# Cointegrated TFP Processes and International Business Cycles

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  - 2 Are cointegrated
  - **(3)** The cointegrating vector is (1, -1): important for balanced growth.

• Based on this evidence, we simulate a standard two-country two-good IRBC model where TFP follow a VECM.

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### The Great Moderation and the Real Exchange Rate



Figure: Standard Deviation of HP-Filtered Data. USA and UK.

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### The Great Moderation and the Real Exchange Rate



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- We derive results that relate RER volatility with the parameters of the VECM.
- We show that the volatility increase can be related to changes in the parameter estimates of the VECM.

 Role of stochastic trends: King, Plosser, Stock, and Watson (1991), Lastrapes (1992), Alvarez and Jermann (2005), Engel and West (2005), Aguiar and Gopinath (2007), Nason and Rogers (2008).

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  - Nontradable goods: Corsetti, Dedola and Leduc (2007, 2008), Benigno, G. and Thoenissen (2007), Dotsey and Duarte (2006).

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- Firms in the intermediate and final goods sectors operate under perfect competition.
- Departure from the literature: TFP processes are C(1,1) and can be characterized with a VECM.

## The Model: Households

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left\{ C\left(s^t\right)^{\tau} \left[1 - L\left(s^t\right)\right]^{1-\tau} \right\}^{1-\sigma}}{1-\sigma}$$

s.t.

$$P\left(s^{t}\right)\left[C\left(s^{t}\right)+X\left(s^{t}\right)\right]+P_{H}\left(s^{t}\right)\overline{Q}\left(s^{t}\right)D\left(s^{t}\right)\leqslant P_{H}\left(s^{t}\right)D\left(s^{t-1}\right)+P\left(s^{t}\right)\left[W\left(s^{t}\right)L\left(s^{t}\right)+R\left(s^{t}\right)K\left(s^{t-1}\right)\right]-P_{H}\left(s^{t}\right)\Phi\left(D\left(s^{t}\right)\right),$$

and

$$\mathcal{K}\left( \mathbf{s}^{t}
ight) =\left( 1-\delta
ight) \mathcal{K}\left( \mathbf{s}^{t-1}
ight) +\mathcal{X}\left( \mathbf{s}^{t}
ight)$$
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#### The Model: Final Goods Producers

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$$\begin{split} \max & \mathcal{P}\left(s^{t}\right) Y\left(s^{t}\right) - \mathcal{P}_{H}\left(s^{t}\right) Y_{H}\left(s^{t}\right) - \mathcal{P}_{F}\left(s^{t}\right) Y_{F}\left(s^{t}\right) \\ & Y\left(s^{t}\right) = \left[\omega^{\frac{1}{\theta}}Y_{H}\left(s^{t}\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}}Y_{F}\left(s^{t}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \end{split}$$

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#### The Model: Intermediate Goods Producers

$$Max \left\{ \begin{array}{c} P_{H}\left(s^{t}\right)\left[Y_{H}\left(s^{t}\right)+Y_{H}^{*}\left(s^{t}\right)\right]-\\ P\left(s^{t}\right)\left[W\left(s^{t}\right)L\left(s^{t}\right)+R\left(s^{t}\right)K\left(s^{t-1}\right)\right] \end{array} \right\}$$
$$Y_{H}\left(s^{t}\right)+Y_{H}^{*}\left(s^{t}\right)=A\left(s^{t}\right)^{1-\alpha}K\left(s^{t-1}\right)^{\alpha}L\left(s^{t}\right)^{1-\alpha}$$

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# The Model: TFP

$$\begin{pmatrix} \Delta \log A \left( s^{t} \right) \\ \Delta \log A^{*} \left( s^{t} \right) \end{pmatrix} = \begin{pmatrix} c \\ c^{*} \end{pmatrix} + \rho(L) \begin{pmatrix} \Delta \log A \left( s^{t-1} \right) \\ \Delta \log A^{*} \left( s^{t-1} \right) \end{pmatrix}$$
$$+ \begin{pmatrix} \kappa \\ \kappa^{*} \end{pmatrix} \left[ \log A \left( s^{t-1} \right) - \gamma \log A^{*} \left( s^{t-1} \right) - \log \xi \right] + \begin{pmatrix} \varepsilon^{a} \left( s^{t} \right) \\ \varepsilon^{a,*} \left( s^{t} \right) \end{pmatrix}$$

- Implies that:
  - $\Delta \log A(s^t)$
  - $\Delta \log A^*(s^t)$ , and
  - $\log A\left(s^{t-1}
    ight) \gamma \log A^*\left(s^{t-1}
    ight)$  are stationary processes.

### The Model: Equilibrium Conditions

$$U_{\mathcal{C}}\left( \mathbf{s}^{t}
ight) =\lambda\left( \mathbf{s}^{t}
ight)$$
 ,

$$rac{U_{L}\left(s^{t}
ight)}{U_{C}\left(s^{t}
ight)}=W\left(s^{t}
ight)$$
 ,

$$\lambda\left(s^{t}
ight)=eta \mathsf{E}_{t}\left\{\lambda\left(s^{t+1}
ight)\left[\mathsf{R}\left(s^{t+1}
ight)+\left(1-\delta
ight)
ight]
ight\}$$
 ,

$$egin{aligned} & \mathcal{K}\left(m{s}^{t}
ight) = \left(1-\delta
ight)\mathcal{K}\left(m{s}^{t-1}
ight) + \mathcal{X}\left(m{s}^{t}
ight)$$
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### The Model: Equilibrium Conditions

$$\overline{Q}\left(s^{t}\right) = \beta E_{t}\left[\frac{\lambda\left(s^{t+1}\right)}{\lambda\left(s^{t}\right)}\frac{\widetilde{P}_{H}\left(s^{t+1}\right)}{\widetilde{P}_{H}\left(s^{t}\right)}\right] - \frac{\Phi'\left[D\left(s^{t}\right)\right]}{\beta}.$$

$$\begin{split} \widetilde{P}_{H}\left(s^{t}\right)\overline{Q}\left(s^{t}\right)D\left(s^{t}\right) &= \widetilde{P}_{H}\left(s^{t}\right)Y_{H}^{*}\left(s^{t}\right)-\widetilde{P}_{F}^{*}\left(s^{t}\right)RER\left(s^{t}\right)Y_{F}\left(s^{t}\right) \\ &+\widetilde{P}_{H}\left(s^{t}\right)D\left(s^{t-1}\right)-\widetilde{P}_{H}\left(s^{t}\right)\Phi\left[D\left(s^{t}\right)\right] \end{split}$$

$$E_{t}\left[\frac{\lambda^{*}\left(s^{t+1}\right)}{\lambda^{*}\left(s^{t}\right)}\frac{\widetilde{P}_{H}\left(s^{t+1}\right)}{\widetilde{P}_{H}\left(s^{t}\right)}\frac{RER\left(s^{t}\right)}{RER\left(s^{t+1}\right)}-\frac{\lambda\left(s^{t+1}\right)}{\lambda\left(s^{t}\right)}\frac{\widetilde{P}_{H}\left(s^{t+1}\right)}{\widetilde{P}_{H}\left(s^{t}\right)}\right]=-\frac{\Phi'\left[D\left(s^{t}\right)-\frac{1}{\beta}\right]}{\beta}$$

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### The Model: Equilibrium Conditions

$$W\left(s^{t}
ight)=(1-lpha)\widetilde{P}_{H}\left(s^{t}
ight)A\left(s^{t}
ight)^{1-lpha}K\left(s^{t-1}
ight)^{lpha}L\left(s^{t}
ight)^{-lpha}$$
 ,

$$R\left(s^{t}
ight)=lpha\widetilde{P}_{H}\left(s^{t}
ight)A\left(s^{t}
ight)^{1-lpha}K\left(s^{t-1}
ight)^{lpha-1}L\left(s^{t}
ight)^{1-lpha}$$
 ,

$$Y_{H}\left(s^{t}
ight)=\omega\widetilde{P}_{H}\left(s^{t}
ight)^{- heta}Y\left(s^{t}
ight)$$
 ,

$$Y_{F}\left(s^{t}
ight)=\left(1-\omega
ight)\left(\widetilde{P}_{F}^{*}\left(s^{t}
ight)\mathsf{RER}\left(s^{t}
ight)
ight)^{- heta}Y\left(s^{t}
ight)$$
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# The Model: Equilibrium Conditions

$$C(s^{t}) + X(s^{t}) = Y(s^{t}),$$

$$Y(s^{t}) = \left[\omega^{\frac{1}{\theta}}Y_{H}(s^{t})^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}}Y_{F}(s^{t})^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

$$Y_{H}(s^{t}) + Y_{H}^{*}(s^{t}) = A(s^{t})^{1-\alpha}K(s^{t-1})^{\alpha}L(s^{t})^{1-\alpha},$$

$$Y_{\mathit{F}}\left( {{{s}^{t}}} 
ight) + Y_{\mathit{F}}^{st}\left( {{{s}^{t}}} 
ight) = {{A}^{st}}\left( {{{s}^{t}}} 
ight)^{1 - lpha } {{\mathcal{K}}^{st}}\left( {{{s}^{t - 1}}} 
ight)^{lpha } {{\mathcal{L}}^{st}}\left( {{{s}^{t}}} 
ight)^{1 - lpha }$$
 ,

and

$$D\left(s^{t}
ight)+D^{*}\left(s^{t}
ight)=0.$$

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#### The Model: Balanced Growth

• Preferences and technology satisfy King, Plosser, and Rebelo (1988) restrictions for the existence of a balanced growth path in the closed economy.

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- But in the open economy we need an additional restriction

$$\begin{split} \widehat{Y}_{F}\left(s^{t}\right) &= (1-\omega)\left[\widetilde{P}_{F}^{*}\left(s^{t}\right)\mathsf{RER}\left(s^{t}\right)\right]^{-\theta}\widehat{Y}\left(s^{t}\right)\frac{A\left(s^{t-1}\right)}{A^{*}\left(s^{t-1}\right)}\\ \end{split}$$
where  $\widehat{Y}_{F}\left(s^{t}\right) &= Y_{F}\left(s^{t}\right)/A^{*}\left(s^{t-1}\right),\ \widehat{Y}\left(s^{t}\right) &= Y\left(s^{t}\right)/A\left(s^{t-1}\right). \end{split}$ 

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here  $\widehat{Y}_{F}\left(s^{t}\right) = Y_{F}\left(s^{t}\right) / A^{*}\left(s^{t-1}\right)$ ,  $\widehat{Y}\left(s^{t}\right) = Y\left(s^{t}\right) / A\left(s^{t-1}\right)$ .

• 
$$\frac{A\left(s^{t-1}
ight)}{A^{*}\left(s^{t-1}
ight)}$$
 is stationary if  $\gamma=1$ 

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• Take U.S. data for real GDP (BEA) and employment (Payroll Survey).

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- Rest of the world: Euro Area, Canada, Japan, Australia, and the UK. Also Mexico and South Korea.
- Aggregate GDPs using PPP-adjusted exchange rates. We aggregate number of employees.
- Follow Heathcote and Perri (2002)

$$\log A(s^{t}) = \left[\log Y(s^{t}) - (1-\alpha)\log L(s^{t})\right] / (1-\alpha)$$
$$\log A^{*}(s^{t}) = \left[\log Y^{*}(s^{t}) - (1-\alpha)\log L^{*}(s^{t})\right] / (1-\alpha)$$



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• Unit root tests: we cannot reject a unit root for the level of (log) TFP processes. We can reject a unit root for their first difference. TFP's are I(1).

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- Using Johansen's test, we cannot reject the existence of one cointegrating relationship. Hence, the TFP processes are C(1,1).
- We estimate the VECM with 2 lags and cannot reject that  $\gamma=1.$
- We run several likelihood ratio tests to test for symmetry.

$$\begin{pmatrix} \Delta \log A \left( s^{t} \right) \\ \Delta \log A^{*} \left( s^{t} \right) \end{pmatrix} = \begin{pmatrix} c \\ c^{*} \end{pmatrix} + \rho^{1} \begin{pmatrix} \Delta \log A \left( s^{t-1} \right) \\ \Delta \log A^{*} \left( s^{t-1} \right) \end{pmatrix}$$
$$+ \rho^{2} \begin{pmatrix} \Delta \log A \left( s^{t-2} \right) \\ \Delta \log A^{*} \left( s^{t-2} \right) \end{pmatrix}$$
$$+ \begin{pmatrix} \kappa \\ \kappa^{*} \end{pmatrix} \left[ \log A \left( s^{t-1} \right) - \gamma \log A^{*} \left( s^{t-1} \right) - \log \xi \right] + \begin{pmatrix} \varepsilon^{a} \left( s^{t} \right) \\ \varepsilon^{a,*} \left( s^{t} \right) \end{pmatrix}$$

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Table 4:	Likelihood	ratio	tests.

Restriction	Likelihood value	Degrees of freedom	p-value
None	744.18	-	-
$\gamma = 1$	743.33	1	0.19
$\kappa = -\kappa^*$	741.71	2	0.09
$c = c^*$	740.43	3	0.06
Symmetry across VAR coefficients	736.51	7	0.032

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Table 5:	Parameter	Estimates,	VECM	model
		1980 — 2007	7	
	с	$0.0071^{st}_{(5.83)}$		
	κ	-0.0045*		
	$o^1$	(-2.65) 0.2041*		
	$\rho_{11}$	(2.97)		
	$ ho_{11}^2$	$\underset{(1.54)}{0.1026}$		
	$ ho_{12}^1$	$\underset{(1.55)}{0.1035}$		
	$\rho_{12}^2$	$-0.1497^{*}$		
	- 12	(-2.40)		

t-statistics in parenthesis.

\* means significance at the 5 percent level

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Image: A mathematical states and a mathem

Table 6: Calibration			
Preferences	eta= 0.99		
	$\mu = 0.34$		
	$\sigma = 2$		
	$\phi=0.01$		
Technology	$\alpha = 0.36$		
	$\delta=0.025$		
	$\omega = 0.9$		
	heta=[0.85,0.62]		

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Iable /a: Results						
Full Sample	σγ	$\sigma_{\mathcal{C}}^+$	$\sigma_X^+$	$\sigma_{N}^{+}$	$\sigma^+_{\it RER}$	$\rho(RER)$
Data	1.25	0.80	3.40	0.91	4.28	0.84
Coint. TFP, $ heta=0.85$	0.81	0.63	2.32	0.28	1.75	0.72
Coint. TFP, $ heta=0.62$	0.70	0.62	2.31	0.28	4.26	0.70
Stat. TFP, $ heta=0.85$	1.19	0.52	2.53	0.32	0.75	0.77
Stat. TFP, $ heta=0.62$	1.12	0.54	2.51	0.31	1.41	0.75

+ denotes relative to output.

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Table 7b: Results					
Full Sample	CORR(Y,N)	CORR(Y,C)	CORR(Y,X)		
Data	0.79	0.81	0.91		
Coint. TFP, $ heta=0.85$	0.94	0.95	0.97		
Coint. TFP, $ heta=0.62$	0.92	0.93	0.95		
Stat. TFP, $ heta=0.85$	0.97	0.93	0.97		
Stat. TFP, $ heta=0.62$	0.97	0.93	0.97		

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• Estimated stationary TFP shocks imply somewhat high persistence and fast spillovers (Heathcote and Perri, 2002).

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- Estimated non-stationary TFP shocks find high persistence (by definition there is one unit root) and slow spillovers.

- Estimated stationary TFP shocks imply somewhat high persistence and fast spillovers (Heathcote and Perri, 2002).
- Estimated non-stationary TFP shocks find high persistence (by definition there is one unit root) and slow spillovers.
- First, we discuss the role of persistence in a stationary model. Then we discuss the role of spillovers in a non-stationary model.

• When persistence increases at home, there is a stronger income effect at home. Hence:

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- When persistence increases at home, there is a stronger income effect at home. Hence:
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#### • Hence higher persistence leads to higher RER volatility.



Figure: Impulse Response to a Home-Country TFP shock. Model with stationary TFP shocks.



Figure: Impulse Response to a Home-Country TFP shock. Model with stationary TFP shocks.

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• Now we switch to the model with VECM shocks:

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• Increased  $\kappa$  implies a stronger "news channel" in the foreign country:

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- Lower production of foreign intermediate good and higher production of home intermediate good lead to RER and TOT apreciation.

• Now we switch to the model with VECM shocks:

$$\begin{array}{lll} \Delta {\pmb{a}}_t & = & -\kappa ({\pmb{a}}_{t-1} - {\pmb{a}}_{t-1}^*) + \varepsilon_t^{\pmb{a}} \\ \Delta {\pmb{a}}_t^* & = & \kappa ({\pmb{a}}_{t-1} - {\pmb{a}}_{t-1}^*) + \varepsilon_t^{\pmb{a},*} \end{array}$$

• Increased  $\kappa$  implies a stronger "news channel" in the foreign country:

- Labor supply and investment decreases, and output in the foreign country decreases on impact.
- Consumption increases, leading to more demand of the home intermediate good.
- Lower production of foreign intermediate good and higher production of home intermediate good lead to RER and TOT apreciation.
- Hence higher speed of convergence leads to lower RER volatility.



Figure: Impulse Response to a Home-Country TFP shock. Model with cointegrated TFP shocks.

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Table 8: Changing $ ho_a$ and $\kappa$				
	SD(RER)	SD(Y)	$SD(RER)^+$	
$ ho_{a}$				
0.9	1.43	1.33	1.07	
0.95	1.96	1.2	1.64	
0.975	2.47	1.06	2.33	
κ				
0.005	1.98	0.64	3.1	
0.05	1.02	0.82	1.25	
0.25	0.71	0.86	0.82	

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• Therefore, one unit root in the joint process of TFP across countries is not enough. We need the second root to be very close to one.

$$\begin{pmatrix} \mathbf{a}_t \\ \mathbf{a}_t^* \end{pmatrix} = \begin{pmatrix} 1-\kappa & \kappa \\ \kappa & 1-\kappa \end{pmatrix} \begin{pmatrix} \mathbf{a}_{t-1} \\ \mathbf{a}_{t-1}^* \end{pmatrix} + \begin{pmatrix} \boldsymbol{\varepsilon}_t^{\mathbf{a}} \\ \boldsymbol{\varepsilon}_t^{\mathbf{a},*} \end{pmatrix}.$$

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- Heathcote and Perri (2008)  $\lambda_1$ ,  $\lambda_2 = 0.91$ . Rel RER volatility: 1.05.

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# A VAR in levels or a VECM?

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- Engle and Granger (1987): small sample improvements from estimating a VECM, estimating a VAR in levels leads to ignoring important constraints that are only satisfied asymptotically.

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#### The Great Moderation and the Real Exchange Rate



Figure: Standard Deviation of HP-Filtered Data. USA and UK.

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#### The Great Moderation and the Real Exchange Rate



Figure: Standard Deviation of HP-Filtered Data. Canada and Australia.

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#### Estimation of the VECM for TFP

Table: Subsample analysis				
	1980 - 1993	1994 - 2007		
с	0.007*	0.008*		
κ	$-0.008^{*}$	-0.003		
$\rho_{11}^{1}$	0.22*	0.13		
$\rho_{11}^{\bar{2}}$	0.07	0.12		
$\rho_{12}^{\bar{1}}$	0.07	0.13		
$ ho_{12}^{\bar{2}^-}$	0.01	-0.36*		

\* means significance at the 5 percent level

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Table 7a: Results						
	σ <sub>Y</sub>	$\sigma_{C}^{+}$	$\sigma_X^+$	$\sigma_N^+$	$\sigma^+_{RER}$	$\rho(RER)$
1980-1993						
Data	1.57	0.80	3.08	0.89	3.97	0.85
Coint. TFP, $ heta=0.85$	1.12	0.63	2.17	0.25	1.33	0.72
Coint. TFP, $ heta=0.62$	0.95	0.65	2.15	0.25	3.17	0.71
1994-2007						
Data	0.83	0.76	4.20	0.96	5.17	0.81
Coint. TFP, $ heta=0.85$	0.64	0.55	2.74	0.38	2.04	0.71
Coint. TFP, $ heta=0.62$	0.62	0.43	3.01	0.42	5.06	0.69

+ denotes relative to output.

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Table 7b: Results					
	CORR(Y,N)	CORR(Y,C)	CORR(Y,X)		
1980-1993					
Data	0.82	0.82	0.93		
Coint. TFP, $ heta=0.85$	0.93	0.96	0.97		
Coint. TFP, $ heta=0.62$	0.91	0.96	0.96		
1994-2007					
Data	0.71	0.76	0.90		
Coint. TFP, $ heta=0.85$	0.89	0.82	0.94		
Coint. TFP, $\theta = 0.62$	0.94	0.78	0.97		

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- If we allow the speed of convergence to the cointegrating vector to change as it does in the data, the model can also explain the observed increase in the relative volatility of the real exchange rate.
- Cointegration of TFP processes should be introduced in larger-scale models (Adolfson et al., 2007)

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