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Monetary policy spillovers, global commodity prices and cooperation

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Abstract

How do monetary policy spillovers complicate the trade-offs faced by central banks face when responding to commodity prices? This question takes on particular relevance when monetary authorities find it difficult to accurately diagnose the drivers of commodity prices. If monetary authorities misdiagnose commodity price swings as being driven primarily by external supply shocks when they are in fact driven by global demand shocks, this conventional wisdom – to look through the first-round effects of commodity price fluctuations – may no longer be sound policy advice.

To analyse this question, we use the multi-country DSGE model of Nakov and Pescatori (2010) which breaks the global economy down into commodity-exporting and non-commodity-exporting economies. In an otherwise conventional DSGE setup, commodity prices are modelled as endogenously changing with global supply and demand developments, including global monetary policy conditions. This framework allows us to explore the implications of domestic monetary policy decisions when there is a risk of misdiagnosing the drivers of commodity prices.

The main findings are: i) monetary authorities deliver better economic performance when they are able to accurately identify the source of the shocks, ie global supply and demand shocks driving commodity prices; ii) when they find it difficult to identify the supply and demand shocks, monetary authorities can limit the deterioration in economic performance by targeting core inflation; and iii) the conventional wisdom approach of responding to global commodity price swings (as external supply shocks when they are truly global demand shocks) results in an excessive procyclicality of global inflation, output and commodity prices. In light of recent empirical studies documenting a significant role of global demand in driving commodity prices, we conclude that the systematic misdiagnoses inherent in the conventional wisdom applied at the country level have contributed to destabilising procyclicality at the global level. These findings support calls for greater attention to global factors in domestic monetary policymaking and highlight potential gains from greater monetary policy cooperation focused on accurate diagnoses of domestic and global sources of shocks.

Keywords: commodity prices, monetary policy, spillovers, global economy.

JEL classification: E52, E61.

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I. Introduction

Over the past decade, global commodity prices have experienced wide swings, reaching historically high levels in the run-up to the Great Recession before plummeting as the global economy collapsed. Prices subsequently rebounded with the global economic recovery but, more recently, commodity prices have fallen again amid significant policy concerns. While challenging, this type of volatility is not a new environment for policymakers. Even though most commodity prices remained broadly stable during the so-called Great Moderation, they were quite volatile in the 1970s amid geopolitical tensions that pushed oil price volatility to then unprecedented levels.

It was the experience of the 1970s that forged the conventional wisdom about how monetary authorities should respond to commodity price fluctuations. Commodity price fluctuations were seen largely as the result of exogenous supply shocks; in such an environment, the conventional wisdom that emerged was that, when facing such swings, monetary authorities should look through the first-round price effects and only respond to the second-round effects on wage and inflation expectations. In practice, this suggested a monetary policy focus on core inflation.

Views about the drivers of global commodity price swings have been evolving, especially in recent years, as a growing body of statistical evidence points to a new interpretation of commodity price swings. Kilian (2009), for example, finds evidence that oil price fluctuations have been increasingly influenced by demand from commodity-hungry emerging market economies (EMEs). The most recent literature has not challenged this view: Kilian and Baumeister (2016) argue that the oil price decline in 2015 should also be ascribed to a slowdown in global economic activity; and Stuermer (2017) and Fukei et al (2018) emphasises the role of demand shocks in a historical perspective. In a similar vein, Sussman and Zohar (2017) report that commodity price fluctuations can be taken as a proxy for global demand. In a broader context, Filardo and Lombardi (2013) note the growing prominence of these global demand shifters for EME inflation dynamics. This new evidence raises doubts about the relevance of the conventional wisdom and may even suggest that the exogenous supply shock view is not only misleading but actually contributing to global economic and financial instability.

The prominence of *endogenous* commodity price swings has important implications for monetary policy, given the central role of monetary policy in influencing aggregate demand. The relationship between monetary policy decisions and endogenous commodity prices implies an important two-way link. Monetary policy decisions influence aggregate demand and hence commodity prices. Indeed, Filardo and Lombardi (2013) and Anzuini et al (2013) report evidence that loose monetary policy has had an impact on commodity prices via the global demand channel.[5] At the same time, commodity price swings influence price stability and hence monetary policy decisions.

This two-way relationship can operate to stabilise the economy under certain conditions and de-stabilise the economy under others. For example, when monetary authorities around the world correctly diagnose the nature of the shocks driving commodity prices and internalise the monetary policy spillovers across national borders, monetary policy can be stabilising. However, when monetary authorities systematically misdiagnose the nature of the shocks driving commodity prices and largely ignore the spillover effects of their collective actions, monetary policy at the global level can be excessively procyclical.

The misdiagnosis risk is particularly high in a world of many central banks with purely domestic monetary policy mandates. Individual countries may think that because they are sufficiently small, they can reasonably ignore the impact of their own policy decisions on the rest of the world. This would be the case if all economies were hit by uncorrelated idiosyncratic shocks.

[5] There is also evidence that US monetary policy may play a special role. Akram (2009) finds that lower interest rates in the United States boost commodity prices through the exchange rate channel. For further discussion about the importance of US exchange rate spillovers, see Hofmann et al (2017).

However, global shocks imply that central banks are likely to respond in a correlated way. So, in the case of a global demand shock, country-level monetary policy responses highlight the potential for a fallacy of size, ie when monetary authorities respond in a similar way to global shocks, the collection of monetary authorities effectively act as if they were a large monetary authority.

The fallacy of size and the potential role of misdiagnosing the drivers of commodity prices also cast light on the shortcomings of the current international monetary system. In this context, questions are being raised about whether monetary authorities are sufficiently internalising monetary policy spillovers and spillbacks. The failure to do so would contribute to both economic and financial stability concerns (Rajan (2014) and Caruana et al (2014)).

From a modelling perspective, this discussion suggests the importance of developing monetary policy models encompassing endogenous commodity prices and monetary authorities that are subject to misdiagnosis risk. To date, the bulk of the theoretical literature has stayed clear of models with endogenous commodity prices (see eg Leduc and Sill (2004), Carlstrom and Fuerst (2006), Montoro (2012), Natal (2012) and Catao and Chang (2015)). Moreover, this literature has generally focused on how a monetary authority should respond to exogenous movements in oil prices, eg whether it is optimal to target core or headline inflation and whether commodity price movements have far-reaching implications for the trade-off between stabilising output and controlling inflation. For example, Blanchard and Gali (2010) have gone so far as to argue that an increase in commodity prices driven by foreign demand can still be treated by a domestic monetary authority as an external supply shock. Such a conclusion is less tenable in models of endogenous commodity prices and correlated monetary policy reaction functions.

Various theoretical papers have addressed the endogeneity of commodity prices in small-scale DSGE models (eg Backus and Crucini (1998), Bodenstein et al (2011) and Nakov and Nuño (2013)). However, these models have generally ignored monetary policy, focusing instead on oil price determination and the frictions affecting it. Nakov and Pescatori (2010) is an early attempt to characterise monetary policy trade-offs in a DSGE model in which oil prices are determined endogenously. Another important contribution to this literature is Bodenstein et al (2012), who highlight, as we do, the importance of identifying the nature of the shocks hitting the economy.

Our model extends this class of models by considering the policy challenges facing a monetary authority when it tries to infer the source of commodity price shocks.[6] Namely, there is a risk that a monetary authority may misdiagnose a commodity price swing as being driven by an external supply shock when it is, in fact, driven by an endogenous global demand shock, and vice versa. In our model, the commodity price is endogenously determined in equilibrium by the interplay of global demand from a commodity-importing country (or region) and commodity supply from two types of commodity-exporting country – one competitive and one monopolistic. In this setting, the optimal monetary policy response to commodity price swings depends on the perception of the underlying drivers of the swings. Unable to fully know the nature of the drivers, the monetary authority infers them via signal extraction, leaving open the possibility of systematic misdiagnosis. The nature and implications of misdiagnosis risk are addressed.

The modelling exercise highlights several policy-relevant implications. First, it is important to distinguish between global demand and supply shocks when a monetary authority responds to commodity prices. The optimal responses to global demand and supply shocks are different. On the one hand, the optimal response to demand shocks is to lean against them fully, a result consistent with a standard New Keynesian closed economy model. On

[6] See Filardo and Lombardi (2013) for a discussion of commodity price misdiagnosis risks in the context of Asian EMEs.

the other hand, the optimal response to commodity supply shocks (ie a decrease in commodity prices) is to look through them.

Second, by looking through the impact on headline inflation, monetary policy does not perfectly stabilise core inflation. In other words, the conventional wisdom of looking through the first-round effects of commodity price swings is not optimal in our model. This is because our model breaks the "divine coincidence" between inflation and output gap stabilisation (eg Blanchard and Gali (2007)), which is a standard feature of DSGE models with exogenous commodity prices. In part, the breaking of such a divine coincidence comes from the assumption of a monopolistic commodity exporter that sets prices by assuming a downward sloping demand curve.

Third, misdiagnosis risk matters in monetary policy. In the case where the monetary authority misinterprets supply-driven increases in commodity prices as demand-driven, the contraction in both output and core inflation is larger than in the case of an accurate diagnosis. This outcome indicates another reason for the breakdown of the divine coincidence in this model (even if the dominant exporter acts as a price taker). This finding underscores the importance of correctly diagnosing the underlying sources of commodity price swings when setting monetary policy. For example, when commodity price fluctuations are driven by global demand shocks, a monetary authority that consistently misdiagnoses them as external supply shocks amplifies cyclical fluctuations and, as a result, destabilises the economy.

Of course, in any uncertain environment, there is a risk that shocks will be misdiagnosed. But in the case of correlated global shocks and domestically-oriented monetary policy mandates, the risk is particularly high. When facing global commodity price swings, there is a natural tendency for a given monetary authority – even in a relatively large country – to treat them as if they were exogenously-determined external supply shocks. Clearly, one lone monetary authority has little, if any, impact on global prices. However, if a sufficient number of monetary authorities were to act in a similar (and uncoordinated) way, their joint behaviour would result in procyclical effect that could destabilise the global economy. This coordination failure supports the case for greater central bank cooperation in a world experiencing wide endogenous commodity price swings.

II. The model

We present a global economic model in which commodity prices are determined endogenously, in the spirit of Nakov and Pescatori (2010).[7] The global economy is split into commodity importers and exporters. The commodity-importing region is treated as one representative country (but this can be extended to a group of importing countries without loss of generality). The commodity-importing country does not produce the commodity itself but uses it both as a consumption good and as an input into the production of final goods. Final goods producers in the commodity-importing country are subject to monopolistic competition and nominal rigidities, and the central bank sets monetary policy using a linear policy rule à la Taylor (1993).

The commodity-exporting region is made up of a dominant commodity-exporting country and a fringe of smaller competitive exporters. These countries produce the commodity using final goods sold by the commodity-importing region. In addition, the commodity-exporting countries buy final goods for their consumption from the commodity-importing country.

On the supply side, the dominant commodity-exporting country has market power and sets prices above marginal cost. The fringe of small exporting countries is similar in structure to the dominant exporting country

[7] Our model deviates from the setup of Nakov and Pescatori (2010) in four key respects in order to better characterise the monetary policy trade-offs: i) we introduce the commodity good into the households' utility function, which can drive a wedge between headline and core inflation in the commodity-importing country; ii) we interpret the commodity as a broad basket of commodities rather than focusing narrowly on oil; iii) we solve the Nash problem for the dominant producer, versus the Ramsey problem, so as to reflect realistic information constraints; and iv) we introduce the possibility of misdiagnosis risk by the monetary authority of the commodity-importing country, which by itself breaks the divine coincidence property of the model.

but operates competitively, taking commodity prices as given. Note that nominal rigidities are absent in the commodity-exporting countries, thereby simplifying the modelling of monetary policy trade-offs; in this model, there is only a role for a monetary authority to conduct countercyclical policy from the perspective of the commodity-importing country.[8]

The rest of this section provides the modelling details.

II.1. Commodity-importing countries

II.1.1 Households

Households are assumed to have the following representative consumer utility function over consumption and labour of the form:

$$U_{t_o} = E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \exp\left(g_t\right) \left[\ln\left(C_t\right) - \frac{L_t^{1+v}}{1+v} \right], \tag{2.1}$$

Where g_t is a preference shock and v is the inverse of the elasticity of labour supply. The consumption basket is defined as a Cobb-Douglas aggregator of the final goods consumption basket $C_{Y,t}$ and the household's demand for the commodity, $\mathfrak{M}_{C,t}$, of the form:

$$C_t = \left(C_{Y,t}\right)^{1-\gamma} \left(\mathfrak{M}_{C,t}\right)^{\gamma}. \tag{2.2}$$

Consumption of final goods, $C_{Y,t}$, is a Dixit-Stiglitz aggregate of a continuum of differentiated goods, $C_{Y,t}(z)$, of the form:

$$C_{Y,t} = \left[\int_0^1 C_{Y,t}(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{2.3}$$

[8] In the model, cross-border financial autarky is assumed, ie there is no borrowing across regions and current accounts are balanced in each period. In addition, trade is assumed to be carried out in a common global currency, suppressing the potential trade-offs arising from exchange rate dynamics. The assumptions streamline the analysis and allow us to highlight the key implications of misdiagnosis risk, which admittedly would be more complex in a richer model.

The representative household takes decisions subject to a standard budget constraint which is given by:

$$C_t = \frac{W_t L_t}{P_t} + \frac{B_{t-1}}{P_t} - \frac{1}{R_t}\frac{B_t}{P_t} + \frac{\Gamma_t}{P_t} + \frac{T_t}{P_t}, \tag{2.4}$$

where W_t is the nominal wage, P_t is the price of the consumption good, B_t is the end of period nominal bond holdings, R_t is the riskless nominal gross interest rate, Γ_t is the share of the representative household's nominal profits, and T_t is net transfers from the government. The first-order conditions for the optimising consumer's problem are:

$$1 = \beta E_t \left[R_t \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{C_{t+1}}{C_t} \right)^{-1} \exp\left(g_{t+1} - g_t \right) \right], \tag{2.5}$$

$$\frac{W_t}{P_t} = C_t L_t^v, \tag{2.6}$$

$$\mathfrak{M}_{C,t} = \gamma \frac{P_t}{P_{\mathfrak{M},t}} C_t, \tag{2.7}$$

$$C_{Y,t} = \left(1 - \gamma\right) \frac{P_t}{P_{Y,t}} C_t, \tag{2.8}$$

$$C_{Y,t}(z) = \left(\frac{P_{Y,t}\left(z\right)}{P_{Y,t}} \right)^{-\varepsilon} C_{Y,t}. \tag{2.9}$$

Equation (2.5) is the standard Euler condition that determines the optimal path of consumption. Equation (2.6) describes the optimal labour supply decision. Equations (2.7), (2.8) and (2.9) are the relative demands for the commodity, the aggregate final good and the differentiated final goods z in the consumption basket.

Substituting equations (2.7) and (2.8) into (2.2) yields the aggregate price level and inflation equations:

$$P_t = \left(P_{Y,t}\right)^{1-\gamma} \left(P_{\mathfrak{M},t}\right)^{\gamma}, \tag{2.10}$$

$$\Pi_t = (\Pi_{Y,t})^{1-\gamma} (\Pi_{\mathfrak{M},t})^{\gamma} \tag{2.11}$$

where $\Pi_t = P_t / P_{t-1}$, $\Pi_{Y,t} = P_{Y,t} / P_{Y,t-1}$ and $\Pi_{\mathfrak{M},t} = P_{\mathfrak{M},t} / P_{\mathfrak{M},t-1}$ are headline, core and commodity inflation, respectively.

Similarly, substituting equation (2.9) in equation (2.3) defines the price level of differentiated final goods:

$$P_{Y,t} = \left[\int_0^1 P_{Y,t}\left(z\right)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}}. \tag{2.12}$$

II.1.2 Final goods producers

Final goods are produced under monopolistic competition in the commodity-importing country using the following Cobb-Douglas technology:

$$Y_t(z) = A_t L_t(z)^{1-\alpha} \mathfrak{M}_{Y,t}\left(z\right)^{\alpha}, \tag{2.13}$$

where $\mathfrak{M}_{Y,t}$ is the aggregate commodity used as an input and α denotes the share of the commodity in the production function. The real commodity price, $Q_t \equiv \frac{P_{\mathfrak{M},t}}{P_t}$, is determined in the world market. Note from equation (2.10) that $Q_t = \left(P_{\mathfrak{M},t} / P_{Y,t} \right)^{1-\gamma}$; that is, the real commodity price is proportional to the inverse of the importing country's terms of trade.

The cost minimisation problem of the firm implies an expression for the real marginal cost:

$$MC_t(z) = \left(\frac{W_t}{P_t} \right)^{1-\alpha} Q_t^{\alpha} / \left[A_t \left(1-\alpha\right)^{1-\alpha} \alpha^{\alpha} \right], \tag{2.14}$$

where $MC_t\left(z\right)$ are the real marginal costs. Note that real marginal costs are the same for all firms producing the final good since the technology has constant returns to scale and factor markets are competitive, ie $MC_t\left(z\right) = MC_t$. By contrast, the first-order conditions for each producer of final goods imply the following demand equations for labour and the commodity:

$$L_t(z) = \left(1-\alpha\right)\frac{MC_t}{W_t/P_t}Y_t(z), \tag{2.15}$$

$$\mathfrak{M}_{Y,t}(z) = \alpha\frac{MC_t}{Q_t}Y_t(z). \tag{2.16}$$

From equation (2.9), the individual demand for the differentiated final goods is:

$$Y_t(z) = \left(\frac{P_{Y,t}(z)}{P_{Y,t}}\right)Y_t, \tag{2.17}$$

where Y_t is the aggregate demand for the final good.

Finally, firm dynamics are influenced by the price setting behaviour and forward-looking expectations. The producers of the final goods set prices assuming a staggered pricing mechanism à la Calvo (1983). Each firm faces an exogenous probability of changing its price given by $(1-\theta)$. A firm that changes its price in period *t* chooses its new price $P_{Y,t}(z)$ to maximise:

$$E_t\sum_{k=0}^{\infty}\theta^k\zeta_{t,t+k}F_{t+k}^{-1}\Gamma_{t+k}\left(z\right), \tag{2.18}$$

where $\zeta_{t,t+k} = \beta^k\left(\frac{C_{t+k}}{C_t}\right)^{-1}$ is the stochastic discount factor, and $F_{t+k} = \frac{P_{Y,\,t+k}}{P_{Y,\,t}}$ is the cumulative effect of core inflation over time.

The equation:

$$\Gamma_t(z) = [(1-\tau)P_{Y,t}(z) - P_t\,MC_t]\left(\frac{P_{Y,t}(z)}{P_{Y,t}}\right)^{-\varepsilon}Y_t \tag{2.19}$$

is the after-tax nominal profit of the supplier of good z with price $P_{Y,t}(z)$. The proportional tax on sale revenues, τ, is assumed to be constant.

The price that solves the firm's problem is given by:

$$\left(\frac{\hat{P}_{Y,t}(z)}{P_{Y,t}}\right) = \frac{\mu E_t\left[\sum_{k=0}^{\infty}\theta^k \zeta_{t,t+k} MC_{t,t+k} F_{t+k}^{\varepsilon} Y_{t+k}\right]}{E_t\left[\sum_{k=0}^{\infty}\theta^k \zeta_{t,t+k} F_{t+k}^{\varepsilon-1} Y_{t+k}\right]}, \tag{2.20}$$

where $\mu \equiv \frac{\varepsilon}{\varepsilon-1} / \left(1-\tau\right)$ is the price mark-up of final goods (net of taxes), and $\hat{P}_{Y,t}\left(z\right)$ is the optimal price level chosen by firm z.

With only a fraction $\left(1-\theta\right)$ of firms changing prices every period, the aggregate price level takes the form:

$$P_{Y,t}^{1-\varepsilon} = \theta P_{Y,t-1}^{1-\varepsilon} + \left(1-\theta\right)\left[\hat{P}_{Y,t}\left(z\right)\right]^{1-\varepsilon}. \tag{2.21}$$

Following Benigno and Woodford (2005), equations (2.20) and (2.21) can be written recursively and simplified by introducing auxiliary variables D_t and N_t of the form:

$$\theta\,\Pi_{Y,t}^{\varepsilon-1} = 1-\left(1-\theta\right)\left(\frac{N_t}{D_t}\right)^{1-\varepsilon}, \tag{2.22}$$

$$D_t = Y_t \,/\, C_t + \theta\beta E_t\left[\left(\Pi_{Y,t+1}\right)^{\varepsilon-1} D_{t+1}\right], \tag{2.23}$$

$$N_t = \mu Y_t MC_t \,/\, C_t + \theta\beta E_t\left[\left(\Pi_{Y,t+1}\right)^{\varepsilon} N_{t+1}\right]. \tag{2.24}$$

Equation (2.22) is derived from the aggregation of prices from individual firms. The ratio N_t/D_t is the optimal relative price $\hat{P}_{Y,t}\left(z\right)/P_{Y,t}$. The three equations (2.22), (2.23) and (2.24) summarise the recursive representation of the non-linear Phillips curve for non-commodity goods, ie core inflation.

II.1.3 Aggregation

Aggregating the demand for labour and the commodity, and the supply of final goods across producers yields:

$$L_t = \int_0^1 L_t(z)dz, \tag{2.25}$$

$$\mathfrak{M}_{Y,t} = \int_0^1 \mathfrak{M}_{Y,t}(z)dz, \tag{2.26}$$

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{2.27}$$

The above, together with equations (2.13), (2.15), (2.16) and (2.17), can be rewritten in the form:

$$L_t = \left(1-\alpha\right)\frac{MC_t}{W_t/P_t} Y_t \Delta_t, \tag{2.28}$$

$$M_{Y,t} = \alpha\frac{MC_t}{Q_t} Y_t \Delta_t, \tag{2.29}$$

$$Y_t = A_t L_t^{1-\alpha}\mathfrak{M}_{Y,t}^{\alpha}/\Delta_t, \tag{2.30}$$

where $\Delta_t = \int_0^1 \left(P_{Y,t}(\mathrm{z})\big/P_{Y,t}\right)^{-\varepsilon} dz$ is a measure of price dispersion. With relative prices differing across firms due to the Calvo staggered price setting mechanism, input usage differs across firms. As a consequence, the price dispersion factor, Δ_t, appears in the aggregate input demand equations. With equation (2.21), the law of motion of Δ_t is of the form:

$$\Delta_t = \left(1-\theta\right)\left(\frac{1-\theta\left(\Pi_{Y,t}\right)^{\varepsilon-1}}{1-\theta}\right)^{\varepsilon/\left(\varepsilon-1\right)} + \theta\Delta_{t-1}\left(\Pi_{Y,t}\right)^{\varepsilon}. \tag{2.31}$$

Equation (2.31) implies that higher inflation increases the dispersion of prices. Moreover, equations (2.28) and (2.29) show that higher price dispersion increases the amount of labour and the commodity needed to produce a given level of output.

II.2. Commodity-exporting countries

The commodity industry is modelled as comprising a dominant commodity exporter and a fringe group of competitive commodity exporters. The relative market share of the imperfectly competitive dominant exporter vis-à-vis the competitive fringe is important because it will influence the transmission channel of monetary policy. Market share can be varied to study the impact of the degree of competitive. That is, the market share of the dominant commodity exporter can range from zero to one, which corresponds to a range of market dynamics from perfect competition to a monopoly situation.

II.2.1 Dominant commodity exporter

The dominant exporting country produces the commodity according to the technology:

$$\mathfrak{M}_t = Z_t I_t^{*,D} \tag{2.32}$$

where Z_t is an exogenous productivity shifter and $I_t^{*,D}$ is the intermediate good used in commodity production (bought from the commodity-importing country). Productivity evolves exogenously according to:

$$\ln Z_t = (1-\rho_z)\ln \overline{Z} + \rho_z \ln Z_{t-1} + \varepsilon_t^z, \tag{2.33}$$

where $\varepsilon_t^z \sim i.i.d.\, N\left(0, \sigma_z^2\right)$. Shocks to Z_t can therefore be interpreted as *global commodity supply shocks*.

The utility function of the household in the dominant commodity exporter country depends only on the consumption of non-commodity final goods:

$$U_{t_0}^{*,D} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \ln(C_t^{*,D}), \tag{2.34}$$

subject to the following period-budget constraint, $P_{Y,t} C_t^{*,\mathrm{D}} = \Gamma_t^{*,\mathrm{D}}$, which equates consumption expenditure to dividends from commodity production, $\Gamma_t^{*,D}$. Note that the dominant exporting firm is modelled as being wholly owned by the household. As such, the representative household's objective of expected utility maximisation can also be recast as maximising the expected present discounted value of profits from commodity production. Profits each period are given by:

$$\frac{\Gamma_t^{*,D}}{P_{Y,t}} = Q_t^{1/(1-\gamma)} \mathfrak{M}_t - I_t^{*,D}. \tag{2.35}$$

The consumption good, $C_t^{*,D}$, and the intermediate good, $I_t^{*,D}$, are demanded by the commodity-exporting countries. The dominant commodity exporter chooses the level of commodity output, $\mathfrak{M}_t$, such that it maximises the expected present discounted utility of the representative household in equation (2.34), subject to demand from the commodity-importing country and supply from the competitive fringe of commodity exporters.[9]

II.2.2 Fringe of competitive commodity exporters

Similarly, the utility function of households in the fringe depends only on the consumption of final goods:

$$U_{t_o}^{*,F} = E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \ln\left(C_t^{*,F}\right), \tag{2.36}$$

subject to the following period-budget constraint, $P_{Y,t} C_t^{*,F} = \Gamma_t^{*,F}$, which equates consumption expenditure to dividends from commodity production, $\Gamma_t^{*,F}$. In terms of production, the fringe is assumed to be a continuum of atomistic firms indexed by $j \in \left[0, \Omega_t\right]$. Each fringe country produces a quantity $X_t\left(j\right)$ of the commodity according to technology of the form:

$$X_t\left(j\right) = \xi\left(j\right) Z_t I_t^{*,F}\left(j\right), \tag{2.37}$$

subject to the capacity constraint:

$$X_t\left(j\right) \in \left[0, \bar{X}\right], \tag{2.38}$$

where $\left[\xi\left(j\right) Z_t\right]^{-1}$ is the marginal cost of country j and is composed of idiosyncratic and aggregate components. The input $I_t^{*,F}\left(j\right)$ is an intermediate good used in commodity production and is bought from the

[9] Further details provided in Appendix D.

commodity-importing country (ie the final goods produced in the commodity-importing country).

The production of the commodity can be sold at the international real price, Q_t, which atomistic exporters take as given. Each country chooses the amount of the commodity to produce in each period so as to maximise profits of the form:

$$\max\left[Q_t X_t\left(j\right) - \frac{\frac{P_{Y,t}}{P_t} X_t\left(j\right)}{\xi\left(j\right) Z_t}\right] \text{ s.t. } X_t\left(j\right) \in \left[0, \overline{X}\right]. \tag{2.39}$$

Supply from the competitive fringe is determined by the marginal cost shocks faced by producers. The idiosyncratic component $1 / \xi(j)$ is assumed to have a uniform distribution, $F\left(1 / \xi(j)\right)$, in the interval from *a* to *b*. Given the total mass of competitive fringe countries, Ω_t, the aggregate amount of the commodity produced by the competitive fringe is given by:

$$X_t \equiv \int_0^{\Omega_t} X_t\left(j\right) d_j = \Omega_t F\left(Q_t Z_t\right). \tag{2.40}$$

As a consequence, total production is given by:

$$X_t = \begin{cases} \Omega_t \overline{X} & Q_t Z_t > b \\ \Omega_t \overline{X} \frac{Q_t Z_t - a}{b-a}, & a < Q_t Z_t \leq b \\ 0 & Q_t Z_t \leq a \end{cases}. \tag{2.41}$$

Assuming $a = 0$ and normalising $b = \overline{X} > 1$ to be sufficiently large, at least some competitive fringe producers are always priced out of the market by the dominant commodity exporter. Under these assumptions, the supply by the fringe of competitive exporters can be written as:

$$X_t = \Omega_t Z_t Q_t. \tag{2.42}$$

II.2.3 Commodity exporters' problem

In the case of a perfectly competitive fringe market of commodity exporters, the equilibrium commodity price is equal to marginal costs:

$$Q_t^{PC} = Z_t^{-1}, \tag{2.43}$$

and the quantity produced is given by the global demand at that price.

In contrast, the dominant commodity exporter maximises the expected present discounted value of the logarithm of profits resulting from the production of the commodity when setting its price. With the dominant commodity exporter having market power, the objection function can be written as:

$$\max_{\{\mathfrak{M}_t\}_{t_0}^{\infty}} E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \ln\left(Q_t^{1/(1-\gamma)} \mathfrak{M}_t - \mathfrak{M}_t \,/\, Z_t \right), \tag{2.44}$$

subject to global demand for the commodity and supply from the competitive fringe:

$$\mathfrak{M}_t = \mathfrak{M}_{C,t} + \mathfrak{M}_{Y,t} - X_t, \tag{2.45}$$

which is equal to:

$$\mathfrak{M}_t = \gamma \frac{1}{Q_t} C_t + \alpha \frac{MC_t}{Q_t} Y_t \Delta_t - \Omega_t Q_t Z_t \tag{2.46}$$

after substituting the commodity demand (equations (2.7) and (2.29)) and supply from the competitive fringe exporters (equation (2.42)) into equation (2.45).

Note that the dominant commodity exporter takes as given the macroeconomic variables of the commodity-importing country $(C_t$, MC_t, Y_t, Ω_t and $\Delta_t)$ but does not completely internalise the feedbacks of its actions on the macroeconomic performance of the commodity-importing country.[10]

The first-order condition of this problem determines the commodity price in real terms:

[10] Nakov and Pescatori (2010) analyse the case in which the dominant commodity exporter completely internalises its actions on the importing country. Technically, they solve the Ramsey problem of the dominant commodity exporter, taking into account the behavioural equations of the importing country. Our Nash solution assumes a more restrictive information assumption which may be seen as being more realistic in a world with incomplete information sharing.

$$Q_t = \Psi_t Z_t^{-1}, \tag{2.47}$$

where $\Psi_t \equiv 1/(1-\eta_t)$ is the commodity market markup and $\eta_t = \mathfrak{M}_t/(\mathfrak{M}_t + 2X_t)$ is the elasticity of substitution of the net global demand for the commodity (in absolute value). Accordingly, the commodity price is a markup over marginal cost, the latter being driven by the process of global commodity supply shocks. In turn, a markup shock can be produced by a shift in agents' preferences towards commodity consumption. We can therefore use it as a proxy for a *demand shock*.

To be sure, various other factors affect the average markup. Firm-specific supply shocks may play a role. In addition, the commodity price markup $\Psi_t \equiv 1 + \frac{\mathfrak{M}_t}{2X_t}$ is an increasing function of the dominant commodity exporter's market share relative to that of the competitive fringe of commodity exporters. The limiting case is when $\mathfrak{M}_t \to 0$ corresponds to perfect competition while $X_t \to 0$ is the case of a single monopolist. However, abstracting from firm-specific shocks as well as from exogenous shifts in the market shares of dominant and competitive commodity exporters, markup shocks can also be produced by shifts in preferences and can therefore proxy for demand shocks.

II.2.4 Market clearing

With the assumption of cross-border financial autarky, bonds are in zero net supply:

$$B_t = 0. \tag{2.48}$$

Total aggregate commodity demand for both consumption and production equals aggregate supply:

$$\mathfrak{M}_{C,t} + \mathfrak{M}_{Y,t} = \mathfrak{M}_t + X_t \tag{2.49}$$

where $\mathfrak{M}_t$ and X_t are the supplies of the commodity produced by the dominant and competitive fringe of commodity-exporting countries. As well, labour supply is set equal to labour demand in the commodity-importing economy.

Substituting this equation (2.49) and the consumption equation (2.8) into the aggregate resource constraint yields the following alternative form for the constraint:

$$\frac{P_{Y,t}}{P_t}C_{Y,t} = \frac{P_{Y,t}}{P_t}Y_t - Q_t\left(\mathfrak{M}_t + X_t\right). \tag{2.50}$$

The aggregate demand for final goods is set equal to aggregate supply:

$$Y_t = C_{Y,t} + C_t^{*,D} + I_t^{*,D} + C_t^{*,F} + I_t^{*,F}, \tag{2.51}$$

which includes final goods consumption in the commodity-importing country and the aggregate consumption and intermediate goods demanded by the dominant and the competitive fringe of the commodity-exporting countries, respectively (the dominant commodity-exporting country is denoted by superscript *D* and the competitive fringe by *F*).

III. Characterising optimal monetary policy

Monetary policy in the commodity-importing country is modelled as a linear Taylor-type rule determined by deviations from model-based benchmarks for output, inflation and the interest rate. In this section, the benchmarks are defined and the implications of alternative policy rules are analysed.[11]

III.1. Output gap benchmarks

Benchmark output gaps can be derived by substituting the equations for labour demand (2.28), labour supply (2.6), aggregate demand (2.50) and commodity demand for production (2.29) into the aggregate production

[11] For completeness, fiscal policy is modelled as a linear rule: $T_t = \tau P_{Y,t} Y_t$; transfers to households are funded by taxing final good producers.

function (2.30). The level of output in terms of marginal costs, the dispersion of prices, productivity and the real commodity price is:[12]

$$Y_t = \left(\frac{A_t}{\Delta_t}\right)^{1/(1-\alpha)} \left[\frac{(1-\alpha)MC_t\,\Delta_t}{(Q_t)^{-\gamma/(1-\gamma)} - \alpha MC_t\,\Delta_t}\right]^{1/(1+v)} \left(\alpha\frac{MC_t\,\Delta_t}{Q_t}\right)^{\alpha/(1-\alpha)}. \quad (3.1)$$

The log-linear approximation of the level of output, in deviations from the steady state is:[13]

$$y_t = \frac{1}{1-\alpha}a_t + \frac{\alpha}{1-\alpha}\left(mc_t - q_t\right) + \frac{1}{1+v}\Upsilon\left(mc_t + \frac{\gamma}{1-\gamma}q_t\right). \quad (3.2)$$

Where $\Upsilon \equiv \left[1 - \frac{\alpha}{\mu}Q^{\gamma/(1-\gamma)}\right]^{-1} \geq 1.$

The l*evel of natural output*, y_t^n, is defined as the level of output consistent with a flexible price equilibrium. In this case, the marginal cost is constant, $MC_t = \mu^{-1}$, and there is an absence of price dispersion, $\Delta_t = 1$. In log-linear terms, the level of natural output, y_t^n, is of the form:

$$y_t^n = \frac{1}{1-\alpha}a_t - \left(\frac{\alpha}{1-\alpha} - \frac{1}{1+v}\frac{\gamma}{1-\gamma}\Upsilon\right)q_t. \quad (3.3)$$

As shown in equation (3.3), commodity price fluctuations have two opposing effects on the level of natural output. In terms of production (i.e. the first term in the parenthesis), an increase in the commodity price has a qualitative effect similar to that of a negative productivity shock; it reduces the level of natural output. In terms of consumption (i.e. the second term in the parenthesis), an increase in the commodity price increases the level of natural output. The latter term reflects an increase in labour due to a negative income effect from a higher commodity price.

12 Derivations provided in Appendix E.

13 Note that a linear approximation of the price dispersion, Δ_t, does not appear in this equation because price dispersion is assumed to have only second-order effects on the dynamics, as shown in Benigno and Woodford (2005).

The *natural output gap*, $\hat{y}_t^n$, measures the difference between the actual and the natural level of output and is given by:

$$\hat{y}_t^n = \left(\frac{\alpha}{1-\alpha} - \frac{1}{1+v} \Upsilon \right) mc_t. \tag{3.4}$$

This implies that responding to the natural output gap is equivalent to responding to real marginal costs, up to a scale factor.

Similarly, *the level of efficient output*, y_t^e, is defined with respect to the efficient allocation, ie flexible prices and no monopolistic distortions in the commodity market or in the final goods market (which implies that $Q_t^e = Z_t^{-1}$ and $\mu^e = 1$):

$$y_t^e = \frac{1}{1-\alpha} a_t + \left(\frac{\alpha}{1-\alpha} - \frac{1}{1+v} \frac{\gamma}{1-\gamma} \Upsilon^e \right) z_t, \tag{3.5}$$

where $\Upsilon^e = \left[1 - \alpha Z^{-\gamma/(1-\gamma)} \right]^{-1}$. The relationship between Υ^e and Υ depends on the extent of monopolistic distortions. Υ^e and Υ are equal only if both markets are perfectly competitive or if the commodity is not used for production (that is, $\alpha = 0$).[14] A key difference is that commodity markup shocks do not affect the level of efficient output: such output is instead affected only by fluctuations associated with supply shocks in the commodity market. As a consequence, a demand-driven increase in the commodity price would leave the benchmark efficient output gap unchanged. However, a negative commodity supply shock would decrease both the natural and efficient output levels, albeit by different amounts.[15]

The *efficient output gap*, $\hat{y}_t^e$, which is defined as the difference between actual output and the efficient level of output, is of the form:

[14] More precisely, $\Upsilon^e > (<) \Upsilon$, if $\Psi^{\gamma/(1-\gamma)} < (>) \mu$, where Ψ and μ are the markups in steady state of the commodity and final goods markets, respectively.

[15] The level of efficient output contracts less than the level of natural output in response to a negative commodity supply shock because the commodity price markup partially offsets the effects of supply shocks on the commodity price.

$$\hat{y}_t^e = \hat{y}_t^n - \left(\frac{\alpha}{1-\alpha} - \frac{1}{1+v}\frac{\gamma}{1-\gamma}\Upsilon\right)\psi_t - \frac{1}{1+v}\frac{\gamma}{1-\gamma}\left(\Upsilon - \Upsilon^e\right)z_t \quad (3.6)$$

where ψ_t is the commodity market markup in log-linear deviations from the steady state. This is the welfare-relevant output gap and is equal to the natural output gap plus a term that depends on the commodity price markup and the commodity supply shock.

III.2. Inflation benchmarks

Both headline and core inflation are determined by the natural output gap, expected inflation and commodity price changes. Expressed in log-linear terms, the equations for headline inflation and core inflation, respectively, are of the form:

$$\pi_t = \pi_{Y,t} + \frac{\gamma}{1-\gamma}\Delta q_t \quad (3.7)$$

$$\pi_{Y,t} = \kappa_y \hat{y}_t^n + E_t \pi_{Y,t+1}. \quad (3.8)$$

Equation (3.7) describes the determinants of headline inflation and equation (3.8) describes core inflation written in the form of a Phillips curve for aggregate final goods.[16] Stabilisation of the natural output gap is equivalent to stabilisation of core inflation. And, in that case, headline inflation would vary proportionally with changes in real commodity prices.

Substituting equation (3.6) into equation (3.8) yields the following expression for the Phillips curve in terms of the efficient output gap:

$$\pi_{Y,t} = \kappa_y \hat{y}_t^e + E_t \pi_{Y,t+1} + u_t, \quad (3.9)$$

where u_t is an endogenous cost-push shock, which is a function of both ψ_t and z_t.

[16] The simplified form of the headline inflation equation is derived by substituting the log-linear version of the inflation equation (2.11) and the log-linear Phillips curve equations (2.22), (2.23) and (2.24) into equation (3.3); and where $\kappa_y \equiv \left((1-\theta)(1-\beta\theta)/\theta\right)/(\alpha/(1-\alpha) - \Upsilon/(1+\nu))$.

In this model, the divine coincidence featured in models with exogenous commodity prices is broken. It is no longer possible to simultaneously stabilise core inflation and the welfare-relevant output gap. The trade-off arises from the impact of commodity price fluctuations on the level of efficient output. An increase in commodity price markups generates a positive cost-push shock, which puts upward pressure on core inflation but lowers the efficient output gap.

III.3. Interest rate benchmarks

The interest rate benchmarks are derived by substituting the equations for the aggregate resources constraint (2.50) and the definition of the price level (2.10) into the IS equation (2.5):

$$\frac{1}{R_t} = \beta E_t \left[\frac{1}{\Pi_{Y,t+1}} \left(\frac{1-\alpha MC_t Q_t^{\gamma/(1-\gamma)} \Delta_t}{1-\alpha MC_{t+1} Q_{t+1}^{\gamma/(1-\gamma)} \Delta_{t+1}} \right) \left(\frac{Y_t}{Y_{t+1}} \right) \exp\left(g_{t+1} - g_t \right) \right]. \quad (3.10)$$

The *natural interest rate* is defined as the rate consistent with flexible final goods pricing in which core inflation is, by definition, zero. In log-linear terms, the natural interest rate can be expressed as:

$$r_t^n = \left(g_t - E_t g_{t+1} \right) - \left(y_t^n - E_t y_{t+1}^n \right) + \frac{\gamma}{1-\gamma} \left(\Upsilon - 1 \right) \left(q_t - E_t q_{t+1} \right). \quad (3.11)$$

Similarly, the *efficient interest rate* is defined in the case where commodity and final goods markets are perfectly competitive and in this model can be written as:

$$r_t^e = \left(g_t - E_t g_{t+1} \right) - \left(y_t^e - E_t y_{t+1}^e \right) - \frac{\gamma}{1-\gamma} \left(\Upsilon^e - 1 \right) \left(z_t - E_t z_{t+1} \right). \quad (3.12)$$

Both the natural and the efficient interest rates respond in the same way to demand shocks, fully leaning against them and neutralising their effects. However, the responses to shocks affecting the commodity price markup are different. The natural interest rate reacts to fluctuations in the actual commodity price, which includes the markup (a commodity price increase driven by a higher markup boosts the natural interest rate), while the efficient interest rate does not respond to changes in the markup.

III.4. Analysing monetary policy rules

This section explores the model dynamics under alternative monetary policy rules, highlighting the differential responses to demand and commodity supply shocks. The baseline policy rule assumes that the monetary authority responds to core inflation and the efficient output gap.[17] It is of the form:

$$r_t = E_{t|t-1}\left[r_t^e + \varphi_{core}\pi_{Y,t} + \varphi_y \hat{y}_t^e\right] + \varphi_{com}\Delta q_t, \tag{3.13}$$

where the relative weights on core inflation and the output gap determine the trade-off between core inflation and output gap stabilisation. The benchmark interest rate also influences the optimal policy setting. We additionally assume that the monetary authority is able to observe inflation and output with a one period lag – hence the presence of the expectations operator – whereas commodity prices are available in real time. Therefore, the central bank uses the forecast (ie the expectation based on the information set available in the previous period) of inflation and output to conduct monetary policy and the observed (contemporaneous) change in the commodity price Δq.

The size of the shocks are standardised to generate a 1% increase in the commodity price under a benchmark scenario; the coefficients of the policy rule have been calibrated to result in $\varphi_{core} = 1.5$, $\varphi_y = 0.5$ and $\varphi_{com} = 0.05$. For most structural parameters, the calibration is in line with those found in the literature (Table 1). For the parameters associated with the commodity market, the choice is a bit more arbitrary owing to the less conventional view of the literature. The share of the commodity in the consumption basket is set to 10%, which roughly matches the share of primary commodity inputs in the

[17] Of course, similar results could be obtained for the natural benchmark. We focus here on the efficient benchmark as it is the one that actually matters for welfare calculations. We will explore alternative and easier-to-compute benchmarks in what follows.

US CPI. For the share of commodities in the production function, we also use 10%, as in Arseneau and Leduc (2013).[18] Finally, the size of the competitive commodity production sector relative to GDP is set at 10%.

Baseline Calibration — Table 1

Structural parameters	*Parameter*	*Value*
Share of commodity in consumption basket	γ	0.1
Share of commodity in production function	α	0.1
Inverse Frisch labour supply elasticity	v	0.5
Price elasticity of substitution	ε	7.66
Quarterly discount factor	β	0.99
Price adjustment probability	θ	0.75
Final goods productivity in steady state	A	1
Commodity productivity in steady state	Z	1
Size of competitive commodity production relative to GDP	X/Y	0.1

As commonly found in the DSGE literature, a markup shock can be fully neutralised by monetary policy: the increase in the policy rate completely offsets any impact on the key macroeconomic variables as well as on those associated with the commodity market. So, we also introduced a more traditional type of demand shock – ie an exogenous shift in aggregate demand. This is displayed in Graph 1: due to the nature of the shock, a change in the interest rate is not sufficient to offset it, and a positive output gap remains open for around six quarters.

Similarly, the monetary authority is unable to offset a commodity supply shock (Graph 2). The policy rate response as well as the macroeconomic impacts are, however, very modest. This confirms the conventional wisdom that monetary authorities should look through the first-round effects of commodity supply shocks.

[18] Given that we focus generically on commodities rather than simply on oil, both values are larger than the 5% commonly used in oil-only models.

III.4.1 A note on the natural benchmark

To simplify the discussion, we do not report the impulse responses for the natural policy rule. It is useful to note, however, that the dynamics associated with a commodity supply shock generate similarities and differences compared to the efficient rule. For both the natural and efficient benchmarks, a negative supply shock increases the commodity's price, boosts headline inflation and reduces the amount of the commodity supplied by the exporters. The levels of natural and efficient output both decline, with the level of efficient output falling more. In the efficient case, the decline in the level of efficient output partially offsets the increase in the commodity's price via a reduction in the markup. The trade-off between inflation and output are different in the two cases. Core inflation and the output gap are completely stabilised in the natural benchmark case. But in the efficient benchmark case, the efficient output gap rises and core inflation declines. This indicates that a monetary authority following an efficient policy rule partially offsets the effects of the higher commodity price on headline inflation with lower core inflation.

III.4.2 Welfare calculations

The performance of the efficient policy rule in equation (3.13) is also assessed in terms of the welfare of the representative household (ie from the commodity-importing country), along with two alternative policy rules that take the general form:

$$r_t = E_{t|t-1}\left[r_t^e + \varphi_{core}\pi_{Y,t}\right] \tag{3.14}$$

And:

$$r_t = E_{t|t-1}\left[r_t^e + \varphi_{head}\pi_t\right] \tag{3.15}$$

For policy rule (3.14), the monetary authority responds only to core inflation. For policy rule (3.15), the response is to headline instead of core inflation.[19]

Expected welfare loss for alternative efficient policy rules
Percent of steady state consumption — Table 2

	(1)	*(2)*	*(3)*	*(4)*	*(5)*
φ_{core}	2.0	–	1.5	1.5	2.0
φ_{head}	–	2.0	–	–	–
φ_{y}	–	–	0.5	0.5	1.0
φ_{com}	–	–	–	0.05	0.1
Welfare loss	-3.8092e-4	-6.0543e-4	-5.9175e-4	-4.9849e-4	-5.2704e-4

Table 2 reports the welfare losses associated with a commodity supply shock, in terms of steady state consumption, using the efficient policy rules. The first column corresponds to the baseline policy rule (3.13), while columns 2 and 3 corresponds to two alternative calibrations of the efficient policy rule (3.14) with $\varphi_{com} = \left\{0.05, 0.1\right\}$, $\varphi_{core} = \left\{1.5, 2\right\}$ and $\varphi_{head} = \left\{0.5, 1\right\}$. The calibration for column 5 is meant to capture the notion that a monetary authority may consider responding somewhat more strongly to the commodity price than a conventional Taylor-type rule would suggest.

The optimal policy rule (column 1) assumes $\varphi_y = 0$ and $\varphi_{com} = 0$, ie there is no response to the efficient output gap and to commodity prices. It provides a welfare loss that can be compared with the other specifications.[20,]

19 Note that both specifications are equivalent when $\varphi_{com} = \varphi_{core} \cdot \gamma / \left(1 - \gamma\right)$. This is the factor by which commodity prices affect headline inflation, as indicated by equation (3.7).

20 The unconditional expected welfare, EW_t is calculated using a second-order solution of the model, where $W_t = U\left(C_t, L_t, g_t\right) + \beta E_t\left(W_{t+1}\right)$ and the welfare cost in terms of steady state consumption is equivalent to $\left\{\exp\left[\left(1-\beta\right)\left(EW_t - W\right)\right] - 1\right\} \times 100$, given the logarithmic preferences of the representative household in the commodity-importing country.

[21] In columns 2, 3, 4 and 5, the policy rules that respond directly to the commodity's price and headline inflation generate larger welfare losses than the policy rule in column 1. This is because the policy rule of column 1 already takes into account the effects of the commodity's price; the additional responses to changes in the commodity price and headline inflation in the alternative policy rules simply generate more volatile interest rates than is optimal. These results highlight the importance of monitoring and responding to core inflation rather than headline inflation. Note, however, that a moderate response to commodity prices (column 4) reduces welfare losses compared to the optimal policy (3.14).

IV. Incomplete information, misdiagnosis and monetary policy spillovers

In practice, monetary authorities do not know in real time which shocks are affecting their jurisdictions: all they can observe are movements in the international prices of commodities. Moreover, the readings on inflation and output will not be available in real time. Given that the optimal responses to demand and supply shocks differ, this complicates the formulation of an appropriate monetary policy reaction,. On top of this, a supply shock would affect the efficient output benchmark, while a demand shock would not, which would generate additional uncertainty on the output gap.

This section considers the challenges arising from misdiagnosis, ie the risk that a monetary authority misinterprets the source of the shocks driving commodity price fluctuations. The optimal monetary policy response to demand and supply shocks differs: the optimal response to demand shocks is to fully neutralise their effects on output and inflation, so that the

[21] The results corresponding to preference shocks are the same under all of the policy rules, and thus are not reported. Also, this policy rule offers positive, albeit small, welfare gains in comparison to the natural policy rule alternative (in which core inflation and the output gap are perfectly stabilised as in Graph 1).

commodity's price is stabilised. By contrast, the optimal response to negative commodity supply shocks is to partially offset their effects on headline inflation by a reduction in both core inflation and the output gap. As a consequence, misdiagnosis of the source of the shocks will lead to a deterioration in economic outcomes.

IV.1. Monetary policy under incomplete information

As pointed out in Section III, the source of the shocks matters for the determination of the benchmark output gap and natural rate. This complicates the real-time problem that policymakers face, ie the inability to observe output and inflation in real time. In practice, policymakers observe commodity prices in a timely fashion but only observe output and inflation with a lag. So there is the risk that a monetary authority will misdiagnose the state of the economy in real time. In this section, we highlight the monetary policy challenges under two types of learning: the first is a classical signal extraction problem in which the monetary authority bases its assessment of commodity demand and supply shocks on past data; and the second in which the monetary authority updates its assessments of the source of the shocks with information about how the economy responds to its monetary policy actions.

IV.2. The signal-extraction problem

In the previous section, the degree of uncertainty faced by the monetary authority is limited to shocks to the benchmarks. In this section, we consider the decision-making problem faced by the monetary authority when it does not observe the supply and demand shocks (z_t and ψ_t) driving the commodity's price. The monetary authority can only infer the shocks from past behaviour. To model this, we assume the commodity price takes the form:

$$q_t = -z_t + \psi_t = H'\xi_t, \tag{4.1}$$

where $H' = \begin{bmatrix} -1 & 1 \end{bmatrix}$ and $\xi_t = [z_t \;\; \psi_t]'$. The unconditional variance of ξ_t is:

$$P \equiv \mathrm{var}(\xi_t) = \begin{bmatrix} \sigma_z^2 & \sigma_{z\psi} \\ \sigma_{\psi z} & \sigma_\psi^2 \end{bmatrix}. \tag{4.2}$$

Given this informational structure, the monetary authority infers the sources of commodity price fluctuations by solving a signal-extraction problem using a Kalman filter, ie:

$$E_t^{ma} \begin{bmatrix} z_t & \psi_t \end{bmatrix}' = Mq_t, \tag{4.3}$$

where $M = PH[H'PH]^{-1}$ is a weighted average of the variances and covariances of z_t and ψ_t; *M* is calculated as:

$$M = \frac{x}{x^2 - 2\rho x + 1} \begin{bmatrix} \rho - x \\ \frac{1}{x} - \rho \end{bmatrix}, \tag{4.4}$$

where $\rho = corr(z_t, \psi_t)$ and $x = \sigma_\psi / \sigma_z$.[22]

Three cases of equation (4.3) help to shed light on the trade-offs facing the monetary authority. In the first case (type A), when $x \to 0$, the volatility of the commodity supply shock is high relative to that of the commodity market markup. In this case, the monetary authority attributes nearly all of the fluctuations to the commodity price markup. That is:

$$\text{if } x \to 0, \qquad E_t^{ma} \begin{bmatrix} z_t & \psi_t \end{bmatrix}' \to \begin{bmatrix} 0 & q_t \end{bmatrix}'.$$

In the case (type B), all the commodity price fluctuations are attributed to the supply shock. That is:

$$\text{if } x \to \infty, \qquad E_t^{ma} \begin{bmatrix} z_t & \psi_t \end{bmatrix}' \to \begin{bmatrix} -q_t & 0 \end{bmatrix}'.$$

In the last case (type C), the monetary authority attributes commodity price fluctuations partially to each component of the commodity price, taking into account the relative volatility and correlation the components, as in

22 See chapter 13 of Hamilton (1994) for the derivation.

equation (4.4). Given the monetary authority's inference of the drivers of commodity price fluctuations, $E_t^{ma}\left(z_t\right)$ and $E_t^{ma}\left(\psi_t\right)$, the monetary authority uses the policy benchmarks, $E_t^{ma}\left(r_t^{ss}\right)$ and $E_t^{ma}\left(\hat{y}_t^{ss}\right)$, in the policy rule (3.13).

IV.3. Implications for the monetary policy model in Section III

We start with the monetary authority's policy rule in Section III:

$$r_t = E_{t|t-1}\left[r_t^e + \varphi_{core}\pi_{Y,t} + \varphi_y \hat{y}_t^e\right] + \varphi_{com}\Delta q_t, \tag{4.5}$$

where φ_{core} and φ_y capture the relative weight of stabilising inflation and the welfare-relevant output gap. Given $E_t^{ma}\left(z_t\right)$ and $E_t^{ma}\left(\psi_t\right)$, the monetary authority can form expectations based on the inferred origin of the shock driving the change in commodity prices Δq. If the monetary authority fails to correctly identify the shocks driving the commodity price fluctuation, a policy error arises. That is:

$$\begin{aligned} r_t &= E_{t|t-1}^{ma}\left[r_t^e + \varphi_\pi \pi_{Y,t} + \varphi_y \hat{y}_t^e\right] + \varphi_{com}\Delta q_t \\ &= E_{t|t-1}\left[r_t^e + \varphi_\pi \pi_{Y,t} + \varphi_y \hat{y}_t^e\right] + \varphi_{com}\Delta q_t + e_t, \end{aligned} \tag{4.6}$$

where:

$$\begin{aligned} e_t \equiv & \left[E_{t|t-1}^{ma}\left(r_t^e\right) - E_{t|t-1}\left(r_t^e\right)\right] + \varphi_{core}\left[E_{t|t-1}^{ma}\left(\pi_{Y,t}\right) - E_{t|t-1}\left(\pi_{Y,t}\right)\right] \\ & + \varphi_y\left[E_{t|t-1}^{ma}\left(\hat{y}_t^e\right) - E_{t|t-1}\left(\hat{y}_t^e\right)\right] \end{aligned}$$

corresponds to a misdiagnosis error, which is an endogenous variable, and $E_{t|t-1}^{ma}$ denotes the expectations under the incorrect diagnosis on the source of the shock. Note that when the monetary authority imputes the change in commodity prices to the wrong type of shock, its estimates of endogenous variables will be incorrect, leading to persistent errors in the interest rate setting.[23] To see this, we augment our dynamic system with equation (4.6) and analyse the implications for the impulse responses.

[23] If the monetary authority correctly identifies the source of the shock, the error is zero.

Graph 3 shows the impulse responses to a commodity supply shock in the misdiagnosis case A. Even though the commodity price is driven by a supply shock, the monetary authority misdiagnoses it as a traditional demand-driven commodity shock. If the monetary authority fails to recognise that an increase in the commodity price is driven by external supply conditions, the consequence is overly tight monetary policy accompanied by an excessive drop in both output and inflation. The commodity price rises less than in the baseline case because of tighter monetary policy. Core and headline inflation both fall because of the economy's slowdown.

Graph 4 shows the impulse responses to a conventional aggregate demand shock in the case of misdiagnosis type B, in which the rise in the commodity's price is mistakenly attributed to a negative commodity supply shock. In this case, the easier monetary policy associated with an attempt to look-through the rise in the commodity price results in higher output and inflation. This type of policy misdiagnosis induces a very procyclical increase in the commodity price.

Graphs 5 and 6 report the results for the case of a misdiagnosis of type C, in which the monetary authority implements an optimally weighted response to the commodity price rise based on the historical correlation of commodity demand and supply shocks as per equation (4.4). The standard deviations of the supply and demand shocks for the Kalman filter are calibrated to the empirical estimates by Filardo and Lombardi (2013), which yields a ratio (x in equation (4.4)) of about 1.5. Consistent with the general conclusions from misdiagnosis types A and B, the monetary authority responds excessively to supply shocks and insufficiently to demand shocks: hence, monetary policy turns out overall to be excessively procyclical.

The results for the different types of diagnosis risk underscore the modelling and policy implications of this informational restriction. On the modelling side, the misdiagnosis risk leads to a breakdown of the divine coincidence found in full information models à la Blanchard and Gali (2007).

On the policy implication side, the difference between equations (3.9) and (4.6) implies a failure to stabilise core inflation in the short run (as is verified in Graphs 3-6). As a consequence, and to the extent to which the monetary authority cannot infer the nature of the shocks perfectly, the policy reaction will inherently tend to amplify fluctuations in commodity prices and the macroeconomy more generally. In the case where the monetary authority misinterprets a rise in the commodity price as supply-driven, the contraction in both output and core inflation would be larger than in the full information case. And, in the case where commodity price fluctuations are driven by global demand, the monetary authority would amplify cyclical fluctuations and, as a result, destabilise the economy. These results underscore the importance of correctly identifying the underlying nature of commodity price shocks. A corollary of this is that if monetary authorities can and do take efforts to learn about the true nature of the shocks, they can improve macroeconomic outcomes. Finally, the results suggest that a monetary authority focused on core inflation would help to stabilise the economy more actively in the presence of misdiagnosis risk than one focused on headline inflation.

IV.4. Cooperation and monetary policy spillovers

This section considers the possibility – if not a natural tendency – of monetary authorities in small economies to treat global demand shocks as external supply shocks. It follows that if all monetary authorities were to misdiagnose the nature of the shocks and respond in a similar manner, their responses would be highly correlated across countries and potentially result in unintended destabilising feedbacks of the type discussed in the previous section, ie this would result in systematically procyclical monetary policy at the global level. A key question is, how strong is the incentive to treat global demand shocks as external supply shocks (ie deliberately misdiagnose the nature of the shock)? In our model, that incentive turns out to be significant.

To assess the relevance of the misdiagnosis incentive, consider a world of incomplete monetary policy cooperation in which monetary authorities act in a manner that is consistent with a Nash policy equilibrium, ie taking the actions of the other countries as given and assuming no monetary policy spillovers. Without loss of generality, also assume that the size of commodity-importing economies is identical. Let one group comprising *N*-1 commodity-importing economies follow the optimal monetary policy rule from Section III. The one remaining commodity-importing economy is then free to deviate from the group optimum, and hence chooses its monetary policy reaction function given the behaviour of the dominant and fringe commodity-exporting countries and *N*-1 commodity-importing countries. As the size of the deviating country gets smaller, the impact of its decisions on the global situation would become smaller and hence the country's monetary authority would act as if global shocks were purely exogenous.[24]

Formalising this logic, assume there are *N* commodity-importing countries indexed by i such that they face a problem similar to that discussed in Section III. The efficient policy rule for each *N*-1 monetary authorities takes the form found in equation (3.13):

$$r_t^i = E_{t|t-1}\left[r_t^{e,i} + \varphi_{core}\pi_{Y,t}^i + \varphi_y \hat{y}_t^{e,i}\right] + \varphi_{com}\Delta q_t, \forall\ i \in \{1,\dots,N-1\}. \quad (4.7)$$

The *N*th commodity-importing economy then optimises the policy rule of the same type, given the optimised rule in equation (4.6):

$$r_t^i = E_{t|t-1}\left[r_t^{e,i} + \varphi_{core}\pi_{Y,t}^i + \varphi_y \hat{y}_t^{e,i}\right] + \varphi_{com}\Delta q_t + e_t, \quad (4.8)$$

where e_t corresponds to the misdiagnosis error.

The extent of the incentive to deviate from the group can be evaluated, in terms of monetary policy implications, by examining the coefficients in the policy rule. For coefficients that are close, the incentive to deviate is small. For large deviations, the incentive is correspondingly large.

[24] See appendix F for a complete derivation of the impact of the demand from one single country *i* on the global commodity price.

As noted above, if one country had an incentive to deviate from the consensus, then each country facing the same situation would have a similar incentive to deviate, which would have implications for the whole group. Moreover, there is a "first mover" advantage, which by itself reinforces the likelihood of non-optimal group behaviour. That is, the incentive for each to deviate implies that the group is likely to collectively act *as if* it were misdiagnosing the true nature of the shock. The result is procyclical monetary policy at the global level. By ignoring monetary policy spillovers and spillbacks, this tendency to act as if global demand shocks were exogenous means that external supply shocks naturally open up potential gains from policy coordination.

V. Learning from past mistakes

In this section, we allow the policymaker to learn from the economy's reaction to its policy decision.[25] This is in contrast to the more elaborate information set in the Section IV, where, the monetary authority diagnosed the nature of the shock at $t=1$ and conditioned the subsequent monetary policy responses on its initial (mis-)diagnosis. For example, if it diagnosed the shock to be a supply shock (even though it was a demand shock), the monetary authority would respond without ever updating its initial diagnosis about the nature of the shock. In this section, the monetary authority is assumed to be able to learn about the initial source of the shock over time and therefore updates its initial diagnosis about the source of the shock. In this way, the impulse

[25] The misperception exercises in the previous section were deliberately designed to be simple: on impact of the shock, the central bank sets the policy rate by only being able to observe the change in commodity prices. While this, in our view, is a realistic assumption – commodity prices can be observed in real time on world markets while GDP and inflation are subject to substantial measurement lags – it implies that as soon as GDP and inflation numbers are released and the central bank finds them at odds with its initial guess of the source of the shock, the central bank would reverse course. The reason for the partial updating in the previous experiments is that monetary policy misperceptions are persistent even after output and inflation are revealed.

responses may converge faster over time to those associated with the full information impulse responses.[26]

The rate of learning will depend on the full range of shocks at the time of the commodity price shock, as well as all the subsequent shocks that may potentially obscure the origins of the commodity price shock. We model this dynamic with Bayesian learning. The setup is as follows. On impact (t=1), the monetary authority has a prior about the source of the shock and makes an inference about it. For our illustrative exercise, the mass of the prior distribution is centered well away from the true shock distribution.

We first focus on the case of a supply shock misinterpreted as a demand shock (similar to misdiagnosis A). We start by setting the monetary authority's prior probability of a demand shock at $P_0(D)=0.99$ and for a supply shock $P_0(S)=0.01$. At the end of the period, the monetary authority observes output and inflation. Since additional shocks may have occurred, the central bank attaches a probability distribution to the outcomes it expects to observe. In other words, the prior probability placed on the *D* scenario (originating shock was demand) induces a prior for the outcomes for output and inflation. As the authorities thought the originating shock was from the demand side, such prior probability distribution is centered around the outcomes that would have occurred if the originating shock was indeed a demand shock, ie around the baseline response of Graph 1. Instead, the originating shock was actually a supply shock, and the actual outcome is the combination of the originating supply shock and the monetary policy shock induced by the misperception. So, the outcomes are drawn from a distribution that is centered around the responses of the misperception A exercise in Graph 3. As the monetary authority observes values of output and inflation that are far from their prior beliefs, it updates its inferences about the initial shock according to the

[26] Note, however, that past "mistakes" during the learning process will influence the state of the economy. Therefore, the impulse responses will generally differ from those in Sections III and IV.

likelihood of what they have observed under the *D* scenario: more formally, the posterior probability at time t=1 is:

$$P_1(D|\pi_1, y_1) = \frac{L(D|\pi_1,y_1)P_0(\mathrm{D})}{L(D|\pi_1,y_1)P_0(\mathrm{D})+L(S|\pi_1,y_1|)P_0(\mathrm{S})},$$

where L(D|π_1,y_1) and L(S|π_1,y_1) are, respectively, the likelihood of an originating demand or supply shock based on observed output and inflation.[27]

This is visualised in Graph 7, which reports the prior distributions for core inflation, the output gap, the likelihood of the observed outcomes and the resulting posterior distribution as well as the implied probability distribution of the policy rate.

The posterior distribution at time t=1 is used as the protoprior for the period t=2 and gets, in turn, updated by the new observations. In general, the updating follows the following recursive equation:

$$P_t(D|\boldsymbol{\pi_t}, \boldsymbol{y_t}) = \frac{L(D|\pi_t, y_t)P_{t-1}(D|\, \boldsymbol{\pi_{t-1}}, \boldsymbol{y_{t-1}})}{L(D|\pi_t, y_t)P_{t-1}(D|\, \boldsymbol{\pi_{t-1}}, \boldsymbol{y_{t-1}}) + L(S|\pi_t, y_t)P_{t-1}(S|\, \boldsymbol{\pi_{t-1}}, \boldsymbol{y_{t-1}})}$$

which by recursive substitution can also be written as:

$$P_t(D|\boldsymbol{\pi_t}, \boldsymbol{y_t}) = \frac{L(D|\pi_t, y_t)P_0(D)\prod_{\tau=1}^{t-1} P_\tau(\boldsymbol{\pi}_\tau, \boldsymbol{y_\tau}|D)}{L(D|\pi_t, y_t)P_0(D)\prod_{\tau=1}^{t-1} P_\tau(\boldsymbol{\pi}_\tau, \boldsymbol{y_\tau}|D) + L(S|\pi_t, y_t)P_0(S)\prod_{\tau=1}^{t-1} P_\tau(\boldsymbol{\pi}_\tau, \boldsymbol{y_\tau}|S)}.$$

This produces a sequence of posterior probabilities that the monetary authority attaches to the original nature of the shock as new information becomes available.

We summarise results up to t=3 for the misperception case A in Graph 7. At time t=1, a supply shock hits the economy but the central bank treats is as a demand shock. The first row of Graph 7 shows the distribution of inflation under the two scenarios. As the likelihood and the prior are still well distinguished, the posterior distribution turns out to be bi-modal. Yet the

27 Note that technically the likelihood function would incorporate all observable variables. Here, we abstract from those variables that do not enter the monetary policy reaction function.

distribution of the output gap (second row) is less precise and the posterior distribution is not bimodal.

At time t=2, the prior and the likelihood are closer to each other, as the monetary authority starts learning about the initial misdiagnosis and responds optimally. The posterior distribution becomes more unimodal. Note, however, that some of the convergence is also due to fairly rapid convergence of the impulse responses under the baseline and the case of misperception A. As a consequence, the interest rate response (bottom row) is somewhere between the prior and the likelihood.

Results for the case of misperception B (Graph 8) also show the benefits of learning. In this case, the initial monetary policy response turns out to be excessively loose. As the monetary authority learns about the mistake and eventually corrects its stance, the resulting policy is procyclical with output and inflation more volatility than if the shock had been diagnosed correct.

This Bayesian exercise suggests that it may be very difficult in real time for a monetary authority to infer an initial misdiagnosis. Of course, if the commodity price shock is very large relative to the other shocks hitting the economy, the learning will be faster as the economy's reaction to the true shock will show through more prominently in the posterior distribution. But for run-of-the-mill shocks of the type calibrated in this paper, the ability of a monetary authority to learn from past mistakes may be constrained by looking simply at the macroeconomic consequences of its policy actions. This exercise suggests that efforts to uncover the initial shocks exploiting microeconomic and the cross-country data would be useful and suggests that better international cooperation and information sharing could prove welfare enhancing.

VI. Conclusions

The main findings of this paper are that: i) monetary authorities deliver better economic performance when they are able to accurately identify the source of the shocks, ie global supply and demand shocks, driving commodity prices; ii) when it is difficult to identify the supply and demand shocks, monetary authorities can limit the deterioration in economic performance by targeting core inflation; and iii) as a cautionary note, if monetary authorities face a risk of misdiagnosing commodity price fluctuations, ie as a result of external (or exogenous) supply shocks when they are truly driven by global demand shocks, their policy actions can contribute unwittingly to increased procyclicality of global inflation, output and commodity prices.

These findings argue for greater prominence to be given to global factors in domestic monetary policymaking. They also support calls for greater monetary policy cooperation, if only to institutionalise the sharing of information about the nature of shocks hitting economies. In the case where *all* monetary authorities initially see shocks hitting their respective economies as arising from external sources, such cooperation might lead to a useful pause and a reality check before acting.[28]

[28] This model can be extended in different ways to cover a wider range of policy challenges, for example, by including final goods production and nominal rigidities in the exporting countries. However, at this stage the model is kept as simple as needed to analyse the basic monetary policy implications for the commodity-importing country from a global perspective. In future work, alternative sources of frictions will be explored to better reflect the full range of policy trade-offs created by commodity price swings.

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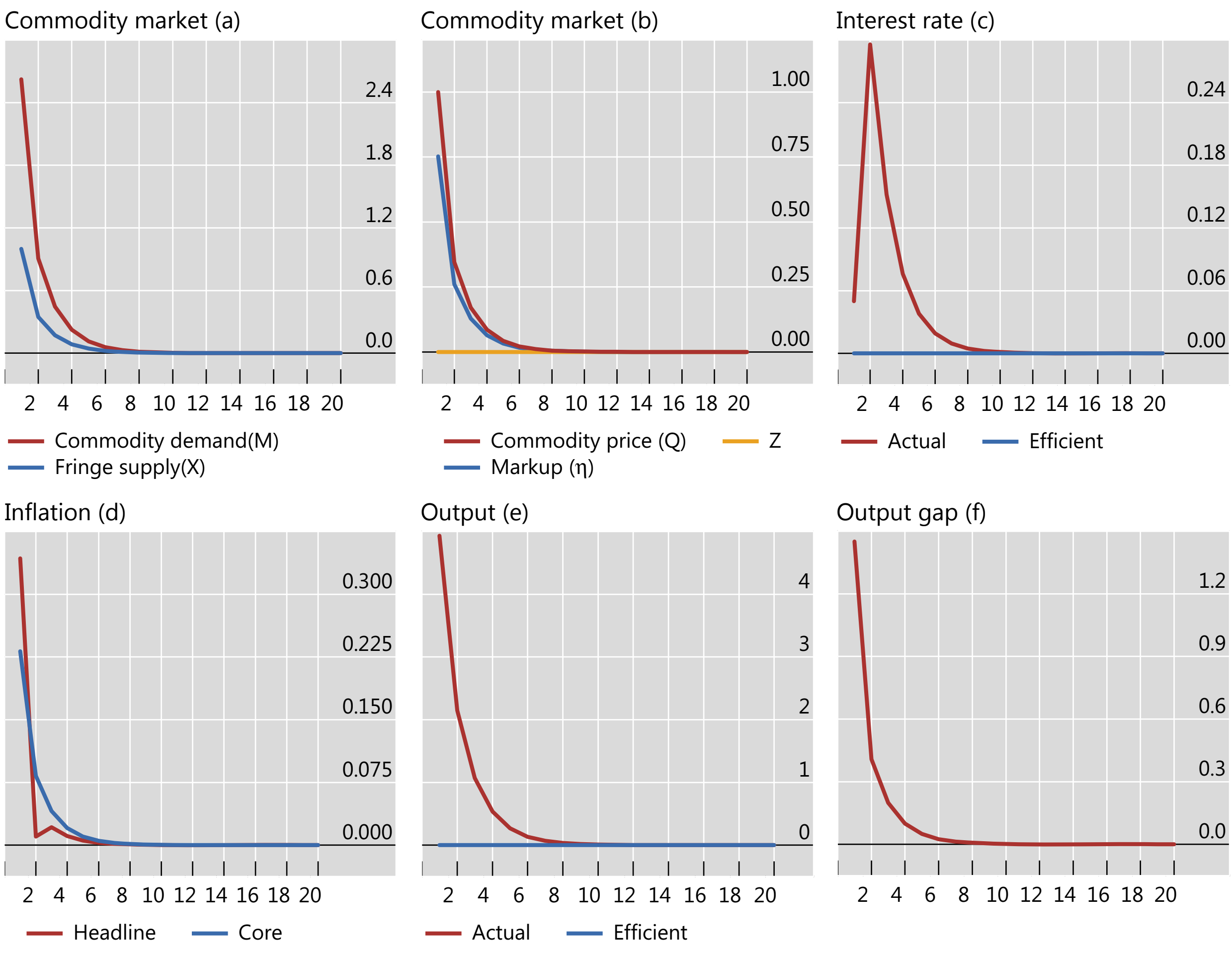
Graphs

Response to a positive aggregate demand shock

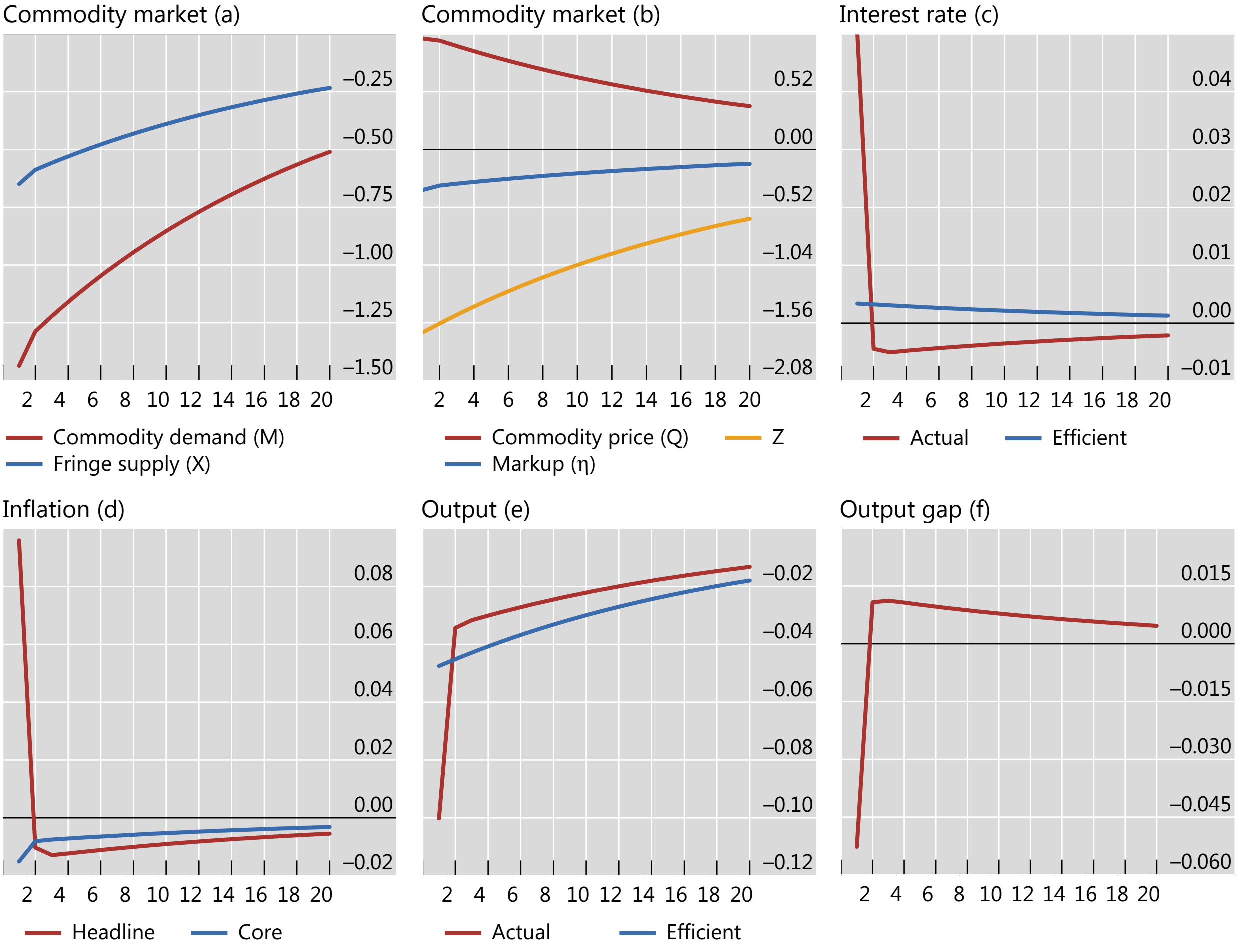
Graph 1



Impulse responses to an aggregate demand shock using the policy rule $r_t = E_{t|t-1}\left(r_t^e + \varphi_{core}\pi_{Y,t} + \varphi_y \hat{y}_t^e\right) + \varphi_q \Delta q_t$, under the efficient benchmark. The shock is calibrated so to generate a 1% increase in commodity prices.

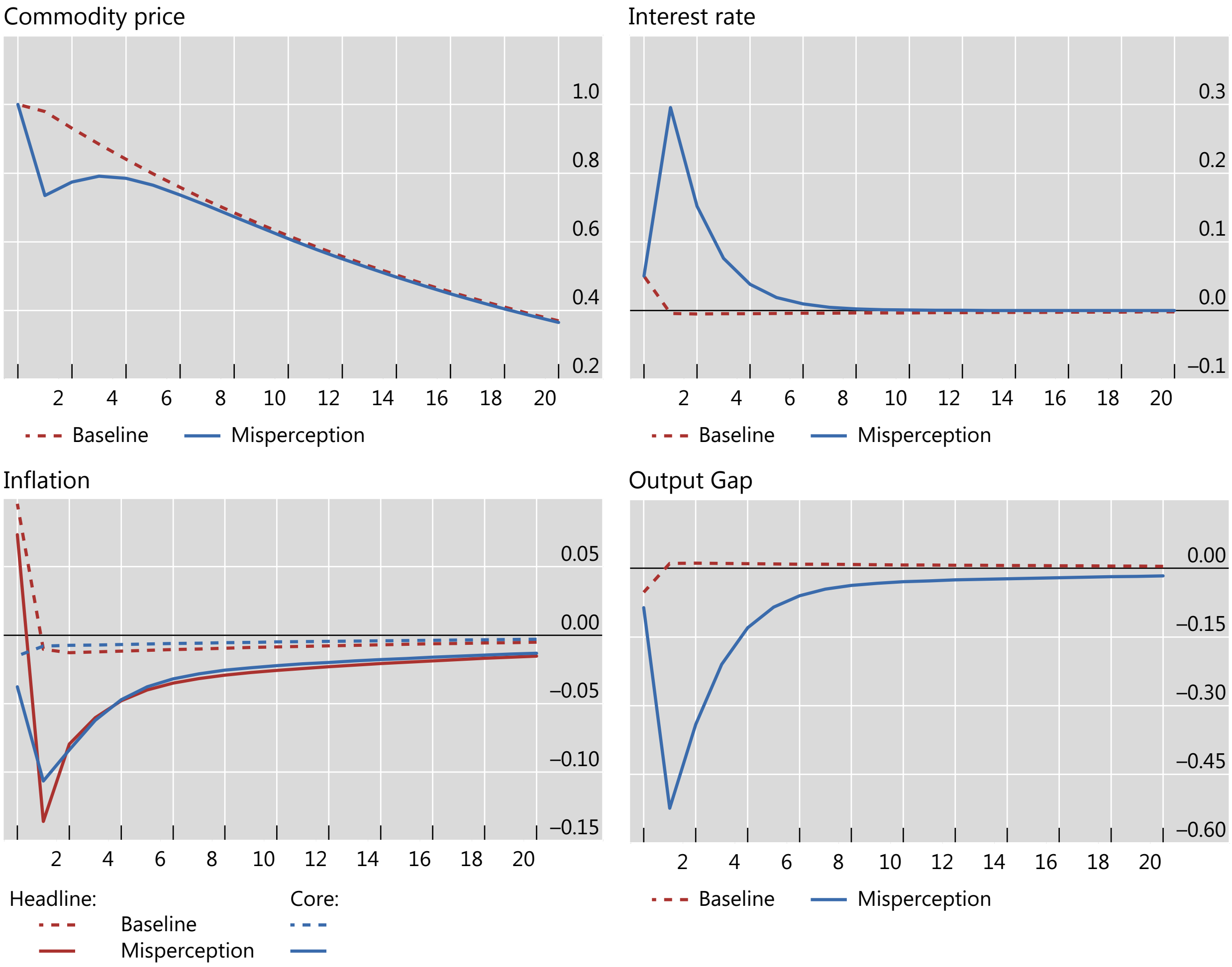
Response to a negative supply shock

Graph 2



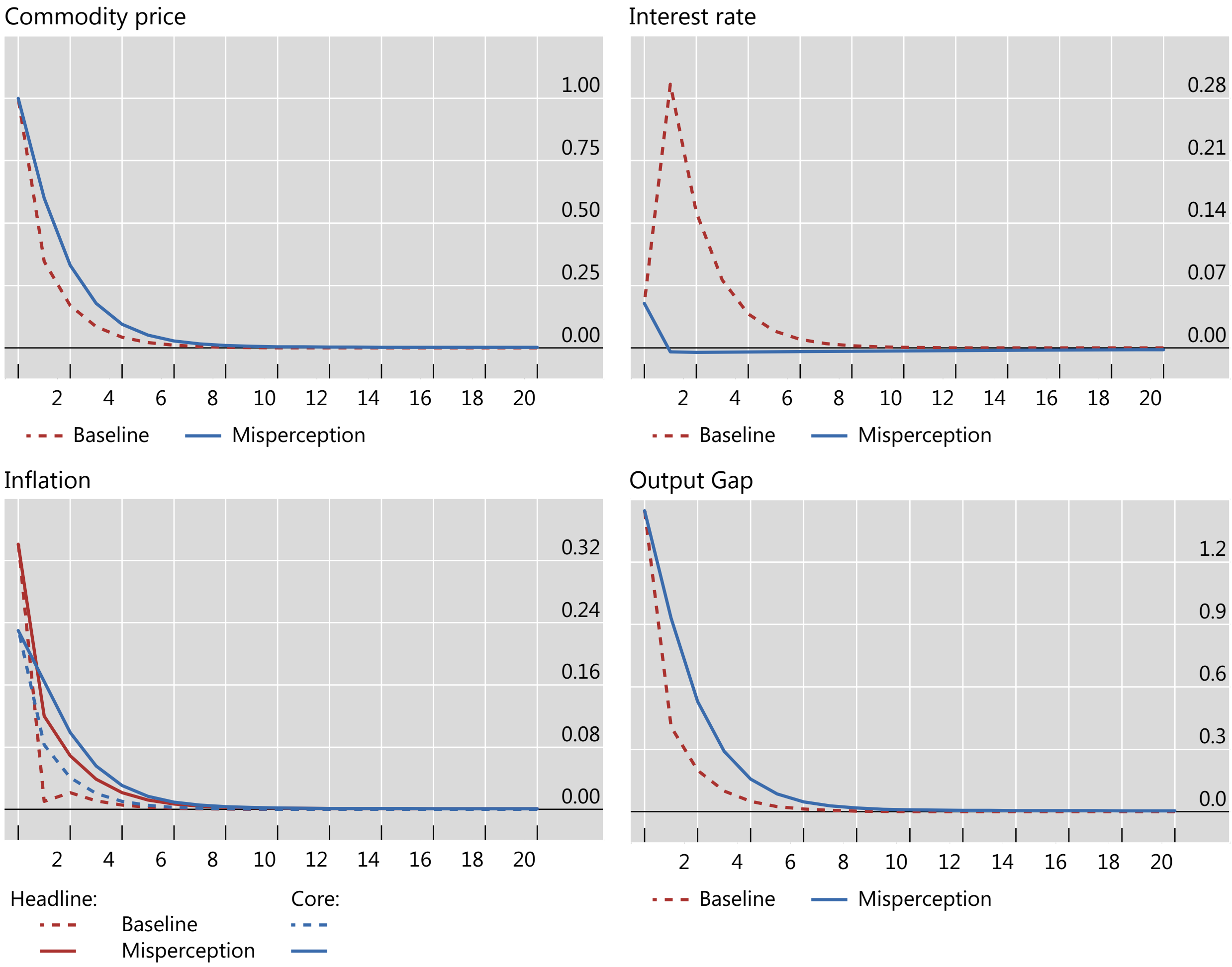
Impulse responses to a (negative) supply shock using the policy rule $r_t = E_{t|t-1}\left(r_t^e + \varphi_{core}\pi_{Y,t} + \varphi_y \hat{y}_t^e\right) + \varphi_q \Delta q_t$, under the efficient benchmark. The shock is calibrated so to generate a 1% increase in commodity prices.

Response to a negative supply shock under misdiagnosis type A — Graph 3



Impulse response to a negative commodity supply shock when the monetary authority attributes all the fluctuation in the commodity price to an aggregate demand shock (misdiagnosis type A), assuming the efficient benchmark policy rule.

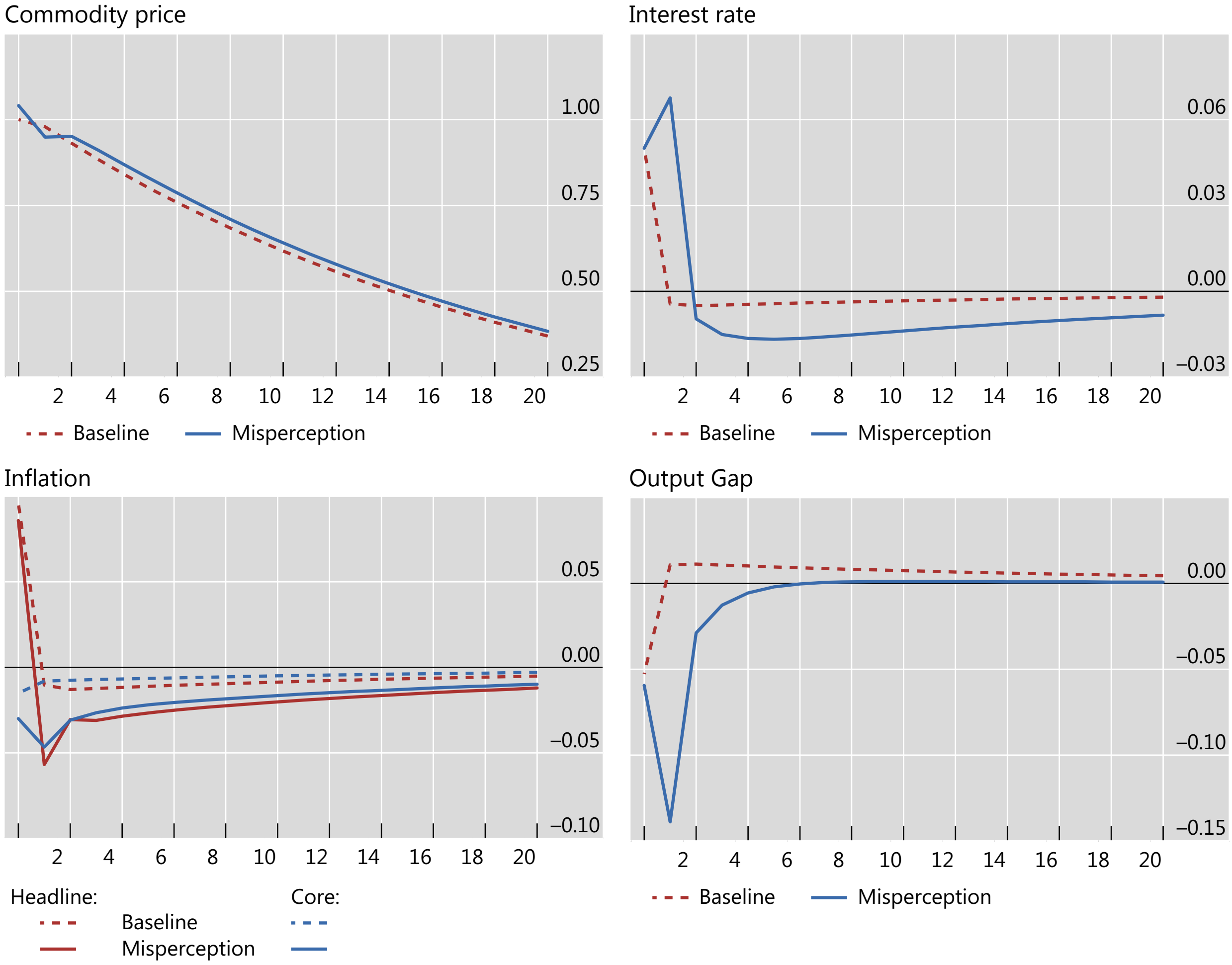
Response to a positive demand shock under misdiagnosis type B — Graph 4



Impulse response to an aggregate demand shock when the monetary authority attributes all the fluctuation in the commodity price to a supply shock (misdiagnosis type B), assuming the efficient benchmark policy rule.

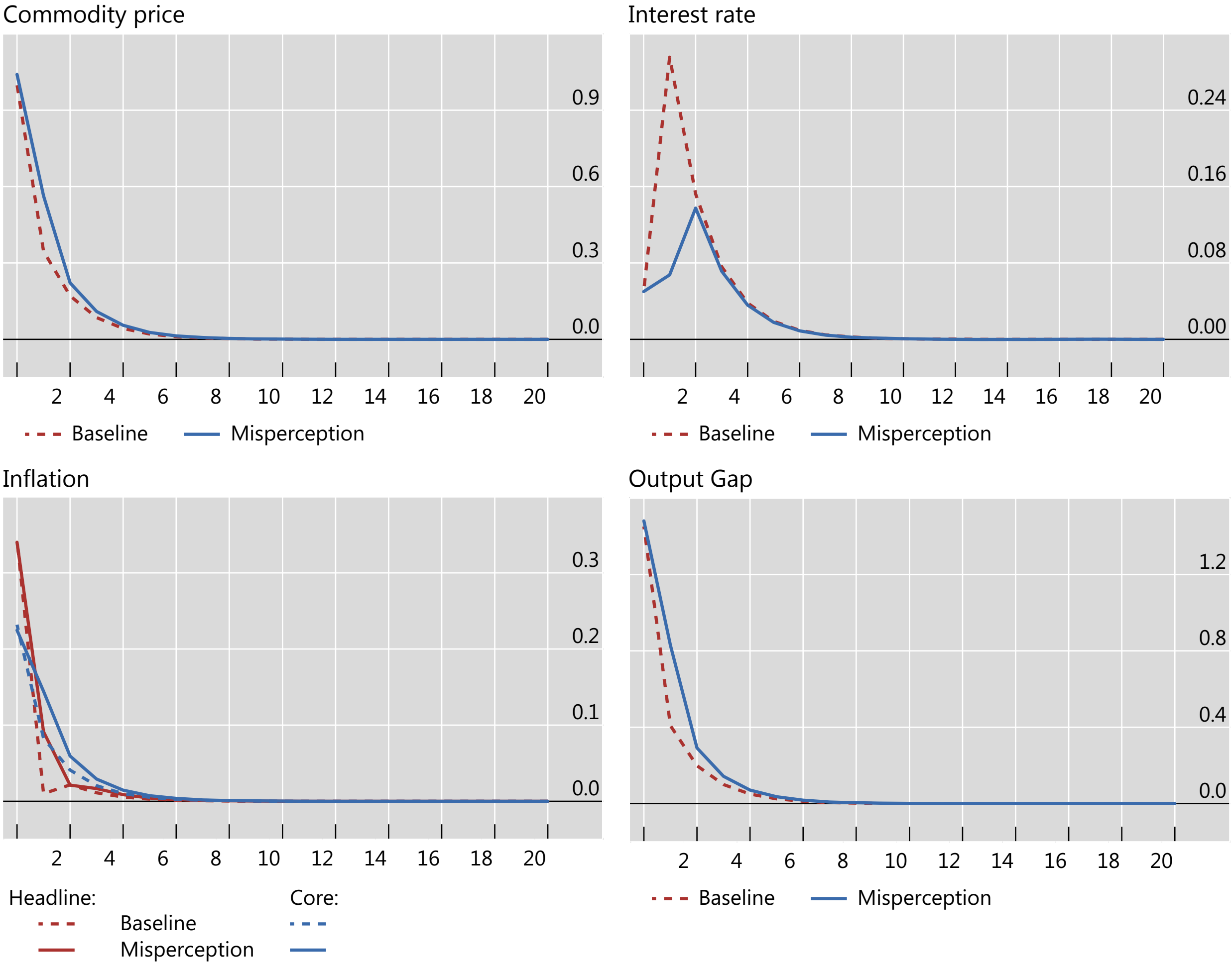
Response to a negative supply shock under misdiagnosis type C

Graph 5



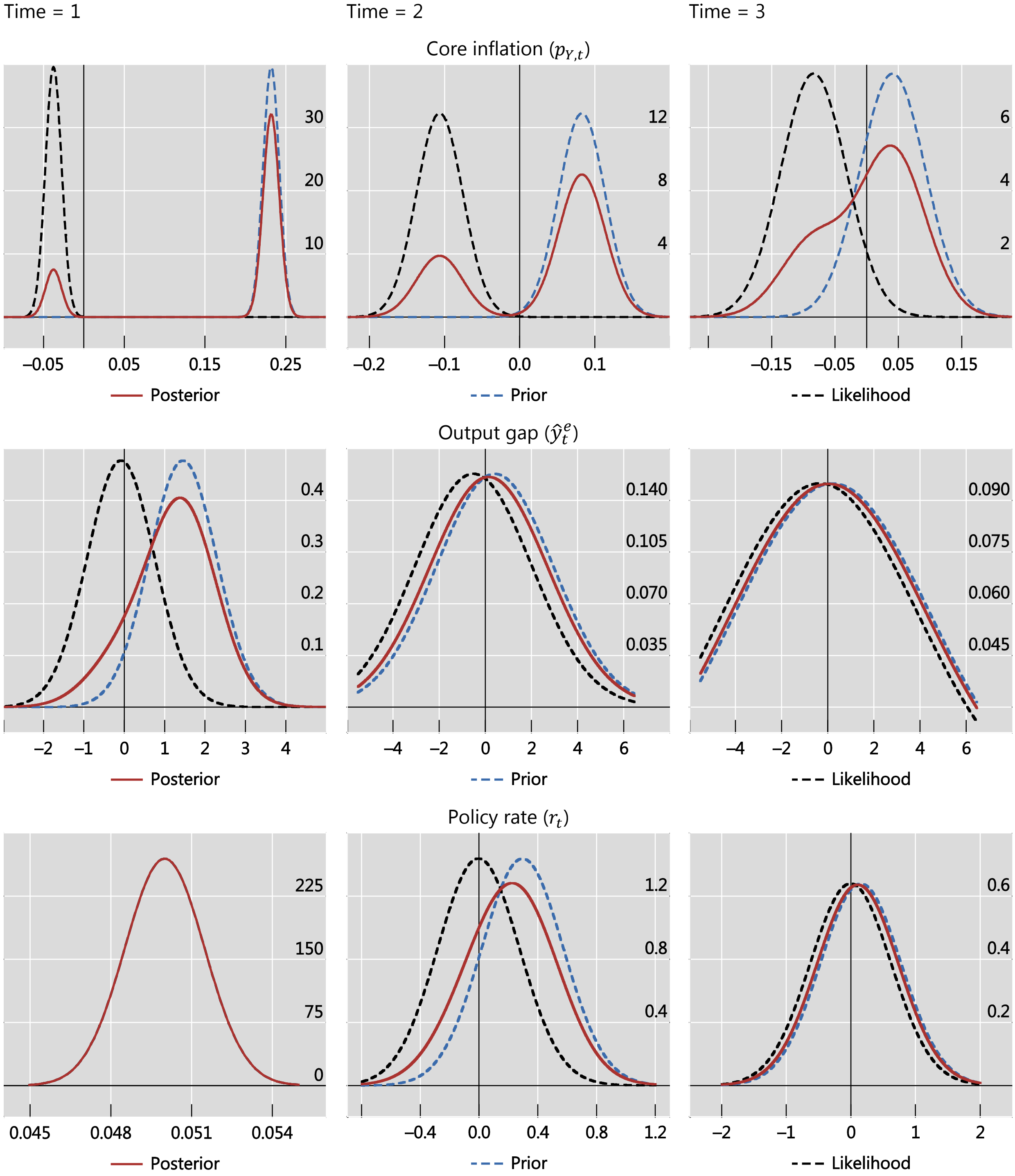
Impulse response to a negative commodity supply shock when the monetary authority attributes the fluctuation in the commodity price proportionally to aggregate demand shock and commodity supply shock, with weights given by the ratio of their standard deviations (misdiagnosis type C), assuming the efficient benchmark policy rule and a 0.5 autoregression coefficient for the shock processes.

Response to a positive demand shock under misdiagnosis type C | Graph 6



Impulse response to an aggregate demand shock when the monetary authority attributes the fluctuation in the commodity price proportionally to aggregate demand shock and commodity supply shock, with weights given by the ratio of their standard deviations (misdiagnosis type C), assuming the efficient benchmark policy rule and a 0.5 autoregression coefficient for the shock processes.

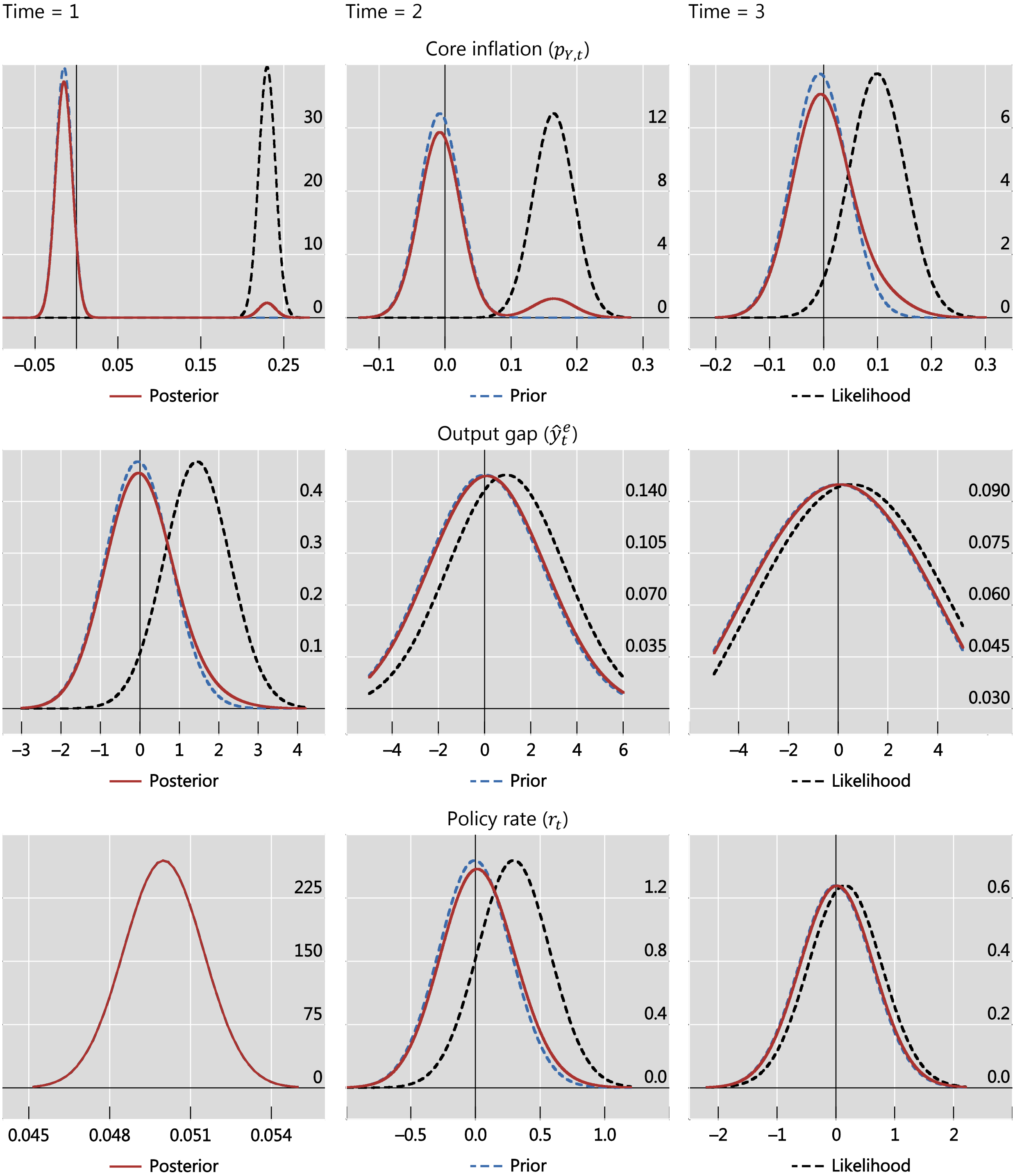
Misperception case A with Bayesian learning Graph 7



Case A: a supply shock misdiagnosed to be a demand shock. Probability distributions for core inflation, the output gap and the policy rate, as perceived by the monetary authority at three different points in time after the initial shock has occurred. "Prior" corresponds to the distribution under the demand shock initially assumed by the monetary authority (*D*); "Likelihood" is the probability distribution of observed variables that actually occur from a combination of the initial supply shock and the monetary policy error induced by misperception (*S*) and "Posterior" is the weighted average of the two, where the weights are given by the posterior probabilities assigned to *D* and *S*.

Misperception case B with Bayesian learning

Graph 8



Case B: a demand shock misdiagnosed to be an (external) supply shock. Probability distributions for core inflation, the output gap and the policy rate, as perceived by the monetary authority at three different points in time after the initial shock has occurred. "Prior" corresponds to the distribution under the supply shock initially assumed by the monetary authority (*S*); "Likelihood" is the probability distribution of observed variables that actually occur from a combination of the initial demand shock and the monetary policy error induced by misperception (*D*) and "Posterior" is the weighted average of the two, where the weights are given by the posterior probabilities assigned to *D* and *S*.

A. Model equations

A.1 Commodity-importing country

A.1.1 Aggregate demand

Resource constraint (AD): $$\frac{P_{Y,t}}{P_t} Y_t = C_t + Q_t \mathfrak{M}_{Y,t}.$$

IS equation: $$1 = \beta E_t \left[R_t \left(\frac{1}{\Pi_{t+1}} \right) \left(\frac{C_{t+1}}{C_t} \right)^{-1} \right].$$

Total consumption: $$C_t = \left(C_{Y,t} \right)^{1-\gamma} \left(\mathfrak{M}_{C,t} \right)^{\gamma}.$$

Relative price final goods/total consumption goods, $\left(P_{Y,t} / P_t \right)$:

$$1 = \left(\frac{P_{Y,t}}{P_t} \right)^{1-\gamma} Q_t^{\gamma}.$$

A.1.2 Aggregate supply

Total inflation: $$\Pi_t = \left(\Pi_{Y,t} \right)^{1-\gamma} \left(\frac{Q_t}{Q_{t-1}} \Pi_t \right)^{\gamma}.$$

Core inflation Phillips Curve:

$$D_t = Y_t \left(C_t \right)^{-1} + \theta \beta E_t \left[\left(\Pi_{Y,t+1} \right)^{\epsilon-1} D_{t+1} \right],$$

$$N_t = \mu Y_t \left(C_t \right)^{-1} MC_t + \theta \beta E_t \left[\left(\Pi_{Y,t+1} \right)^{\epsilon} N_{t+1} \right],$$

$$\theta \left(\Pi_{Y,t} \right)^{\epsilon-1} = 1 - (1-\theta) \left(\frac{N_t}{D_t} \right)^{1-\epsilon}.$$

Price dispersion (only second-order effects):

$$\Delta_t = \left(1 - \theta \right) \left(\frac{N_t}{D_t} \right)^{-\varepsilon} + \theta \Delta_{t-1} \left(\Pi_{Y,t} \right)^{\varepsilon}.$$

Marginal costs: $$MC_t = \left(\frac{W_t}{P_t}\right)^{1-\alpha} \left(Q_t\right)^{\alpha} / \left[A_t \left(1-\alpha\right)^{1-\alpha} \alpha^{\alpha}\right].$$

A.1.3 Labour market

Labour supply: $$\frac{W_t}{P_t} = C_t L_t^{v}.$$

Labour demand: $$L_t = \left(1-\alpha\right)\frac{MC_t}{W_t/P_t} Y_t \Delta_t.$$

A.1.4 Monetary policy

$$R_t = \bar{R}\left(\Pi_t\right)^{\varphi_{head}} \left(\Pi_{Yt}\right)^{\varphi_{core}} \left(\frac{Q_t}{Q_{t-1}}\right)^{\varphi_{com}}.$$

A.2 Commodity exporters

A.2.1 Dominant exporter

Total commodity demand: $$\mathfrak{M}_t = \mathfrak{M}_{C,t} + \mathfrak{M}_{Y,t} - X_t.$$

Commodity demand (consumption): $$\mathfrak{M}_{C,t} = \gamma \frac{1}{Q_t} C_t.$$

Commodity demand (production): $$\mathfrak{M}_{Y,t} = \alpha \frac{MC_t}{Q_t} Y_t \Delta_t.$$

A.2.2 Fringe exporters

Commodity supply by fringe: $$X_t = \Omega_t Q_t Z_t.$$

Commodity supply:

a) Perfect competition (PC): $$Q_t^{PC} = Z_t^{-1};$$

b) Imperfect competition (IC): $$Q_t^{IC} = \Psi Z_t^{-1} \text{ and } \Psi_t = 1 + \frac{\mathfrak{M}_t}{2X_t}$$

.

B. Linearised model equations

B.1 Commodity-importing country

B.1.1 Aggregate demand

Resource constraint (AD): $\frac{P_Y}{P}\left(t_{y,t}+y_t\right)=\frac{C}{Y}\left(c_t\right)+Q\frac{\mathfrak{M}_Y}{Y}\left(q_t+\mathfrak{m}_{y,t}\right)$

IS equation: $c_t-E_t c_{t+1}=-\left(r_t-E_t\pi_{t+1}\right)$

Total consumption: $c_t=\left(1-\gamma\right)c_{y,t}+\gamma\mathfrak{m}_{c,t}$

Relative prices: $T_{y,t}=P_{Y,t}\ /\ P_t$ and $0=\left(1-\gamma\right)t_{y,t}+\gamma q_t$

B.1.2 Aggregate supply

Total inflation: $\pi_t=\left(1-\gamma\right)\pi_{y,t}+\gamma\left(q_t-q_{t-1}+\pi_t\right)$

Core inflation Phillips Curve: $\pi_{y,t}=\kappa mc_t+\beta E_t\pi_{y,t+1}$

Marginal costs: $mc_t=\left(1-\alpha\right)wp_t+\alpha q_t-a_t$

B.1.3 Labour market

Labour supply ($WP_t=W_t/P_t$): $wp_t=c_t+vl_t$

Labour demand: $l_t=mc_t-wp_t+y_t$

B.1.4 Monetary policy

$$r_t=\varphi_{head}\,\pi_t+\varphi_{core}\,\pi_{y,t}+\varphi_{com}\left(q_t-q_{t-1}\right)$$

B.2 Commodity exporters

Total commodity demand: $\frac{\mathfrak{M}}{Y}\mathfrak{m}_t=\frac{\mathfrak{M}_C}{Y}\mathfrak{m}_{c,t}+\frac{\mathfrak{M}_Y}{Y}\mathfrak{m}_{y,t}-\frac{X}{Y}x_t$

Commodity consumption demand: $\mathfrak{m}_{c,t}=c_t-q_t$

Commodity production demand: $\mathfrak{m}_{y,t}=mc_t+y_t-q_t$

Supply by competitive fringe: $x_t=\omega_t+z_t+q_t$

Commodity supply:

a) Perfect competition (PC): $q_t^{PC} = -z_t$

b) Imperfect competition (IC): $q_t^{IC} = -z_t + \psi_t$ and

$$\psi_t = \left(\frac{\Psi - 1}{\Psi}\right)\left(\mathfrak{m}_t - x_t\right)$$

Exogenous variables: $\left(z_t, \omega_t, a_t, g_t\right)$

C. Steady-state equations

The equations determining the steady state of the commodity sector are:

$$\Psi = 1 + \frac{\mathfrak{M}/Y}{2X/Y}$$

$$Q = \Psi Z^{-1}$$

$$\frac{P_Y}{P} = Q^{-\gamma/(1-\gamma)}$$

$$\frac{X}{Y} = \frac{\Omega Z}{Y} Q, \quad given \ \frac{\Omega Z}{Y}$$

$$\frac{\mathfrak{M}}{Y} = \left[\gamma \frac{P_Y}{P} + (1-\gamma)\frac{\alpha}{\mu}\right]\frac{1}{Q} - \frac{X}{Y}.$$

This system of equations depends on the parameterisation of $(\alpha, \gamma, \frac{\Omega Z}{Y}, \mu, Z)$.

Also, with zero steady-state inflation, the following steady-state equations (in log-linear form) complete the model:

$$\Pi = 1$$

$$MC = \frac{1}{\mu}$$

$$\Delta = 1$$

$$\frac{C}{Y} = \frac{P_Y}{P} - \frac{\alpha}{\mu}$$

$$L = \left[\frac{1-\alpha}{\mu} / \left(\frac{C}{Y}\right)\right]^{1/(1-v)}$$

$$Y = \left[A\left(\frac{\alpha}{\mu}\frac{1}{Q}\right)^{\alpha}\right]^{1/(1-\alpha)} L\,.$$

D. Dominant commodity exporter problem

For the dominant commodity exporter, the optimisation problem can be written as a series of intra-temporal decisions. Under the assumption that the dominant commodity exporter does not fully internalise of the actions of the other exporters (i.e. taking as given the macroeconomic variables (C_t, MC_t, Y_t, Δ_t and Ω_t) of the commodity-importing country), the problem can be written as:

$$\max_{\mathfrak{M}_t} \ln\left(Q^{1/(1-\gamma)}\mathfrak{M}_t - \mathfrak{M}_t / Z_t\right)$$

$$\text{s.t.}\, Q_t = h(\mathfrak{M}_t),$$

where this corresponds to equations (2.44), (2.45) and (2.46). The first-order condition of this problem is:

$$Q_t = \left[Z_t^{-1}\frac{1}{1-\zeta^{-1}\eta_t}\right]^{\zeta},$$

where $\eta_t \equiv -\partial \ln Q_t / \partial \ln \mathfrak{M}_t = -h'(\mathfrak{M}_t)\mathfrak{M}_t) / Q_t$ is the elasticity of commodity demand (in absolute value) and $\zeta = 1-\gamma$.

The demand for the commodity by the dominant commodity producer takes the form:

$$\mathfrak{M}_t = \frac{1}{Q_t}\mathfrak{D}_t - Q_t\mathfrak{E}_t,$$

where $\mathfrak{D}_t \equiv \left(\gamma C_t + \alpha MC_t Y_t \Delta_t\right)$ and $\mathfrak{E}_t \equiv \Omega_t Z_t$.

The inverse demand function is:[29]

$$Q_t = \frac{1}{2}\frac{\sqrt{\mathfrak{M}^2 + 4\mathfrak{D}_t\mathfrak{E}_t} - \mathfrak{M}}{\mathfrak{E}_t}.$$

29 The other solution, $Q_t = -\frac{1}{2}\frac{\sqrt{\mathfrak{M}_t^2 - 4\mathfrak{E}\mathfrak{D}_t} + \mathfrak{M}_t}{\mathfrak{E}_t}$, is ruled out because it would imply a negative value for the commodity price.

Given this, the elasticity of demand for the commodity is:

$$\eta_t = -\frac{f'\left(\mathfrak{M}_t\right)\mathfrak{M}_t}{Q_t} = \frac{\mathfrak{M}_t}{\sqrt{\mathfrak{M}_t^2 + 4\mathfrak{D}_t\mathfrak{E}_t}} = \frac{\mathfrak{M}_t}{\sqrt{\mathfrak{M}_t^2 + 4\left(\mathfrak{M}_t + X_t\right)X_t}} = \frac{\mathfrak{M}_t}{\mathfrak{M}_t + 2X_t}$$

Finally, the commodity price mark-up is:

$$\Psi_t = \frac{1}{1-\eta_t} = 1 + \frac{\mathfrak{M}_t}{2X_t}.$$

E. Deriving the benchmark level for final goods output

This appendix derives the benchmark level of final goods output as a function of variables MC_t, Δ_t and Q_t, starting from the aggregate production function: $Y_t = A_t L_t^{1-\alpha} \mathfrak{M}_{Y,t}{}^{\alpha}$.

First, the labour input, L_t, is recast in terms of these variables of interest. Substituting the labour supply equation (2.6) into the aggregate lab Δ_t, our demand equation (2.28) and solving for L_t yields:

$$L_t = \left[(1-\alpha) \frac{MC_t}{C_t} Y_t \, \Delta_t \right]^{1/(1+v)}.$$

Replacing C_t using the aggregate demand equation (2.50) along with equations (2.7), (2.8) and (2.49) yield the following form:

$$L_t = \left[(1-\alpha) \frac{MC_t}{\frac{P_{Y,t}}{P_t} Y_t - Q_t \mathfrak{M}_{Y,t}} Y_t \, \Delta_t \right]^{1/(1+v)}.$$

Substituting the commodity demand for output (2.29), making use of equation (2.10), to solve for Q_t achieves the desired result:

$$L_t = \left[\frac{(1-\alpha) MC_t}{(Q_t)^{-\gamma/(1-\gamma)} - \alpha MC_t \Delta_t} \Delta_t \right]^{1/(1+v)}.$$

Second, the commodity demand in equation (2.28) is plugged into the aggregate output equation (2.29), giving the following form for Y_t:

$$Y_t = \left(\frac{A_t}{\Delta_t} \right)^{1/(1-\alpha)} L_t \left(\alpha \frac{MC_t}{Q_t} \Delta_t \right)^{\alpha/(1-\alpha)}.$$

Finally, substituting the labour input from the equation above completes the derivation:

$$Y_t = \left(\frac{A_t}{\Delta_t} \right)^{1/(1-\alpha)} \left[\frac{(1-\alpha) MC_t}{(Q_t)^{-\gamma/(1-\gamma)} - \alpha \, MC_t \, \Delta_t} \Delta_t \right]^{1/(1+v)} \left(\alpha \frac{MC_t}{Q_t} \Delta_t \right)^{\alpha/(1-\alpha)}.$$

F. The impact of a small country on commodity prices

Consider N countries indexed by j. If they are all equal in the production technology the demand for commodity of each country is:

$$M_{Y,t}^{j} = \alpha^{j} \frac{MC_t^j}{P_{m,t}} P_{Y,t}^{j} Y_t^j \Delta_t^j$$

If all countries do not deviate from the cooperative equilibrium, the world demand for commodity ($\bar{M}_{Y,t}^{W}$) is:

$$\bar{M}_{Y,t}^{W} = \sum_{j=1}^{N} \alpha^{j} \frac{MC_t^j}{P_{m,t}} P_{Y,t}^{j} Y_t^j \Delta_t^j = N\alpha^{j} \frac{MC_t^j}{P_{m,t}} P_{Y,t}^{j} Y_t^j \Delta_t^j$$

Solving for the commodity price:

$$\bar{P}_{m,t} = \frac{N\alpha^{j} MC_t^j P_{Y,t}^{j} Y_t^j \Delta_t^j}{\bar{M}_{Y,t}^{W}}$$

If one country (i) deviates, the world demand ($M_{Y,t}^{W}$) becomes:

$$M_{Y,t}^{W} = \sum_{j=1}^{N-1} \alpha^{j} \frac{MC_t^j}{P_{m,t}} P_{Y,t}^{j} Y_t^j \Delta_t^j + \alpha^{i} \frac{MC_t^i}{P_{m,t}} P_{Y,t}^{i} Y_t^i \Delta_t^i$$

$$= (N-1)\alpha^{j} \frac{MC_t^j}{P_{m,t}} P_{Y,t}^{j} Y_t^j \Delta_t^j + \alpha^{i} \frac{MC_t^i}{P_{m,t}} P_{Y,t}^{i} Y_t^i \Delta_t^i$$

Dividing for the cooperative equilibrium ($\bar{M}_{Y,t}^{W}$):

$$\frac{M_{Y,t}^{W}}{\bar{M}_{Y,t}^{W}} = \frac{(N-1)\alpha^{j} \frac{MC_t^j}{P_{m,t}} P_{Y,t}^{j} Y_t^j \Delta_t^j}{N\alpha^{j} \frac{MC_t^j}{\bar{P}_{m,t}} P_{Y,t}^{j} Y_t^j \Delta_t^j} + \frac{\alpha^{i} \frac{MC_t^i}{P_{m,t}} P_{Y,t}^{i} Y_t^i \Delta_t^i}{N\alpha^{j} \frac{MC_t^j}{\bar{P}_{m,t}} P_{Y,t}^{j} Y_t^j \Delta_t^j}$$

With simple algebra:

$$\frac{M_{Y,t}^{W}}{\bar{M}_{Y,t}^{W}} = \frac{(N-1)}{N} \frac{P_{m,t}}{\bar{P}_{m,t}} + \frac{\alpha^{i} \frac{MC_t^i}{P_{m,t}} P_{Y,t}^{i} Y_t^i \Delta_t^i}{N\alpha^{j} \frac{MC_t^j}{\bar{P}_{m,t}} P_{Y,t}^{j} Y_t^j \Delta_t^j}$$

When $N \to \infty$, $\frac{M_{Y,t}^{W}}{\bar{M}_{Y,t}^{W}} \to \frac{P_{m,t}}{\bar{P}_{m,t}} = 1$ or equivalently $P_{m,t} \to \bar{P}_{m,t}$. Therefore, a deviation from the cooperative equilibrium of one small country does not influence the global commodity price.