Cyclical Fluctuations, Co-movement and the Role of External Shocks in Latin America

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1 Banco Central de Reserva del Perú

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Cyclical Fluctuations, Co-movement and the Role of External Shocks in Latin America *

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Abstract

This paper compares the business cycles across five Latin American economies (Brazil, Chile, Colombia, Mexico and Peru) for the period 1997-2014. We estimate a Multi-Country VAR through Bayesian Methods, where the model takes into account dynamic interdependencies and time-varying parameters (Canova and Ciccarelli, 2009). We present regional and country-specific indicators of real economic activity. We find a significant common regional component, as well as significant country specific indicators, meaning that there exists some synchronization across business cycles, but at the same time there is some heterogeneity across these economies. We find some heterogeneity before the international financial crisis of 2008 and a more significant co-movement after that date. Furthermore, we explore the transmission at different dates of domestic (country specific) and external shocks such as Chinese GDP growth. Overall, we find that the transmission of both domestic and external shocks are somewhat stable after the Inflation Targeting adoption.

JEL Classification: C11, C33, E32

Key words: Business Cycles, Panel Vector Autoregressions, Latin American economies, Bayesian Methods

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1 Introduction

Latin American business cycles are a relevant component for global projections, according to Carabenciov et al. (2013). Still, this region is tremendously vulnerable to global shocks such as the Great Recession in the US (2008-2008) or the Chinese economic growth boom starting last decade, given that most of them are commodity exporters. Moreover, since there exists partial financial dollarization, to some extent it is natural and reasonable to assume that these economies are vulnerable to exchange rate shocks, e.g. external monetary policy shocks. On the other hand, main economies in this region have adopted the Inflation Targeting framework as a monetary policy scheme, which is considered an important institutional change, since it has provided a good signal for expectations formation and expectations anchoring. Furthermore, several trade agreements have been signed during the last twenty years. As we can see, Latin America data seems to be a good laboratory for testing the relevance of each of the mentioned facts.

This paper compares the business cycles across main Latin American economies (Brazil, Chile, Colombia, Mexico and Peru) for the period 1997-2014. We focus our attention in these five economies because they share some common features, such as the application of Inflation Targeting, floating exchange rates, and because economic activity in these economies highly depends on external shocks, such as growth of US, Europe and China and price fluctuations, i.e. Oil and Commodities. There exists several studies that try to model Latin American Business Cycles\textsuperscript{1}. However, all these institutional changes can be considered as structural breaks, and thus we cannot use standard time series tools in order to assess a good data fit. Thus, in this paper we estimate a Multi-Country VAR through Bayesian Methods, where the model takes into account dynamic inter-dependencies and time-varying parameters (Canova and Ciccarelli, 2009)\textsuperscript{2}, since we want to exploit the interaction among Latin American countries and also take into account the institutional changes present in the sample of analysis. Having said that, it is important to remark that the sample span covers two crisis episodes (1998 and 2008) and also

\textsuperscript{1}Early literature for Business Cycles Latin America and other emerging economies can be found in Tienda et al. (1987), De-Gregorio (1992), Loayza et al. (2001), Izquierdo et al. (2008), Astorga (2010), Chang and Fernández (2010), García-Cicco et al. (2010), Aiolfi et al. (2011), Camacho and Pérez-Quiros (2013), among others.

\textsuperscript{2}See also Canova and Ciccarelli (2013).
captures the Inflation Targeting adoption of these five countries, between 1997 and 2002.

We present regional and country-specific indicators of real economic activity, where we find a significant common regional component, as well as significant country specific indicators, meaning that there exists some synchronization across business cycles, but at the same time there is some heterogeneity across these economies. We also find some heterogeneity before the international financial crisis of 2008 and a more significant co-movement after that date. Furthermore, we explore the transmission at different dates of domestic (country specific) and external shocks such as Chinese GDP growth. Overall, we find that the transmission of both domestic and external shocks are somewhat stable after the Inflation Targeting adoption.

The document is organized as follows: section 2 describes the econometric model, section 3 describes the estimation procedure, section 4 discusses the main results, and section 5 concludes.

2 The Multi-Country Panel VAR model

This section closely follows Canova and Ciccarelli (2009). We specify a Multi-Country model with lagged inter-dependencies and time varying parameters. We abstract from the possible presence of Stochastic Volatility, since the current setup is already computationally demanding.\footnote{See also Canova et al. (2007), Canova and Ciccarelli (2012) and Canova et al. (2012).}

2.1 The setup

The statistical model employed in this paper has the form:

\[
y_{it} = D_{it}(L) Y_{t-1} + F_{it}(L) Z_t + c_t + e_{it}
\]

where \( i = 1, \ldots, N \) refers to countries and \( t = 1, \ldots, T \) refers to time periods. In addition, \( y_{it} \) is a \( M \times 1 \) vector of endogenous variables for each country \( i \) and \( Y_t = (y_{1t}', y_{2t}', \ldots, y_{Nt}')' \).

We define the polynomials

\[
D_{it}(L) = D_{it,1} + D_{it,2}L + \cdots + D_{it,p}L^{p-1}
\]
\[ F_{it}(L) = F_{it,0} + F_{it,1}L + \cdots + F_{it,q}L^q \]

where \( D_{it,j} \) are \( M \times NM \) matrices for each lag \( j = 1, \ldots, p \). Moreover, \( Z_t \) is a \( M_2 \times 1 \) vector of exogenous variables common to all countries and \( F_{it,j} \) are \( M \times M_2 \) matrices for each lag \( j = 0, \ldots, q \), \( e_{it} \) is a \( M \times 1 \) vector of intercepts and \( e_{it} \) is a \( M \times 1 \) vector of random disturbances.

Notice that cross-unit lagged inter-dependencies are allowed whenever the \( NM \times NM \) matrix \( D_t(L) = [D_{1t}(L), D_{2t}(L), \ldots, D_{Nt}(L)]' \) is not block diagonal. Notice also that coefficients in (1) are allowed to vary over time and that dynamic relationships are unit-specific. All these features add realism to the econometric model. However, this comes at the cost of having an extremely large number of parameters to estimate (we have \( k = NMp + M_2(1 + q) + 1 \) parameters per equation). For that reason, we specify a more parsimonious representation of the latter model in order to proceed to the estimation.

Equation (1) can be rewritten in a compact form as

\[ Y_t = W_t \delta_t + E_t, \quad E_t \sim N(0, \Omega) \]  

(2)

where \( W_t = I_{NM} \otimes X_t' \); \( X_t' = (Y_{t-1}', Y_{t-2}', \ldots, Y_{t-p}', Z_t', Z_{t-1}', \ldots, Z_{t-q}', 1)' \); \( \delta_t = (\delta_{1,t}', \delta_{2,t}', \ldots, \delta_{N,t}')' \) and \( \delta_t \) are \( Mk \times 1 \) vectors containing, stacked, the \( M \) rows of matrix \( D_{it} \) and \( F_{it} \), while \( Y_t \) and \( E_t \) are \( NM \times 1 \) vectors. Notice that since \( \delta_t \) varies with cross-sectional units in different time periods, it is impossible to estimate it using classical methods. Even in the case of constant coefficients, the amount of degrees of freedom needed to conduct proper inference is tremendously large. For that reason, Canova and Ciccarelli (2009) suggest to reduce the dimensionality of this problem as follows:

\[ \delta_t = \Xi_1 \theta_{1t} + \Xi_2 \theta_{2t} + \Xi_3 \theta_{3t} + \Xi_4 \theta_{4t} + u_t \]  

(3)

where \( \Xi_1, \Xi_2, \Xi_3, \Xi_4 \) are matrices of dimensions \( NMk \times 1 \), \( NMk \times N \), \( NMk \times M \), \( NMk \times 1 \) respectively. \( \theta_{1t} \) captures movements in coefficients that are common across countries and variables; \( \theta_{2t} \) captures movements in coefficients which are common across countries; \( \theta_{3t} \) captures movements in coefficients which are common across variables; \( \theta_{4t} \) captures movements in coef-
coefficients which are common across exogenous variables. Finally, $u_t$ captures all the un-modeled features of the coefficient vector$^4$.

The factorization (3) significantly reduces the number of parameters to be estimated. In other words, it transforms an over-parametrized panel VAR into a parsimonious SUR model, where the regressors are averages of certain right-hand side variables. In fact, substituting (3) in (2) we have

$$Y_t = \sum_{i=1}^{4} W_{it} \theta_{it} + v_t$$

where $W_{it} = W_t \Xi_i$ capture respectively, common, country specific, variable specific and exogenous specific information present in the data, and $v_t = E_t + W_t u_t$.

To complete the model, we specify $\theta_t = [\theta'_{1t}, \theta'_{2t}, \theta'_{3t}, \theta'_{4t}]'$ so that we have the law of motion:

$$\theta_t = \theta_{t-1} + \eta_t, \quad \eta_t \sim N(0, B_t)$$

where $B_t$ is block-diagonal with:

$$B_t = \gamma_1 B_{t-1} + \gamma_2 B$$

where $\gamma_1$ and $\gamma_2$ are scalars and $B$ is block-diagonal matrix.

To summarize, the empirical model has the state-space form:

$$Y_t = (W_t \Xi) \theta_t + v_t$$

$$\theta_t = \theta_{t-1} + \eta_t$$

where $v_t \sim N(0, \sigma_t)$; $\sigma_t = (1 + \sigma^2 X'_t X_t)$ and $\eta_t \sim N(0, B_t)$. To compute the posterior distributions, we need prior densities for the parameters $(\Omega, \sigma^2, B, \theta_0)$.

### 2.2 Priors

Following the references we set conjugated priors, i.e. such that the posterior distribution has the same shape as the likelihood function. In particular, given the normality assumption for

$^4$See details in Canova and Ciccarelli (2009).
the shocks, the variance and covariance parameters have an Inverse Gamma distribution \(^5\) or Inverse Wishart distribution for the multivariated case. In addition, since we are going to use the kalman filter and smoother for simulating the posterior distribution of latent factors, it is reasonable to assume that the initial point as normally distributed.

\[
p(\Omega^{-1}) = W_i(z_1, Q_1)
\]

\[
p(\sigma^2) = IG\left(\frac{\zeta}{2}, \frac{\zeta s^2}{2}\right)
\]

\[
p(b_i) = IG\left(\frac{\varpi_0}{2}, \frac{\delta_0}{2}\right), \quad i = 1, \ldots, 4
\]

\[
p(\theta_0) = N(\bar{\theta}_0, R_0)
\]

where the latter implies a prior for \(\theta_t = N(\theta_{t-1|t-1}, R_{t-1|t-1} + B_t)\).

### 2.3 Posterior Distribution

The posterior distribution of model parameters is the efficient combination of prior information with the observed data. Denote the parameter vector as

\[
\psi = \left(\Omega^{-1}, \{b_i\}_{i=1}^4, \sigma^2, \{\theta_t\}_{t=1}^T\right)
\]

Given the normality assumption of the error term \(\nu_t\), the likelihood function of the Multi-Country Panel VAR model (4) is equal to

\[
L(Y^T | \psi) \propto \left(\prod_{t=1}^T \sigma_t^{-NM/2}\right) |\Omega|^{-T/2} \exp \left[-\frac{1}{2} \sum_{t=1}^T (Y_t - W_t\Xi\theta_t)^{\prime} (\sigma_t\Omega)^{-1} (Y_t - W_t\Xi\theta_t)^\prime\right]
\]

where \(Y^T = (Y_1, Y_2, \ldots, Y_T)\) denotes the data, and \(\sigma_t = \left(1 + \sigma^2 X_t^\prime X_t\right)\).

Using the Bayes’ rule, we have the posterior distribution

\[
p(\psi | Y^T) \propto L(Y^T | \psi) p(\psi)
\]

\(^5\)See e.g. Zellner (1971) and Koop (2003).
In the next section we will explain how to obtain the optimal estimates of model parameters in a tractable way. So far, we have identified our object of interest, and the next step is to proceed to the estimation.

3 Bayesian Estimation

3.1 A Gibbs Sampling routine

Analytical computation of the posterior distribution (8) is impossible. However, we can factorize $p(\psi | Y^T)$ into different parameter blocks according to (6). The latter allows us to specify the cycle:

1. Simulate $\{\theta_t\}_{t=1}^T$ from $p(\theta_t | Y^T, \psi_{-\theta_t})$ such that

$$\theta_t | Y^T, \psi_{-\theta_t} \sim N(\bar{\theta}_{t|T}, \bar{R}_{t|T}), \ t \leq T \quad (9)$$

2. Simulate $\Omega^{-1}$ from $p(\Omega^{-1} | Y^T, \psi_{-\Omega})$ such that

$$\Omega^{-1} | Y^T, \psi_{-\Omega} \sim W_i \left( z_1 + T_i \left[ \sum_t (Y_i - W_t \Xi \theta_t) (Y_i - W_t \Xi \theta_t)' \sigma_t + Q^{-1} \right]^{-1} \right) \quad (10)$$

3. Simulate $b_i$ from $p(b_i | Y^T, \psi_{-b_i})$ such that

$$b_i | Y^T, \psi_{-b_i} \sim IG \left( \frac{\varpi_i}{2}, \sum_t (\theta_i^{t} - \theta_i^{t-1})' (\theta_i^{t} - \theta_i^{t-1}) + \delta_0 \right) \quad (11)$$

where $\xi_t = \gamma_1 \frac{1}{1 - \gamma_1}.$

4. Simulate $\sigma^2$ from $p(\sigma^2 | Y^T, \psi_{-\sigma^2})$ such that

$$\sigma^2 | Y^T, \psi_{-\sigma^2} \propto L(Y^T | \psi) p(\sigma^2) \quad (12)$$

where $\bar{\theta}_{t|T}$ and $\bar{R}_{t|T}$ are the one-period ahead forecasts of $\theta_t$ and the variance-covariance matrix of the forecast error, respectively, calculated through the Kalman Smoother, as described
in Chib and Greenberg (1995). We also have $\varpi^1 = T + \varpi_0$, $\varpi^2 = TM + \varpi_0$, $\varpi^3 = TN + \varpi_0$, $\varpi^4 = T + \varpi_0$.

The posterior of $\sigma^2$ is simulated using a Random-Walk Metropolis-Hastings step, since it is non-standard. That is, at each iteration $l$ we draw a candidate $(\sigma^2)^*$ according to

$$(\sigma^2)^* = \exp \left[ \ln (\sigma^2)^{l-1} + c_\sigma \varepsilon \right]$$

with $\varepsilon \sim N(0,1)$ and $c_\sigma$ is a parameter for scaling the variance of the proposal distribution. In particular, this is chosen such that the acceptance rate is between $0.2 - 0.4$. Moreover, the acceptance probability at each draw $l$ is given by:

$$\alpha = \min \left\{ \frac{L \left( (\sigma^2)^*, \psi_{\sigma^2}^l \mid Y^T \right) p \left( (\sigma^2)^{l-1} \mid (\sigma^2)^* \right)}{L \left( (\sigma^2)^{l-1}, \psi_{\sigma^2}^l \mid Y^T \right) p \left( (\sigma^2)^{l-1} \mid (\sigma^2)^* \right)} \right\}^{-1}$$

where we take into account the fact that the proposal distribution is not symmetric.

Under regularity conditions, cycling through the conditional distributions (9) – (10) – (11) – (12) will produce draws from the limiting ergodic distribution.

### 3.2 Estimation setup

We run the presented Gibbs sampler for $K = 150,000$ draws and discard the first 100,000 in order to minimize the effect of initial values. Moreover, in order to reduce the serial correlation across draws, we set a thinning factor of 10, i.e. given the remaining 50,000 draws, we take 1 every 10 and discard the remaining ones. As a result, we have 5,000 draws for conducting inference. Priors are calibrated using a training sample based on the first five years of data. Specific details about the Data Description and how we conduct inference and assess convergence can be found in Appendices A and B respectively. We set $\varpi_0 = 10^6$, $\delta_0 = 1$, $z_1 = NM + 5$, $Q_1 = \text{diag} \left( Q_{11}, \ldots, Q_{1N} \right)$ where $Q_{ii}$ is the residual covariance matrix of the time invariant VAR for the $i$-th country, $\zeta = 1$, $s^2 = \hat{\sigma}^2$ where $\hat{\sigma}^2$ is the average of the estimated variances of $NM$ independent $AR(p)$ models. Moreover, $\bar{\theta}_0 = \hat{\theta}_0$ is the OLS estimation of the time-invariant

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6See also Kim and Nelson (1999).
version of the model and $R_0 = I_{\text{dim}(\theta_t)}$. Given the calibrated value of $c_\sigma$, the acceptance rate of the metropolis-step is around 0.38. Finally, we set $\gamma_1 = 0$ and $\gamma_2 = 1$, meaning that $\eta$ has a constant variance.

3.3 Impulse responses computation

In this section we explain how we compute the dynamic responses at different points in time using the presented model. We define the Impulse Responses as follows: let the expression

$$IR_j^h (t, h) = E \left( Y_{t+h} \mid F^1_t \right) - E \left( Y_{t+h} \mid F^2_t \right), \quad h = 1, 2, \ldots$$

be the response of vector $Y_t$ to a shock in variable $j$ of size $\delta$ at date $t$. Where

$$F^1_t = \{ Y^t, \theta_{t+1}^{t+h}, S_t, J_t, \xi_{J,t}, \xi_{-J,t}, \xi_{t+1}^{t+h} \}$$

$$F^2_t = \{ Y^t, \theta_{t+1}^{t+h} + S_t, J_t, \xi_{t+1}^{t+h} \}$$

$$S_t = (\Omega, B_t) ; \quad \Omega = J_t J_t'$$

and where

$$\theta_{t+1}^{t+h} = [\theta_{t+1}', \theta_{t+2}', \ldots, \theta_{t+h}']'$$

$$Y_{t+1}^{t+h} = [Y_{t+1}', Y_{t+2}', \ldots, Y_{t+h}']'$$

In order to forecast $Y_{t+h}$ and $\theta_{t+h}$, we use the equations (4) and (5), respectively. We repeat this procedure for a subset of random draws from the posterior distribution, and for different dates. Then we collect the draws and compute the median value and relevant percentiles.

4 Results

4.1 Data and variables selection

For each country we use year-to-year growth rates of real variables such as GDP, Consumption, Investment and public expenditures. As in Canova et al. (2007), we include different expenditure
components in order to better capture GDP dynamics. Moreover, as in Canova et al. (2012), we also include domestic price indexes in order to control for variation in nominal variables. As exogenous variables, we include the annual GDP growth rate of US, European Union and China. In addition, we include the growth rate of WTI Oil prices and Commodity prices. Our main data sources are the International Financial Statistics (IFS) from the International Monetary Fund (IMF) and the FRED Database. The sample of analysis covers the period 1997Q1-2014Q3. Following the references, data is demeaned and standardized.

4.2 Model Comparison

Our Baseline specification considers one lag for domestic and exogenous variables, i.e. $p = 1$ and $q = 1$. We also include a common component, a country-specific component, a variable-specific component and an exogenous component. The purpose of this section is to compare this Baseline specification with alternative ones. To do so, a good practice in Bayesian Econometrics is to compute the Marginal Likelihood for each model. That is, we need to integrate out the posterior distribution across the parameter space, and see to what extent a given model is a good representation of the data, i.e. the model with a higher marginal likelihood will be the best one. The marginal likelihood for each model $M_i$ is

$$f (Y^T | M_i) = \int L (\psi_j | Y^T, M_i) P (\psi_j | M_i) \, dj$$

Given the scales, it is better to compute the log-marginal likelihood $\ln f (Y^T | M_i)$, and this is estimated using a standard harmonic mean estimator.
Results are shown in Table 1. We observe that our baseline model is preferred with respect to alternative specifications. In particular, if we drop the Country-specific or the Variable-specific component, then these models have a diminished marginal likelihood. Moreover, if we add more lags to the domestic or external component, we also observe models with a diminished marginal likelihood.

### 4.3 Analysis of Latin American cycles

First of all, Figure 1 depicts the Common Regional Indicator. This indicator is significant for the whole sample given the 68% confidence bands. This indicator is representative for the region, since it captures the recession episodes of 1998 and 2008-2009 and also the commodity boom episode, pre- and post- financial crisis.
Then, Figure 3 depicts the individual country indicators. We also observe significant components for the whole sample given the 68% confidence bands for all countries, meaning that heterogeneity is also relevant for our analysis. If we compare the common indicator with the individual country indicators, we observe that the periods with a higher synchronization are in-
timately related with the crisis episodes. That is, we have a first evidence of significant regional movements during crisis and, on the other hand, evidence of significant individual country effects in periods of calm. Moreover, we compare the country indicators with the real GDP growth for each case in Figure 2.

It is important to remark that we have also included a variable component and an exogenous component in our baseline specification. Therefore, we are not omitting information when presenting these results.

Figure 3: Posterior distribution of Country-specific Indicators; median value and 68% bands

4.4 The transmission of GDP shocks within Latin American countries

We now turn to another dimension of our results. Since the model has time varying parameters and dynamic inter-dependencies, it is possible to explore the transmission of different shocks in the system. In particular, we are interested in the interaction among these five economies, and for that reason we consider a Brazilian GDP shock as a benchmark. Given the dynamic inter-dependencies, we expect a transmission effect from one country to another in the same region. Moreover, given the time variation, we explore to what extent the transmission of shocks have changed over time.
Figure 4 depicts the dynamic effects of a Brazilian GDP shocks at different points in time. We see a positive effect to the rest of the economies. In addition, we observe that the transmission effects from Brazilian economy to the remaining countries have diminished over time, i.e. effects were larger (and more uncertain) in 1998 with respect to the last decade. One possible interpretation of this result is the fact that Latin American economies are less vulnerable after the Inflation Targeting adoption. In fact, during the 2008 crisis we do not observe the large effects of 1998, and we observe less uncertainty in the results, given the confidence bands.
4.5 The transmission of external GDP shocks across Latin American countries

In this section we also explore the transmission of external shocks, given that Latin American economies are also subject to the fluctuation of big economies such as China, United States or Europe. Figure 5 depicts the transmission of a shock to Chinese GDP at different dates. We also observe a positive and persistent effect, but we also observe that responses in 1998 are much more unstable and uncertain with respect to the dates after 2000. We observe some differences in the responses across dates, with a particular peak in 2008, but given the confidence bands we cannot conclude that responses are significantly different. It remains to be explored the sensitivity of these results with respect to the amount of time variation given \textit{a priori}, i.e. the parameter $\varpi_0 = 10^6$. 
Figure 5: Response of GDP to a unit shock on Chinese GDP; median value and 68% bands

5 Concluding Remarks

We have estimated a Multi-Country VAR through Bayesian Methods, where the model takes into account dynamic inter-dependencies and time-varying parameters (Canova and Ciccarelli, 2009). The model is parsimonious and allows us to construct coincident indicators and to explore the transmission across the region of domestic and external shocks at different dates.

We find a significant common regional component, as well as significant country specific indicators, meaning that there exists some synchronization across business cycles, but at the
same time there is some heterogeneity across these economies. We find some heterogeneity before the international financial crisis of 2008 and a more significant co-movement after that date. Furthermore, we explore the transmission at different dates of domestic (country specific) and external shocks such as Chinese GDP growth. Overall, we find that the transmission of both domestic and external shocks are somewhat stable after the Inflation Targeting adoption.
References


A The posterior distribution of hyper-parameters

![Posterior Distribution of $b$](image1)

Figure 6: Posterior Distribution of $b$

![Posterior Draws of $b$](image2)

Figure 7: Posterior Draws of $b$
B Data Description

- For each country we use year-to-year growth rates of real variables such as GDP, Consumption, Investment and public expenditures.

- As in Canova et al. (2012), we also include domestic price indexes in order to control for variation in nominal variables.

- As exogenous variables, we include the annual growth rate of US GDP, European Unions
GDP, Chinas GDP and the growth rate of WTI Oil prices. At the end of the day, the sample of analysis covers the period 1997Q1-2014Q3.

- Following the references, data is demeaned and standardized.

![Graphs of Y, C, I, G, and P for Brazilian Data](image1)

**Figure 10: Brazilian Data**

![Graphs of Y, C, I, G, and P for Chilean Data](image2)

**Figure 11: Chilean Data**
Figure 12: Colombian Data

Figure 13: Mexican Data
Figure 14: Peruvian Data

Figure 15: Exogenous Data