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Abstract

One of the key elements for inflation targeting regime is the right identification of inflationary or disinflationary pressures through the output gap. In this paper we provide an estimation of the Peruvian output gap using a multivariate unobserved component (MUC) model, relying on an explicit short run relation between the output gap and inflation rate (Phillips Curve) and structural restrictions over output dynamics. The results show that the MUC output gap estimate is less sensible to end of sample problems and exhibits closer dynamics with the inflation process than the standard output gap estimates.

JEL classification: E32, E31, C51, C52.
Keywords: Output gap, Inflation, unobserved component model.

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1. Motivation

One of the key elements for the implementation of Inflation Targeting regime is the right identification of inflationary or disinflationary pressures. It is important to have a reliable indicator of these pressures because the central bank will use it for guiding its monetary policy to achieve its inflation target. The central bank will engage on tight (expansive) policy whenever the indicator signs inflationary (disinflationary) pressures that risks achieving its target.

In general, the indicator used is the output gap. This variable tries to measure the short run pressures of marginal costs over inflation generated by a demand expansion and an inaccurate distribution of the productive factors of the economy. Unfortunately, the output gap is an unobservable variable and its value must be inferred from the information contained in other economic variables. To this respect, the estimation of the output gap has been the focus of considerable research effort of many central banks[1].

The most common techniques are based on univariate filters, which only use gross domestic product (GDP) information[2]. These methodologies assume that output is an isolate process from the rest of macroeconomic time series. In most of the cases, this simplicity implies a high degree of uncertainty in the output gap measure, specially at the end of the sample[3]. Moreover, in the cases that other relevant variables have affected output gap, these univariate approaches do not allow to identify them[4], thus disturbing the decisions of monetary policy[5].

As an alternative, different multivariate methods have been developed, each one is based on a particular theory and implementation technique. One of the most common multivariate methods is the Production Function approach, which consists on a neoclassical production function with different inputs, generally capital stock, labor

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[1]See for example, Benes and N'Diaye (2002) and Butler (1996).

[2]See for example, Hodrick and Prescott (1997), Beveridge and Nelson (1981), Baxter and King (1995) and Harvey and Jaeger (1993).

[3]Several studies have addressed this problem in univariate methods. For example, Orphanides and van Norden (1999) studied uncertainty in US output gap estimation process and Gruen *et. al.* (2002) do the same for Australian GDP. Their results confirm that end of problem is the principal source of uncertainty affecting output gap estimation.

[4]For example, Haltmaier (2001) uses cyclical indicators to adjust Japanese output gap estimates derived from the Hodrick and Prescott filter over the most recent period.

[5]Smets (2002) and Gaudich and Hunt (2000) found that the bigger the uncertainty surrounding output gap estimates, the smaller the reaction of monetary policy to it.

force and total factor productivity (Solow residual). Often, researchers attempting to apply this technique use an univariate method to estimate the trend of productivity[6]. As a consequence, uncertainty remains on this component affecting the output gap reliability.

Another way to impose structural restrictions is using the SVAR identification of Blanchard and Quah (1989). The SVAR output gap is the component not affected by permanent shocks and related to the employment rate or inflation in a transitory way. This method has several limitations, it is not accurate to identify permanent and transitory shocks and its performance could be undermined by omitted variable problems[7].

More recently, a new group of multivariate methods use unobserved component models, which combine structural relationships with properties of statistical filters. Their main characteristic is that they include an explicit relation between output gap and inflation (Phillips Curve), and/or between the output gap and the unemployment rate (Okun's law). Several authors have used multivariate techniques based on unobserved component models, whose estimation is carried out via the Kalman filter algorithm[8]. This approach benefits from correlation in the data and model structure, mixing this information according to the lowest prediction error. This technique has been successfully applied, increasing the accuracy and reliability of output gap estimations.[9]

In order to show the limitations of univariate methods in figure (1) we plot the annual variation of the core Consumer Price Index and the output gap, estimated with the univariate Hodrick-Prescott (HP) filter for the quarterly period 1992-2003. The inflation process in Peru presents two episodes. The first one (1992-1994) is characterized by a continuous disinflation process from high (more than 80 percent during 1992) to moderate inflation rates (around 20 percent in 1994). In the second episode (1995-2003), the inflation rate continues decreasing, but at a lower pace, moving from moderate (around 11 percent in 1995) to low inflation rates (one-digit inflation in 1997 and lower than 5 percent since 1999).

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[6]See for example, Miller (2004a) and Texeira (2002).

[7]See for a technical details van Norden (1995), Blanchard and Quah (1989) and Cerra and Chaman (2000).

[8]For example, see De Brouwer (1998), Scott (2000b), Camba-Mendez and Rodriguez-Palenzuela and Benes (2001) and N'Diaye (2002). Alternatively, Laxton and Tetlow (1992) and Hirose and Kamada (2001) propose a multivariate Hodrick and Prescott filter.

[9]See for example, Rünstler (2002). Although, it worths to say that the improvement depends on the structure imposed, and calibration or parameters estimation, see Butler (1996).

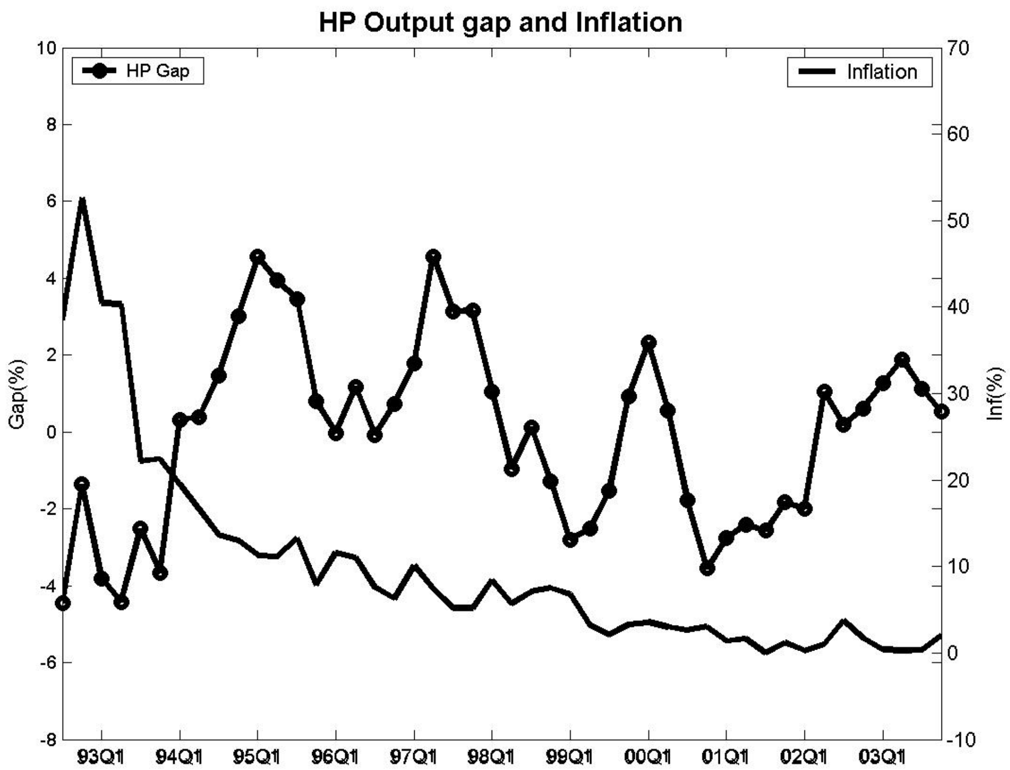

Figure 1. Output gap corresponds to Hodrick and Prescott estimate with smoothing parameter of 1600. The inflation rate is calculated on quarterly base (annualized).

On the other hand, the HP output gap during the first episode (specifically in 1994-1996) indicates high excess demand conditions, which implies the presence of strong inflationary pressures. Nevertheless, this result does not seem to be in line with the persistent decline in inflation along the nineties. Another similar episode to highlight is observed at the end of the sample, where inflation is relatively stable but the HP output gap is positive, indicating inflationary pressures. In this context, the results obtained with the HP filter do not permit to analyze and explain correctly the evolution of the inflation, particularly during periods where output was growing significantly and inflation was falling or stable. This univariate technique only captures the output process, without taking into account any structure or the dynamics process of other important macroeconomic variables.

Given that Peruvian economy is a small open economy, many other variables (for example, imported inflation) are critical in understanding inflation dynamics. Figure (2), plots the Core CPI inflation and imported inflation rates[10]. Core inflation has been evolving together with imported inflation except in two remarkable cases: 1994-1996 and 1998-1999. During the former period, inflation is higher than imported inflation suggesting that some inflationary pressures might have restrained the total pass-through. The opposite happens in the second case: imported inflation is higher than core inflation, and this coincides with a weak output phase. This analysis suggests that imported inflation is a key variable that have to be considered in the determination of the output gap.

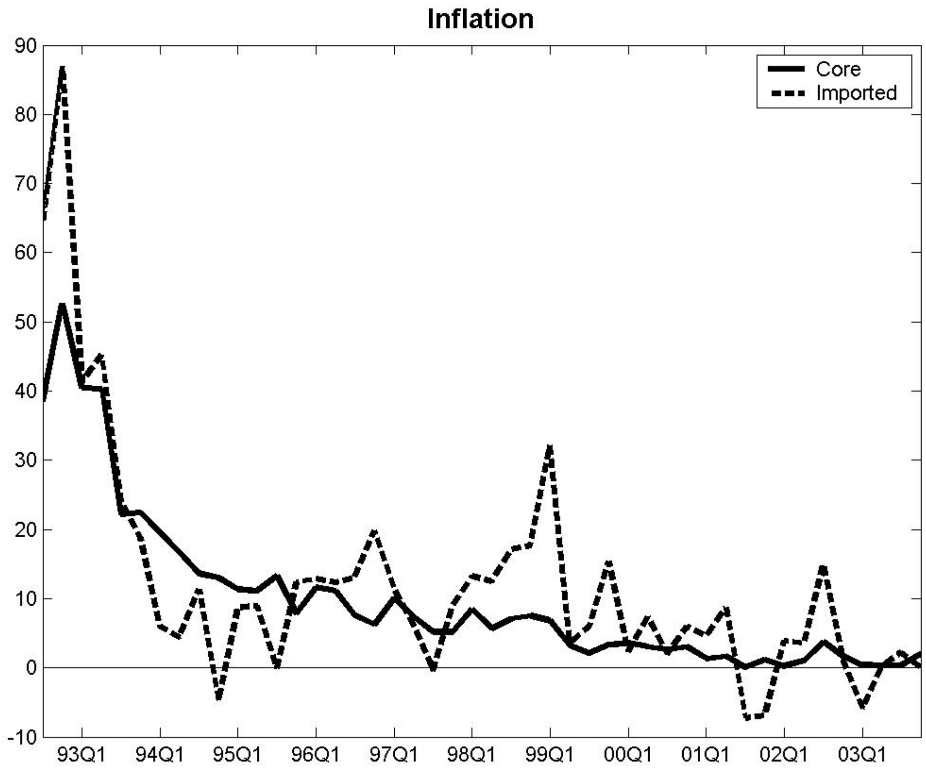

Figure 2. Quarterly core inflation and imported inflation. Imported inflation is computed by PPP condition.

[10]Imported inflation is calculated from US inflation and nominal exchange rate depreciation (appreciation).

For the Peruvian case, most of the studies had based on univariate filters[11]. Given the advantages of multivariate unobserved component models, the aim of this paper is to provide an estimation of the output gap using this technique. The model employed relies on an explicit short run relation between the output gap and inflation rate (Phillips Curve) and structural restrictions over output dynamics. We estimate the model via Kalman Filter for the period 1992-2003.

The results show that the multivariate unobserved component output gap (MUC) is less sensible to end of sample problems and presents a better relation with the Peruvian inflation process than other estimates, calculated with the Hodrick-Prescott filter and the production function approach. In particular, in periods of high output growth together with disinflationary or stable inflation environments, MUC output gap is lower than the ones obtained with the alternative methods mentioned. Besides, MUC identification is quite related to pass-through effect from imported prices to consumer prices. In particular, whenever imported inflation was higher (lower) than domestic inflation, the system found a negative (positive) output gap. Furthermore, the diagnostic statistics report that MUC estimate is more reliable than other alternatives and increases out of sample predictive power for inflation.

The document is organized in the following form. In the second section, the structure of the model used, as well as its implementation and the data, are explained and analyzed. In next section, we present the most important features of MUC estimate, and some of its properties: updating properties and inflation forecasts power. Finally, in the fourth section, we conclude.

2. The model

We use a semi-structural model for a small open economy. The system is based on three behavioral equations:

1. Uncovered interest parity.
2. Phillips Curve.
3. Aggregate demand.

The uncovered interest parity allows us to estimate the permanent and transitory

[11] For example, Cabredo and Valdivia (1999), Caballero and Gallegos (2001) and Miller (2004a) compare different output gap estimates using those techniques. Their results indicate that production function output gap is the best indicator of inflation pressures in Peru.

components of real interest rate and real exchange rate. Combining the gaps of real interest rate and real exchange rate, we construct a real monetary condition index[12]. Taking this index as an exogenous variable, we use the aggregate demand equation and Phillips curve to calculate the output gap related to the evolution of real activity and inflation. The model takes the following form,

$$y_t = \bar{y}_t + \hat{y}_t \qquad (1)$$

$$B(L)\hat{y}_t = \kappa RMCI_t + \eta_t^y \qquad \eta_t^y \sim N(0,\sigma_{\eta y}^2) \qquad (2)$$

$$\pi_t = \tilde{\pi}_t + \varepsilon_t^\pi \qquad \varepsilon_t^\pi \sim N(0,\sigma_{\varepsilon\pi}^2) \qquad (3)$$

$$\tilde{\pi}_t = \alpha_1 \tilde{\pi}_{t-1} + \alpha_2 \tilde{\pi}_t^m + (1-\alpha_1-\alpha_2)\tilde{\pi}_{t,t+1}^e + \gamma \hat{y}_t + \eta_t^\pi \qquad (4)$$
$$\eta_t^\pi \sim N(0,\sigma_{\eta\pi}^2)$$

From equation (1), output y_t (in logarithms) is decomposed into potential output $\bar{y}_t$ and the output gap $\hat{y}_t$. The second equation describes the output gap dynamics influenced by the real monetary condition index, $RMCI_t$. The lag polynomial is defined by $B(L) = 1-\beta$, which represent an AR(1) stationary process. Equation (3) decomposes the CPI core inflation, π_t, into its forecastable component[13], $\tilde{\pi}_t$, and an stochastic shock η_t^π. The underlying inflation is modeled using a Phillips curve for a small open economy, equation (4). According to this equation, this measure is influenced by its own inertia, imported inflation $\tilde{\pi}_t^m$, inflation expectations $\tilde{\pi}_{t,t+1}^e$ and the output gap $\hat{y}_t$.

Potential output $\bar{y}_t$ follows a random walk process with a stochastic slope μ_t. The

[12]This index captures the general orientation of monetary policy affecting the aggregate demand with the aim to control the inflation rate, see Dennis (1997) for technical discussion. Given the trends in the data, the index is constructed using gaps instead of levels. The details are shown in appendix A.
[13]Forescastable inflation may be interpreted as a measure of underlying or trend inflation, which is filtered from high frequency fluctuations. Arguably, a central bank should be responsible primarily for development in underlying inflation and not for high frequency inflation, which is unable to control.

slope is modeled as an stationary autoregressive process with constant, $\bar{\mu}$, reflecting the growth rate of potential output in steady state[14].

$$\bar{y}_t = \bar{y}_{t-1} + \mu_t \qquad (5)$$

$$\mu_t = \phi\mu_{t-1} + (1-\phi)\bar{\mu} + \eta_t^u \qquad \eta_t^u \sim N(0,\sigma^2_{\eta\mu}) \qquad (6)$$

The model is completed by the assumption that stochastic shocks $\varepsilon_t^\pi, \eta_t^\pi, \eta_t^y$, and η_t^u are normally and independently distributed and mutually uncorrelated.

2.1. Inflation expectations

One important issue is the measurement of inflation expectations. Typically, the New Keynesian Phillips curve stresses on forward looking behavior in the price setting process[15]. Particularly, the forward-looking component on inflation is quite important during disinflation episodes[16].

On the other hand, empirical work usually assumes totally backward looking expectations[17], implying that the output gap has permanent effects on the level of inflation rate[18].

Confronting this trade off between theoretical and empirical grounds, we consider a simple error correction mechanism for inflation expectations which allows us to incorporate the deceleration on Peruvian inflation without assuming totally backward looking expectations[19],

$$\tilde{\pi}^e_{t,t+1} = \bar{\pi}_{t+1} + (\tilde{\pi}_t - \bar{\pi}_t) = \tilde{\pi}_t + \Delta\bar{\pi}_{t+1} \qquad (7)$$

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[14]A local linear trend model for potential output was proved. The results indicate that a steady state growth rate of potential output reduces end of sample revisions. For a technical discussion of local level and local linear trend models see Harvey (1993).
[15]See Calvo (1983) and Clarida *et. al.* (2002)
[16]See Mankiw and Reis (2001).
[17]See for example, Rünstler (2002).
[18]Also, this structure implies that the level of inflation rate is unforecastable, which is a clear contradiction to inflation targeting policy framework.
[19]A more suitable technique could be related to adaptative learning expectations, see Evans and Honkapohja (2001).

where $\tilde{\pi}_t$ is underlying inflation, $\tilde{\pi}^e_{t,t+1}$ represent inflation expectations over next quarter, and $\bar{\pi}_t$ is interpreted as inflation target rate[20]. Given (7), if underlying inflation is higher (lower) than the target, inflation expectations raises (decreases). If inflation is aligned to the target, expectations do not change. Considering this structure, replace (4) with[21],

$$\tilde{\pi}_t = \frac{\alpha_1}{\alpha_1+\alpha_2}\tilde{\pi}_{t-1} + \frac{\alpha_2}{\alpha_1+\alpha_2}\tilde{\pi}^m_t + \frac{(1-\alpha_1-\alpha_2)}{\alpha_1+\alpha_2}\Delta\bar{\pi}_{t+1} + \frac{\gamma}{\alpha_1+\alpha_2}\hat{y}_t + \eta^\pi_t \qquad (8)$$

We assume that in the long run, real exchange rate depreciation is zero and inflation target is constant. Thus, there is not a relationship between output gap and the inflation in the steady state[22].

2.2. The state space form

For estimation, the model must be put in its state space form, which comprises two equations[23]. Measurement equation (9) relates observations x_t at time $t, t = 1,...,T,$ to the unobserved state vector α_t[24]. Transition equation (10) denotes the stochastic dynamic behavior governing the state vector.

$$x_t = Z\alpha_t + \varepsilon_t \qquad (9)$$

$$A_0\alpha_t = c + A_1\alpha_{t-1} + B\varphi_t + R_0\eta_t \qquad (10)$$

where:

$x_t = \begin{bmatrix}\pi_t & \Delta y_t\end{bmatrix}'$ is the observable vector,

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[20]Formally, inflation targeting was adopted as a monetary policy framework in Peru in 2002. Nevertheless, the Central Reserve Bank of Peru has been announcing inflation targets since 1994. See Rossini (2001) for details.

[21]Although the error term has changed, we maintain its nomenclature.

[22]This also implies that in the long run inflation rate is constant and output gap is zero.

[23]Because output has a unit root, the model was stationarized differencing output equation. Differentiation permits direct calculation of Kalman Filter initial conditions from the model structure and data, without applying diffuse priors for initial conditions, which modifies severely the results at the beginning of the sample.

[24]We present an structural version of state equation, which incorporates contemporaneous effects between underlying inflation and output gap. To get the autoregressive form, invert the left matrix of the state system.

$\alpha_t = \begin{bmatrix} \tilde{\pi}_t & \hat{y}_t & \hat{y}_{t-1} & \mu_t \end{bmatrix}'$ is the state vector,

$\varphi_t = \begin{bmatrix} \pi^m_{t-1} & \Delta\overline{\pi}_{t+1} & RMCI_t \end{bmatrix}'$ is the exogenous vector

$\varepsilon_t = \begin{bmatrix} \varepsilon^\pi_t & 0 \end{bmatrix}'$ and $\eta_t = \begin{bmatrix} \eta^\pi_t & \eta^y_t & 0 & \eta^\mu_t \end{bmatrix}'$ are innovation vectors.

Matrices are given by:

$$Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix}$$

$$A_0 = \begin{bmatrix} 1 & \frac{-\gamma}{\alpha_1+\alpha_2} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ (1-\phi)\overline{\mu} \end{bmatrix},$$

$$A_1 = \begin{bmatrix} \frac{\alpha_1}{\alpha_1+\alpha_2} & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \phi \end{bmatrix},$$

$$B = \begin{bmatrix} \frac{\alpha_2}{\alpha_1+\alpha_2} & \frac{1-\alpha_1-\alpha_2}{\alpha_1+\alpha_2} & 0 \\ 0 & 0 & \kappa \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Innovations η_t and ε_t are mutually uncorrelated and have diagonal covariance matrices. Both are modeled as multivariate gaussian distributions. Matrices A_0, A_1, B, R_0 and vector c depend on unknown hyperparameters[25]. After fixing hyperparameters, prediction, updating and smoothing algorithms are applied.

To get the usual state space representation, take into account the following equalities,
$T = A_0^{-1}A_1$, is the transition matrix,
$d = c + A_0^{-1}B\varphi_t$, summarizes exogenous variables,
$R = A_0^{-1}R_0$.

2.3. Calibration

The model (1)-(6) incorporates several hyperparameters, coefficients $\{\beta$, κ, α_1, α_2, γ, ϕ, $\overline{\mu}\}$ and variances $\{\sigma^2_{\varepsilon\pi}$, $\sigma^2_{\eta\pi}$, $\sigma^2_{\eta y}$, $\sigma^2_{\eta\mu}\}$. This hyperparameters can be estimated using maximum likelihood procedure. However there are several issues with this approximation. First, for the sample selected, the inflation rate shows a persistent dynamics can be explained by a non-stationary homogenous component in the stochastic dynamic equation of inflation[26]. Second, the quarterly sample used is too short to permit a reliable econometric estimation. Third, we suspect that structural breaks, due to institutional changes and structural reforms in Peru, could prevent a suitable econometric identification[27].

As an alternative, we choose to calibrate the model using external information. In order to get priors, a Phillips curve and a aggregate demand function, similar to equations (2) and (8), were estimated econometrically using Hodrick and Prescott output gap[28]. The chart 1 reports the selected values.

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[25]The next section focus on the criteria utilized for hyperparameters calibration.
[26] This phenomenon may invalidate any econometric estimation. For a technical discussion, see Enders (1995) chapter 1.
[27]There exist some evidence about structural breaks in Peruvian data, see Quispe (1999). In general, structural breaks could distort inflation - output relationship, see Clark and McCracken (2003).
[28]We took these results with caution because the presence of persistent dynamics on inflation can invalidate statistical inference and also the use of an incorrect output gap measure distort coefficient values. Nevertheless, the estimations give useful information about the parameter values and their uncertainty. In particular, we found that $\{\gamma, \kappa\}$ are blurred by tremendous uncertainty.

Chart 1: Model parameters

Parameter	Calibrated value
Inflation persistence (α_1)	0.70
Pass through (α_2)	0.15
Output gap to inflation (γ)	0.70
Output gap persistence (β)	0.70
RMCI to output gap (κ)	0.10
Potential slope persistence (ϕ)	0.80
Potential quarterly growth $(\overline{\mu})$	0.01

In the Phillips curve, we calibrate the parameter α_1 in 0.7. The inflation elasticity to output gap (γ) is calibrated in 0.7. This value is higher than the ones found for other countries[29]. However, it reflects the low sacrifice ratio during the disinflation process in the last ten years[30]. Additionally, we set the pass-through effect from imported inflation over CPI core inflation captured by α_2 in 0.15, according to those found by Miller (2004b) and Winkelried (2004).

For the output gap equation we use the econometric estimation to set the inertia parameter β in 0.7, the effect of the real monetary condition index κ in 0.1 and the value of ϕ in 0.8. The steady state growth rate of potential output was fixed in 4 per cent (annualized), according to the mean growth rate of potential output calculated using the production function approach[31].

All variances, except that for the growth rate of potential output, were normalized. For filtering process, we have to identify the permanent and transitory components of output. The signal extraction problem is basically related to the variance ratio between

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[29]For Germany 0.40 and United States 0.44, see Ball (1994), and Czech Republic 0.22, see Benes *et. al.* (2002).
[30]See Zegarra (2000).
[31]See Miller (2004a).

growth rate of potential output and output gap[32], $\sigma^2_{\eta\mu}/\sigma^2_{\eta y}$. We set this value to 1/64. This smoothes potential output and increases the relation between the cyclical component of output and inflation.

2.4. The data

We use quarterly data form the Central Reserve Bank of Peru. The sample spans from 1992 to 2003. We utilized the real GDP calculated using 1994 prices. Inflation is represented by core CPI inflation and nominal exchange rate by soles/US$ parity. As an international interest rate we use monthly LIBOR rate. External inflation is approximated by United States CPI inflation rate. Imported inflation is constructed using PPP condition: $\pi_t^m = \pi_t^{US} + \Delta e_t$, where π_t^{US} is US CPI inflation and Δe_t is the exchange rate depreciation (appreciation).

The real exchange rate is measured by the imported prices index deflated by core consumer prices index. On the other hand, the ex-post real interest rate is measured as: $r_t = i_t - \pi_t^{core}$, where i_t is the annualized interbank interest rate and π_t^{core} is year-to-year core inflation rate. Real monetary condition index is constructed with real interest rate and real exchange rate gaps. The risk premium is calculated as the uncovered interest parity condition residual.

Finally, the inflation target rate is the HP filtered of core inflation, restricted to the last announced target (2.5 percent) as a final level prior since 2002[33].

3. Results

In this section, first we describe the MUC output gap, comparing it to HP filtered and the production function estimates. Then, properties of revisions in the output gap estimates and inflation forecast performance are discussed. Finally, we evaluate which output gap estimate improves inflation predictability. The results indicate that the MUC estimate shows more relation with the Peruvian inflation process, reduces end of sample uncertainty and improves inflation forecast.

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[32]Signal extraction problem is practically intractable without imposing some ad-hoc restrictions, see Quah (1992) for a technical discussion. For example, the direct estimation of variance ratio between transitory and permanent component of a time series tend to differ to those recommended by Hodrick and Prescott, see for example, Blith *et. al.* (2001).

[33]For a discussion of prior's inclusion on Hodrick and Prescott filter, see St. Amant y van Norden (1997).

3.1. Output gap estimates

Panel (a) of figure (3) plots the MUC output gap estimate using the multivariate unobserved component approach, based on the model defined by (1)-(6) and (8).

According to the results, Peruvian output gap has fluctuated inside the range of -7 to 2 per cent. Four periods of inflationary pressures can be identified: 1994Q2-1995Q4, 1997Q1-1997Q4, 1999Q4-2000Q2, and more recently 2002Q2-2002Q4. The first two periods have been the most outstanding and the longest, reaching levels near 2 per cent. With regard to the disinflationary pressures' episodes, they have been longer and have presented a higher average magnitude than inflationary ones. Four periods have been also identified: 1992Q3-1994Q2, 1996Q1-1996:Q4, 1998Q2-1999Q4, and 2000Q4-2002Q1, being the first one the most significant, reaching values near to -7 per cent.

Panel (b) of figure (3) plots quarterly underlying inflation and imported inflation. Both series show a high correlation during the ninety's. However, this relation breaks in two remarkable periods: 1994-1995 and 1998-1999. In the first one, underlying inflation is higher than imported inflation. At the same time, a positive output gap is identified, explaining the incomplete pass-through. The opposite happens in the second period: underlying inflation is lower than imported inflation, phenomenon accompanied with a negative output gap[34].

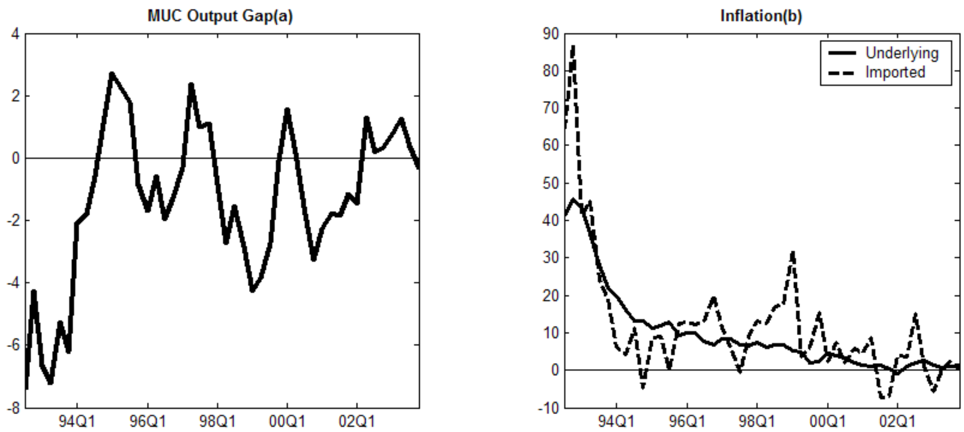

Figure 3. Output gap correspond to MUC smoother estimate. Underlying inflation is computed using semi-structural model and imported inflation is computed by PPP condition.

[34]This kind of non linear pass-through has been discussed recently in Winkelried (2004).

Figure (4) displays the MUC output gap estimate alongside to the Hodrick-Prescott filtered (HP) and the production function (PF) estimates.

MUC, HP and PF output gap estimates are very similar for the entire sample. However, our estimate is lower than the alternatives in two periods: 1994-1997 and at the end of the sample (2003). The most remarkable feature about those episodes is that they combine high output growth rates with a disinflationary process (1994-1997) or stable inflation environment (2003). On one hand, HP and PF methods tends to link the output gap evolution with the economic cycle, even when this cycle had not affected the inflation rate. On the other hand, MUC estimate is influenced not only by output behavior but also by domestic and imported inflation.

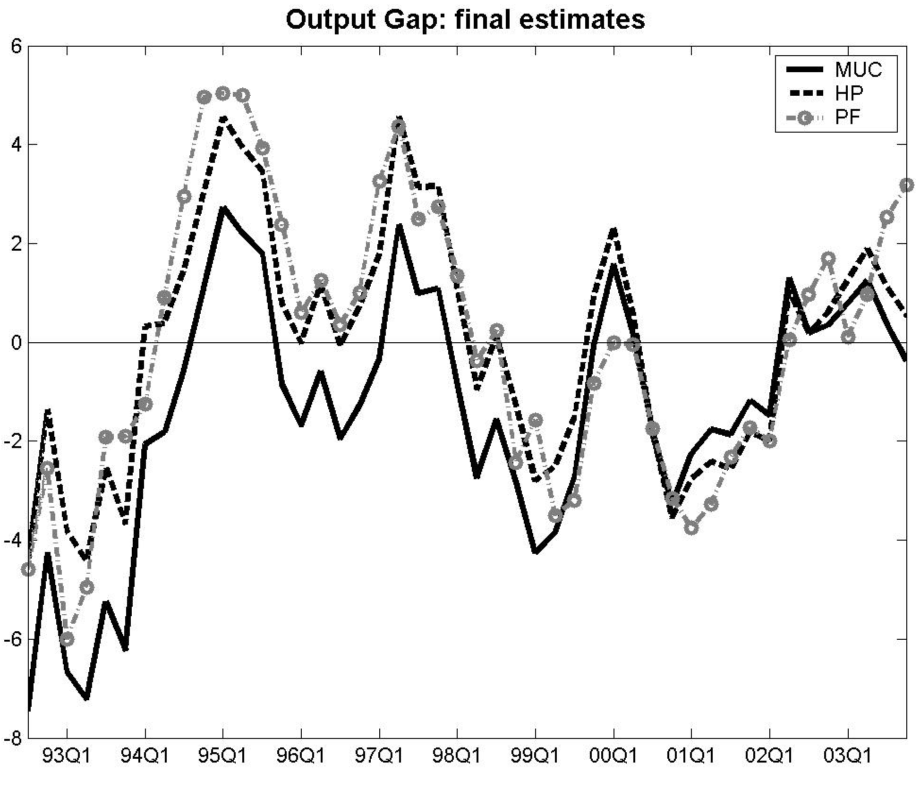

Figure 4. Smoother output gap estimate.

3.2. Properties of revisions: end of sample problem

In this section, we analyze the updating properties of MUC, HP and PF estimates. Measuring output gap revisions due to additional observations is one way of evaluating the uncertainty surrounding different methods[35]. In fact, the higher uncertainty around output gap estimate, the lower is the sensitivity of monetary authority reactions to it[36].

In order to compare the reliability of each method we calculated the real-time estimates and the final estimates of the output gap[37]. The uncertainty that each method introduce in the output gap estimation is determined by the comparison between the final estimate and the real time estimate.

The results of this exercise are shown in the figure (5). The left panel plots the real-time and final estimates calculated with MUC, HP and PF methods. It is evident that revisions of the MUC estimate of the real-time output gap in response to new data are much smaller than those of HP and PF. In the right panel, the scatter graphs between real-time and final estimates are presented. Those graphs allow to have a clearer picture of the uncertainty degree surrounding each method. The graphs are divided in 4 areas: Areas I and III show the points where the final and real-time estimates provide contradictory signals, while areas II and IV present those occasions in which both estimations give similar signals. The results indicate that MUC estimates are grouped around the 45° line (areas II and IV), while HP and PF provide contradictory signals (areas I and II).

With the aim of quantifying the uncertainty degree, we calculated the correlation coefficients and concordance indices[38]. Additionally, we test the reliability of output gap estimates using the Pesaran and Timmermann (1992) test[39]. The results are

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[35]See Gruen *et.al.* (2002) and Orphanides and van Norden (1999).
[36]See Gaudich and Hunt (2000) and Smets (2002).
[37]The real time estimates correspond to updated state estimate in Kalman Filter recursion, conditional to past and current observation of the published data available. This means that we are not considering the ex-post revisions of published data, see for example Orphanides and van Norden (1999). On the other hand, final estimates are equivalent to smoother estimates in Kalman Filter, reflecting all available information to forecast sequentially observable variables. To get updated estimates of Hodrick and Prescott filtered we use its state space representation, see Scott (2000b).
[38]The concordance index is simply a non parametric statistic method that measures the time proportion in which two time series are in the same state. Thus, the degree of concordance will be 1 if both output gap measures have the same sign for a determined period. By contrast, it will take a zero value if the sign of both measures (final and real-time) are always opposite. For more details of this indicator see McDermott and Scott (1999).
[39]Quoted in Camba-Mendez and Rodriguez-Palenzuela (2001).

summarized in chart 2. The correlation coefficients indicate that the output gaps calculated (final and real-time) with MUC present higher co-movements (0.65) than those obtained with HP (0.26) and PF (0.16). In the same way, the concordance statistic indicates that real-time and final estimates with MUC provides similar signals (0.73), better than HP (0.63) and PF (0.52) do. Moreover, the application of the Pesaran and Timmermann test shows that the acceptance probability of similar signals in the case of MUC is 70.89 per cent, in contrast with the 0.01 per cent and 0.00 per cent of HP and FP, respectively. Those results suggest that the multivariate approach provides more reliable estimates[40].

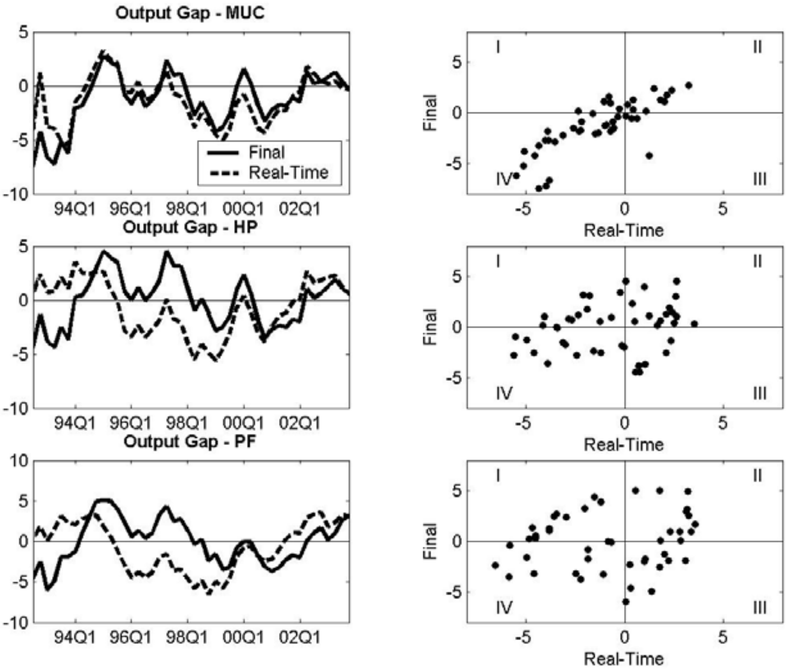

Figure 5. Final estimates correspond to smoother estimates. Real-Time estimates correspond to updated estimates.

[40]A variety of studies evaluates the reliability of different univariate and multivariate methods, comparing the real-time estimates relative to final ones. Butler (1996), Conway and Hunt (1997), Camba-Mendez and Rodriguez-Palenzuela (2001), De Brouwer (1998) and Scott (2000a) compare the updating properties of different output gaps measures for Canada, European Union, United States, Australia and New Zealand. Their results suggest that multivariate output gaps estimates are statistically more reliable. Rünstler (2002) concentrated only on unobservable components methods, univariate and multivariate. As well as in the preceding cases, his study indicates that bigger information levels increase the confidence of the output gap estimate in European Union.

Chart 2: Evaluation statistics

	MUC	HP	PF
Correlation coefficient	0.65	0.26	0.16
Concordance index	0.73	0.63	0.52
Pesaran and Timmerman test*	70.89%	0.01%	0.00%

*Acceptance probability of the null hypothesis that the sign of real-time and final estimates is the same

What explains these results? Because future data always contains relevant information to the current decomposition of transitory and permanent shocks, the most recent estimates of output gap will invariably change as the persistence characteristics of past shocks become more apparent. With structural restrictions, the MUC approach exploits the correlation in the data, guiding the output gap estimation at the end of the sample.

3.3. Inflation forecast

The predictive power of the output gap for inflation through the short run supply curve (Phillips curve) is an essential precondition for the economic validity of any output gap estimate. This section test the information content of different real-time output gap estimates as a leading indicator for future inflation change. For this purpose, we analyze the following regression,

$$\Delta\tilde{\pi}_t = \theta\,\hat{y}_{t|t} + \sum_{i=1}^{k} \Delta\tilde{\pi}_{t-i} + \varepsilon_t \qquad (11)$$

where $\hat{y}_{t|t}$ is the real time output gap estimate and $\tilde{\pi}_t$ is the underlying inflation measure calculated from Kalman Filter recursion[41].

We apply this equation on real-time output gap estimates computed with different methods: MUC, HP and PF. Additionally an ARIMA regression is estimated, which is taken as a benchmark. In all cases, order lags k is found from Akaike criterion minimization.

We evaluate out-sample performance using the following steps. First, the equation (11)

[41]The use of inflation changes eliminates the excessive persistence on inflation. Econometrically, this approach is optimal since improves the short run forecast.

and ARIMA equation are estimated for the sample selected. Second, the out of sample forecasts of inflation changes over the next four quarters are computed. Third, another observation to the sample is added and the first two steps are applied. We start this procedure with a sample from 1992Q1 to 1997Q4, expanding it until 2002Q2.

Chart 3 reports the mean square error of forecast for underlying inflation using different output gap estimates in relation to the benchmark equation.

Chart 3: Out-sample forecasts performance

MSE-ratio (in percentages)	Forecast Horizon			
	+1	+2	+3	+4
MUC	57	55	65	67
HP	66	73	77	84
PF	75	77	78	85

MSE-ratio denotes the mean square error of the inflation forecast relative to MSE of the random walk forecast. MSE's of the out-of-sample random walk forecast are given 1.56, 1.61, 1.76, and 1.29 for 1,2,3,4 quarters ahead, respectively. Initial sample: 1992Q1 –1997Q4.

Out-of-sample forecasts using output gap estimates improve substantially on the random walk forecast at all the four quarters ahead. However, the improvement varies across different methodologies considered. MUC output gap increases inflation predictability more than HP and PF gaps estimates do. Additionally, the forecast performance of the MUC estimate is nearly stable as the forecast horizon increases. In that sense, HP and PF perform worse, showing MSE-ratios increments as the forecast horizon is expanded. At best, the four quarter forecast of HP and PF improve only slightly relative to the random walk model.

4. Final remarks

With the objective of improving output gap measurement in Peru, we develop a semi-structural model for a small open economy. The model was estimated as a multivariate unobserved component model using the Kalman Filter technique. The system incorporates explicitly a short run relation between output gap and inflation process through a Phillips Curve and also adds some other structural restrictions over potential output dynamics. The model parameters were calibrated using external information sources. Our results indicate that the MUC output gap estimate outperforms alternatives such as the HP filter or PF estimates.

The results indicate that the MUC output gap is quite similar to alternatives measures.

However, in periods of high output growth rate together with a disinflation or stable inflation context, our estimate indicate lower demand pressures than other estimates do. In particular, at the end of the sample (characterized by an environment of stable inflation), HP and FP are biased toward excess demand conditions. Besides, MUC output gap identification is quite related to pass-through effect from imported prices to consumer prices. In particular, whenever imported inflation was higher (lower) than domestic inflation, the system found a negative (positive) output gap.

Furthermore, we studied updating properties comparing the smoother estimates and the updated estimates of the three competitive approaches. The diagnostic statistics report that MUC estimate is the most reliable of the group. Finally we explore the out-sample predictive power for inflation of different output gap estimates. The results indicate that the MUC estimates forecast better inflation changes, confirming the essential precondition for the economic validity of any output gap estimate.

The advantages above-mentioned prove the importance of adding structural information on output gap calculation. For monetary policy purposes, this outcome could imply a significant uncertainty reduction and could improve future inflation control. Given that, a future research agenda could be oriented to explore additional cyclical indicators to improve output decomposition, in that sense, we recommend Rünstler (2002). Further, as the model presented here was calibrated, uncertainty involved in this process must be quantified. Regarding to this, Bayesian analysis of posterior densities of hyperparameters as in Harvey *et. al.* (2002) could be implemented.

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Technical appendix

Appendix A: The real monetary condition index

A real monetary condition index summarizes the main transmission channels of monetary policy: real interest rate and real exchange rate channels[42]. The index is calculated as a linear combination of real interest rate and real exchange rate gaps,

$$RMCI_t = -\theta \hat{r}_t + (1-\theta)\hat{q}_t \qquad \text{(A.1)}$$

where $\hat{r}_t$ and $\hat{q}_t$ are the gaps of real interest rate and real exchange rate, respectively. The coefficient θ measures the relative importance of the real interest[43]. A positive (negative) real monetary condition index implies an expansionary (contractionary) monetary policy stance.

Real interest rate and real exchange rate gaps were computed using a Kalman filtering technique. The model is based on uncovered interest parity condition.

$$r_t - r_t^* = 4\Delta q_t + \rho_t \qquad \text{(A.2)}$$

where r_t is the domestic real interest rate; r_t^* is the external real interest rate; q_t is the real exchange rate (in logarithms) and ρ_t represents the risk premium level. We can decompose every variable in the UIP equation into transitory (gap) and trend components.

$$r_t - 4\Delta q_t - \rho_t = \hat{r}_t + \bar{r}_t - 4\Delta\hat{q}_t - 4\Delta\bar{q}_t - \hat{\rho}_t - \bar{\rho}_t$$

$$r_t - 4\Delta q_t - \rho_t = \left[\hat{r}_t - 4\Delta\hat{q}_t - \hat{\rho}_t\right] + \left[\bar{r}_t - 4\Delta\bar{q}_t - \bar{\rho}\right]$$

Taking UIP as a cointegration relation implies that the real interest rate, the change in real exchange rate and the risk premium move together around a long run equilibrium[44].

$$\bar{r}_t - \bar{r}_t^* = 4\Delta\bar{q}_t + \bar{\rho}_t$$

Considering the above-mentioned, we rearrange the UIP equation as,

$$x_t = \left[\hat{r}_t - 4\Delta\hat{q}_t - \hat{\rho}_t\right] + \bar{x}_t$$

where $x_t = r_t - 4\Delta q_t - \rho_t$

[42]See Dennis (1997). Typically, monetary condition indexes are calculated in level. However, to be consistent to the semi-structural model employed, we center the monetary condition index detrending its component.

[43]A higher value of θ indicates that real interest rate channel is more important than real exchange rate channel. Therefore, a higher real depreciation (appreciation) is required to off set the effects of real interest rate increment (reduction).

[44]In this long run equilibrium, external real interest rate is taking as an exogenous variable, which do not adjust to any domestic disequilibrium.

We need to specify the stochastic laws of motion of the real interest rate, the real exchange rate and the risk premium. These three variables follow a local level model $\left(z_t = r_t, \Delta q_t, \rho_t\right)$[45] as,

$$z_t = \bar{z}_t - \hat{z}_t$$

$$\bar{z}_t = \bar{z}_{t-1} + \bar{\eta}_t \qquad \bar{\eta}_t \sim N\left(0, \sigma^2_{\bar{\eta}}\right)$$

$$\hat{z}_t = \hat{\eta}_t \qquad \hat{\eta}_t \sim N\left(0, \sigma^2_{\hat{\eta}}\right)$$

where $\bar{z}_t$ and $\hat{z}_t$ represent the permanent (trend) and transitory (gap) component, respectively. To compute the gaps we calibrate the signal extraction ratio between transitory and permanent shocks.

Figure (6) plots the real monetary condition index calculated. We set $\theta =$ **0.9**[46]. The results indicate two stages of monetary policy stance. Expansionary during 1992Q3-1995Q4 and 2001Q1-2003Q4 and contractionary during 1996Q1-2001Q4.

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[45]For technical discussion of local linear trend models and local level models, see Harvey (1993).

[46]To calibrate this parameter we estimate equation (2) for HP output gap with the real interest rate and the change in real exchange rate in gaps separately. The results show that the real interest rate gap is more important in output gap determination than real exchange rate depreciation (appreciation) gap. Furthermore, we explore alternative specifications with primary and non-primary sectors instead of the total GDP. We found that the real interest rate gap has more effect on non-primary output gap, while the real exchange rate depreciation (appreciation) gap has more effect on primary. Given that the contribution of non-primary sector on total GDP is more than 60% (in average), the results obtained with total GDP output gap are reasonable.

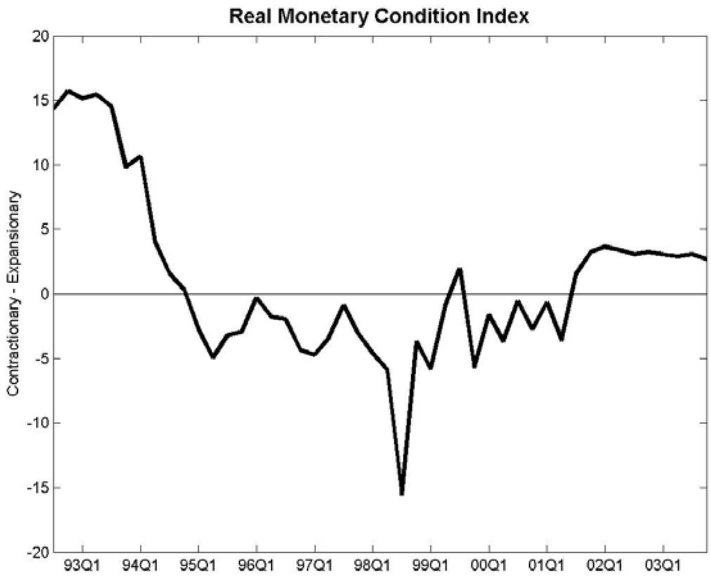

Figure 6. Real monetary conditions index is computed as a linear combination of the real interest rate and the real exchange rate gaps.