & \equiv\left[\begin{array}{ll}
R_{i, t} & T_{i, t} \\
S_{i, t} & G_{i, t}
\end{array}\right] ; i=1, \ldots, p \\
C_{t} & \equiv\left[\begin{array}{ll}
C_{11, t} & C_{12, t} \\
C_{21, t} & C_{22, t}
\end{array}\right]=\left[\begin{array}{cc}
C_{t}^{n p} & \mathbf{0} \\
\mathbf{0} & C_{t}^{p}
\end{array}\right] ; \Sigma_{t} \equiv\left[\begin{array}{cc}
\Sigma_{t}^{n p} & \mathbf{0} \\
\mathbf{0} & \Sigma_{t}^{p}
\end{array}\right]
\end{align*}
$$

so that the model can be re-expressed as a Structural VAR with time-varying coefficients:

$$
\begin{equation*}
A_{t} \mathbf{y}_{t}=c_{t} D_{t}+A_{1, t} \mathbf{y}_{t-1}+\ldots+A_{p, t} \mathbf{y}_{t-p}+C_{t} v_{t} \tag{3}
\end{equation*}
$$

However, the economic model expressed in its structural form cannot be directly estimated without additional assumptions. That is, in order to identify the vector of structural shocks $\left[v_{t}^{n p \prime}, v_{t}^{p \prime}\right]^{\prime}$ we should ask which form take matrices $A_{t}$ and $C_{t}$, which are governed by structural parameters. We will describe the structural model in subsection 2.3 but first we describe the setup in detail in next subsection.

### 2.2 Basic setup

Consider a vector of $M$ variables $\mathbf{y}_{t(M \times 1)}$ with data available for $T$ periods. I assume that the data generating process for $\mathbf{y}_{t}$ is the reduced-form version of the model (3), i.e. a $\operatorname{VAR}(p)$ process such that:

$$
\begin{equation*}
\mathbf{y}_{t}=B_{0, t} D_{t}+B_{1, t} \mathbf{y}_{t-1}+\ldots+B_{p, t} \mathbf{y}_{t-p}+u_{t} ; \quad t=1, \ldots, T \tag{4}
\end{equation*}
$$

where $B_{0, t} \equiv A_{t}^{-1} c_{t}$ is a matrix of coefficients on a $\bar{M} \times 1$ vector of deterministic variables $D_{t}$ and $B_{i, t} \equiv A_{t}^{-1} A_{i, t} ; i=1, \ldots, p$ are $M \times M$ matrices containing the coefficients on the
lags of the endogenous variables and the error term is distributed as $u_{t(M \times 1)} \sim N\left(0, \Omega_{t}\right)$, where $\Omega_{t(M \times M)}$ is a symmetric, positive definite, full rank matrix for every $t$. Equation (4) is a reduced form and the error terms $u_{t}$ do not have an economic interpretation. Let the structural shocks be $\varepsilon_{t} \sim N\left(0, I_{M}\right)$ and the mapping between these shocks and their reduced form counterpart is

$$
\begin{equation*}
u_{t}=A_{t}^{-1} C_{t} \Sigma_{t} \varepsilon_{t} \tag{5}
\end{equation*}
$$

where $A_{t(M \times M)}, C_{t(M \times M)}$ and $\Sigma_{t(M \times M)}$ are defined in (2). In order to be in line with the notation of previous subsection, we should note that $\varepsilon_{t} \equiv \Sigma_{t}^{-1} v_{t}$ is the normalized version of the structural shocks. I substitute (5) into (4) so that we get structural form of the $\operatorname{VAR}(p)$ model:

$$
\begin{equation*}
\mathbf{y}_{t}=X_{t}^{\prime} B_{t}+A_{t}^{-1} C_{t} \Sigma_{t} \varepsilon_{t} \tag{6}
\end{equation*}
$$

The matrix of regressors is $X_{t}^{\prime}=I_{M} \otimes\left[D_{t}^{\prime}, \mathbf{y}_{t-1}^{\prime}, \ldots, \mathbf{y}_{t-p}^{\prime}\right]$ is a $M \times K$ matrix where $D_{t}$ potentially includes a constant term, trends, seasonal dummy variables, etc and $K=M \times$ $\bar{M}+p \times M^{2}$. Parameter blocks $\left(B_{t}, A_{t}, C_{t}^{-1}, \sigma_{t}\right)$ are treated as latent variables that evolve as independent random walks:

$$
\begin{align*}
B_{t} & =B_{t-1}+v_{t}  \tag{7}\\
\alpha_{t} & =\alpha_{t-1}+\zeta_{t}  \tag{8}\\
\widetilde{c}_{t} & =\widetilde{c}_{t-1}+\varrho_{t}  \tag{9}\\
\log \left(\sigma_{t}\right) & =\log \left(\sigma_{t-1}\right)+\eta_{t} \tag{10}
\end{align*}
$$

where $B_{t(K \times 1)}=\left[\operatorname{vec}\left(B_{0, t}\right)^{\prime}, \operatorname{vec}\left(B_{1, t}\right)^{\prime}, \ldots, v e c\left(B_{p, t}\right)^{\prime}\right]^{\prime}$ is a $K \times 1$ vector; $\alpha_{t}$ and $\widetilde{c}_{t}$ denote free parameters of matrices $A_{t}$ and $C_{t}^{-1}$, respectively ${ }^{5}$. In addition, $\sigma_{t(M \times 1)}$ is the main diagonal of

[^4]$\Sigma_{t}$. Finally, the covariance matrix for the error vector is:
\[

V=\operatorname{Var}\left(\left[$$
\begin{array}{c}
\varepsilon_{t}  \tag{11}\\
v_{t} \\
\zeta_{t} \\
\varrho_{t} \\
\eta_{t}
\end{array}
$$\right]\right)=\left[$$
\begin{array}{ccccc}
I_{M} & 0 & 0 & 0 & 0 \\
0 & Q & 0 & 0 & 0 \\
0 & 0 & S_{a} & 0 & 0 \\
0 & 0 & 0 & S_{\widetilde{c}} & 0 \\
0 & 0 & 0 & 0 & W
\end{array}
$$\right]
\]

The model presented captures time variations of different parameter blocks: i) lag structure (7), ii) structural parameters (8) and (9) and iii) structural variances (10). In other words, the model is capable of capturing the sources of potential structural changes, i.e. drifting coefficients $\left(B_{t}, \alpha_{t}, \widetilde{c}_{t}\right)$ or stochastic volatility $\left(\sigma_{t}\right)$ without imposing prior information about specific dates or number of structural breaks. In particular, in the process of identifying parameters that affect the policy stance and represent the operating procedures, a subset of $\left(\alpha_{t}, \widetilde{c}_{t}\right)$ will have a major relevancy.

### 2.3 A Structural VAR model with an Interbank Market

Bernanke and Mihov (1998) presented a semi-structural VAR model that characterizes Federal Reserve's operating procedures. The purpose of this section is to present an extension of the mentioned model in order to take into account different dimensions of UMP together with conventional policy.

Consider the vector of variables $\mathbf{y}_{t}=\left[x_{t}, \pi_{t}, \Delta \operatorname{Pcom}_{t}, T R_{t}, F F R_{t}, N B R_{t}\right]^{\prime}$, where $x_{t}$ represents output growth rate, $\pi_{t}$ represents the inflation rate, $\Delta P \operatorname{com}_{t}$ is the growth rate of an index of commodity prices, $T R_{t}$ is the total amount of reserves that banks hold at the Central Bank, $F F R_{t}$ is the Federal Funds Rate in annual terms ${ }^{6}$ and $N B R_{t}$ is the total amount of nonborrowed reserves (portion of reserves different from borrowed reserves at the discount window,

[^5]i.e. $N B R=T R-B R)$. Regarding the model specification, we re-write equation (5) as follows ${ }^{7}$
$$
A_{t} u_{t}=C_{t} v_{t}
$$
where we need to recall that $v_{t}=\Sigma_{t} \varepsilon_{t}$ is the re-scaled vector of structural shocks and $u_{t}$ is the vector of reduced-form innovations. Moreover, recall the system partition described in subsection 2.1. That is, there is a non-policy block and a policy block:
\[

\left[$$
\begin{array}{cc}
A_{11, t} & A_{12, t}  \tag{12}\\
A_{21, t} & A_{22, t}
\end{array}
$$\right]\left[$$
\begin{array}{c}
u_{t}^{n p} \\
u_{t}^{p}
\end{array}
$$\right]=\left[$$
\begin{array}{ll}
C_{11, t} & C_{12, t} \\
C_{21, t} & C_{22, t}
\end{array}
$$\right]\left[$$
\begin{array}{c}
v_{t}^{n p} \\
v_{t}^{p}
\end{array}
$$\right]
\]

Within the non-policy block we can find output growth, inflation, and commodity prices growth that is, $u_{t}^{n p}=\left[u_{t}^{x}, u_{t}^{\pi}, u_{t}^{c}\right]^{\prime}$ and $v_{t}^{n p}=\left[v_{t}^{x}, v_{t}^{\pi}, v_{t}^{c}\right]^{\prime}$. the system has $M=6$ variables, where we denote the number of non-policy variables as $M_{n p}=\operatorname{dim} u_{t}^{n p}=3$. The policy block contains the remaining variables of the system, i.e. $M_{p}=M-M_{n p}=\operatorname{dim} u_{t}^{p}=3$, and we will call them policy instruments. Here below we specify a sub-system of equations for the portion of $u_{t}^{p}$ that is orthogonal to the non-policy block $u_{t}^{n p}$. The set of assumptions embedded in the system of equations above can be summarized as follows:

1. Non-policy block: First, recall that non-policy variables only react to policy changes with some delay, i.e. according to (2), we have $A_{12, t}=\mathbf{0}_{\left(M_{p} \times M_{n p}\right)}, \forall t$. The intuition behind this assumption is that the private sector considers lagged stance of policy as state variable. That is, output growth and inflation will not change in the same quarter after an innovation in a particular instrument that belongs to $u_{t}^{p}$. Moreover, following Bernanke and Mihov (1998), I will keep this non-policy block unmodeled and I will just assume that

[^6]$A_{11, t}$ is lower triangular, so that
\[

A_{11, t}=\left[$$
\begin{array}{ccc}
1 & 0 & 0 \\
\alpha_{x, t}^{\pi} & 1 & 0 \\
\alpha_{x, t}^{c} & \alpha_{\pi, t}^{c} & 1
\end{array}
$$\right]
\]

The ordering in this block is an open question and that is why the model is called semistructural $^{8}$. Moreover, I also assume that innovations in $u_{t}^{n p}$ will affect the policy block in the same quarter, i.e. $A_{21, t}$ is an unrestricted $M_{n p} \times M_{p}$ matrix of potentially non-zero parameters (see Appendix A.4). In addition, according to (2), we have $C_{12, t}=\mathbf{0}_{\left(M_{p} \times M_{n p}\right)}$ and $C_{21, t}=\mathbf{0}_{\left(M_{n p} \times M_{p}\right)}, \forall t$. We also assume that $C_{11, t}=\mathbf{I}_{M_{p}}$ is the identity matrix, which means that structural shocks $v_{t}^{x}, v_{t}^{\pi}$ and $v_{t}^{c}$ only affect output growth, inflation and commodity growth on impact, i.e. there are no cross-effects on impact.

Turning to the policy block, the next three equations have the aim to describe the Interbank Market of Reserves. That is, each period $t$ banks have to meet their reserve requirements determined by the Fed. The sum of the level of reserves across banks determines the term "Total Reserves" denoted by $T R_{t}$. Moreover, these reserves pay an interest that is closely related to the Federal Funds Rate, $F F R_{t}$ and a result the latter is a relevant indicator for the demand of reserves. In order to meet their reserve requirements banks have two alternatives: they could get liquidity from the Discount Window ("Borrowed Reserves", $B R_{t}$ ) or through Open Market Operations ("Non-Borrowed Reserves", $N B R_{t}$ ). Banks have a pool of assets that are used as a collateral in order to get liquidity through the mentioned alternatives. To close the model, we assume that the Federal Reserve regulates liquidity by deciding the amount of reserves that is going to be injected through Open Market Operations. I will now proceed to describe the structural equations that conform the Interbank Market. In line with equation (12), I express the model in terms of reduced-form and structural innovations:

- Demand for Reserves equation: It represents the total demand for reserves of banks.

[^7]In particular, the portion of $u_{t}^{T R}$ which is orthogonal to the non-policy block depends negatively on the Federal Funds Rate's innovation $u_{t}^{F F R}$ and $v_{t}^{d}$ represents a shock to reserves' demand.

$$
\begin{equation*}
u_{t}^{T R}=-\alpha_{1, t}^{d} u_{t}^{F F R}+v_{t}^{d} \tag{13}
\end{equation*}
$$

- Demand for discount window operations equation: Borrowed Reserves (BR) is the portion of reserves obtained through the discount window. They depend positively on the Federal Funds Rate's innovations $u_{t}^{F F R}$ and $v_{t}^{b}$ represents a shock to the discount window operations' demand, a potential source of fluctuation that could become relevant in episodes of financial stress or under BR targeting.

$$
\begin{equation*}
u_{t}^{B R}=u_{t}^{T R}-u_{t}^{N B R}=\alpha_{1, t}^{b} u_{t}^{F F R}+v_{t}^{b} \tag{14}
\end{equation*}
$$

- Federal Reserve equation: It represents the money supply process, i.e. providing enough liquidity through open market operations in order clear the money market. The portion of $u_{t}^{N B R}$ which is orthogonal to the non-policy block responds contemporaneously to shocks in spreads, demand for total and borrowed reserves. Every other action unrelated with the mentioned shocks is called an exogenous monetary policy shock, $v_{t}^{s}$.

$$
\begin{equation*}
u_{t}^{N B R}=\phi_{t}^{d} v_{t}^{d}+\phi_{t}^{b} v_{t}^{b}+v_{t}^{s} \tag{15}
\end{equation*}
$$

Equations (13) and (14) and (15) also appear in the benchmark version from Bernanke and Mihov (1998) and our contribution is the addition of time variation in structural parameters plus the inclusion of the shadow interest rates. From the latter reference we import the idea that each of the three variables is considered as a monetary policy instrument controlled by the Fed. In this framework we abstract for the role of Fed's monetary policy communication (i.e. FOMC meeting Releases, Minutes, Speeches, etc.) as well as for interest paid by reserves. In this regard, it is worth to say that our strategy is safe in terms of specification, since we are allowing structural parameters from both policy and non-policy blocks as well as structural variances to vary over time, i.e. since the model is an approximation that could be potentially
misspecified (indeed, the model is linear and not micro-founded), it is likely that posterior estimates of structural parameters will vary across sub-samples. Therefore, it is even better to allow for continuous drifting parameters ${ }^{9}$.

Recall (12) and consider now the sub-system of equations for the portion of $u_{t}^{p}$ that is orthogonal to the non-policy block $u_{t}^{n p}$, i.e. $A_{22, t} u_{t}^{p}=C_{22, t} v_{t}^{p}$. The system is ${ }^{10}$ :

$$
\underbrace{\left[\begin{array}{ccc}
1 & \alpha_{1, t}^{d} & 0 \\
1 & -\alpha_{1, t}^{b} & -1 \\
0 & 0 & 1
\end{array}\right]}_{A_{22, t}} \times\left[\begin{array}{c}
u_{t}^{T R} \\
u_{t}^{F F R} \\
u_{t}^{N B R}
\end{array}\right]=\underbrace{\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
\phi_{t}^{d} & \phi_{t}^{b} & 1
\end{array}\right]}_{C_{22, t}} \times\left[\begin{array}{c}
v_{t}^{d} \\
v_{t}^{b} \\
v_{t}^{s}
\end{array}\right]
$$

The latter system can be solved for structural shocks, i.e. $v_{t}^{p}=C_{22, t}^{-1} A_{22, t} u_{t}^{p}$. In particular, the bottom equation of this system corresponds to the monetary policy shock $v_{t}^{s}$, that is:

$$
\begin{equation*}
v_{t}^{s}=-\left(\phi_{t}^{b}+\phi_{t}^{d}\right) u_{t}^{T R}+\left(\phi_{t}^{b} \alpha_{1, t}^{b}-\phi_{t}^{d} \alpha_{1, t}^{d}\right) u_{t}^{F F R}+\left(\phi_{t}^{b}+1\right) u_{t}^{N B R} \tag{16}
\end{equation*}
$$

The intuition behind equation (16) is that monetary policy actions can be expressed as a linear combination of innovations of different instruments. That is, policy actions are not only characterized as interest rate innovations as it is commonly suggested in the literature. Therefore, (16) can be used to evaluate the Monetary Policy Stance.

Because we are allowing coefficients to vary over time, the weight of each instrument will turn to be time-varying as well. These weights are nonlinear functions of structural parameters that come from the estimated SVAR model, as it is explicit in (16). As a result, this characterization of policy actions is robust to changes in the operating procedures during the sample of analysis. For instance, consider the case of monetary policy conducted by interest rate setting. If the ZLB is binding, then the FFR will no longer be the policy instrument, at least temporarily. As a result, the Fed will re-design its operating procedures putting more weight on other instruments

[^8]and assigning zero weight to the $\mathrm{FFR}^{11}$.
The sub-system has $M_{p}=3$ variables. Therefore the variance covariance matrix of the reduced form error terms $u_{t}^{p}$ will have $3 \times(3+1) / 2=6$ parameters. On the other hand, the vector of structural parameters has 7 elements, i.e.
$$
\theta_{t}=\left(\alpha_{1, t}^{d}, \alpha_{1, t}^{b}, \phi_{t}^{d}, \phi_{t}^{b}, \sigma_{t}^{d}, \sigma_{t}^{b}, \sigma_{t}^{s}\right)^{\prime}
$$

These parameters are actually latent variables, however we set identifying restrictions to be in line with the SVAR literature (see Canova and Pérez Forero (2015) for details). Thus, to achieve identification it is necessary to impose an additional restriction. Following Bernanke and Mihov (1998), we can focus our attention on equation (16) and set restrictions such that monetary policy is associated with a particular instrument, i.e. set restrictions such that all the brackets are equal to zero except the one associated with our instrument of interest. Nevertheless, since our sample covers a period where the Fed had different chairmen plus UMP, we have less incentives in restricting our attention to a particular instrument. Instead, we want to capture changes in operating procedures, which means that all the brackets in (16) should be potentially different from zero. For that reason we set

$$
\begin{equation*}
\alpha_{1, t}^{d}=0 \tag{17}
\end{equation*}
$$

This restriction is in line with Strongin (1995) and Bernanke and Mihov (1998), and it means that the demand of Total Reserves is inelastic with respect to the Federal Funds Rate, though we have to take into account that this argument was used in a context where the Federal Funds Rate was strictly positive. Our system of equation will be exactly identified after imposing (17) and, as mentioned above, it implies that we are still free to consider different instruments of monetary policy at given time. We also test the sensitivity and plausibility of this identification restrictions in section.

In general, the estimation of the path for the structural parameters and the consequent

[^9]computation of $v_{t}^{s}$ in (16) will give us a measure of the monetary policy stance that internalizes changes in operating procedures, i.e. time varying weights will shed light on the relative importance of each of the four instruments at each point in time. On the other hand, the estimated paths for the variances will shed light on the relative importance of each of the shocks.

## 3 Bayesian Estimation

The purpose of this section is to describe the procedure in which the parameters of the statistical model described in (6) are simulated through Markov Chain Monte Carlo methods. In particular, we are interested in the posterior distribution of latent variables described in (7), (8), (9) and (10). I will adopt a Bayesian perspective following Primiceri (2005)'s Multi-move Gibbs Sampling procedure. Moreover, we will sample structural coefficients from (8) and (9) introducing two Metropolis-type steps, as suggested by Canova and Pérez Forero (2015).

### 3.1 Data description

The time series used for the experiment are in monthly frequency and were taken from the FRED Database and from the Federal Reserve Board website ${ }^{12}$. From the second database I took Total Reserves of aggregated depository institutions, Nonborrowed Reserves of aggregated depository institutions. The sample horizon is 1959:M1-2016:M9 (683 obs.), i.e. it includes the whole period of the Great Recession.

Industrial Production, Consumer Price Index and the Commodity Price Index were expressed in annual growth rates. Federal Funds Rate is expressed in annual terms, including the period of the shadow rate. Besides, to induce stationarity, Total and Nonborrowed Reserves were standardized using the mean and standard deviation of Non-Borrowed Reserves for a window of 48 months. Alternatively, Bernanke and Mihov (1998) divide Total and Non-Borrowed reserves by the average of Total Reserves using a window of 36 months. However this approach is not useful for inducing stationarity anymore, given the recent changes in reserves ${ }^{13}$.

[^10]
### 3.2 Priors and setup

Priors are shown in Table 1 and they are chosen to be conjugated. As a result the posterior distribution will be Normal and Inverted-Wishart for each corresponding case. Moreover, block $B^{T}$ 's posterior distribution is truncated for stationary draws. That is, the associated companion form of the VAR (4) is computed for each draw of $B^{T}$ and it is discarded if it does not satisfy the stability condition for each period $t=1, \ldots, T$. The latter procedure is captured by the indicator function $I_{B}($.$) . In addition, I calibrate the prior for initial states of structural parameters using$ the first $\tau=120$ observations (1959:M1 - 1969:M12) as a training sample, i.e. we estimate $(\overline{\mathbf{B}}, \overline{\mathbf{V B}})$ via OLS and $\left(\bar{\alpha}^{\prime}, \overline{\mathbf{c}}^{\prime}, \bar{\sigma}^{\prime}\right)^{\prime}$ via Maximum Likelihood ${ }^{1415}$.

Moreover, I calibrate $k_{Q}^{2}=0.5 \times 10^{-4}, k_{W}^{2}=1 \times 10^{-4}$, following Primiceri (2005), but also I calibrate $k_{S_{a}}^{2}=k_{S_{\tilde{c}}}^{2}=1 \times 10^{-1}$ to allow for significant time variation. Finally, lag length is set to $p=2$ as in the latter references.

Table 1: Priors

| $B_{0} \sim N(\overline{\mathbf{B}}, \overline{\mathbf{V B}})$ | $Q \sim I W\left(k_{Q}^{2} \cdot \overline{\mathbf{V B}}, \tau\right)$ |
| :---: | :---: |
| $\alpha_{0} \sim N\left(\bar{\alpha}, I_{\operatorname{dim} \alpha}\right)$ | $S_{a \sim} \sim I W\left(k_{S_{a}}^{2} \cdot I_{\operatorname{dim} \alpha}, 1+\operatorname{dim} \alpha\right)$ |
| $\widetilde{c}_{0} \sim N\left(-\overline{\mathbf{c}}, I_{\operatorname{dim} \widetilde{c})}\right.$ | $S_{\widetilde{c} \sim} \sim I W\left(k_{S_{\tilde{c}}}^{2} \cdot I_{\operatorname{dim} \widetilde{c}}, 1+\operatorname{dim} \widetilde{c}\right)$ |
| $\log \left(\sigma_{0}\right) \sim N\left(\log (\bar{\sigma}), I_{M}\right)$ | $W_{i} \sim I G\left(k_{W}^{2}, 1 / 2\right), i=1, \ldots, M$ |

Since it is assumed that blocks $\left(B^{T}, \alpha^{T}, \tilde{c}^{T}, \sigma^{T}\right)$ follow random walks according to equations (7), (8), (9) and (10), we use the mean and the variance of the priors of $B_{0}, \alpha_{0}, \widetilde{c}_{0}, \log \left(\sigma_{0}\right)$ to initialize the Kalman Filter at each iteration. The sampling procedure is described in the next subsection.

[^11]
### 3.3 Sampling parameter blocks

We have to sample parameter blocks ( $B^{T}, \alpha^{T}, \tilde{c}^{T}, \sigma^{T}, s^{T}, V$ ) and we do it sequentially using the logic of Gibbs Sampling (see Chib (2001)). The block $s^{T}$ is an auxiliary one used as an intermediate step for sampling $\sigma^{T}$, see Kim et al. (1998). The sampling algorithm is as follows:

1. Set an initial value for $\left(B_{0}^{T}, \alpha_{0}^{T}, \widetilde{c}_{0}^{T}, \sigma_{0}^{T}, s_{0}^{T}, V_{0}\right)$ and set $i=1$.
2. Draw the reduced form coefficients $B_{i}^{T}$ from $p\left(B_{i}^{T} \mid \alpha_{i-1}^{T}, \widetilde{c}_{i-1}^{T}, \sigma_{i-1}^{T}, s_{i-1}^{T}, V_{i-1}\right) \cdot I_{B}\left(B_{i}^{T}\right)$.
3. Draw the structural parameters $\alpha_{i}^{T}$ from $p\left(\alpha_{i}^{T} \mid B_{i}^{T}, \tilde{c}_{i-1}^{T}, \sigma_{i-1}^{T}, s_{i-1}^{T}, V_{i-1}\right)$.
4. Draw the structural parameters $\widetilde{c}_{i}^{T}$ from $p\left(\widetilde{c}_{i}^{T} \mid B_{i}^{T}, \alpha_{i}^{T}, \sigma_{i-1}^{T}, s_{i-1}^{T}, V_{i-1}\right)$.
5. Draw volatilities $\sigma_{i}^{T}$ from $p\left(\sigma_{i}^{T} \mid B_{i}^{T}, \alpha_{i}^{T}, \widetilde{c}_{i}^{T}, s_{i-1}^{T}, V_{i-1}\right)$.
6. Draw the indicator $s_{i}^{T}$ from $p\left(s_{i}^{T} \mid B_{i}^{T}, \alpha_{i}^{T}, \tilde{c}_{i}^{T}, \sigma_{i}^{T}, V_{i-1}\right)$.
7. Draw the hyper-parameters $V_{i}$ from $p\left(V_{i} \mid \alpha_{i}^{T}, \tilde{c}_{i}^{T}, y_{i}^{T}, \sigma_{i}^{T}, s_{i}^{T}\right)$.
8. Set $B_{i}^{T}, \alpha_{i}^{T}, \tilde{c}_{i}^{T}, \sigma_{i}^{T}, s_{i}^{T}, V_{i}$ as the initial value for the next iteration. If $i<N$, set $i=i+1$ and go back to 2 , otherwise stop.

The indicator function $I_{B}($.$) truncates the posterior distribution of B^{T}$ for draws that exhibit a stationary companion form for $t=1, \ldots, T$. I perform $N=100,000$ draws discarding the first 50,000 and I store one every 100 draws for the last 50,000 to reduce serial correlation. Specific details regarding the sampling algorithm are described in Canova and Pérez Forero (2015) and also in Appendix A. 2 and B for the diagnosis of convergence.

## 4 The Stance of Monetary Policy

Using equation (16) and the identification restrictions (17); replacing the reduced form innovations by the data included in the VAR; and renaming the resulting index as the monetary policy stance (MPS), the expression of interest is:

$$
\begin{equation*}
M P S_{t}=-\left(\phi_{t}^{b}+\phi_{t}^{d}\right) u_{t}^{T R}+\left(\phi_{t}^{b} \alpha_{1, t}^{b}\right) u_{t}^{F F R}+\left(\phi_{t}^{b}+1\right) u_{t}^{N B R} \tag{18}
\end{equation*}
$$

We compute the posterior distribution for $M P S_{t}$ at each point in time using posterior estimates of the parameters.


Figure 1: Posterior distribution of the Monetary Policy Stance, median value and 90 percent confidence bands

Figure 1 depicts the Monetary Policy Stance for the period 1974-2016. To the best of our knowledge, this is the first time that the path of the policy stance index is produced with error bands. The Stance of Monetary Policy is loose for a first half of the decade of 1970s, a situation that is reverted in late 1970s. Moreover, we can see an even tighter stance at the beginning of Volcker's period, i.e. the so-called Volcker's disinflation experiment (1980-1982)
with a subsequent loose stance. Then, Volcker's period ends with a relatively tight stance . Greenspan's first five years (1987-1992) exhibit a tight stance. On the other hand, a large episode of loose stance (1992-1999) is observed. Then we have a tight stance for a short period (1999-2003). Last Greenspan's years (2003-2005) exhibit a relatively loose stance. The stance turns to be tight when Bernanke's period starts until the outbreak of the Great Recession in 2007:Q4, when stance turns to be loose again. Part of the explanation of the reversion in this pattern is the implementation of UMPs, when the stance turns to be relatively loose at the end of our sample (2011-2016). This result is also in line with Beckworth (2011), who claim that the Monetary Policy Stance was relatively tight in 2008. Moreover, although the scales are different and besides the fact that we provide error bands for the estimated values, the estimated index qualitatively replicates the periods of loose and tight monetary policy suggested by Bernanke and Mihov (1998) and Boschen and Mills (1991). Therefore, the contribution of our analysis is precisely the quantification of the uncertainty and the addition of the recent period of unconventional monetary policies.


Figure 2: Monetary Policy Stance and NBER recession dates (shaded areas)

It is important to associate the monetary policy stance with each appointed chairman. In addition, we show the same stance but including the NBER recession dates in shaded areas in Figure 2. The first aspect that needs to be mentioned is the one related with Volcker's disinflation period (1980-1982), where we can observe a sharp fall in the stance in late 1980 related with a sharp fall in long term interest rates ${ }^{16}$. In general, we observe a raise in our MPS index during recessions, meaning that the usual reaction is to implement an expansionary monetary policy in order to revert the negative state of the economy.

[^12]
## 5 The Transmission Mechanism of Monetary Policy revisited

In this section we explore the transmission of monetary policy shocks $\left(\varepsilon_{t}^{s}\right)$ on the interbank market and the aggregate economy. As a consequence of the continuous time variation of parameter values, it will be possible to trace impulse responses along the time dimension and explore their evolution over time. Let the impulse response function be

$$
\begin{equation*}
\frac{\partial y_{t+j}}{\partial \varepsilon_{t}}=F_{j}\left(\left\{B_{i}\right\}_{i=t}^{t+j}, A_{t}, C_{t}^{-1}, \Sigma_{t}\right) ; \quad j=0,1, \ldots \tag{19}
\end{equation*}
$$

where $F_{j}($.$) depends on the companion form matrix of (4) for periods t, t+1, \ldots, t+j$ and the blocks $A_{t}, C_{t}^{-1}, \Sigma_{t}$ and depends on when the shock occurs. The exact form of equation (19) is

$$
\frac{\partial y_{t+j}}{\partial \varepsilon_{t}}=E_{t}\left[\mathbf{J}\left(\begin{array}{l}
j-1  \tag{20}\\
k=0
\end{array} \mathbf{A}_{t+j-k}^{c}\right) \mathbf{J}^{\prime} H_{t}\left(\widetilde{\varepsilon}_{t}-\bar{\varepsilon}_{t}\right)\right]
$$

where $H_{t}=A_{t}^{-1} C_{t} \Sigma_{t}$ and $\mathbf{A}_{t}^{c}$ is the companion form matrix of (4). Details on the derivation of equation (20) can be found in Canova and Ciccarelli (2009).

Figure 3 depicts the response of each variable after an expansionary policy shock $\left(\varepsilon_{t}^{s}\right)$ in 1996, a date that we associate with normal times. First, output growth and inflation exhibits a humpshaped response where the peaks are 6 and 12 quarters after the shock. The expansionary policy produces an increase in Total Reserves (TR) as well as a liquidity effect, i.e. Non-Borrowed Reserves (NBR) and Federal Funds Rate (FFR) move in opposite directions after the shock occurs.


Figure 3: Responses to Monetary Policy shocks in 1996, 90 percent confidence interval

Figure 4 depicts the response of each variable after an expansionary policy shock $\left(\varepsilon_{t}^{s}\right)$ in 2012, a date that is associated with the aftermath of the last financial crisis. As it is noticeable, the transmission mechanism of monetary policy has been altered. In particular, the response of inflation is stronger and more persistent. As a matter of fact, the response of the FFR turns to be small and almost insignificant, since the target rate is in an interval close to zero, i.e. between 0 and 0.25 basis points.


Figure 4: Responses to Monetary Policy shocks in 2012, 90 percent confidence interval

Given this evidence of changes in the monetary transmission mechanism, we want to dig into a particular issue of interest, named the Vanishing liquidity effect. According to the SVAR literature on monetary policy shocks ${ }^{17}$, there is evidence of a temporary negative reaction of interest rates that vanishes in the long run after an expansionary shock, i.e. an increase in money supply. This pattern is manifested, in particular, when Non-Borrowed Reserves are included in the SVAR model instead of money aggregates such as M1 or M2. As a result, our model is capable of reproducing this feature and, furthermore, is capable of saying that this pattern has been changing over time. Figure 5 depicts the evolution of the response of the Federal Funds Rate, where it can be observed a strong response previous to late 1980s, and then a decreasing response over time until hitting the Zero Lower Bound (ZLB) in 2009.

[^13]

Figure 5: Response of the Federal Funds Rate to an expansionary policy shock

Overall, there is evidence of time-varying monetary transmission mechanism; this instability can be associated with changes in monetary policy conduction with different operating procedures. Note also that our result controls for changes in the private sector behavior, since we allow the parameters of non-policy block to vary over time as well, and these changes are correlated with changes in the policy design (see Canova and Pérez Forero (2015)).

## 6 The Systematic and Non-systematic components of Monetary Policy

The recent literature of Time varying coefficients (TVC), Stochastic Volatility and Monetary Policy claims for more volatile policy shocks in early 1980s, a date that is associated with changes in monetary policy conduction and the use of atypical instruments (see Primiceri (2005), Sims and Zha (2006), Justiniano and Primiceri (2008), Canova et al. (2008), Canova and Gambetti (2009) among others). However, we believe that part of this result is driven by the fact that
most of these models only allow for a single monetary policy instrument, i.e. the short term interest rate. On the other hand, as we pointed out above, here we identify monetary policy shocks including different instruments and controlling for changes in the systematic component. For instance, recall the policy rule equation (15):

$$
u_{t}^{N B R}=\phi_{t}^{d} v_{t}^{d}+\phi_{t}^{b} v_{t}^{b}+v_{t}^{s}
$$

The systematic component is represented by the policy coefficients $\phi_{t}=\left(\phi_{t}^{d}, \phi_{t}^{b}\right)^{\prime}=-\widetilde{c}_{t}$ and the non-systematic one is governed by the shock $v_{t}^{s} \sim N\left(0,\left(\sigma_{t}^{s}\right)^{2}\right)$. We will call the evolution of $\phi_{t}$ as changes in the systematic component and the evolution of $\sigma_{t}^{s}$ as changes in the non-systematic one.

Figure 6 depicts the evolution of the components of vector $\phi_{t}$. In particular, we observe a relatively unstable behavior of $\phi_{t}^{d}$ starting in 2000 and magnified during the crisis episode. That is, we clearly observe a change in the operating procedures during this era, which is in line with the motivation of this paper, i.e. quantifying the implementation of Unconventional Monetary Policies. In particular, the changing reaction function of the Fed reflects the fact that liquidity demand shocks became more important during the crisis episode. On the other hand, regarding the discount window shocks, our results reflect that they were significant during the 1970s and early 1980s, and now the became less important for the Fed.


Figure 6: Policy rule coefficients, median value and 90 percent c.i.

On the other hand, Figure 7 shows the evolution of the non-systematic component. In line with the mentioned references, we observe a high level of volatility in 1970s and early 1980s, but until 1985. On the other hand, we observe a second episode of high volatility starting in 2007, i.e. an episode related with the Great Financial Crisis. This jump is probably capturing the other dimensions of Unconventional Monetary Policy not related with Reserves and affecting future interest rates, e.g. Direct Financial Intermediation as in Gertler and Karadi (2011). For a thorough description of different dimensions of UMPs see also Reis (2009), Borio and Disyatat (2010), Cúrdia and Woodford (2011), Cecioni et al. (2011) among others.


Figure 7: Standard Deviation of Monetary Policy shocks

Finally, two things are worth to recall. In first place, the setup of the model is such that the variances of structural shocks evolve independently, i.e. matrix $W$ in (11) is set as diagonal. Therefore, we cannot attribute the latter result to the fact that these shocks were correlated. As a matter of fact, by definition these structural shocks are orthogonal to any other source of disturbance. Second, we are capturing the portion of structural change that can be considered as
non-systematic, i.e. different than changes in the operating procedures represented by changes in parameters of matrices $A_{t}$ and $C_{t}$.

## 7 Concluding Remarks

We have estimated the monetary policy stance index for the U.S. economy for the period 19742016, taking into account the time-varying operating procedures. To the best of our knowledge, this is the first paper that presents a monetary policy stance index with error bands. Moreover, we identify periods of loose and tight monetary policy, in line with previous literature, and we also quantify the relative importance of instruments in this process. For the last decade, we can observe on average a loose stance, a situation that is reverted before the outbreak of the Great Recession, with a tight period governed by the drop in Non-Borrowed reserves and a subsequent loose stance after the implementation of Unconventional Monetary Policies starting in 2009. Furthermore, the identified monetary policy shocks exhibit an important change in their transmission mechanism, especially for the period related with the Great Recession.

Overall, this paper presents a powerful approach that is capable of capturing different episodes regarding monetary policy design, especially the so-called Unconventional Monetary Policies. Even more important is the fact that we present here a monetary policy stance index with error bands, which we expect to be useful for both policy makers and researchers.

It remains to be explored the role of communication in Unconventional Monetary Policies, the announcement of future paths of interest rates (forward guidance) and monetary authority reputation. We believe that these type of issues should be explored in a richer setup and we leave it for future agenda.

## A Sampling Parameter blocks

This section takes an extended version of the algorithm described in Canova and Pérez Forero (2015). We describe the sampling procedure for parameter blocks ( $\left.B^{T}, \alpha^{T}, \widetilde{c}^{T}, \Sigma^{T}, s^{T}, V\right)$ and we do it sequentially using the logic of Gibbs Sampling (see Chib (2001)). We emphasize how to sample blocks $\left(\alpha^{T}, \widetilde{c}^{T}\right)$ and we refer the reader to Primiceri (2005)'s Appendix A for specific details regarding sampling blocks $\left(B^{T}, \Sigma^{T}, s^{T}, V\right)$.

## A. 1 Setting the State Space form for matrices $A_{t}$ and $C_{t}^{-1}$

Consider the state space model generated after sampling the reduced-form coefficients $\widehat{B}_{t}$. From (6) let

$$
A_{t}\left(y_{t}-X_{t}^{\prime} \widehat{B}_{t}\right)=A_{t} \widehat{y}_{t}=C_{t} \Sigma_{t} \varepsilon_{t}
$$

Then the state-space form can be written as

$$
\begin{gather*}
\widetilde{y}_{t}=Z_{\alpha, t} \alpha_{t}+C_{t} \Sigma_{t} \varepsilon_{t}  \tag{21}\\
\alpha_{t}=\alpha_{t-1}+\zeta_{t} \tag{22}
\end{gather*}
$$

where $\widetilde{y}_{t}$ and $Z_{\alpha, t}$ are defined in subsection A.4, $\alpha_{t}$ are the free elements in $A_{t}$ and $\operatorname{Var}\left(\zeta_{t}\right)=S_{a}$.
Similarly, consider the following state space model generated after sampling the vector $\alpha_{t}$. From (6) let

$$
C_{t}^{-1} A_{t}\left(y_{t}-X_{t}^{\prime} \widehat{B}_{t}\right)=C_{t}^{-1} \widetilde{y}_{t}=\vec{y}_{t}=\Sigma_{t} \varepsilon_{t}
$$

Then the state-space form can be written as

$$
\begin{gather*}
\vec{y}_{t}=Z_{\widetilde{c}, t} \widetilde{c}_{t}+\Sigma_{t} \varepsilon_{t}  \tag{23}\\
\widetilde{c}_{t}=\widetilde{c}_{t-1}+\varrho_{t} \tag{24}
\end{gather*}
$$

where $Z_{\widetilde{c}, t}$ is also defined in subsection A.4, $\widetilde{c}_{t}$ are the free elements in $C_{t}^{-1}$ and $\operatorname{Var}\left(\varrho_{t}\right)=S_{\widetilde{c}}$.

## A. 2 The algorithm

In this section we closely follow Canova and Pérez Forero (2015)'s extension to Primiceri (2005)'s procedure. Let $\left\{\left\{s_{l, t}\right\}_{t=1}^{T}\right\}_{l=1}^{M}$ be a discrete indicator variable which takes $j=1, \ldots, k$ possible values. The procedure has 8 steps and 6 sampling blocks:

1. Set an initial value for $\left(B_{0}^{T}, \alpha_{0}^{T}, \widetilde{c}_{0}^{T}, \Sigma_{0}^{T}, s_{0}^{T}, V_{0}\right)$ and set $i=1$.
2. Draw $B_{i}^{T}$ from from $p\left(B_{i}^{T} \mid \alpha_{i-1}^{T}, \widetilde{c}_{i-1}^{T}, \Sigma_{i-1}^{T}, s_{i-1}^{T}, V_{i-1}\right) \cdot I_{B}\left(B_{i}^{T}\right)$ using kalman smoothed estimates $B_{t \mid T}$ obtained from the system (6) - (7) and compute $\widehat{y}_{i}^{T}$, where $I_{B}($.$) truncates$ the posterior distribution to insure the stability of the companion form.
3. Draw $\alpha_{i}^{T}$ from

$$
\begin{align*}
p\left(\alpha_{i}^{T} \mid \widehat{y}_{i}^{T}, \widetilde{c}_{i-1}^{T}, \Sigma_{i-1}^{T}, s_{i-1}^{T}, V_{i-1}\right)= & p\left(\alpha_{i, T} \mid \widehat{y}_{i}^{T}, \widetilde{c}_{i-1, T}, \Sigma_{i-1, T}, s_{i-1, T}, V_{i-1}\right) \times  \tag{25}\\
& \prod_{t=1}^{T-1} p\left(\alpha_{i, t} \mid \alpha_{i, t+1}, \widehat{y}_{i}^{t}, \widetilde{c}_{i-1, t}, \Sigma_{i-1, t}, s_{i-1, t}, V_{i-1}\right) \\
\propto & p\left(\alpha_{i, T} \mid \widehat{y}_{i}^{T}, \widetilde{c}_{i-1, T}, \Sigma_{i-1, T}, s_{i-1, T}, V_{i-1}\right) \times \\
& \prod_{t=1}^{T-1} p\left(\alpha_{i, t} \mid \widehat{y}_{i}^{t}, \widetilde{c}_{i-1, t}, \Sigma_{i-1, t}, s_{i-1, t}, V_{i}\right) \times \\
& f_{t+1}\left(\alpha_{i, t+1} \mid \alpha_{i, t}, \widetilde{c}_{i-1, t}, \Sigma_{i-1, t}, s_{i-1, t}, V_{i-1}\right)
\end{align*}
$$

4. Draw $\widetilde{c}_{i}^{T}$ from

$$
\begin{align*}
p\left(\widetilde{c}_{i}^{T} \mid \vec{y}_{i}^{T}, \alpha_{i}^{T}, \Sigma_{i-1}^{T}, V_{i-1}\right)= & p\left(\widetilde{c}_{i, T} \mid \vec{y}_{i}^{T}, \alpha_{i, T}, \Sigma_{i-1, T}, s_{i-1, T}, V_{i-1}\right)  \tag{26}\\
& \times \prod_{t=1}^{T-1} p\left(\widetilde{c}_{i, t} \mid \widetilde{c}_{i, t+1}, \vec{y}_{i}^{t}, \alpha_{i, t}, \Sigma_{i-1, t}, s_{i-1, t}, V_{i-1}\right) \\
\propto & p\left(\widetilde{c}_{i, T} \mid \vec{y}_{i}^{T}, \alpha_{i, T}, \Sigma_{i-1, T}, s_{i-1, T}, V_{i-1}\right) \\
& \times \prod_{t=1}^{T-1} p\left(\widetilde{c}_{i, t} \mid \vec{y}_{i}^{t}, \alpha_{i, t}, \Sigma_{i-1, t}, s_{i-1, t}, V_{i-1}\right) \\
& \times f_{t+1}\left(\widetilde{c}_{i, t+1} \mid \widetilde{c}_{i, t}, \alpha_{i, t}, \Sigma_{i-1, t}, s_{i-1, t}, V_{i-1}\right)
\end{align*}
$$

5. Draw $\Sigma_{i}^{T}$ using a log-normal approximation to their distribution as in Kim et al. (1998). After sampling $\left(B_{i}^{T}, \alpha_{i}^{T}, \widetilde{c}_{i}^{T}\right)$, the state space is linear but the error term is not normally
distributed. To see this, note that given $\left(B_{i}^{T}, \alpha_{i}^{T}, \widetilde{c}_{i}^{T}\right)$, the state space system is

$$
\widehat{C}_{t}^{-1} \widehat{A}_{t} \widetilde{y}_{t}=y_{t}^{* *}=\Sigma_{t} \varepsilon_{t}
$$

and (10). Consider the $i-t h$ equation $y_{i, t}^{* *}=\sigma_{i, t} \varepsilon_{i, t}$, where $i=1, \ldots, M, \sigma_{i, t}$ is the $i-$ th element in the main diagonal of $\Sigma_{t}$ and $\varepsilon_{i, t}$ is the $i$-th element of $\varepsilon_{t}$.

$$
\begin{equation*}
y_{t}^{*}=\log \left[\left(y_{i, t}^{* *}\right)^{2}+\bar{c}\right] \approx 2 \log \left(\sigma_{i, t}\right)+\log \varepsilon_{i, t}^{2} \tag{27}
\end{equation*}
$$

Then where $\bar{c}$ is a small constant. Since $\varepsilon_{i, t}$ is Gaussian, $\log \varepsilon_{i, t}^{2}$ is $\log \left(\chi^{2}\right)$ distributed and we approximate this distribution by a mixture of normals. Since conditional on $s_{t}$, the model is linear and Gaussian, standard Kalman Filter recursions can be used to draw $\left\{\Sigma_{t}\right\}_{t=1}^{T}$ from the system (27) - (10). To ensure independence across structural variances, each element of the sequence $\left\{\sigma_{i, t}\right\}_{i=1}^{M}$ is sampled assuming that the covariance matrix $W$ is diagonal.
6. Draw the indicator of mixture of normals $s_{i}^{T}$. Conditional on $\Sigma_{i}^{T}, y_{t}^{*}$, and given $l$ and $t$, we draw $u \sim U(0,1)$ and compare it with the discrete distribution of $s_{l, t}$ which is given by

$$
\begin{aligned}
P\left(s_{l, t}=j \mid y_{l, t}^{*}, \log \left(\sigma_{l, t}\right)\right) & \propto q_{j} \times \phi\left(\frac{y_{l, t}^{*}-2 \log \left(\sigma_{l, t}\right)-m_{j}+1.2704}{v_{j}}\right) ; \\
j & =1, \ldots, k ; l=1, \ldots, M
\end{aligned}
$$

where $\phi($.$) is the probability density function of a normal distribution, and the argument$ of this function is the standardized error term $\log \varepsilon_{l, t}^{2}$ (see Kim et al. (1998)). Then we assign $s_{l, t}=j$ iff $P\left(s_{l, t} \leq j-1 \mid y_{l, t}^{*}, \log \left(\sigma_{l, t}\right)\right)<u \leq P\left(s_{l, t} \leq j \mid y_{l, t}^{*}, \log \left(\sigma_{l, t}\right)\right)$.
7. Draw $V_{i}$ from $P\left(V_{i} \mid \alpha_{i}^{T}, \tilde{c}_{i}^{T}, y_{i}^{T}, \Sigma_{i-1}^{T}, s_{i-1}^{T}\right)$ using definitions (7) - (10). The covariance matrix $V_{i}$ is sampled assuming that each block follows an independent Wishart distribution.
8. Set $B_{i}^{T}, \alpha_{i}^{T}, \widetilde{c}_{i}^{T}, \Sigma_{i}^{T}, s_{i}^{T}, V_{i}$ as the initial value for the next iteration and set $i=i+1$.

Repeat 2 to 7 if $i<N$, otherwise stop.

The complete cycle of draws is repeated $N=N b+N d$ times and the first $N b$ draws are discarded to ensure convergence in distribution. Because the draws are generally serially correlated, one every $n t h i n$ of the last $N d$ draws is used for inference.

## A. 3 The details in steps 3 and 4

This subsection follows closely Canova and Pérez Forero (2015). For steps 3 and 4 we use a Metropolis step to determine whether a draw from a proposal distribution is retained or not. We only illustrate the case of sampling vector $\alpha_{t}$, since sampling vector $\widetilde{c}_{t}$ will be completely symmetric following the Multi-move Gibbs Sampling logic. The densities $p\left(\alpha_{t} \mid \widehat{y}^{t}, \widetilde{c}_{t}, \Sigma_{t}, s, V\right)$ are obtained applying the Extended Kalman Smoother (see subsection A.5) to the original system $(21)-(22)$. To draw $\alpha_{i}^{T}$ given $\widehat{y}_{i}^{T}, \tilde{c}_{i-1}^{T}, \Sigma_{i-1}^{T}, s_{i-1}^{T}, V_{i-1}$, we proceed as follows:

1. If $i=0$, take an initial value $\alpha_{0}^{T}=\left\{\alpha_{0, t}\right\}_{t=1}^{T}$. If not,
2. Given $\widehat{y}_{i}^{T}, \widetilde{c}_{i-1}^{T}, \Sigma_{i-1}^{T}, s_{i-1}^{T}, V_{i-1}$, set the state space form and compute $\left\{\alpha_{t \mid t+1}^{*(i)}\right\}_{t=1}^{T}$ and $\left\{P_{t \mid t+1}^{*(i)}\right\}_{t=1}^{T}$ using the EKS where $\left\{P_{t \mid t+1}^{*}\right\}_{t=1}^{T}$ denotes the covariance matrix of $\left\{\alpha_{t \mid t+1}^{*}\right\}_{t=1}^{T}$.
3. Generate a candidate draw $z^{T}=\left\{z_{t}\right\}_{t=1}^{T}$, where for each $t=1, \ldots, T p_{* \alpha}\left(z_{t}\right)=N\left(\alpha_{t \mid t+1}^{*(i)}, r P_{t \mid t+1}^{*(i)}\right)$, and $r$ is a constant. Let $p_{* \alpha}\left(z^{T}\right)=\prod_{t=1}^{T} p_{* \alpha}\left(z_{t}\right)$.
4. Compute $\theta=\frac{p\left(z^{T}\right)}{p\left(\alpha_{i-1}^{T}\right)}$ where $p($.$) is the RHS of (25) using the EKS approximation. Draw$ a $v \sim U(0,1)$. Set $\alpha_{i}^{T}=z^{T}$ if $v<\omega$ and set $\alpha_{i}^{T}=\alpha_{i-1}^{T}$ otherwise, where

$$
\omega \equiv\left\{\begin{array}{cc}
\min \{\theta, 1\}, & \text { if } I_{\alpha}\left(z^{T}\right)=1 \\
0, & \text { if } I_{\alpha}\left(z^{T}\right)=0
\end{array}\right.
$$

and $I_{\alpha}($.$) is a truncation indicator.$

Finally, steps 2 to 4 in this sub-loop are repeated every time step 3 of the main loop is executed.

## A. 4 The identified system

Recall the expression

$$
A_{t} \widehat{y}_{t}=C_{t} \Sigma_{t} \varepsilon_{t}
$$

The expression of the measurement equation is possible to obtain since vec $\left(A_{t} \widehat{y}_{t}\right)=v e c\left(C_{t} \Sigma_{t} \varepsilon_{t}\right)$. Then, using the fact that $A_{t(M \times M)}, \widehat{y}_{t(M \times 1)}, \Sigma_{t(M \times M)}, \varepsilon_{t(M \times 1)}$, then

$$
\operatorname{vec}\left(A_{t} \widehat{y}_{t}\right)=\operatorname{vec}\left(I_{M} A_{t} \widehat{y}_{t}\right)=\left(\widehat{y}_{t}^{\prime} \otimes I_{M}\right) \operatorname{vec}\left(A_{t}\right)
$$

and also

$$
\operatorname{vec}\left(C_{t}\left(\Sigma_{t} \varepsilon_{t}\right)\right)=\left(\left(\Sigma_{t} \varepsilon_{t}\right)^{\prime} \otimes I_{M}\right) \operatorname{vec}\left(C_{t}\right)=\left(I_{1} \otimes C_{t}\right) \operatorname{vec}\left(\Sigma_{t} \varepsilon_{t}\right)=C_{t} \Sigma_{t} \varepsilon_{t}
$$

since $A_{t} \widehat{y}_{t}$ and $C_{t} \Sigma_{t} \varepsilon_{t}$ are already column vectors ${ }^{18}$. On the other hand, following Amisano and Giannini (1997) and Hamilton (1994), we also know that the matrix of the SVAR can be decomposed as follows

$$
\begin{gather*}
\operatorname{vec}\left(A_{t}\right)=S_{A} \alpha_{t}+s_{A}  \tag{28}\\
\operatorname{vec}\left(C_{t}^{-1}\right)=S_{C} F\left(c_{t}\right)+s_{C} \tag{29}
\end{gather*}
$$

where $S_{A\left(M^{2} \times \operatorname{dim} \alpha\right)}, s_{A\left(M^{2} \times 1\right)}, S_{C\left(M^{2} \times \operatorname{dim} F(c)\right)}$ and $s_{C\left(M^{2} \times 1\right)}$ are matrices filled by ones and zeros. Moreover, $\alpha_{t}, c_{t}$ and $F\left(c_{t}\right)$ are the vectors of free parameters in $A_{t}, C_{t}$ and $C_{t}^{-1}$, respectively and $F():. R^{\operatorname{dim}(c)} \rightarrow R^{\operatorname{dim} F(c)}$ is in general a nonlinear invertible function. That is, we sample the vector $\left\{F\left(c_{t}\right)\right\}_{t=1}^{T}$ and if and only if $F($.$) is invertible, then we can recover$ $\left\{c_{t}\right\}_{t=1}^{T}=\left\{F^{-1}\left[F\left(c_{t}\right)\right]\right\}_{t=1}^{T}$. We will denote $\widetilde{c}_{t}=F\left(c_{t}\right)$. Collecting all the results we get

$$
\left(\widehat{y}_{t}^{\prime} \otimes I_{M}\right)\left(S_{A} \alpha_{t}+s_{A}\right)=C_{t} \Sigma_{t} \varepsilon_{t}
$$

[^14]Rewriting this equation

$$
\begin{gathered}
\left(\widehat{y}_{t}^{\prime} \otimes I_{M}\right) S_{A} \alpha_{t}+\left(\widehat{y}_{t}^{\prime} \otimes I_{M}\right) s_{A}=C_{t} \Sigma_{t} \varepsilon_{t} \\
\left(\widehat{y}_{t}^{\prime} \otimes I_{M}\right) s_{A}=-\left(\widehat{y}_{t}^{\prime} \otimes I_{M}\right) S_{A} \alpha_{t}+C_{t} \Sigma_{t} \varepsilon_{t}
\end{gathered}
$$

The state space form is now

$$
\begin{gathered}
\widetilde{y}_{t}=Z_{\alpha, t} \alpha_{t}+C_{t} \Sigma_{t} \varepsilon_{t} \\
\alpha_{t}=\alpha_{t-1}+\zeta_{t}
\end{gathered}
$$

where

$$
\begin{align*}
\widetilde{y}_{t} & \equiv\left(\widehat{y}_{t}^{\prime} \otimes I_{M}\right) s_{A}  \tag{30}\\
Z_{\alpha, t} & \equiv-\left(\widehat{y}_{t}^{\prime} \otimes I_{M}\right) S_{A} \tag{31}
\end{align*}
$$

On the other hand, given $\alpha_{t}$, we proceed in the following way

$$
\begin{gathered}
\operatorname{vec}\left(C_{t}^{-1} A_{t} \widehat{y}_{t}\right)=\operatorname{vec}\left(\left(\Sigma_{t} \varepsilon_{t}\right)\right)=\Sigma_{t} \varepsilon_{t} \\
\left(\left(A_{t} \widehat{y}_{t}\right)^{\prime} \otimes I_{M}\right) \operatorname{vec}\left(C_{t}^{-1}\right)=\Sigma_{t} \varepsilon_{t} \\
\left(\left(A_{t} \widehat{y}_{t}\right)^{\prime} \otimes I_{M}\right)\left(S_{C} \widetilde{c}_{t}+s_{C}\right)=\Sigma_{t} \varepsilon_{t} \\
\left(\left(A_{t} \widehat{y}_{t}\right)^{\prime} \otimes I_{M}\right) s_{C}=-\left(\left(A_{t} \widehat{y}_{t}\right)^{\prime} \otimes I_{M}\right) S_{C} \widetilde{c}_{t}+\Sigma_{t} \varepsilon_{t}
\end{gathered}
$$

The state space form is now

$$
\begin{gathered}
\vec{y}_{t}=Z_{C, t} \widetilde{c}_{t}+\Sigma_{t} \varepsilon_{t} \\
\widetilde{c}_{t}=\widetilde{c}_{t-1}+\varrho_{t}
\end{gathered}
$$

where

$$
\begin{gather*}
\vec{y}_{t} \equiv\left(\left(A_{t} \widehat{y}_{t}\right)^{\prime} \otimes I_{M}\right) s_{C}  \tag{32}\\
Z_{C, t} \equiv-\left(\left(A_{t} \widehat{y}_{t}\right)^{\prime} \otimes I_{M}\right) S_{C} \tag{33}
\end{gather*}
$$

Moreover, for the specific case of the model presented, we have that $\widehat{y}_{t}$ denotes the residuals for the first stage and also the matrices

$$
A_{t}=\left[\begin{array}{cccccc}
1 & 0 & 0 & 0 & 0 & 0  \tag{34}\\
\alpha_{x, t}^{\pi} & 1 & 0 & 0 & 0 & 0 \\
\alpha_{x, t}^{c} & \alpha_{\pi, t}^{c} & 1 & 0 & 0 & 0 \\
\alpha_{x, t}^{T R} & \alpha_{\pi, t}^{T R} & \alpha_{c, t}^{T R} & 1 & \alpha_{1, t}^{d} & 0 \\
\alpha_{x, t}^{F F R} & \alpha_{\pi, t}^{F F R} & \alpha_{c, t}^{F F R} & -1 / \alpha_{1, t}^{b} & 1 & 1 / \alpha_{1, t}^{b} \\
\alpha_{x, t}^{N B R} & \alpha_{\pi, t}^{N B R} & \alpha_{c, t}^{N B R} & 0 & 0 & 1
\end{array}\right]
$$

and

$$
C_{t}=\left[\begin{array}{llllll}
1 & 0 & 0 & 0 & 0 & 0  \tag{35}\\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & \phi_{t}^{d} & \phi_{t}^{b} & 1
\end{array}\right]
$$

therefore, $M=6, \operatorname{dim} \alpha=13$. As it is evident, $C_{t}$ is a lower-triangular matrix with the main diagonal governed by ones, i.e. a unitriangular matrix. Moreover, $C_{t}^{\prime}$ will be unitriangular as well and in this case it can also be classified as a Frobenius matrix ${ }^{19}$. The inverse of a Frobenius matrix $X$ is exactly $X^{-1}=-X$. Thus, provided by the fact that $\left[C_{t}^{\prime}\right]^{-1}=\left[C_{t}^{-1}\right]^{\prime}$, we have that $\left[C_{t}^{\prime}\right]^{-1}=\left[C_{t}^{-1}\right]^{\prime}=-C_{t}^{\prime} \Rightarrow C_{t}^{-1}=-C_{t}$, hence

$$
C_{t}^{-1}=\left[\begin{array}{cccccc}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & -\phi_{t}^{d} & -\phi_{t}^{b} & 1
\end{array}\right]
$$

[^15]and $\operatorname{dim} F(c)=\operatorname{dim} \widetilde{c}=2$. Therefore, it turns out that in this particular case the function $F($. is actually a linear transformation, i.e. $\widetilde{c}_{t}=F\left(c_{t}\right)=-c_{t}$. That is, as long as $C_{t}^{\prime}$ is Frobenius matrix, $F($.$) will be linear with a well defined F^{-1}($.$) .$

As a result, we need to define matrices $S_{A}, s_{A}, S_{C}, s_{C}$ filled by 0 s and 1 s . These matrices, together with the column vectors

$$
\alpha_{t}=\left[\begin{array}{c}
\alpha_{x, t}^{\pi}, \alpha_{x, t}^{P c o m}, \alpha_{x, t}^{T R}, \alpha_{x, t}^{N B R}, \alpha_{x, t}^{F F R}, \alpha_{\pi, t}^{P c}, \alpha_{\pi, t}^{T R}, \\
\alpha_{\pi, t}^{N B R}, \alpha_{\pi, t}^{F F R}, \alpha_{P c, t}^{T R}, \alpha_{P c, t}^{N B R}, \alpha_{P c, t}^{F R}, \alpha_{1, t}^{b}
\end{array}\right]^{\prime}
$$

and

$$
\widetilde{c}_{t}=\left[-\phi_{t}^{d},-\phi_{t}^{b}\right]^{\prime}
$$

are set such that equations (28) and (29) hold exactly.

## A. 5 Extended Kalman Smoother

This subsection follows closely Canova and Pérez Forero (2015). Consider the nonlinear system

$$
\begin{gathered}
y_{t}=z_{t}\left(\alpha_{t}, \varepsilon_{t}\right) \\
\alpha_{t}=t_{t}\left(\alpha_{t-1}, \eta_{t}\right)
\end{gathered}
$$

with $\operatorname{Var}\left(\varepsilon_{t}\right)=Q_{t}^{\varepsilon}$ and $\operatorname{Var}\left(\eta_{t}\right)=Q_{t}^{\eta}$. To apply the EKS to our system of equations, we first compute the jacobians:

$$
\begin{align*}
& \widehat{Z}_{\alpha, t}=\frac{\partial z_{t}}{\partial \alpha_{t}}\left(\widehat{a}_{t \mid t-1}, 0\right) ; \widehat{Z}_{\varepsilon, t}=\frac{\partial z_{t}}{\partial \varepsilon_{t}}\left(\widehat{a}_{t \mid t-1}, 0\right)  \tag{36}\\
& \widehat{T}_{\alpha, t}=\frac{\partial t_{t}}{\partial \alpha_{t}}\left(\widehat{a}_{t \mid t-1}, 0\right) ; \widehat{T}_{\eta, t}=\frac{\partial t_{t}}{\partial \eta_{t}}\left(\widehat{a}_{t \mid t-1}, 0\right)
\end{align*}
$$

and we predict the mean and variance at each $t=1, \ldots, T$ :

$$
\widehat{a}_{t \mid t-1}=t_{t-1}\left(\widehat{a}_{t-1 \mid t-1}, 0\right)
$$

$$
P_{t \mid t-1}=\widehat{T}_{\alpha, t} P_{t-1 \mid t-1} \widehat{T}_{\alpha, t}^{\prime}+\widehat{T}_{\eta, t} Q_{t}^{\eta} \widehat{T}_{\eta, t}^{\prime}
$$

The Kalman gain is computed:

$$
\begin{gathered}
\Omega_{t}=\widehat{Z}_{\alpha, t} P_{t \mid t-1} \widehat{Z}_{\alpha, t}^{\prime}+\widehat{Z}_{\varepsilon, t} Q_{t}^{\varepsilon} \widehat{Z}_{\varepsilon, t}^{\prime} \\
K_{t}=P_{t \mid t-1} \widehat{Z}_{\alpha, t}^{\prime} \Omega_{t}^{-1}
\end{gathered}
$$

As new information arrives, estimates of $\alpha_{t}$ and variance are updated according to

$$
\begin{gathered}
\widehat{a}_{t \mid t}=\widehat{a}_{t \mid t-1}+K_{t}\left[y_{t}-z_{t}\left(\widehat{a}_{t \mid t-1}, 0\right)\right] \\
P_{t \mid t}=P_{t \mid t-1}-P_{t \mid t-1} \widehat{Z}_{\alpha, t}^{\prime} \Omega_{t}^{-1} \widehat{Z}_{\alpha, t} P_{t \mid t-1}^{\prime}
\end{gathered}
$$

To smooth the estimates, set $\alpha_{T \mid T}^{*}=\widehat{a}_{T \mid T}, P_{T \mid T}^{*}=P_{T \mid T}$ and , for $t=T-1, \ldots, 1$, compute

$$
\begin{gathered}
\alpha_{t \mid t+1}^{*}=\widehat{a}_{t \mid t}+P_{t \mid t} \widehat{T}_{\alpha, t}^{\prime} P_{t+1 \mid t}^{-1}\left(\alpha_{t+1 \mid t+2}^{*}-t_{t-1}\left(\widehat{a}_{t \mid t}, 0\right)\right) \\
P_{t \mid t+1}^{*}=P_{t \mid t}-P_{t \mid t} \widehat{T}_{\alpha, t}^{\prime} P_{t+1 \mid t}^{-1} \widehat{T}_{\alpha, t} P_{t \mid t-1}^{\prime}
\end{gathered}
$$

To start the iterations we use fixed values $\widehat{a}_{1 \mid 0}=\bar{\alpha}_{N \times 1}$ and $P_{0 \mid 0}=I_{N}$.
Finally, notice that the original $t_{t}($.$) and z_{t}($.$) are used for computing prediction and up-$ dating equations of $\alpha_{t}$.

## B Diagnosis of convergence of the Markov Chain to the Ergodic Distribution

Following Geweke (1992), Primiceri (2005) and Baumeister and Benati (2013) among others, we check for the autocorrelation properties of the different blocks of the Markov Chain via the inefficiency factor. Let the Relative Numerical Efficiency (RNE) be:

$$
\begin{equation*}
R N E=\frac{1}{2 \pi} \frac{1}{S(0)} \int_{-\pi}^{\pi} S(\omega) d \omega \tag{37}
\end{equation*}
$$

where $S($.$) is the spectral density of an element of the Markov Chain. The inefficiency factor$ $I F=1 / R N E$, can be interpreted as an estimate of $\left(1+2 \sum_{k=1}^{\infty} \rho_{k}\right)$, where $\rho_{k}$ denotes the autocorrelation of $k$-th order. Thus, high values of $I F$ indicate strong serial correlation across draws. We use the MATLAB file coda.m from James P. Lesage toolbox to calculate the IF. We set a $4 \%$ tapered window for the estimation of the spectral density at frequency zero and take values around or below 20 as cut-off point (and values below are considered as satisfactory). Figure 8 depicts the $I F$ value for each parameter of the model. Overall, serial correlation across draws does not seem to be an issue.


Figure 8: Inefficiency Factor IF for each parameter in the model

## References

Amisano, G. and C. Giannini (1997). Topics in Structural VAR Econometrics. Springer.
Baumeister, C. and L. Benati (2013). Unconventional monetary policy and the great recession - estimating the impact of a compression in the yield spread at the zero lower bound. International Journal of Central Banking 165, 165-212.

Beckworth, D. (2011). The stance of US monetary policy: An alternative measure. Applied Economics Letters 18(2), 121-125.

Bernanke, B. S. and A. S. Blinder (1992). The federal funds rate and the channels of monetary transmission. American Economic Review 82(4), 901-921.

Bernanke, B. S., J. Boivin, and P. Eliasz (2005). Measuring the effects of monetary policy: A factor-augmented vector autoregressive (FAVAR) approach. Quarterly Journal of Economics 120(1), 387-422.

Bernanke, B. S. and I. Mihov (1998). Measuring monetary policy. Quarterly Journal of Economics 113(3), 869-902.

Blinder, A. S. (2010). Quantitative easing: Entrance and exit strategies. CEPS Working Paper 204.

Borio, C. and P. Disyatat (2010). Unconventional monetary policies: An appraisal. The Manchester School 78(1), 53-89.

Boschen, J. F. and L. O. Mills (1991). The effects of countercyclical monetary policy on money and interest rates: an evaluation of evidence from fomc documents. No 91-20, Working Papers from Federal Reserve Bank of Philadelphia.

Canova, F. (2007). Methods for Applied Macroeconomic Research. Princeton University Press.
Canova, F. and M. Ciccarelli (2009). Estimating multicountry var models. International Economic Review 50(3), 929-959.

Canova, F. and G. De Nicoló (2002). Monetary disturbances matter for business fluctuations in the g-7. Journal of Monetary Economics 49, 1131-1159.

Canova, F. and L. Gambetti (2009). Structural changes in the US economy: Is there a role for monetary policy? Journal of Economic Dynamics and Control 33, 477-490.

Canova, F., L. Gambetti, and E. Pappa (2008). The structural dynamics of u.s. output and inflation: What explains the changes? Journal of Money, Credit and Banking 40(2-3), 369-388.

Canova, F. and F. J. Pérez Forero (2015). Estimating overidentified, nonrecursive, time-varying coefficients structural vector autoregressions. Quantitative Economics 6, 359-384.

Cecioni, M., G. Ferrero, and A. Secchi (2011). Unconventional monetary policy in theory and in practice. Banca D'Italia: Questioni di Economia e Finanza (Occasional papers) No 102.

Chang, Y., S.-B. Kim, and F. Schorfheide (2010). Labor-market heterogeneity, aggregation, and the lucas critique. NBER Working Paper No. 16401.

Chen, H., V. Cúrdia, and A. Ferrero (2012). The macroeconomic effects of large-scale asset purchase programmes. The Economic Journal 122(564), F289-F315.

Chib, S. (2001). Markov Chain Monte Carlo Methods: Computation and Inference, Chapter 57, pp. 3569-3649. J.J. Heckman and E.E. Leamer (ed.) Handbook of Econometrics.

Christiano, L. and M. Eichenbaum (1992). Identification and the Liquidity Effect of a Monetary Policy Shock. in A. Cukierman, Z. Hercowitz and L. Leiderman, eds., Political Economy, Growth, and Business Cycles (Cambridge MA: MIT Press).

Christiano, L., M. Eichenbaum, and C. Evans (1999). Monetary Policy Shocks: What Have We Learned and to What End? Handbook of Macroeconomics, Ed.1,Vol. 1,Ch.II. Edited by J.B. Taylor and M. Woodford. Elsevier Science B.V. All rights reserved.

Christiano, L. J. and M. Eichenbaum (1991). Identification and the liquidity effect of a monetary policy shock. NBER Working Paper No 3920.

Cogley, T. and T. J. Sargent (2005). Drifts and volatilities: Monetary policies and outcomes in the post WWII u.s. Review of Economic Dynamics 8(2), 262-302.

Cogley, T. and T. Yagihashi (2010). Are DSGE approximating models invariant to shifts in policy? The B.E. Journal of Macroeconomics 10(1). Article 27.

Cosimano, T. F. and D. W. Jansen (1988). Federal reserve policy, 1975-1985: An empirical analysis. Journal of Macroeconomics 10(1), 27-47.

Cosimano, T. F. and R. G. Sheehan (1994). The federal reserve operating procedure, 1984-1990: An empirical analysis. Journal of Macroeconomics 16(4), 573-588.

Cúrdia, V. and M. Woodford (2011). The central-bank balance sheet as an instrument of monetary policy. Journal of Monetary Economics 58, 54-79.

Del Negro, M. (2003). Discussion of cogley and sargent's "drifts and volatilities: Monetary policy and outcomes in the post WWII u.s.". Federal Reserve Bank of Atlanta. Working Paper 2003-26.

Gertler, M. and P. Karadi (2011). A model of unconventional monetary policy. Journal of Monetary Economics 58, 17-34.

Geweke, J. (1992). Evaluating the accuracy of samplingbased approaches to the calculation of posterior moments, Chapter 4, pp. 169-193. Bayesian Statistics - Bernardo, J. M., Berger, J. O., Dawid, A. P. and Smith, A. F. M. (eds.). Oxford University Press.

Goodfriend, M. and R. G. King (2005). The incredible volcker disinflation. Journal of Monetary Economics 52, 981-1015.

Hamilton, J. D. (1994). Time Series Analysis. Princeton University Press.

Hamilton, J. D. and J. Wu (2012). The effectiveness of alternative monetary policy tools in a zero lower bound environment. Journal of Money, Credit and Banking 44 (1), 3-46.

Justiniano, A. and G. E. Primiceri (2008). The time-varying volatility of macroeconomic fluctuations. American Economic Review 98(3), 604-641.

Kilian, L. (2012). Structural vector autoregressions. Unpublished manuscript.

Kim, S., N. Shephard, and S. Chib (1998). Stochastic volatility: Likelihood inference and comparison with ARCH models. The Review of Economic Studies 65(3), 361-393.

Leeper, E., C. Sims, and T. Zha (1996). What does monetary policy do? Brookings Papers on Economic Activity 2.

Lenza, M., H. Pill, and L. Reichlin (2010). Monetary policy in exceptional times. European Central Bank, Working Paper Series No 1253.

Lütkepohl, H. (2005). New Introduction to Multiple Time Series Analysis. Springer.

Magnus, J. R. and H. Neudecker (2007). Matrix Differential Calculus with Applications in Statistics and Econometrics (Third Edition ed.). Wiley Series in Probability and Statistics. John Wiley and Sons.

Peersman, G. (2011). Macroeconomic effects of unconventional monetary policy in the euro area.

Primiceri, G. (2005). Time varying structural vector autoregressions and monetary policy. Review of Economic Studies 72, 821-852.

Reis, R. (2009). Interpreting the unconventional u.s. monetary policy of 2007-09: Comments and discussion. Brookings Papers on Economic Activity, 119-165.

Rubio-Ramírez, J. F., D. F. Waggoner, and T. Zha (2010). Structural vector autoregressions: Theory of identification and algorithms for inference. Review of Economic Studies 77, 665696.

Sims, C. A. (1986). Are forecasting models usable for policy analysis? Federal Reserve Bank of Minneapolis, Quarterly Review, 2-16.

Sims, C. A. and T. Zha (2006). Were there regime switches in u.s. monetary policy? American Economic Review 96(1), 54-81.

Strongin, S. (1995). The identification of monetary policy disturbances: Explaining the liquidity puzzle. Journal of Monetary Economics 35, 463-497.

Uhlig, H. (2005). What are the effects of monetary policy on output? results from an agnostic identification procedure. Journal of Monetary Economics 52, 381-419.

Williams, J. C. (2011). Unconventional monetary policy: Lessons from the past three years. FRBSF Economic Letter 2011-31 (October 3).

Williams, J. C. (2012a). The federal reserves unconventional policies. FRBSF Economic Letter 2012-34 (November 13).

Williams, J. C. (2012b). Monetary policy, money, and inflation. FRBSF Economic Letter 2012-21 (July 9).

Wu, J. C. and F. D. Xia (2015). Measuring the macroeconomic impact of monetary policy at the zero lower bound. Chicago Booth Research Paper No. 13-77.


[^0]:    *This paper is a modified version of the chapter 2 of my dissertation titled Essays in Structural Macroeconometrics. I would like to thank Fabio Canova and Kristoffer Nimark for their advising during my PhD at Universitat Pompeu Fabra. I would also like to thank Christian Matthes, Luca Gambetti, Marek Jarocinski, seminar participants of Universitat Pompeu Fabra (2011-2013), seminar participants at the Central Reserve Bank of Peru, CEF 2017 participants at New York and LACEA-LAMES 2017 participants at Buenos Aires and Marco Vega for their helpful comments and suggestions. The views expressed are those of the author and do not necessarily reflect those of the Central Bank of Peru. All remaining errors are mine.
    ${ }^{\dagger}$ Head of Monetary Programming Department, Central Reserve Bank of Peru (BCRP), Jr. Santa Rosa 441, Lima 1, Perú; Email address: fernando.perez@bcrp.gob.pe

[^1]:    ${ }^{1}$ See also Sims (1986) and Leeper et al. (1996).
    ${ }^{2}$ See Rubio-Ramírez et al. (2010) and Kilian (2012) for recent surveys related with SVARs and identification.

[^2]:    ${ }^{3}$ See Williams (2011), Williams (2012b) and Williams (2012a) for a detailed description of UMP. See also Borio and Disyatat (2010) and Cecioni et al. (2011) for a thorough survey of the different dimensions of UMPs.

[^3]:    ${ }^{4}$ Reis (2009), Blinder (2010), Lenza et al. (2010) and Hamilton and Wu (2012) present the main characteristics of UMPs, emphasizing the role of yield curve spreads as a powerful indicator that summarizes both credit policy as well as the expectations of future paths for interest rates (Forward Guidance).

[^4]:    ${ }^{5}$ The reason of why we are focused on $C_{t}^{-1}$ instead of $C_{t}$ is for computational convenience after the construction of the State-Space model (see Appendix A. 4 for details). For instance, denote $\bar{c}_{t}$ as the vector of free parameters of $C_{t}$. Then, if $C_{t}$ is lower-triangular with ones in the main diagonal (which is indeed the case here), then the set of free parameters of $C_{t}^{-1}$ will be simply the vector $\widetilde{c}_{t}=-\bar{c}_{t}$. Thus, recovering the original parameters will be straightforward.

[^5]:    ${ }^{6}$ We use the shadow interest rate estimated by Wu and Xia (2015) for the period 2008-2015.

[^6]:    ${ }^{7}$ According to Amisano and Giannini (1997) and Lütkepohl (2005) (ch. 9), the model presented in is one version of the $A B$ model. See the mentioned references for details.

[^7]:    ${ }^{8}$ However, Cogley and Sargent (2005) as well as Primiceri (2005) find that changing the ordering of these variables did not affect their results significantly.

[^8]:    ${ }^{9}$ See Cogley and Yagihashi (2010), Chang et al. (2010) and Canova and Pérez Forero (2015) for more details about this issue.
    ${ }^{10}$ We have considered that now $\sqrt{\operatorname{var}\left(v_{t}^{b}\right)}=\sigma_{t}^{b} /\left|\alpha_{1, t}^{b}\right|$. See also ?) for a similar approach using Bernanke and Mihov (1998)'s model.

[^9]:    ${ }^{11}$ See for instance Reis (2009) and Blinder (2010).

[^10]:    ${ }^{12}$ https://fred.stlouisfed.org, https://www.federalreserve.gov/releases/h3/current/,
    ${ }^{13}$ We have also tried to regress Total and Non-Borrowed Reserves onto a constant and linear trend with breaks in the third quarter of 2008 . However, the obtained residuals are extremely volatile.

[^11]:    ${ }^{14}$ The MATLAB code csminwel.m from professor C. Sims website is used (http://sims.princeton.edu/yftp/optimize/mfiles/). I have chosen randomly 100 different starting points in order to find a global maximum.
    ${ }^{15}$ Alternatively, we could use a Minnesota-style prior for calibrating $(\overline{\mathbf{B}}, \overline{\mathbf{V B}})$ (see Del Negro (2003), Canova (2007) (ch.10), among others), but we do not cover this issue here.

[^12]:    ${ }^{16}$ See Goodfriend and King (2005).

[^13]:    ${ }^{17}$ See. e.g. Christiano and Eichenbaum (1991), Strongin (1995), Christiano et al. (1999), among others.

[^14]:    ${ }^{18}$ In general, we have applied the property $A B d=\left(d^{\prime} \otimes A\right) \operatorname{vec}(B)$, where $A$ is an $m \times n$ matrix, $B$ is an $n \times q$ matrix and $d$ is a ( $q \times 1$ ) vector. See details in Magnus and Neudecker (2007), chapter 2, pp. 35.

[^15]:    ${ }^{19} \mathrm{http}: / /$ en.wikipedia.org/wiki/Frobenius_matrix

