Inferring inflation expectations from fixed-event forecasts

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Inferring inflation expectations from fixed-event forecasts∗

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Abstract

Often, expected inflation measured by surveys are available only as fixed-event forecasts. Even though these surveys do contain information of a complete term structure of expectations, direct inferences about them are troublesome. Records of fixed-event forecasts through time are associated with time-varying forecast horizons, and there is no straightforward way to interpolate such figures. This paper proposes an adaptation of the measurement model of Kozicki and Tinsley (2012) [“Effective use of survey information in estimating the evolution of expected inflation”, Journal of Money, Credit and Banking, 44(1), 145-169] to suit the intricacies of fixed-event data. Using the Latin American Consensus Forecasts, the model is estimated to study the behavior of inflation expectations in four inflation targeters (Chile, Colombia, Mexico and Peru). For these countries, the results suggest that the announcement of credible inflation targets has been instrumental in anchoring long-run expectations.

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1 Introduction

Expectations about future inflationary outcomes play a key role in macroeconomic analysis. For instance, the determination of aggregate prices in modern macroeconomic models is often summarized in expectations-augmented Phillips curves. Also, real interest rates, which provide a rough measure of the monetary policy stance once compared to a suitable reference level, can be computed by deducting inflationary expectations from the nominal interest rate. Finally, many countries conduct their monetary policy within an explicit inflation targeting regime, and the pillar of such a monetary arrangement is to maintain expectations anchored to a predetermined target.

Economic surveys, where a number of participants are asked to produce forecasts of future inflation, provide a direct source of information on expected inflation.\(^1\) As illustrated in Figure 1, two types of data structures emerge.\(^2\) The first one corresponds to “rolling-event” forecasts (REFs, henceforth), such as those recorded in the Livingston Survey for the US or the Gallop Poll for the UK. Here, the survey collects \(h\)-period ahead forecasts, so every new release of the survey the “event” to be forecast “rolls” forward. In other words, in a REF the horizon \(h\) is fixed, and the target date is always separated \(h\) periods from the forecast origin. The bulk of the literature that uses empirical measures of expectations from surveys, either to assess their rationality (e.g. Davies and Lahiri, 1999) or to use them in econometric models (e.g. Mehra and Herrington, 2008; Mavroeidis et. al., 2014), is based on this type of measures. This is so because expectations in theoretical macroeconomic models are formulated as REFs, and also because of the widespread popularity in applied work of sources such as the Livingston Survey.

The second data structure corresponds to “fixed-event” forecasts (FEFs, henceforth) which, in contrast to REFs, has received scant attention despite the fact that FEF data are available for a much larger number of countries. The “event” to be forecast is often an annual rate, for current and subsequent years. Specifically, following Figure 1(b), in period \(t_1\) survey participants are asked to forecast inflation for period \(t_1 + h_1\), an \(h_1\)-period ahead prediction; later on, in period \(t_2 > t_1\) they are asked for a forecast of inflation for the same date, which now corresponds to an \(h_2\)-period ahead forecast, where \(h_2 = h_1 - (t_2 - t_1) < h_1\). The forecast event is kept fixed throughout, while the forecasting horizon shrinks as the time line approaches the event. The highly reputed company Consensus Economics Inc. conducts the Consensus Forecasts monthly poll among forecasters working in the private sector (in more than 70 countries), compiling their predictions of a range of economic variables, and reporting them in a fixed-event format. Other widespread sources of FEFs of global and country-specific economic activity are the IMF’s World Economic Outlook, the World Bank’s Global Economic Prospects, the OECD’s Economic Outlook and polls conducted by Central Banks.

Notwithstanding its availability, FEFs are an unexploited resource to describe the evolution of expected inflation.\(^3\) We conjecture that this is so because the time-varying nature of the forecast horizons makes comparisons over time troublesome. As suggested, it seems more natural to conceptualize expectations in macroeconomic models as REFs (for instance, to estimate an expectations-augmented Phillips curve or to compute real interest rates). And even though a survey that registers FEFs for horizons \(h_A\) and \(h_B\) (see

\(^1\) An alternative approach is to deduce expected inflation from the difference between nominal and indexed bond yields. Such an approximation, nonetheless, is often biased and more volatile than survey measures, as it also captures factors that are not directly linked to inflation expectations, such as movements of risk or liquidity premia. Consistently with this critique, Ang et. al. (2007) and Chernov and Mueller (2012) find that survey measures categorically outperform financial measures in forecasting inflation.

\(^2\) We focus our analysis on quantitative (i.e. point) forecasts. A large literature, in contrast, favors using qualitative or directional forecasts (“would inflation be higher or lower?”) arguing that survey respondents are unable to produce trustworthy numerical predictions. Thus, the mapping from a qualitative perception to a precise quantitative figure is left to the econometrician, using either the so-called “probability” or “regression” approach (see Pesaran and Wealey, 2006, for a comprehensive survey). This assumes, of course, that the econometrician is better suited to perform such mapping than the survey respondent. This may be true for consumer surveys, but the argument is weakened when the respondents are experts or informed professionals, as with the surveys used in this study.

\(^3\) Since the seminal contribution of Nordhaus (1987), FEFs have become popular in testing forecasts rationality or optimality (e.g. Clements, 1997; Bakhshi et. al., 2005; Timmermann, 2007). FEFs are in fact ideal for such purposes. Its evolution over time provides a stream of revisions whose correlation against various information sets can be directly assessed. A further advantage is that rationality can be tested without having the data of the target variable.
Figure 1. Rolling-event and fixed-event forecasts

(a) Rolling-event forecasts

(b) Fixed-event forecasts

Figure 1) does contain information for expectations at any intermediate horizon $h_A \leq h \leq h_B$ (for instance, 12-month ahead expectations are implicitly contained in current and next year forecasts), there is no obvious way to interpolate such figures. The purpose of this paper is to develop an empirical model to explicitly infer expectations from data on actual inflation and FEFs.

The model is a version of the shifting-endpoint model advanced in Kozicki and Tinsley (1998, 2001, 2012), suitably modified to deal with the intricacies of the FEF data structure. We reckon that our modifications to the Kozicki and Tinsley’s framework widens its scope and should be useful in practice, especially to analyze expectations in countries for which only FEFs are available. Importantly, even though the model is fitted to irregularly sampled observations associated to time-varying forecast horizons, it is capable to produce a coherent and complete term structure of inflation expectations. Thus, the model provides an interpolation method that is internally consistent with inflation dynamics and survey information.

As stressed in Kozicki and Tinsley (1998, 2001), predictions derived from time-series models of inflation are likely to be poor proxies of expected inflation. Such predictions are based on historical data only and fail to accommodate structural changes in inflation dynamics that may be well reflected in survey measures. On the other hand, forecasts from surveys are often responsive to the latest inflation observations for short-term horizons, and relatively unresponsive for long-term horizons. This pattern is difficult to reconcile with model-based multiperiod forecasts. If the model fitted to the inflation process exhibits strong mean reversion to a fixed level, forecasts for all horizons would be too insensitive to recent inflation, whereas if the inflation model is persistent (say, because it contains a unit root), then forecasts would be excessively sensitive to inflation news.

The shifting-endpoint model is a compromise whose multiperiod forecasts both reflect the main dynamic features of the inflation process, and also account for the behavior of available survey data. The endpoint is an unobservable variable that measures the perceptions of long-run inflation held by economic agents at a given point in time, and that directly affects actual inflation and its forecasts for arbitrary horizons. Survey records are taken as error-ridden versions of such forecasts, and so the inferred inflation expectations would exploit all available information from inflation and surveys.

The remaining of the document is organized as follows. Section 2 presents the shifting-endpoint model, discusses its main properties and describes some variants. The model can be represented in state space form, and the moving nature of the forecast horizons in FEFs implies that this representation is time-varying. The Kalman filter can be used for estimation and inference, even when survey expectations are sampled less frequently than inflation. Section 3 presents empirical results for four Latin American countries, using data from the Latin American Consensus Forecasts survey. The evolution of the predicted endpoints reveals the key role that expectations have played in these countries, first to reduce inflation to single-digit levels and then to keep it stable, as well as the importance of the Central Bank’s credibility in anchoring long-run expectations. Finally, section 4 presents concluding remarks.

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4 This is indeed the case in our empirical application below. Survey measures in all the Latin American countries of our sample correctly anticipated the trend decline in inflation during the 1990s. This is remarkable, since purely backward looking or adaptive expectations would tend to systematically over-predict inflation in such a context.
2 Econometric framework

Next, we describe the workings of the shifting-endpoint model for inflation and discuss its statistical treatment. We also present model variants that can be evaluated within the same analytical framework. In what follows, \( x_t | s \) denotes the expectation of a random variable \( x_t \) formed by economic agents conditional on the information up to and including period \( s \).

2.1 Shifting-endpoint model for inflation

The limiting conditional forecast of inflation is given by

\[
\mu_t = \lim_{h \to \infty} \pi_h | t ,
\]

(1)

where \( t \) denotes the time subscript of the information set on which expectations are conditioned. Thus, given information up to and including period \( t \), \( \mu_t \) measures the perceived level at which inflation would eventually stabilize, i.e. the endpoint.

We assume that long-run expectations are formed in a weakly rational manner, in the sense of Nordhaus (1987): changes in perception are unpredictable or, more formally, \( \mu_t \) is a martingale with respect to its own past. In other words, if agents can anticipate future changes to their long-run perceptions, then such changes should be immediately incorporated in their current perceptions, as in the law of iterated expectations. This behavior can be satisfactorily modelled by assuming that the endpoint follows a random walk,

\[
\mu_t = \mu_{t-1} + \nu_t ,
\]

(2)

where \( \nu_t \) is an innovation satisfying \( \nu_t | s = 0 \) for \( s < t \). The endpoint \( \mu_t \) is treated as an unobservable variable, and the main purpose of the analysis is to infer about its state using inflation and survey data.

The notion of a varying endpoint for expected inflation can be easily accommodated in a parametric forecasting model. Let \( \pi_t \) denote inflation at period \( t \) and suppose that the expectations in period \( t \) are formed using information up to and including period \( t - 1 \). Define also \( \Phi(L) = \phi_1 + \phi_2 L + \cdots + \phi_{p-1} L^{p-2} + \phi_p L^{p-1} \) as a polynomial in the lag operator \( L \), and assume that the roots of \( 1 - \Phi(L) \) lie outside the unit circle (we make some allowance for a unit root in \( 1 - \Phi(L) \) later). Inflation dynamics are captured by

\[
\pi_t = \Phi(L) \pi_{t-1} + (1 - \Phi(1)) \mu_{t-1} + \epsilon_t ,
\]

(3)

where \( \epsilon_t \) is an inflation shock, assumed to be uncorrelated with \( \nu_t \) at all lags and leads. Under the aforementioned assumptions, \( \pi_t - \mu_{t-1} \), the deviation of inflation from its latest perceived long-run level, is a zero-mean stationary process. By construction, given the information of period \( t - 1 \), inflation is expected to converge to \( \mu_{t-1} \) as the forecasting horizon increases. Thus, the dynamic specification (3) allows us to disentangle the effects of the shifting-endpoint from short-run fluctuations in inflation, see section 2.5.

2.2 Incorporating survey data

It is convenient to write the forecasting model in companion form. Let

\[
z_t = \begin{bmatrix} \pi_t \\ \pi_{t-1} \\ \vdots \\ \pi_{t-p+1} \end{bmatrix}
\]

and

\[
C = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_{p-1} & \phi_p \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix},
\]

(4)

where \( z_t \) is a \( p \)-vector with current and past inflation data and \( C \) is the companion matrix. Let also \( I_p \) be the \( p \times p \) identity matrix, \( I_p \) be a \( p \)-vector of ones and \( e_1 \) be a unit \( p \)-vector with a one in the first element and
remaining elements zero. Then, (3) corresponds to the first row of the companion form

\[ z_t = Cz_{t-1} + (I_p - C)\mu_{t-1} + e_t. \]  

Multistep forecasts of inflation based on this model are \( \pi_{t+h|i-1} = e_t'z_{t+h|i-1} \), where

\[ z_{t+h|i-1} = C^{h+1}z_{t-n} + (I_p - C^{h+1})\mu_{t-1}. \]  

Survey data registered in period \( t \) represent participants’ forecasts conditional on inflation up to and including period \( t - 1 \). Moreover, consider that the survey collects information for \( m \) different forecasts \( F_{it} \), each associated to a different horizon that may be time varying \( h_{it} \) \((i = 1,2,\ldots,m)\). Taking the inflation forecast from the shifting-endpoint model as an approximation of the survey expectation, we may write

\[ F_{it} = \pi_{t+h_{it}|i-1} + e_{it}, \]  

where \( e_{it} \) is an approximation error that captures discrepancies between the implicit (and unknown) forecasting model of survey participants and the shifting-endpoint model, as well as possible measurement errors in survey data. It is important to note that the errors in (7) do not reflect differences between actual inflation and forecasts. This is because both the model-based forecasts and survey records are conditioned on \( t - 1 \), which implies that \( \text{cov}(e_t, e_{it}) = 0 \) for \( i = 1,2,\ldots,m \). Thus, unlike forecast errors, there is no reason to expect them to be serially correlated; hence, \( \text{cov}(e_{it}, e_{js}) = 0 \) for \( t \neq s \) and any \( i \) or \( j \).

On the other hand, revisions of forecasts of different target dates made at the same time are likely to be highly correlated (see Clements, 1997; Bakhshi et al., 2005, for a detailed discussion). News at period \( t \) that lead to a revision in the forecast at horizon \( h_{it} \) are also likely to produce a revision in the forecast at horizon \( h_{ij} \) \((i \neq j)\). Following (7), even though an important part of these comovements would be accounted for by changes in the endpoint, we expect the approximation errors to be contemporaneously correlated \( \sigma_{ij} = \text{cov}(e_{it}, e_{jt}) \neq 0 \). Note that whether \( \sigma_{ij} \) is zero or not is a straightforward testable hypothesis that can be formally addressed.

### 2.3 State-space form

The statistical treatment of the model is based on its state-space representation (see Harvey, 1989, ch. 5). The Kalman filter yields linear least squares predictions of the unobserved endpoint \( \mu_t \) based on current and past observations, along with their corresponding mean square errors.\(^5\) Moreover, given the parameters of the model \((\phi_1,\ldots,\phi_p, \nu_t, \text{ and } e_t, \text{ and the covariances among } e_{1t},\ldots,e_{mt})\), a Gaussian likelihood function can be evaluated from the one-step-ahead prediction errors produced by the Kalman filter. This function can be maximized numerically, thereby providing (quasi) maximum likelihood estimates of the unknown parameters.

The law of motion of the unobservable endpoint (2) constitutes the scalar transition (or state) equation. Upon stacking the shifting endpoint model (3), the conditional forecasts (6) and their relationship with various data points from surveys (7), we obtain the measurement equations

\[ y_t = A_tz_{t-1} + B_t\mu_{t-1} + w_t, \]  

where \( y_t \) is an \((m + 1)\)-vector that contains current date information on inflation and surveys, \( A_t \) and \( B_t \) are, respectively, an \((m + 1) \times p\) matrix and \((m + 1)\)-vector of coefficients, and \( w_t \) is an \((m + 1)\)-vector of measurement errors. The entries of \( A_t \) and \( B_t \) depend on \( h_{it} \) and thus may be time-varying, capturing the shrinking nature of the forecasting horizon for FEFs. This is the most important difference with respect to the model analyzed by Kozicki and Tinsley (2001, 2012).

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\(^5\) If desired, a refined prediction conditioned on all the available information in the sample can be obtained by means of a smoothing algorithm that uses the output of the Kalman filter.
More explicitly, let \( a(h) = e_i C^{h+1} \) (a \( p \)-row vector) and \( b(h) = 1 - a(h)i_p \) (a scalar). Then,

\[
\begin{bmatrix}
\pi_t \\
F_{1t} \\
F_{2t} \\
\vdots \\
F_{mt}
\end{bmatrix}
= \begin{bmatrix}
a(0) \\
\vdots \\
a(h_{mt})
\end{bmatrix}
\begin{bmatrix}
z_{t-1} \\
\vdots \\
z_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
b(0) \\
\vdots \\
b(h_{mt})
\end{bmatrix}
\begin{bmatrix}
\mu_{t-1} \\
\vdots \\
\mu_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_t \\
\varepsilon_t \\
\varepsilon_t \\
\varepsilon_t
\end{bmatrix},
\] (9)

where each vector and system matrix in (8) is implicitly defined.

It is worth mentioning that although measurement equations (9) are designed to accommodate \( m \) different survey responses at different horizons, they are flexible enough to adapt to many other structures. One case would be if the surveys contain both REFs and FEFs: for REFs the horizons are fixed \( h_{1t} = h_i \), whereas for FEFs the forecasting horizons will vary with \( t \) in a deterministc fashion. For instance, in surveys such as the Consensus Forecasts, the data from month \( M_t \) refer to forecasts by the end of year \( i \), with \( i = 1 \) being the current year, so the forecasting horizons (in months) evolve deterministically as \( h_{1t} = 12i - (M_t - 1) \).

Surveys may also report averages of forecasts over \( R \) different horizons. In this case, the corresponding measurement equation would read \( F_t = \frac{1}{R} \sum_{r=1}^{R} [a(h_{rt})z_{t-1} + b(h_{rt})\mu_{t-1}] + \varepsilon_t \). We did attempt to incorporate information of this kind in our empirical application below, but with marginal effects on the results. Thus, we ignore such forecasts and stick to the model (9) as it stands.

On the other hand, we have implicitly interpreted \( F_{1t}, \ldots, F_{mt} \) as aggregate responses for different forecasting horizons. This is just for expositional convenience, since (9) can handle responses made by different agents, by simply allowing subindex \( i \) to denote a forecaster/horizon pair. For instance, for two forecasters and two horizons, \( F_{1t}, F_{2t} \) can represent the results for the first participant, and \( F_{3t}, F_{4t} \) for the second, with \( h_{1t} = h_{3t} \) and \( h_{2t} = h_{4t} \) indicating that there are only two target horizons. In this case, more structure to the measurement error covariances, in the spirit of Davies and Lahiri (1999), may be appropriate. For instance, by letting \( \varepsilon_{iit} = R_t + H_t + \text{error}_{iit} \), where \( R_t \) and \( H_t \) are, respectively, respondent and horizon effects.

### 2.4 Missing data

Generally, survey data are available less frequently than inflation data (which is assumed to be available for all periods in the sample), so for certain periods some of the observations in the \( y_t \) vector will be missing. The treatment of missing observations is perhaps one of the clearest advantages of using the Kalman filter to process time-series models, as the filter requires only minor modifications to deal with such a problem (see Harvey, 1989, p. 144).

Consider the case when only \( \tilde{m}_t < m \) forecasts in \( y_t \) are available at period \( t \), and let \( D_t \) be the \((\tilde{m}_t+1) \times (m+1)\) selection matrix that collects non-missing elements, so for instance \( y_t^* = D_t y_t \) is an \((\tilde{m}_t + 1)\)-vector that contains observable data. Note that \( D_t \) is formed by \( \tilde{m}_t + 1 \) rows of the identity matrix of order \( m + 1 \). Then, upon premultiplying the measurement equations (8) by \( D_t \),

\[
y_t^* = A_t^* z_{t-1} + B_t^* \mu_{t-1} + w_t^*.
\] (10)

where \( A_t^* \), \( B_t^* \) and \( w_t^* \) contain the rows of \( A_t \), \( B_t \) and \( w_t \), respectively, that correspond to the available observations in \( y_t^* \). The state variable \( \mu_t \) is not affected by the transformation, so the Kalman filter can be applied normally to the observable system (10) in period \( t \). The missing observations will not contribute to neither the state predictions nor to the likelihood function.

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* A further extension of the model allows expectations to be formed at different moments. The measurement equation for an expectation formed with information up to period \( t - n \) (\( n \geq 1 \)) is \( F_t = a(h_t + n - 1)z_{t-n} + b(h_t + n - 1)\mu_{t-n} + \varepsilon_t \). Thus (8) becomes \( y_t = A_t Z_t + B_t \mu_t + w_t \), where \( Z_t \) and \( \mu_t \) stack, respectively, the lags of \( z_t \) and \( \mu_t \) associated to each information set. The number of columns of the now sparse matrices \( A_t \) and \( B_t \) should vary accordingly.
2.5 Signal extraction and the term structure of expectations

From (6) and (7), we can readily verify that

\[ \pi_{t+h | t-1} = a(h)z_{t-1} + b(h)\mu_{t-1}, \tag{11} \]

so that expected inflation is estimated as a linear combination of the latest realizations of inflation \( z_{t-1} \) and the perceived long-run level \( \mu_{t-1} \). Note \( \lim_{h \to \infty} a(h) = 0 \) and \( \lim_{h \to \infty} b(h) = 1 \). Thus, as \( h \) increases expectations converge from a short-term forecast dominated by recent history to the endpoint (1). By the same token, equation (7) implies that (roughly) \( F_{it} \approx \pi_{t-1} + \epsilon_{it} \) for small \( h_{it} \), and \( F_{it} \approx \mu_{t-1} + \epsilon_{it} \) for large \( h_{it} \). Therefore, in the process of extracting the signal \( \mu_{t} \), more importance is given to long-term survey expectations (when available), even though inflation data and shorter-term expectations contain also information about this hidden state. Kozicki and Tinsley (2012) provide a more detailed discussion on the information content of the observable variables. See also the discussion surrounding Table 3 below.

These properties of the model are illustrated in Figure 2. For \( \Phi(L) = \Phi(1) = 0.85 \), the figure shows the response of inflation and expectations formed with information up to period \( t - 1 \), to the two shocks of the system. An inflation shock \( \epsilon_0 = 1 \) – panel (a) – produces a persistent deviation in the inflation rate from the endpoint. Given the transitory nature of the shock, for short-run horizons (say, \( h \leq 6 \)) expected inflation reflects the temporarily higher current inflation; in contrast, expectations for longer horizons (say, \( h \geq 12 \)) are virtually unaffected, as agents correctly anticipate that the influence of the shock would vanish almost completely by the end of the forecast horizon. On the other hand, expectations are sensitive to shifts in the endpoint \( \nu_0 = 1 \), panel (b). Such a shock causes a sluggish response in inflation, which displays a smooth transition towards its new long-run level, that in turn affects short-run expectations; on the other hand, the shift passes-through immediately and almost completely to long-run expectations, as agents can anticipate that the change in the endpoint will be completely transferred to inflation by the end of the forecast horizon.

The state-space form includes error-ridden versions of (11) for time-varying horizons where survey data is available. Thus, even though survey data may be limited to infrequently sampled observations and to selected horizons, once the model is estimated, it can be used to construct a complete term structure of expected inflation. Put it differently, in practice we only observe some points of the responses shown in Figure 2, but the model allows us to interpolate the entire profile of responses in a coherent, data-consistent way.

2.6 Model variants

In our empirical exploration, we also analyze three variants of the model within the same state space framework, in order to highlight the most important features of the shifting-endpoint model that uses...
both inflation and survey data. In particular, the model’s ability to simultaneously explain inflation and expectations data under competing specifications.

The first variant assumes a constant endpoint (CE), \( \mu_t = \mu \). This can be achieved by setting \( \text{var}(\nu_t) = 0 \) and treating the initial condition \( \mu_0 = \mu \) as an additional parameter to be estimated. Provided that \( \Phi(1) < 1 \), inflation is treated as a stationary, mean-reverting process.

In contrast, the second variant imposes a unit root (UR) to the autoregressive polynomial in (3), \( \Phi(1) = 1 \). In this case \( C^h i_p = i_p \) for any \( h > 0 \), and the companion form (5) reduces to \( z_t = Cz_{t-1} + e_t \epsilon_t \). Since it turns out that \( B_t = 0 \) in all measurement equations, \( \mu_t \) is not identified and cannot be computed with the Kalman filter. However, \( \mu_t \) can be reformulated to be the limiting forecast which continues to exist: it can be verified that \( \lim_{h \to \infty} C^h = \bar{C} \), where \( \bar{C} \bar{C} = \bar{C} \), and so \( \mu_t = e_1' \bar{C} z_t \). The endpoint is a moving average of the most recent inflation observations, and Kozicki and Tinsley (1998) show that, following the definition in (1), it corresponds to the permanent component of the Beveridge-Nelson decomposition of the inflation equation. The Kalman filter can still be used to evaluate the likelihood function in this case, under \( \text{var}(\nu_t) = 0 \).

The last variant is a model of inflation that ignores the information from surveys. The result is a univariate generalization of the local level (LL) model described in Harvey (1989, ch. 2), where inflation is decomposed as the sum of a random walk (the endpoint) and a zero-mean stationary component. This variant can be easily treated by considering a single measurement equation, the first row in (9), or by making the variances of the approximation errors arbitrarily large, \( \text{var}(\epsilon_{1t}) = \kappa \to \infty \) for all \( i \).

3 Application to Latin American countries

After a long history of high inflation, during the 1990s many Latin American countries adopted a series of reforms that would eventually bring inflation down to single-digit levels (see, inter alia, Mishkin and Savastano, 2001). Even though experiences may differ in the detail, many commonalities across countries can be identified (see Corbo and Schmidt-Hebbel, 2001; Quispe-Agnoli, 2001, for comprehensive surveys). First, to facilitate the reduction of inflation, and to isolate monetary policy from political pressures, these countries granted independence to their Central Banks at an early stage of the stabilization effort. Then, economic authorities would adopt a rudimentary form of inflation targeting, typically by simply announcing numerical targets (or official forecasts), as a first attempt to anchor market expectations. The process of disinflation would be gradual as the Central Banks improved its credibility and built reputation as inflation targets. Once the conditions to consolidate price stability were reached, the Central Banks would adopt a fully-fledged inflation targeting regime, characterized by the announcement of long-run targets and the abandonment of any other nominal anchor (typically, currency depreciation or money growth).

We estimate the shifting-endpoint model of section 2 using data from four successful Latin American inflation targeters: Chile, Colombia, Mexico (all of them adopted the regime in 1999) and Peru (who adopted the regime in 2002). Following Vega and Winkelried (2005), the Central Banks began announcing numerical targets long before the definite adoption date: 1991 in Chile, 1995 in Colombia and Mexico, and 1994 in Peru. Thus, by analyzing the evolution of the estimated endpoints, our empirical application aims to assess the role of expectations in the initial disinflation and the subsequent periods of price stability.

3.1 Data

Monthly inflation corresponds to the percent variation of the officially targeted Consumer Price Index. These data and the numerical targets come from each Central Bank’s website.

Survey data, on the other hand, are extracted from the well-known Latin American Consensus Forecasts reports. These reports are available bi-monthly (alternate, even months) between April 1993 and April 2001.

Brazil is also an important Latin American inflation targeter. Its inflationary experience is different from those of the listed countries; enough to deserve a special treatment. Hence, for the same of brevity, the results for the Brazilian case (which are available upon request) are not reported.
and monthly thereafter (since May 2001). Each report surveys a number of prominent financial and economic analysts and publishes their individual forecasts as well as simple descriptive statistics. The focus of our analysis is on the mean (i.e. the “consensus”) forecast \(F_{1t}\). The surveys are conducted by the middle of the month, after the inflation figure for the previous month is released. Thus, in agreement with the shifting-endpoint model assumptions, in period \(t\) expectations are formed with information up to period \(t-1\).

The typical issue of the Latin American Consensus Forecasts provides forecasts for current-year and next-year inflation. Denoting the year of the survey as year 1, these forecasts correspond to the measurements \(F_{1t}\) and \(F_{2t}\). We refer to these variables jointly as “short-term” forecasts. In addition, the April and October issues include also “long-term” (up to 10 years ahead) forecasts. Forecasts for years 3 to 6 \((F_{3t}, F_{4t}, F_{5t}, F_{6t})\) are explicitly published, whereas forecasts for longer horizons (from year 7 to year 11) are reported as an average. In practice, such averages show little variation with respect to \(F_{6t}\), and are thus discarded from our analysis with no consequential effects on our results. Thus, depending on the month, the measurement equations have a minimum of \(m = 2\) horizons and a maximum of \(m = 6\). As mentioned earlier, the forecasting horizons evolve deterministically as \(h_{it} = 12i - (M_t - 1)\), where \(M_t\) denotes the month of the survey.

Our sample spans from February 1997 to December 2013. Thus, for each country, the estimations are based on 203 observations on inflation, 178 observations on “short-term” forecasts and 34 observations on “long-term” forecasts.

Figure 3 displays some of the data. The first row plots the evolution of the FEFs as they approach the inflation outturn (to avoid clutter only the events 2004, 2007, 2010 and 2013 are displayed). For a given event, each point corresponds to a different survey. The second row presents the data as they enter the regression model: the short-term forecasts \(F_{1t}\) and \(F_{2t}\), and the longest-term forecast \(F_{6t}\), all of them reported at the same time \(t\). Each month is associated to different forecast horizons, so the strong swings in \(F_{1t}\) and \(F_{2t}\) are due to a change in the event to be forecast (say, \(h_{1t} = 1\) in December and then \(h_{1t} = 12\) in January).

A preliminary analysis of these survey provides prima facie evidence of the adequacy of the shifting-endpoint model to explain the behavior of measured expectations. An important implication of the model is that the deviation of inflation from the endpoint is expected to be a mean-reverting process around zero. Also, short-term forecasts would be influenced by recent news in inflation, whereas long-term forecasts should be determined by the endpoint. Consider the regression equation

\[ F_{it} - F_{6t} = \alpha_t + \beta_t(\pi_{t-1} - F_{6t}) + \text{error}_t, \]

where \(\pi_{t-1}\) denotes the latest inflation figure (the forecast origin), and \(F_{6t}\) is the forecast corresponding to the longest horizon available, which serves as a rough proxy of the moving endpoint. The shifting-endpoint model is to be regarded as a reasonable description of how expectations are formed if both \(\alpha_t\) and \(\beta_t\) approach zero as \(i\) increases. This is exactly the pattern that emerges in Table 1. The estimates decrease with \(i\) in all cases, and often loose statistical significance for \(i \geq 3\) onwards (note that the sample size for these estimations is constrained by the 34 observations available for long-term forecasts, so the results should be taken as indicative rather than conclusive).

### 3.2 Estimation results

Next, we present the estimation results of the shifting-endpoint model (SE), and its variants (CE, UR and LL). We use a diffuse prior (i.e. setting the initial state variance to a very large number) to initialize the Kalman filter. In all cases, the lag-length of the AR model for inflation is set to \(p = 13\), which is the value that minimizes the Schwarz information criterion. Also, the null hypothesis that the approximation errors in (7) are not contemporaneously correlated was contrasted with an LR test, and categorically rejected in all instances. Hence, these covariances are estimated unrestrictedly.

For each model, Table 2 presents the sum of the estimated autoregressive coefficients \(\Phi(1)\) as a measure of persistence of inflation around the endpoint, which is restricted to \(\Phi(1) = 1\) in the UR model; the estimated
**Figure 3. Inflation, selected fixed-event forecasts and moving horizon forecasts**

Notes: The dots in the figure represent forecasts that are irregularly sampled (long-term forecasts in all the sample period, and short-term forecasts until April 2001). To ease visualization, these dots are connected with linearly interpolated values represented by discontinuous lines (the interpolations are not used in the estimation). Within each row, the scale of the vertical axis is the same for Chile/Peru and Colombia/Mexico.
Table 1. Mean reversion of forecasts

<table>
<thead>
<tr>
<th></th>
<th>$F_1 - F_6$</th>
<th>$F_2 - F_6$</th>
<th>$F_3 - F_6$</th>
<th>$F_4 - F_6$</th>
<th>$F_5 - F_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chile</td>
<td>$\alpha$ 0.257 (0.147)*</td>
<td>0.181 (0.070)*</td>
<td>0.083 (0.047)*</td>
<td>0.055 (0.039)</td>
<td>0.013 (0.023)</td>
</tr>
<tr>
<td></td>
<td>$\beta$ 0.565 (0.119)*</td>
<td>0.116 (0.035)*</td>
<td>0.026 (0.027)</td>
<td>0.007 (0.021)</td>
<td>0.004 (0.016)</td>
</tr>
<tr>
<td>Colombia</td>
<td>$\alpha$ 0.525 (0.201)*</td>
<td>0.451 (0.194)*</td>
<td>0.276 (0.180)</td>
<td>0.166 (0.163)</td>
<td>0.036 (0.096)</td>
</tr>
<tr>
<td></td>
<td>$\beta$ 0.696 (0.090)*</td>
<td>0.413 (0.110)*</td>
<td>0.262 (0.128)*</td>
<td>0.159 (0.112)</td>
<td>0.066 (0.048)</td>
</tr>
<tr>
<td>Mexico</td>
<td>$\alpha$ 0.101 (0.113)</td>
<td>0.005 (0.085)</td>
<td>0.028 (0.054)</td>
<td>0.019 (0.037)</td>
<td>0.016 (0.034)</td>
</tr>
<tr>
<td></td>
<td>$\beta$ 0.732 (0.058)*</td>
<td>0.395 (0.032)*</td>
<td>0.204 (0.020)*</td>
<td>0.096 (0.016)*</td>
<td>0.037 (0.017)*</td>
</tr>
<tr>
<td>Peru</td>
<td>$\alpha$ 0.422 (0.156)*</td>
<td>0.357 (0.143)*</td>
<td>0.201 (0.099)*</td>
<td>0.077 (0.053)</td>
<td>0.015 (0.046)</td>
</tr>
<tr>
<td></td>
<td>$\beta$ 0.544 (0.111)*</td>
<td>0.191 (0.095)*</td>
<td>0.059 (0.062)</td>
<td>0.045 (0.032)</td>
<td>0.015 (0.032)</td>
</tr>
</tbody>
</table>

Notes: Least squares estimates of equation (12), using the 34 available observations for long-term forecasts. HAC standard errors in parenthesis. *" denotes coefficients different from zero at a 5% significance level.

standard deviation of the inflation shock, std($\epsilon_t$); the estimated standard deviation of the endpoint shock, std($\Delta \mu_t$), which is restricted to std($\Delta \mu_t$) = 0 in the CE and UR models; and, to save space, the average standard deviation of the approximation errors of the $m = 6$ survey measures, std($\epsilon_t$). In addition, as measures of fit, the Table presents the root mean square error (RMSE) of the one-step ahead predictions produced by the Kalman filter for the observed data, i.e. inflation and survey forecasts. In the case of the LL model, we proceed in two steps: first, the inflation parameters were estimated using only the first measurement equation; second, the remaining output in the Table was obtained conditional on the first-stage estimates. Figure 4 shows the predicted endpoints, evaluated at the maximum likelihood estimates.

There are various similarities in the estimations across countries. In all cases, the UR model does a relatively good job in explaining actual inflation, but performs rather poorly when it comes to survey data. This finding is related to the mean-reverting properties of expectations discussed in Table 1 which, by construction, the UR model overlooks. In this respect, the LL model constitutes an improvement upon the UR model. Figure 4 reveals that the predicted endpoint in the LL model can be regarded as a de-noised and smoother version of the moving average endpoint of the UR model; inflation expectations revert quickly from the observed data to such an estimated trend (the sum of autoregressive coefficients is $\Phi(1) \approx 0.5$ in all cases), which improves the model’s performance to account for the variability of short-term forecasts. Long-term forecasts, nonetheless, are also poorly predicted in the LL model, as survey data appear to be much less sensitive to inflation news than what is implied in the LL model.

On the other hand, in the CE model the sum of autoregressive coefficients $\Phi(1)$ is significantly higher than in the LL and SE models. In the latter specifications, some of the persistency in observed inflation is attributed to the dynamics of a time-varying mean, and some to short-run deviations from this mean. Since inflation in the CE model is assumed to revert to an imposed constant level, the process is unsurprisingly estimated as highly persistent. Thus, short-term forecasts (especially for the current year) are similar to those of the UR model. However, with the exception of Colombia, the performance of the CE in fitting survey data improves rapidly and dramatically as the forecast horizon increases. A unit increase in $i$ (the year index) implies an increase of $h_{i+1,t} - h_{it} = 12$ months in the associated forecast horizon, so despite the large values of $\Phi(1)$, current inflation becomes less influential even for small values of $i$. As a result, the CE estimates turn out to be closer to the sample average of long-term forecasts than to the sample average of observed inflation (see Figure 4), and the CE model does remarkably well in explaining long-term forecasts in Chile, Mexico and Peru. For these countries, inflation and expectations have been fluctuating (albeit, persistently) within a relatively narrow range for most of the sample period. In contrast, inflation trends downwards during all the sample period in the Colombian case, making the CE model unsuitable.

The SE model is a compromise between the LL and CE models. Survey information points out to a smoother endpoint than the LL model, as shown in Figure 4, which is manifested in a reduction of the standard deviation...
Figure 4. Predicted endpoints by model variant

Notes: CE: Constant endpoint model; UR: Unit root model; LL: Local level model; SE: Shifting-endpoint model. For the CE, the figure displays the estimated value of $\mu$. For the UR, the moving average endpoint $\mu_t = e_1' \bar{C}z_t$ (see section 2.6). Only point estimates are reported for these models to avoid overloading the graphs. For the LL and SE models, the figure shows the smoothed predictions along with a 95% confidence interval. The scale of the vertical axis is the same for Chile/Peru and Colombia/Mexico.
of the endpoint shocks from $\text{std}(\Delta \mu_t) \approx 0.25$ to $\text{std}(\Delta \mu_t) \approx 0.17$ and, more importantly, to significant increases in the noise-to-signal ratios $q = \text{var}(\epsilon_t) / \text{var}(\Delta \mu_t)$ from $q \approx 1$ to $q \in [2.8, 6.7]$. Given the actual inflation persistence, a smoother endpoint is traded with an increase in $\Phi(1)$, from $\Phi(1) \approx 0.5$ to $\Phi(1) \approx 0.7$. With this, the SE model accounts for short-term forecasts variability as much as the LL model, while clearly outperforming the LL model to explain long-term forecasts. When compared to the CE model, the SE model performs considerably better when it comes to short-term forecast but, except in the Colombian case, it is outperformed when predicting long-term forecasts. However, the improvement in the short-term fit of the SE model seems to more than compensate the moderate deterioration in fitting longer-term expectations.

Table 2. Estimation results

<table>
<thead>
<tr>
<th>Country</th>
<th>Model specification</th>
<th>RMSE Inflation 1</th>
<th>RMSE Inflation 2</th>
<th>RMSE Inflation 3</th>
<th>RMSE Inflation $\geq 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chile</td>
<td>CE</td>
<td>0.379</td>
<td>0.473</td>
<td>0.38</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>UR</td>
<td>1.000</td>
<td>1.417</td>
<td>0.39</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>LL</td>
<td>0.544</td>
<td>1.038</td>
<td>0.46</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>SE</td>
<td>0.677</td>
<td>0.844</td>
<td>0.43</td>
<td>0.57</td>
</tr>
<tr>
<td>Colombia</td>
<td>CE</td>
<td>0.309</td>
<td>1.322</td>
<td>0.33</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>UR</td>
<td>1.000</td>
<td>1.570</td>
<td>0.34</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>LL</td>
<td>0.480</td>
<td>1.122</td>
<td>0.43</td>
<td>0.41</td>
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<tr>
<td></td>
<td>SE</td>
<td>0.713</td>
<td>0.925</td>
<td>0.36</td>
<td>0.53</td>
</tr>
<tr>
<td>Mexico</td>
<td>CE</td>
<td>0.278</td>
<td>0.798</td>
<td>0.28</td>
<td>0.60</td>
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<tr>
<td></td>
<td>UR</td>
<td>1.000</td>
<td>1.750</td>
<td>0.29</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>LL</td>
<td>0.564</td>
<td>1.263</td>
<td>0.34</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>SE</td>
<td>0.790</td>
<td>0.987</td>
<td>0.30</td>
<td>0.51</td>
</tr>
<tr>
<td>Peru</td>
<td>CE</td>
<td>0.334</td>
<td>0.745</td>
<td>0.34</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>UR</td>
<td>1.000</td>
<td>1.037</td>
<td>0.35</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>LL</td>
<td>0.511</td>
<td>0.819</td>
<td>0.41</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>SE</td>
<td>0.693</td>
<td>0.606</td>
<td>0.39</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Notes: Maximum likelihood estimates, using data from February 1997 to December 2013. All models use $p = 13$. CE: Constant endpoint model; UR: Unit root model; LL: Local level model; SE: Shifting-endpoint model. $\Phi(1)$: sum of the autoregressive coefficients, $\Phi(1) = \sum_{i=1}^{6} \phi_i$, (robust standard errors in parentheses) restricted to $\Phi(1) = 1$ in the UR model; std($\epsilon_t$): standard deviation of the endpoint shock; std($\Delta \mu_t$): standard deviation of the endpoint shock, restricted to std($\Delta \mu_t$) = 0 in the CE and UR models; std($\epsilon_t$): average standard deviation of the approximation errors of the $m = 6$ survey measures; “RMSE” is the root mean square error of the one-step ahead predictions produced by the Kalman filter inflation and survey forecasts for years 1 (current), 2 (next), 3 and $\geq 4$ (subsequent) (the column labeled “$\geq 4$” shows the average RMSE for the remaining 3 long-term forecasts).
Table 3. Kalman gains in the Peruvian SE model

<table>
<thead>
<tr>
<th>Month</th>
<th>Inflation</th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$F_3$</th>
<th>$F_4$</th>
<th>$F_5$</th>
<th>$F_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>February 2001</td>
<td>0.1036</td>
<td>0.1767</td>
<td>0.1661</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>March 2001</td>
<td>0.1187</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>April 2001</td>
<td>0.0654</td>
<td>0.1410</td>
<td>0.0720</td>
<td>0.0167</td>
<td>0.0364</td>
<td>0.0669</td>
<td>0.1364</td>
</tr>
<tr>
<td>September 2006</td>
<td>0.0640</td>
<td>0.0985</td>
<td>0.0947</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>October 2006</td>
<td>0.0528</td>
<td>0.0992</td>
<td>0.0550</td>
<td>0.0160</td>
<td>0.0220</td>
<td>0.0568</td>
<td>0.1034</td>
</tr>
<tr>
<td>March 2013</td>
<td>0.0621</td>
<td>0.1047</td>
<td>0.0993</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>April 2013</td>
<td>0.0492</td>
<td>0.1062</td>
<td>0.0542</td>
<td>0.0126</td>
<td>0.0274</td>
<td>0.0504</td>
<td>0.1028</td>
</tr>
</tbody>
</table>

Notes: The Kalman gain measures how prediction errors of the observed data are weighted in order to get an updated prediction of $\mu_t$. The table shows these weights for selected dates. The gain is set to zero for missing values. Survey data are not available for March 2001, whereas long-term forecasts are only available in April and October.

level close to the gain of $F_1$. Interestingly, since the prediction errors are typically larger for $F_6$ than for $F_1$ (see the RMSE results in Table 2), the absolute magnitude of the correction would also be larger for $F_6$ than for $F_1$. Thus, when $F_6$ is observed, the Kalman filter updates the endpoint prediction towards it.

3.3 Inflation targeting

Figure 5 shows the evolution of the predicted endpoints along with the inflation targets for each country shifted forwards one year. For instance, Chile’s inflation target was 3.5 percent for 2000, and this value appears during 1999 in the figure.\(^8\) The purpose is to illustrate the joint evolution of the endpoint and the announcements of the inflation targets, and thus we are assuming that the Central Banks have announced their targets one-year in advance. This was the actual practice during disinflation; however, all Central Banks have progressively increase the horizon for the targets as inflation have reached long-run levels.

During disinflation, roughly until 2002, the announcements have clearly served as a benchmark for private economic agents’ long-run forecasts. Shocks to the endpoints bears an almost one-to-one correspondence to changes in the announced targets.

From 2002 onwards, long-run expectations have lied within the target ranges in almost all instances. The remarkable exception is year 2008, especially for Chile and Peru. During 2007 and most of 2008, these economies where hit by a sequence of large food price and oil shocks that deviated inflation from its target. The shocks were persistent enough so that they affected long-run perceptions. However, the endpoint returned rapidly to the inflation targets by late 2008 as the result of both aggressive increases in the monetary policy rates, and the start of the global financial crisis. It has remained within the target range ever since.

Given these dynamics, our results support the conclusion that the countries in our sample are good examples of successful inflation targeting experiences (see Corbo and Schmidt-Hebbel, 2001). Their Central Banks have managed to establish a credible regime of stable inflation with anchored expectations.

4 Closing remarks

Fixed-event forecasts provide a widespread, yet unexplored, source of inflation expectations in many countries. The main difficulty is that the very structure of the FEFs, especially the fact that they correspond to moving forecast horizons, hinders their direct applicability in empirical work. To overcome this hindrance, and to infer about the term structure of inflation expectations from FEFs, we have proposed an extended version of the shifting-endpoint model of Kozicki and Tinsley (2012). Even though the resulting model

\(^8\) Chile announced point targets until 2000; thereafter, the inflation targets have been published as ranges (actually, central values surrounded by a symmetric tolerance level). The same occurred in Colombia and Mexico until 2002. Peru has announced target ranges during all the sample period.
is time-varying, it can be easily handled with the Kalman filter, even for irregularly sampled survey expectations. By fitting the shifting-endpoint model to Latin American data, we conclude that it is able to jointly account for the behavior of actual inflation and survey records, often outperforming competing specifications. Our empirical exploration also suggests that survey FEFs provide a valuable source of information on expected inflation, complementary to that contained in historical records of inflation.

Given the availability of FEFs, exploring alternative methods to readily and effectively use such data in econometric models is likely to have important practical implications. We hope our analysis to be a meaningful contribution to this promising research agenda.

References


