# Cointegrated TFP Processes and International Business Cycles*

Pau Rabanal† Juan F. Rubio-Ramírez‡ Vicente Tuesta§

July 2, 2008

**Abstract**

A central puzzle in international macroeconomics is that observed real exchange rates are highly volatile. Standard International Real Business Cycle (IRBC) models cannot reproduce this fact when calibrated using conventional parameterizations, and can only generate one fourth of the real exchange rate volatility observed in the data. Typically, IRBC models are solved assuming that total factor productivity (TFP) processes are stationary. In this paper, we first show that TFP processes for the U.S. and the "rest of the world" have a unit root, are cointegrated, and can be jointly characterized with a Vector Error Correction Model (VECM). Then, we explore the implications of extending an otherwise standard international real business cycle model that allows for cointegrated technology shocks. We show that the model can account for the high

*We thank Gabriel Rodríguez for very useful comments.

†Research Department, Caixa d'Estalvis i Pensions de Barcelona. Email: prabanal@lacaixa.es.

‡Corresponding author. Duke University and Research Department, Federal Reserve Bank of Atlanta. Email: juan.rubio-ramirez@duke.edu.

§Research Department, Central Reserve Bank of Peru, and CENTRUM Católica. Email: vicente.tuesta@bcrp.gob.pe

real exchange rate volatility observed in the data without having to rely on any particular nominal or real friction. Also, we show that the increase of relative volatility of the real exchange rate with respect to output in the last 20 years can be explained by changes in the parameter estimates of the VECM.

JEL Classification: E32, F32, F33, F41

Keywords: International Business Cycles, Real Exchange Rates, Cointegration.

# 1. Introduction

A central puzzle in international macroeconomics is that observed real exchange rates are highly volatile. Standard International Real Business Cycle (IRBC) models cannot reproduce this fact when calibrated using conventional parameterizations. For instance, Heathcote and Perri (2002) simulate a two-country two-good economy with total factor productivity (TFP) shocks and find that the model can only explain less than a fourth of the observed volatility in real exchange rates for U.S. data. An important feature of their model, following the seminal work of Backus, Kehoe, and Kydland (1992), is that it considers stationary TFP shocks.

In this paper we provide evidence that TFP processses for the U.S. and a sample of main industrialized trade partners have a unit root, are cointegrated, and have cointegrating vector $(1, -1)$. Motivated by this empirical finding, we introduce technology shocks that display stochastic trends and that are cointegrated into an otherwise standard two-country two-good model. This specification of TFP across countries solves a large part of the real exchange rate volatility puzzle without affecting the good match for domestic variables. In particular, we show that our model can generate a real exchange rate volatility more than four times larger than an equivalent model with stationary shocks.

The intuition behind our result is as follows. In the standard model with no spillovers, when productivity increases at home, output, consumption, investment, and labor increase, while the marginal cost decreases. As output at home increases, the demand for intermediate goods produced in the foreign country also increases. Provided that the elasticity of substitution between home and foreign intermediate goods is low enough, output, consumption, investment, and labor also increase abroad. Therefore, marginal costs increase in the foreign country and the real exchange rate depreciates. As the persistence of TFP shocks increases, home country households feel richer and supply less labor and capital. This has several effects. First, it lowers the initial increase of home output, second, it causes a larger decrease of home marginal costs (and therefore, a larger real exchange rate depreciation), and third, as home households want higher consumption, the demand for foreign intermediate goods increases. This leads to a larger increase of labor and investment abroad. Hence, marginal costs abroad suffer a larger increase and the real exchange rate depreciates even further. As a result, higher persistence in TFP shocks leads to a higher relative volatility of the real exchange rate with respect to output.

When spillovers of TFP shocks across countries are introduced in the model, a "news" channel arises. This channel has the opposite effect than the one described above. As foreign country households know that productivity will eventually increase in their country, they feel richer and supply less labor and capital but demand more consumption goods. Thus, marginal costs decrease abroad and increase at home and the real exchange rate would tend to depreciate less when spillovers are faster. Therefore, faster spillovers in TFP shocks leads to a lower relative volatility of the real exchange rate with respect to output. Since we estimate higher persistence and slower spillovers of TFP shocks than traditionally found in the literature, the real exchange rate is more volatile in our calibrated model.

Another very well documented empirical fact is the substantial decline in the volatility of most U.S macroeconomic variables during the last 20 years. That change in the cyclical volatility is know as the "Great Moderation".[1] In this paper, we report that, for most industrialized countries, the Great Moderation has not affected the real exchange rate as strongly as it has affected output. As a result, the ratio of real exchange rate volatility to output volatility has increased. We also show that the increase in the relative volatility of the real exchange rate of the U.S. dollar coincides in time with a weakening of the cointegrating relationship of TFP shocks between the U.S. and the "rest of the world".[2] More importantly, we confirm that if we allow for a fading in the cointegrating relationship of the size estimated in the data, the model can jointly account for the observed increase in the relative volatility of the real exchange rate and the substantial decline in the volatility of output.

Our paper relates to two important strands of the literature. On the one hand, it connects with the literature stressing the importance of stochastic trends to explain economic fluctuations. King, Plosser, Stock, and Watson (1991) find that a common stochastic trend explains the comovements of main U.S. real macroeconomic variables. Lastrapes (1992) reports that fluctuations in real and nominal exchanges rates are due primarily to permanent real shocks. Engel and West (2005) show that real exchange rates manifests near–random walk behavior if

[1]Some early discussion of the Great Moderation can be found in Kim and Nelson (1999). A discussion of different interpretations for this phenomenon and some international evidence can be found in Stock and Watson (2002) and Stock and Watson (2007), respectively.

[2]In section 4 we describe the set of countries that compose our "rest of the world" definition.

TFP processes are random walk and the discount factor is near one. Nason and Rogers (2008) generalize this hypothesis to a larger class of models. Aguiar and Gopinath (2007) show that trend shocks are the primary source of fluctuations in emerging economies. Alvarez and Jermann (2005) and Corsetti, Dedola and Leduc (2007) highlight the importance of persistent disturbances to explain asset prices and real exchange rates fluctuations respectively.

On the other hand, our paper also links to the literature analyzing different mechanisms to understand real exchange rate fluctuations. Some recent papers study the effects of monetary shocks and nominal rigidities. Chari, Kehoe and McGrattan (2002) are able to explain real exchange rate volatility in a monetary model with sticky prices and a high degree of risk aversion. Benigno (2005) focuses on the role of interest rate inertia and asymmetric nominal rigidities across countries. Other papers use either non-tradable goods, pricing to market or some form of distribution costs (see Corsetti, Dedola and Leduc 2007, 2008; Benigno and Thoenissen 2007, and Dotsey and Duarte 2006). Our model only includes tradable goods with home bias. Our choice is guided by evidence that the relative price of tradable goods has large and persistent fluctuations that explain most of the real exchange rate volatility (see, Engel 1993 and 1999). Fluctuations of the relative price of nontradable goods accounts for, at most, one third of the real exchange rate volatility (see Betts and Kehoe 2006, Burstein, Eichenbaum and Rebelo 2006, and Rabanal and Tuesta 2007).

The rest of the paper is organized as follows. Section 2 documents the increase of the real exchange rate volatility with respect to output for most industrialized countries. In Section 3 we present the model with cointegrated TFP shocks. In Section 4 we report estimates for the law of motion of the (log) TFP processes of the United States and a "rest of the world" aggregate. In Section 5 we present the main findings from simulating the model, leaving Section 6 for concluding remarks.

## 2. The Great Moderation and Real Exchange Rate Volatility

In this section, we present evidence that in the period known as "the Great Moderation", the relative volatility of the real exchange rate (measured as the Real Effective Exchange Rate) with respect to output (measured as real GDP) has increased in the United States, the United Kingdom, Canada, and Australia. In Figures 1 and 2 we present the standard deviation of the HP-filtered output, the standard deviation of the HP-filtered real exchange rate, and the ratio of the two for these four countries. We compute the standard deviation of rolling windows of 40 quarters.[3]

Let us first focus on the US economy. Figure 1 shows a substantial decline in the volatility of output, from 2.3 percent standard deviation in the window 1973:1-1982:4, to 0.8 percent in the window 1997:3-2007:2. This decline in output volatility is what is typically referred to as "The Great Moderation". The volatility of the real exchange rate has experienced a different path: the standard deviation was at about 4.5 percent for the window 1973:1-1982:4; thereafter, it increased to values above 7 percent for the window 1980:1-1989:4, and declined to a value of 4.3 percent for the window 1997:3-2007:2.

So what is the behavior of the ratio of volatilities between the two series? The ratio has increased in a non-monotonic way from 1.96 percent to 4.5 percent in the period we study. Hence, the volatility of the real exchange rate has more than doubled relative to that of output.

What has been the experience with the other main currencies? As Figures 1 and 2 show, the pattern that arises with the United Kingdom, Canada, and Australia is also quite similar: dramatic declines in the volatility of output, erratic behavior of the absolute volatility of the real exchange rate, and dramatic increases in the relative volatility of the real exchange rate with respect to output.

Having presented some evidence for the main industrialized countries, in this paper we only focus on the relationship between the U.S. economy and the "rest of the world". Hence, we build a two-country, two-good model that we calibrate using standard parameters of the IRBC literature, and estimated parameters of a Vector Error Correction Model (VECM) using TFP processes for the U.S. and a "rest of the world" aggregate. In Section 5 we show that it is possible to explain the observed increase in relative volatility of the real exchange rate with respect to output with changes in the estimated parameters of the VECM.

[3] Therefore, the first data point in 1982:4 denotes the standard deviation of HP-filtered output for the period 1973:1-1982:4. The last data point 2007:2 denotes the standard deviation of HP-filtered output between 1997:3-2007:2.

## 3. The Model

In this section, we present a standard two-country two-good IRBC model similar to the one described in Heathcote and Perri (2002). The main difference with respect to the standard IRBC literature is the definition of the stochastic processes for TFP. In that literature, the TFP processes of the two countries are assumed to be stationary or trend stationary in logs and they are modelled as a VAR.[4] In this paper, we consider instead (log) TFP processes that are cointegrated of order C(1,1). This implies that (log) TFP processes are integrated of order one but a linear combination is stationary. According to the Granger Representation Theorem,[5] our C(1,1) assumption is equivalent to defining a VECM for the law of motion of the log differences of the TFP processes. The VECM is defined in more detail in section 3.2.3. Our cointegration assumption has strong and testable implications for the data. The empirical evidence supporting our assumption will be presented in section 4.

In each country, a single final good is produced by a representative competitive firm that uses intermediate goods in the production process. These intermediate goods are imperfect substitutes of each other and can be purchased from representative competitive intermediate goods producers in both countries. Intermediate goods producers use local capital and labor in the production process. The final good is locally consumed and invested by the consumers. The stock of local capital good can only be increased using the domestic final good. Thus, all trade between the countries is in intermediate goods. In addition, consumers trade across countries an uncontingent international riskless bond, that is denominated in units of domestic intermediate goods. No other financial asset is available. In each period of time $t$, the economy experiences one of many finitely events $s_t$. We denote by $s^t = (s_0, ..., s_t)$ the history of events up through period $t$. The probability, as of period 0, of any particular history $s^t$ is $\pi(s^t)$ and $s_0$ is given.

In the remaining of this section, we describe the households problem, the intermediate and final goods producers problems, and the VECM process. Then, we explain the market

[4] See Backus, Kehoe and Kydland (1992), Kehoe and Perri (2002), and Heathcote and Perri (2002). Interestingly, Baxter and Crucini (1995) estimate a VECM using TFP processes for the United States and Canada, but dismiss this evidence when simulating their model.

[5] See Engel and Granger (1987).

clearing and equilibrium. Finally, we discuss the conditions for the existence of a balanced growth path, and explain how to transform the variables in the model to achieve stationarity.

### 3.1. Households

In this subsection, we describe the decision problem faced by home-country households. The problem faced by foreign-country households is similar, and hence it is not presented. The representative household of the home country solves

$$\max_{\{C(s^t),L(s^t),X(s^t),K(s^t),D(s^t)\}} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi\left(s^t\right) \frac{\left\{C\left(s^t\right)^{\tau}\left[1-L\left(s^t\right)\right]^{1-\tau}\right\}^{1-\sigma}}{1-\sigma} \tag{1}$$

subject to the following budget constraint

$$\begin{gathered} P\left(s^t\right)\left[C\left(s^t\right)+X\left(s^t\right)\right]+P_H\left(s^t\right)\overline{Q}\left(s^t\right)D\left(s^t\right) \leqslant \\ P\left(s^t\right)\left[W\left(s^t\right)L\left(s^t\right)+R\left(s^t\right)K\left(s^{t-1}\right)\right]+P_H\left(s^t\right)D\left(s^{t-1}\right)-P_H\left(s^t\right)\Phi\left(D\left(s^t\right)\right), \end{gathered} \tag{2}$$

and the law of motion for capital

$$K\left(s^t\right)=(1-\delta)K\left(s^{t-1}\right)+\varpi\left[\frac{X\left(s^t\right)}{K\left(s^{t-1}\right)}\right]X\left(s^t\right), \tag{3}$$

The following notation is used: $\beta \in (0,1)$ is the discount factor, $L\left(s^t\right) \in (0,1)$ are hours worked in the home country, $C\left(s^t\right) \geq 0$ are units of consumption of the final good, $X\left(s^t\right) \geq 0$ are units of investment, $K\left(s^t\right) \geq 0$ is the capital level in the home country at the beginning of period $t+1$. $P\left(s^t\right)$ is the price of the home final good, which will be defined below, $W\left(s^t\right)$ is the hourly wage in the home country and $R\left(s^t\right)$ is the home country rental rate of capital, where both factor inputs prices are measured in units of the final good. $P_H\left(s^t\right)$ is the price of the home intermediate good, $D\left(s^t\right)$ denotes the holdings of the internationally traded riskless bond that pays one unit of home intermediate good in period $t+1$ regardless of the state of nature, and $\overline{Q}\left(s^t\right)$ is its price, measured in units of the home intermediate good. Finally, the function $\varpi$ represents the cost of adjusting the capital units measured in units of final good and the function $\Phi$ is an arbitrarily small cost of holding bonds measured in units of the home intermediate good.[6]

Our results hold for any convex adjustment cost functions $\varpi$ and $\Phi$ such that $\varpi\left(\frac{\bar{X}}{\bar{K}}\right)=0$ and $\varpi'\left(\frac{\bar{X}}{\bar{K}}\right)=0$ and $\Phi\left(0\right)=0$ and $\Phi'\left(0\right)=0$. We assume that the function $\varpi$ representing the cost of adjusting the capital takes the form $\varpi\left(\frac{X\left(s^{t}\right)}{K(s^{t-1})}\right)=\left[1-\frac{\psi}{2}\left(\frac{X\left(s^{t}\right)}{K(s^{t-1})}-\frac{\bar{X}}{\bar{K}}\right)^{2}\right]$. Thus, there is a cost of adjusting the capital stock when the investment-capital ratio differs from its steady state value, $\frac{\bar{X}}{\bar{K}}$.

We assume, following the existing literature, that the function $\Phi$ takes the following functional form $\Phi\left[D\left(s^{t}\right)\right]=\frac{\phi}{2}A(s^{t-1})\left[\frac{D\left(s^{t}\right)}{A(s^{t-1})}\right]^{2}$. Note that we need to include the level of TFP in the home country, $A(s^{t-1})$ in the adjustment cost function, both dividing $D\left(s^{t}\right)$ and multiplying $\left[\frac{D\left(s^{t}\right)}{A(s^{t-1})}\right]^{2}$. The reason is that since $A(s^{t-1})$ is an integrated process, hence $D\left(s^{t}\right)$ will grow at the rate of growth of $A(s^{t-1})$ along the balanced growth path, making the ratio $\frac{D\left(s^{t}\right)}{A(s^{t-1})}$ stationary. Also, since all home real variables will also grow at the rate of growth of $A(s^{t-1})$ along the balanced growth path, we need to make the adjustment cost (measured in units of home intermediate good) also grow at the same rate in order to induce stationarity. As it was the case for the cost of adjusting the capital level, the cost of holding debt grows quadratically as the normalized debt level moves away from its steady state level of zero.[7]

### 3.2. Firms

#### 3.2.1. Final good producers

The final good in the home country, $Y\left(s^{t}\right)$ is produced using home intermediate goods, $Y_{H}\left(s^{t}\right)$, and foreign intermediate goods, $Y_{F}\left(s^{t}\right)$, with the following technology:

$$Y\left(s^{t}\right)=\left[\omega^{\frac{1}{\theta}}Y_{H}\left(s^{t}\right)^{\frac{\theta-1}{\theta}}+\left(1-\omega\right)^{\frac{1}{\theta}}Y_{F}\left(s^{t}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{4}$$

[6] The $\Phi(\cdot)$ cost is introduced to ensure stationarity of the level of $D(s^{t})$ in IRBC models with incomplete markets, as discussed by Schmitt-Grohé and Uribe (2003).

[7] Our results will also hold for any other assumption for the steady state level of debt, but assuming balanced trade in the steady-state is convenient to solve for the levels of imports and exports in both economies.

where $\omega$ denotes the fraction of home intermediate goods that are used for the production of the home final good and $\theta$ controls the elasticity of substitution between home and foreign intermediate goods. Therefore, the representative final goods producer in the home country solves the following problem

$$\max_{Y(s^t)\geq 0, Y_H(s^t)\geq 0, Y_F(s^t)\geq 0} P\left(s^t\right) Y\left(s^t\right) - P_H\left(s^t\right) Y_H\left(s^t\right) - P_F^*\left(s^t\right) Y_F\left(s^t\right)$$

subject to the production function (4).

### 3.2.2. Intermediate Goods Producers

The representative intermediate goods producer in the home country uses home labor and capital in order to produce home intermediate goods and sells her product to both the home and foreign final good producers. Taking prices of all goods and factor inputs as given, she maximizes profits. Hence, she solves

$$\underset{L(s^t)\geq 0, K(s^{t-1})\geq 0}{Max} P_H\left(s^t\right)\left[Y_H\left(s^t\right) + Y_H^*\left(s^t\right)\right] - P\left(s^t\right)\left[W\left(s^t\right) L\left(s^t\right) + R\left(s^t\right) K\left(s^{t-1}\right)\right]$$

subject to the production function

$$Y_H\left(s^t\right) + Y_H^*\left(s^t\right) = A\left(s^t\right)^{1-\alpha} K\left(s^{t-1}\right)^{\alpha} L\left(s^t\right)^{1-\alpha} \tag{5}$$

where $Y_H(s^t)$ is the amount of home intermediate good sold to the home final good producers, $Y_H^*(s^t)$ is the amount of home intermediate good sold to the foreign final good producers, and $A(s^t)$ is an stochastic process affecting TFP of home intermediate goods producers, that we will characterize below.

### 3.2.3. The Processes for TFP

As mentioned above, we depart from the standard assumption in the IRBC literature and consider processes for both $\log A(s^t)$ and $\log A^*(s^t)$ that are cointegrated of order $C(1,1)$. Equivalently, we specify the following VECM for the law of motion driving the log differences of TFP processes for both the home and the foreign country

$$\begin{pmatrix} \Delta \log A(s^t) \\ \Delta \log A^*(s^t) \end{pmatrix} = \begin{pmatrix} c \\ c^* \end{pmatrix} + \rho^1 \begin{pmatrix} \Delta \log A(s^{t-1}) \\ \Delta \log A^*(s^{t-1}) \end{pmatrix} + \rho^2 \begin{pmatrix} \Delta \log A(s^{t-2}) \\ \Delta \log A^*(s^{t-2}) \end{pmatrix} \tag{6}$$
$$+ \begin{pmatrix} \kappa \\ \kappa^* \end{pmatrix} \left[\log A\left(s^{t-1}\right) - \gamma \log A^*\left(s^{t-1}\right) - \log \xi\right] + \begin{pmatrix} \varepsilon^a(s^t) \\ \varepsilon^{a,*}(s^t) \end{pmatrix}$$

where $\rho^1$ and $\rho^2$ are $2 \times 2$ coefficient matrices, $(1, -\gamma)$ is called the cointegrating vector, $\xi$ is the constant in the cointegrating relationship, $\varepsilon^a(s^t) \sim N(0, \sigma^{\varepsilon})$ and $\varepsilon^*(s^t) \sim N(0, \sigma^{\varepsilon,*})$ and independent of each other, $\Delta$ is the first-difference operator, and either $\kappa$ or $\kappa^*$ are different from zero and of the right sign.[8]

This VECM representation implies that deviations of today's log differences of TFP with respect to its mean value depend not only on lags of home and foreign log differences of TFP but also on a function of the ratio of lag home and foreign TFP, $A(s^{t-1}) / \left[\xi A^*(s^{t-1})^{\gamma}\right]$. Thus, if the ratio $A(s^{t-1}) / A^*(s^{t-1})^{\gamma}$ is larger than its long run value, $\xi$, then $\kappa < 0$ and $\kappa^* > 0$ will imply that $\Delta \log A(s^t)$ would fall and $\Delta \log A^*(s^t)$ would rise, driving both series towards their long run equilibrium values. The VECM representation also implies that $\Delta \log A(s^t)$, $\Delta \log A^*(s^t)$, and $\log A(s^{t-1}) - \gamma \log A^*(s^{t-1}) - \log \xi$ are stationary processes.

### 3.3. Market Clearing

The model is closed with the following market clearing conditions in the final good markets

$$C\left(s^t\right) + X\left(s^t\right) = Y\left(s^t\right), \tag{7}$$

$$C^*\left(s^t\right) + X^*\left(s^t\right) = Y^*\left(s^t\right), \tag{8}$$

and the bond markets

$$D\left(s^t\right) + D^*\left(s^t\right) = 0. \tag{9}$$

[8] Here we restrict ourselves to a VECM with two lags. This assumption is motivated by the empirical results to be presented in section 4, where only two lags are significant. In any case, we recognize that the Granger Representation theorem states that the series are C(1,1) if and only if there exists a VECM representation of some lag lenght. In our theoretical model we could use any finite number of lags.

### 3.4. Equilibrium

#### 3.4.1. Equilibrium Definition

Now we are ready to define the *equilibrium* for this economy. Given our law of motion for (log) TFP shocks defined by (6), an equilibrium for this economy is a set of allocations for home consumers, $C(s^t)$, $L(s^t)$, $X(s^t)$, $K(s^t)$, and $D(s^t)$ and foreign consumers, $C^*(s^t)$, $L^*(s^t)$, $X^*(s^t)$, $K^*(s^t)$, and $D^*(s^t)$, allocations for home and foreign intermediate good producers, $Y_H(s^t)$, $Y_H^*(s^t)$, $Y_F(s^t)$ and $Y_F^*(s^t)$, allocations for home and foreign final good producers, $Y(s^t)$ and $Y^*(s^t)$, intermediate goods prices $P_H(s^t)$ and $P_F^*(s^t)$, final goods prices $P(s^t)$ and $P^*(s^t)$, rental prices of labor and capital in the home and foreign country, $W(s^t)$, $R(s^t)$, $W^*(s^t)$, and $R^*(s^t)$ and the price of the bond $\overline{Q}(s^t)$ such that $(i)$ given prices household allocations solve the households' problem; $(ii)$ given prices, intermediate good producers allocations solves the intermediate good producers' problem; $(iii)$ given prices final good producers allocations solves the final good producers' problem; $(iv)$ and markets clear.

#### 3.4.2. Equilibrium Conditions

At this point, it is useful to define the following relative prices: $\widetilde{P}_H(s^t) = \frac{P_H(s^t)}{P(s^t)}$, $\widetilde{P}_F^*(s^t) = \frac{P_F^*(s^t)}{P^*(s^t)}$ and $RER(s^t) = \frac{P^*(s^t)}{P(s^t)}$. Note that $\widetilde{P}_H(s^t)$ is the price of home intermediate goods in terms of home final goods, $\widetilde{P}_F^*(s^t)$ is the price of foreign intermediate goods in terms of foreign final goods, that appears in the foreign country's budget constraint, and $RER(s^t)$ is the real exchange rate between the home and foreign countries. In our model the law of one price holds, hence, we have that $P_H(s^t) = P_H^*(s^t)$ and $P_F(s^t) = P_F^*(s^t)$.

Let us now determine the equilibrium conditions implied by the first order conditions of households, intermediate and final goods producers in both countries, as well as the relevant laws of motion, production functions, and market clearing conditions. The marginal utility of consumption and labor supply are given by:

$$U_C\left(s^t\right) = \lambda\left(s^t\right), \tag{10}$$

$$\frac{U_L(s^t)}{U_C(s^t)} = W\left(s^t\right), \tag{11}$$

where $U_x$ denotes the partial derivative of the utility function $U$ with respect to variable $x$. The first order condition with respect to investment is given by:

$$\lambda\left(s^t\right) = \mu\left(s^t\right)\left[\varpi\left(\frac{X\left(s^t\right)}{K\left(s^{t-1}\right)}\right) + \varpi'\left(\frac{X\left(s^t\right)}{K\left(s^{t-1}\right)}\right)\frac{X\left(s^t\right)}{K\left(s^{t-1}\right)}\right], \tag{12}$$

where $\frac{\mu\left(s^t\right)}{\lambda\left(s^t\right)}$ is the shadow value of investment goods in terms of consumption goods, also referred to in the literature as "Tobin's Q". Absent adjustment costs to capital, the ratio is constant and equal to one. The first order condition with respect to capital delivers an intertemporal condition that relates the relative price of investment goods to the rental rate of capital and the depreciation rate, including the role of marginal adjustment costs:

$$\mu\left(s^t\right) = \beta\sum_{s^{t+1}}\pi\left(s^{t+1}|s^t\right)\left\{R\left(s^{t+1}\right)\lambda\left(s^{t+1}\right) + \mu\left(s^{t+1}\right)\left[(1-\delta) - \varpi'\left(\frac{X\left(s^{t+1}\right)}{K\left(s^t\right)}\right)\left(\frac{X\left(s^{t+1}\right)}{K\left(s^t\right)}\right)^2\right]\right\}, \tag{13}$$

where $\pi\left(s^{t+1}|s^t\right) = \frac{\pi\left(s^{t+1}\right)}{\pi\left(s^t\right)}$ is the conditional probability of $s^{t+1}$ given $s^t$.

Finally, the law of motion of capital is:

$$K\left(s^t\right) = (1-\delta)\,K\left(s^{t-1}\right) + \varpi\left(\frac{X\left(s^t\right)}{K\left(s^{t-1}\right)}\right)X\left(s^t\right), \tag{14}$$

The analogous expressions for the foreign country are as follows. All starred variables denote the foreign-country analogous to the home-country variables (i.e. $C$ is consumption of the final home good, then $C^*$ denotes consumption of the final foreign good, and so on).

$$U_{C^*}\left(s^t\right) = \lambda^*\left(s^t\right), \tag{15}$$

$$\frac{U_{L^*}\left(s^t\right)}{U_{C^*}\left(s^t\right)} = W^*\left(s^t\right), \tag{16}$$

$$\lambda^*\left(s^t\right) = \mu^*\left(s^t\right)\left[\varpi\left(\frac{X^*\left(s^t\right)}{K^*\left(s^{t-1}\right)}\right) + \varpi'\left(\frac{X^*\left(s^t\right)}{K^*\left(s^{t-1}\right)}\right)\frac{X^*\left(s^t\right)}{K^*\left(s^{t-1}\right)}\right], \tag{17}$$

$$\mu^{*}\left(s^{t}\right)=\beta\sum_{s^{t+1}}\pi\left(s^{t+1}|s^{t}\right)\left\{R^{*}\left(s^{t+1}\right)\lambda^{*}\left(s^{t+1}\right)+\mu^{*}\left(s^{t+1}\right)\left[(1-\delta)-\varpi'\left(\frac{X^{*}\left(s^{t+1}\right)}{K^{*}\left(s^{t}\right)}\right)\left(\frac{X^{*}\left(s^{t+1}\right)}{K^{*}\left(s^{t}\right)}\right)^{2}\right]\right\}, \tag{18}$$

and

$$K^{*}\left(s^{t}\right)=(1-\delta)K^{*}\left(s^{t-1}\right)+\varpi\left(\frac{X^{*}\left(s^{t}\right)}{K^{*}\left(s^{t-1}\right)}\right)X^{*}\left(s^{t}\right). \tag{19}$$

The optimal choice by households of the home country delivers the following expression for the price of the riskless bond:

$$\overline{Q}\left(s^{t}\right)=\beta\sum_{s^{t+1}}\pi\left(s^{t+1}|s^{t}\right)\frac{\lambda\left(s^{t+1}\right)}{\lambda\left(s^{t}\right)}\frac{\widetilde{P}_{H}\left(s^{t+1}\right)}{\widetilde{P}_{H}\left(s^{t}\right)}-\frac{\Phi'\left[D\left(s^{t}\right)\right]}{\beta}. \tag{20}$$

The risk-sharing condition is given by the optimal choice by the households of both countries for the riskless bond:

$$\sum_{s^{t+1}}\pi\left(s^{t+1}|s^{t}\right)\left[\frac{\lambda^{*}\left(s^{t+1}\right)}{\lambda^{*}\left(s^{t}\right)}\frac{\widetilde{P}_{H}\left(s^{t+1}\right)}{\widetilde{P}_{H}\left(s^{t}\right)}\frac{RER\left(s^{t}\right)}{RER\left(s^{t+1}\right)}-\frac{\lambda\left(s^{t+1}\right)}{\lambda\left(s^{t}\right)}\frac{\widetilde{P}_{H}\left(s^{t+1}\right)}{\widetilde{P}_{H}\left(s^{t}\right)}\right]=-\frac{\Phi'\left[D\left(s^{t}\right)\right]}{\beta} \tag{21}$$

From the intermediate goods producers maximization problems, we obtain labor and capital are paid their marginal product, where the rental rate of capital and the real wage are expressed in terms of the final good in each country:

$$W\left(s^{t}\right)=(1-\alpha)\widetilde{P}_{H}\left(s^{t}\right)A\left(s^{t}\right)^{1-\alpha}K\left(s^{t-1}\right)^{\alpha}L\left(s^{t}\right)^{-\alpha}, \tag{22}$$

$$R\left(s^{t}\right)=\alpha\widetilde{P}_{H}\left(s^{t}\right)A\left(s^{t}\right)^{1-\alpha}K\left(s^{t-1}\right)^{\alpha-1}L\left(s^{t}\right)^{1-\alpha}, \tag{23}$$

$$W^{*}\left(s^{t}\right)=(1-\alpha)\widetilde{P}_{F}^{*}\left(s^{t}\right)A^{*}\left(s^{t}\right)^{1-\alpha}K^{*}\left(s^{t-1}\right)^{\alpha}L^{*}\left(s^{t}\right)^{-\alpha}, \tag{24}$$

and

$$R^{*}\left(s^{t}\right)=\alpha\widetilde{P}_{F}^{*}\left(s^{t}\right)A^{*}\left(s^{t}\right)^{1-\alpha}K^{*}\left(s^{t-1}\right)^{\alpha-1}L^{*}\left(s^{t}\right)^{1-\alpha}. \tag{25}$$

From the final goods producers maximization problem, we obtain the demands of intermediate goods, that depend on their relative price:

$$Y_{H}\left(s^{t}\right)=\omega\widetilde{P}_{H}\left(s^{t}\right)^{-\theta}Y\left(s^{t}\right), \tag{26}$$

$$Y_F\left(s^t\right) = (1-\omega)\left(\widetilde{P}_F^*\left(s^t\right) RER\left(s^t\right)\right)^{-\theta} Y\left(s^t\right), \tag{27}$$

$$Y_H^*\left(s^t\right) = (1-\omega)\left(\frac{\widetilde{P}_H\left(s^t\right)}{RER\left(s^t\right)}\right)^{-\theta} Y^*\left(s^t\right), \tag{28}$$

and

$$Y_F^*\left(s^t\right) = \omega \widetilde{P}_F^*\left(s^t\right)^{-\theta} Y^*\left(s^t\right). \tag{29}$$

Finally, goods, inputs and bonds markets clear. Thus

$$C\left(s^t\right) + X\left(s^t\right) = Y\left(s^t\right), \tag{30}$$

$$C^*\left(s^t\right) + X^*\left(s^t\right) = Y^*\left(s^t\right), \tag{31}$$

$$Y\left(s^t\right) = \left[\omega^{\frac{1}{\theta}} Y_H\left(s^t\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} Y_F\left(s^t\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{32}$$

$$Y^*\left(s^t\right) = \left[\omega^{\frac{1}{\theta}} Y_F^*\left(s^t\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} Y_H^*\left(s^t\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{33}$$

$$Y_H\left(s^t\right) + Y_H^*\left(s^t\right) = A\left(s^t\right)^{1-\alpha} K\left(s^{t-1}\right)^{\alpha} L\left(s^t\right)^{1-\alpha}, \tag{34}$$

$$Y_F\left(s^t\right) + Y_F^*\left(s^t\right) = A^*\left(s^t\right)^{1-\alpha} K^*\left(s^{t-1}\right)^{\alpha} L^*\left(s^t\right)^{1-\alpha}, \tag{35}$$

and

$$D\left(s^t\right) + D^*\left(s^t\right) = 0. \tag{36}$$

The law of motion of the level of debt

$$\begin{aligned} \widetilde{P}_H\left(s^t\right)\overline{Q}\left(s^t\right)D\left(s^t\right) &= \widetilde{P}_H\left(s^t\right)Y_H^*\left(s^t\right) - \widetilde{P}_F^*\left(s^t\right)RER\left(s^t\right)Y_F\left(s^t\right) \\ &\quad + \widetilde{P}_H\left(s^t\right)D\left(s^{t-1}\right) - \widetilde{P}_H\left(s^t\right)\Phi\left[D\left(s^t\right)\right] \end{aligned} \tag{37}$$

is obtained using (2) and the fact that intermediate and final goods producers at home make zero profits. Finally, the productivity shocks follow the VECM described in section 3.2.3.

### 3.5. Balanced Growth and the Restriction on the Cointegrating Vector

Equations (10) to (37) and the VECM process for (log) TFP characterize the equilibrium in this model. Since we assume that both $\log A(s^t)$ and $\log A^*(s^t)$ are integrated processes, we need to normalize the equilibrium conditions in order to get a stationary system more amenable to study. Following King, Plosser and Rebelo (1998) we divide the home-country variables that have a trend by the lagged domestic level of TFP, $A(s^{t-1})$, and the foreign-country variables that have a trend by the lagged foreign level of TFP, $A^*(s^{t-1})$. In the appendix A.1, we detail the full set of normalized equilibrium conditions. (See equations (41) to (68))

For the model to have balanced growth we require some restrictions on preferences, production functions, and the law of motion of productivity shocks. The restrictions on preferences and technology of King, Plosser, and Rebelo (1988) are sufficient for the existence of balanced growth in a closed economy Real Business Cycle (RBC) model. However, in our two-country model, an additional restriction on the cointegrating vector is needed if the model is to exhibit balanced growth. In particular, we need the ratio $A(s^{t-1})/A^*(s^{t-1})$ to be stationary.

In order to understand why the international dimension of the model requires this additional restriction, let us focus, for example, at equation (58) of set of normalized equilibrium conditions. This equation is the normalized condition for the demand of imported foreign-produced intermediate goods by the home country,

$$\widehat{Y}_F\left(s^t\right) = (1-\omega)\left[\widetilde{P}_F^*\left(s^t\right) RER\left(s^t\right)\right]^{-\theta} \widehat{Y}\left(s^t\right) \frac{A\left(s^{t-1}\right)}{A^*\left(s^{t-1}\right)}$$

where $\widehat{Y}_F(s^t) = Y_F(s^t)/A^*(s^{t-1})$ while $\widehat{Y}(s^t) = Y(s^t)/A(s^{t-1})$. Since $\widetilde{P}_F^*(s^t)$ and $RER(s^t)$ are stationary, if the ratio between $A(s^{t-1})$ and $A^*(s^{t-1})$ were to be non-stationary, the ratio between $\widehat{Y}_F(s^t)$ and $\widehat{Y}(s^t)$ would also be non-stationary and balanced growth would not exist. A similar argument must hold for the following normalized equilibrium conditions: $(59)$, $(63)$, $(64)$, $(67)$, and $(68)$.

Our VECM implies that the ratio between $A(s^{t-1})$ and $A^*(s^{t-1})^{\gamma}$ is stationary. Therefore, a sufficient condition for balanced growth is that the parameter $\gamma$ equals one or, equivalently, that the cointegrating vector equals $(1, -1)$.

## 4. Estimation of the VECM

In this section, after describing our constructed TFP series for the U.S. and the "rest of the world", we perform three exercises. First, we show that our assumption that the TFP processes are cointegrated of order C(1,1) cannot be rejected in the data. By the Granger Representation Theorem this implies that our VECM specification is valid. Second, we also show that the restriction imposed by balanced growth, i.e. that the parameter $\gamma$ is equal to one, can not be rejected in the data either. Finally, we estimate the parameters driving our VECM in order to simulate our model in next section.

### 4.1. Data

In order to estimate our VECM we use data for the U.S. and an aggregate for the "rest of the world". For the U.S., we obtain quarterly output data from the Bureau of Economic Analysis, and employment data from the Payroll Survey from 1973:1 to 2007:3. The "rest of the world" aggregate contains nominal output and employment data for the 12 countries of the Euro Area (using Eurostat and the Area Wide Model dataset maintained at the European Central Bank), the United Kingdom, Canada, Japan, and Australia (using national sources data). This group accounts for about 50 percent of the basket of currencies that the Federal Reserve uses to construct the real exchange rate for the U.S. Dollar. Given some restrictions on employment data necessary to build TFP shocks our sample period for the "rest of the world" goes from 1980:1 to 2007:3. Ideally, one would want to include additional countries that represent an important and increasing share of trade with the United States, such as China, but long quarterly output and employment figures are not available.[9]

We aggregate the nominal outputs of the "rest of the world" using PPP exchange rates to convert each national nominal output to current U.S. dollars, and then use the output

[9] We also included in our definition of the rest of the world Mexico and South Korea, which resulted in a shortening of the starting point to 1982:3, which is when the Korean employment series starts. The results were similar including these two countries, but to take advantage of the longer time series in our subsample analysis, we decided to exclude them.

deflator of the United States to convert the "rest of the world" nominal output to constant U.S. dollars. We obtain aggregate "rest of the world" employment data by simply aggregating the number of employees in each country.

Since capital stock series are not available at a quarterly frequency for most countries, we estimate the TFP shock as follows

$$\log A\left(s^t\right) = \left[\log Y\left(s^t\right) - (1-\alpha)\log L\left(s^t\right)\right] / (1-\alpha)$$

$$\log A^*\left(s^t\right) = \left[\log Y^*\left(s^t\right) - (1-\alpha)\log L^*\left(s^t\right)\right] / (1-\alpha)$$

where $\alpha$ is the capital share of output. Heathcote and Perri (2002) use a similar approach when constructing TFP series for the United States and a "rest of the world" aggregate. Figure 3 plots the constructed (log) TFP processes for the U.S. and "rest of the world". Note that TFP processes exhibit similar patterns and the distance between both series appears to be constant across time. This preliminary evidence indicates some potential of cointegration between both TFP series. In the next subsection we verify statistically this first pass evidence.

### 4.2. Integration and Cointegration Properties

In this section, we present evidence supporting our assumption that the (log) TFP processes for the U.S. and "rest of the world" are cointegrated of order C(1,1). First, we will empirically support the unit root assumption for the univariate processes. Second, we will test for the presence of cointegrating relationships using the Johansen (1991) procedure. Both the trace and the maximum eigenvalue methods support the existence of a cointegrating vector.

Univariate analysis of the log TFP processes for the U.S. and "rest of the world" strongly indicates that both series can be characterized by unit root processes with drift. Table 1 presents results for the U.S. log TFP process using the following commonly applied unit root tests: augmented Dickey and Fuller (Dickey and Fuller, 1979, and Said and Dickey 1984), the DF-GLS and the optimal point statistic ($P_T GLS$) both of Elliott et al. (1996), and the modified $MZ_\alpha$, $MZ_t$, and MSB of Ng and Perron (2001). The lag length is chosen using the modified Akaike Information criterion (MAIC) as Ng and Perron (1995) recommend.[10]

[10] We have tried different maximum lag lenghts with similar outcomes. At the end we choose to report

In each case a constant and a trend are included in the specification and data from 1973:1 to 2007:4 is used. Table 1 also presents the same unit root test results for the "rest of the world" log TFP process using data from 1980:1 to 2007:3. None of the test statistics are even close to rejecting the null hypothesis of unit root at the 5 percent critical value and only the augmented Dickey and Fuller t-test rejects at the 10 percent critical value for the U.S. Using the same statistics, unit root tests on the first difference of the log TFP processes for the U.S. and "rest of the world" are stationary. For the U.S. all the tests reject the null hypothesis of unit root at the one percent critical value. For the "rest of the world" augmented Dickey and Fuller, $P_T GLS$, and MSB reject the null hypothesis of unit root at the five percent critical value while the DF-GLS and $MZ_\alpha$ tests rejects at the 10 percent value.

Once we have presented evidence that strongly indicates that the (log) TFP for the U.S. and "rest of the world" are well characterized by integrated processes of order one, we now focus on presenting evidence supporting our assumption that the processes are cointegrated. Table 2 presents some statistics calculated from an unrestricted VAR with five lags and a trend for the two-variables system $[\log A(s^t), \log A^*(s^t)]$ for the sample period 1980:3 to 2007:3.[11] Table 2 shows the two eigenvalues with the largest absolute value for the VAR implied by the point estimates. If $\log A(s^t)$ and $\log A^*(s^t)$ share one common stochastic trend (balanced growth), the estimated VAR has to have a single eigenvalue equal to one and all other eigenvalues have to be less than one. As shown in Table 2, point estimates are in accord with this prediction: the highest eigenvalue equals one while the second highest is less than one. But this is not a formal test of cointegration. Table 3 reports results from the unrestricted cointegration rank test using the trace and the maximum eigenvalue methods as defined by Johansen (1991). The cointegration tests are run for the sample period 1981:2 to 2007:3 and assume a constant in the cointegrating vector. Clearly, the data strongly supports a single eigenvalue.

---

results with a maximum of two lag lengths because of the short sample we have available.

[11] We choose the number of lags using the AIC criterion.

### 4.3. The VECM model

In the last subsection, we presented evidence that $\log A(s^t)$ and $\log A^*(s^t)$ are cointegrated of order C(1,1). In this subsection we provide four additional results. First, we show that the null hypothesis of $\gamma = 1$ can not be rejected by the data using a likelihood ratio test. This is very important because a cointegrating vector $(1, -1)$ implies that the balanced growth path hypothesis cannot be rejected.

In the IRBC literature, it is typically assumed that the coefficients driving TFP processes are symmetric across countries. Thus, we also use the likelihood ratio test to present evidence supporting the following three null hypothesis: (1) whether the coefficients related to the speed of adjustment in the cointegrating vector are equal and of opposite sign, i.e. $\kappa = -\kappa^*$, (2) whether the coefficients of the constant terms are the same, i.e. $c = c^*$, and (3) we also check for symmetry in the coefficients of the VAR. Since the lag coefficient matrices are

$$\rho^1 = \begin{pmatrix} \rho_{11}^1 & \rho_{12}^1 \\ \rho_{21}^1 & \rho_{22}^1 \end{pmatrix} \text{ and } \rho^2 = \begin{pmatrix} \rho_{11}^2 & \rho_{12}^2 \\ \rho_{21}^2 & \rho_{22}^2 \end{pmatrix},$$

the restrictions we check are: $\rho_{11}^1 = \rho_{22}^1$, $\rho_{11}^2 = \rho_{22}^2$, $\rho_{12}^1 = \rho_{21}^1$, and $\rho_{12}^2 = \rho_{21}^2$.

Finally, after imposing the above described restrictions, i.e. balanced growth, symmetric constant terms, symmetric speed of adjustment parameters, and symmetric coefficients of the VAR, we estimate our VECM.

In Table 4, we present the outcome of the four likelihood ratio tests. Note that the tests are incremental. The first important result is that the restriction that the cointegrating vector is $(1, -1)$, i.e. $\gamma = 1$, is not rejected by the data. Second, we can not reject that the coefficients on the speed of adjustment are the same in absolute value across countries. Third, we can not reject that the constant term is equal across countries. Finally, the symmetry in the coefficients restriction is marginally rejected by the data at the five percent level. The above evidence allows us to follow the usual practice in the literature and simulate our model with all the restrictions in place.

In the final step, we estimate a restricted VECM. The estimated restricted model delivers the parameter estimates reported in Table 5. The results are as follow. First, it is worth noting that the coefficient of the speed of adjustment, while significant, is quantitatively small denoting that TFP processes converge slowly over time. This result is going to be important in order to explain our results. Second, the coefficient on the own first lag implies significant but low autocorrelation. The crossed second lag is also significant. Third, the rest of the coefficients are not significant. Finally, we estimate the standard deviation of the innovations $\sigma^{\varepsilon}$ and $\sigma^{\varepsilon,*}$ to be around 0.0082. When simulating our model, we calibrate the stochastic process using the point estimates reported in Table 5 for the significant parameters, including those for $\sigma^{\varepsilon}$ and $\sigma^{\varepsilon,*}$.

## 5. Results

### 5.1. Parameterization

Our baseline parameterization follows Heathcote and Perri (2002) closely. Table 6 summarizes the parameter values. The discount factor $\beta$ is set equal to 0.99 which implies an annual rate return of capital of 4 percent. We set the consumption share, $\tau$, equal to 0.34 and the coefficient of risk aversion, $\sigma$, equal to 2. Backus, Kehoe and Kydland (1995) assume the same value for the latter parameter. We assume a cost of bond holdings, $\phi$, of 100 basis points (0.01). Parameters on technology are fairly standard in the literature. Thus, the depreciation rate, $\delta$, is set to a quarterly value of 0.025, the capital share of output is set to $\alpha = 0.36$, home bias for domestic intermediate goods is set to $\omega = 0.9$, which implies the observed import/output ratio in steady state. We set the adjustment cost of investment such that the volatility of investment over GDP is roughly as in the data. We assume two possible values for the elasticity of substitution between intermediate goods, $\theta = 0.85$ and $\theta = 0.62$. The first value is based on Heathcote and Perri (2002), the second one is used by Corsetti, Dedola and Leduc (2008). The baseline technology process is calibrated as described in Table 5. For the stationary case, we set the parameters of the TFP shocks as in Heathcote and Perri (2002).

### 5.2. Matching Real Exchange Rate Volatility

In this subsection we analyze the performance of our model in generating enough real exchange rate volatility. Results are shown in Table 7a. In order to perform the simulation, we solve the model taking a log-linear approximation around the steady state, and calculate the moments of the HP-filtered artificial data.[12]

The first and second rows of Table 7a report the results of the economy with cointegrated TFP and high and low values for the trade elasticity, $\theta$, respectively. For comparison with a model such as the one in Heathcote and Perri (2002), we also report the results for the economy with stationary TFP shocks in the next two rows (we use Heathcote and Perri estimates for stationary TFP shocks).[13] Overall, models with cointegrated shocks generate higher relative volatility of the real exchange rate with respect to output than models with stationary TFP. Note that with high trade elasticity and cointegrated TFP shocks the relative volatility of the real exchange rate more than doubles with respect to the model with stationary shocks (1.75 versus 0.75). We go from explaining less than 20 percent of the observed relative volatility of the real exchange rate to explain more than 40 percent. As expected for lower values for the trade elasticity the relative volatility of the real exchange rate increases under both the stationary and cointegrated models. The striking finding is that the model with cointegrated TFP shocks and elasticity equal to 0.62 is able to closely match the relative volatility of the real exchange rate (4.26 in the model versus 4.28 in the data) while the model with stationary shocks and the same elasticity can only get to 1.41 (which only represents about 30 percent of the fluctuation in the data). Interestingly, even though the model with cointegrated TFP shocks improves significantly in matching the real exchange rate volatility, it does not affect other unconditional moments. Both the stationary and the cointegrated TFP models display very similar volatilities of consumption, hours, and investment relative to output. Also both models display similar cross-correlations between consumption, hours, and investment relative and output and autocorrelations of real exchange rates (Tables 7a and 7b).

[12] One might question the use of the Hodrick-Prescott filter in a model without a stochastic trend. In any case, in order to calculate unconditional moments of a nonstationary series we need to normalize the variables. We also want to replicate patterns studied in the international business cycle literature. We want emphasize the fact that the stochastic trend process generates much of the RER variance at business cycle frequencies.

[13] In particular, Heathcote and Perri use the following calibrated VAR(1) process:

$$
\begin{aligned}
a_t &= \rho_a a_{t-1} + \rho_a^* a_{t-1}^* + \varepsilon_t^a \\
a_t^* &= \rho_a a_{t-1}^* + \rho_a^* a_{t-1} + \varepsilon_t^{a,*}
\end{aligned}
$$

where $\rho_a = 0.97$, $\rho_a^* = 0.025$, $Var(\varepsilon_t^a) = Var(\varepsilon_t^{a,*}) = 0.0073^2$, and $corr(\varepsilon_t^a, \varepsilon_t^{a,*}) = 0.29$.

### 5.3. Intuition

In this subsection we explain why our results differ from those obtained with more traditional calibrations of TFP processes. There are two forces driving relative volatility of the real exchange rate with respect to output. In particular, the model needs high persistence and low spillovers of the TFP processes across countries in order to get high relative volatility of the real exchange rate. Our VECM estimates imply higher persistence and slower spillover than typically assumed in the literature and therefore we succeed in matching the high relative volatility of the real exchange rate. Let us now explain how persistence affects relative volatility of real exchange rate. Later we will explain how spillovers also affect it.

In order to understand the effects of the persistence, we simulate our model assuming the following processes for TFP

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \varepsilon_t^a, \\ a_t^* &= \rho_a a_{t-1}^* + \varepsilon_t^{a,*} \end{aligned}$$

where the innovations are uncorrelated and there are no spillovers across countries. In Figures 4 and 5 we present the impulse responses to a home TFP shock for different values of $\rho_a = [0.9, 0.95, 0.975]$ using the same calibration as Table 6 for the rest of the parameters (for these Figures we use $\theta = 0.62$). As Table 8 shows, as we increase the persistence parameter, the relative volatility of the HP-filtered real exchange rate with respect to HP-filtered output also increases.

For any given persistence coefficient, when productivity increases at home, output, consumption, investment, and labor increase, while the price of the domestically produced intermediate good decreases (real wages and the rental rate of capital increase but the marginal cost decreases because of the productivity improvement). As final output at home increases, the demand for intermediate goods produced in the foreign country also increases. Provided that the elasticity of substitution between home and foreign intermediate goods is low enough, output, consumption, investment, and labor increase abroad. Therefore, marginal costs increase in the foreign country and the real exchange rate depreciates.

As the persistence of TFP shocks increases, the initial impact and the persistence of the response of the real exchange rates are larger. In Table 8, we confirm that the standard deviation of HP-filtered real exchange rate increases with $\rho_a$. However, the response of output is different. Its response is initially smaller, but more persistent. Thus, by just looking at the impulse response in Figure 4, the effect on volatility is uncertain. Table 8 shows that this two conflicting forces lead to a lower standard deviation of HP-filtered output as persistence increases. Hence, the relative volatility of real exchange rate with respect to output increases.

What is the mechanism behind this result? With a higher persistence of TFP shocks, home country households suffer a larger income effect and therefore supply less labor and capital. This income effect has two implications in the home country. First, it lowers the initial increase of output. Second, it causes a larger decrease of marginal costs and, therefore, a larger real exchange rate depreciation. In addition, this income effect leads to home households demanding more consumption goods. Thus, the demand for foreign intermediate goods increases because home final goods producers substitute away from domestic intermediate goods. This leads to an increase of labor, investment abroad, and marginal costs abroad: the real exchange rate depreciates even further.

In order to analyze the effects of spillover changes on the relative volatility of real exchange rate we assume the following simple VECM model:

$$\begin{aligned} \Delta a_t &= -\kappa(a_{t-1} - a^*_{t-1}) + \varepsilon^a_t \\ \Delta a^*_t &= \kappa(a_{t-1} - a^*_{t-1}) + \varepsilon^{a,*}_t \end{aligned}$$

where the innovations are uncorrelated and $\kappa$ represents the speed of adjustment to the cointegrating relationship. Note that we have switched to a model with one unit root. The reasons behind this choice are twofold. First, it allows a clear mapping to our estimated VECM while changing $\kappa$. Second, the behavior of a model with TFP shocks driven by a VAR as the one used above with persistence coefficient arbitrarily close to one is numerically very similar to a model with TFP shocks driven by a VECM with speed of convergence coefficient arbitraly close to zero.

In Figures 6 and 7 we present the impulse responses to a home TFP shock for different values of $\kappa = [0.005, 0.05, 0.25]$. As before, we use the same calibration as Table 6 for the rest of the parameters and we fix $\theta = 0.62$. Now the foreign TFP process also responds over time to a home TFP increase due to the cointegrating relationship. The larger is $\kappa$, the faster is the response of foreign TFP to home TFP shocks. The most important consequence of considering cointegration (and therefore, spillovers) is the fact that there is a "news" channel effect as foreign country households anticipate the future increase of foreign TFP. When $\kappa = 0.005$ (slow speed of convergence), the mecanism at work is very similar to that of Figures 4 and 5 with $\rho_a = 0.975$ because the "news" channel is quantitatively very small. As $\kappa$ increases, the "news" channel becomes more important as the foreign households feel the income effect associated to it.

When productivity increases at home and spillovers are faster, foreign country households know that productivity will increase sooner in their country. Hence, because of an income effect, they supply less labor and capital but demand more consumption goods than they would do if spillovers were slower. Thus, the demand for home intermediate goods increases because foreign final goods producers substitute away from domestic intermediate goods. As a consequence, home households supply more labor and capital and the initial rise of home output increases. As marginal costs increase at home and decrease abroad, the real exchange rate would tend to depreciate less as the speed of convergence increases. Therefore, faster spillovers in TFP shocks leads to a lower relative volatility of the real exchange rate with respect to output. Table 8 confirms this intuition. As the speed of convergence increases, the volatility of HP-filtered output increases, and the volatility of HP-filtered real exchange rate decreases. Hence, the relative volatility of the real exchange rate with respect to output decreases.

Note that in the case of $\kappa = 0.25$, the relative standard deviation is 0.48, which is much lower than the values obtained under stationary TFP shocks, despite the fact that the VECM has one unit root. Hence, having cointegrated TFP shocks is not enough to solve the real exchange rate puzzle: a very slow speed of convergence is also necessary. Note that we can write the VECM as a VAR in levels as follows:

$$\begin{pmatrix} a_t \\ a_t^* \end{pmatrix} = \begin{pmatrix} 1-\kappa & \kappa \\ \kappa & 1-\kappa \end{pmatrix} \begin{pmatrix} a_{t-1} \\ a_{t-1}^* \end{pmatrix} + \begin{pmatrix} \varepsilon_t^a \\ \varepsilon_t^{a,*} \end{pmatrix}. \tag{38}$$

where the eigenvalues of the VAR are $\lambda_1 = 1$, $\lambda_2 = 1 - 2\kappa$.[14] Therefore, a small $\kappa$ means that we need both one unit root and that the second eigenvalue is very close to one. In fact, in the simple VECM with $\kappa = 0.005$, the two eigenvalues are $\lambda_1 = 1$ and $\lambda_2 = 0.99$ (while our point estimate for $\kappa$ is 0.0045, see Table 5).

Why are the results using our VECM estimates so different from the ones obtained using Heathcote and Perri's calibration? Their estimated VAR for joint TFP processes has eigenvalues equal to $\lambda_1 = 0.995$ and $\lambda_2 = 0.945$ (see footnote 13) While one of the eigenvalues is very close to one (which would imply high relative volatility of the real exchange rate given the results reported in Table 8), the second eigenvalue is farther away from one (implying fast spillovers), which matters for real exchange rate volatility. Finally, Heathcote and Perri (2002) calibrate the correlation of innovations to TFP shocks of 0.29, which acts as a contemporaneous spillover, further reducing the relative volatility of the real exchange rate for the reasons we have just explained.

In the view of (38), the question is if it would possible to solve the real exchange rate volatility puzzle using TFP shocks driven by a VAR in levels with one lag instead of a VECM. In principle, it would be possible as long as we calibrate the law of motion of the VAR so that the two eigenvalues are very close to one. The problem is that when we estimate a VAR as one described above using our data set we find that the two eigenvalues are 0.999 and 0.952, and a correlation between innovations of 0.16. Using this point estimates the relative volatility of the real exchange rate with respect to output is 1.67. Higher than the one reported by Heathcote and Perri (2002) but much smaller than the value reported in Table 7a.

### 5.4. Matching the Increase in Real Exchange Rate Volatility

As described in section 2, the volatility of the real exchange rate with respect to the volatility of output has increased in the last decades for most industrialized economies. If we focus on the U.S., the increase seems to be dated around the early-mid 90's. As Table 7a shows, the volatility of the real exchange rate has gone from below four times the volatility of output during the period 1980:1 to 1993:4 to more than five times during the period 1994:1 to 2007:3. Using U.S. and "rest of the world" data, in this section we present evidence that relates a

[14] Also note that if $\kappa = 0$, then the VAR has two unit roots, consistent with two independent random walks.

decrease on the speed of convergence to the cointegrating relationship, i.e. lower $\kappa$, with the increase on the relative volatility of the real exchange rate.

In order to present our evidence, we estimate our VECM for two non-overlapping sub-samples[15]. The first sample goes from 1980:1 to 1993:4, while the second sub-sample goes from 1994:1 to 2007:3. We split the sample such that we have the same number of observations in both sub-samples. Using data from 1980:1 to 1993:4, we observe three significant changes with respect to the full sample estimates. The value of the speed of adjustment term is larger in absolute value, the first own lag is also somewhat larger, and the second crossed-lag is close to zero. In particular $\kappa$ moves from $-0.0045$ to $-0.0077$, making the speed of converge faster and $\rho_{11}^1$ moves from 0.2041 to 0.2203, increasing the autocorrelation of the process. Also, the standard deviation of the stochastic process for the U.S., $\sigma$, is estimated to be 0.010 while standard deviation for the "rest of the world", $\sigma^*$, is estimated to be 0.0081.

In the second sub-sample, 1994:1 to 2007:3, the estimated speed of adjustment coefficient dramatically decreases with respect to both the full sample and first sub-sample: the point estimate is $-0.0029$. This means that the catching up process is much slower in the second part of the sample. In addition, the second crossed-lag coefficient gets larger and negative: $\rho_{12}^2 = -0.4124$. In this case, the first own lag moves close to zero. These results indicates that the comovement between total factor productivities in the post-1994 period is characterized by short-run negative comovement and slow return to the long-run level. Finally, the standard deviations $\sigma$ and $\sigma^*$ are estimated to be 0.0062 and 0.0086 respectively. Our sub-sample estimates of $\sigma$ and $\sigma^*$ reflect both our sample period and the countries that we include in the "rest of the world". While the big drop in $\sigma$ across sub-sample reveals the reduction in output volatility that the U.S. experiences during the 80's (see Kim and Nelson, 1999, and McConnell and Perez-Quirós, 2000), the stable $\sigma^*$ exposes that this was not the case for most of the countries in our definition of the "rest of the world" during the considered period. This second finding is in line with those in Stock and Watson (2005).

We now simulate the model under the estimates of the VECM for each of the sub-samples. Tables 7a and 7b report the results. Our results indicates that the change in the estimates of the VECM across samples is an important force behind the increase in the relative volatility

[15] We assume that the cointegrating relationship is the same across samples.

of the real exchange rate. While in the data the relative real exchange rate volatility increases by 30 percent across samples our simulations show that the model generates relative volatility increases of more than 50 percent for both low and high values of $\theta$.

### 5.5. The "Backus-Smith Puzzle"

How does the model perform in terms of the correlation between the real exchange rate and the ratio of consumption across countries? As the last column of Table 7b shows, our model implies that the correlation between the real exchange rate and relative consumption is very close to one, whereas in the data this correlation is negative but close to zero. This discrepancy between the models and the data is known as the "Backus-Smith puzzle".[16] The failure in accounting for the "Backus- Smith puzzle" is typical in standard IRBC models. In recent papers, Corsetti, Dedola and Leduc (2007) and Benigno and Thoenissen (2007) show that adding non tradable goods to a traditional IRBC model helps to solve the "Backus-Smith puzzle".

As we have shown in Figures 6 and 7, in our model a domestic TFP shock induces an increase in home consumption relative to foreign consumption and at the same time causes a real exchange rate depreciation. Hence, it is hard for our model to generate a negative correlation between real exchange rate and relative consumption unless another source of fluctuations is considered. One option is to introduce taste shocks that affect the marginal utility of consumption and allow to break the risk sharing condition implied by the model. Following this line of research, Heathcote and Perri (2008) introduce taste shocks and show how this simple device accomplishes the objective. We have introduced this type of shock in our framework and obtained a negative correlation between relative consumptions and the real exchange rate.

Since it is difficult to measure taste shocks in the data, we also consider another avenue. In particular we introduce investment-specific technology shocks, as in Greenwood, Hercowitz, and Krusell (1997). Thus, we change equation (3), and its foreign country counterpart, to

$$K\left(s^{t}\right)=(1-\delta) K\left(s^{t-1}\right)+V\left(s^{t}\right) X\left(s^{t}\right) \tag{39}$$

[16] Chari, Kehoe and McGrattan (2002) coin this problem as the consumption-real exchange rate anomaly.

and

$$K^* \left(s^t\right) = (1 - \delta)\, K^* \left(s^{t-1}\right) + V^* \left(s^t\right) X^* \left(s^t\right) \tag{40}$$

where both $\log V\left(s^t\right)$ and $\log V^*\left(s^t\right)$ that are cointegrated of order $C(1,1)$ and follow the same VECM process as the TFP shocks introduced in section 6.

Ideally, we would like to estimate the VECM process for the investment-specific technology shocks. In the literature, these shocks have been proxied by the quality-adjusted relative price of investment goods with respect to the price of consumption goods.[17] While the quality-adjusted relative price of investment goods is available for the U.S., it is not for most other countries in our "rest of the world" definition. Hence, we cannot estimate a VECM. In order to ilustrate the potential of this shock to solve the "Backus-Smith puzzle", we calibrate the VECM process for the investment-specific technology shocks using the same parametrization obtained for the TFP shock (see Table 5). Since we do not have a way to determine the relative importance of these two shocks, in Table 9 we report simulations of the model letting the standard deviation of the investment-specific technology shocks change from one to three times that of the TFP shock. Given the estimates reported in the literature, this appears to be a plausible range.[18]

As it can be observed in Table 9, as investment-specific technology shocks become more important, there are good and bad news. The good news is that the correlation between relative consumptions and real exchange rate drops dramatically as the importance of investment-specific technology shocks grows. When we only consider TFP shocks the correlation is 0.97, but when the standard deviation of the investment-specific technology shocks is three times the standard deviation of TFP shocks the correlation becomes negative and very similar to that in the data. The bad news is that the relative volatility of the real exchange rate with respect to output declines, but only mildly. It goes from 4.26 when only TFP are considered to 3.82 when the standard deviation of the investment-specific technology shocks is three times the standard deviation of TFP shocks.

Why investment-specific technology shocks do the job? As an investment-specific technology shock hits the home country, investment increases and consumption decreases at home.

[17] See Fisher (2005).

[18] See Justiniano and Primiceri (2007) and Fernández-Villaverde and Rubio-Ramírez (2007).

Through the labor supply condition, real wages also drop at home. The marginal cost of home-produced intermediate goods drops because the decline in real wages more than compensates the increase in the rental rate of capital. As home-produced intermediate goods become cheaper, final good producers in the foreign country substitute away from locally produced intermediate goods. This reduces the labor supply and investment abroad while increasing consumption, real wages, and the rental rate of capital abroad. Therefore, the price of intermediate goods produced in the foreign country increases reinforcing the real exchange rate depreciation. As a result, the model generates a negative correlation between the consumption ratio across countries and the real exchange rate.

## 6. Concluding Remarks

In this paper, we document two empirical facts. First, that TFP processes of the U.S. and the "rest of the world" are cointegrated with cointegrating vector $(1, -1)$ and second, that the relative volatility of the real exchange rate with respect to output has increased in the United States, the United Kingdom, Canada, and Australia during the last 20 years.

Then, we have shown that introducing cointegrated TFP processes in an otherwise standard IRBC model increases the ability of the model to explain real exchange rate volatility, without affecting the fit to other second moments of the data. We have also documented that if we allow the speed of convergence to the cointegrating vector to change as it does in the data, the model can also explain the observed increase in the relative volatility of the real exchange rate.

For future research, it would be interesting to introduce cointegrated TFP processes in medium-scale open economy macroeconomic models, that typically include more frictions and try to match a larger set of domestic and international variables.[19] Also, it would be desirable to investigate if investment-specific technology shocks are cointegrated across countries, and their role in international business cycles models.

[19] See Adolfson et al. (2007).

## References


[1] Adolfson, M., S. Laseen, J. Lindé and M. Villani, 2007, "Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through," forthcoming, *Journal of International Economics*

[2] Aguiar, M. and G. Gopinath, 2007, "Emerging Market Business Cycles: The Cycle is the Trend" *Journal of Political Economy,* Vol. 115, No. 1, 69 -102.

[3] Alvarez F. and U. Jermann, 2005, "Using Asset Prices to Measure the Persistence of the Marginal Utility of Wealth" , *Econometrica,* 1977-2016.

[4] Backus, D., P. Kehoe and F. Kydland, 1992, "International Business Cycles," *Journal of Political Economy,* Vol. 100, No. 41, 745 -775.

[5] Benigno, G. and C. Thoenissen, 2006, "Consumption and Real Exchange Rates with Incomplete Markets and Non-Traded Goods," forthcoming, *Journal of International Money and Finance.*

[6] Baxter, M. and M. Crucini, 1995, "Business Cycles and the Asset Structure of Foreign Trade," *International Economic Review*, Vol. 36, No. 4, 821-854.

[7] Betts, C. and T. Kehoe, 2006, "Real Exchange Rate Movements and the Relative Price of Nontradable Goods," *Journal of Monetary Economics* (forthcoming).

[8] Burnstein, A., M. Eichenbaum, and S. Rebelo, 2006, "How Important are Non Tradable Goods Prices as a Source of Cyclical Fluctuations in the Real Exchange Rates?," *Japan and the World Economy.*

[9] Chari, V.V. , P. Kehoe and E. McGrattan, 2002, "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?," *Review of Economic Studies,* Vol. 69, 533-563.

[10] Corsetti, G., L. Dedola and S. Leduc, 2007, "International Risk Sharing and the Transmission of Productivity Shocks," forthcoming, *Review of Economic Studies.*

[11] Corsetti, G., L. Dedola and S. Leduc, 2008, "DSGE Models of High Exchange Rate Volatility and Low Passthrough," forthcoming, *Journal of Monetary Economics.*

[12] Dickey, A. and W. Fuller, 1979, "Distribution of the Estimators for Autoregressive Time Series with a Unit Root," *Journal of the American Statistical Association*, 74, 427–431.

[13] Dotsey, M. and M. Duarte, 2007, "Non Traded Goods, Market Segmentation and Exchange Rates," mimeo, University of Toronto.

[14] Elliott, G., T. Rothenberg and J. Stock, 1996, "Efficient Tests for an Autoregressive Unit Root," *Econometrica,* Vol. 64, No. 4, 813-836.

[15] Elliott, G. and E. Pesavento, 2006 "On the Failure of PPP for Bilateral Exchange Rates After 1973", *Journal of Money, Credit, and Banking*, Vol. 38, No. 6 ,1405-1430.

[16] Engel, C., 1993, "Real Exchange Rates and Relative Prices: An Empirical Investigation," *Journal of Monetary Economics,* 35-50.

[17] Engel, C., 1999, "Accounting for U.S. Real Exchange Rate Changes," *Journal of Political Economy,* Vol. 107, No. 3, 507-538.

[18] Engel C. and K. West, 2005, "Exchange Rates and Fundamentals", *Journal of Political Economy,* Vol. 107, No. 3, 485-517.

[19] Engle R. and C. Granger, 1987, "Co-Integration and Error Correction: Representation, Estimation, and Testing," *Econometrica,* Vol. 55, No. 2, 251-276.

[20] Greenwood, J., Z. Hercowitz, and Per Krusell, Per, 1997, "Long-Run Implications of Investment-Specific Technological Change," *American Economic Review*, vol. 87(3), pages 342-62, June.

[21] Heathcote, J., and F. Perri, 2002, "Financial Autarky and International Business Cycles," *Journal of Monetary Economics,* Vol. 49/3, 601-627.

[22] Heathcote, J., and F. Perri, 2008, "The International Diversification Puzzle is not as Bad as You Think," manuscript, University of Minnesota.

[23] Johansen, S., 1991, "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models," *Econometrica* 59, 1551-1580.

[24] Justiniano, A., and B. Preston, 2006, "Can Structural Small Open Economy Models Account for the Influence of Foreign Disturbances?," Manuscript, Columbia University and Board of Governors of the Federal Reserve System.

[25] Kehoe, P. and F. Perri, 2002, "International Business Cycles with Endogenous Incomplete Markets," *Econometrica* 70:3, 907-928.

[26] Kim, C., and C. Nelson, 1999, "Has the U.S. Economy Become More Stable? A Bayesian Approach Based on a Markov Switching Model of the Business Cycle," *The Review of Economic and Statistics,* 81 (4) 608-616.

[27] King, R., C., Plosser and S., Rebelo, 1988, "Production, Growth and the Business Cycle," *Journal of Monetary Economics,* 21, 195-232.

[28] King, R., C. Plosser, J. Stock and M. Watson, 1991, "Stochastic Trends and Economic Fluctuations" *The American Economic Review,* Vol. 81, No. 4, 819-840.

[29] Lastrapes, W., 1992, "Source of Fluctuations in Real and Nominal Exchange Rates" *Review of Economic and Statistics,* vol. 74 (3), 530-539.

[30] Lubik T. and F. Schorfheide, 2006, "A Bayesian Look at New Open Economy Macroeconomics," *2005 NBER Macroeconomics Annual*, MIT Press.

[31] McConnell, M. abd G. Perez-Quiros, 2000, "Output Fluctuations in the United States: What Has Changed since the Early 1980's?," *American Economic Review*, vol. 90(5), 1464-1476.

[32] Nason J. and J. Rogers, 2008, "Exchange Rates and Fundamentals: A Generalization" *Federal Reserve Bank of Atlanta working paper 2008-16.*

[33] Ng, S. and P. Perron, 1995, "Unit Root Tests in ARMA Models with Data-Dependent Methods for the Selection of the Truncation Lag," *Journal of the American Statistical Association,* Vol. 90, 268-281.

[34] Ng, S. and P. Perron, 2001, "LAG Length Selection and the Construction of Unit Root Tests with Good Size and Power," *Econometrica* 69 (6), 1519–1554.

[35] Rabanal, P. and V. Tuesta, 2007, "Non Traded Goods and the Real Exchange Rates,"La Caixa Working Paper 03/2007.

[36] Stock, J., and M. Watson, 2002, "Has the Business Cycle Changed and Why?," *NBER Macroeconomics Annual, MIT Press.*

[37] Stock, J., and M. Watson, 2005, "Understanding Changes in International Business Cycle Dynamics," *Journal of the European Economic Association,* vol 3, issue 5, 968-1006.

[38] Robert G. King, Charles I. Plosser, James H. Stock and Mark W. Watson, 1991, "Stochastic Trends and Economic Fluctuations" *The American Economic Review*, Vol. 81, No. 4, 819-840.

[39] Said, S. and A. Dickey, 1984, "Testing for Unit Roots in Autoregressive-Moving Average Models of Unknown Order," *Biometrika,* vol 71, issue 3, 599-607.

[40] Schmitt-Grohé, St. and M. Uribe, 2003, "Closing Small Open Economy Models," *Journal of International Economics,* 61, 163-185.

Table 1: Unit Root tests for TFP

| log U.S. TFP | | | | |
|---|---|---|---|---|
| | Level | | First Difference | |
| Method | t-statistic | p-value | t-statistic | p-value |
| ADF | -3.29 | 0.07 | -3.03 | 0.13 |
| DF-GLS | -1.30 | 0.2 | -1.16 | 0.25 |
| $P_T$-GLS | 31.98 | 6.81* | 49.64 | 6.82* |
| $MZ_\alpha$ | -3.49 | -14.2** | -2.00 | -14.2** |
| $MZ_t$ | -1.23 | -2.6** | -0.97 | -2.6** |
| MSB | 0.35 | 0.19** | 0.48 | 0.18** |

| log "Rest Of The World" TFP | | | | |
|---|---|---|---|---|
| | Level | | First Difference | |
| | t-statistic | p-value | t-statistic | p-value |
| ADF | -7.11 | 0.00 | -3.89 | 0.02 |
| DF-GLS | -2.76 | 0.00 | -6.95 | 0.07 |
| $P_T$-GLS | 1.75 | 0.00 | 4.66 | 6.8* |
| $MZ_\alpha$ | -51.5 | 0.00 | -14.77 | -14.2** |
| $MZ_t$ | -5.06 | 0.00 | -2.48 | -2.62** |
| MSB | 0.09 | 0.00 | 0.16 | 0.19** |

Notes: ADF stands for Augmented Dickey-Fuller Test. DF-GLS stands for Elliott-Rothenberg-Stock detrended residuals test statistic. $P_T$-GLS stands for Elliott-Rothenberg-Stock Point-Optimal test statistic. $MZ_\alpha$, $MZ_t$, and MSB stands for the class of modified tests analyzed in Ng-Perron (2001). p-values for the ADF test are one-sided p-values as in MacKinnon (1996). p-values for the DF-GLS test are as in Elliott-Rothenberg-Stock (1996, Table 1). * This value do not represent the p-values but the critical values of test at the 10 percent as reported in Elliott-Rothenberg-Stock (1996) Table 1. ** These values do not represent the p-values but the asymptotic critical values of test at the 10 percent as reported in Ng-Perron (2001) Table 1.

Table 2: Cointegration Statistics I

| Real | Imaginary | Modulus |
|---|---|---|
| 1.01 | 0 | 1.01 |
| 0.97 | 0 | 0.97 |
| 0.77 | 0.28 | 0.82 |
| 0.77 | 0.28 | 0.82 |
| Log likelihood | | 752.51 |

Table 3: Cointegration Statistics II: Johansen's test

| Number of Vectors | Eigenvalue | Trace | p-value | Max-Eigenvalue | p-value |
|---|---|---|---|---|---|
| 0 | 0.15 | 19.06 | 0.01 | 18.21 | 0.01 |
| 1 | 0.00 | 0.855 | 0.35 | 0.00 | 0.35 |

Note:p-values as reported in MacKinnon-Haug-Michelis (1999)

Table 4: Likelihood ratio tests

| Restriction | Likelihood value | Degrees of freedom | p-value |
|---|---|---|---|
| None | 744.18 | - | - |
| $\gamma = 1$ | 743.33 | 1 | 0.19 |
| $\kappa = -\kappa^*$ | 741.71 | 2 | 0.09 |
| $c = c^*$ | 740.43 | 3 | 0.06 |
| Symmetry across VAR coefficients | 736.51 | 7 | 0.032 |

Table 5: VECM model

| | 1980 − 2007 |
|---|---|
| $c$ | $0.0071^{*}$ (5.83) |
| $\kappa$ | $-0.0045^{*}$ ($-2.65$) |
| $\rho_{11}^{1}$ | $0.2041^{*}$ (2.97) |
| $\rho_{11}^{2}$ | 0.1026 (1.54) |
| $\rho_{12}^{1}$ | 0.1035 (1.55) |
| $\rho_{12}^{2}$ | $-0.1497^{*}$ ($-2.40$) |

t-statistics in parenthesis.

* denotes significance at the 5 percent level.

Table 6: Calibration

| | | |
|---|---|---|
| Preferences | Discount factor | $\beta = 0.99$ |
| | Consumption share | $\tau = 0.34$ |
| | Risk aversion | $\sigma = 2$ |
| | Cost of bond holdings | $\phi = 0.01$ |
| Technology | Capital share | $\alpha = 0.36$ |
| | Depreciation rate | $\delta = 0.025$ |
| | Home bias | $\omega = 0.9$ |
| | Elasticity of substitution between intermediate goods | $\theta = [0.85, 0.62]$ |
| | Adjustment cost of investment | varies |

Table 7a: Results

| Full Sample | $SD(Y)$ | $SD(C)^+$ | $SD(X)^+$ | $SD(N)^+$ | $SD(RER)^+$ | $\rho(RER)$ |
|---|---|---|---|---|---|---|
| Data | **1.25** | **0.80** | **3.40** | **0.91** | **4.28** | **0.84** |
| Cointegrated TFP, $\theta = 0.85$ | 0.81 | 0.63 | 2.32 | 0.28 | 1.75 | 0.72 |
| Cointegrated TFP, $\theta = 0.62$ | 0.70 | 0.62 | 2.31 | 0.28 | 4.26 | 0.70 |
| Stationary TFP, $\theta = 0.85$ | 1.19 | 0.52 | 2.53 | 0.32 | 0.75 | 0.77 |
| Stationary TFP, $\theta = 0.62$ | 1.12 | 0.54 | 2.51 | 0.31 | 1.41 | 0.75 |
| **1980-1993** | | | | | | |
| Data | **1.57** | **0.80** | **3.08** | **0.89** | **3.97** | **0.85** |
| Cointegrated TFP, $\theta = 0.85$ | 1.12 | 0.63 | 2.17 | 0.25 | 1.33 | 0.72 |
| Cointegrated TFP, $\theta = 0.62$ | 0.95 | 0.65 | 2.15 | 0.25 | 3.17 | 0.71 |
| **1994-2007** | | | | | | |
| Data | **0.83** | **0.76** | **4.20** | **0.96** | **5.17** | **0.81** |
| Cointegrated TFP, $\theta = 0.85$ | 0.64 | 0.55 | 2.74 | 0.38 | 2.04 | 0.71 |
| Cointegrated TFP, $\theta = 0.62$ | 0.62 | 0.43 | 3.01 | 0.42 | 5.06 | 0.69 |

$^+$ denotes relative to output.

Table 7b: Results

| Full Sample | $CORR(Y,N)$ | $CORR(Y,C)$ | $CORR(Y,X)$ | $CORR(RER,C/C^*)$ |
|---|---|---|---|---|
| Data | **0.79** | **0.81** | **0.91** | **-0.04** |
| Cointegrated TFP, $\theta = 0.85$ | 0.94 | 0.95 | 0.97 | 0.95 |
| Cointegrated TFP, $\theta = 0.62$ | 0.92 | 0.93 | 0.95 | 0.97 |
| Stationary TFP, $\theta = 0.85$ | 0.97 | 0.93 | 0.97 | 0.99 |
| Stationary TFP, $\theta = 0.62$ | 0.97 | 0.93 | 0.97 | 0.99 |
| | | | | |
| 1980-1993 | | | | |
| Data | **0.82** | **0.82** | **0.93** | **-0.10** |
| Cointegrated TFP, $\theta = 0.85$ | 0.93 | 0.96 | 0.97 | 0.95 |
| Cointegrated TFP, $\theta = 0.62$ | 0.91 | 0.96 | 0.96 | 0.94 |
| | | | | |
| 1994-2007 | | | | |
| Data | **0.71** | **0.76** | **0.90** | **0.12** |
| Cointegrated TFP, $\theta = 0.85$ | 0.89 | 0.82 | 0.94 | 0.98 |
| Cointegrated TFP, $\theta = 0.62$ | 0.94 | 0.78 | 0.97 | 0.95 |

Table 8: Changing $\rho_a$ and $\kappa$

| | $SD(RER)$ | $SD(Y)$ | $SD(RER)^+$ |
|---|---|---|---|
| $\rho_a$ | | | |
| 0.9 | 1.43 | 1.33 | 1.07 |
| 0.95 | 1.96 | 1.2 | 1.64 |
| 0.975 | 2.47 | 1.06 | 2.33 |
| $\kappa$ | | | |
| 0.005 | 1.98 | 0.64 | 3.1 |
| 0.05 | 1.02 | 0.82 | 1.25 |
| 0.25 | 0.71 | 0.86 | 0.82 |

$^+$ denotes relative to output.

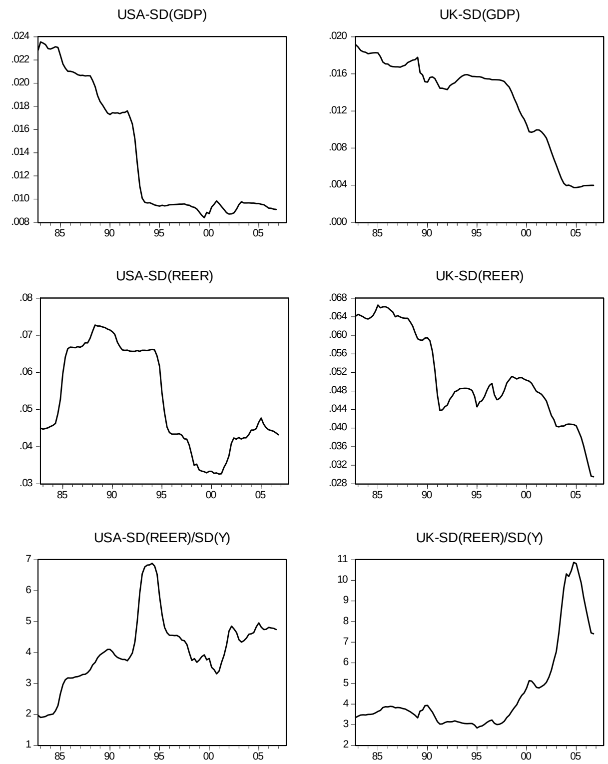


Figure 1: Standard Deviation of HP-Filtered Data. USA and UK.

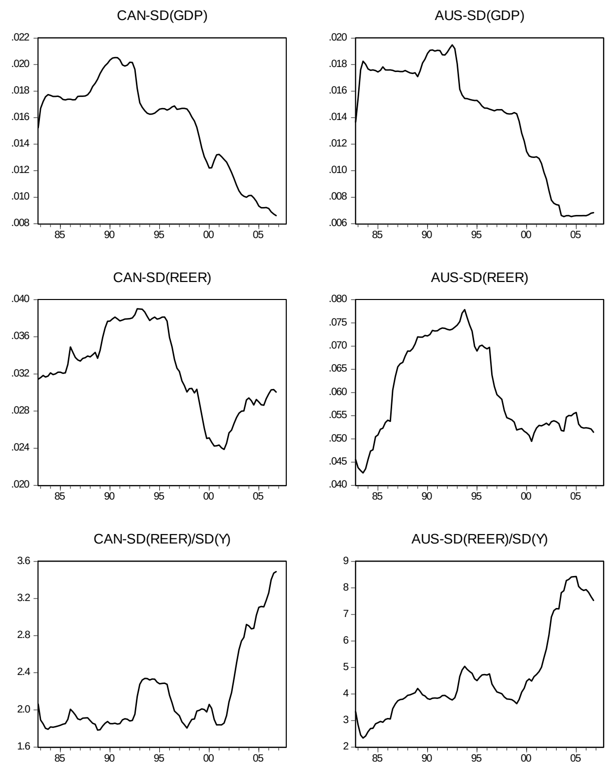


Figure 2: Standard Deviation of HP-Filtered Data. Canada and Australia.

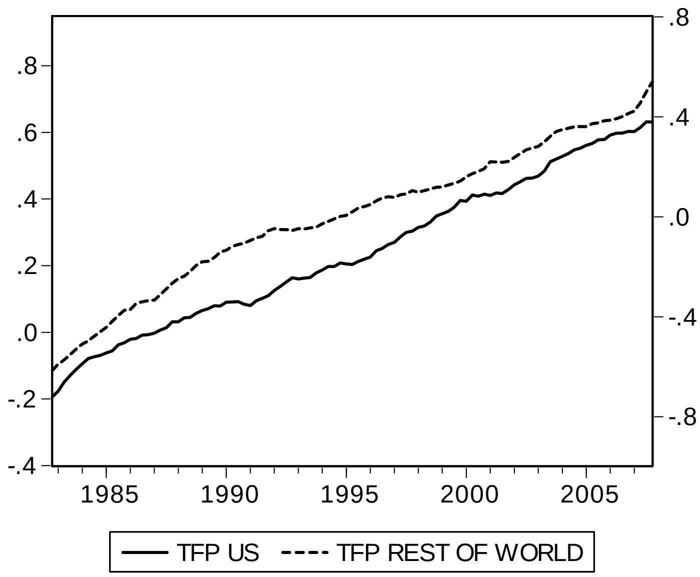


Figure 3: TFP processes for the US and the “rest of the world”.

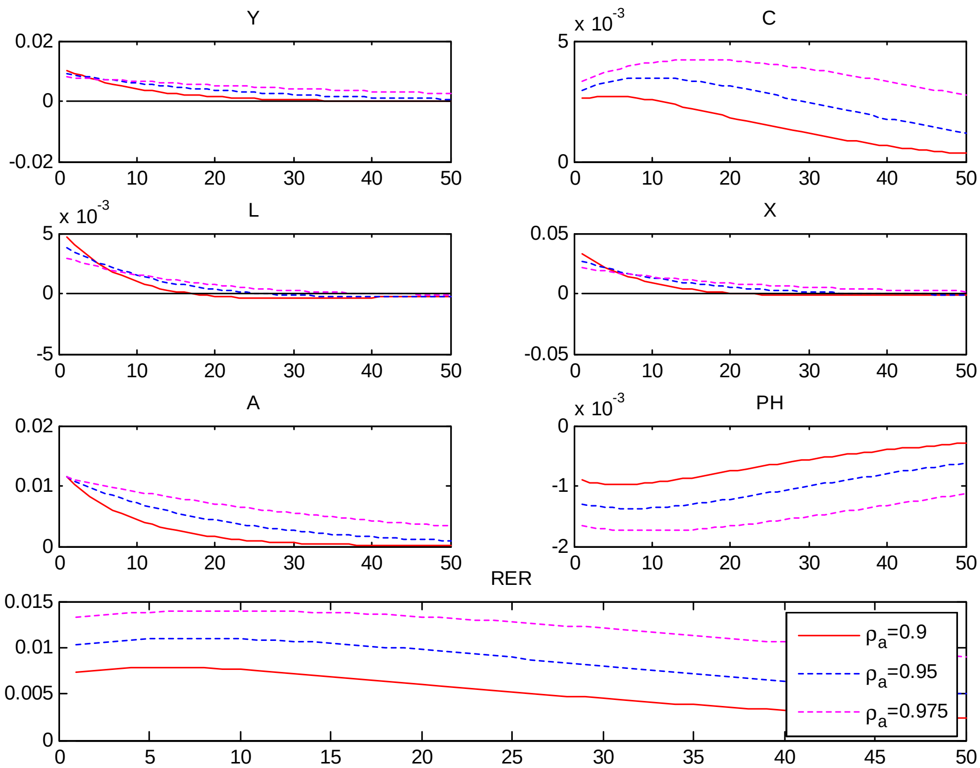


Figure 4: Impulse Response to a Home-Country TFP shock. Model with stationary TFP shocks.

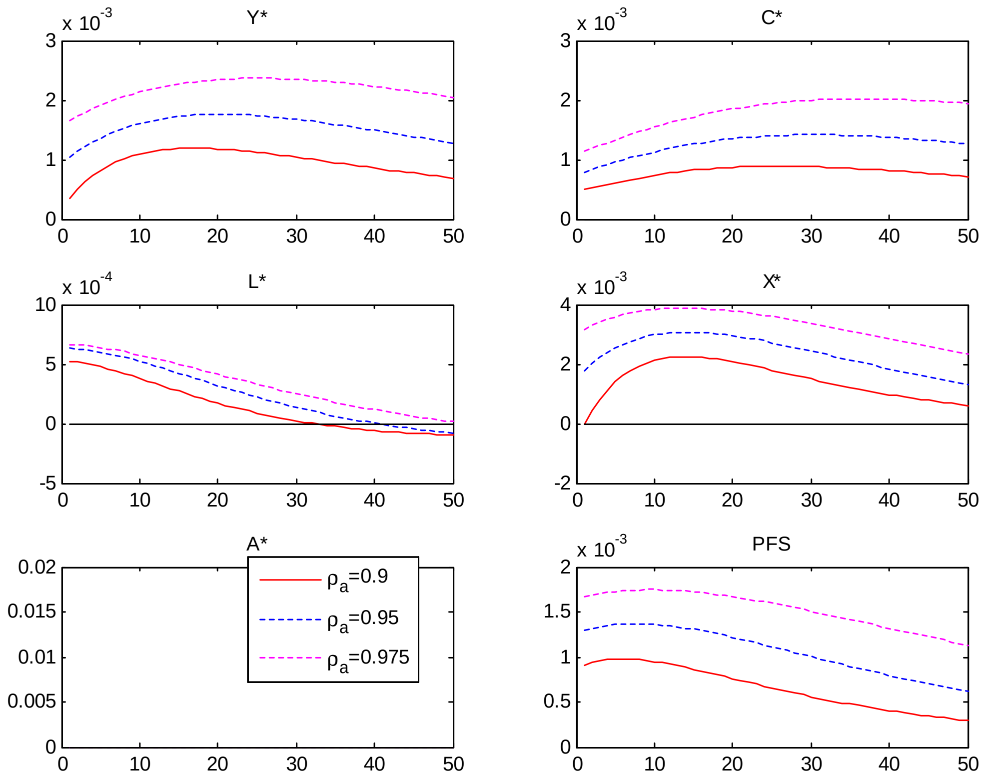


Figure 5: Impulse Response to a Home-Country TFP shock. Model with stationary TFP shocks.

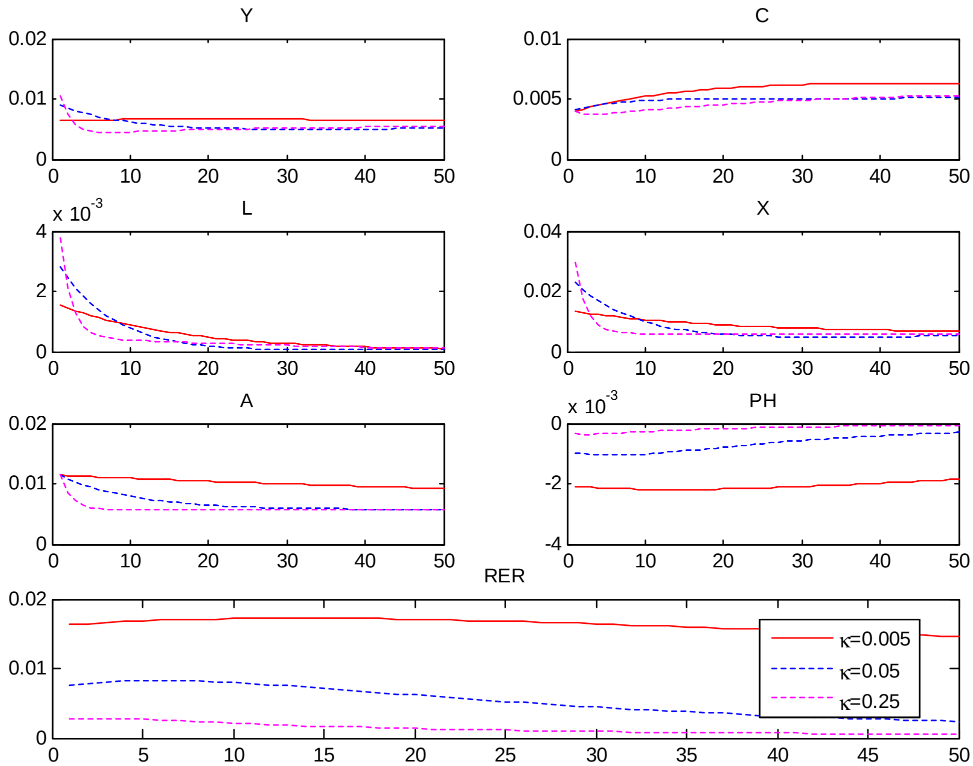


Figure 6: Impulse Response to a Home-Country TFP shock. Model with cointegrated TFP shocks.

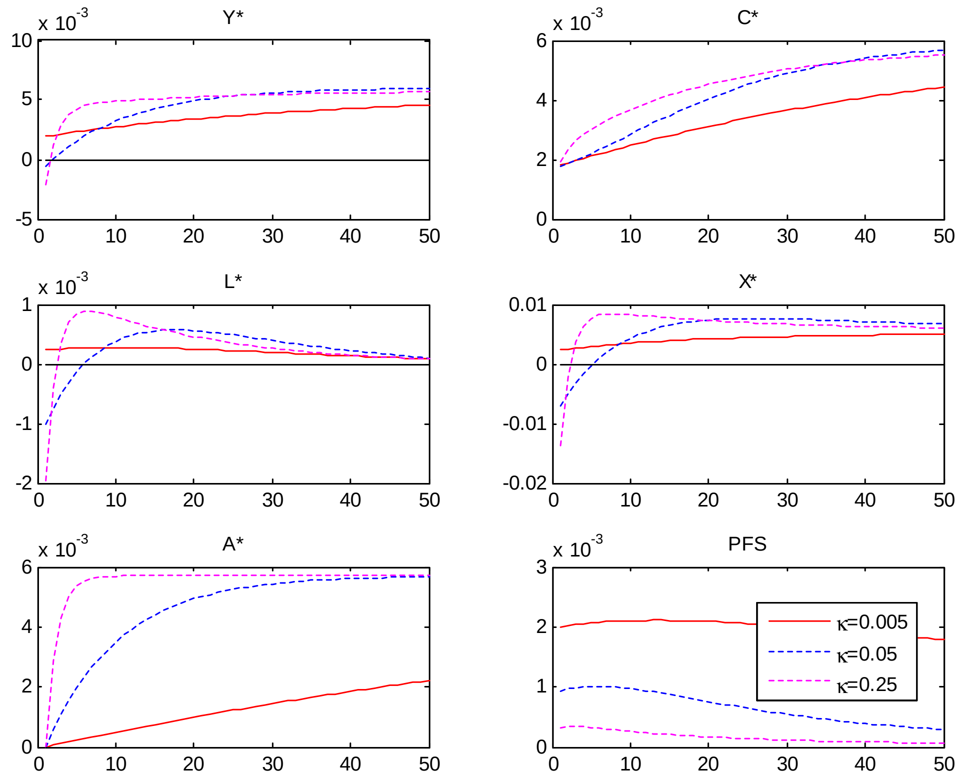


Figure 7: Impulse Response to a Home-Country TFP shock. Model with cointegrated TFP shocks.

# A. Appendix

## A.1. Normalized Equilibrium Conditions

Equations (10) to (37) characterize equilibrium in this model. Since both $\log A(s^t)$ and $\log A^*(s^t)$ are integrated, we now normalize the above described system in order to get a stationary system more amenable to study. Additional restrictions the VECM defining the law of motion of the technological processes are required if the model is to exhibit balanced growth. Those restrictions are described in next subsection.

Let us first define the following normalized variables $\widehat{Y}_H(s^t) = \frac{Y_H(s^t)}{A(s^{t-1})}$, $\widehat{Y}^*_H(s^t) = \frac{Y^*_H(s^t)}{A(s^{t-1})}$, $\widehat{Y}_F(s^t) = \frac{Y_F(s^t)}{A^*(s^{t-1})}$, $\widehat{Y}^*_F(s^t) = \frac{Y^*_F(s^t)}{A^*(s^{t-1})}$, and $\widehat{K}(s^{t-1}) = \frac{K(s^{t-1})}{A(s^{t-1})}$, $\widehat{K}^*(s^{t-1}) = \frac{K^*(s^{t-1})}{A^*(s^{t-1})}$, $\widehat{Y}(s^t) = \frac{Y(s^t)}{A(s^{t-1})}$, $\widehat{Y}^*(s^t) = \frac{Y^*(s^t)}{A^*(s^{t-1})}$, $\widehat{C}(s^t) = \frac{C(s^t)}{A(s^{t-1})}$, $\widehat{C}^*(s^t) = \frac{C^*(s^t)}{A^*(s^{t-1})}$, $\widehat{X}(s^t) = \frac{X(s^t)}{A(s^{t-1})}$, $\widehat{X}^*(s^t) = \frac{X^*(s^t)}{A^*(s^{t-1})}$, $\widehat{W}(s^t) = \frac{W(s^t)}{A(s^{t-1})}$, $\widehat{W}^*(s^t) = \frac{W^*(s^t)}{A^*(s^{t-1})}$, $\widehat{D}(s^t) = \frac{D(s^t)}{A(s^{t-1})}$, $\widehat{D}^*(s^t) = \frac{D^*(s^t)}{A^*(s^{t-1})}$, $\widehat{\lambda}(s^t) = \lambda(s^t) A(s^{t-1})^{1-\gamma(1-\sigma)}$, $\widehat{\lambda}^*(s^t) = \lambda^*(s^t) A^*(s^{t-1})^{1-\gamma(1-\sigma)}$, $\widehat{\mu}(s^t) = \mu(s^t) A(s^{t-1})^{1-\gamma(1-\sigma)}$, and $\widehat{\mu}^*(s^t) = \mu^*(s^t) A^*(s^{t-1})^{1-\gamma(1-\sigma)}$. Then, the stationary first order conditions are

$$U_C(s^t) = \widehat{\lambda}(s^t), \tag{41}$$

$$\frac{U_L(s^t)}{U_C(s^t)} = \widehat{W}(s^t), \tag{42}$$

$$\widehat{\lambda}(s^t) = \widehat{\mu}(s^t)\left[\phi\left(\frac{\widehat{X}(s^t)}{\widehat{K}(s^{t-1})}\right) + \phi'\left(\frac{\widehat{X}(s^t)}{\widehat{K}(s^{t-1})}\right)\frac{\widehat{X}(s^t)}{\widehat{K}(s^{t-1})}\right], \tag{43}$$

$$\left(\frac{A(s^t)}{A(s^{t-1})}\right)^{1-\gamma(1-\sigma)}\widehat{\mu}(s^t) = \beta\sum_{s^{t+1}}\pi(s^{t+1}/s^t)\left\{\begin{array}{c} R(s^{t+1})\widehat{\lambda}(s^{t+1}) \\ +\widehat{\mu}(s^{t+1})\left[(1-\delta) - \phi'\left(\frac{\widehat{X}(s^{t+1})}{\widehat{K}(s^t)}\right)\left(\frac{\widehat{X}(s^{t+1})}{\widehat{K}(s^t)}\right)^2\right]\end{array}\right\}, \tag{44}$$

$$\widehat{K}(s^t) = (1-\delta)\widehat{K}(s^{t-1})\frac{A(s^{t-1})}{A(s^t)} + \phi\left(\frac{\widehat{X}(s^t)}{\widehat{K}(s^{t-1})}\right)\widehat{X}(s^t)\frac{A(s^{t-1})}{A(s^t)}, \tag{45}$$

$$U_{C^*}(s^t) = \widehat{\lambda}^*(s^t), \tag{46}$$

$$\frac{U_{L^*}\left(s^t\right)}{U_{C^*}\left(s^t\right)} = \widehat{W}^*\left(s^t\right), \tag{47}$$

$$\widehat{\lambda}^*\left(s^t\right) = \widehat{\mu}^*\left(s^t\right)\left[\phi\left(\frac{\widehat{X}^*\left(s^t\right)}{\widehat{K}^*\left(s^{t-1}\right)}\right) + \phi'\left(\frac{\widehat{X}^*\left(s^t\right)}{\widehat{K}^*\left(s^{t-1}\right)}\right)\frac{\widehat{X}^*\left(s^t\right)}{\widehat{K}^*\left(s^{t-1}\right)}\right], \tag{48}$$

$$\left(\frac{A^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}\right)^{1-\gamma(1-\sigma)}\widehat{\mu}^*\left(s^t\right) = \beta\sum_{s^{t+1}}\pi\left(s^{t+1}/s^t\right)\left\{\begin{array}{c} R^*\left(s^{t+1}\right)\widehat{\lambda}^*\left(s^{t+1}\right) \\ +\widehat{\mu}^*\left(s^{t+1}\right)\left[(1-\delta)-\phi'\left(\frac{\widehat{X}^*\left(s^{t+1}\right)}{\widehat{K}^*\left(s^t\right)}\right)\left(\frac{\widehat{X}^*\left(s^{t+1}\right)}{\widehat{K}^*\left(s^t\right)}\right)^2\right]\end{array}\right\}, \tag{49}$$

$$\widehat{K}^*\left(s^t\right) = (1-\delta)\widehat{K}^*\left(s^{t-1}\right)\frac{A^*\left(s^{t-1}\right)}{A^*\left(s^t\right)} + \phi\left(\frac{\widehat{X}^*\left(s^t\right)}{\widehat{K}^*\left(s^{t-1}\right)}\right)\widehat{X}^*\left(s^t\right)\frac{A^*\left(s^{t-1}\right)}{A^*\left(s^t\right)}, \tag{50}$$

$$\overline{Q}\left(s^t\right) = \beta\sum_{s^{t+1}}\pi\left(s^{t+1}/s^t\right)\frac{\widehat{\lambda}\left(s^{t+1}\right)}{\widehat{\lambda}\left(s^t\right)}\left(\frac{A\left(s^{t-1}\right)}{A\left(s^t\right)}\right)^{1-\gamma(1-\sigma)}\frac{\widetilde{P}_H\left(s^{t+1}\right)}{\widetilde{P}_H\left(s^t\right)} - \Phi'\left(D\left(s^t\right)\right), \tag{51}$$

$$\sum_{s^{t+1}}\pi\left(s^{t+1}/s^t\right)\left[\begin{array}{c}\frac{\widehat{\lambda}^*\left(s^{t+1}\right)}{\widehat{\lambda}^*\left(s^t\right)}\frac{\widetilde{P}_H\left(s^{t+1}\right)}{\widetilde{P}_H\left(s^t\right)}\frac{RER\left(s^t\right)}{RER\left(s^{t+1}\right)}\left(\frac{A\left(s^t\right)}{A\left(s^{t-1}\right)}\frac{A^*\left(s^{t-1}\right)}{A^*\left(s^t\right)}\right)^{1-\gamma(1-\sigma)} \\ -\frac{\widehat{\lambda}\left(s^{t+1}\right)}{\widehat{\lambda}\left(s^t\right)}\frac{\widetilde{P}_H\left(s^{t+1}\right)}{\widetilde{P}_H\left(s^t\right)}\end{array}\right] = -\frac{\Phi'\left(D\left(s^t\right)\right)}{\beta}, \tag{52}$$

$$\widehat{W}\left(s^t\right) = (1-\alpha)\widetilde{P}_H\left(s^t\right)\widehat{K}\left(s^{t-1}\right)^{\alpha}L\left(s^t\right)^{-\alpha}\left(\frac{A\left(s^t\right)}{A\left(s^{t-1}\right)}\right)^{1-\alpha}, \tag{53}$$

$$R\left(s^t\right) = \alpha\widetilde{P}_H\left(s^t\right)\widehat{K}\left(s^{t-1}\right)^{\alpha-1}L\left(s^t\right)^{1-\alpha}\left(\frac{A\left(s^t\right)}{A\left(s^{t-1}\right)}\right)^{1-\alpha}, \tag{54}$$

$$\widehat{W}^*\left(s^t\right) = (1-\alpha)\widetilde{P}_F^*\left(s^t\right)\widehat{K}^*\left(s^{t-1}\right)^{\alpha}L^*\left(s^t\right)^{-\alpha}\left(\frac{A^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}\right)^{1-\alpha}, \tag{55}$$

$$R^*\left(s^t\right) = \alpha\widetilde{P}_F^*\left(s^t\right)\widehat{K}^*\left(s^{t-1}\right)^{\alpha-1}L^*\left(s^t\right)^{1-\alpha}\left(\frac{A^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}\right)^{1-\alpha}, \tag{56}$$

$$\widehat{Y}_H\left(s^t\right) = \omega\widetilde{P}_H\left(s^t\right)^{-\theta}\widehat{Y}\left(s^t\right), \tag{57}$$

$$\widehat{Y}_F\left(s^t\right) = (1-\omega)\left(\widetilde{P}_F^*\left(s^t\right)RER\left(s^t\right)\right)^{-\theta}\widehat{Y}\left(s^t\right)\frac{A\left(s^{t-1}\right)}{A^*\left(s^{t-1}\right)}, \tag{58}$$

$$\widehat{Y}_H^*\left(s^t\right) = (1-\omega)\left(\frac{\widetilde{P}_H\left(s^t\right)}{RER\left(s^t\right)}\right)^{-\theta}\widehat{Y}^*\left(s^t\right)\frac{A^*\left(s^{t-1}\right)}{A\left(s^{t-1}\right)}, \tag{59}$$

$$\widehat{Y}_F^*\left(s^t\right) = \omega\widetilde{P}_F^*\left(s^t\right)^{-\theta}\widehat{Y}^*\left(s^t\right). \tag{60}$$

$$\widehat{C}\left(s^t\right) + \widehat{X}\left(s^t\right) = \widehat{Y}\left(s^t\right), \tag{61}$$

$$\widehat{C}^*\left(s^t\right) + \widehat{X}^*\left(s^t\right) = \widehat{Y}^*\left(s^t\right), \tag{62}$$

$$\widehat{Y}\left(s^t\right) = \left[\omega^{\frac{1}{\theta}}\widehat{Y}_H\left(s^t\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}}\widehat{Y}_F\left(s^t\right)^{\frac{\theta-1}{\theta}}\left(\frac{A^*\left(s^{t-1}\right)}{A\left(s^{t-1}\right)}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{63}$$

$$\widehat{Y}^*\left(s^t\right) = \left[\omega_F^{\frac{1}{\theta}}\widehat{Y}_F^*\left(s^t\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}}\widehat{Y}_H^*\left(s^t\right)^{\frac{\theta-1}{\theta}}\left(\frac{A\left(s^{t-1}\right)}{A^*\left(s^{t-1}\right)}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{64}$$

$$\widehat{Y}_H\left(s^t\right) + \widehat{Y}_H^*\left(s^t\right) = \widehat{K}\left(s^{t-1}\right)^{\alpha}\left(\widehat{L}\left(s^t\right)\frac{A\left(s^t\right)}{A\left(s^{t-1}\right)}\right)^{1-\alpha}, \tag{65}$$

$$\widehat{Y}_F\left(s^t\right) + \widehat{Y}_F^*\left(s^t\right) = \widehat{K}^*\left(s^{t-1}\right)^{\alpha}\left(\widehat{L}^*\left(s^t\right)\frac{A^*\left(s^t\right)}{A^*\left(s^{t-1}\right)}\right)^{1-\alpha}, \tag{66}$$

$$\widehat{D}\left(s^t\right) + \widehat{D}^*\left(s^t\right)\frac{A^*\left(s^{t-1}\right)}{A\left(s^{t-1}\right)} = 0, \tag{67}$$

and

$$\begin{aligned}\widetilde{P}_H\left(s^t\right)\overline{Q}\left(s^t\right)\widehat{D}\left(s^t\right) &= \widetilde{P}_H\left(s^t\right)\widehat{Y}_H^*\left(s^t\right) - \widetilde{P}_F^*\left(s^t\right)RER\left(s^t\right)\widehat{Y}_F\left(s^t\right)\frac{A^*\left(s^{t-1}\right)}{A\left(s^{t-1}\right)} \\ &+\widetilde{P}_H\left(s^t\right)\widehat{D}\left(s^{t-1}\right)\frac{A\left(s^{t-2}\right)}{A\left(s^{t-1}\right)} - \widetilde{P}_H\left(s^t\right)\frac{\Phi\left(D\left(s^t\right)\right)}{A\left(s^{t-1}\right)}.\end{aligned} \tag{68}$$

Finally, the productivity shocks do not need to be normalized. Also, note that our functional form $\Phi\left[D\left(s^t\right)\right] = \frac{\phi}{2}A(s^{t-1})\left[\frac{D\left(s^t\right)}{A\left(s^{t-1}\right)}\right]^2$ implies that $\Phi\left[\widehat{D}\left(s^t\right)\right]/A\left(s^{t-1}\right)$ and $\Phi'\left[\widehat{D}\left(s^t\right)\right]$ are stationary. This is important to make normalized equations (51) to (52) stationary.