BANCO CENTRAL DE RESERVA DEL PERÚ

An Application of a Short Memory Model with Random Level Shifts to the Volatility of Latin American Stock Market Returns

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DT. N° 2015-004
Serie de Documentos de Trabajo
Working Paper series
Julio 2015

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Abstract

Empirical research indicates that the volatility of stock return time series have long memory. However, it has been demonstrated that short memory processes contaminated with random level shifts can often be confused as being long memory. Often this feature is referred to as spurious long memory. This paper represents an empirical study of the random level shift (RLS) model using the approach of Lu and Perron (2010) and Li and Perron (2013) for the volatility of daily stocks returns data for five Latin American countries. The RLS model consists of the sum of a short term memory component and a level shift component, where the level shift component is governed by a Bernoulli process with a shift probability α. The estimation results suggest that the level shifts in the volatility of daily stocks returns data are infrequent but once they are taken into account, the long memory characteristic and the GARCH effects disappear. An out-of-sample forecasting exercise is also provided.

JEL Classification: C22.

Keywords: Returns, Volatility, Long Memory, Random Level Shifts, Kalman Filter, Forecasting, Latin America.

Resumen

La evidencia empírica indica que la volatilidad de las series de retornos bursátiles (o financieras en general) poseen la característica de larga memoria. Sin embargo, de otro lado, existe evidencia que ha mostrado que los procesos de memoria corta contaminados con cambios de nivel aleatorios o esporádicos a menudo pueden ser confundidos con procesos de larga memoria en cuyo caso se dice que esta larga memoria es espuria. En este caso se tiene prcesos con memoria larga espúria. Este trabajo representa un estudio empírico del modelo de cambio de nivel aleatorio (RLS), utilizando el enfoque de Lu y Perron (2010) y Li y Perron (2013) para la volatilidad de los retornos bursátiles diarios de cinco países de América Latina. El modelo RLS consiste en la suma de un componente de memoria corta y un componente de cambio de nivel aleatorios, el cual se rige por un proceso de Bernoulli con una probabilidad α. Los resultados de las estimaciones sugieren que los cambios de nivel son poco frecuentes, pero una vez que se tienen en cuenta, la característica de larga memoria y los efectos GARCH desaparecen. También se proporciona un ejercicio de pronóstico fuera de muestra.

Classificación JEL: C22.

Palabras Claves: Retornos, Volatilidad, Larga Memoria, Cambios de Nivel Aleatorios, Filtro de Kalman, Predicción, América Latina.

An Application of a Short Memory Model with Random Level Shifts to the Volatility of Latin American Stock Market Returns[1]

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1 Introduction

There are two important stylized facts that are found in returns from financial market variables such as stock and exchange rate: the long memory behavior of the volatility of returns and the presence of GARCH effects. Fractionally integrated processes have become a standard class of models to describe the long memory features of economic and financial time series data. Let $\{y_t\}_{t=1}^{T}$ be a stationary time series. Let $\gamma_y(\tau)$ be the autocovariance function of y_t, so y_t has long memory if $\gamma_y(\tau) = c(\tau)\tau^{2d-1}$, for $\tau \Rightarrow \infty$, where $c(\tau)$ is a smooth variation function. It implies that the autocorrelation function (ACF) decays to a hyperbolic rate[3]. On the other hand, $\{y_t\}_{t=1}^{T}$ has spectral density function $f_y(\omega)$ in the frequency ω, so y_t has long memory if $f_y(w) = g(\omega)\omega^{-2d}$, for $\omega \Rightarrow 0$, where $g(\omega)$ is a smooth variation function in a neighborhood of the origin, which means that for all real numbers t, it is verified that $g(t\omega)/g(\omega) \Rightarrow 1$ for $w \Rightarrow 0$. When $d > 0$, the spectral density function is growing for frequencies that are increasingly close to the origin. The rate of divergence to infinity depends on the given value of the parameter d.

A vast literature exists in estimating the long memory parameter d. Granger and Joyeux (1980) developed by first time the notion of fractional integration in terms of an infinite filter corresponding to the expansion of $(1 - L)^d$, where L is the lag operator. When this expansion is applied to a white noise, then we get a series with long memory. Then, Hosking (1981) developed the ARFIMA(p,d,q) model that generalizes the autoregressive integrated moving average processes incorporating fractional values for the integration parameter d. When $0 < d < 0.5$, the fractional integration process has long memory and when $-0.5 < d < 0.5$ the series is stationary. Geweke and Porter-Hudak (1983) show that the asymptotic distribution of the integration parameter d has a Normal distribution based on a linear regression of the log-periodogram with a deterministic regressor; see also Robinson (1995).

In the context of GARCH models, Baillie et al. (1996) proposed the FIGARCH model, where the fractional integration parameter determines that shocks to the conditional variance disappear

[1] This paper is drawn from the Thesis of Roxana Tramontana Tocto at the Department of Economics, Pontificia Universidad Católica del Perú. We thank useful comments of Paul Castillo (Central Bank of Peru), Rodolfo Cermeño (CIDE), Jiawen Xu (Shangai University of Finance and Economics), Zhongjun Qu and Pierre Perron (Boston University), Patricia Lengua Lafosse (PUCP), Mauricio Zevallos (Universidad Estadual de Campinas, Brazil) and participants at the XXXII Meeting of the Central Bank of Peru (2014). We also thank Marco Vega (BCRP) and useful comments of an anonymous Referee. Any remaining errors are our responsibility.

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[3] A practical definition of long memory is to state that the sum of the autocorrelations is infinite; that is, $\lim_{T \Rightarrow \infty} \sum_{j=-T}^{T} |\rho_j| = \infty$.

at a hyperbolic rate of decay. This characteristic allows the temporal dependencies in financial market volatility to be explained. Bollerslev and Mikkelsen (1996) extended the model to the fractional integration exponential GARCH (FIEGARCH). In both cases, the fractional parameter is significant and asymmetries are identified. Ding et al. (1993) estimates the ARCH model taking into account the squared returns and absolute returns and show the existence of long memory. Then, the authors propose the asymmetric power GARCH model (APARCH) allowing the long memory parameter in the volatility and the asymmetry parameter. Finally, Lobato and Savin (1998) apply a semiparametric test to detect the presence of long memory in the daily S&P500 stock returns and their squares. The short memory null hypothesis is not rejected for the level of stock returns while the null hypothesis is rejected for the squared and absolute returns. However, the authors argue that the finding results could be spurious in the squared stock returns due to the nonstationarity of the series and in the absolute values due to the aggregation.

It has been demonstrated in studies that structural break processes and processes with nonlinear features can often be confused as being long memory. Often this feature is referred to as spurious long memory. A steadily growing literature has developed with emphasis on whether it is possible to empirically discriminate between true long memory processes (or fractionally integrated processes) and spurious long memory processes. Perron (1989, 1990) has shown that when there is a contaminated stationary process with structural breaks, the sum of the autoregressive coefficients is biased to the unit. Diebold and Inoue (2001) show that the change of stochastic regime is easily confused with long memory, even asymptotically, provided that the probabilities of structural breaks are small; see also Engle and Smith (1999). Using Monte Carlo simulation, they emphasize the relevance of the theory in finite samples and make it clear the confusion is not merely a theoretical question, but a real possibility in the empirical applications in economic and finance time series. Gourieroux and Jasiak (2001) find that nonlinear time series with infrequent linear breaks could have long memory on the basis of the estimation of the correlogram instead of the estimation of the fractional parameter. This findings show that these series and not the fractionally integrated processes with *i.i.d.* innovations would generate the hyperbolic decay of the autocorrelogram. Granger and Hyung (2004) show that the slow decay in autocorrelation and others properties of fractionally integrated models are generated by occasional breaks. The authors show that not taking into account the breaks causes the presence of long memory in the ACF and the fractional parameter estimated using the method of Geweke and Porter-Hudak (1983) is biased. Mikosch and Stărică (2004a) provide the theoretical basis to explain two stylized facts that are observed in the logarithm of returns: long range dependence in volatility and the integrated GARCH (IGARCH) if it assumed that the data are not stationary. The simulations show that the time series with changing unconditional variance produce estimates of the long memory parameter d that may be erroneously interpreted as evidence of long memory under the assumption os stationarity. There is evidence that the characteristic of long range dependence is caused by feasible structural changes in the logarithm of stock market returns. Also Mikosch and Stărică (2004b) propose a goodness of fit test that shows the similarity between the spectral density of a GARCH process and the logarithm of stock market returns that detect changes in the structure of the data that are related to changes in the unconditional variance. These changes would induce long range dependence in the ACF of absolute returns; see also Stărică and Granger (2005).

Perron and Qu (2010) perform an analysis of various statistics when the underlying model is a short memory process with random level shifts rather than a fractionally integrated process. They analyze the estimates of the ACF, the periodogram, and the log-periodogram. The results show that a short memory process with level shifts is a good candidate for modeling volatility. The estimates clearly follow a pattern that would be obtained if the underlying process was short memory with level shifts. Lu and Perron (2010) estimate a random level shift (RLS) model that consists of the sum of a short term process and a level shift component, where the shift component are governed by a Bernoilli process with a shift probability α. The estimation method transforms the model into linear state-space equations with a mixture of Normal innovations to apply the Kalman filter. The results show that there is reduced evidence of correlation in the remaining noise; therefore, there is no evidence of long memory. On the other hand, once the level shifts found are introduced into a GARCH model and applied to the series, any evidence of GARCH effects disappear. For predictions outside the sample of squared returns, in most cases the RLS model has a better performance than the GARCH (1,1) model and than the fractionally integrated GARCH. Similar results are found by Li and Perron (2013) but applied to two exchange rates.

Empirical studies for financial variables in Latin America are very scarce. This essay forms part of a research agenda suggested in Humala and Rodríguez (2013). The main aim of this paper is to estimate a RLS model to the volatilities of returns of five Latin American financial markets following Lu and Perron (2014) and Li and Perron (2013). The results show that the probability of level shifts is small but is responsible for the presence of long memory in the volatilities of the series analyzed. Having estimated the probability of level shifts, the exact number of such level shifts can be calculated. Thus, the component obtained as a difference between the volatility series and the level shifts possesses an ACF that indicates an absence of long memory. Therefore, we show that short memory processes contaminated with random level shifts can be confused as being long memory in the data considered. Finally, an exercise of out-of-sample forecasting shows that the RLS model has better performance than traditional models for modeling long memory such as the ARFIMA (p,d,q) models.

This paper is structured as follows: Section 2 presents the RLS model and some details related to the estimation algorithm. Section 3 presents the empirical results, which are divided into two aspects: the effects of the level shift component on long memory and on GARCH components. Section 4 discusses the performance of the RLS in terms of prediction, while Section 5 presents the conclusions.

2 The Model

We utilize a simple mixture model that is a combination of a short memory process that depends on a Binomial distribution. Following the notation of Lu and Perron (2014), the RLS model is specified as follows:

$$\begin{aligned} y_t &= a + \tau_t + c_t, \\ \tau_t &= \tau_{t-1} + \delta_t, \\ \delta_t &= \pi_t \eta_t, \end{aligned} \tag{1}$$

where a is a constant, τ_t is the level shift component and c_t is the short memory process, π_t, is a Binomial variable, which takes the value of 1 with probability α and the value of 0 with probability $(1-\alpha)$. In this way, the third expression in (1), when π_t assumes the value of 1, a random level shift η_t occurs with a distribution $\eta_t \sim i.i.d.N(0, \sigma_\eta^2)$. The short memory process (in its general form) is defined by the process $c_t = C(L)e_t$, with $e_t \sim i.i.d.\ N(0, \sigma_e^2)$ and $E|e_t|^r < \infty$ for values $r > 2$ and where $C(L) = \sum_{i=0}^{\infty} c_i L^i$, $\sum_{i=0}^{\infty} i|c_i| < \infty$ y $C(1) \neq 0$. Likewise, it is assumed that π_t, η_t and c_t are mutually independent. Based on the results of Lu and Perron (2010) and Li and Perron (2013), even when it would be worthwhile to consider the component e_t as a random variable (noise), in this paper we model this component as an AR(1) process; that is, $c_t = \phi c_{t-1} + e_t$[4].

In comparison with the Hamilton's Markov-Switching model (1989), this model does not limit the magnitude of level shifts, meaning that any number of patterns is possible. Moreover, the probability 0 or 1 does not depend on past facts, unlike the Markov model. Note that the process δ_t can be described as $\delta_t = \pi_t \eta_{1t} + (1 - \pi_t)\eta_{2t}$, with $\eta_{it} \sim i.i.d.N(0, \sigma^2_{\eta_i})$ for $i = 1, 2$ and $\sigma^2_{\eta_1} = \sigma^2_{\eta}$, $\sigma^2_{\eta_2} = 0$. The first difference model, with the object of eliminating the autoregressive process of the level shift component, only depends on the Binomial process: $\Delta y_t = \tau_t - \tau_{t-1} + c_t - c_{t-1} = c_t - c_{t-1} + \delta_t$, and passing to the state space form, the measurement and transition equations are obtained, respectively: $\Delta y_t = c_t - c_{t-1} + \delta_t$, $c_t = \phi c_{t-1} + e_t$. In matrix form we have $\Delta y_t = HX_t + \delta_t$ and $X_t = FX_{t-1} + U_t$, where, $X_t = [c_t, c_{t-1}]$, $F = \begin{bmatrix} \phi & 0 \\ 1 & 0 \end{bmatrix}$, $H = [1, -1]'$. In this case, the first row of the matrix F shows the coefficient ϕ of the autoregressive part of the short memory component. Moreover, U is a Normally distributed vector of dimension 2 with mean 0 and variance: $Q = \begin{bmatrix} \sigma^2_e & 0 \\ 0 & 0 \end{bmatrix}$. In comparison with the standard state space model, the major difference in the current model is that the distribution of δ_t is a mixture of Normal differences with variance σ^2_η and 0, occurring with probabilities α and $1 - \alpha$, respectively[5].

The model set out above is a special case of the models employed in Wada and Perron (2006) and Perron and Wada (2009). In this case, there only exist shocks that affect the level of the series, with the restriction imposed that the variance of one of the components of the mixture of distributions is zero. The basic input for the estimation is the increase of the states through realizations of the mixture at time t so that the Kalman filter can be used to form the conditional likelihood function to the realizations of the states. The latent states are eliminated from the final likelihood expression by adding on all possible realizations of the states. In consequence, despite its fundamental differences, the model takes a structure that is similar to Hamilton's Markov-Switching model (1994). Let $Y_t = (\Delta y_1, ..., \Delta y_t)$ the vector of available observations at time t and denote the vector of parameters by $\theta = [\sigma^2_\eta, \alpha, \sigma^2_e, \phi]$. By adopting the notation used in Hamilton (1994), $\mathbf{1}$ represents a (4×1) vector of ones, the symbol $\odot$ denotes element-by-element multiplication, $\widehat{\xi}^{ij}_{t|t-1} = vec(\widetilde{\xi}_{t|t-1})$ with the (i, j)th element of $\widetilde{\xi}_{t|t-1}$ being $\Pr(s_{t-1} = i, s_t = j|Y_{t-1}; \theta)$ and $\omega_t = vec(\widetilde{\omega}_t)$ with the (i, j)th element of $\widetilde{\omega}_t$ being $f(\Delta y_t|s_{t-1} = i, s_t = j, Y_{t-1}; \theta)$ for $i, j \in \{1, 2\}$. Thus, we have $s_t = 1$ when $\pi_t = 1$; that is, a level shift occurs. Using the same notation as Lu and Perron (2010), the logarithm of the likelihood function is $\ln(L) = \sum_{t=1}^{T} \ln f(\Delta y_t|Y_{t-1}; \theta)$, where $f(\Delta y_t|Y_{t-1}, \theta) = \sum_{i=1}^{2} \sum_{j=1}^{2} f(\Delta y_t|s_{t-1} = i, s_t = j, Y_{t-1}, \theta) \Pr(s_{t-1} = i, s_t = j|Y_{t-1}, \theta) \equiv \mathbf{1}'(\widehat{\xi}_{t|t-1} \odot \omega_t)$. By applying conditional probability rules, the Bayes rule and the independence of

[4]We opted for an AR(1) specification but if the coefficient ϕ is statistically insignificant, $c_t = e_t$. Estimates with longer lags for the AR process showed no significance of the respective parameters. This is consistent with the statements by the RLS model because if the persistence or long memory in the volatility of the series analyzed is mainly explained by rare or sporadic level shifts, then the short-memory component contains little persistence or it is a noise. This justifies c_t is modeled as a noise or maximum as an AR (1) process.

[5]Note that this model could be extended to model the short memory component as an ARMA process.

s_t with respect to past realizations, we have $\widetilde{\xi}^{ki}_{t|t-1} = \Pr(s_{t-2} = k, s_{t-1} = i|Y_{t-1};\theta)$. The evolution of $\widehat{\xi}_{t|t-1}$ can be expressed as:

$$\begin{bmatrix} \widetilde{\xi}^{11}_{t+1|t} \\ \widetilde{\xi}^{21}_{t+1|t} \\ \widetilde{\xi}^{12}_{t+1|t} \\ \widetilde{\xi}^{22}_{t+1|t} \end{bmatrix} = \begin{bmatrix} \alpha & \alpha & 0 & 0 \\ 0 & 0 & \alpha & \alpha \\ 1-\alpha & 1-\alpha & 0 & 0 \\ 0 & 0 & 1-\alpha & 1-\alpha \end{bmatrix} \begin{bmatrix} \widetilde{\xi}^{11}_{t|t} \\ \widetilde{\xi}^{21}_{t|t} \\ \widetilde{\xi}^{12}_{t|t} \\ \widetilde{\xi}^{22}_{t|t} \end{bmatrix},$$

which is equal to $\widehat{\xi}_{t+1|t} = \Pi\widehat{\xi}_{t|t}$ with $\widehat{\xi}_{t|t} = \frac{(\widehat{\xi}_{t|t-1}\odot\omega_t)}{\mathbf{1}'(\widehat{\xi}_{t|t-1}\odot\omega_t)}$. In consequence, the conditional likelihood function for Δy_t follows the following Normal density:

$$\widetilde{\omega}^{ij}_t = f(\Delta y_t|s_{t-1} = i, s_t = j, Y_{t-1}, \theta) = \frac{1}{\sqrt{2\pi}}|f^{ij}_t|^{-1/2}\exp(-\frac{v^{ij\prime}_t(f^{ij}_t)^{-1/2}v^{ij}_t}{2}), \tag{2}$$

where v^{ij}_t is the prediction error and f^{ij}_t is its variance, and these terms are defined as: $v^{ij}_t = \Delta y_t - \Delta y^i_{t|t-1} = \Delta y_t - E[\Delta y_t|s_t = i, Y_{t-1};\theta]$, and $f^{ij}_t = E(v^{ij}_t v^{ij\prime}_t)$. The best predictions for the state variable and its respective conditional variance in $s_{t-1} = i$ are $X^i_{t|t-1} = FX^i_{t-1|t-1}$, and $P^i_{t|t-1} = FP^i_{t-1|t-1}F' + Q$, respectively.

The measurement equation is $\Delta y_t = HX_t + \delta_t$, where the error δ_t has mean 0 and a variance that can take values $R_1 = \sigma^2_\eta$ with probability α or $R_2 = 0$ with probability $(1-\alpha)$. Thus, the prediction error is $v^{ij}_t = \Delta y_t - HX^i_{t|t-1}$ and its variance is $f^{ij}_t = HP^i_{t|t-1}H' + R_j$. In this way, given that $s_t = j$ y $s_{t-1} = i$ and using update formulas, we have $X^{ij}_{t|t} = X^i_{t|t-1} + P^i_{t|t-1}H'(HP^i_{t|t-1}H' + R_j)^{-1}(\Delta y_t - HX^i_{t|t-1})$ and $P^{ij}_{t|t-1} = P^i_{t|t-1} - P^i_{t|t-1}H'(HP^i_{t|t-1}H' + R_j)^{-1}HP^i_{t|t-1}$. With the objective of reducing the dimensionality problem in the estimation, Lu and Perron (2010) use the re-collapsing procedure suggested by Harrison and Stevens (1976). By doing so, ω^{ij}_t is unaffected by the history of the states before time $t-1$. We have four possible states corresponding to $S_t = 1$ when $(s_t = 1, s_{t-1} = 1)$, $S_t = 2$ when $(s_t = 1, s_{t-1} = 2)$, $S_t = 3$ when $(s_t = 2, s_{t-1} = 2)$ and $S_t = 4$ when $(s_t = 2, s_{t-1} = 2)$ and the matrix Π is defined as before. Taking the definitions of ω_t, $\widehat{\xi}_{t|t}$, $\widehat{\xi}_{t+1|t}$, the group of conditional probabilities and one-period forward predictions, the same structure as a version of Hamilton's Markov model (1994) is obtained. Nonetheless, the EM algorithm cannot be utilized. This is because the mean and the variance in the function of conditional density are non-linear functions of the parameters θ and of the past realizations $\{\Delta y_{t-j}; j \geq 1\}$. Likewise, the conditional probability of being in a given regime $\widehat{\xi}_{t|t}$, it is not separable from the conditional densities ω_t. For further details, see Lu and Perron (2010), Li and Perron (2013), and Wada and Perron (2006).

Once the point estimate of α is obtained, a possible path is the use of a smoothed estimate of the level shift component τ_t. Nonetheless, in this context of abrupt structural changes, the conventional smothers perform poorly. Instead of this, we use the method proposed by Bai and Perron (1998, 2003) to obtain the dates on which the level shifts occur as well as the means (averages) inside each segment. Thus, we use the estimate α to obtain an estimate of the number of level shifts and the Bai and Perron method (1998, 2003) to obtain the estimates of break dates that globally minimize the following squared residuals: $\sum_{i=1}^{m+1}\sum_{t=T_{i-1}+1}^{T_i}[y_t - \mu_i]^2$, where m is the number of breaks,

T_i $(i = 1, 2, ...; m)$ are the break dates with $T_0 = 0$ and $T_{m+1} = T$ and μ_i $(i = 1, 2, ..., m+1)$ are the means (averages) inside each regime, which can be estimated once the break dates are estimated. This method is efficient and can manage a large number of observations; see Bai and Perron (2003) for further details[6].

3 Empirical Results

We use five daily series that are the volatilities of the returns of the major Latin American stock markets: MERVAL (Argentina) from 05/01/1988 to 13/06/2013 (6284 observations), IBOV (Brazil) from 02/01/1992 to 13/06/2013 (5303 observations), IPSA (Chile) from 03/01/1989 to 13/06/2013 (6097 observations), MEXBOL (Mexico) from 20/01/1994 to 13/06/2013 (4840 observations), and IGBVL (Peru) from 03/01/1990 to 13/06/2013 (5832 observations). The returns series are generated as $r_t = \ln(P_t) - \ln(P_{t-1})$, where P_t is the closing price index of the respective stock market. Following recent literature (see Lu and Perron (2010), Li and Perron (2010), Xu and Perron (2010), among others), we model log-absolute returns[7]. When returns are zero or close to it, the log-absolute transformation implies extreme negative values. Using the estimation method described in Section 2.1, these outliers would be attributed to the level shifts component and thus bias the probability of shifts upward. To avoid this inconvenient, we bound absolute returns away from zero by adding a small constant, i.e., we use $y_t = \log(|r_t| + 0.001)$, a technique introduced to the stochastic volatility literature by Fuller (1996). The results are robust to alternative specifications, for example using another value for this so-called offset parameter, deleting zero observations, or replacing them by a small value. Another important comment is the fact that we use daily returns as opposed to realized volatility series constructed from intra-daily high-frequency data which has recently become popular. It is true that realized volatility series are less noisy measure of volatility. However, it is problematic in the current context for he following reasons: (i) such series are typically available for short span. Given the fact that the level shifts will be relatively rare, it is imperative to have a long span of data in order to made reliable estimates of the probability of occurrence of the level shifts; (ii) such series are available only for specific assets as opposed to market indices. Because the goal of the RLS model is to allow for particular events affecting overall markets, using specific asset would confound such market-wide events with idiosyncratic ones associated with the particular asset used; (iii) we are interested to re-evaluate the adequacy of ARFIMA and GARCH models applied to daily returns when taking into account the possibility of level shifts. Therefore, it is important to have estimates of these level shifts for squared daily returns which are equivalent to those estimated using log-absolute returns.

The Figure 1 shows the returns series for five economies while Table 1 shows the descriptive statistics of the returns and the volatilities. In Figure 1, the frequent high variation grouping of returns in periods of international or local crisis can be observed. The values of the descriptive

[6] Note that since the model allows for consecutive level shifts, we set the minimum segment length to only one observation.

[7] Using this measure has two advantages: (i) it does not suffer from a non-negativity constraint as do, for example, absolute or squared returns. Actually, it is a similar argument as used in the EGARCH(1,1) model proposed by Nelson (1991): the dependent variable is $\log(\sigma_t^2)$ in order to avoid the problems of negativity when the dependent variable is σ_t^2 as in the standard GARCH models and other relatives models; (ii) there is no loss relative to using square returns in identifying level shifts since log-absolute returns is a monotonic transformation. It is true that log-absolute returns are quite noisy but it is not problematic since the algorithm used is robust to the presence of noise.

statistics show a mean close to zero. On the other hand, the standard deviation is different, and the markets of Argentina, Brazil, and Peru are the most volatile. Figure 2 shows the ACF for the five series for 2,000 lags. In all cases, the long memory evidence is clear.

3.1 Effects of Level Shifts on Long Memory and ARFIMA Models

The estimated parameters are set out in Table 2[8] and correspond to the standard deviation of the level shift component σ_η, the probability of a level shift α, the standard deviation of the stationary component σ_e and the autoregressive ϕ of the specification $AR(1)$ for c_t. All estimated coefficients can be seen to be significant.

The estimate of ϕ is not significant for the cases of Argentina and Brazil. In the other cases, though significant, it is small. On the other hand, the probability of level shift is small in all cases considered. Therefore, given T and the estimates of α, we can find the number of level shifts for each country: Argentina has 25 breaks, Brazil has 53 breaks, Chile has 49 breaks, Mexico has 29 breaks and Peru has 26 breaks. These values indicate that level shifts are rare and occur with a duration of 222, 98, 124, 161 and 216 days on average for Argentina, Brazil, Chile, Mexico and Peru, respectively[9].

The Figure 3 presents the smoothed (Gaussian kernel) series of the level shift component and the level shift component with dates estimated by the method of Bai and Perron (1998, 2003). The smoothed estimates are erratic, even though they closely follow the changes in the mean of the series as indicated by the method of Bai and Perron (1998, 2003). Figure 3 shows the grouping of the dates of level shifts where volatility has experienced strong variations in short periods of time. Moreover, Figure 3 shows that the main dates of level shifts are similar across the five countries. These level shifts proceed from two sources. The first is of external origin and comes due to crises occurring in other countries: the Asian, Russian and Mexican crises that affected the volatilities of the five markets analyzed. Another important external factor was the international crisis (2008) that affected all the economies analyzed. The second is of domestic origin and includes level shifts caused by primary election periods, monetary crises that trigger periods of high inflation, as well as social events. All these factors contribute to the presence of level shifts in the volatility series. Argentina and Brazil experience successive level shifts at the start of their samples due to high inflationary processes that have occurred in these countries. The continuous level shifts in Brazil go on until 1998. The greatest level shifts in Chile are discerned in 1990-1996. In the case of Peru, level shifts in 1990, 1996, and 1998 are appreciated[10]. The year 2008 clearly affects all economies.

The ACF was estimated by leaving aside the estimated level component using the algorithm of Bai and Perron (2003). What is observed (Figure 4) is that all traces of long memory have disappeared[11]. A complementary way of analyzing the long memory characteristic is by estimating

[8] Given that all components of the vector of states are stationary, we will initialize the vector of states and its variance matrix by its unconditional expected values: $X_{0|0} = (0,0)'$ and $P_{0|0} = \begin{bmatrix} \sigma_e^2 & 0 \\ 0 & 0 \end{bmatrix}$. With the aim of avoiding the problem of local maximums, we re-estimate the model using a long group of random initial values and select the related estimates with the highest likelihood after finding convergence.

[9] Observing the distribution of the level shifts, we find that the minimum occurrence of the level shifts are 2, 1, 3, 4, and 3 days for Argentina, Brazil, Chile, Mexico and Peru, respectively. Their maximum values are, respectively: 1010, 750, 614, 734 and 1715 days.

[10] A more detailed account of the Peruvian case can be found in Ojeda Cunya and Rodríguez (2014).

[11] Similar results are obtained when we use the smoothed estimates of the level shift component.

models ARFIMA(0,d,0) and ARFIMA (1,d,1) for the volatility series and the volatility series excluding the level component[12]. In the case of ARFIMA(0,d,0) model estimates, the results indicate the same message observed in the ACF in Figure 4. The estimates of the fractional parameter (d) indicate a long term behavior, which is a stylized fact frequently mentioned in the literature. However, the short term component shows fractional parameter estimates $\widehat{d}$ with values that are positive but too reduced to imply long memory or in other cases is negative implying anti persistence. The results are similar for the case of the ARFIMA(1,d,1) model. The volatility series show a positive and significant fractional parameter. Moreover, the parameters ϕ (autoregressive) and θ (moving averages) are small but significant. In the case of volatility adjusted by the level shift component estimated by Bai and Perron (1998, 2003), the fractional parameter show a value that is negative and very close to zero by allowing confirmation that once the level shifts are taken into account, the long memory behavior is eliminated.

3.2 Level Shift Effect in the GARCH and CGARCH Models

Given that GARCH models -as well as ARFIMA models -are frequently used to model volatility, we estimate GARCH(1,1) and CGARCH models. The GARCH model is formulated as:

$$\begin{aligned} \widetilde{r}_t &= \sigma_t \epsilon_t, \quad (3) \\ \sigma_t^2 &= \mu + \beta_1 \widetilde{r}_{t-1}^2 + \beta_2 \sigma_{t-1}^2, \quad (4) \end{aligned}$$

where $\widetilde{r}_t$ are returns discounted by their mean, ϵ_t is a distribution *i.i.d.* t-Student with mean 0 and variance 1. The CGARCH model is specified as follows:

$$\begin{aligned} \widetilde{r}_t &= \sigma_t \varepsilon_t, \quad (5) \\ (\sigma_t^2 - n_t) &= \beta_1(\widetilde{r}_{t-1}^2 - n_{t-1}) + \beta_2(\sigma_{t-1}^2 - n_{t-1}), \quad (6) \\ n_t &= \mu + \rho(n_{t-1} - \mu) + \psi(\widetilde{r}_{t-1}^2 - \sigma_{t-1}^2). \quad (7) \end{aligned}$$

The important coefficients are β_1 and β_2, which indicate the presence of conditional heteroskedasticity effects. The parameter μ is a constant to which n_t converges, which represents the long term component of time-varying volatility. Moreover, the equation (6) represents the transitory component of volatility. In addition, the parameter ρ measures the persistence of shocks in the permanent component of the equation (7), while persistence is measured by $(\beta_1 + \beta_2)$ in the equation (4) and the transitory component in the equation (6).

On the other hand, a CGARCH model is estimated but increased with auxiliary variables (dummy):

$$\begin{aligned} \widetilde{r}_t &= \sigma_t \varepsilon_t \quad (8) \\ (\sigma_t^2 - n_t) &= \beta_1(\widetilde{r}_{t-1}^2 - n_{t-1}) + \beta_2(\sigma_{t-1}^2 - n_{t-1}) \quad (9) \\ n_t &= \mu + \rho(n_{t-1} - \mu) + \psi(\widetilde{r}_{t-1}^2 - \sigma_{t-1}^2) + \sum_{i=2}^{m+1} D_{i,t}\gamma_i, \quad (10) \end{aligned}$$

where $D_{i,t} = 1$ if t pertains to the regime i, and 0 if this does not pertain to the regime, with $t \in \{T_i + 1, ..., T_{i+1}\}$ and T_i $(i = 1, ..., m)$ with the dates of the level shifts which are estimated by

[12] The results are available upon request.

the Bai and Perron method (1998, 2003). The coefficients γ_i are estimated alongside the parameters of the GARCH model and indicate the size of the level shifts.

The results are presented in Table 3. The parameters β_1 and β_2 in the GARCH model are significant for the five markets analyzed. The parameter β_2 is raised and fluctuates between 0.729 (Perú) and 0.913 (Argentina). The sum of β_1 and β_2 is close to the unit indicating that the effect of the shocks slowly decline, which is a characteristic frequently found in the estimates. The estimates imply that half-lives of the shocks are around 77, 29, 99 and 43 days for Brazil, Chile, Mexico and Peru, respectively. In the case of the Argentina, the sum of the coefficients is the unity implying a half-life of the shocks equal to infinity days.

In the CGARCH model we find two scenarios for the estimates of β_1 and β_2. For the case of Argentina, Brazil and Mexico these coefficients are not significant. On the other hand, for Chile and Peru they are significant but their sum is reduced to around 0.50 (Chile) and 0.80 (Peru). Overall, the half-life of the shocks is reduced. This fact is interesting as it shows that in the CGARCH model, on dividing the volatility into a long term component and another short term component, the coefficients β_1 and β_2 are not important, with the so-called GARCH effects disappearing. Nonetheless, the coefficients linked to the long term (ρ, ψ) are highly significant. In particular, it is important to note that the parameter ρ is located very close to the unit. This indicates high persistence in all financial markets analyzed. In fact the half-lives of the shocks are 77, 41, 87 and 173 days for Brazil, Chile, Mexico and Peru, respectively. In the case of Argentina we find again a half-life of the shocks equal to infinity days.

However, since the level shifts are taken into consideration under the form of dummy variables, the parameters β_1 and β_2 are not significant for all series except for the case of Chile. Nonetheless, the sum of both coefficients is much less than the unit. On the other hand it can be seen that the parameter ρ lowers its value drastically from 0.73 (Perú) to 0.26 (Chile). This shows that even when this parameter is significant,the impact of the shocks declines more rapidly than when the level shifts are not considered. The half-lives of the shocks are 1.5, 1.1, 0.52, 1.0, and 2.2 days for Argentina, Brazil, Chile, Mexico and Peru, respectively[13].

Moreover, we analyze the sensitivity of the results using the smoothed estimator of the trend component. This is done by replacing the term $\sum_{i=2}^{m+1} D_{i,t}\gamma_i$ with the smoothed estimator (Gaussian kernel) of the level shift component. The results are similar to those obtained previously. The parameters β_1 and β_2 are not significant (this time including Chile) and the value of the parameter ρ -even if it is significant- decreases drastically by reducing the power of the permanent effect of the equation.

Some conclusions can be ventured thus far: (i) the RLS model with a stationary AR(1) component providing an adequate description of the data; (ii) the level shift component is important and explains both the long memory aspect and the presence of conditional heteroskedasticity as they are generally perceived in the literature. As a final test, we will look at whether the RLS model provides reasonable predictions compared with some traditional models.

4 Forecasting

In this section the RLS model is assessed in comparison with ARFIMA models, with respect to prediction capacity. The predictions are based on the approximation of Varneksov and Perron

[13] Using the smoothed estimate of the component of level shifts, the estimates are very similar: 2.7, 3.0, 1.5, 2.7 and 2.1 for Argentina, Brazil, Chile, Mexico and Peru, respectively.

(2014). In this way, the τ-periods forward predictions are given by:

$$\widehat{y}_{t+\tau|t} = y_t + HF^{\tau}[\sum_{i=1}^{2}\sum_{j=1}^{2}\Pr(s_{t+1}=j)\Pr(s_t=i|Y_t)X^{ij}_{t|t}], \quad (11)$$

where $E_t(y_{t+\tau}) = \widehat{y}_{t+\tau|t}$ is the prediction of volatility in time $t+\tau$, conditional to information on time t, and the matrices F and H are defined as before and the prediction horizons are $\tau = 1, 5, 10, 20, 50$ and 100. Moreover, as a criteria for measuring prediction confidence, we use the mean squared forecast error (MSFE) proposed by Hansen and Lunde (2006) and defined by:

$$MSFE_{\tau,i} = \frac{1}{T_{out}}\sum_{t=1}^{T_{out}}(\overline{\sigma}^2_{t,\tau} - \overline{y}_{t+\tau,i|t})^2, \quad (12)$$

where T_{out} is the number of predictions $\overline{\sigma}^2_{t,\tau} = \sum_{s=1}^{\tau} y_{t+s}$, and $\overline{y}_{t+\tau,i|t} = \sum_{s=1}^{\tau}\widehat{y}_{t+s,i|t}$, with i representing each model. The evaluation and comparison are performed using 5% of the model confidence set (MCS) proposed by Hansen et al. (2011). The MCS allows better evaluations of the models than can be done with comparisons between pairs of models. One of the advantages of this procedure is that the evaluations are performed by taking into account the limitations of the data. This means that if the data are clear, then a single model will be selected; while the data are not sufficiently informative, a MCS with various models would be the result. In these cases we can establish that more than one model offers a good prediction, which cannot be established using other kinds of comparisons.

To perform the predictions, the observations from 02/01/2006 up to the end of the sample were retained. This period includes the international crisis and can serve to verify whether the RLS model is a good predictor. The results are presented in Table 4, and lead to the conclusion that the RLS model is included within 5% of the MCS for practically all prediction horizons.

5 Conclusions

In this paper, we estimate a RLS model using the approach of Lu and Perron (2010) and Li and Perron (2013) for the volatilities of the financial returns of five Latin American economies. Even though we have less observations in comparison with developed countries, our results are conclusive and in line with the findings of Lu and Perron (2010). The estimation results show that the probability of level shifts is small but is responsible for the presence of long memory in the volatilities of the series analyzed. Having estimated the probability of level shifts, the exact number of such level shifts can be calculated. Thus, the component obtained as a difference between the volatility series and the level shifts possesses an ACF that indicates an absence of long memory. Therefore, we show that short memory processes contaminated with random level shifts can be confused as being long memory in the data considered. The estimates of autoregressive conditional heteroskedasticity models discounted by level shifts shows that these components are artificially introduced by level shifts because the estimates of the fractional parameter is negative or close to zero. Finally, an exercise of out-of-sample forecasting shows that the RLS model has better performance than traditional models for modeling long memory such as the models ARFIMA (p,d,q).

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Table 1. Descriptive Statistics

Country	Mean	sd	Maximum	Minimum	Skewness	Kurtosis	Jarque-Bera	Prob
				Returns				
Argentina	0.002	0.033	0.329	-0.757	-0.574	56.744	756638.8	0.000
Brazil	0.003	0.046	0.693	-0.693	0.703	105.558	2544970	0.000
Chile	0.000	0.012	0.118	-0.077	0.182	8.694	8270.115	0.000
Mexico	0.001	0.016	0.122	-0.143	-0.020	9.595	8770.366	0.000
Perú	0.001	0.017	0.143	-0.132	0.519	11.088	16159.29	0.000
				Volatility				
Argentina	-4.434	1.112	-0.277	-6.908	-0.271	2.799	87.557	0.000
Brazil	-4.502	1.173	-0.365	-6.901	-0.275	2.995	73.195	0.000
Chile	-4.992	0.845	-2.128	-6.908	-0.151	2.538	77.484	0.000
Mexico	-4.797	0.894	-1.937	-6.908	-0.161	2.604	52.561	0.000
Peru	-4.858	0.951	-1.931	-6.908	-0.027	2.622	35.376	0.000

Table 2. Estimates of the RLS Model

Country	σ_η	α	σ_e	ϕ	Likelihood
Argentina	1.122^a	0.004^a	0.964^a		8868.144
	(0.131)	(0.001)	(0.009)		
Brazil	0.425^a	0.010^a	0.881^a		6965.685
	(0.118)	(0.006)	(0.009)		
Chile	0.612^a	0.008^a	0.778^a	0.080^a	7299.642
	(0.150)	(0.004)	(0.007)	(0.014)	
Mexico	0.520^a	0.006^a	0.830^a	0.025^a	6067.754
	(0.157)	(0.004)	(0.009)	(0.015)	
Perú	0.875^a	0.004^a	0.842^a	0.115^a	7421.854
	(0.128)	(0.002)	(0.008)	(0.015)	

Standard errors are in parentheses; ^{a,b,c} denote significance at the 1.0%, 5.0% and 10.0%, respectively.

Table 3. Estimates of GARCH and CGARCH Models

Model	parameter	value	s.e.	p-values
		Argentina		
GARCH	β_1	0.092	0.005	0.000
	β_2	0.913	0.004	0.000
CGARCH	β_1	-0.002	0.012	0.900
	β_2	-0.516	5.312	0.922
	ρ	1.000	6.18	0.000
	ψ	0.079	0.004	0.000
CGARCH (using τ_t from Bai and Perron)	β_1	-0.018	0.011	0.107
	β_2	-0.671	0.117	0.000
	ρ	0.631	0.036	0.000
	ψ	0.015	0.016	0.000
CGARCH (using smoothed estimated of τ_t)	β_1	0.013	0.012	0.259
	β_2	0.004	0.600	0.995
	ρ	0.770	3.82	0.000
	ψ	0.212	0.009	0.000
		Brazil		
GARCH	β_1	0.095	0.008	0.000
	β_2	0.896	0.008	0.000
CGARCH	β_1	-0.026	0.018	0.143
	β_2	-0.179	0.644	0.781
	ρ	0.991	0.004	0.000
	ψ	0.100	0.009	0.000
CGARCH (using τ_t from Bai and Perron)	β_1	0.005	0.028	0.865
	β_2	0.037	3.154	0.991
	ρ	0.543	0.049	0.000
	ψ	0.075	0.026	0.004
CGARCH (using smoothed estimated of τ_t)	β_1	-1.394	7.658	0.856
	β_2	2.170	7.729	0.779
	ρ	0.795	0.036	0.000
	ψ	1.446	7.660	0.852

Table 3 (continues). Estimates of GARCH and CGARCH Models

Model	parameter	value	s.e.	p-values
		Chile		
GARCH	β_1	0.169	0.013	0.000
	β_2	0.807	0.012	0.000
CGARCH	β_1	0.138	0.022	0.000
	β_2	0.359	0.109	0.001
	ρ	0.983	0.005	0.000
	ψ	0.120	0.014	0.000
CGARCH (using τ_t from Bai and Perron)	β_1	-1.095	0.484	0.024
	β_2	1.321	0.551	0.016
	ρ	0.261	0.049	0.000
	ψ	1.235	0.489	0.011
CGARCH (using smoothed estimated of τ_t)	β_1	-0.039	0.406	0.924
	β_2	0.515	1.676	0.759
	ρ	0.625	0.087	0.000
	ψ	0.246	0.401	0.540
		Mexico		
GARCH	β_1	0.084	0.008	0.000
	β_2	0.909	0.008	0.000
CGARCH	β_1	0.016	0.017	0.341
	β_2	-0.650	0.507	0.199
	ρ	0.992	0.004	0.000
	ψ	0.082	0.008	0.000
CGARCH (using τ_t from Bai and Perron)	β_1	0.037	0.041	0.365
	β_2	0.018	0.528	0.974
	ρ	0.515	0.022	0.000
	ψ	0.060	0.038	0.120
CGARCH (using smoothed estimated of τ_t)	β_1	-0.740	1.905	0.698
	β_2	1.484	1.986	0.455
	ρ	0.774	0.006	0.000
	ψ	0.811	1.910	0.671

Table 3 (continues). Estimates of GARCH and CGARCH Models

Model	parameter	value	s.e.	p-values
		Peru		
GARCH	β_1	0.255	0.010	0.000
	β_2	0.729	0.008	0.000
CGARCH	β_1	0.232	0.023	0.000
	β_2	0.583	0.043	0.000
	ρ	0.996	0.004	0.000
	ψ	0.104	0.022	0.000
CGARCH (using τ_t from Bai and Perron)	β_1	0.023	0.093	0.804
	β_2	0.426	1.169	0.716
	ρ	0.730	0.040	0.000
	ψ	0.230	0.094	0.014
CGARCH (using smoothed estimated of τ_t)	β_1	-0.024	0.018	0.171
	β_2	0.024	0.448	0.957
	ρ	0.723	0.000	0.000
	ψ	0.306	0.021	0.000

Table 4. Comparison of Forecasts ($\widehat{y}_{t+\tau|t}$)

Argentina	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
RLS	0.75	4.73	12.53	37.64	214.88	886.66
	(1.00*)	(1.00*)	(1.00*)	(1.00*)	(1.00*)	(1.00*)
ARFIMA(0,d,0)	0.94	7.32	21.13	65.67	322.63	1096.05
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	1.23	14.53	50.05	181.17	1025.11	3833.36
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Brazil						
RLS	0.69	3.92	10.04	31.05	175.25	773.83
	(1.00*)	(1.00*)	(1.00*)	(1.00*)	(1.00*)	(1.00*)
ARFIMA(0,d,0)	0.93	8.37	26.76	92.74	498.01	1792.57
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.88	7.22	22.14	74.18	383.24	1341.15
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Chile						
RLS	0.45	4.08	11.91	40.69	260.54	1002.64
	(1.00*)	(1.00*)	(1.00*)	(1.00*)	(1.00*)	(0.000)
ARFIMA(0,d,0)	0.70	6.31	18.68	58.95	268.86	769.50
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(1.00*)
ARFIMA(1,d,1)	0.71	6.29	18.56	58.47	266.00	760.18
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Mexico						
RLS	0.57	4.07	11.90	40.19	231.50	886.56
	(1.00*)	(1.00*)	(1.00*)	(1.00*)	(1.00*)	(1.00*)
ARFIMA(0,d,0)	0.79	7.08	22.73	77.58	394.14	1319.17
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.79	7.11	22.83	77.93	395.49	1322.16
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Peru						
RLS	0.49	4.35	11.94	31.94	142.01	620.40
	(1.00*)	(1.00*)	(1.00*)	(1.00*)	(1.00*)	(1.00*)
ARFIMA(0,d,0)	0.80	6.86	19.72	58.01	271.13	927.40
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.80	6.87	19.84	58.91	283.42	1002.74
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Numbers are the MSFE; p-values of the MCS are reported in parentheses; * denotes that the model belongs to the 5% of the MCS of Hansen et al. (2011) comparing between all models.

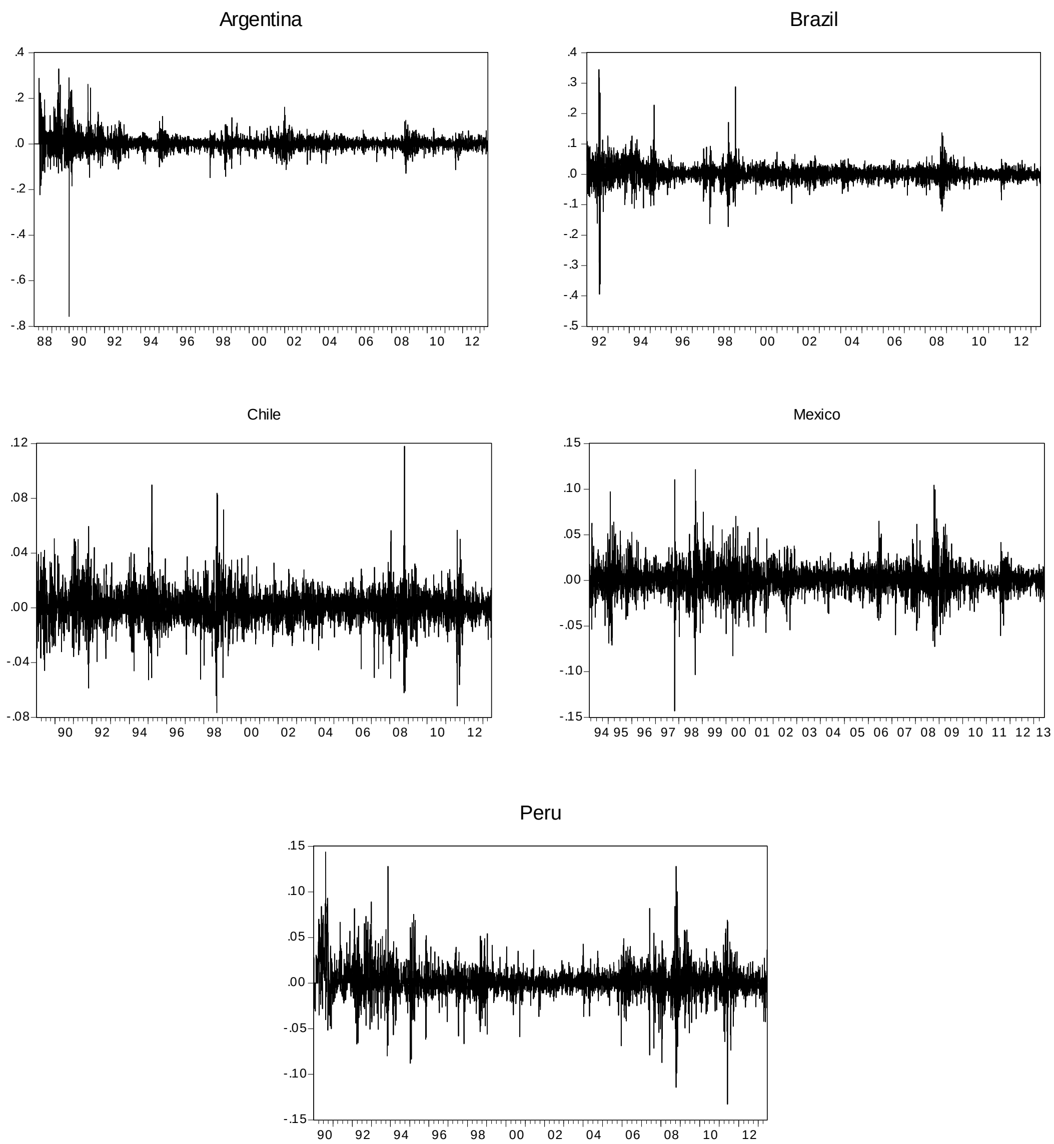

Figure 1. Stock Returns Series

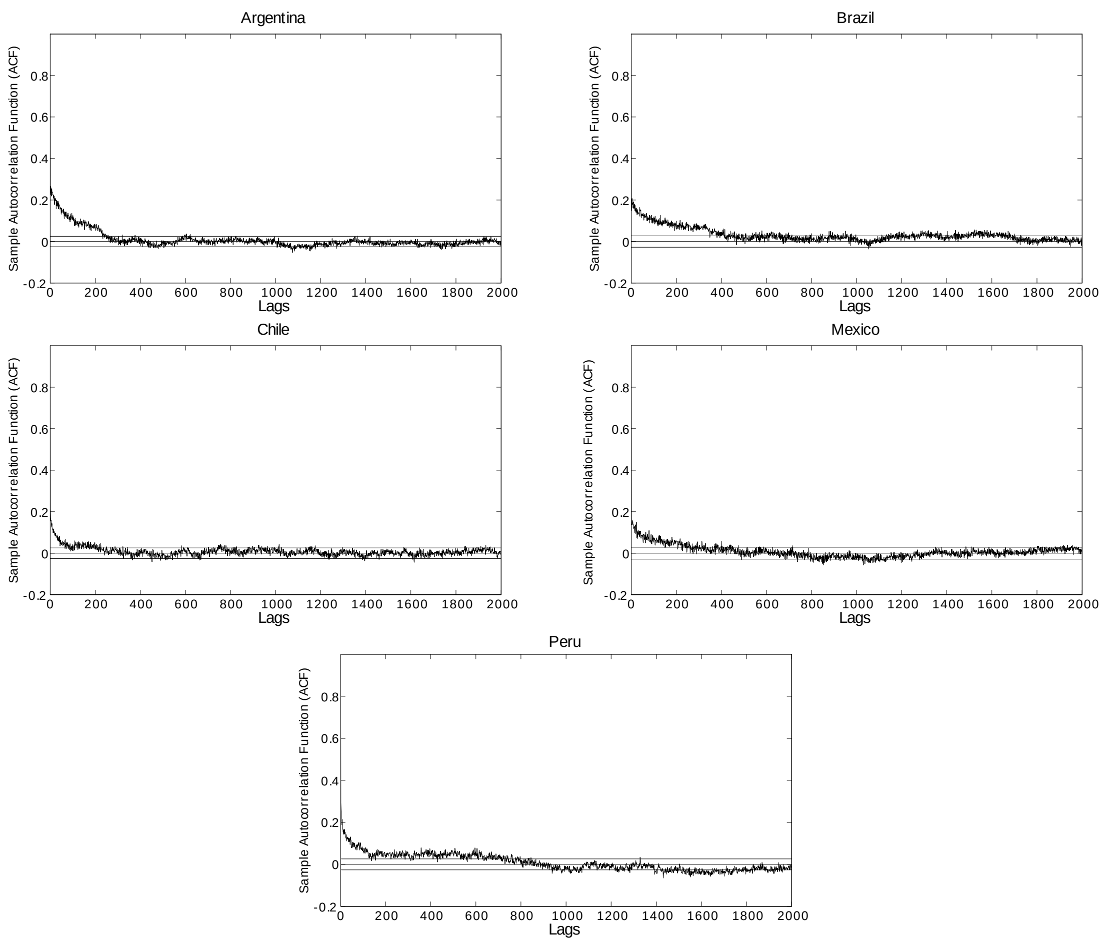

Figure 2. Sample ACF of Returns Volatility Series

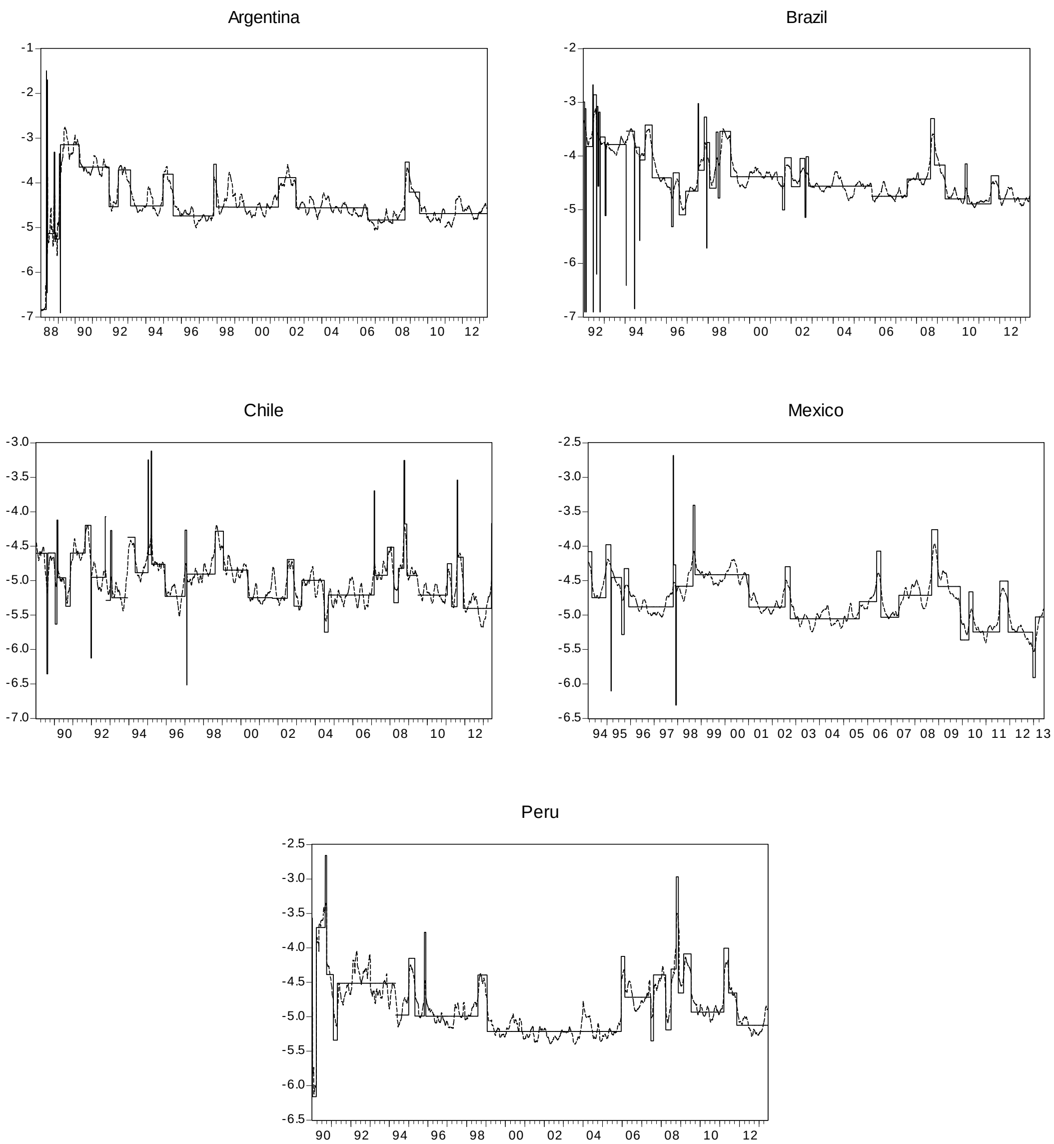

Figure 3. Level Shift Component τ_t estimated by Bai and Perron (2003): solid line and Smoothed Level Shift Component: dotted line

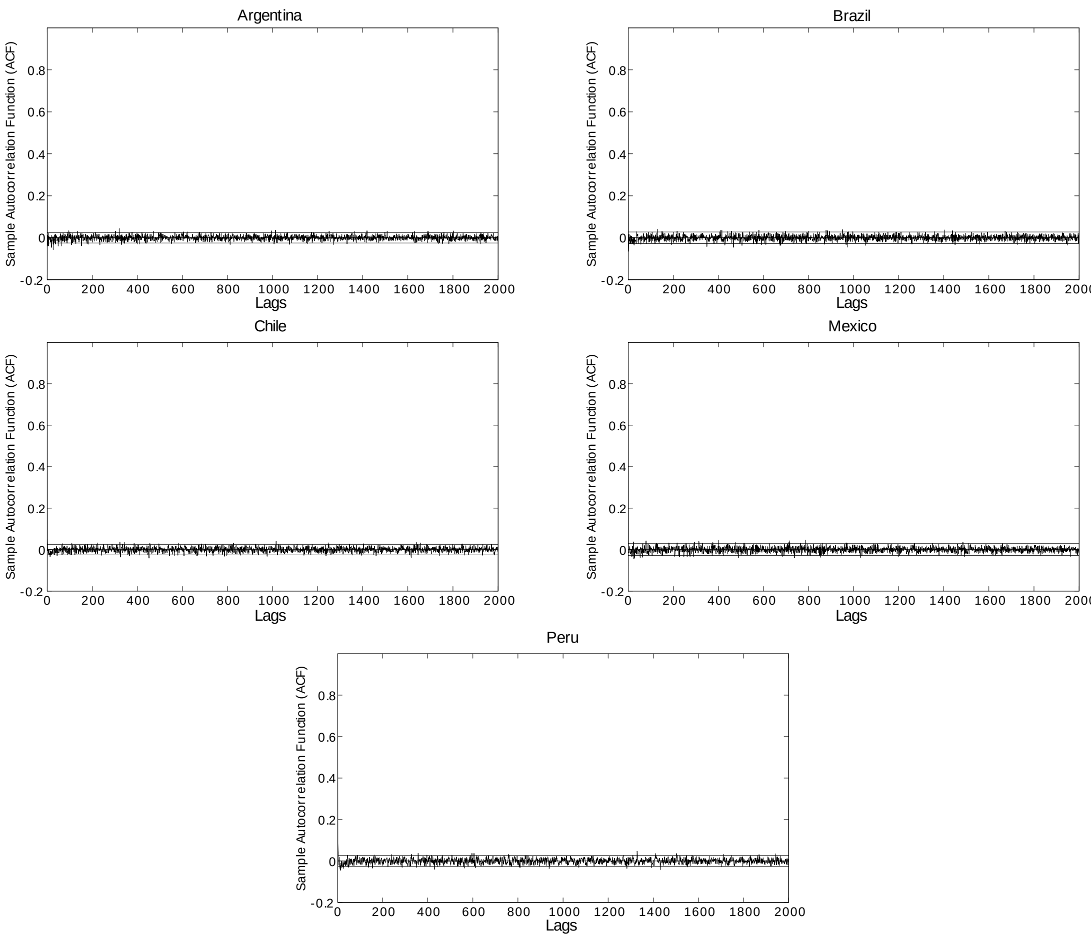

Figure 4. Sample ACF of Residuals Series