Aiming for the Bull’s Eye:
Uncertainty and Inertia in Monetary Policy

Maria Demertzis    Nicola Viegi¹

De Nederlandsche Bank

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¹University of KwaZulu-Natal
Introduction

Motivation

- Brainard Uncertainty introduces a discrepancy:
  - the role of policy is reduced (attenuation effect)
  - the role for expectations is increased

- Standard application of RE: expectations act as a "jump" variable

- Assume Differential Information (Morris and Shin 2006)

Our Contribution

- Two-Step (TS) algorithm
- Simulations: when is TS policy $\triangleright$ BR policy
The Model

New-Keynesian Economy:

\[ \pi_t = \beta E_t \pi_{t+1} + \alpha y_t + \varepsilon_t \]  

\[ y_t = E_t y_{t+1} - \gamma (i_t - E_t \pi_{t+1}) + \xi_t \]  

\[ \varepsilon_{t+1} = \rho \varepsilon_t + \nu_{t+1}, \quad 0 < \rho < 1 \]

Central Bank Preferences:

\[ L_t = E_t \sum_{i=0}^{\infty} \frac{1}{2} \beta^i \left\{ (\pi_{t+i} - \pi^*)^2 + y_{t+i}^2 \right\} \]
The Set-up

Assumptions

- IS: identity - $y$ : intermediate target,
- $\beta = 1$
- Discretionary Case

Certainty

$$\min_y L = \frac{1}{2} E \left\{ (\pi_t - \pi^*)^2 + y_t^2 \right\}$$

Brainard Uncertainty ($\pi_t = \beta E_t \pi_{t+1} + \alpha_t y_t + \epsilon_t$, $\alpha_t \rightarrow \bar{\alpha}$, $\sigma_\alpha^2$)

$$\min_y L = \frac{1}{2} E \left\{ (\bar{\pi}_t - \pi^*)^2 + y_t^2 \left(1 + \sigma_\alpha^2\right) \right\}$$
Representing the Solution

Structural Form:

\[ y_t = \ldots \pi^* - \ldots E_t \pi_{t+1} - \ldots \varepsilon_t \]  \hspace{1cm} (3)
\[ \pi_t = \ldots \pi^* + \ldots E_t \pi_{t+1} + \ldots \varepsilon_t \]  \hspace{1cm} (4)

Reduced Form:

\[ y_t = \ldots \pi^* - \ldots \varepsilon_t \]  \hspace{1cm} (5)
\[ \pi_t = \ldots \pi^* + \ldots \varepsilon_t \]  \hspace{1cm} (6)
### Table 1: The Role of Policy and Expectations

<table>
<thead>
<tr>
<th></th>
<th>$\pi^*$</th>
<th>$E_t \pi_{t+1}$</th>
<th>$\varepsilon_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Output</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Certainty</td>
<td>$\frac{\alpha}{1+\alpha^2}$</td>
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</tr>
<tr>
<td>Brainard Uncertainty</td>
<td>$\frac{1+\tilde{\alpha}^2+\sigma^2_{\tilde{\alpha}}}{1+\tilde{\alpha}^2+\sigma^2_{\tilde{\alpha}}}$</td>
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</tr>
<tr>
<td><strong>Inflation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Certainty</td>
<td>$\frac{\tilde{\alpha}^2}{1+\tilde{\alpha}^2}$</td>
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### Table 2: The Role of Policy and Expectations - Inflation

<table>
<thead>
<tr>
<th></th>
<th>( \pi^* )</th>
<th>...</th>
<th>( \varepsilon_t )</th>
</tr>
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<tbody>
<tr>
<td><strong>Certainty</strong></td>
<td>1</td>
<td>...</td>
<td>( \frac{1}{1+\alpha^2-\rho} )</td>
</tr>
<tr>
<td><strong>Brainard Uncertainty</strong></td>
<td>1</td>
<td>...</td>
<td>( \frac{1+\sigma_\alpha^2}{\bar{\alpha}^2+(1+\sigma_\alpha^2)(1-\rho)} )</td>
</tr>
</tbody>
</table>
In the presence of Multiplicative Uncertainty:
- Policy does less (Brainard Attenuation Effect)
- Expectations do more (Enhanced contribution)

But,
- Standard RE eliminates this shift of emphasis

Extension:
- Look for alternative Expectations
Alternative Expectations

Differential Information Morris and Shin (AER, 2006):

- **Set-up**
Alternative Expectations
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  \[ \pi_t = \begin{cases} 
  \pi_0 & \text{for } t = 0 \\
  \pi^* & \text{for } t \geq 1 
  \end{cases} \]
Alternative Expectations

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- \( \mu(<1) \rightarrow \pi^* \quad (1-\mu) \rightarrow \pi_0 \quad \mu_s \rightarrow 1, \text{ as } t \rightarrow \infty \)
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  \[ \bar{E}_1 \bar{E}_2 \ldots \bar{E}_t (\pi_{t+h}) = \left(1 - \prod_{s=1}^{t} \mu_s\right) \pi_0 + \left(\prod_{s=1}^{t} \mu_s\right) \pi^* \]
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  - \( \lim_{k \to \infty} \bar{E}_{t-k} \bar{E}_{t-k+1} \ldots \bar{E}_t (\pi_{t+1}) = \pi_0 \)
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  \end{cases} \)
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  - \( \mu_s \rightarrow 1, \text{ as } t \rightarrow \infty \)
  - \( \bar{E}_1 \bar{E}_2 \ldots \bar{E}_t (\pi_{t+h}) = \left(1 - \prod_{s=1}^{t} \mu_s\right) \pi_0 + \left(\prod_{s=1}^{t} \mu_s\right) \pi^* \)
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- **Intuition**

Monetary Policy is a coordination game
For coordination games, common knowledge is important

Research (De Nederlandsche Bank) Monetary Policy under Uncertainty 11/06 9 / 18
Alternative Expectations
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- **Intuition**
  - Monetary Policy is a coordination game
  - For coordination games, **common knowledge** is important
Alternative Expectations (2)
Differential Information Morris and Shin (2006):

Reduced Form Solutions:

\[
\pi^C_E_t = \frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2} \pi^* + \frac{1}{1 + \bar{\alpha}^2} \pi_0 + \frac{1}{1 + \bar{\alpha}^2} \varepsilon_t
\]  

\[
\pi^B_R_t = \frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2 + \sigma^2_{\alpha}} \pi^* + \frac{1 + \sigma^2_{\alpha}}{1 + \bar{\alpha}^2 + \sigma^2_{\alpha}} \pi_0 + \frac{1 + \sigma^2_{\alpha}}{1 + \bar{\alpha}^2 + \sigma^2_{\alpha}} \varepsilon_t
\]

Limiting Case:

\[
\lim_{\sigma^2_{\alpha} \to \infty} \pi_t = \pi_0 + \varepsilon_t
\]

This implies that in the presence of uncertainty, it becomes increasingly difficult for policy to achieve its objective and the system is characterised by full inertia.
Two-Step Algorithm

1st Step: Calculate $\theta$

$$\theta$$

$$\pi_0 \quad \pi^* \quad \pi^* + \theta$$

2nd Step: Calculate policy action

$$\pi = \pi^*$$
"Applying a two-step procedure in which $\theta$ is contingent on the shocks that hit the economy, the existing uncertainty and the inflation target, neutralises the ex ante effects of uncertainty on the policy rules."

\[
\pi_t = \frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2} \pi^* + \frac{1}{1 + \bar{\alpha}^2} (E_t \pi_{t+1} + \epsilon_t)
\]

Inflation Expectations

\[E_t \pi_{t+1} = \left[ \left( 1 - \prod_{s=1}^{t} \mu_s \right) \pi_0 + \left( \prod_{s=1}^{t} \mu_s \right) \pi^* \right]\]

and as $\mu_s < 1$, and therefore, $E_t \pi_{t+1} = \pi_0$
A Comparison: Inflation Outcome

Table 3: The Role of Policy and Expectations

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</tr>
<tr>
<td>Two-Step</td>
<td>$\frac{\tilde{\alpha}^2}{1+\tilde{\alpha}^2}$</td>
<td>$\frac{1}{1+\tilde{\alpha}^2}$</td>
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</table>
Numerical Simulations

Set-up
\[ \pi_0 = 0, \quad \pi^* = 1, \quad \beta = 0.99, \quad \alpha \sim N \left(0.5, \sigma^2_\alpha\right), \quad \rho = 0.8 \]

Instruments
\[
\begin{align*}
    y_t^{BR} &= \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma^2_\alpha} \pi^* - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma^2_\alpha} E_t \pi_{t+1} - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma^2_\alpha} \epsilon_t \\
    y_t^{TS} &= \frac{\bar{\alpha}}{1 + \bar{\alpha}^2} \pi^* - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2} E_t \pi_{t+1} - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2} \epsilon_t
\end{align*}
\]

Simulations
\[ \pi_t^j = \beta E_t \pi_{t+1} + \alpha_i y_t^j + \epsilon_t \quad j = BR, TS \]
Output Gap

Figure: Output Gap - Typical Path
Figure: Inflation - Typical Path
10,000 stochastic simulations

\[ L_{j,t} = \frac{1}{2} \left\{ \left( \pi_t^j - \pi^* \right)^2 + \left( y_t^j \right)^2 \right\} \]

Table 4. First Period Losses

<table>
<thead>
<tr>
<th>CV</th>
<th>( L_{BR} )</th>
<th>( L_{TS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>11.9</td>
<td>12.1</td>
</tr>
<tr>
<td>1</td>
<td>11.6</td>
<td>12.2</td>
</tr>
<tr>
<td>1.5</td>
<td>11.4</td>
<td>12.7</td>
</tr>
</tbody>
</table>

\[ \sum_{t=1}^{n} \beta^t L_{j,t} \]

Table 5. Cum. Losses (\( n = 10 \))

<table>
<thead>
<tr>
<th>CV</th>
<th>( L_{BR} )</th>
<th>( L_{TS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>260.7</td>
<td>252.7</td>
</tr>
<tr>
<td>1</td>
<td>316.6</td>
<td>292.1</td>
</tr>
<tr>
<td>1.5</td>
<td>573.1</td>
<td>1032.0</td>
</tr>
</tbody>
</table>
Conclusions

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- small levels of uncertainty: TS
Conclusions

- With uncertainty, policy does less, expectations do more
- If expectations are "wrong", then policy is ineffective
- Two-Step algorithm that aims at hitting the target
- Sacrifices the first period, optimises end result
- small levels of uncertainty: TS
- high levels of uncertainty: BR