Aiming for the Bull's Eye: Uncertainty and Inertia in Monetary Policy*

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Abstract

We study the implications of uncertainty on the Central Bank's ability to achieve its objectives. Assuming multiplicative uncertainty in a standard forward looking model, we show that while the role for policy is reduced in line with Brainard's attenuation effect, at the same time, the contribution of expectations to the final outcome increases. In the context of a discretionary set-up, if individuals are subject to differential information according to which expectations exhibit inertia, then the system takes a long time to align with the CB's inflation objective following a shock. With this in mind, we thus look for an algorithm that concentrates on removing inertia and thus bring expectations back on target. To this end, we put forward a two-step algorithm in which the inflation target is state contingent. The Central Bank sets (as an auxiliary step) a variable inflation target that depends on both the degree of uncertainty, as well as the shocks that occur each time. We show that such an algorithm increases the level of variability in the system but for small levels of uncertainty the cumulative benefits of pinning down expectations more than compensate the costs of having to overuse the instrument. We demonstrate this through Monte Carlo Simulations.

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1 Introduction

The benefits of inflation targeting in the Svensson (1999) sense amount to providing a nominal anchor for the private sector to infer policies with, in order to formulate expectations with greater accuracy. For the Central Bank (CB), on the other hand, inflation targeting provides an implicit commitment mechanism which increases its cost of deviating from announced targets and hence discourages it from doing so. The economy on the whole benefits from greater transparency because it leads to greater credibility and by consequence to effective monetary policies. From a political economy standpoint therefore, the literature associates the concept of inflation targeting with greater transparency and hence with more credible and effective policies. By the same token, a central bank that fails to achieve the target that it sets (and announces) will be penalised with a loss in credibility and hence a subsequent reduction in the ability to pursue its objectives. "It appears that for monetary policy makers, announcements alone are not enough; the only way to gain credibility is to earn it", (Bernanke and Mishkin, 1997).

Our paper is motivated from the importance that inflation targeting puts on achieving the monetary objective announced. We analyse this issue in the context of an economy that is characterised by parameter uncertainty as modelled by Brainard, (1967). In describing the attenuation effect he puts forward, we observe that as the contribution of policy to the final inflation outcome reduces in the presence of multiplicative uncertainty, that of expectations increases proportionally to the prevailing degree of the specified uncertainty. Naturally, if the Central Bank operates under commitment, expectations are anchored by the level of inflation the Central Bank aims at. In a discretionary environment where expectations are parametric to the CB's actions, it is then the assumption of rational expectations (RE) that "forces" the private sector to adjust their expectations such that the outcome is consistent with the intentions of the Central Bank. The discrepancy in the way policy and expectations affect the final outcome is therefore redundant. However, if one departs from a rigid application of the RE paradigm, then multiplicative uncertainty can seriously compromise the CB's ability to attain its objectives. To demonstrate this we allow private agents to be subject to differential information (Morris and Shin, 2006). This implies that even if a very small proportion of people are backward-looking in the way they form expectations, their beliefs prevail and average expectations are also backward-looking. This in turn prevents policy from closing the gap between current inflation and the CB's objective. We will add to that, that this inability is made worse in the presence of uncertainty and therefore, attaining the target becomes increasingly more difficult. We analyse two issues: *first*, if there is some value in attaining the target as inflation targeting proponents advocate, then we aim to find an algorithm that will both manage to achieve it on average, as well as still operate in an optimisation framework, so that the procedure remains transparent to the public. We thus identify a two-step algorithm according to which, in the first step, the central bank deviates from the target in order to reactivate the instrument and only in the second step, does

it aim for the actual target itself. The two-step procedure amounts therefore, to the Central bank aiming *for* the bull's eye and not directly *at* it. *Second*, we identify the conditions of uncertainty under which such an algorithm can prove superior in welfare terms to the Brainard result. This is important in an inflation targeting framework as announcing a target that is unlikely to be achieved is not necessarily increasing one's credibility (Posen, 2002).
The paper is organised as follows. Sections 2 discusses the model under certainty and multiplicative uncertainty respectively. Section 3 introduces the concept of differential information and how it affects private agents' expectations. Section 4 then derives a two-step algorithm to inflation targeting and with the aid of numerical simulations, section 5 discusses when such an algorithm is beneficial. Section 6 concludes.

2 The Role of Expectations

Most of the attempts to examine the effects of uncertainty in a dynamic framework rely on a backward-looking set-up (Söderstöm, 2002, Srour, 1999). The somehow surprising result, from the point of view of inflation targeting proponents, is that uncertainty in the structure of the economy implies that achieving the target is not optimal as it may lead to instability in the system. This seems at odds with the general perception that the main advantage of inflation targeting is that it stabilises expectations (see the empirical evidence in Johnson, 2002 and Levin et al, 2003). The contradiction is only apparent: controlling expectations is relevant only if private sector expectations are an important determinant of the economic outcome. Thus the right set-up to test the effect of model uncertainty on the Central Bank's ability to achieve its inflation objective is a forward-looking model. We apply therefore, a standard New Keynesian model as described in Clarida Gali and Getler (1999) and Woodford (2004) and used in a similar context by Giannoni (2002) in which expectations play again a central role (Woodford, 2003). Following Clarida, Gali and Getler (1999), the economy is thus described by the following pair of log-linear relations in deviation from their steady state:

$$\pi_t = \beta E_t \pi_{t+1} + \alpha y_t + \varepsilon_t \quad (1)$$
$$y_t = E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right) + \xi_t \quad (2)$$

where (1) is an expectations-augmented "AS" relation in which present inflation is a function of private sector inflation expectations one period ahead, and (2) is an intertemporal "IS" relation. Notation follows convention and coefficients satisfy, $\alpha, \gamma > 0$. Supply shocks are uncorrelated autoregressive processes, i.e.:

$$\varepsilon_{t+1} = \rho \varepsilon_t + v_{t+1}, \qquad 0 < \rho < 1$$

and v_{t+1} has a zero mean and constant variance. We solve under the AS constraint in which output gap is considered the intermediate instrument. We thus abstract, for simplicity, from the issue of actual monetary policy transmission. The Central Bank minimises the following objective function:

$$\min_y L_t = \frac{1}{2} E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left\{ (\pi_{t+\tau} - \pi^*)^2 + y_{t+\tau}^2 \right\} \tag{3}$$

and the discount factor is equal to one, i.e. $\beta = 1$[1]. The central bank announces that it aims at a level of inflation equal to π^*, known as the inflation target. In evaluating policy, expectations are treated as parametric (Currie and Levine 1999) and the time-consistent discretionary solution reduces to a period-by-period optimization of the loss function under and (1) and (2), or in other words:

$$\min_y L = \frac{1}{2} E \left\{ (\pi_t - \pi^*)^2 + y_t^2 \right\} \tag{4}$$

Assume now that there is limited knowledge about the monetary transmission mechanism. Similar to Brainard's contribution this is represented by coefficient α in the AS equation being stochastic i.e.: $\alpha_t \to N\left(\bar{a}, \sigma_\alpha^2\right)$[2] and has come to be knows as Brainard Uncertainty. The existence of such uncertainty implies that the objective function (3) can now be expressed in terms of the first and second moment of the uncertain terms (see Appendix B for a detailed derivation):

$$\min_y L = \frac{1}{2} E \left\{ (\bar{\pi}_t - \pi^*)^2 + y_t^2 \left(1 + \sigma_\alpha^2\right) \right\} \tag{5}$$

where $\bar{\pi}_t = E_t \pi_{t+1} + \bar{\alpha} y_t + \varepsilon_t$. The term $y_t^2 \sigma_\alpha^2$ now represents the extra cost that the CB incurs as a result of facing uncertainty in the parameter structure of the model. Again expectations are treated as parametric and the discretionary solution reduces the problem to a period-by-period optimization of the loss function (5) subject to (1). We compare next how the existence of such uncertainty affects the results. We optimise thus respectively (4) and (5). The structural representation of the results with **Certainty** (when α is treated as a parameter) is then:

$$y_t^{CE} = \frac{\alpha}{1+\alpha^2}\pi^* + \frac{\alpha}{1+\alpha^2} E_t \pi_{t+1} + \frac{\alpha}{1+\alpha^2} \varepsilon_t \tag{6}$$

$$\pi_t^{CE} = \frac{\alpha^2}{1+\alpha^2}\pi^* + \frac{1}{1+\alpha^2} E_t \pi_{t+1} + \frac{1}{1+\alpha^2} \varepsilon_t \tag{7}$$

By contrast, the results under **Brainard Uncertainty** are:

[1] See Appendix A for the general solution of what follows when $0 < \beta < 1$.

[2] Note, that uncertainty in α is qualitatively equivalent to uncertainty in γ when one wishes to examine the impact of supply shocks. Walsh 2003 shows that if the central bank had an interest rate term in its objective function and examined the effect of a quasi-demand shock (real interest rate shock), then the uncertainty in either α or γ would have qualitatively different implications on policy implementation.

$$y_t^{BR} = \frac{\alpha}{1+\alpha^2+\sigma_\alpha^2}\pi^* + \frac{\alpha}{1+\alpha^2+\sigma_\alpha^2}E_t\pi_{t+1} + \frac{\alpha}{1+\alpha^2+\sigma_\alpha^2}\varepsilon_t \quad (8)$$

$$\pi_t^{BR} = \frac{\alpha^2}{1+\alpha^2+\sigma_\alpha^2}\pi^* + \frac{1+\sigma_\alpha^2}{1+\alpha^2+\sigma_\alpha^2}E_t\pi_{t+1} + \frac{1+\sigma_\alpha^2}{1+\alpha^2+\sigma_\alpha^2}\varepsilon_t \quad (9)$$

We observe that the existence of parameter uncertainty implies the following things:

- As uncertainty increases ($\sigma_\alpha^2 \to \infty$), the instrument ($y_t$) is used less and less, in line with Brainard's classical attenuation effect, from (8).
- As a consequence, the relative contribution of policy to the inflation outcome (coefficient of π^* in (9)) reduces in uncertainty.
- By contrast, that of expectations (coefficient of $E_t\pi_{t+1}$ in (9)) increases in the level of prevailing uncertainty. At the limit when uncertainty is infinite ($\sigma_\alpha^2 \to \infty$), it is straightforward to show that the target of the Central Bank becomes irrelevant and all that matters is private sector expectations (naturally shocks always play a role).

Note that these observations hold for the *structural* representation of the solutions. Moving next to the *reduced* form solutions where expectations are solved for with recursive substitution, the discrepancy in the way uncertainty affects the role of policy and expectations is eliminated. The inflation outcome is now, respectively for the two cases:

$$\tilde{\pi}_t^{CE} = \pi^* + \frac{1}{1+\alpha^2-\rho}\varepsilon_t \quad (10)$$

$$\tilde{\pi}_t^{BR} = \pi^* + \frac{1+\sigma_\alpha^2}{\bar{\alpha}^2+(1+\sigma_\alpha^2)(1-\rho)}\varepsilon_t \quad (11)$$

and the Central Bank is able to achieve its objective π^* (but for the supply error and its persistence). Imposing rational expectations in a standard way ensures therefore, that expectations $E_t\pi_{t+1}$ act as a 'jump' variable that always moves to compensate for any shortcomings in the policy action and bring inflation in line with the objective. But this also obfuscates the effect of uncertainty on the relative importance of expectations. We attempt to address this issue next.

3 The Formation of Expectations

For the relevance of expectations in the presence of uncertainty to come to the fore we need to identify an expectation formation process that prevents this immediate adjustment to the desired level. To this end, we apply the concept of *Differential Information* as presented by Morris and Shin (2006). This latter formulation is an attractive way of capturing this issue because it allows for individuals to have different information about the relevant level of inflation, while still operating within the context of rational expectations.

3.1 Differential Information and Expectations

The concept of *Differential Information* builds on previous work by Morris and Shin (Morris and Shin, 2002a and 2002b). When applied to monetary policy the idea behind this is as follows: the current state of inflation is π_0 but the Central Bank wants to move it to a new level π^*. Inflation state π at time t evolves in the following way:

$$\pi_t = \begin{cases} \pi_0 & \text{for } t = 0 \\ \pi^* & \text{for } t \geq 1 \end{cases}$$

However, in the eyes of those who form expectations, the assumption of differential information implies that at time $t \geq 1$ only a proportion of the private sector μ knows the value of π_t and automatically adjusts expectations to that level (π^*), whereas, $1 - \mu$ does not. The authors assume that the proportion of people, $(1 - \mu)$, that are effectively "unaware" of this target is very small to start with (such that μ is very close but not quite one) and diminishes from period to period, such that eventually everyone adjusts their expectation in line with the target. This latter assumption implies that their approach is consistent with Rational Expectations. The authors then derive the way expectations are formed for the two groups through forward iterations (described in detail in Appendix C). They show that expected inflation h periods ahead is

$$\bar{E}_1 \bar{E}_2 ... \bar{E}_t \left(\pi_{t+h}\right) = \left(1 - \prod_{s=1}^{t} \mu_s\right) \pi_0 + \left(\prod_{s=1}^{t} \mu_s\right) \pi^* \tag{12}$$

Note that in the absence of differential information ($\mu = 1$) iterated expectations collapse to the single expectation at time t, which under RE is equal to π^*. However for $\mu < 1$, the higher order expectations for a given timing $t+h$ (where $h > 0$) depends on the limiting property of $\prod_{s=1}^{t} \mu_s$. And if μ_s approaches one only when $t \to \infty$, then it follows that $\prod_{s=1}^{t} \mu_s \to 0$ (even if μ_s is very close to one to start with) and the current level of inflation π_0 prevails in (12). The intuition behind this stems from the fact that monetary policy is an information game between the central bank and the private agents but also between the private agents themselves[3]. The element of coordination between the agents is important in the process of forming expectations and second-guessing how others think is crucial to one's decision. In the presence of differential information, those that are aware of the target are also aware that there is a very small minority that is not and will therefore form expectations according to the current level of inflation. Knowing that, the desire to coordinate "forces" them to match their expectations to those of the least informed group, namely at π_0. The existence of such small minority of people implies that π^* can therefore never be attained. The effect of such an assumption is that while the economy is forward-looking, the system effectively operates as though it were backward-looking.

[3] See Demertzis and Viegi (2006) for a detailed description of monetary policy as an information game.

In order to allow for differential information in the context of our model, inflation expectation $E_t\pi_{t+1}$ is now proxied by its higher order equivalent, i.e.:

$$\bar{E}_{t-k}\bar{E}_{t-k+1}...\bar{E}_t\left(\pi_{t+1}\right) = \left(1 - \prod_{s=t-k}^{t}\mu_s\right)\pi_0 + \left(\prod_{s=t-k}^{t}\mu_s\right)\pi^* \tag{13}$$

What (13) then tells us is that the longer it takes for μ to converge to 1 (k is a large number), the closer the expectations remains to the original level of inflation π_0. Substituting then (13) in (9) we can now see that this effect is exacerbated in the presence of uncertainty.

$$\pi_t = \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi^* + \frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\left[\left(1 - \prod_{s=t-k}^{t}\mu_s\right)\pi_0 + \left(\prod_{s=t-k}^{t}\mu_s\right)\pi^*\right] \tag{14}$$
$$+\frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\varepsilon_t$$

For demonstration purposes only we will consider here the limit case when $k \to \infty$, or in other words, when agents take a very long time to update their beliefs and expectations are therefore, purely backward-looking:

$$\lim_{k\to\infty}\bar{E}_{t-k}\bar{E}_{t-k+1}...\bar{E}_t\left(\pi_{t+1}\right) = \pi_0 \tag{15}$$

Substituting then (15) into (14) produces:

$$\pi_t = \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi^* + \frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi_0 + \frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\varepsilon_t \tag{16}$$

This shows that the system exhibits inertia and therefore, the ability of the Central Bank to achieve its objectives is seriously hindered. The inflation outcome will therefore, not close the whole distance between π_0 and π^*. It is then straight forward to illustrate from (16) the role of uncertainty in exacerbating this discrepancy. Even if a little differentiated information is introduced, the existence of uncertainty emphasises the role of expectations in terms of determining the outcome and de-emphasises that of policy. At the limit when uncertainty is infinite, the central bank is unable to move away from the current level of inflation. In other words,

$$\lim_{\sigma_\alpha^2\to\infty}\pi_t = \pi_0 + \varepsilon_t \tag{17}$$

This implies that in the presence of uncertainty, it becomes increasingly difficult for policy to achieve its inflation objective and the system is characterised by even greater inertia than is due to just differential information.

4 Two-Step Inflation Targeting

We have thus shown so far that the presence of Brainard uncertainty exacerbates the degree of inertia due to differential information and at the limit immobilises policy. In turn, this also implies that the inflation objective becomes more and more difficult to achieve, which lends itself to the question of what use is a quantitative objective if it is seldom achieved. Our objective as a consequence, is to identify an algorithm that, given the set-up assumed, reactivates the relevance of policy and aims explicitly at attaining the target. Achieving that, we can then evaluate whether and when such an algorithm actually does better that the Brainard solution shown above and why.

We derive an algorithm based on the following rationale. We know from (16) that the attenuated policy will close only part of the distance between the current level of inflation π_0 and the target π^*, say $k(\pi^* - \pi_0)$, where $k < 1$ (and reducing in the level of prevailing uncertainty σ_α^2). Closing the full distance requires then that one aims at a target further away than the desired level, say $(\pi^* + \theta)$ such that $k\left[(\pi^* + \theta) - \pi_0\right])$ is the distance covered by the attenuated policy rule that will land at precisely π^*. We do this in two steps: we first derive a policy rule as a function of the overshooting target $(\pi^* + \theta)$ and in the second step then identify the value of θ that matches the degree of "overshooting" to the proportion of the distance that needs to be covered.

4.1 An optimisation framework

We describe next the two-step procedure reduces the impact of Brainard uncertainty in greater detail.

STEP 1

In the first step, and after the shock has occurred, the monetary policy authority identifies the optimal policy rule as a function of an auxiliary target $(\pi^* + \theta)$. Formally this means optimising the following objective function instead of (5),

$$\min_y E\left(L\right) = \frac{1}{2}\left\{\left[\bar{\pi}_t - (\pi^* + \theta)\right]^2 + y_t^2\left(1 + \sigma_\alpha^2\right)\right\} \qquad (18)$$

subject to the supply curve (1). The optimal rule derived following the optimisation of (18) is a function of θ:

$$\hat{y}_t = \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}\left(\pi^* + \theta\right) - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}\left(E_t\pi_{t+1} + \varepsilon_t\right) \qquad (19)$$

and therefore inflation is:

$$\hat{\pi}_t = \frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}\left(\pi^* + \theta\right) + \frac{1 + \sigma_\alpha^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}\left(E_t\pi_{t+1} + \varepsilon_t\right) \qquad (20)$$

The above two equations imply that for a given level of uncertainty, the CB will choose to deviate, at first instance, from its ultimate target π^* by a parameter θ.

STEP 2

The CB now chooses θ in full knowledge of the extent of uncertainty and the size of the shock and aims to maximise the probability of achieving its true objectives. In other words, since inflation expectations move away from the target as uncertainty increases, the deviation term θ will move to close that gap. In that respect θ is therefore, an auxiliary step, necessary to make full use of the information available to the bank. The derived rules from Step 1 (19) and (20) are now substituted into the objective function of the Central Bank:

$$\min_{\theta} E\left(L\right) = \frac{1}{2} E_t \left[\left(\hat{\pi}_t - \pi^*\right)^2 + \hat{y}^2 \right] \tag{21}$$

to produce

$$\min_{\theta} E\left(L\right) = f(\theta, \sigma_a^2, \hat{y}_t, \hat{\pi}_t) \tag{22}$$

Given the rule derived in Step 1, the CB chooses now the degree of overshooting θ required, (contingent on the economy's past history and the perceived uncertainty of the transmission of policies), that will get her closer to π^*, i.e.:

$$\theta(\sigma_\alpha^2) = \arg\min_{\theta} E\left(L\right)$$

which in its analytical form is

$$\theta = \frac{\sigma_a^2}{1+\bar{\alpha}} \left[\pi^* - E_t \pi_{t+1} - \varepsilon_t\right] \tag{23}$$

As uncertainty decreases, the deviations from π^* decrease as well, such that at the limit they become zero, i.e.

$$\lim_{\sigma_a^2 \to 0} (\theta) = 0$$

Proposition 1 *Applying a two-step procedure in which θ is contingent on the shocks that hit the economy, the existing uncertainty and the inflation target, neutralises the ex ante effects of uncertainty on the policy rules.*

Proof 1: Substituting the analytical solutions for θ (23) into (19) and (20) produces the *two-step* target rules that a Central Bank needs to apply under uncertainty.

$$y_t^{TS} = \frac{\bar{\alpha}}{1+\bar{\alpha}^2} \pi^* - \frac{\bar{\alpha}}{1+\bar{\alpha}^2} \left(E_t \pi_{t+1} + \varepsilon_t\right) \tag{24}$$

$$\pi_t^{TS} = \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2} \pi^* + \frac{1}{1+\bar{\alpha}^2} \left(E_t \pi_{t+1} + \varepsilon_t\right) \tag{25}$$

The rules achieved are similar to those attained with no uncertainty, (30) and (31) (with α replaced by $\bar{\alpha}$). This demonstrates that by varying the target optimally, neutralises the presence of uncertainty in the transmission process[4]. This however, is an *ex ante* result. As we will show next, this happens at the expense of using y_t more actively, thereby introducing greater variability in the system. In a period-by-period optimisation Brainard's attenuated policy is by construction optimal. However, this policy is not guaranteed to be optimal if one was interested in the dynamic properties (i.e. cumulative effects) of the rule, where the benefits of hitting the target more often may on aggregate compensate for the increased volatility. We will show this in the sections of simulations.

Before doing that, we impose again differential information by applying (15) in (24) and (25). The TS algorithm then implies the following results for output and inflation respectively

$$\tilde{y}_t^{TS} = \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2}\pi^* - \frac{\bar{\alpha}}{1+\bar{\alpha}^2}\pi_0 - \frac{\bar{\alpha}}{1+\bar{\alpha}^2}\varepsilon_t \tag{26}$$

$$\tilde{\pi}_t^{TS} = \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2}\pi^* + \frac{1}{1+\bar{\alpha}^2}\pi_0 + \frac{1}{1+\bar{\alpha}^2}\varepsilon_t \tag{27}$$

equivalent to the solution under no uncertainty but for the first moment of α replacing the actual value. It is important to examine next, whether reactivating the instrument compensate for the variability introduced, and under which conditions.

5 The merits of the TS procedure

The nature of the problem the Central Bank faces under uncertainty is encapsulated in (5) and the **optimal** solution to that problem is given by (8) and (9). In which sense then is it sensible to advocate the merits of the TS algorithm? The answer to the latter question comes from the way the game is set-up. The discretionary nature of our set-up explained earlier on, implies that we face a period-by-period game. In any given period therefore that the CB needs to move its instrument in reaction to a shock, the losses obtained by applying the BR rule are by definition lower than that of any other rule, including the TS. Hence in that period the advantage of getting closer to the target attained by the TS algorithm does not compensate for the extra variability introduced in the system. However, the dynamic nature of the adjustment to the shock implies that multiple periods are required before the system converges to the objective. From the second period onwards however, the benefits can potentially outweigh

[4] Our approach is in fact equivalent to introducing an extra instrument while the number of targets remains the same. As Hughes Hallett (1989) mentions "...all the instruments will be needed to combat uncertainty even when there are only a few targets compared to the number of instruments".

the costs such that on aggregate and for small degrees of variance the TS algorithm does better. We demonstrate this next through Monte Carlo simulations but show how this is done also analytically in Appendix D.

5.1 Numerical Simulations

We design the simulations as follows: the solutions for output for the two different regimes Brainard (BR) and Two-Step (TS).

$$\begin{aligned} y_t^{BR} &= \frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi^* - \frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}E_t\pi_{t+1} - \frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}\varepsilon_t \\ y_t^{TS} &= \frac{\bar{\alpha}}{1+\bar{\alpha}^2}\pi^* - \frac{\bar{\alpha}}{1+\bar{\alpha}^2}E_t\pi_{t+1} - \frac{\bar{\alpha}}{1+\bar{\alpha}^2}\varepsilon_t \end{aligned}$$

are substituted in the equation for prices

$$\pi_t^j = \beta E_t\pi_{t+1} + \alpha_i y_t^j + \varepsilon_t \qquad j = BR,TS$$

where parameter α_i is drawn from a distribution $N\left(\bar{\alpha},\sigma_\alpha^2\right)$. Further, expectations are backward looking and errors exhibit a certain degree of persistence, i.e.:

$$\begin{aligned} E_t\pi_{t+1} &= \pi_{t-1} \\ \varepsilon_t &= \rho\varepsilon_{t-1} + v_t \qquad v_t \simeq N(0,1) \end{aligned} \tag{28}$$

As we operate in a discretionary framework we calculate losses period-by-period as measured by

$$\begin{aligned} L_{BR,t} &= \frac{1}{2}\left\{\left(\pi_t^{BR}-\pi^*\right)^2 + y_{BR,t}^2\right\} \\ L_{TS,t} &= \frac{1}{2}\left\{\left(\pi_t^{TS}-\pi^*\right)^2 + y_{TS,t}^2\right\} \end{aligned}$$

We then calculate the cumulative losses for a certain number of years (n), discounted by the appropriate discount factor, i.e.

$$\sum_{t=1}^{n}\beta^t L_{j,t} \qquad \forall \quad j = BR,TS \quad \text{and} \quad n = 10 \tag{29}$$

We apply the following parameterisation[5]:

[5] Note that for $\beta < 1$ then the rules become

$$\begin{aligned} y_t^{BR} &= \frac{\overline{\alpha}}{1+\overline{\alpha}^2+\sigma_\alpha^2}\pi^* - \frac{\overline{\alpha}}{1+\overline{\alpha}^2+\sigma_\alpha^2}\beta E_t\pi_{t+1} - \frac{\overline{\alpha}}{1+\overline{\alpha}^2+\sigma_\alpha^2}\beta\varepsilon_t \\ y_t^{TS} &= \frac{\overline{\alpha}}{1+\overline{\alpha}^2}\pi^* - \frac{\overline{\alpha}}{1+\overline{\alpha}^2}\beta E_t\pi_{t+1} - \frac{\overline{\alpha}}{1+\overline{\alpha}^2}\beta\varepsilon_t \end{aligned}$$

$$\begin{aligned}\beta &= 0.99 \\ \alpha &\simeq N\left(0.5, \sigma_\alpha^2\right) \\ \rho &= 0.8 \\ \pi^* &= 1, \quad \pi_0 = 0\end{aligned}$$

The average value of α applied is somewhat higher than what exists in the literature, where it ranges from a minimum of 0.024 in Woodford (1999) to a maximum of 0.3 in McCallum and Nelson (1999). However, as the qualitative nature of the results is dependent only on the coefficient of variation of α (CV$\equiv \frac{\sigma_\alpha}{\bar{\alpha}}$) and not its mean, the choice of numerical value for α is done purely for presentation purposes. The updating of expectations in equation (28) is consistent with Morris and Shin (2006) definition of expectations inertia. The model is similar to the model used by Svensson (1999) and Söderstrom (2002), although the timing of policy responses and effectiveness is different. In the model applied, the policy response is contemporaneous to the supply shock and to the realisation of inflation. The lag response of the system to the policy action observed is due to the inertia in expectations formation imposed and there is no built-in lag in the monetary transmission mechanism. In the first period the economy is subjected both to a supply shock ε_t and a policy shift from π_0 to π^*. Note that the numerical value of the target does not influence the qualitative nature of the results. We then apply the optimal targeting rule and calculate the impulse response functions for y and π. Cumulative losses for ten periods are calculated in deviation from the targets. Before presenting a detailed welfare analysis, figures (1) and (2) show a typical path for output and inflation produced by the simulations.

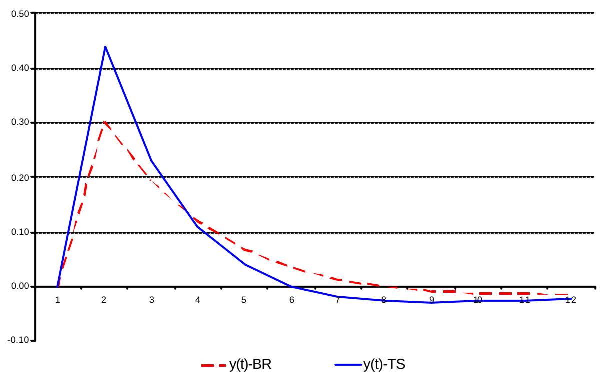

Figure 1: Output Gap - Typical Path

Figure 1 shows a typical path of y, the instrument in our targeting rule. To achieve the inflation target the economy is subjected to higher real variability in the early periods of the policy plan, than in the *two-step* regime. As can be seen in figure 2, once inflation and inflation expectations converge towards the target, the *two-step* policy rule produces both lower real variability as well as a path of inflation closer to the target, relative to the cautious Brainard policy rule.

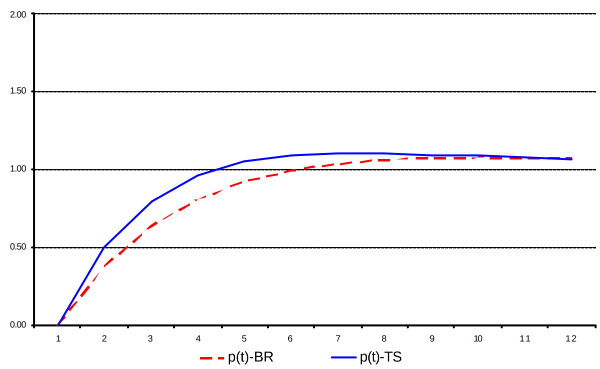

Figure 2: Inflation - Typical Path

Table 2 presents then the average cumulative losses of 10,000 stochastic simulations for the two regimes for different degrees of uncertainty captured by the coefficient of variation. Cumulative losses are lower in the TS regime for coefficient of variations equal to 0.5 and 1. For higher than that levels of uncertainty, the gains in convergence no longer compensate for the early losses, relative to Brainard's cautious approach.

Table 2. Cumulative Losses ($n = 10$)

CV	$L_{BR}(*10)$	$L_{TS}(*10)$
0.5	260.7	252.7
1	316.6	292.1
1.5	573.1	1032.0

Table 3 instead shows the first period losses for the two policy regime. This is also confirmed analytically in appendix B. In order to stabilise the system around the target, the two steps regime introduces greater variability in the early periods, thus increasing early losses[6].

[6] This also means that the results are a function of the discount rate applied. A very myopic policy maker will be always cautious

Table 3. First Period Losses

CV	$L_{BR}(*10)$	$L_{TS}(*10)$
0.5	11.9	12.1
1	11.6	12.2
1.5	11.4	12.7

It is the case therefore, that when evaluating the benefits of two regimes in terms of their dynamic properties, then there exist levels of uncertainty when it is better to ignore the prevailing level of uncertainty and aim to achieve the objectives set. The variability introduced as result is more than compensated by the benefits of achieving them.

6 Conclusions

Our motivation stems from the observation that the role of policy is reduced in the presence of multiplicative uncertainty and that of expectations is increased. If there are reasons to believe that inflation expectations cannot automatically adjust to be in line with what the Bank intends, then in the presence of Brainard uncertainty this can seriously hinder the ability of the Central Bank to achieve its objectives. Our intention then was to find a way of reducing the detrimental effect of uncertainty on the Central Banks' ability to get to its objectives.We assume first that expectations are subject to differentiated information as put forward by Morris and Shin (2006). The private sector then requires sometime to learn what level of inflation the Central Bank aims to achieve and only gradually therefore converges to it. This introduces by itself inertia to the system which is worsened in the presence of multiplicative uncertainty. We identify then a two-step algorithm that aims to reintroduce the relative relevance of policy. This has the advantage that the Central Bank is able to achieve its objectives quicker, but at the expense of introducing greater variability in the system. Our simulation section then shows that in a one-period framework, Brainard does indeed better on average. However, as any shock requires multiple periods before it peters out, we evaluate the cumulative benefits of the two algorithms. We thus show that as the TS algorithm attains the targets quicker, there are levels of uncertainty where the benefits of hitting the "bull's eye" outweigh the costs of greater variability. Furthermore, as this regime is done within an optimisation framework that accounts for the level of prevailing uncertainty, the rules derived are easier to communicate to the public and are in line with the levels of transparency required by modern monetary policy.

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APPENDICES

A General Solutions

We derive here the general solutions with the discount factor.

$$L = \frac{1}{2}E\left\{(\beta E_t\pi_{t+1} + \alpha y_t + \varepsilon_t - \pi^*)^2 + y_t^2\right\}$$

The discretionary solution of the problem is therefore the following:

$$\begin{aligned}
\frac{\partial L}{\partial y} &= \alpha\left(\beta E_t\pi_{t+1} + \alpha y_t + \varepsilon_t - \pi^*\right) + y_t = 0 \\
y_t\left(1+\alpha^2\right) &= -\alpha\left(\beta E_t\pi_{t+1} + \varepsilon_t - \pi^*\right) \\
y_t &= \frac{\alpha}{1+\alpha^2}\pi^* - \frac{\alpha}{1+\alpha^2}\left(\beta E_t\pi_{t+1} + \varepsilon_t\right) \qquad (30)
\end{aligned}$$

Substituting (30) in (1), we obtain the discretionary level of inflation:

$$\begin{aligned}
\pi_t &= \beta E_t\pi_{t+1} + \alpha\left[\frac{\alpha}{1+\alpha^2}\pi^* - \frac{\alpha}{1+\alpha^2}\left(\beta E_t\pi_{t+1} + \varepsilon_t\right)\right] + \varepsilon_t \\
&= \frac{\alpha^2}{1+\alpha^2}\pi^* + \frac{1+\alpha^2-\alpha^2}{1+\alpha^2}\beta E_t\pi_{t+1} + \frac{1}{1+\alpha^2}\varepsilon_t
\end{aligned}$$

or

$$\pi_t = \frac{\alpha^2}{1+\alpha^2}\pi^* + \frac{1}{1+\alpha^2}\beta E_t\pi_{t+1} + \frac{1}{1+\alpha^2}\varepsilon_t \qquad (31)$$

This equation can be solved forward to obtain (under $\varepsilon_{t+1} = \rho\varepsilon_t$ and calling $\frac{1}{1+\alpha^2} = A$) a solution for inflation

$$\begin{aligned}
\pi_t &= A\alpha^2\pi^* + A\beta E_t\pi_{t+1} + A\varepsilon_t \\
\pi_{t+1} &= A\alpha^2\pi^* + A\beta E_t\pi_{t+2} + A\rho\varepsilon_t
\end{aligned}$$

then it follows that

$$\begin{aligned}
\pi_t &= A\alpha^2\pi^* + A\beta\left(A\alpha^2\pi^* + A\beta E_t\pi_{t+2} + A\rho\varepsilon_t\right) + A\varepsilon_t \\
&= A\alpha^2\left[1 + \beta A + \beta^2A^2 + \ldots\right]\pi^* + A\varepsilon_t\left[1 + A\beta\rho + A^2\beta^2\rho^2 + \ldots\right]
\end{aligned}$$

The two geometric series inside the square brackets are respectively equal to:

$$\begin{aligned}
1 + A\beta + A^2\beta^2 + \ldots &= \frac{1}{1-A} \\
1 + A\beta\rho + A^2\beta^2\rho^2 + \ldots &= \frac{1}{1-A\rho}
\end{aligned}$$

Therefore, equilibrium inflation is equal to

$$\pi_t = \frac{A\alpha^2}{1-A\beta}\pi^* + \frac{A}{1-A\beta\rho}\varepsilon_t$$

Substituting for $A = \frac{1}{1+\alpha^2}$, we obtain

$$\begin{aligned}
\pi_t &= \frac{\frac{1}{1+\alpha^2}\alpha^2}{1-\frac{1}{1+\alpha^2}\beta}\pi^* + \frac{\frac{1}{1+\alpha^2}}{1-\frac{1}{1+\alpha^2}\beta\rho}\varepsilon_t \\
&= \frac{\frac{\alpha^2}{1+\alpha^2}}{\frac{1+\alpha^2-\beta}{1+\alpha^2}}\pi^* + \frac{\frac{1}{1+\alpha^2}}{\frac{1+\alpha^2-\beta\rho}{1+\alpha^2}}\varepsilon_t
\end{aligned}$$

and therefore,

$$\pi_t = \frac{\alpha^2}{1+\alpha^2-\beta}\pi^* + \frac{1}{1+\alpha^2-\beta\rho}\varepsilon_t \tag{32}$$

Note that for $\beta = 1$ the result collapse to (10). Similarly under uncertainty when the objective function is now

$$L = \frac{1}{2}E\left\{\left(\beta E_t\pi_{t+1} + \bar{\alpha}y_t + \varepsilon_t - \pi^*\right)^2 + y_t^2\left(1+\sigma_\alpha^2\right)\right\}$$

Like above, we solve under the AS constraint only and then identify the i that is implied by the aggregate demand curve. The FOC is:

$$\begin{aligned}
\frac{\partial L}{\partial y} = \bar{\alpha}\left(\beta E_t\pi_{t+1} + \bar{\alpha}y_t + \varepsilon_t - \pi^*\right) + y_t\left(1+\sigma_\alpha^2\right) = 0 \\
y_t\left(1+\sigma_\alpha^2+\bar{\alpha}^2\right) = -\bar{\alpha}\left(\beta E_t\pi_{t+1} + \varepsilon_t - \pi^*\right) \\
y_t = \frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi^* - \frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}\left(\beta E_t\pi_{t+1} + \varepsilon_t\right)
\end{aligned} \tag{33}$$

Substituting (33) in (1), we obtain the discretionary level of inflation:

$$\begin{aligned}
\pi_t &= \beta E_t\pi_{t+1} + \bar{\alpha}\left[\frac{\bar{\alpha}}{1+\bar{\alpha}+\sigma_\alpha^2}\pi^* - \frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}\left(\beta E_t\pi_{t+1} + \varepsilon_t\right)\right] + \varepsilon_t \\
&= \frac{\bar{\alpha}^2}{1+\bar{\alpha}+\sigma_\alpha^2}\pi^* + \frac{1+\bar{\alpha}^2+\sigma_\alpha^2-\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\beta E_t\pi_{t+1} + \frac{1+\bar{\alpha}^2+\sigma_\alpha^2-\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\varepsilon_t
\end{aligned}$$

and therefore,

$$\pi_t = \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi^* + \frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\beta E_t\pi_{t+1} + \frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\varepsilon_t \tag{34}$$

To solve for expectations, under the assumption of rational expectations, we iterate the equation forward (and assume just like above that $\varepsilon_{t+1} = \rho\varepsilon_t$ and calling $\frac{1}{1+\bar{\alpha}^2+\sigma_\alpha^2} = \Psi$)

$$\begin{aligned}
\pi_t &= \Psi\bar{\alpha}^2\pi^* + \Psi\left(1+\sigma_\alpha^2\right)\beta E_t\pi_{t+1} + \Psi\left(1+\sigma_\alpha^2\right)\varepsilon_t \\
\pi_{t+1} &= \Psi\bar{\alpha}^2\pi^* + \Psi\left(1+\sigma_\alpha^2\right)\beta E_t\pi_{t+2} + \Psi\left(1+\sigma_\alpha^2\right)\rho\varepsilon_t
\end{aligned}$$

then we can substitute and iterate forward

$$\begin{aligned}
\pi_t &= \Psi\bar{\alpha}^2\pi^* + \left(1+\sigma_\alpha^2\right)\Psi\beta\left[\Psi\bar{\alpha}^2\pi^* + \Psi\left(1+\sigma_\alpha^2\right)\beta E_t\pi_{t+2} + \Psi\left(1+\sigma_\alpha^2\right)\rho\varepsilon_t\right] + \Psi\left(1+\sigma_\alpha^2\right)\varepsilon_t \\
\pi_t &= \Psi\bar{\alpha}^2\left[1+\left(1+\sigma_\alpha^2\right)\beta\Psi + \left(1+\sigma_\alpha^2\right)^2\beta^2\Psi^2 ...\right]\pi^* + \Psi\left(1+\sigma_\alpha^2\right)\left[1+\Psi\beta\left(1+\sigma_\alpha^2\right)\rho + \Psi^2\beta^2\left(1+\sigma\right.\right.
\end{aligned}$$

The two geometric series inside quadratic brackets are equal to

$$\begin{aligned}
\left[1+\left(1+\sigma_\alpha^2\right)\beta\Psi + \left(1+\sigma_\alpha^2\right)^2\beta^2\Psi^2 ...\right] &= \frac{1}{1-\left(1+\sigma_\alpha^2\right)\beta\Psi} \\
\left[1+\Psi\beta\left(1+\sigma_\alpha^2\right)\rho + \Psi^2\beta^2\left(1+\sigma_\alpha^2\right)^2\rho^2\right] &= \frac{1}{1-\Psi\left(1+\sigma_\alpha^2\right)\beta\rho}
\end{aligned}$$

Therefore, equilibrium inflation is equal to

$$\pi_t = \frac{\Psi\bar{\alpha}^2}{1-\left(1+\sigma_\alpha^2\right)\beta\Psi}\pi^* + \frac{\Psi\left(1+\sigma_\alpha^2\right)}{1-\left(1+\sigma_\alpha^2\right)\Psi\beta\rho}\varepsilon_t$$

Substituting back the value for $\Psi = \frac{1}{1+\bar{\alpha}^2+\sigma_\alpha^2}$ we obtain

$$\begin{aligned}
\pi_t &= \frac{\frac{1}{1+\bar{\alpha}^2+\sigma_\alpha^2}\bar{\alpha}^2}{1-\left(1+\sigma_\alpha^2\right)\beta\frac{1}{1+\bar{\alpha}^2+\sigma_\alpha^2}}\pi^* + \frac{\frac{1}{1+\bar{\alpha}^2+\sigma_\alpha^2}\left(1+\sigma_\alpha^2\right)}{1-\frac{1}{1+\bar{\alpha}^2+\sigma_\alpha^2}\left(1+\sigma_\alpha^2\right)\beta\rho}\varepsilon_t \\
&= \frac{\frac{\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}}{\frac{1+\bar{\alpha}^2+\sigma_\alpha^2-\beta-\beta\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}}\pi^* + \frac{\frac{\left(1+\sigma_\alpha^2\right)}{1+\bar{\alpha}^2+\sigma_\alpha^2}}{\frac{1+\bar{\alpha}^2+\sigma_\alpha^2-\left(1+\sigma_\alpha^2\right)\beta\rho}{1+\bar{\alpha}^2+\sigma_\alpha^2}}\varepsilon_t
\end{aligned}$$

and therefore,

$$\pi_t = \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2-\beta-\beta\sigma_\alpha^2}\pi^* + \frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2-\left(1+\sigma_\alpha^2\right)\beta\rho}\varepsilon_t \tag{35}$$

In both case we see that the smaller the discount factor (and therefore, the higher the degree of impatience) the longer it takes to come to the target π^*.

B The Objective Function with Uncertainty in α

Under uncertainty, where $\alpha_t \rightarrow N\left(\bar{\alpha}, \sigma_\alpha^2\right)$, losses conditional on shocks ε, we can express the objective function of the CB in terms of the moments of α.

$$
\begin{aligned}
L &= \frac{1}{2}E\left\{(\pi_t-\pi^*)^2+y_t^2\right\}\\
&= \frac{1}{2}E\left(\pi_t-\pi^*\right)^2+\frac{1}{2}E\left(y_t^2\right)\\
&= \frac{1}{2}E\left\{E_t\pi_{t+1}+\alpha y_t+\varepsilon_t-\pi^*\right\}^2+\frac{1}{2}E\left(y_t^2\right)\\
&= \frac{1}{2}E\Big\{\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)^2+\left(\alpha y_t\right)^2\\
&\quad +2\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)\left(\alpha y_t\right)\Big\}+\frac{1}{2}E\left(y_t^2\right)\\
&= \frac{1}{2}\Big\{\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)^2+E\left(\alpha y_t\right)^2\\
&\quad +2\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)E\left(\alpha y_t\right)\Big\}+\frac{1}{2}E\left(y_t^2\right)
\end{aligned}
$$

but since $E\left(\alpha y_t\right)^2=y_t^2E\left(\alpha\right)^2$and $E\left(\alpha^2\right)=\sigma_\alpha^2+\bar{\alpha}^2$ then,

$$
\begin{aligned}
L &= \frac{1}{2}\Big\{\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)^2+y_t^2E\left(\alpha\right)^2\\
&\quad +2\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)\bar{\alpha}y_t\Big\}+\frac{1}{2}E\left(y_t^2\right)\\
&= \frac{1}{2}\Big\{\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)^2\\
&\quad +y_t^2\left(\sigma_\alpha^2+\bar{\alpha}^2\right)+2\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)\bar{\alpha}y_t\Big\}+\frac{1}{2}E\left(y_t^2\right)\\
&= \frac{1}{2}\Big\{\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)^2+y_t^2\bar{\alpha}^2\\
&\quad +2\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)\bar{\alpha}y_t+y_t^2\sigma_\alpha^2\Big\}+\frac{1}{2}E\left(y_t^2\right)
\end{aligned}
$$

From this, it follows that

$$\bar{\pi}_t\equiv E(\pi_t)=E_t\pi_{t+1}+\bar{\alpha}y_t+\varepsilon_t$$

and therefore,

$$
\begin{aligned}
L &= \frac{1}{2}\left\{\left(\bar{\pi}_t-\pi^*\right)^2+y_t^2\sigma_\alpha^2+y_t^2\right\}\\
&= \frac{1}{2}\left\{\left(\bar{\pi}_t-\pi^*\right)^2+y_t^2\left(\sigma_\alpha^2+1\right)\right\}
\end{aligned}
$$

C Introducing Differential Information

We introduce next the concept of Differential Information as presented by Morris and Shin (2006) which builds on some of their previous work (Morris and Shin, 2002a and 2002b). When applied to monetary policy, the idea behind this is as follows: the current state of inflation is π_0 but the Central Bank wants to move it to a new state π^* within a given time horizon. It announces therefore, the desired level of inflation π^*, which under the assumption of differential information, is not automatically believed. A proportion of the private sector μ will automatically adjust expectations to that level at time t, whereas, $1-\mu$ will not. In the eyes of those who form expectations, inflation state π_t therefore evolves in the following way:

$$\pi_t = \begin{cases} \pi_0 & \text{for } t \leq 0 \\ \pi^* & \text{for } t \geq 1 \end{cases}$$

If agents are subject to differential information, only a proportion μ_t of the agents know the true value of π_t at time $t \geq 1$. This proportion is increasing over time, such that eventually $\mu_t = 1$, and therefore, all agents learn what the true value of π_t is in the sufficiently distant future (as $t \rightarrow \infty$). Note that this latter assumption implies that as agents eventually learn the value of the state variable, this set-up is consistent with Rational Expectations. Moreover, MS assume that μ_1 is very close to one to start with, such that the informational friction is sufficiently small by comparison to the occasion of no differential information. Looking at the agents individually, it is then the case that informed agents form expectations as:

$$E_{i,t}\begin{bmatrix} \pi_0 \\ \pi_{t+h} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \tag{36}$$

whereas uniformed agents form expectations as:

$$E_{i,t}\begin{bmatrix} \pi_0 \\ \pi_{t+h} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \tag{37}$$

The assumption therefore, is that everyone knows the current or past state of inflation. Then, since a proportion μ_t are informed at time t, average expectation for inflation is

$$\bar{E}_t\begin{bmatrix} \pi_0 \\ \pi_{t+h} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1-\mu_t & \mu_t \end{bmatrix}\begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \tag{38}$$

Similarly,

$$\begin{aligned}
\bar{E}_{t-1}\bar{E}_t\begin{bmatrix} \pi_0 \\ \pi_{t+h} \end{bmatrix} &= \bar{E}_{t-1}\begin{bmatrix} 1 & 0 \\ 1-\mu_t & \mu_t \end{bmatrix}\begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 \\ 1-\mu_t & \mu_t \end{bmatrix}\bar{E}_{t-1}\begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 \\ 1-\mu_t & \mu_t \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1-\mu_{t-1} & \mu_{t-1} \end{bmatrix}\begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 \\ 1-\mu_{t-1}\mu_t & \mu_{t-1}\mu_t \end{bmatrix}\begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix}
\end{aligned}$$

In a forward looking world, the present is a function of the sequence of all future expectations. In other words, to derive the appropriate expectation one needs to iterate this forward such that

$$\bar{E}_1\bar{E}_2...\bar{E}_t\begin{bmatrix}\pi_0\\ \pi_{t+h}\end{bmatrix}=\begin{bmatrix}1 & 0\\ 1-\prod\limits_{s=1}^{t}\mu_s & \prod\limits_{s=1}^{t}\mu_s\end{bmatrix}\begin{bmatrix}\pi_0\\ \pi^*\end{bmatrix}$$

and therefore,

$$\bar{E}_1\bar{E}_2...\bar{E}_t\left(\pi_0\right) = \pi_0 \tag{39}$$

$$\bar{E}_1\bar{E}_2...\bar{E}_t\left(\pi_{t+h}\right) = \left(1-\prod_{s=1}^{t}\mu_s\right)\pi_0+\left(\prod_{s=1}^{t}\mu_s\right)\pi^* \tag{40}$$

This implies that the higher order expectation for a given timing $t+h$ (where $h>0$) depends on the limiting property of $\prod\limits_{s=1}^{t}\mu_s$.

D Ex Ante losses comparison

Static losses are evaluated based on

$$L=\frac{1}{2}E\left\{\left(\bar{\pi}_t-\pi^*\right)^2+y_t^2\left(1+\sigma_\alpha^2\right)\right\}$$

The solutions for Brainard are:

$$\begin{aligned}
y_{BR,t} &= \frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi^*-\frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi_0-\frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}\varepsilon_t\\
\pi_{BR,t} &= \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi^*+\frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi_0+\frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\varepsilon_t
\end{aligned}$$

and similarly for the TS solution:

$$\begin{aligned}
y_{TS,t} &= \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2}\pi^*-\frac{\bar{\alpha}}{1+\bar{\alpha}^2}\pi_0-\frac{\bar{\alpha}}{1+\bar{\alpha}^2}\varepsilon_t\\
\pi_{TS,t} &= \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2}\pi^*+\frac{1}{1+\bar{\alpha}^2}\pi_0+\frac{1}{1+\bar{\alpha}^2}\varepsilon_t
\end{aligned}$$

Substituting then the solutions to the objective functions we calculate losses for any given shock ε_t:

$$\begin{aligned}
L_{BR,t} &= \frac{\left(1+\sigma_\alpha^2\right)\left[\varepsilon_t+\pi_0-\pi^*\right]^2}{2\left(1+\bar{\alpha}^2+\sigma_\alpha^2\right)}\\
L_{TS,t} &= \frac{\left[1+\bar{\alpha}^2\left(1+\sigma_\alpha^2\right)\right]\left[\varepsilon_t+\pi_0-\pi^*\right]^2}{2\left(1+\bar{\alpha}^2\right)^2}
\end{aligned}$$

When are losses for Brainard bigger than for TS?

$$L_{BR,t} = \frac{\left(1+\sigma_\alpha^2\right)\left[\varepsilon_t + \pi_0 - \pi^*\right]^2}{2\left(1+\bar{\alpha}^2+\sigma_\alpha^2\right)} > L_{TS,t} = \frac{\left[1+\bar{\alpha}^2\left(1+\sigma_\alpha^2\right)\right]\left[\varepsilon_t + \pi_0 - \pi^*\right]^2}{2\left(1+\bar{\alpha}^2\right)^2}$$

and therefore,

$$\begin{aligned}
\frac{\left(1+\sigma_\alpha^2\right)\left[\varepsilon_t + \pi_0 - \pi^*\right]^2}{2\left(1+\bar{\alpha}^2+\sigma_\alpha^2\right)} &> \frac{\left[1+\bar{\alpha}^2\left(1+\sigma_\alpha^2\right)\right]\left[\varepsilon_t + \pi_0 - \pi^*\right]^2}{2\left(1+\bar{\alpha}^2\right)^2} \\
\bar{\alpha}^2 &> \bar{\alpha}^2 + \sigma_\alpha^2
\end{aligned}$$

This is never true for $\sigma_\alpha^2 > 0$ and therefore $L_{BR,t} < L_{TS,t}$ holds for $t = 1$ always.

D.1 Cumulative Losses

However, for $t = 2$, in other words, in the second period after the shock has occurred, as TS is more aggressive it will have managed to close more of the distance between actual inflation and the target i.e. $\pi_1^{TS} - \pi^* < \pi_1^{BR} - \pi^*$ and therefore $\left[\varepsilon_t + \pi_1^{TS} - \pi^*\right]^2 < \left[\varepsilon_t + \pi_1^{BR} - \pi^*\right]^2$. This implies that in the next period losses with Brainard can be worse if the following holds.

$$\begin{aligned}
\frac{\left(1+\sigma_\alpha^2\right)\left[\varepsilon_t + \pi_1^{BR} - \pi^*\right]^2}{2\left(1+\bar{\alpha}^2+\sigma_\alpha^2\right)} &> \frac{\left[1+\bar{\alpha}^2\left(1+\sigma_\alpha^2\right)\right]\left[\varepsilon_t + \pi_1^{TS} - \pi^*\right]^2}{2\left(1+\bar{\alpha}^2\right)^2} \\
\frac{\left(1+\sigma_\alpha^2\right)\left(1+\bar{\alpha}^2\right)^2}{\left(1+\bar{\alpha}^2+\sigma_\alpha^2\right)\left[1+\bar{\alpha}^2\left(1+\sigma_\alpha^2\right)\right]} &> \frac{\breve{\pi}_1^{TS}}{\breve{\pi}_1^{BR}}
\end{aligned}$$

where $\breve{\pi}_1^{TS} = \left[\varepsilon_t + \pi_1^{TS} - \pi^*\right]^2$ and $\breve{\pi}_1^{BR} = \left[\varepsilon_t + \pi_1^{BR} - \pi^*\right]^2$. In general, for any period n this condition is

$$\frac{\left(1+\sigma_\alpha^2\right)\left(1+\bar{\alpha}^2\right)^2}{\left(1+\bar{\alpha}^2+\sigma_\alpha^2\right)\left[1+\bar{\alpha}^2\left(1+\sigma_\alpha^2\right)\right]} > \frac{\pi_{n-1}^{TS}}{\pi_{n-1}^{BR}}$$

We demonstrate through simulations for which values for the coefficient of variation this happens and then compare the cumulative losses implied by the two methods.