Aiming for the Bull's Eye:

Uncertainty and Inertia in Monetary Policy

Maria Demertzis   Nicola Viegi[1]

De Nederlandsche Bank

November 2006

[1] University of KwaZulu-Natal

Introduction

Motivation

- Brainard Uncertainty introduces a discrepancy:
  - the role of policy is reduced (attenuation effect)
  - the role for expectations is increased
- Standard application of RE: expectations act as a "jump" variable
- Assume Differential Information (Morris and Shin 2006)

Our Contribution

- Two-Step (TS) algorithm
- Simulations: when is TS policy $\succ$ BR policy

The Model

New-Keynesian Economy:

$$\pi_t = \beta E_t \pi_{t+1} + \alpha y_t + \varepsilon_t \quad (1)$$

$$y_t = E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right) + \xi_t \quad (2)$$

$$\varepsilon_{t+1} = \rho \varepsilon_t + v_{t+1}, \qquad 0 < \rho < 1$$

Central Bank Preferences:

$$L_t = E_t \sum_{i=0}^{\infty} \frac{1}{2} \beta^i \left\{ \left(\pi_{t+i} - \pi^* \right)^2 + y_{t+i}^2 \right\}$$

The Set-up

Assumptions

- IS: identity - y : intermediate target,
- $\beta = 1$
- Discretionary Case

Certainty

$$\min_{y} L = \frac{1}{2} E \left\{ (\pi_t - \pi^*)^2 + y_t^2 \right\}$$

Brainard Uncertainty ($\pi_t = \beta E_t \pi_{t+1} + \alpha_t y_t + \varepsilon_t, \quad \alpha_t \rightarrow \bar{\alpha}, \sigma_\alpha^2$)

$$\min_{y} L = \frac{1}{2} E \left\{ (\bar{\pi}_t - \pi^*)^2 + y_t^2 \left(1 + \sigma_\alpha^2\right) \right\}$$

Representing the Solution

Structural Form:

$$y_t = (\ldots)\,\pi^* - (\ldots)\,E_t\pi_{t+1} - (\ldots)\,\varepsilon_t \quad (3)$$

$$\pi_t = (\ldots)\,\pi^* + (\ldots)\,E_t\pi_{t+1} + (\ldots)\,\varepsilon_t \quad (4)$$

Reduced Form:

$$y_t = (\ldots)\,\pi^* - (\ldots)\,\varepsilon_t \quad (5)$$

$$\pi_t = (\ldots)\,\pi^* + (\ldots)\,\varepsilon_t \quad (6)$$

Table 1: The Role of Policy and Expectations

		π^*	$E_t\pi_{t+1}$	ε_t
	Output			
Certainty		$\frac{\alpha}{1+\alpha^2}$	$\frac{\alpha}{1+\alpha^2}$	$\frac{\alpha}{1+\alpha^2}$
Brainard Uncertainty		$\frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}$	$\frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}$	$\frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}$
	Inflation			
Certainty		$\frac{\alpha^2}{1+\alpha^2}$	$\frac{1}{1+\alpha^2}$	$\frac{1}{1+\alpha^2}$
Brainard Uncertainty		$\frac{\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}$	$\frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}$	$\frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}$

Table 2: The Role of Policy and Expectations - Inflation

	π^*	...	ε_t
Certainty	1	...	$\frac{1}{1+\alpha^2-\rho}$
Brainard Uncertainty	1	...	$\frac{1+\sigma_\alpha^2}{\bar{\alpha}^2+(1+\sigma_\alpha^2)(1-\rho)}$

Summary

- In the presence of Multiplicative Uncertainty:
  - Policy does less (Brainard Attenuation Effect)
  - Expectations do more (Enhanced contribution)

But,

- Standard RE eliminates this shift of emphasis

Extension:

- Look for alternative Expectations

Alternative Expectations

Differential Information Morris and Shin (AER, 2006):

- *Set-up*

Alternative Expectations

Differential Information Morris and Shin (AER, 2006):

- *Set-up*
  - $\pi_t = \begin{cases} \pi_0 & \text{for } t = 0 \\ \pi^* & \text{for } t \geq 1 \end{cases}$

Alternative Expectations

Differential Information Morris and Shin (AER, 2006):

- *Set-up*
  - $\pi_t = \begin{cases} \pi_0 & \text{for } t = 0 \\ \pi^* & \text{for } t \geq 1 \end{cases}$
  - $\mu(< 1) \longrightarrow \pi^* \qquad (1 - \mu) \longrightarrow \pi_0 \qquad \mu_s \rightarrow 1, \text{ as } t \rightarrow \infty$

- *Set-up*
  - $\pi_t = \begin{cases} \pi_0 & \text{for } t = 0 \\ \pi^* & \text{for } t \geq 1 \end{cases}$
  - $\mu(< 1) \longrightarrow \pi^* \qquad (1-\mu) \longrightarrow \pi_0 \qquad \mu_s \rightarrow 1, \text{ as } t \rightarrow \infty$
  - $\bar{E}_1 \bar{E}_2 ... \bar{E}_t \left(\pi_{t+h}\right) = \left(1 - \prod_{s=1}^{t} \mu_s\right) \pi_0 + \left(\prod_{s=1}^{t} \mu_s\right) \pi^*$

Alternative Expectations

Differential Information Morris and Shin (AER, 2006):

- *Set-up*
  - $\pi_t = \begin{cases} \pi_0 & \text{for } t = 0 \\ \pi^* & \text{for } t \geq 1 \end{cases}$
  - $\mu(<1) \longrightarrow \pi^* \qquad (1-\mu) \longrightarrow \pi_0 \qquad \mu_s \rightarrow 1, \text{ as } t \rightarrow \infty$
  - $\bar{E}_1 \bar{E}_2 ... \bar{E}_t \left(\pi_{t+h}\right) = \left(1 - \prod_{s=1}^{t} \mu_s\right) \pi_0 + \left(\prod_{s=1}^{t} \mu_s\right) \pi^*$
  - $\lim_{k\rightarrow\infty} \bar{E}_{t-k} \bar{E}_{t-k+1} ... \bar{E}_t \left(\pi_{t+1}\right) = \pi_0$

Alternative Expectations

Differential Information Morris and Shin (AER, 2006):

- *Set-up*
  - $\pi_t = \begin{cases} \pi_0 & \text{for } t = 0 \\ \pi^* & \text{for } t \geq 1 \end{cases}$
  - $\mu(< 1) \longrightarrow \pi^* \qquad (1-\mu) \longrightarrow \pi_0 \qquad \mu_s \to 1$, as $t \to \infty$
  - $\bar{E}_1 \bar{E}_2 ... \bar{E}_t \left(\pi_{t+h}\right) = \left(1 - \prod_{s=1}^{t} \mu_s\right) \pi_0 + \left(\prod_{s=1}^{t} \mu_s\right) \pi^*$
  - $\lim_{k\to\infty} \bar{E}_{t-k} \bar{E}_{t-k+1} ... \bar{E}_t \left(\pi_{t+1}\right) = \pi_0$
- *Intuition*

Alternative Expectations

Differential Information Morris and Shin (AER, 2006):

- *Set-up*
  - $\pi_t = \begin{cases} \pi_0 & \text{for } t = 0 \\ \pi^* & \text{for } t \geq 1 \end{cases}$
  - $\mu(< 1) \longrightarrow \pi^* \qquad (1-\mu) \longrightarrow \pi_0 \qquad \mu_s \rightarrow 1, \text{ as } t \rightarrow \infty$
  - $\bar{E}_1 \bar{E}_2 ... \bar{E}_t \left(\pi_{t+h}\right) = \left(1 - \prod_{s=1}^{t} \mu_s\right) \pi_0 + \left(\prod_{s=1}^{t} \mu_s\right) \pi^*$
  - $\lim_{k\rightarrow\infty} \bar{E}_{t-k} \bar{E}_{t-k+1} ... \bar{E}_t \left(\pi_{t+1}\right) = \pi_0$
- *Intuition*
  - Monetary Policy is a coordination game

Alternative Expectations

Differential Information Morris and Shin (AER, 2006):

- *Set-up*
  - $\pi_t = \begin{cases} \pi_0 & \text{for } t = 0 \\ \pi^* & \text{for } t \geq 1 \end{cases}$
  - $\mu(< 1) \longrightarrow \pi^* \qquad (1-\mu) \longrightarrow \pi_0 \qquad \mu_s \rightarrow 1$, as $t \rightarrow \infty$
  - $\bar{E}_1 \bar{E}_2 ... \bar{E}_t \left(\pi_{t+h}\right) = \left(1 - \prod_{s=1}^{t} \mu_s\right) \pi_0 + \left(\prod_{s=1}^{t} \mu_s\right) \pi^*$
  - $\lim_{k \to \infty} \bar{E}_{t-k} \bar{E}_{t-k+1} ... \bar{E}_t \left(\pi_{t+1}\right) = \pi_0$
- *Intuition*
  - Monetary Policy is a coordination game
  - For coordination games, **common knowledge** is important

Alternative Expectations (2)

Differential Information Morris and Shin (2006):

Reduced Form Solutions:

$$\pi_t^{CE} = \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2}\pi^* + \frac{1}{1+\bar{\alpha}^2}\pi_0 + \frac{1}{1+\bar{\alpha}^2}\varepsilon_t \quad (7)$$

$$\pi_t^{BR} = \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi^* + \frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi_0 + \frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\varepsilon_t \quad (8)$$

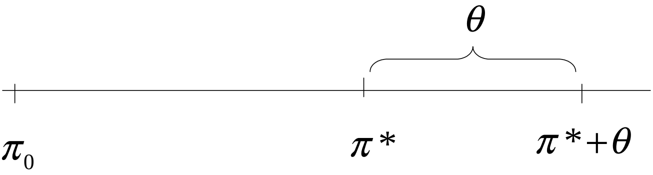
Limiting Case:

$$\lim_{\sigma_\alpha^2 \to \infty} \pi_t = \pi_0 + \varepsilon_t$$

This implies that in the presence of uncertainty, it becomes increasingly difficult for policy to achieve its objective and the system is characterised by full inertia.

Two-Step Algorithm

1st Step: Calculate θ



2nd Step: Calculate policy action

$$\pi = \pi^*$$

Two-Step (2)

"Applying a two-step procedure in which θ is contingent on the shocks that hit the economy, the existing uncertainty and the inflation target, neutralises the ex ante effects of uncertainty on the policy rules"

$$\pi_t = \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2}\pi^* + \frac{1}{1+\bar{\alpha}^2}\left(E_t \pi_{t+1} + \varepsilon_t\right)$$

Inflation Expectations

$$E_t \pi_{t+1} = \left[\left(1 - \prod_{s=1}^{t} \mu_s\right)\pi_0 + \left(\prod_{s=1}^{t} \mu_s\right)\pi^*\right]$$

and as $\mu_s < 1$, and therefore, $E_t \pi_{t+1} = \pi_0$

A Comparison: Inflation Outcome

Table 3: The Role of Policy and Expectations

	π^*	π_0	ε_t
Certainty	$\frac{\alpha^2}{1+\alpha^2}$	$\frac{1}{1+\alpha^2}$	$\frac{1}{1+\alpha^2}$
Brainard Uncertainty	$\frac{\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}$	$\frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}$	$\frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}$
Two-Step	$\frac{\bar{\alpha}^2}{1+\bar{\alpha}^2}$	$\frac{1}{1+\bar{\alpha}^2}$	$\frac{1}{1+\bar{\alpha}^2}$

Numerical Simulations

Set-up

$$\pi_0 = 0, \qquad \pi^* = 1, \qquad \beta = 0.99, \qquad \alpha \simeq N\left(0.5, \sigma_\alpha^2\right), \qquad \rho = 0.8$$

Instruments

$$\begin{aligned}
y_t^{BR} &= \frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi^* - \frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}E_t\pi_{t+1} - \frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}\varepsilon_t \\
y_t^{TS} &= \frac{\bar{\alpha}}{1+\bar{\alpha}^2}\pi^* - \frac{\bar{\alpha}}{1+\bar{\alpha}^2}E_t\pi_{t+1} - \frac{\bar{\alpha}}{1+\bar{\alpha}^2}\varepsilon_t
\end{aligned}$$

Simulations

$$\pi_t^j = \beta E_t\pi_{t+1} + \alpha_i y_t^j + \varepsilon_t \qquad j = BR, TS$$

Output Gap

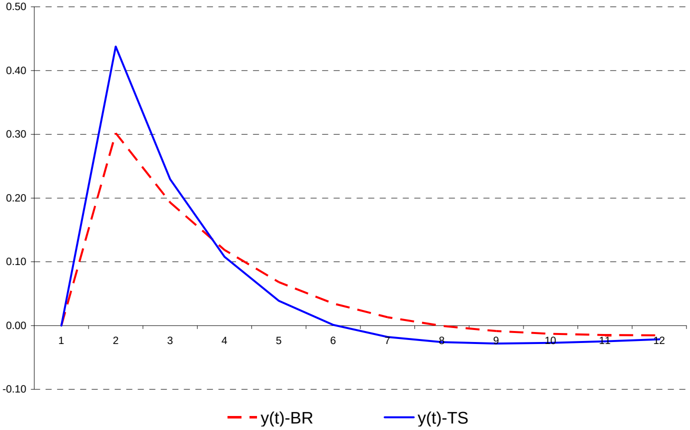


Figure: Output Gap - Typical Path

Inflation

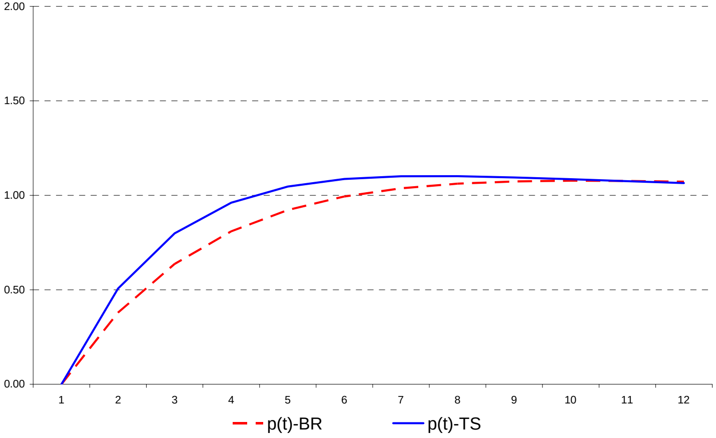


Figure: Inflation - Typical Path

Expected Losses

10,000 stochastic simulations

$$L_{j,t} = \frac{1}{2}\left\{\left(\pi_t^j - \pi^*\right)^2 + \left(y_t^j\right)^2\right\}$$

Table 4. First Period Losses

CV	L_{BR}	L_{TS}
0.5	11.9	12.1
1	11.6	12.2
1.5	11.4	12.7

$$\sum_{t=1}^{n} \beta^t L_{j,t}$$

Table 5. Cum. Losses ($n = 10$)

CV	L_{BR}	L_{TS}
0.5	260.7	252.7
1	316.6	292.1
1.5	573.1	1032.0

Conclusions

- With uncertainty, policy does less, expectations do more

Conclusions

- With uncertainty, policy does less, expectations do more
- If expectations are "wrong", then policy is ineffective

Conclusions

- With uncertainty, policy does less, expectations do more
- If expectations are "wrong", then policy is ineffective
- Two-Step algorithm that aims at hitting the target

Conclusions

- With uncertainty, policy does less, expectations do more
- If expectations are "wrong", then policy is ineffective
- Two-Step algorithm that aims at hitting the target
- Sacrifices the first period, optimises end result

Conclusions

- With uncertainty, policy does less, expectations do more
- If expectations are "wrong", then policy is ineffective
- Two-Step algorithm that aims at hitting the target
- Sacrifices the first period, optimises end result
- small levels of uncertainty: TS

Conclusions

- With uncertainty, policy does less, expectations do more
- If expectations are "wrong", then policy is ineffective
- Two-Step algorithm that aims at hitting the target
- Sacrifices the first period, optimises end result
- small levels of uncertainty: TS
- high levels of uncertainty: BR