Uncovering Inflation Nonlinearities with KANs

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BCRP and PUCP

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An overview

1. What do we do here?
   Explore functional nonlinearities in the Phillips curve

2. How do we do it?
   We use KANs which deliver interpretable mappings from inputs to outputs

3. What do we find?
   - Data reveals linearity in PCs
   - But also nonlinearities, specially from exchange rate changes

Contents

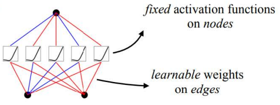
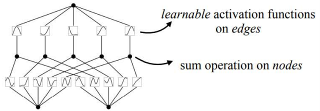
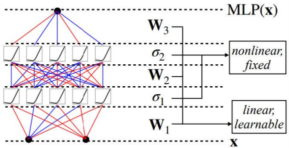
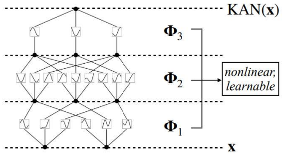
Introduction

- Phillips curve.
  - Close to 70 years of history
  - Ongoing debate
  - Building block of macro models
  - Endogeneity problem
- Nonlinearities.
  - **Convex nonlinearity**
  - Threshold effects
  - Piecewise linearity
  - Time-varying parameters

Contents

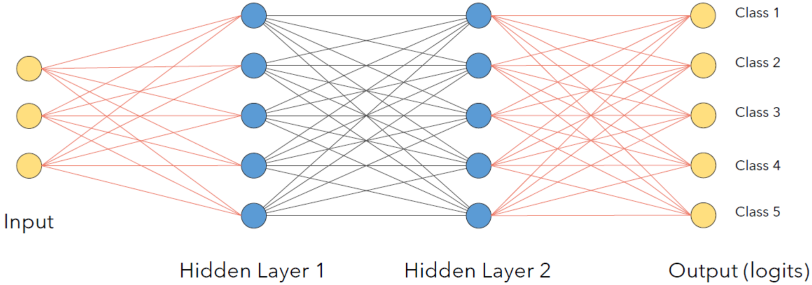
KANs

Kolmogorov-Arnold Networks

Model	**Multi-Layer Perceptron (MLP)**	**Kolmogorov-Arnold Network (KAN)**
Theorem	**Universal Approximation Theorem**	**Kolmogorov-Arnold Representation Theorem**
Formula (Shallow)	$f(\mathbf{x}) \approx \sum_{i=1}^{N(\epsilon)} a_i \sigma(\mathbf{w}_i \cdot \mathbf{x} + b_i)$	$f(\mathbf{x}) = \sum_{q=1}^{2n+1} \Phi_q \left(\sum_{p=1}^{n} \phi_{q,p}(x_p) \right)$
Model (Shallow)	**(a)** 	**(b)** 
Formula (Deep)	$\mathrm{MLP}(\mathbf{x}) = (\mathbf{W}_3 \circ \sigma_2 \circ \mathbf{W}_2 \circ \sigma_1 \circ \mathbf{W}_1)(\mathbf{x})$	$\mathrm{KAN}(\mathbf{x}) = (\mathbf{\Phi}_3 \circ \mathbf{\Phi}_2 \circ \mathbf{\Phi}_1)(\mathbf{x})$
Model (Deep)	**(c)** 	**(d)** 

MLP

Multilayer perceptron



KAN

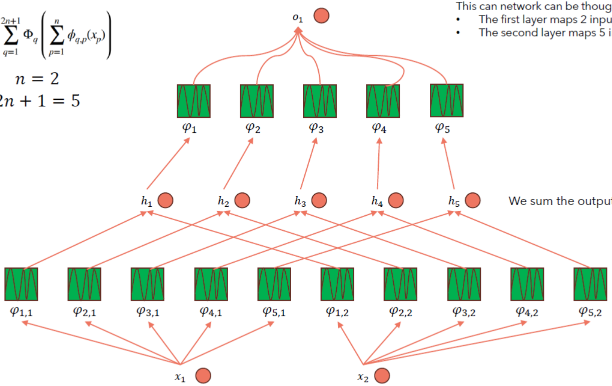
Structure

$$f(\mathbf{x}) = \sum_{q=1}^{2n+1} \Phi_q \left(\sum_{p=1}^{n} \phi_{q,p}(x_p) \right)$$

$$n = 2$$
$$2n + 1 = 5$$

This can network can be thought of as two layers applied in sequence:

- The first layer maps 2 input features into 5 output features.
- The second layer maps 5 input features into 1 output feature.



We sum the output of the learnable functions

Instead of having learnable weights, we have **learnable functions**

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DATA

1. Monthly and quarterly data.
2. Span: monthly(2004:01-2020:04), quarterly (2002Q1-2019Q4)
3. Variables: Core inflation, y-on-y GDP growth, exchange rate change, one-year-ahead inflation expectations, unemployment rate Lima M.
4. Instruments: lagged values, terms of trade, Chilean and Colombian peso depreciation to USD, Geopolitical risk index, Cooper and Gold prices

Procedure

We want to estimate:

$$\pi_t = f(\pi_{t-1}, E_t \pi_{t+1}, \Delta s_t, x_t) \tag{1}$$

And we use instruments:

$$\pi_t = f(\pi_{t-1}, \hat{\pi}_t^e, \hat{\Delta} s_t, \hat{x}_t) \tag{2}$$

Namely, we use the vector $(\pi_{t-1}, \hat{\pi}_t^e, \hat{\Delta} s_t, \hat{x}_t)$ as input in the KAN network.

KAN setting

- We use the pykan python library of Liu et.al (2024)
- We estimate KAN models of the form [4,k,1]: 4 inputs, k nodes in hidden layer and 1 output.
- The simplest model is [4,1]: 4 inputs, 1 output
- Grids size = 10, k = 3, train ratio = 80%

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Results [1]

Monthly KAN with GDP growth

Model	Specification	train loss	test loss
$[4,1]$	$0.092\pi_{t-1} + 0.024\hat{\Delta s}_t + 0.118\hat{\pi}_t^e - 0.008\hat{x}_t - 0.108$	0.106	0.11
$[4,1,1]$	$0.231\pi_{t-1} + 0.018\hat{\Delta s}_t + 0.090\hat{\pi}_t^e - 0.004\hat{x}_t - 0.083$	0.111	0.09
$[4,2,1]$	$0.179\pi_{t-1} + 0.013\hat{\Delta s}_t + 0.096\hat{\pi}_t^e - 0.007\hat{x}_t - 0.079$	0.107	0.107
$[4,3,1]$	$0.182\pi_{t-1} + 0.018\hat{\Delta s}_t + 0.115\hat{\pi}_t^e - 0.007\hat{x}_t - 0.127$	0.108	0.103
$[4,4,1]$	$0.160\pi_{t-1} + 0.024\hat{\Delta s}_t + 0.080\hat{\pi}_t^e - 0.007\hat{x}_t - 0.034$	0.097	0.142

Results [1]

Monthly linear regression with GDP growth

Table: Linear regression with output growth as activity indicator

	Dependent variable:
	π_t
π_{t-1}	0.2^{**}
	(0.1)
$\hat{\Delta s}$	0.02^{*}
	(0.01)
$\hat{\pi}_t^e$	0.1^{***}
	(0.02)
$\hat{x}_t$	-0.01^{**}
	(0.003)
Constant	-0.1^{*}
	(0.04)
Observations	195
R^2	0.3
Adjusted R^2	0.3
Residual Std. Error	0.1 (df = 190)
F Statistic	19.0^{***} (df = 4; 190)
Note:	*p<0.1; **p<0.05; ***p<0.01

Results [2]

Monthly KAN with unemployment rate

Model	Specification	train loss	test loss
$[4, 1]$	$0.200\pi_{t-1} + 0.029\hat{\Delta} s_t + 0.063\hat{\pi}_t^e - 0.043\hat{x}_t + 0.319$	0.099	0.133
$[4, 1, 1]$	$0.155\pi_{t-1} + 0.015\hat{\Delta} s_t + 0.103\hat{\pi}_t^e - 0.052\hat{x}_t + 0.300$	0.106	0.112
$[4, 2, 1]$	$0.212\pi_{t-1} + 0.019\hat{\Delta} s_t + 0.077\hat{\pi}_t^e - 0.052\hat{x}_t + 0.356$	0.111	0.089
$[4, 3, 1]$	$0.218\pi_{t-1} + 0.026\hat{\Delta} s_t + 0.076\hat{\pi}_t^e - 0.062\hat{x}_t + 0.437$	0.113	0.077
$[4, 4, 1]$	$0.173\pi_{t-1} + 0.015\hat{\Delta} s_t + 0.095\hat{\pi}_t^e - 0.047\hat{x}_t + 0.283$	0.108	0.100

Results [2]

Monthly linear regression with unemployment rate

	Dependent variable:
	π_t
π_{t-1}	0.176**
	(0.071)
$\hat{\Delta} s_t$	0.021*
	(0.012)
$\hat{\pi}_t^e$	0.086***
	(0.018)
$\hat{x}_t$	−0.052**
	(0.021)
Constant	0.336*
	(0.178)
Observations	195
R^2	0.3
Adjusted R^2	0.3
Residual Std. Error	0.1 (df = 190)
F Statistic	19.0*** (df = 4; 190)
Note:	*p<0.1; **p<0.05; ***p<0.01

Results [3]

Quarterly KAN with GDP growth

Model	Specification	train loss	test loss
[4, 1]	$0.370\pi_{t-1} + 0.109\hat{\Delta s}_t + 0.098\hat{\pi}_t^e + 0.058\hat{x}_t - 0.018$	0.143	0.143
[4, 1, 1]	$0.232\pi_{t-1} + 0.111\hat{\Delta s}_t + 0.175\hat{\pi}_t^e + 0.007\hat{x}_t - 0.15$	0.146	0.131
[4, 2, 1]	$0.232\pi_{t-1} + 0.111\hat{\Delta s}_t + 0.175\hat{\pi}_t^e + 0.007\hat{x}_t - 0.15$	0.146	0.131
[4, 3, 1]	$0.232\pi_{t-1} + 0.111\hat{\Delta s}_t + 0.175\hat{\pi}_t^e + 0.007\hat{x}_t - 0.15$	0.146	0.131

Result [3]

Quarterly linear regression with GDP growth

	Dependent variable:
	π_t
π_{t-1}	0.317***
	(0.105)
$\hat{\Delta} s_t$	0.100***
	(0.036)
$\hat{\pi}_t^e$	0.148**
	(0.062)
$\hat{x}_t - \bar{x}$	0.017
	(0.072)
Constant	−0.118
	(0.147)
Observations	71
R^2	0.629
Adjusted R^2	0.607
Residual Std. Error	0.147 (df = 66)
F Statistic	28.023*** (df = 4; 66)
Note:	*p<0.1; **p<0.05; ***p<0.01

Results [4]

Quarterly KAN with unemployment rate

Model	Specification	train loss	test loss
[4, 1]	$0.173\pi_{t-1} + 0.103\hat{\Delta s}_t + 0.173\hat{\pi}^e_t - 0.037\hat{x}_t - 0.123$	0.142	0.127
[4, 1, 1]	$0.173\pi_{t-1} + 0.103\hat{\Delta s}_t + 0.173\hat{\pi}^e_t - 0.037\hat{x}_t - 0.123$	0.142	0.127
[4, 2, 1]	$0.173\pi_{t-1} + 0.103\hat{\Delta s}_t + 0.173\hat{\pi}^e_t - 0.037\hat{x}_t - 0.123$	0.142	0.127
[4, 3, 1]	$0.165\pi_{t-1} + 27.844\sin(0.037\hat{\Delta s}_t - 1.482) + 0.149\hat{\pi}^e_t - 0.043\hat{x}_t + 27.664$	0.139	0.106
[4, 4, 1]	$0.173\pi_{t-1} + 0.103\hat{\Delta s}_t + 0.173\hat{\pi}^e_t - 0.037\hat{x}_t - 0.123$	0.142	0.127

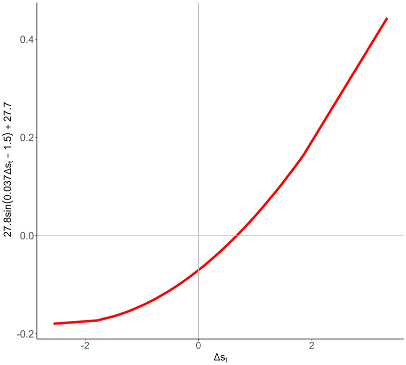
Results [4]

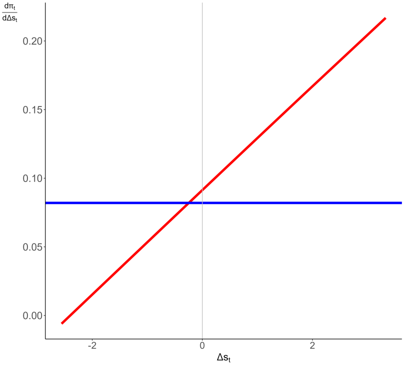
Quarterly linear regression with unemployment rate

	Dependent variable:	
	π_t	
	(1)	(2)
π_{t-1}	0.241**	0.218**
	(0.097)	(0.094)
$\hat{\Delta} s_t$	0.082***	
	(0.024)	
$\hat{\pi}_t^e$	0.154**	0.123**
	(0.059)	(0.059)
$\hat{x}_t - \bar{x}$	−0.036**	−0.041**
	(0.018)	(0.017)
$\sin(0.037\hat{\Delta} s_t - 1.482)$		26.839***
		(6.420)
Constant	−0.100	26.707***
	(0.143)	(6.471)
Observations	71	71
R^2	0.651	0.675
Adjusted R^2	0.630	0.656
Residual Std. Error (df = 66)	0.142	0.137
F Statistic (df = 4; 66)	30.827***	34.327***
Note:	*p<0.1; **p<0.05; ***p<0.01	

Nature of nonlinearity

Passthrough





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Conclusions

1. KAN framework helps discover functional nonlinearities
2. Phillips curve with unemployment rate is worth looking at
3. Quarterly Phillips curve exhibit some form of nonlinearity
   - but not in the mapping from activity or the unemployment rate but from the exchange rate changes to inflation
   - Pass-through coefficient depends on the exchange rate change and is asymmetric
4. In general, linearity as well as some forms of nonlinearity are prevalent in the data.

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