OPTIMAL MONETARY POLICY WITH REAL TIME SIGNAL EXTRACTION FROM THE BOND MARKET

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Abstract. Monetary policy is conducted in an environment of uncertainty. This paper sets up a model where the central bank uses real time data from the bond market together with standard macroeconomic indicators to estimate the current state of the economy more efficiently, while taking into account that its own actions influence the bond market and therefore what it observes. The timeliness of bond market data allows for quicker responses of monetary policy to disturbances compared to the case when the central bank has to rely solely on collected aggregate data. The information content of the term structure creates a link between the bond market and the macro economy that is novel to the literature. To quantify the importance of the bond market as a source of information, the model is estimated on data for the United States and Australia using Bayesian methods. The empirical exercise suggests that there is some information in US bonds with maturities of less than one year that can help the Federal Reserve to identify shocks to the economy on a timely basis. Australian bond prices seem to be less informative than their US counterparts.

Keywords: Monetary Policy, Imperfect Information, Bond Market, Term Structure of Interest Rates

1. Introduction

Hayek (1945) famously argued that market economies are more efficient than planned economies because of markets’ ability to efficiently use information dispersed among market participants. According to Hayek, prices in competitive markets reflect all information that is known to anyone and competitive markets can potentially allocate resources more efficiently. In most western economies there is now little planning and almost all prices are determined by market forces without interference from any central authority. However, there is one important exception: the market for short term nominal debt where central banks borrow and lend at fixed interest rates. In the presence of nominal frictions in product or wage markets, this practise can improve welfare by reducing the volatility of inflation and output. Hayek’s insight, though formulated in a more general setting of a planned economy, was that even a central bank that shares the objective of the representative agent may not be able to implement an optimal stabilizing policy due to incomplete information. In this paper, the central bank would implement an optimal stabilizing policy if it knew the state of the economy with certainty, and any deviation from...
optimal policy is due only to information imperfections. Under this assumption we demonstrate how the central bank can make use of Hayek’s insight and use the market for debt of longer maturities as a source of information that makes a more efficient estimation of the state of the business cycle possible, and thus reduces deviations from optimal policy. That this is close to how some central banks think about and use the term structure is illustrated by a quote by the Chairman (then Governor) of the Federal Reserve Board, Ben Bernanke:

“To the extent that financial markets serve to aggregate private-sector information about the likely future course of inflation, data on asset prices and yields might be used to validate and perhaps improve the Fed’s forecasts.”

The suggestion that the bond market can provide information that is valuable to policy makers is thus not news to the policy makers themselves. Rather, the contribution of the present paper is to provide a coherent framework for analyzing and estimating the interaction between information contained in the term structure and the monetary policy making process. In the model presented below the central bank set interest rates in an uncertain environment, where the yield curve is informative about the state of the economy and thus also informative about the desired interest rate. This has the consequence that the macro economy is not independent of the term structure. The only direct effect of interest rates on the macro economy is from the expected path of the short rate set by the central bank to aggregate demand, as is standard in the New-Keynesian literature. However, there is also an indirect feedback from rates on longer maturity bonds to the macro economy through an information channel. The mechanism is the following. Bonds are traded daily and the affine form of the bond pricing function makes the bond pricing equation with macro factors formally equivalent to a linear measurement of the state of economy. The term structure can thus be used as a more timely indicator of the state of the economy than collected aggregate information that is available only with delay and sometimes significant measurement error. A movement in the term structure signals a shift in the underlying macro factors that induces the central bank to re-evaluate what the optimal short term interest rate should be. The shift in the term structure thus feeds into a change in demand through the change in the short term interest rate.

In the present model the policy makers exploit the fact that bond market participants’ expectations about the future are revealed by the term structure. As pointed out by Bernanke and Woodford (1997), letting monetary policy react mechanically to expectations may lead to a situation where expectations become uninformative about the underlying state and no equilibrium exists. They further argue that “targeting expectations” by policymakers can not be a substitute to structural modelling. In the proposed framework below, the information in the term structure is complementary to other information and firmly connected to an underlying structural model. Policymakers then avoid the potential pitfalls of a pure “expectations targeting” regime.

There is a large literature on the informational content of the term structure. Mostly, it has focused on whether the term structure, often modeled as the spread

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between short and long rates, can help predict future outcomes of macro variables. This literature thus investigates whether the component of the yield curve that is orthogonal to other macro variables adds predictive power to forecasting models of inflation and GDP growth. A recent paper in this vein is Ang, Piazzesi and Wei (2003) who finds that the short interest rate performs better than any term spread in predicting GDP growth. The negative correlation between the orthogonal component of short interest rates and future output found by Ang et al is consistent with the findings of the VAR literature on the real effects of monetary policy shocks. In the VAR literature on the transmission mechanism of monetary policy, shocks are defined to be exactly the component of interest rates that is orthogonal to other macro variables (e.g. Christiano, Eichenbaum and Evans (2001)). In this paper we deviate from the previous literature on the informational content of the term structure by asking if there is information in the term structure that can be used in the monetary policy process when the transmission mechanism is assumed to be known.

There are two potentially important types of information that could be revealed by the term structure that the present model is silent about. Goodfriend (1998) discusses the Federal Reserves responses to "inflation scares" in the 1980s, which he defines as increases in the long term yields. He interprets these as doubts by market participants about the Federal Reserves commitment to fighting inflation. The present paper does not address questions about central bank credibility, but takes a perfectly credible central bank with a publicly known inflation target as given. The model presented here is also not suited to analyzing or interpreting market perceptions of the reasons for a change in the monetary policy stance, as done by Ellingsen and Söderström (2001). The policymakers’ relative preferences for stabilizing inflation or the output gap are assumed to be publicly known. In this paper, we restrict our attention to what the term structure can tell us about the state of the business cycle.

The practical relevance of any information contained in the term structure is ultimately an empirical question. When bond markets are noisy, observing the term structure is not very informative. In order to quantify the informational content of the term structure the variances of the non-fundamental shocks in the term structure are estimated simultaneously with the structural parameters of the macro economy. The estimation methodology is similar to recent work by Hårdahl, Tristani and Vestin (2004) who estimate the term structure dynamics jointly with a small empirical macro model where the central bank is assumed to be perfectly informed. Hårdahl et al impose only a no arbitrage condition on the pricing of bonds while in this paper the bond pricing function and the dynamics of the macro economy are derived from the same underlying utility function. This makes the analysis more stringent, but it comes at the cost of an empirically less flexible bond pricing function.

In the next section a model is presented where the central bank extracts information from the term structure about unobservable shocks while recognizing that its own actions influences the term structure itself. In Section 3 the model is estimated to quantify the potential of the yield curve as a source of information. Section 4 concludes.

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2For example Harvey (1990), Mishkin (1990) and Estrella and Mishkin (1998).
2. THE MACRO ECONOMY, THE TERM STRUCTURE AND MONETARY POLICY UNDER IMPERFECT INFORMATION

This section presents a model where the central bank extracts information about the state of the economy from the term structure of interest rates. Movements in the term structure will then have a direct impact on the macro economy through its effect on the central bank’s estimate of the state and therefore also on the setting of the policy instrument. This means that the macro economy, the term structure model and the filtering problem of the central bank have to be solved simultaneously which makes the model different from other recent papers, for example Hordahl et al (2004) and Bekaert, Cho and Moreno (2003), where the macro economic model can be solved separately from the term structure. Though the complete model has to be solved simultaneously, it is still useful to divide the description of the model into three parts. The macro economy is described first without specifying an explicit interest rate function, but merely noting that it is set by the central bank to minimize a loss function that in principle could be derived from micro foundations. The filtering problem of the central bank is then solved, taking the term structure model as given. Finally, in the last part the term structure model is derived.

2.1. THE MACRO ECONOMY. We use a standard business cycle model of the macro economy with monopolistically competitive firms that sell differentiated goods. Prices are set according to the Calvo mechanism, with a fraction of firms using a rule of thumb rather than optimizing as in Gali an Gertler (1999). Households supply labor and consume goods. In addition to their own current consumption, they also care about the lagged aggregate consumption level.

2.1.1. HOUSEHOLDS AND FIRMS. Consider a representative household \( j \in (0, 1) \) that wishes to maximize the discounted sum of expected utility

\[
E_t \left\{ \sum_{s=0}^{\infty} \beta^s U(C_{t+s}(j), N_{t+s}(j)) \right\} \tag{1}
\]

where \( \beta \in (0, 1) \) is the household’s subjective discount factor and the period utility function in consumption \( C_t \) and labor \( N_t \) is given by

\[
U(C_t(j), N_t(j)) = \frac{\left( H_t^{-\eta} \right)^{1-\gamma}}{(1-\gamma)} - \frac{N_t(j)^{1+\varphi}}{1+\varphi}. \tag{2}
\]

The variable \( H_t \)

\[
H_t = \int C_{t-1}(j) \, dj \tag{3}
\]

is a reference level of consumption that we interpret as external habits that makes marginal utility of consumption an increasing function of lagged aggregate consumption. The habit specification helps to explain the inertial movement of aggregate output as well as the procyclicality of asset prices.\(^3\) Differentiated goods indexed by \( i \in (0, 1) \) are produced with a technology that is linear in labor and subject to a persistent productivity shock \( A_t \)

\[
Y_t(i) = A_t N(i) \tag{4}
\]

\(^3\)See Campbell and Cochrane (1999) for the implications of habits for asset prices.
that follow an AR(1) process in logs
\[ a_t = \rho a_{t-1} + \varepsilon_t^a \]  
\[ \varepsilon_t^a \sim N(0, \sigma_{\varepsilon}^2). \]

Firms set prices according to the Calvo (1983) mechanism where a fraction \( \theta \) of firms reset their price in a given period. Of the firms resetting their price, a fraction \( 1 - \omega \) optimize their price decision and take into account that their price may be effective for more than one period while a fraction \( \omega \) of price setters use a “rule of thumb” as in Gali and Gertler (1999). Price setters using a “rule of thumb” set their price equal to last period’s average reset price plus the lagged inflation rate.

### 2.1.2. The linearized model

The linearized structural equations are given by equations (7) - (8)

\[ y_t = \frac{\gamma}{\gamma - \eta + \gamma \eta} E_t y_{t+1} + \frac{-\eta (1 - \gamma)}{\gamma - \eta + \gamma \eta} y_{t-1} - \frac{1}{\gamma - \eta + \gamma \eta} [\mu_t - E_t \pi_{t+1}] + \varepsilon_t^y \]  
\[ \pi_t = \frac{\beta \theta}{\theta + \omega (1 - \theta (1 - \beta))} E_t \pi_{t+1} + \frac{\omega}{\theta + \omega (1 - \theta (1 - \beta))} \pi_{t-1} + \frac{(1 - \omega)(1 - \theta)(1 - \beta)}{\theta + \omega (1 - \theta (1 - \beta))} mc_t + \varepsilon_t^\pi \]

where \( \{y_t, \pi_t, \mu_t\} \) is real output, inflation and the short nominal interest rate in period \( t \). Marginal cost in period \( t \), \( mc_t \), can be found by equating the marginal utility of consuming the real wage paid for an additional unit of labor with the household’s disutility of providing the additional unit of labor. The real marginal cost then equals the market clearing real wage divided by productivity

\[ mc_t = (\varphi + \gamma) y_t + \eta (1 - \gamma) y_{t-1} - (1 + \varphi) a_t. \]

where the relationship

\[ n_t = y_t - a_t \]

was used to substitute out labor supply. Potential output, \( \overline{y}_t \), defined as the level of output that is compatible with no acceleration in inflation then is

\[ \overline{y}_t = \frac{\eta (1 - \gamma)}{\varphi + \gamma} y_{t-1} + \frac{1 + \varphi}{\varphi + \gamma} a_t. \]

The short term interest rate is set by a monetary authority to minimize the expected value of the loss function

\[ L_t = E_t \left\{ \sum_{k=0}^{\infty} \beta^k \left[ \lambda_y (y_{t+k} - \overline{y}_{t+k})^2 + \pi_{t+k}^2 + \lambda_i (i_{t+k} - i_{t+k-1})^2 \right] \right\}. \]

The weights \( \lambda_y \) and \( \lambda_i \) can be chosen such that the loss function (12) is a second order approximation of the utility function of the representative agent.\(^4\) However, we do not necessarily want to impose this restriction when we estimate the model.

The equations (5), (7), (8) and (9) can be written more compactly as

\[
\begin{bmatrix}
X_{1,t+1} \\
E_t X_{2,t+1}
\end{bmatrix}
= A
\begin{bmatrix}
X_{1,t} \\
X_{2,t}
\end{bmatrix}
+ B i_t + C \varepsilon_t
\]

\(^4\)See Amato and Laubach (2004).
where
\[ X_{1,t} = [a_t, y_{t-1}, \pi_{t-1}, \varepsilon_t^y, \varepsilon_t^\pi, \Delta_it, v_t]^T \]  
\[ X_{2,t} = [y_t, \pi_t]^T \]  
\[ \varepsilon_t = [\varepsilon_t^y, \varepsilon_t^\pi, \varepsilon_t^v] . \]

The vector \( v_t \) in \( X_{1,t} \) are (potentially serially correlated) measurement errors that follow
\[ v_t = Wv_{t-1} + e_v^t \]

The measurement errors have not yet been introduced to the model but are included in the state definition already here in order to keep notation compact and consistent throughout the paper.

### 2.2. Monetary Policy and Real Time Signal Extraction.
Monetary policy operates in an uncertain environment where some state variables are only observed with error and delay and some variables, like productivity and thus potential output, are not observed at all. Variables that are not observable but relevant for monetary policy have to be inferred from the variables that are observable. In such a setting, Svensson and Woodford (2003) show that a form of certainty equivalence holds. That is, with a quadratic objective function and linear constraints, the optimal interest rate can be expressed as a linear function of the central bank’s estimate of the state \( X_{1,t|t} \)
\[ X_{1,t|t} = E_t [X_{1,t} | I_t] \]
where \( I_t \) is the information set of the central bank at time \( t \). The coefficients in the policy function are then the same as they would be if the central bank could observe the predetermined state perfectly. The coefficient vector \( F \) of the optimal interest function
\[ i_t = FX_{1,t|t} \]
can thus be found by standard full information methods, for instance by the algorithm in Söderlind (1999). Here we describe how the central bank can apply the Kalman filter to estimate the state \( X_{1,t} \). The affine function that maps the predetermined state into bond prices, characterized by the matrices \( Q_1 \) and \( Q_2 \), are taken as given and deriving the equilibrium dynamics of the model is then a straightforward application of the procedure in Svensson and Woodford (2003).

Partition the coefficient matrices in (13) conformably to the predetermined and forward looking variables and substitute in the interest rate function to get
\[ \begin{bmatrix} X_{1,t+1} \\ E_tX_{2,t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} FX_{1,t|t} + C\xi_t . \]

The equilibrium dynamics of the model can then be described by equations (20)-(24)
\[ X_{1,t} = HX_{1,t-1} + JX_{1,t-1|t-1} + C_1\xi_t \]  
\[ X_{2,t} = G^1X_{1,t} + (G - G^1)X_{1,t|t} \]  
\[ X_{1,t|t} = X_{1,t|t-1} + K [Z_t - L_1X_{1,t|t-1} - L_2X_{1,t|t}] \]  
\[ Z_t = z + L_1X_{1,t} + L_2X_{1,t|t} \]  
\[ Y_t = q + Q_1X_{1,t} + Q_2X_{1,t|t} \]
where \( Z_t \) is the vector of variables that are observable to the central bank and \( Y_t \) is a vector of bond yields of different maturities. The system of equations (20)-(24) can be written solely as functions of the actual state, the central bank’s estimate of the state and the shock vectors \( \varepsilon_t \). The coefficient matrices \( G \) and \( G^1 \) are derived in Svensson and Woodford (2003) and satisfy equations (25) and (26)

\[
G = (A_{22} - GA_{12})^{-1} \{-A_{21} + GA_{11} + (GB_1 - B_2)F\} \quad (25) \\
G^1 = A_{22}^{-1} \{-A_{21} + [G^1 + (G - G^1)KL]H\} \quad (26)
\]

where the following definitions were used

\[
H \equiv A_{11} + A_{12}G^1 \quad (27) \\
J \equiv B_1F + A_{12}(G - G^1) \cdot \quad (28)
\]

The Kalman gain matrix \( K \) is given by

\[
K = PL_1^{-1}(L_1PL_1^{-1})^{-1} \quad (29) \\
P = H(P - PL_1^{-1}(L_1PL_1^{-1})L_1P)H' + \Sigma_{\varepsilon\varepsilon} \quad (30)
\]

where \( P \) is the one period ahead forecast error, \( \Sigma_{\varepsilon\varepsilon} \) is the covariance matrix of the shocks \( \{\varepsilon_t^0, \varepsilon_t^y, \varepsilon_t^\pi, \varepsilon_t^v\} \). The coefficient matrices \( G, G^1 \), the Kalman gain \( K \) and the one period forecast error \( P \) have to be determined jointly by finding a fixed point of the system described by the equations (25),(26),(29) and (30). Before we can solve for the equilibrium dynamics we need to specify the selection matrices \( L_1 \) and \( L_2 \) in the observation equation (23). We thus have to decide what the central bank can observe.

2.2.1. Variables observable by the central bank. The central bank observes bond yields contemporaneously, while output and inflation are only observable with a one period lag. This is a compromise that is necessary due to the division of time into discrete periods that do not conform to the exact delays of data releases, though it does capture some broad features of data availability. Data on real GDP are released with a significant delay while bond prices are observed every day that bonds are traded. The compromise is the observation of the price level. In most countries, CPI data are released the month after observation so the one quarter lag is thus too long for most countries.\(^5\) We can write the measurement equation (23) as

\[
Z_t = \begin{bmatrix} y_{t-1} \\ x_{t-1} \\ Y_t \end{bmatrix} + \begin{bmatrix} v_t^y \\ v_t^x \\ 0 \end{bmatrix} \quad (31)
\]

and the matrices \( L_1 \) and \( L_2 \) are then given by

\[
L_1 = \begin{bmatrix} 0_{2\times 1} & I_2 & 0_{2\times 4} & I_2 & 0_{2\times 3} \\ Q_1 & \end{bmatrix} \quad (32) \\
L_2 = \begin{bmatrix} 0_{2\times 12} \\ Q_2 \end{bmatrix}. \quad (33)
\]

The information set of the central bank is given by

\[
I_t^h = \{A, B, C, Q_1, Q_2, \Sigma_{\varepsilon\varepsilon}, Z_{t-s} \mid s \geq 0\} \quad (34)
\]

that is, in addition to observing the vector \( Z_t \), the central bank also knows the structure of the economy.

\(^5\)One exception is Australia where data on the CPI is collected quarterly.
2.3. The Law of Motion for the State of the Economy. In full information models, the relevant state for the pricing of bonds is the same as the state of the economy. In the present model the central bank cannot observe the state of the economy with certainty and uses the Kalman filter to estimate it. The central bank’s information set is a subset of the information set of the bond market participants. This assumption allows us to model bond market participants as if they know the central bank’s estimate of the state. We define the extended state $X_t$ as

$$X_t = \begin{bmatrix} X_{1,t} & X_{1,t|t} \end{bmatrix}'$$

(35)

and we want to find a system of the form

$$X_t = M X_{t-1} + N \varepsilon_t$$

(36)

$$Y_t = q + Q X_t$$

(37)

that is, we conjecture that yields $Y_t$ are an affine function of the extended state $X_t$. We start by substituting the observation equation (23) into the central bank’s updating equation (22) to get

$$X_{1,t|t} = X_{1,t|t-1} + K \left[ L_1 X_{1,t} + L_2 X_{1,t|t} - L_1 X_{1,t|t-1} - L_2 X_{1,t|t} \right]$$

(38)

Using equations (20)-(22) and the definitions (27) and (28) and rearranging we get

$$X_{1,t|t} = \left[ (H + J) + K L_1 J - K L_1 (H + J) \right] X_{1,t-1|t-1}$$

$$+ K L_1 H X_{1,t-1|t-1} + K L_1 \varepsilon_t$$

(39)

The matrices $K$, $L_1$ and $L_2$ depend on the coefficients in the conjectured term structure function (37) and the covariance matrix of the structural shocks and measurement errors $\Sigma_{zz}$. Combining equation (20) and (39) we get the conjectured form from equation (36)

$$X_t = \begin{bmatrix} H & J \\ K L_1 H & [(H + J) + K L_1 J - K L_1 (H + J)] \end{bmatrix} X_{t-1}$$

$$+ \begin{bmatrix} C_1 \\ K L_1 C_1 \end{bmatrix} \varepsilon_t$$

(40)

2.4. The Term Structure and the State of the Economy. In this section we derive the law of motion for the nominal stochastic discount factor that is used to price the bonds from the utility function of the representative household. However, the framework presented here is general enough to accommodate any affine asset pricing function and it is thus not necessary to impose that the macro model and the bond pricing function are determined by the same underlying micro foundations. Define the nominal stochastic discount factor $M_{t+1}$ as

$$E_t M_{t+1} \equiv E_t \beta \frac{U_{ct+1} P_t}{U_{ct} P_{t+1}}.$$  

(41)

where $U_{ct}$ is the marginal utility of consumption in period $t$. If we assume that the distribution of $M_{t+1}$ is log normal, that is, if $m_{t+1} = \log M_{t+1}$ and

$$m_{t+1} \sim N(\bar{m}_{t+1}, \sigma_m^2)$$

(42)

then the expected value of $m_{t+1}$ is

$$E_t m_{t+1} = \bar{m}_{t+1} - \frac{\sigma_m^2}{2}.$$  

(43)
Plugging in the utility function (2) into (41) and (43) we get
\[ E_t \bar{m}_{t+1} = -\gamma E_t c_{t+1} + (\gamma - \eta) c_t + \eta (1 - \gamma) c_{t-1} - E_t \pi_{t+1} \] (44)
where \( \bar{m} \) denotes the deviation of \( m \) from its mean. Using that in equilibrium
\[ E_t \bar{m}_{t+1} + i_t = 0 \] (45)
must hold and that the interest rate \( i_t \) is a function of the state \( X_t \)
\[ i_t = F X_t \] (46)
\[ F = \begin{bmatrix} 0_{1x7} & F \end{bmatrix} \] (47)
we get the following expression for the the expected value of \( m_{t+1} \)
\[ E_t m_{t+1} = \bar{\beta} - F X_t - \frac{V' \Sigma V}{2} \] (48)
The vector \( V' \) determines how the variance of the structural and non-fundamental shocks \( \Sigma \) is mapped into the conditional variance of the stochastic discount factor. \( V' \) is a function of the structural parameters of the model and given by
\[ V' = \begin{bmatrix} -\gamma & -1 \end{bmatrix} \begin{bmatrix} G^1 & G^1 - G^1 \end{bmatrix} \begin{bmatrix} C_1 & 0 \\ KL_1 C_1 & K \end{bmatrix} \] (49)
\[ \Sigma = \begin{bmatrix} \Sigma_{xx} & 0 \\ 0 & \Sigma_{vv} \end{bmatrix} \] (50)
The log of the price at time \( t \) of a nominal bond paying one dollar in period \( t + n \) will then be
\[ \log P^n_t = \bar{A}_n + \bar{B}_n' X_t \] (51)
where the constant \( \bar{A}_n \) and the vector \( \bar{B}_n \) are given by the recursive relations
\[ \bar{A}_n = -\bar{i} + \bar{A}_{n-1} - \bar{B}_{n-1}' V + \frac{1}{2} \bar{B}_{n-1}' \Sigma V \Sigma V' \bar{B}_{n-1} \] (52)
\[ \bar{B}_n = -F + M' \bar{B}_{n-1} \] (53)
starting from
\[ \bar{A}_1 = -\bar{i} \] (54)
\[ \bar{B}_1 = -F' \] (55)
where \( \bar{i} \) is the average short interest rate. To find the vector of yields of selected maturities \( \gamma_t \) collect the appropriate constants \( \bar{A}_n \) and vectors \( \bar{B}_n \) as
\[ \gamma_t = \begin{bmatrix} -\bar{A}_1 \\ \vdots \\ -\frac{1}{n} \bar{A}_n \end{bmatrix} + \begin{bmatrix} -\bar{B}_1 \\ \vdots \\ -\frac{1}{n} \bar{B}_n \end{bmatrix} X_t + v_t^{\gamma} \] (56)
\[ E v_t^{\gamma} v_t^{\gamma'} = \Sigma v v' \] (57)
where the yield of an \( n \) periods to maturity bond is found by dividing the price by \( n \). Equation (56) has a dual interpretation. On one hand it can be used to express bond yields as a function of the state and the vector of shocks to the term structure, \( v_t^{\gamma} \), are then residuals, i.e. the component of the yields that cannot be explained by the dynamics of the stochastic discount factor. A small variance of \( v_t^{\gamma} \) should then be interpreted as that the term structure model provide a good fit of the observed yields. Equation (56) can also be viewed as a measurement equation where the observable bond yields \( \gamma_t \) tell us something about the unobservable state \( X_t \). The
vector of shocks $\nu^Y_t$ are then measurement errors and when the variance of the innovations to $\nu^Y_t$ are small, the signal-to-noise ratio is high and the term structure is informative about the state of the economy. In the special case of the rank of $[-B_1 \ldots -\frac{1}{n}B_n]'$ being equal to the dimension of the state and $\Sigma_{\nu^Y} = 0$, the model replicates the full information dynamics, since the state can then be backed out perfectly from the term structure. In the opposite case, when the variances of $\Sigma_{\nu^Y}$ are very large, the model will replicate the dynamics when the central bank can only observe imperfect but direct measures of the lagged aggregate variables.

In practice, the measurement errors on the longer maturity bonds turn out to be strongly serially correlated. By including the measurement errors in the state, it is straightforward to estimate their persistence together with the rest of the model’s coefficients. The precision of the yields as a measure of the state will then depend on the variance of the innovations $e^v_t$ to the measurement error process (17), rather than the variance of the measurement errors themselves.

Partitioning the stacked vectors $-\frac{1}{n}B_n$ appropriately and since $\nu^Y_t \in \mathcal{X}_t$, we can write (56) in the desired form (37)

$$\mathcal{Y}_t = \mathbf{q} + \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \mathcal{X}_t$$

(58)

where

$$\begin{bmatrix} Q_1 & Q_2 \end{bmatrix} = \begin{bmatrix} -B_1 \\ \vdots \\ -\frac{1}{n}B_n \end{bmatrix} + \begin{bmatrix} 0 & I & 0 \end{bmatrix}.$$ 

This completes the description of the model.

3. The Dynamics of the Estimated Model

The implications of the letting the central bank extract information from the term structure depend on the magnitude of the noise in the bond market. There is little information in the term structure when bond prices are very noisy, and including it in the information set of the central bank then has little effect on the dynamics of inflation and output. It is therefore of interest to quantify the variances in the model. The parameters of the model are estimated by Bayesian methods using quarterly data for the US ranging from 1982:Q1 to 2005:Q4 and for Australia ranging from 1991:Q1 to 2005:Q3. The first 8 observations were used as a convergence sample for the Kalman filter. The shorter sample period for Australia is motivated by the fact that Australia experienced a large downward shift in the level of inflation in the early 1990’s. Preliminary estimations suggested that including this shift in the sample may bias the results. First, it makes the estimates more sensitive to the detrending method used. Second, including a non-typical period when inflation is brought down by contractionary monetary policy may bias the estimates of the preference parameters of the Reserve Bank of Australia to make it appear more averse to inflation volatility than is actually the case.

The interest rates included are the Federal Funds rate and secondary market rates for 6 and 12 month Treasury Bills for the US and the Cash Rate and the 180 day Bank Bill Rate and the 12 month Treasury Bond Rate for Australia. For both the US and Australia, non-farm real GDP and CPI (excluding food and energy) are used as a measure of output and to calculate quarterly inflation rates. The GDP data series were detrended using the Hodrick-Prescott filter and the CPI and interest rate series were demeaned. By using demeaned data, we do not
fully exploit the information the yield curve about the structural parameters of the model. However, estimating the model in level form to also match the average slope of the yield curve in US data led to large reductions in overall fit. For Australia, there is an additional data problem since the 6 month bonds and 1 year bonds are of different quality and the privately backed 6 month Bank Bills have a higher average yield than the government backed 1 year Treasury Bonds.

The priors on the structural parameters are reported in Table 1 and 2 in the Appendix. We impose rather tight priors on the degree of price stickiness \( \theta \) which can be measured directly, and the discount factor \( \gamma \) which can be backed out from average real interest rates. The priors on the rest of the behavioral parameters are more dispersed. The prior distributions of the variances of the structural shocks of the model are uniform over the interval \((0,1)\) and the prior distributions of the variances of the measurement errors are uniform over the interval \((0,\sigma_{z_t}^2)\) where \(\sigma_{z_t}^2\) is the variance of the corresponding observable time series.

All yields except for the short interest rate are assumed to be affected by serially correlated measurement errors/non-fundamental shocks. The variance of the non-fundamental shocks and their persistence is estimated together with the other parameters of the model. We estimate two sets of measurement errors on output and inflation. The first, which we may call the econometric measurement errors, are motivated by the fact that the output and inflation measures that we use do not correspond exactly to the model's concept of output and inflation as well as that data on GDP may include errors introduced through the data collection process. The variance of the econometric measurement errors are denoted \(\sigma_{vy}^2\) and \(\sigma_{v\pi}^2\).

We do not want to impose that the time series that we use to estimate the model corresponds exactly to the information that enters into the filtering problem of the central bank. We therefore allow for a second set of measurement errors on the lagged output and inflation measure that the model’s central bank observes. These should be interpreted more generally as the central bank’s misperceptions of the lagged aggregate variables and their variances are denoted by \(\sigma_{vycb}^2\) and \(\sigma_{v\pi cb}^2\). The estimated variances of both the econometric measurement errors and the measurement errors of the central banks’ observation of lagged output and inflation are reported in Table 1 and 2 in the Appendix where more details about the estimation procedure also can be found.

The posterior estimated of the behavioral parameters of households and firms are quite similar across the US and Australia. The exceptions are the curvature of the disutility of supplying labor which is lower in the US than in Australia and the preference for output gap stabilization and interest rate smoothing which are estimated to be lower in Australia than in the US. Our primary interest lies in quantifying the variances of the non-fundamental shocks to the term structure since it is these parameters that govern how informative the term structure is as a measure of the current state of the business cycle. The posterior estimates of these variances are larger for Australia than for the US and the consequences of this are analyzed in the next section.

3.1. Impulse Responses and the Role of Term Structure Information. Figures 1 and 2 below illustrate the impulse responses of output (solid line), inflation (dashed line) and the short interest rate (dotted line) for the US and Australia to productivity, demand and cost push shocks. The estimated impulse responses look similar across the two countries with one important exception: The response of
the short interest rate in the impact period. In the US, short interest rates rise immediately when the economy is hit by either a demand or cost push shock while in Australia, the short interest rate move only marginally in the impact period. The short rate does not respond in the impact period to productivity shocks in neither of the two countries.

The different responses of the short interest rate to demand and cost push shocks in the US and in Australia can be explained by the different estimated variances of the non-fundamental shocks to the term structure. In Figure 3 the impulse responses to the same shocks as above are plotted for Australia, but with the variance of the non-fundamental shocks to the term structure set to zero. The Australian central bank then has more accurate information and we can see that the response of the short rate in the impact period is larger. Another interesting difference between Figure 2 and 3 that illustrates the non-trivial interaction between information in the term structure and equilibrium responses to shocks is that even though the initial response of the short rate to the cost push shock is larger without noise in the term structure, the second period response is much stronger when the
term structure is noisy. When the term structure is noisy, the short interest rate fails to respond immediately to the cost push shock, but forward looking households expect interest rates to rise in the future and lowers consumption immediately. In the second period, when the impact period fall in output and increase in inflation are observed by the central bank, this is partly interpreted as a negative productivity shock which causes the central bank to raise interest rates more than is justified solely by the increase in inflation from the cost push shock. When the term structure is less noisy, the central bank attributes a larger proportion of what it observes to the correct sources.

The analysis above emphasizes the beneficial aspects of responding to the term structure, but since the term structure is noisy this means that sometimes the central bank will inadvertently respond to non-fundamental shocks. The responses of output, inflation and the short interest rate in the US and Australia to a unit non-fundamental shock to the 6 month bond rate is plotted in Figure 4. An increase in the 6 month yield signals that a shock has occurred that the market believes that the central bank will want to respond to by raising interest rates so the short interest
rate in both the US and Australia increases in response to the non-fundamental shock. This leads to a fall in both output and inflation. The response is stronger in the US than in Australia since historically, the variance of the noise has been lower in the US, which leads the Fed to attribute a larger fraction of what it observes to fundamental sources rather than noise. That the central bank responds to a non-fundamental increase in long rates by raising short rates does not imply that the model permits self fulfilling expectation shocks. The reason is twofold. Firstly, since the term structure is known by the central bank to be noisy, policymakers will not put enough weight on the observation of bond yields to respond so strongly that a unit increase in a long bond rate justifies it self through the expectations hypothesis. Secondly, the initial increase of the short rate will lower output and inflation. Ceteris paribus, this means that in the next period when the output and inflation outcomes of the impact period are observed, the central bank will want to lower interest rates. These two forces rule out self fulfilling expectations shocks to longer maturity yields.
3.2. Variance Decomposition. The estimated model can be used to decompose historical variances of the endogenous variables into their exogenous sources. Table 3 and 4 report the variance decompositions evaluated at the posterior modes as well as the 95% confidence intervals. In the US, output variance is primarily caused by productivity shocks which also explain more than half of the variance of US inflation. Demand shocks in the US appear to not be very important for output variance, but explain more than half the variance of the short interest rate which suggest that the Federal Reserve has successfully raised interest rates to counter exogenous demand pressure during the sample period.

Productivity shocks are important also in Australia and explain almost half of both output and inflation volatility, while demand shocks explains almost the entire other half. The posterior mode estimates suggest that cost push shocks are not a major source of macroeconomic variability in neither the US nor Australia, though the confidence intervals are quite wide in the case of Australia.
same two sources, but the magnitude of their influence on short interest rates is much smaller: Misperceptions about lagged output is responsible for only 1% of another 6%. Bank misperceptions about lagged output and inflation and the non-fundamental shocks to the 6 month yields, which explain 17% of the variability and Wouters (2004). That a large portion of the “policy shocks” in the US comes in the US is comparable to what full information studies have found, e.g. Smets and Wouters (2004). That a large portion of the “policy shocks” in the US comes to the macro economy is the short interest rate. In the present model, all non-systematic changes in monetary policy are attributed to misperceptions of the state of the economy by the central bank. The four non-structural shocks thus provide a way to interpret what in a full information model would be termed "monetary policy shocks”.

Perhaps the more interesting part of Table 3 and 4 are the four columns to the right. These report the proportion of the variances of output, inflation and interest rates of different maturities that are caused by non-structural shocks, i.e. by central bank misperceptions about lagged output and inflation and the non-fundamental shocks to the term structure. The channel through which these shocks affect the macro economy is the short interest rate. In the present model, all non-systematic changes in monetary policy are attributed to misperceptions of the state of the economy by the central bank. The four non-structural shocks thus provide a way to interpret what in a full information model would be termed "monetary policy shocks”.

For the US, the model attributes policy shocks to come from either misperceptions of lagged output, which explains 11% of short interest rate variability, and non-fundamental shocks to the 6 month yields, which explain 17% of the variability short interest rates. For Australia, policy shocks appear to come mainly from the same two sources, but the magnitude of their influence on short interest rates is much smaller: Misperceptions about lagged output is responsible for only 1% of short rate variability and non-fundamental shocks to the 6 month yield explains another 6%.

<table>
<thead>
<tr>
<th>Table 3 Variance Decomposition US (2.5%-97.5%)</th>
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<tr>
<td>$\xi_t$</td>
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<td>$y_t$</td>
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<td>$\pi_t$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
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</tr>
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<td></td>
</tr>
<tr>
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<tr>
<td>$\gamma_t^4$</td>
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<table>
<thead>
<tr>
<th>Table 4 Variance Decomposition Australia (2.5%-97.5%)</th>
</tr>
</thead>
<tbody>
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<td>$\xi_t^a$</td>
</tr>
<tr>
<td>$y_t$</td>
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<tr>
<td>$\pi_t$</td>
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<td></td>
</tr>
<tr>
<td>$i_t$</td>
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<tr>
<td></td>
</tr>
<tr>
<td>$\gamma_t^2$</td>
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<td></td>
</tr>
<tr>
<td>$\gamma_t^4$</td>
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<tr>
<td></td>
</tr>
</tbody>
</table>

This magnitude of interest rate variance that is attributable to “policy shocks” in the US is comparable to what full information studies have found, e.g. Smets and Wouters (2004). That a large portion of the “policy shocks” in the US comes
from the bond market suggest that there may have been episodes where misperceptions about the state of the economy may have been shared by the bond market participants. These non-fundamental shocks feed into output and inflation through the short term interest rate, and it may appear as if responding to these shocks is not optimal. However, since the central bank is assumed to respond with statistically optimal weights to movements in the term structure, the benefits of having more accurate estimates of the state on average outweighs the cost of occasionally responding to “false alarms”.

3.3. The (lack of) information in longer maturity yields. In the variance decomposition tables, we could see that the fraction of the variance of yields that is explained by non-fundamental shocks is larger for the 1 year bonds than for 6 month bonds and for the US, almost all information in the term structure about the state of the economy is in the observation of the 6 month yield. Excluding the 1 year yield hardly changes the responses to shocks of output, inflation and short interest rates at all. The model was also estimated including 2 year and 5 year yields in the vector of variables that the central bank can observe. These longer yields held virtually no information about the state and their variance was almost entirely explained by non-fundamental shocks. That longer maturity bonds are uninformative about the state of the economy is the dual of the fact that standard macro models are not very good at explaining the movements of long maturity yields (see for instance Gurkaynak, Sack and Swanson (2005)). If the business cycle model is not very good at explaining long yields, then movements in long yields will not be very informative about the current state of the business cycle. This is not to say that there is no correlation between long yields and the business cycle, but rather that if we want to interpret this correlation using structural models that can guide policy, then we need to develop a better understanding of what drives long maturity yields.

3.4. Robustness. In the baseline estimation above it was assumed that the central banks had an explicit interest rate smoothing objective. An alternative explanation of inertial interest rate changes could be that central banks move slowly because they want to accumulate more information before they move. The model was re-estimated after imposing no interest rate smoothing, that is, by setting $\lambda_i = 0$. This lead to reductions in the marginal likelihoods for both the US and Australia and the corresponding posterior odds ratios do not support the hypothesis that $\lambda_i = 0$.

In the estimation, it was also imposed that the central banks of both the US and Australia used the information in the term structure to set policy. If this assumption is incorrect, it would bias the estimates of the noise in the term structure upwards. The log likelihood for both the US and Australia decreases slightly when we re-estimate the model without assuming that the central banks use the information in the term structure. The change for the US is a larger, probably because the assumption that the central bank observes the term structure should make less of a difference for Australia where the term structure is relatively more noisy. Though the change in marginal likelihood is small, the direction of the change lends some empirical support for the assumption that the Fed uses the information in the term structure to set policy, in addition to anecdotal evidence in the form of quotes from Federal Reserve officials.

\footnote{More details on posterior mode estimates and likelihoods for the alternative specifications are available from the author upon request.}
4. Conclusions

This paper has presented a general equilibrium model of monetary policy where the central bank operates in an uncertain environment and uses information contained in the term structure to estimate the underlying state of the economy more efficiently. This set up creates a link between the term structure and the macro economy that is novel to the literature. A movement in the term structure signals that a change in the short term interest rate set by the central bank may be desirable, which when implemented in turn affects aggregate demand.

Söderlind and Svensson (1997) warn that “central banks should not react mechanically to [market expectations]” since this may lead to a situation of “the central bank chasing the market, and the market simultaneously chasing the central bank”. This argument is formalized in Bernanke and Woodford (1997) where the authors show that if central banks react to market expectations, a situation with a multiplicity of equilibria or where no equilibrium exists may arise. In this paper we have argued that there may be benefits from systematic reactions to market expectations, but with some important qualifications. The non-existence of equilibria arises in the model of Bernanke and Woodford because the central bank can extract the underlying state perfectly from observing the expectations of the private sector. Inflation will thus always be on target. But if inflation is always on target and private agents only care about accurate inflation forecasts there is no incentive for the private sector to pay a cost to be informed about the underlying shock, and observing expectations will not reveal any information. The model here differs because, to the extent that there is noise in the bond market, the central bank cannot extract the underlying shock perfectly. Thus there will always exist a cost of information gathering that is small enough to make it profitable for the private sector to acquire information about the underlying shock, even if private agents only cared about having accurate inflation forecasts. Additionally, in this model the forecasting problem of private agents involves more than accurately forecasting inflation since bond prices depend on real factors through the stochastic discount factor as well as the price level. In so far as the real discount factor is affected by the underlying state, agents will have an incentive to collect information about it, regardless of the behavior of inflation.

Ultimately, the informational content of the term structure is an empirical question. The model presented here provides a coherent framework within which any information about the state of the economy that is contained in the term structure can be quantified in a general equilibrium setting. The model explicitly takes into account that the central bank may use the information in the term structure to set policy and therefore influences what it observes. The model was estimated on US and Australian data using Bayesian methods. The empirical exercise suggests that there is some information in the US term structure that allow the Federal Reserve to respond to shocks in a timely manner while the Australian term structure appears to be more noisy and less informative for the monetary policy process. This difference may be explained by the fact that Australia is a small and relatively open economy and hence difficult to represent using a closed economy model.

Most of the information in the US term structure about the state of the business cycle could be found in yields with maturities of less than one year. That longer maturity bonds are uninformative about the state of the economy is a consequence of the fact that the standard macro models are not very good at explaining
movements in long maturity yields (see for instance Gurkaynak, Sack and Swanson (2005)). If a business cycle model is not very good at explaining long yields, then movements in long yields will not be very informative about the current state of the business cycle as defined by the model at hand. There is reduced form evidence that long maturity yields are correlated with the business cycle, with perhaps the most famous link being between an inverted yield curve and the onset of a recession (Harvey (1993)) which suggest that there may be potentially useful information also in the long end of the term structure. However, the behaviour of long term interest rates are still quite poorly understood, and has even recently been referred to as a "conundrum". Until a better understanding of what drives long rates is achieved, we may have to settle for looking at the short end of the yield curve to find clues about the current state of the business cycle.

References


7See Greenspan (2005).
The model can be put in compact form

\[
\begin{bmatrix}
X_{1,t+1} \\
E_{t}X_{2,t+1}
\end{bmatrix}
= A \begin{bmatrix}
X_{1,t} \\
X_{2,t}
\end{bmatrix} + B \epsilon_t + C \epsilon_t
\]

(59)

\[
X_{1,t} = [a_t, y_{t-1}, \pi_{t-1}, \pi_t, \pi_t, \mu_i, \epsilon_t]'
\]

(60)

\[
X_{2,t} = [y_t, \pi_t]'
\]

(61)

\[
\epsilon_t = [\epsilon_t, \epsilon_t, \epsilon_t, \epsilon_t, \epsilon_t]'.
\]

(62)

where the coefficient matrices \(A, B, \) and \(C\) are given by

\[
A = A_0^{-1} A_1,
B = A_0^{-1} B_1,
C = A_0^{-1} \begin{bmatrix}
C_1 \\
0_{2 \times 1}
\end{bmatrix}
\]

\[
A_0 = \begin{bmatrix}
I_5 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & I_5 & 0 & 0 \\
0 & 0 & 0 & \mu_{\phi} & \phi & 0 \\
0 & 0 & 0 & 0 & \mu_{\pi_f} & 0
\end{bmatrix}
\]

(63)
\[
A_1 = \begin{bmatrix}
\rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

where

\[
\begin{align*}
\mu_{yf} & \equiv \frac{\gamma}{\gamma - \eta + \gamma \eta}, & \mu_{yb} & \equiv -\frac{\eta (1 - \gamma)}{\gamma - \eta + \gamma \eta} \\
\phi & \equiv \frac{1}{\gamma - \eta + \gamma \eta}, & \mu_{\pi f} & \equiv \frac{\beta \theta}{\theta + \omega (1 - \theta (1 - \beta))}, & \mu_{\pi b} & \equiv \frac{\omega}{\theta + \omega (1 - \theta (1 - \beta))} \\
\kappa & \equiv \frac{(1 - \omega)(1 - \theta)(1 - \theta \beta)}{\theta + \omega (1 - \theta (1 - \beta))} \\
W & = \begin{bmatrix} 0 & 0 & 0 & 0 \\
0 & \rho y_2 & 0 \\
0 & 0 & \rho y_4 \end{bmatrix}
\end{align*}
\]

\[
B_1 = \begin{bmatrix} 0_{5 \times 1} \\
0_{6 \times 1} \\
\phi \\
0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 & 0 \\
0_{2 \times 1} & 0_{2 \times 1} & 0_{2 \times 1} \\
0 & 1 & 0 \\
0 & 0 & 1 \end{bmatrix}
\]

\[
\mathcal{L}_t = E_t \left[ \sum_{k=0}^{\infty} \beta^k \left[ \lambda_y (y_{t+k} - \eta_{t+k})^2 + \pi_{t+k}^2 + \lambda_i (i_t - i_{t-1})^2 \right] \right]
\]
Appendix B. The likelihood function

Form a state space system of the AR(1) process of the state $X_t$

\[
\begin{align*}
\ddot{X}_t &= M \ddot{X}_{t-1} + \nu_t \\
\ddot{Z}_t &= \tilde{\mu} + D \ddot{X}_t + v_t \\
\dot{\Sigma}_{vv} &= \ddot{\Sigma}_{vv}
\end{align*}
\] (67)

where $\ddot{Z}_t$ is the vector of variables that are observable and $\ddot{\Sigma}_{vv}$ is the covariance matrix of the measurement errors given by

\[
\ddot{\Sigma}_{vv} = \begin{bmatrix}
\sigma^{2}_{v_y} & 0 & 0 \\
0 & \sigma^{2}_{v_z} & 0 \\
0 & 0 & 0
\end{bmatrix}
\] (70)

Construct the innovation series $\{u_t\}_{t=0}^T$ from the innovation representation

\[
\begin{align*}
\dddot{X}_{t+1} &= \tilde{\mu} + M \dddot{X}_t + Ku_t \\
\dddot{Z}_t &= D \dddot{X}_t + u_t
\end{align*}
\] (71, 72)

by rearranging to

\[
u_t = \dddot{Z}_t - D \dddot{X}_t
\] (73)

where $K$ is the Kalman gain matrix

\[
\begin{align*}
K &= PD'(DPD' + \ddot{\Sigma}_{vv})^{-1} \\
P &= M(P - PD'(DPD' + \ddot{\Sigma}_{vv})^{-1}DP)M' + N \Sigma \varepsilon N'.
\end{align*}
\] (74, 75)

The log likelihood $L(\dddot{Z} | \Theta)$ of observing the data $\dddot{Z}$ for a given set of parameters $\Theta$ can then be computed as

\[
L(\dddot{Z} | \Theta) = -0.5 \sum_{t=0}^{T} [p \ln(2\pi) + \ln |\Omega| + u_t' \Omega^{-1} u_t] \\
\] (76)

where

\[
\Omega = MPM' + \ddot{\Sigma}_{vv}.
\] (77)

The posterior mode $\hat{\Theta}$ is then given by

\[
\hat{\Theta} = \arg \max L(\Theta) + L(\dddot{Z} | \Theta)
\] (78)

where $L(\Theta)$ denotes the log of the prior likelihood of the parameters $\Theta$. The posterior mode was found using Bill Goße’s simulated annealing minimizer (available at http://cook.rfe.org/). The posterior standard deviations were calculated using Gary Koop’s Random Walk Metropolis-Hastings distribution simulator (available at http://www.wiley.co.uk/koopbayesian/).
### Appendix C. Parameter Estimates

#### Table 1 US

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior mode</th>
<th>Prior s.e.</th>
<th>Distribution</th>
<th>Posterior s.e.</th>
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<td>$\omega$</td>
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<td>beta</td>
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$L(\hat{\Theta})$: 3.8

$L(Z \mid \hat{\Theta})$: 2272.9

$L(\hat{\Theta} \mid Z)$: 2276.7
Table 2 Australia

<table>
<thead>
<tr>
<th></th>
<th>Prior mode</th>
<th>Prior s.e.</th>
<th>Distribution</th>
<th>Posterior s.e.</th>
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$L(\Theta) = 1.8$

$L(Z \mid \Theta) = 1149.3$

$L(\hat{\Theta} \mid Z) = 1151.1$
Appendix D. Additional figures

Figure 5. US data and model’s one-step-ahead forecasts

Figure 6. Australian data and model’s one-step-ahead forecasts
Figure 7. IRF Productivity Shock US

Figure 8. IRF Demand Shock US

Figure 9. IRF Cost Push Shock US
**Figure 10.** IRF Central Bank measurement error on output shock US

**Figure 11.** IRF 6 Month Rate Non-Fundamental shock US

**Figure 12.** IRF Productivity Shock Australia
Figure 13. IRF Demand Shock Australia

Figure 14. IRF Cost Push Shock Australia

Figure 15. IRF 6 Month Rate Non-Fundamental shock Australia