The Capital Puzzle

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Motivation

- **Can a central bank tighten monetary policy and the real interest rate fall under monetary dominance?**

- Textbook New Keynesian (NK) models are about general equilibrium, but a model still needs **a story to tell**

- Common view of the transmission of monetary policy is through a **real interest rate channel**:

$$\uparrow \varepsilon_t^m \quad \Rightarrow \quad \uparrow i_t \quad \Rightarrow \quad \underbrace{\uparrow r_t}_{\text{if prices are sticky}} \quad \Rightarrow \quad \downarrow c_t \quad \Rightarrow \quad \downarrow y_t \quad \Rightarrow \quad \downarrow \pi_t$$

(1) Monetary policy rule

(2) Fisher equation

(3) IS curve

(4) Phillips curve

- This mechanism assumes monetary dominance
  - Some sort of Taylor Principle, e.g., $\nu > 1$

$$\hat{i}_t = \nu \pi_t + \xi_t^m$$

endogenous (systematic) | exogenous

- It is a textbook result that i_t may fall, but the rest remains valid:
  - $\uparrow \varepsilon_t^m \Rightarrow$
    - if ν is high enough, the endogenous component may numerically exceed the exogenous one with the opposite sign, but still $\uparrow r_t$

Both results are consistent with the existence of the real interest rate channel

Motivation

- **Can a central bank tighten monetary policy, and the real interest rate fall under monetary dominance?**

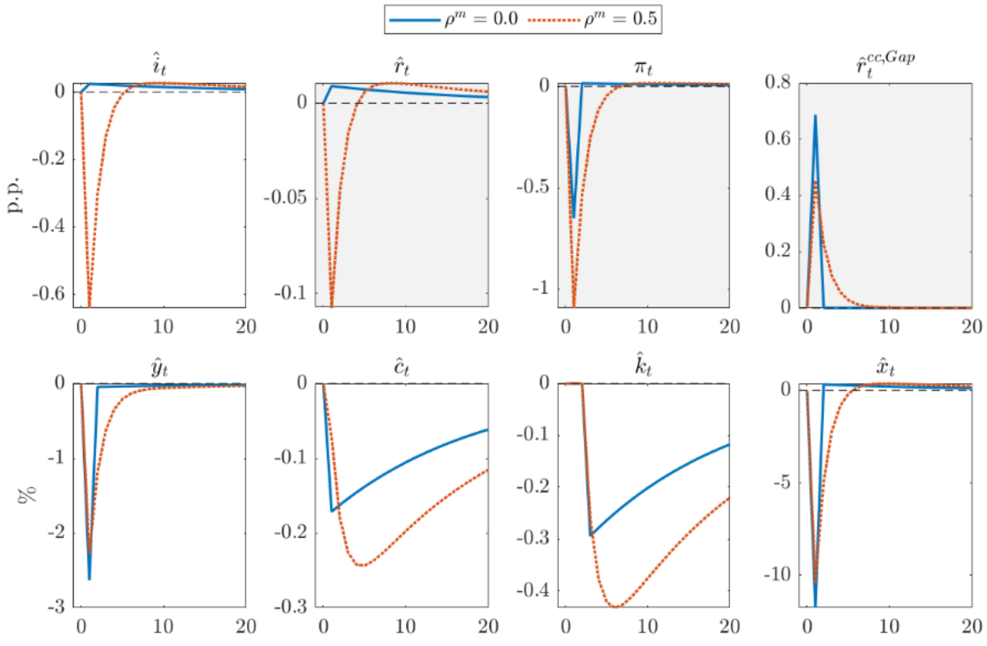
- **Rupert and Šustek (2019)**
  - Added **endogenous capital** to the NK model
  - Allowed monetary shock to be **persistent** ($\rho^m > 0$)
  - **Found:** real interest rates may move in **any direction** after a contractionary monetary shock (I call the **capital puzzle**)

- In the canonical model, that would not happen because it was like a model with **infinite capital adjustment costs**

- They argue that the real interest rate channel is observational, **not structural**
  - subject to policy choices (ρ^m), thus, not immune to the Lucas critique

$$\uparrow \varepsilon_t^m \;\Rightarrow\; \uparrow i_t \;\Rightarrow\; \downarrow \pi_t \;\Rightarrow\; \underbrace{\downarrow y_t}_{\text{if prices are sticky}} \;\Rightarrow\; \downarrow c_t \;\Rightarrow\; \underbrace{?r_t}_{\text{depends on the presence of capital and calibration}}$$

Mr. Wicksell



Note: hat variables are deviations from the zero-inflation-target steady state. Nominal interest rate ($\hat{i}_t$), real interest rate ($\hat{r}_t$), inflation (π_t), real interest rate gap with constant capital ($\hat{r}_t^{cc,Gap}$), output ($\hat{y}_t$), consumption ($\hat{c}_t$), capital at the beginning of period ($\hat{k}_t$), and investment ($\hat{x}_t$).

Figure 1: Impulse response function to a positive monetary shock in a canonical New Keynesian model augmented with endogenous capital

Is it really the case that the channel is not structural?

Preview of results

endogenous response of the policy rule

- The real interest rate channel **is structural**:

$$\uparrow \varepsilon_t^m \Rightarrow \uparrow i_t \Rightarrow \underbrace{\uparrow r_t}_{\text{if prices are sticky}} \Rightarrow \underbrace{\downarrow\downarrow c_t}_{\text{if capital sinks}} \Rightarrow \downarrow\downarrow y_t \Rightarrow \downarrow\downarrow \pi_t \Rightarrow \downarrow i_t \Rightarrow \downarrow r_t$$

- The flow continues onward with shrinking and oscillating amplitude until convergence to the in-period equilibrium, as induced by monetary dominance.

- The identification problem can be mitigated by smoothing interest rates in the policy rule
  - What central banks, arguably, all do

- The sign of $r_t^{Gap} = r_t - r_t^*$ (**RIRG**) can be misleading after a strong shock (e.g., pandemic, financial crisis, or trade war)

- As r_t^{Gap} depends on state variables, after a shock, r_t and r_t^* do not reflect the same states.
  - Comparing apples to oranges (as warned by Woodford (2003))
  - **Alternative:** calculate state-invariant gap, $r_t^{Gap,cc}$, sign-consistent and a better predictor of future π_t in the United States from 1965 to 2019

The literature in 5 strands

- **The (real) interest rate channel:**
  - Wicksell (1898, 1907), Woodford (2003), Neiss and Nelson (2003), Laubach and Williams (2003)

- **Suspicion about the real interest rate channel is not new:**
  - Kimball (1995): “Neo-Monetarist” model (RBC + sticky prices + monetary aggregates rule)
    - A monetary expansion would raise real interest rate under “plausible parameters”
    - The "implausible" scenario would occur if either adjustment costs were "too high" or convergence back to the long-run equilibrium after a monetary shock was "too fast"

- **Rupert and Sustek’s odd choice of ingredient:**
  - Brault and Kahn (2022) : Contemporary medium-scale models use investment adjustment costs.

- **Interest-rate smoothing is *business as usual*:**
  - Empirics: Clarida et al. (1999), Coibion and Grodnichenko (2012), Smets and Wouters (2007).
  - Theory: Goodfriend (1987), Sack and Wieland (2000) and Woodford (2003)
  - Survey: Amaral et al. (2025)

- **But not all agree (data-dependency vs. inertia and forward guidance):**
  - Rudebusch (2006) and Carrillo et al. (2018)

Outline

- The (reverse) mechanics of New-Keynesian models
- Interest-rate smoothing
- Revisiting the real interest rate gap (RIRG)
- Can Smets and Wouters (2007) do it?
- Forecasting inflation with the real interest rate gaps
- The US monetary policy history under the lens of RIRGs
- Conclusion

The (reverse) mechanics of New-Keynesian models

A state-space representation of the **conventional state-variant real interest rate gap**

- $\{A, B, C, D\}$ are matrices of coefficients
- $\{s_t\}$ is a matrix with a **given** sequence of actual and expected states
- ϵ_t is a vector of exogenous shocks
- ξ_t is a vector of persistent exogenous shocks, where each follows an AR(1) process

Endogenous State Variable	Shock Persistence	$\hat{r}_t$	$\hat{r}_t^n$	$\hat{r}_t^{Gap}$
no	no	$\hat{r}_t = B\epsilon_t$	$\hat{r}_t^n = D\epsilon_t$	$(B-D)\epsilon_t$
no	yes	$\hat{r}_t = B\xi_t$	$\hat{r}_t^n = D\xi_t$	$(B-D)\xi_t$
yes	no	$\hat{r}_t = A\hat{s}_{t-1} + B\epsilon_t$	$\hat{r}_t^n = C\hat{s}_{t-1}^n + D\epsilon_t$	$A\hat{s}_{t-1} - C\hat{s}_{t-1}^n + (B-D)\epsilon_t$
		$A = \frac{\mathbb{E}_t \hat{r}_{t+1}}{\hat{s}_t}$	$C = \frac{\mathbb{E}_t \hat{r}_{t+1}^n}{\hat{s}_t^n}$	$\mathbb{E}_t \hat{r}_{t+1} \frac{\hat{s}_{t-1}}{\hat{s}_t} - \mathbb{E}_t \hat{r}_{t+1}^n \frac{\hat{s}_{t-1}^n}{\hat{s}_t^n} + \boxed{(B-D)\epsilon_t}$
yes	yes	$\hat{r}_t = A\hat{s}_{t-1} + B\xi_t$	$\hat{r}_t^n = C\hat{s}_{t-1}^n + D\xi_t$	$\left(\mathbb{E}_t \hat{r}_{t+1} - B\rho_m \xi_t\right)\frac{\hat{s}_{t-1}}{\hat{s}_t} - \left(\mathbb{E}_t \hat{r}_{t+1}^n - D\rho_m \xi_t\right)\frac{\hat{s}_{t-1}^n}{\hat{s}_t^n} + (B-D)\xi_t$
		$A = \frac{\mathbb{E}_t \hat{r}_{t+1} - B\rho_m \xi_t}{\hat{s}_t}$	$C = \frac{\mathbb{E}_t \hat{r}_{t+1}^n - D\rho_m \xi_t}{\hat{s}_t^n}$	$\mathbb{E}_t \hat{r}_{t+1} \frac{\hat{s}_{t-1}}{\hat{s}_t} - \mathbb{E}_t \hat{r}_{t+1}^n \frac{\hat{s}_{t-1}^n}{\hat{s}_t^n} + \underbrace{\left(B - B\rho_m \frac{\hat{s}_{t-1}}{\hat{s}_t} - D + D\rho_m \frac{\hat{s}_{t-1}^n}{\hat{s}_t^n}\right)}_{} \xi_t$

Table 1: State-variant definition

sign depends on elasticity of state variables

That's where the capital puzzle is born!

The (reverse) mechanics of New-Keynesian models

A state-space representation of **Woodford (2003)'s state-consistent real interest rate gap**

- $\{A, B, C, D\}$ are matrices of coefficients
- $\{s_t\}$ is a matrix with a **given** sequence of actual and expected states
- ϵ_t is a vector of exogenous shocks
- ξ_t is a vector of persistent exogenous shocks, where each follows an AR(1) process

Endogenous State Variable	Shock Persistence	$\hat{r}_t$	$\hat{r}_t^{n,cons}$	$\hat{r}_t^{Gap,cons}$
no	no	$\hat{r}_t = B\epsilon_t$	$\hat{r}_t^n = D\epsilon_t$	$(B-D)\,\epsilon_t$
no	yes	$\hat{r}_t = B\xi_t$	$\hat{r}_t^n = D\xi_t$	$(B-D)\,\xi_t$
yes	no	$\hat{r}_t = A\hat{s}_{t-1} + B\epsilon_t$ $A = \frac{\mathbb{E}_t \hat{r}_{t+1}}{\hat{s}_t}$	$\hat{r}_t^n = C\hat{s}_{t-1} + D\epsilon_t$ $C = \frac{\mathbb{E}_t \hat{r}^n_{t+1}}{\hat{s}_t}$	$(A-C)\,\hat{s}_{t-1} + (B-D)\,\epsilon_t$ $\mathbb{E}_t\,\hat{r}_{t+1}\frac{\hat{s}_{t-1}}{\hat{s}_t} - \mathbb{E}_t\,\hat{r}^n_{t+1}\frac{\hat{s}_{t-1}}{\hat{s}_t} + \boxed{(B-D)\,\epsilon_t}$
yes	yes	$\hat{r}_t = A\hat{s}_{t-1} + B\xi_t$ $A = \frac{\mathbb{E}_t \hat{r}_{t+1} - B\rho_m\xi_t}{\hat{s}_t}$	$\hat{r}_t^n = C\hat{s}_{t-1} + D\xi_t$ $C = \frac{\mathbb{E}_t \hat{r}^n_{t+1} - D\rho_m\xi_t}{\hat{s}_t}$	$\left(\mathbb{E}_t\,\hat{r}_{t+1} - B\rho_m\xi_t\right)\frac{\hat{s}_{t-1}}{\hat{s}_t} - \left(\mathbb{E}_t\,\hat{r}^n_{t+1} - D\rho_m\xi_t\right)\frac{\hat{s}_{t-1}}{\hat{s}_t} + (B-D)\,\xi_t$ $\left(\mathbb{E}_t\,\hat{r}_{t+1} - \mathbb{E}_t\,\hat{r}^n_{t+1}\right)\frac{\hat{s}_{t-1}}{\hat{s}_t} + \underbrace{\left(B - B\rho_m\frac{\hat{s}_{t-1}}{\hat{s}_t} - D + D\rho_m\frac{\hat{s}_{t-1}}{\hat{s}_t}\right)}_{\text{sign depends on elasticity of state variables}}\xi_t$

Table 2: State-consistent definition

Interest-rate smoothing

$$i_t = \rho^i i_{t-1} + \left(1 - \rho^i\right)(i + \nu\pi_t) + \xi_t^m$$

- Interest-rate smoothing can deliver IRFs with the sign consistent with the real interest rate channel within the empirically relevant parameter range

- This finding largely minimizes the identification problem from an empirical perspective and provides new insight into its mechanics

Table 5: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.0$

	$\rho^i = 0$	$\rho^i = 0.1$	$\rho^i = 0.2$	$\rho^i = 0.3$	$\rho^i = 0.4$	$\rho^i = 0.5$	$\rho^i = 0.6$	$\rho^i = 0.7$	$\rho^i = 0.8$	$\rho^i = 0.9$	$\rho^i = 0.95$	$\rho^i = 0.99$
$\rho^m = 0$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.1$	-	-	-	-	-	-	-	+	+	+	+	+
$\rho^m = 0.2$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m = 0.3$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m = 0.4$	-	-	-	-	-	-	-	-	-	-	+	+
$\rho^m = 0.5$	-	-	-	-	-	-	-	-	-	-	+	+
$\rho^m = 0.6$	-	-	-	-	-	-	-	-	-	-	+	+
$\rho^m = 0.7$	-	-	-	-	-	-	-	-	-	-	+	+
$\rho^m = 0.8$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m = 0.9$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m = 0.95$	-	-	-	-	-	-	-	-	+	+	+	+
$\rho^m = 0.99$	-	-	-	-	-	+	+	+	+	+	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases.

Interest-rate smoothing

$$i_t = \rho^i i_{t-1} + \left(1 - \rho^i\right)(i + \nu \pi_t) + \xi_t^m$$

- Interest-rate smoothing can deliver IRFs with the sign consistent with the real interest rate channel outside of the empirically relevant parameter range if there is a little bit of adjustment costs

- This finding also largely minimizes the identification problem from an empirical perspective and provides new insight into its mechanics

Table 6: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.1$

	$\rho^i=0$	$\rho^i=0.1$	$\rho^i=0.2$	$\rho^i=0.3$	$\rho^i=0.4$	$\rho^i=0.5$	$\rho^i=0.6$	$\rho^i=0.7$	$\rho^i=0.8$	$\rho^i=0.9$	$\rho^i=0.95$	$\rho^i=0.99$
$\rho^m=0$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.1$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.2$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.3$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.4$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.5$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.6$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.7$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.8$	-	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.9$	-	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.95$	-	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.99$	-	+	+	+	+	+	+	+	+	+	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases.

Interest-rate smoothing $i_t = \rho^i i_{t-1} + \left(1-\rho^i\right)(i+\nu\pi_t) + \xi_t^m$

- The mechanics of smoothing
  - Solving by the **undetermined coefficients method**

I assume $\hat{c}_t = a_0\hat{k}_t + a_1\xi_t^m + a_2\hat{i}_{t-1}$; $\pi_t = b_0\hat{k}_t + b_1\xi_t^m + b_2\hat{i}_{t-1}$; $\hat{y}_t = d_0\hat{k}_t + d_1\xi_t^m + d_2\hat{i}_{t-1}$; $\hat{k}_{t+1} = f_0\hat{k}_t + f_1\xi_t^m + f_2\hat{i}_{t-1}$

$$\hat{r}_t = \mathbb{E}_t\hat{c}_{t+1} - \hat{c}_t$$

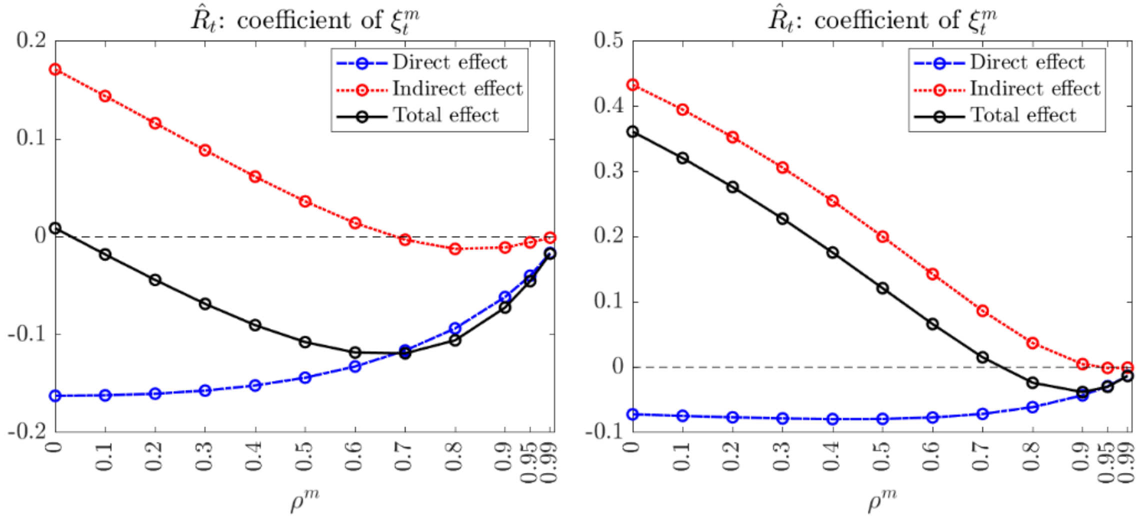
$$= \underbrace{\left(a_0 f_0 - a_0 + a_2\left(1-\rho^i\right)\nu b_0\right)\hat{k}_t + \left(a_0 f_2 - a_2 + a_2\rho^i + a_2\left(1-\rho^i\right)\nu b_2\right)\hat{i}_{t-1}}_{=0 \text{ at the shock}}$$

$$+\left(\underbrace{\underbrace{\rho^m a_1 - a_1}_{\text{ex-smoothing}} + \underbrace{a_2\left(1-\rho^i\right)\nu b_1 + a_2}_{\text{smoothing}}}_{\text{indirect effect of capital}} \quad \underbrace{+a_0 f_1}_{\text{direct effect of capital}}\right)\xi_t^m$$

$$\frac{\partial\hat{r}_t}{\partial\xi_t^m} = \underbrace{\underbrace{\left(\rho^m - 1\right)\frac{\partial\hat{c}_t}{\partial\xi_t^m}}_{\text{ex-smoothing}} + \underbrace{\left(1+\left(1-\rho^i\right)\nu\frac{\partial\hat{\pi}_t}{\partial\xi_t^m}\right)\frac{\partial\hat{c}_t}{\partial i_{t-1}}}_{\text{smoothing}}}_{\text{indirect effect of capital}} \underbrace{+\frac{\partial\hat{c}_t}{\partial\hat{k}_t}\frac{\partial\hat{k}_{t+1}}{\partial\xi_t^m}}_{\text{direct effect of capital} < 0}$$

Interest-rate smoothing

- The mechanics of smoothing



(a) $\kappa = 0$ (b) $\kappa = 0.1$

Figure 3: Decomposition of the effect of capital on $\hat{r}_t$ from a monetary shock when $\rho^i = 0$

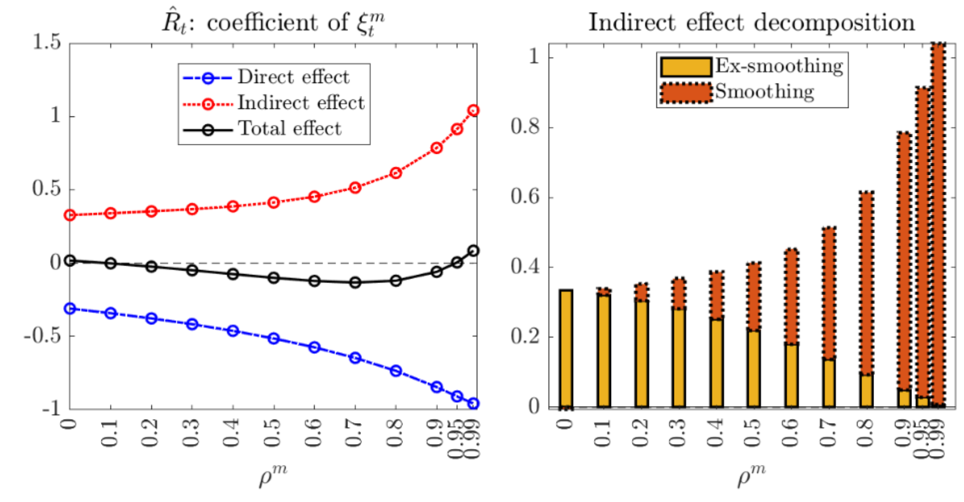


Figure 4: Decomposition of the effect of capital on $\hat{r}_t$ from a monetary shock when $\rho^i = 0.5$ and $\kappa = 0$

No capital puzzle!

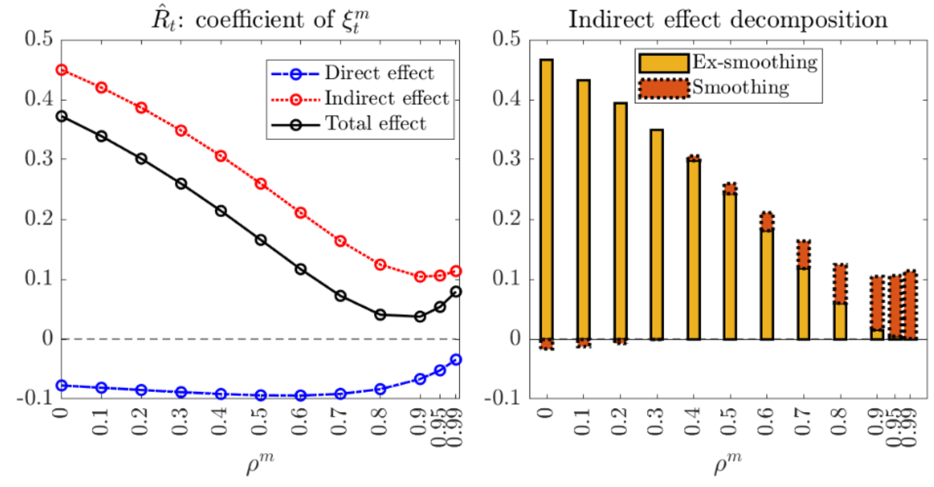


Figure 5: Decomposition of the effect of capital on $\hat{r}_t$ from a monetary shock when $\rho^i = 0.1$ and $\kappa = 0.1$

Revisiting the real interest rate gap (RIRG)

- From the definition of $\hat{r}_t^{Gap}$, I define the state-invariant $\hat{r}_t^{Gap,cc}$
  - The gap as if endogenous state variables (capital) were kept fixed:
  - Not the same as capital does not exist! Capital affects levels and elasticities, but not dynamics!

$$\hat{r}_t^{Gap} = \hat{r}_t - \hat{r}_t^n$$

$$\hat{r}_t^n = \hat{r}_t^{n,cc} + \left(\eta_{nk}\hat{k}_t^n + \eta_{nkk}\varepsilon_t^m\right)$$

$$\hat{r}_t = \hat{r}_t^{cc} + \left(\eta_k\hat{k}_t + \eta_{kk}\varepsilon_t^m\right)$$

$$\hat{r}_t^{Gap,cc} = \hat{r}_t^{cc} - \hat{r}_t^{n,cc}$$

$$\hat{r}_t^{Gap,cc} = \left(\hat{r}_t - \eta_k\hat{k}_t - \eta_{kk}\varepsilon_t^m\right) - \left(\hat{r}_t^n - \eta_{nk}\hat{k}_t^n - \eta_{nkk}\varepsilon_t^m\right)$$

$$\hat{r}_t^{Gap,cc} = \hat{r}_t^{Gap} - \eta_k\hat{k}_t + \eta_{nk}\hat{k}_t^n - \eta_{kk}\varepsilon_t^m + \eta_{nkk}\varepsilon_t^m$$

$$\hat{r}_t^{Gap} = \underbrace{\hat{r}_t^{Gap,cc}}_{\text{state-invariant effect}} \underbrace{+\eta_k\hat{k}_t - \eta_{nk}\hat{k}_t^n + \eta_{kk}\varepsilon_t^m - \eta_{nkk}\varepsilon_t^m}_{\text{state-variant effect}}$$

As the only shock of the model is monetary, we know that $\hat{r}_t^{Gap,cc} = \hat{r}_t^{cc}$, $\eta_{nkk} = 0$, and $\hat{k}_t^n = 0$.

$$\hat{r}_t^{Gap} = \underbrace{\hat{r}_t^{cc}}_{\text{state-invariant effect}} + \underbrace{\eta_k\hat{k}_t + \eta_{kk}\varepsilon_t^m}_{\text{state-variant effect}} \tag{21}$$

Revisiting the real interest rate gap (RIRG)

- I build the counterfactual model in which the state variable is always fixed ex-ante:

$$\left.\begin{aligned} -\hat{c}_t^{cc} &= -\mathbb{E}_t \hat{c}_{t+1}^{cc} + \nu \pi_t^{cc} + \xi_t^m - \mathbb{E}_t \pi_{t+1}^{cc} \\ \pi_t^{cc} &= \Psi \left(\frac{\eta + \alpha}{1 - \alpha} \hat{y}_t^{cc} + \hat{c}_t^{cc} \right) + \beta \mathbb{E}_t \pi_{t+1}^{cc} \\ \hat{y}_t^{cc} &= \frac{\bar{c}}{\bar{y}} \hat{c}_t^{cc} \end{aligned}\right\}$$

$$\hat{r}_t^{cc} = \underbrace{\left(1 + \frac{1}{\underbrace{\frac{1}{(\nu - \rho^m)\Omega} - 1}_{>-1}} \right)}_{\geq 0} \epsilon_t^m$$

+/- sign of (row variable) at the contractionary monetary shock

Table 1: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.0$

	$\rho^m = 0$	$\rho^m = 0.1$	$\rho^m = 0.2$	$\rho^m = 0.3$	$\rho^m = 0.4$	$\rho^m = 0.5$	$\rho^m = 0.6$	$\rho^m = 0.7$	$\rho^m = 0.8$	$\rho^m = 0.9$	$\rho^m = 0.95$	$\rho^m = 0.99$
r^{Gap}	+	-	-	-	-	-	-	-	-	-	-	-
$r^{cc,Gap}$	+	+	+	+	+	+	+	+	+	+	+	+

Table 2: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.1$

	$\rho^m = 0$	$\rho^m = 0.1$	$\rho^m = 0.2$	$\rho^m = 0.3$	$\rho^m = 0.4$	$\rho^m = 0.5$	$\rho^m = 0.6$	$\rho^m = 0.7$	$\rho^m = 0.8$	$\rho^m = 0.9$	$\rho^m = 0.95$	$\rho^m = 0.99$
r^{Gap}	+	+	+	+	+	+	+	+	-	-	-	-
$r^{cc,Gap}$	+	+	+	+	+	+	+	+	+	+	+	+

Table 3: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.2$

	$\rho^m = 0$	$\rho^m = 0.1$	$\rho^m = 0.2$	$\rho^m = 0.3$	$\rho^m = 0.4$	$\rho^m = 0.5$	$\rho^m = 0.6$	$\rho^m = 0.7$	$\rho^m = 0.8$	$\rho^m = 0.9$	$\rho^m = 0.95$	$\rho^m = 0.99$
r^{Gap}	+	+	+	+	+	+	+	+	+	-	-	-
$r^{cc,Gap}$	+	+	+	+	+	+	+	+	+	+	+	+

Table 4: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.5$

	$\rho^m = 0$	$\rho^m = 0.1$	$\rho^m = 0.2$	$\rho^m = 0.3$	$\rho^m = 0.4$	$\rho^m = 0.5$	$\rho^m = 0.6$	$\rho^m = 0.7$	$\rho^m = 0.8$	$\rho^m = 0.9$	$\rho^m = 0.95$	$\rho^m = 0.99$
r^{Gap}	+	+	+	+	+	+	+	+	+	+	-	-
$r^{cc,Gap}$	+	+	+	+	+	+	+	+	+	+	+	+

Revisiting the real interest rate gap (RIRG)

A state-space representation of **the state-invariant real interest rate gap**

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Endogenous State Variable	Shock Persistence	$\hat{r}_t^{cc}$	$\hat{r}_t^{n,cc}$	$\hat{r}_t^{Gap,cc}$
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no	yes	$\hat{r}_t^{cc} = B\xi_t$	$\hat{r}_t^{n,cc} = D\xi_t$	$\boxed{(B-D)\,\xi_t}$

Table 3: State-invariant definition

Can Smets and Wouters (2007) do it?

- Sweeping the parameters with very low adjustment costs (estimated US $\kappa^i = 5.4882$)

Table 8: Smets and Wouters (2007)'s parameter sweep with $\kappa^i = 0.005$

	$\rho^i=0$	$\rho^i=0.1$	$\rho^i=0.2$	$\rho^i=0.3$	$\rho^i=0.4$	$\rho^i=0.5$	$\rho^i=0.6$	$\rho^i=0.7$	$\rho^i=0.8$	$\rho^i=0.9$	$\rho^i=0.95$	$\rho^i=0.99$
$\rho^m=0$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.1$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.2$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.3$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.4$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.5$	+	+	+	-	-	-	-	-	-	+	+	+
$\rho^m=0.6$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m=0.7$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m=0.8$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m=0.9$	-	-	-	-	-	-	+	+	+	+	+	+
$\rho^m=0.95$	-	-	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.99$	+	+	+	+	+	+	+	+	+	+	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases.

Table 9: Smets and Wouters (2007)'s parameter sweep with $\kappa^i = 0.01$

	$\rho^i=0$	$\rho^i=0.1$	$\rho^i=0.2$	$\rho^i=0.3$	$\rho^i=0.4$	$\rho^i=0.5$	$\rho^i=0.6$	$\rho^i=0.7$	$\rho^i=0.8$	$\rho^i=0.9$	$\rho^i=0.95$	$\rho^i=0.99$
$\rho^m=0$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.1$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.2$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.3$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.4$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.5$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.6$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.7$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.8$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.9$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.95$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m=0.99$	+	+	+	+	+	+	+	+	+	+	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases.

Forecasting inflation with the real interest rate gaps

- Simulating **SW (2007)** calibrated at the estimated posterior mode with all shocks turned on (250,000 periods; 50,000 burn-in)

With **realistic adjustment costs**,
all gap measures are sign-consistent and predict similarly well

Table 13: Forecasting $\Delta_4\pi_{t+4}$ under realistic investment adjustment costs

Forecasting 4-period ahead annual inflation									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
$\Delta_4\hat{\pi}_{t-4}$	0.705***	0.700***	0.717***	0.703***	0.714***	0.708***	0.711***	0.714***	0.684***
	(0.00158)	(0.00158)	(0.00172)	(0.00157)	(0.00153)	(0.00152)	(0.00152)	(0.00158)	(0.00181)
$\hat{r}^{Gap}_{t-4}$		-0.0860***				-0.103***		0.0940***	0.161***
		(0.00159)				(0.00154)		(0.0161)	(0.0162)
$\hat{r}^{Gap,cons}_{t-4}$			-0.0178***						0.0439***
			(0.000976)						(0.00133)
$\hat{r}^{Gap,cc}_{t-4}$				-0.0928***			-0.103***	-0.196***	-0.312***
				(0.00157)			(0.00152)	(0.0159)	(0.0162)
$\hat{r}_{t-4}$					-0.809***	-0.850***	-0.835***	-0.821***	-0.807***
					(0.00681)	(0.00676)	(0.00674)	(0.00716)	(0.00715)
Constant	0.00331	0.00324	0.00370	0.00325	0.00785**	0.00801**	0.00794**	0.00788**	0.00683**
	(0.00327)	(0.00325)	(0.00327)	(0.00325)	(0.00317)	(0.00313)	(0.00313)	(0.00313)	(0.00312)
Observations	199,993	199,993	199,993	199,993	199,993	199,993	199,993	199,993	199,993
Adjusted R-squared	0.498	0.505	0.498	0.506	0.531	0.541	0.541	0.541	0.544
Root Mean Square Error	1.465	1.454	1.463	1.452	1.415	1.400	1.399	1.399	1.395

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

With **negligible adjustment costs**,
state-variant gap predicts better, but only other gaps are sign-consistent

Table 14: Forecasting $\Delta_4\pi_{t+4}$ under negligible investment adjustment costs

Forecasting 4-period ahead annual inflation									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
$\Delta_4\hat{\pi}_{t-4}$	0.690***	0.699***	0.691***	0.678***	0.684***	0.692***	0.674***	0.683***	0.684***
	(0.00162)	(0.00156)	(0.00161)	(0.00158)	(0.00160)	(0.00154)	(0.00157)	(0.00152)	(0.00152)
$\hat{r}^{Gap}_{t-4}$		3.865***				3.688***		3.553***	3.615***
		(0.0302)				(0.0300)		(0.0295)	(0.0297)
$\hat{r}^{Gap,cons}_{t-4}$			-0.0139***						-0.00802***
			(0.000503)						(0.000487)
$\hat{r}^{Gap,cc}_{t-4}$				-0.168***			-0.449***	-0.420***	-0.405***
				(0.00166)			(0.00487)	(0.00470)	(0.00478)
$\hat{r}_{t-4}$					0.126***	0.108***	-0.303***	-0.293***	-0.284***
					(0.00171)	(0.00165)	(0.00495)	(0.00478)	(0.00481)
Constant	0.00331	0.00189	0.00365	0.00328	0.00255	0.00131	0.00504	0.00369	0.00380
	(0.00351)	(0.00338)	(0.00350)	(0.00342)	(0.00346)	(0.00334)	(0.00339)	(0.00328)	(0.00327)
Observations	199,993	199,993	199,993	199,993	199,993	199,993	199,993	199,993	199,993
Adjusted R-squared	0.477	0.516	0.479	0.502	0.491	0.526	0.511	0.544	0.545
Root Mean Square Error	1.570	1.509	1.567	1.531	1.549	1.494	1.517	1.465	1.464

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Combining all measures improve inflation forecasting

Forecasting inflation with the real interest rate gaps

- Simulating **SW (2007)** calibrated at the estimated posterior mode with historical data from 1966Q1 to 2004Q4

All gap measures are sign-consistent but the state-invariant predicts better

Table 15: Forecasting $\Delta_4\pi_{t+4}$ from 1966Q1 to 2004Q4

Forecasting 4-period ahead annual inflation: from from 1966Q1 to 2004Q4

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
$\Delta_4\hat{\pi}_{t-4}$	0.855***	0.830***	0.961***	0.723***	0.862***	0.836***	0.721***	0.590***	0.609***
	(0.0414)	(0.0390)	(0.0495)	(0.0399)	(0.0404)	(0.0373)	(0.0372)	(0.0437)	(0.0552)
$\hat{r}^{Gap}_{t-4}$		-0.208***				-0.224***		0.440***	0.451***
		(0.0425)				(0.0409)		(0.0890)	(0.0914)
$\hat{r}^{Gap,cons}_{t-4}$			-0.0859***						-0.0131
			(0.0237)						(0.0231)
$\hat{r}^{Gap,cc}_{t-4}$				-0.210***			-0.229***	-0.525***	-0.522***
				(0.0285)			(0.0269)	(0.0649)	(0.0652)
$\hat{r}_{t-4}$					-0.447***	-0.522***	-0.590***	-0.629***	-0.626***
					(0.146)	(0.135)	(0.122)	(0.114)	(0.114)
Constant	0.115	-0.242**	-0.143	-0.534***	0.114	-0.272**	-0.592***	-0.747***	-0.759***
	(0.100)	(0.119)	(0.120)	(0.123)	(0.0978)	(0.114)	(0.116)	(0.112)	(0.114)
Observations	156	156	156	156	156	156	156	156	156
Adjusted R-squared	0.733	0.768	0.753	0.802	0.747	0.787	0.827	0.850	0.850
Root Mean Square Error	1.194	1.114	1.150	1.029	1.163	1.067	0.960	0.894	0.896

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Combining multiple gaps improve inflation forecasting

Forecasting inflation with the real interest rate gaps

- Simulating **SW (2007)** calibrated at the estimated posterior mode with historical data in subsamples

All gap measures are sign-consistent but the state-invariant predicts better

Table 16: Forecasting $\Delta_4\pi_{t+4}$ from 1966Q1 to 1979Q2

Forecasting 4-period ahead annual inflation: from from 1966Q1 to 1979Q2

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
$\Delta_4\hat{\pi}_{t-4}$	0.626***	0.800***	0.692***	0.770***	0.640***	0.811***	0.768***	0.419***	0.389***
	(0.0944)	(0.0974)	(0.130)	(0.0843)	(0.0962)	(0.0978)	(0.0844)	(0.102)	(0.110)
$\hat{r}^{Gap}_{t-4}$		-0.445***				-0.519***		2.662***	2.352***
		(0.122)				(0.141)		(0.558)	(0.709)
$\hat{r}^{Gap,cons}_{t-4}$			-0.0382						0.0561
			(0.0515)						(0.0784)
$\hat{r}^{Gap,cc}_{t-4}$				-0.505***			-0.547***	-2.800***	-2.595***
				(0.104)			(0.113)	(0.481)	(0.562)
$\hat{r}_{t-4}$					0.387	-0.525	-0.416	0.951**	1.081**
					(0.480)	(0.496)	(0.433)	(0.461)	(0.498)
Constant	1.245***	0.113	1.023**	-0.331	1.379***	-0.257	-0.604	-0.386	-0.165
	(0.286)	(0.403)	(0.415)	(0.404)	(0.332)	(0.534)	(0.494)	(0.415)	(0.519)
Observations	54	54	54	54	54	54	54	54	54
Adjusted R-squared	0.448	0.553	0.443	0.614	0.444	0.554	0.614	0.731	0.728
Root Mean Square Error	1.419	1.276	1.425	1.186	1.424	1.275	1.187	0.990	0.995

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 17: Forecasting $\Delta_4\pi_{t+4}$ from 1984Q1 to 2004Q4

Forecasting 4-period ahead annual inflation: from from 1984Q1 to 2004Q4

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
$\Delta_4\hat{\pi}_{t-4}$	0.721***	0.739***	0.765***	0.701***	0.676***	0.726***	0.809***	0.833***	0.780***
	(0.0654)	(0.0629)	(0.0741)	(0.0590)	(0.0812)	(0.0802)	(0.0762)	(0.0751)	(0.0816)
$\hat{r}^{Gap}_{t-4}$		-0.106***				-0.104***		0.142**	0.0986
		(0.0363)				(0.0377)		(0.0625)	(0.0677)
$\hat{r}^{Gap,cons}_{t-4}$			-0.0202						0.0305
			(0.0162)						(0.0194)
$\hat{r}^{Gap,cc}_{t-4}$				-0.101***			-0.137***	-0.232***	-0.239***
				(0.0224)			(0.0275)	(0.0498)	(0.0495)
$\hat{r}_{t-4}$					0.122	0.0353	-0.312**	-0.495***	-0.516***
					(0.130)	(0.129)	(0.144)	(0.162)	(0.161)
Constant	-0.135**	-0.276***	-0.195**	-0.461***	-0.169**	-0.283***	-0.492***	-0.561***	-0.545***
	(0.0569)	(0.0729)	(0.0741)	(0.0887)	(0.0673)	(0.0769)	(0.0879)	(0.0910)	(0.0907)
Observations	84	84	84	84	84	84	84	84	84
Adjusted R-squared	0.592	0.627	0.595	0.670	0.592	0.622	0.684	0.700	0.705
Root Mean Square Error	0.510	0.488	0.508	0.459	0.510	0.491	0.449	0.438	0.434

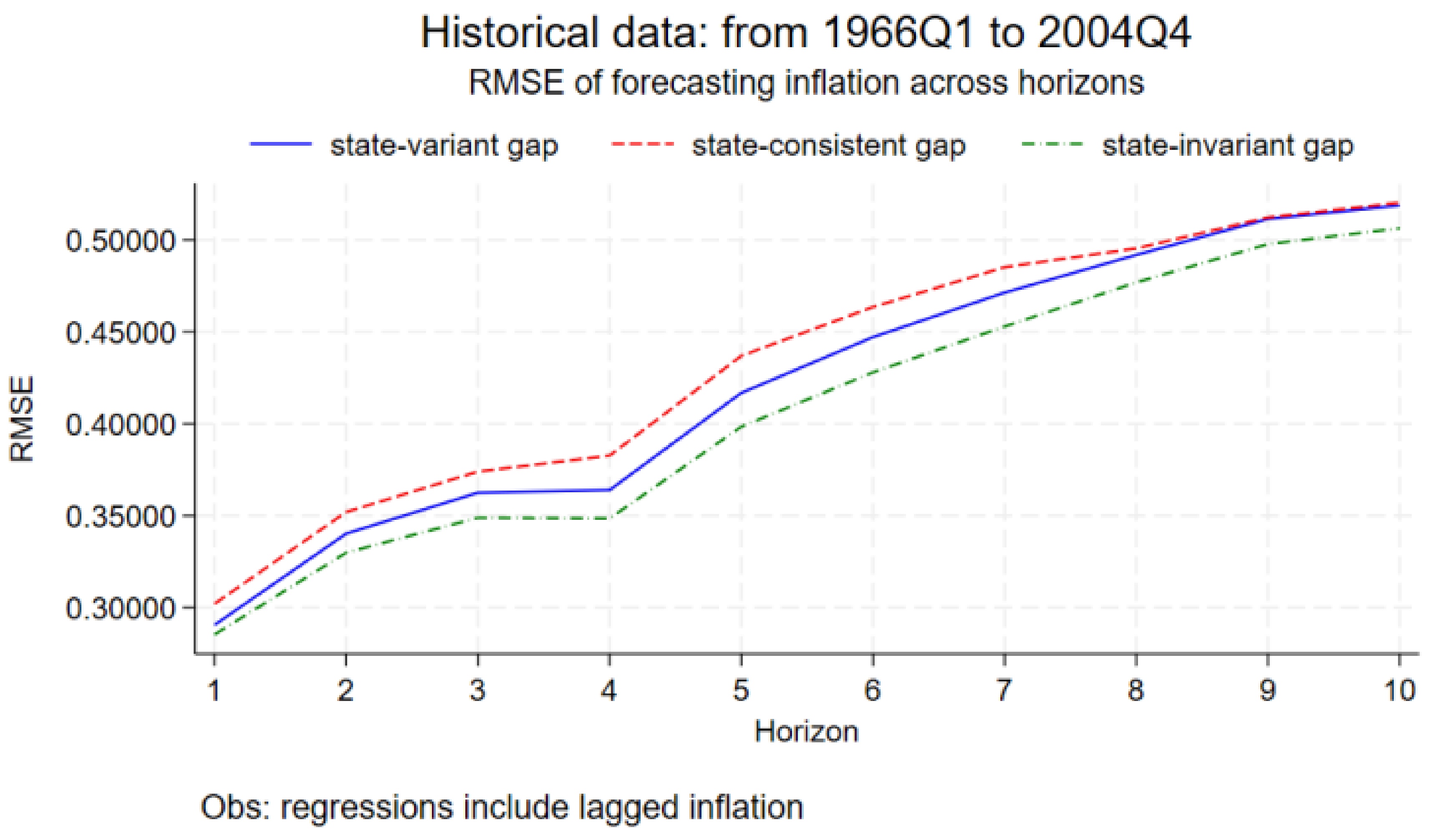
Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Combining multiple gaps improve inflation forecasting

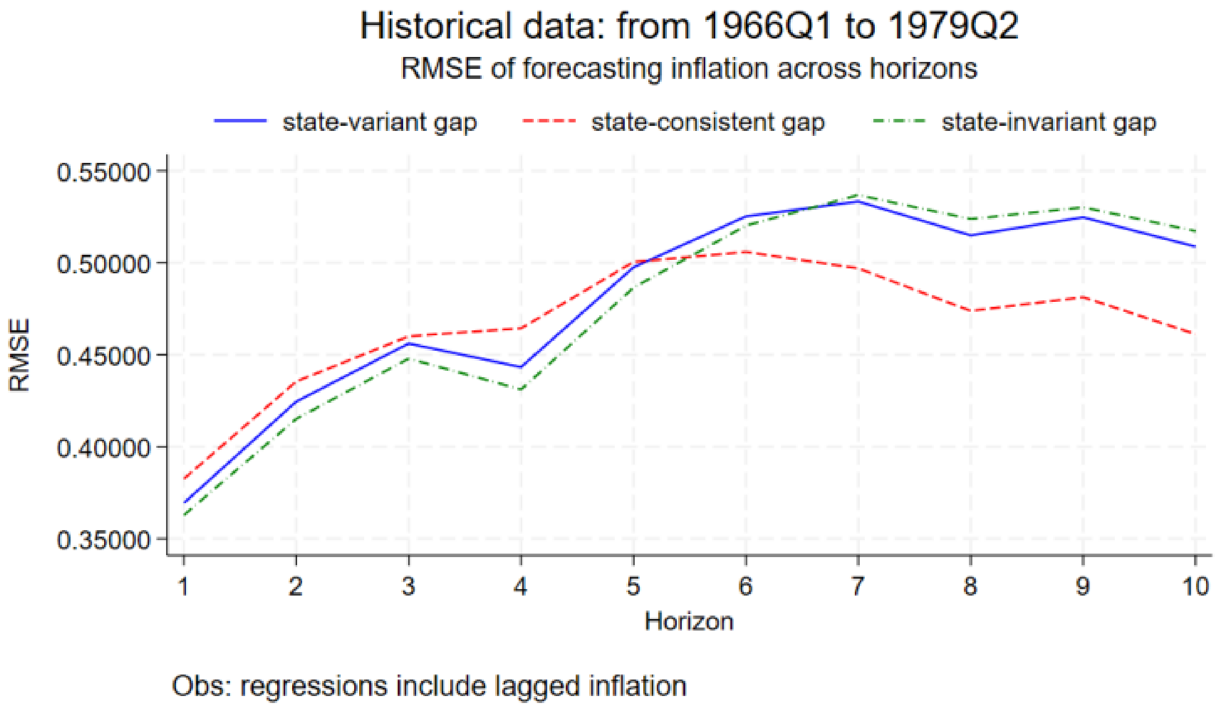
Forecasting inflation with the real interest rate gaps

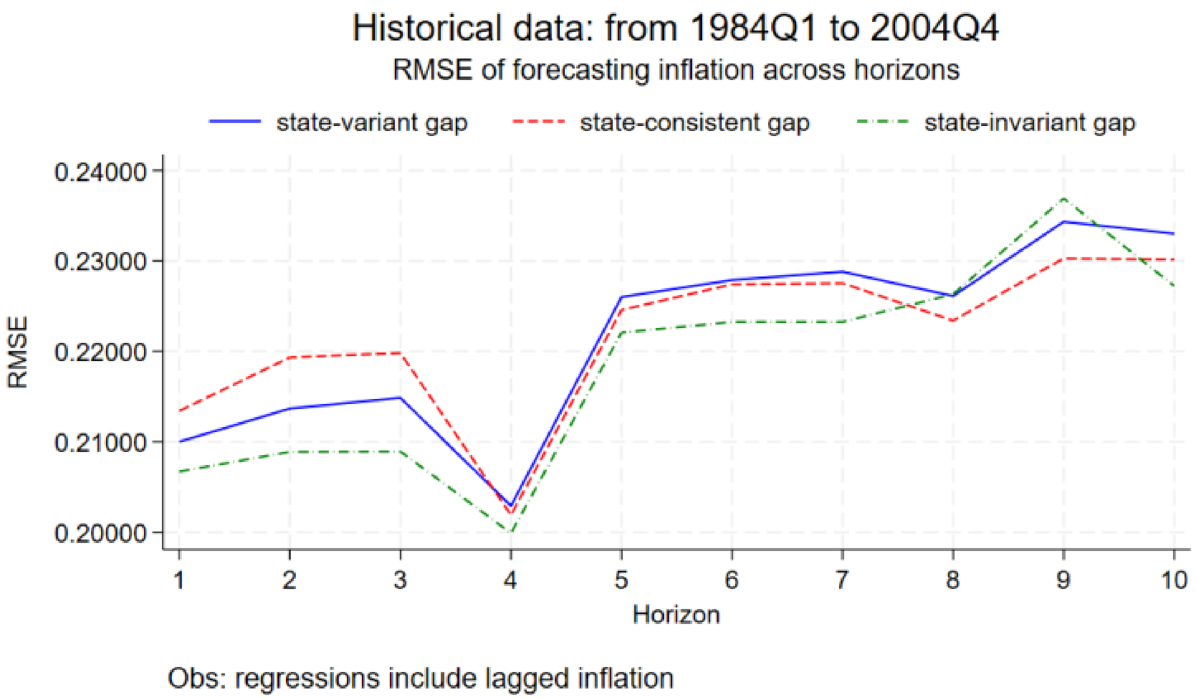
- Simulating **SW (2007)** calibrated at the estimated posterior mode with historical data from 1966Q1 to 2004Q4



Forecasting inflation with the real interest rate gaps

- Simulating **SW (2007)** calibrated at the estimated posterior mode with historical data from 1966Q1 to 2004Q4





Forecasting inflation with the real interest rate gaps

- Simulating **SW (2024)** calibrated at the estimated posterior mode with historical data from 1965Q1 to 2019Q4

All gap measures are sign-consistent but the state-invariant predicts better

Table 19: Forecasting $\Delta_4\pi_{t+4}$ from 1965Q1 to 2019Q4

Forecasting 4-period ahead annual inflation: from from 1965Q1 to 2019Q4

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
$\Delta_4\hat{\pi}_{t-4}$	0.869***	0.818***	0.866***	0.822***	0.886***	0.833***	0.841***	0.857***	0.839***
	(0.0341)	(0.0349)	(0.0343)	(0.0326)	(0.0356)	(0.0366)	(0.0338)	(0.0354)	(0.0390)
$\hat{r}^{Gap}_{t-4}$		-0.393***				-0.383***		0.238	0.0871
		(0.0921)				(0.0922)		(0.163)	(0.214)
$\hat{r}^{Gap,cons}_{t-4}$			-0.0600						-0.0940
			(0.0702)						(0.0860)
$\hat{r}^{Gap,cc}_{t-4}$				-0.350***			-0.354***	-0.486***	-0.416***
				(0.0586)			(0.0582)	(0.107)	(0.125)
$\hat{r}_{t-4}$					-0.199	-0.159	-0.225**	-0.260**	-0.223*
					(0.121)	(0.118)	(0.112)	(0.115)	(0.120)
Constant	0.179**	0.158*	0.170*	0.169**	0.154*	0.138	0.140*	0.144*	0.128
	(0.0893)	(0.0861)	(0.0901)	(0.0829)	(0.0903)	(0.0871)	(0.0836)	(0.0834)	(0.0847)
Observations	216	216	216	216	216	216	216	216	216
Adjusted R-squared	0.751	0.769	0.750	0.785	0.752	0.770	0.788	0.789	0.790
Root Mean Square Error	1.107	1.065	1.107	1.027	1.102	1.063	1.020	1.017	1.016

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

- When combining different RIRGs, the sign of the state-variant turns positive
- Only the state-invariant is significant

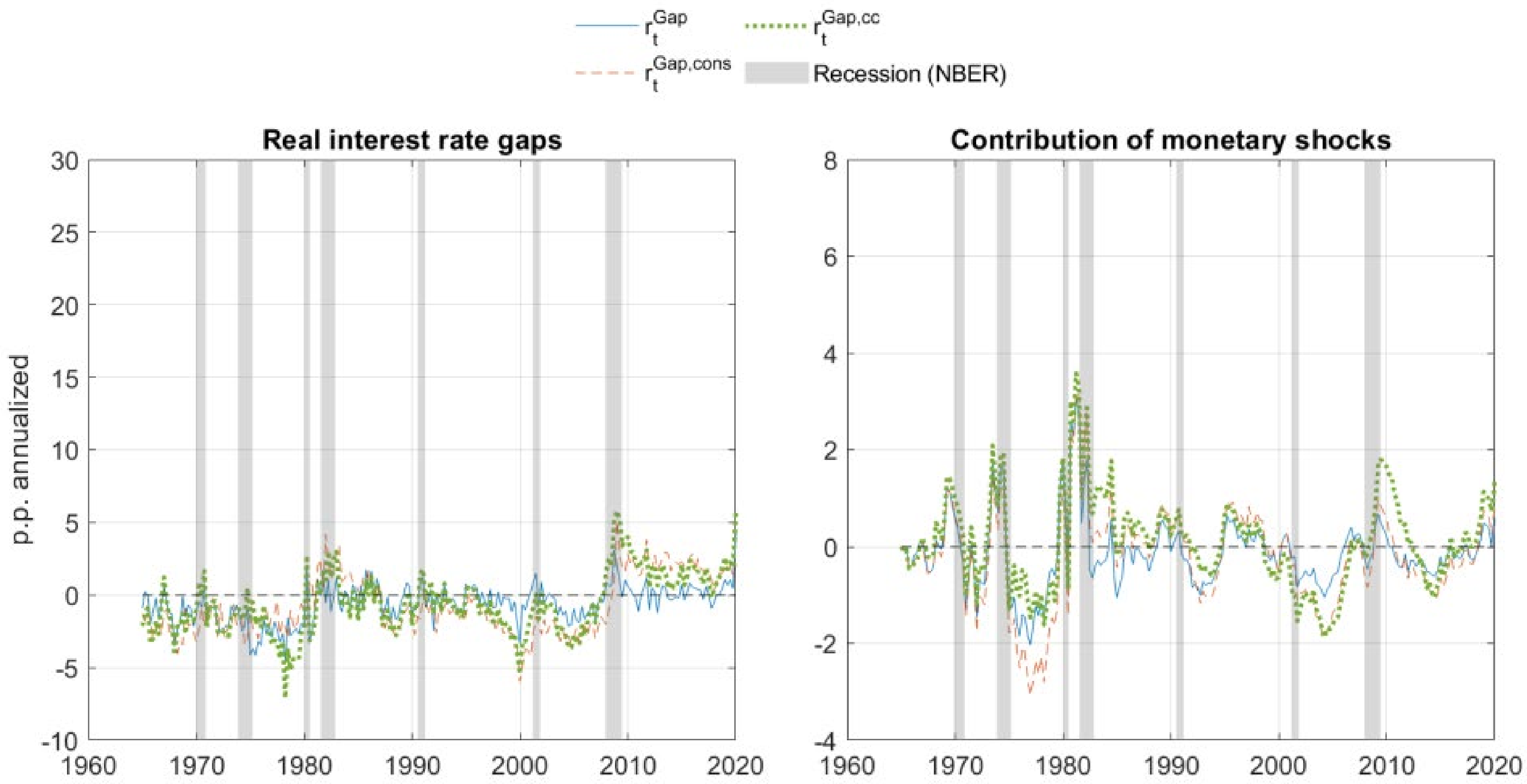
Combining multiple gaps improve inflation forecasting

The US monetary policy history under the lens of RIRGs

- It is possible to discern the rough trends of recent US monetary policy history by tracking the evolution of estimated RIRGs.
- Smets and Wouters (2024) provides a narrative similar to the one constructed using a broader range of sources.
- All RIRGs exhibit similar trends, but the state-invariant gap appears to provide stronger signals during turning points.

- **Great Inflation (1960s – 1970s):** RIRGs reveals a consistently loose monetary policy stance, especially in the second half of the 1970s, contributing to high and volatile inflation.

- **Volcker Disinflation (1979 – ~1985):** The abrupt tightening under Chairman Paul Volcker is reflected in the sudden reverse of the RIRGs, marking a decisive shift in monetary policy.



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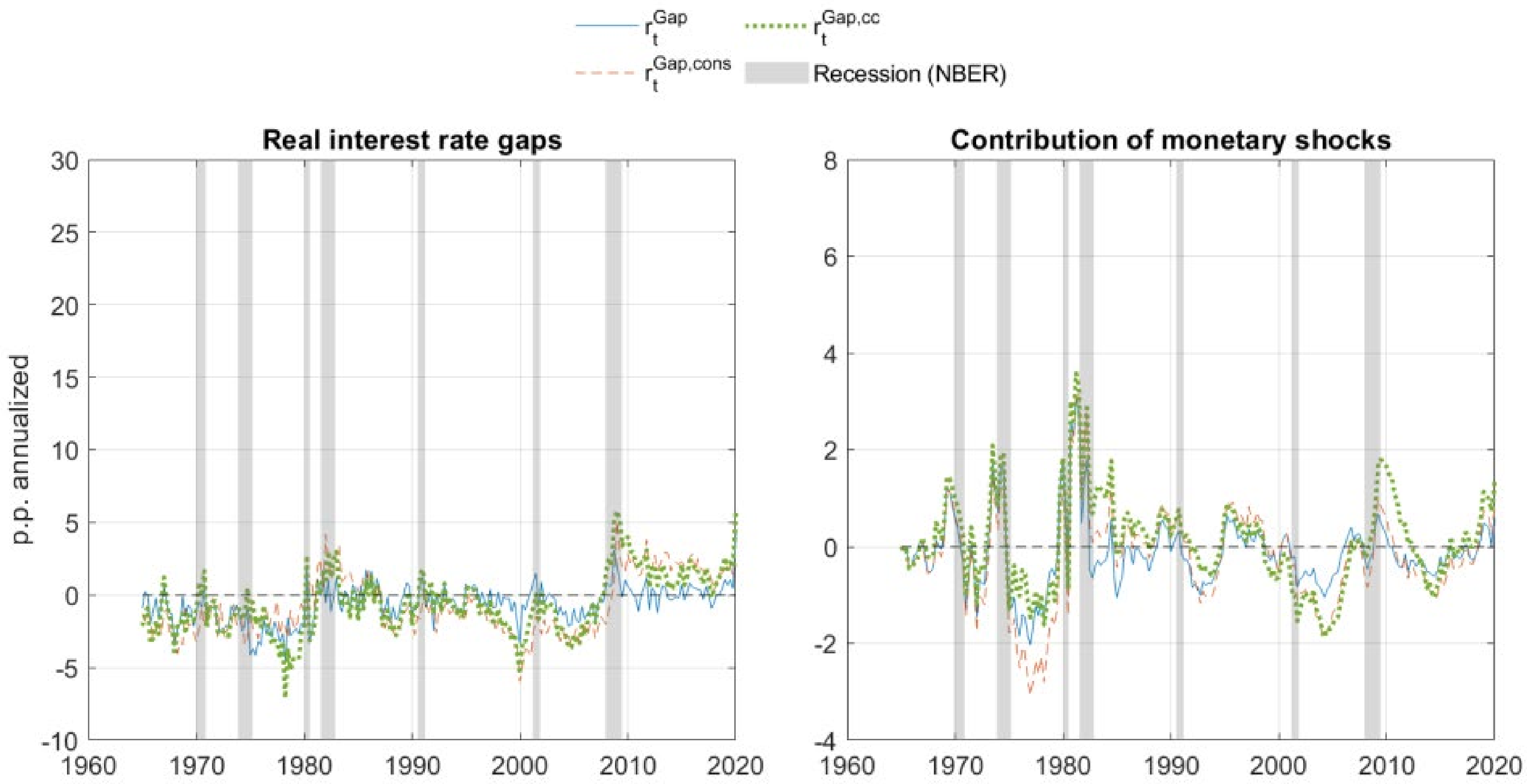
- **Great Moderation (~1985 – 2007):** RIRGs show a prolonged period of moderate looseness, coinciding with stable inflation and growth rates, and a clearer loosing contribution from monetary policy in the 2000s.

- **Post-GFC (2008–2019):** RIRGs capture the involuntary tightening caused by the effective lower bound on policy rates and the preponderance of negative natural interest rates, highlighting the challenges faced by the Fed.



The US monetary policy history under the lens of RIRGs

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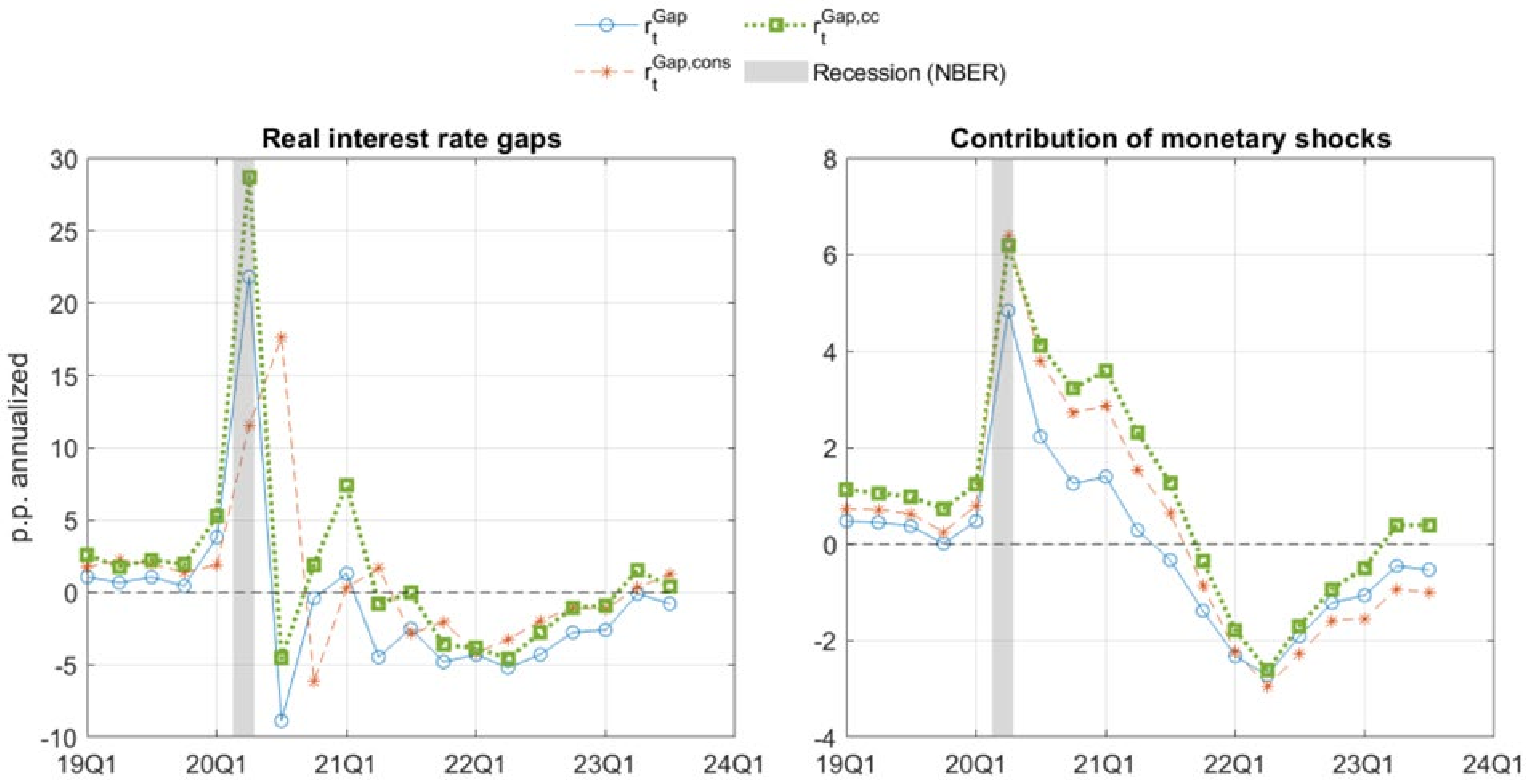
- All RIRGs exhibit similar trends, but the state-invariant gap appears to provide stronger signals during turning points.

- **COVID-19 Pandemic (2020–2023):** The RIRGs document the intense and involuntary contraction of monetary policy stance following the pandemic shock, as investment collapsed and the effective lower bound of the policy rate was binding.



Conclusion

- **Key Findings:**
  - The real interest rate channel **is structural** in New Keynesian models
  - The **capital puzzle arises** from a **high capital elasticity to shocks (the phenomenon is broader than capital)**
  - The **identification problem** can be mitigated by **interest-rate smoothing**
  - The **state-invariant RIRG** is **sign-consistent** with the **monetary policy stance**
    - **And** was a better predictor of inflation in the United States from 1965 to 2019

- **Implications:**
  - The capital puzzle highlights the **need to reconsider how the stance of monetary policy is measured** and communicated, particularly during periods of significant economic disruption
  - **Reconciling short- and long-run estimates of r-star** becomes crucial in such contexts
  - The **state-invariant RIRG** may be used by central banks for **gauging the monetary policy stance and predicting inflation**
  - The relationship between inflation and real interest rates is **more robust** than with nominal interest rates
  - Acceptable to identify **VAR models** with the **same sign restriction** on the real interest rate's response to a monetary shock, more so in the presence of **interest-rate smoothing**

Thank you!