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State Contingent Assets, Financial Crises and Pecuniary Externalities in Models with Collateral Constraints

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State Contingent Assets, Financial Crises and Pecuniary Externalities in Models with Collateral Constraints

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Abstract

This paper analyzes the effect of developing financial markets for contingent assets on the degree of risk sharing and its ability to reduce spillover effects due to credit externalities. In an environment with persistent shocks and collateral constraints, state contingent assets allow for partial hedging against income fluctuations, which reduce the need for precautionary savings and lower the incidence of financial crises. In addition, these assets reduce the size of spillover effects of individual leverage on the valuation of the collateral constraint of other agents, but is not able to fully correct the pecuniary externality. Private agents take too little state contingent debt by ignoring the larger hedging properties of this instrument against the debt deflation mechanism.

Este trabajo analiza el impacto de los instrumentos financieros indexados en la exposición al riesgo y la posibilidad de reducir los efectos de contagio de las externalidades crediticias. Los instrumentos financieros indexados otorgan cobertura parcial contra las fluctuaciones en los ingresos de los deudores, lo que reduce la necesidad de ahorro por motivo de precaución y la incidencia de crisis financieras. Asimismo, estos activos reducen el efecto contagio del apalancamiento individual sobre la valuación del colateral de otros agentes, pero no es capaz de corregir completamente la externalidad pecuniaria. Así, los agentes privados eligen una proporción muy baja de deuda indexada al ignorar la cobertura que brinda este instrumento contra el mecanismo de deflación de la deuda.

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1 Introduction

There has been great interest in the benefits and policy implications of financial innovation as a possible source of insurance against the vulnerabilities faced by emerging countries and the likelihood of financial crises and default episodes. Different types of state contingent financial instruments and their theoretical benefits have been discussed in the literature, as a way to stabilize capital flows to emerging market economies and to provide a better source of hedging against macroeconomic risks.[1]

This work analyzes the implications of diversifying the external liability portfolio of private agents by having access to financial instruments with state contingent payments, in the spirit of instruments such as GDP-linked bonds and future contracts. We focus on its effect in terms of risk sharing, the frequency of financial crises and especially on the size of spillover effects of individual debt on the valuation of collateral of other agents in the economy.

In an environment with a single bond with a fixed interest rate, private agents only have one instrument to engage in risk sharing in two margins: inter-temporally between current and future consumption and intra-temporally between consumption in periods of high and low endowment. Higher debt allows for inter-temporal risk sharing for highly impatient agents, but does not allow for risk sharing across states. Risk sharing is improved when there is access to an additional state contingent asset. In an environment with i.i.d. shocks, full risk sharing can be achieved by using the state contingent bond to perfectly hedge against income fluctuations and using the regular bond to engage in inter-temporal risk sharing. With persistent shocks, it is only possible to partially hedge against the income shock, so that agents accumulate some precautionary savings, but in a smaller amount compared to the case with bonds only.

In terms of the frequency of financial crises, access to state contingent debt lowers the likelihood of default and financial crises, as repayment becomes less costly in bad states. When a country faces a bad shock, a GDP-linked bond reduces the amount of resources needed for debt rollover. As financial

[1] Shiller (1993) presents an analysis of the benefits of developing financial markets for bonds with payments linked to the level of GDP as a way to hedge against macroeconomic risks in a closed economy setup. For references in the open economy literature, Williamson (2005) analyzes the benefits of having access to growth indexed bonds that could be traded in international financial markets as a way to reduce the probability of sharp reversals in capital flows.

crises are related to high levels of debt, there is a lower probability of crisis and smoother drops in consumption. Lower frequency of financial crises benefits the borrower because crises are extremely costly in terms of the reduction in output and consumption and loss of access to financial markets.[2]

The main contribution of this work is to analyze the normative implications of the interaction between the risk sharing properties of contingent assets and the spillover effects on other agents in the economy of an individual agent taking excessive debt. State contingent debt service provides additional insurance through its effect on the valuation of non-tradable collateral of all agents. A smaller capital outflow in bad times, due to lower interest payments under the GDP-linked bond, leads to a milder real depreciation. This reduces the size of spillover effects through the debt deflation mechanism, as a milder depreciation reduces the fall in the valuation of non-tradable collateral of all other agents in the economy.

We present a qualitative analysis of the effects of having access to state contingent instruments in a two-good endowment small open economy with limited access to international financial markets. In an environment with i.i.d. shocks, agents can fully insure against income shocks by using the state contingent asset and borrow up to the collateral constraint using the non-state contingent bond to increase current consumption. There is no pecuniary externality as it is optimal to borrow at the binding collateral constraint due to the impatience factor.

However, agents cannot fully hedge against persistent income shocks. Partial exposure to income fluctuations creates a need for accumulating precautionary savings, but in a smaller magnitude than in an environment with bonds only. A constrained social planner considers the spillover effect of diversifying the debt portfolio towards the state contingent bond, as it provides additional insurance by relaxing the collateral constraint in bad states without tightening them in good states. State contingent repayment reduces capital outflows, dampens the real exchange depreciation and reduces the size of the spillover effect on collateral valuation.

From a normative perspective, private agents do not internalize the insurance properties of the

[2] See Eichengreen (2004)

state contingent instrument in terms of relaxing the collateral constraint. Higher individual borrowing reduces the price of non-tradable collateral and therefore tightens the collateral constraint of other agents. A private agent would therefore accumulate less precautionary savings than a constrained social planner. Compared to the environment with bonds only, partial insurance through the state contingent asset dampens the size of the pecuniary externality.

We also present a quantitative analysis of the benefits of having access to financial instruments with state contingent interest payments and its effect on risk sharing, the frequency of financial crises and the reduction in the pecuniary externality that arises through the valuation of collateral. We calculate the optimal holding of both types of assets and the frequency of financial crises in the decentralized equilibrium and compare them to the solution of a constrained social planner who internalizes spillover effects of individual borrowing on the collateral valuation of other borrowers. We calculate the size of the spillover effect in an environment with access to non-state contingent bonds only and in one where agents have access to both non-state contingent and state contingent bonds.

Our results show that private agents take higher levels of total debt and face a lower frequency of financial crises when they have access to both state contingent and non-state contingent bonds. Comparing the decentralized equilibria with and without state contingent bonds shows that more stable debt service reduces the requirement for new borrowing in bad times, hence being financially constrained less often.[3]

By comparing the size of the spillover effects on the valuation of collateral, we find that the difference in the distribution of debt and the frequency of financial crises between the decentralized equilibrium and the constrained social planner's problem is smaller in an environment with pro-cyclical debt service.[4]This is related to the milder incidence of exchange rate depreciations on the collateral constraint in bad states.

[3]Caballero and Panageas (2008) show that having access to future contracts lowers the need for precautionary savings by roughly 10 percent of GDP.

[4]Given the presence of financial frictions, as mentioned in Korinek (2010), even a complete set of state contingent bonds would not help to overcome the collateral constraint and achieve a socially optimal outcome. However, there might be certain types of instruments that reduce the likelihood of hitting the collateral constraint, and are able to get a smaller distortion in the debt choice compared to the one of a credit constrained social planner.

Related Literature. This work is related to the literature on the effects of having access to markets for indexed bonds. Durdu (2009) studies the effect of indexing debt to GDP and solves for the optimal degree of indexation, as there is a trade-off between income fluctuations and interest rate fluctuations. We extend this model to consider the effect of pecuniary externalities in this environment with endogenous collateral constraints, where a mismatch in the valuation of total income and debt denominated in tradable goods creates an additional channel that amplifies the cost and frequency of financial crises.

Borensztein and Mauro (2004) present a detailed study on the benefits of indexed bonds and a discussion on the concerns around choosing the optimal variable for indexing debt, as well as the implications of developing markets for liquid trade of this type of instrument. Consistent with this ersult, we find that indexed debt reduces the volatility of the debt-to-GDP ratio and the likelihood of financial crises. During a period of low GDP, debt repayments fall and a lower value of debt is required to rollover previously contracted debt. As financial crises are related to high levels of debt, this financial instrument lowers the probability of crises and reduces the amplification effect on consumption, the real exchange rate depreciation and the reversal in capital flows.

On the theoretical side, this work is related to models of incomplete markets and limited enforcement, such as Kehoe and Perri (2002), where agents have access to a full set of state contingent assets. However, due to the existence of an enforcement constraint, where borrowing is limited to prevent default, agents are not able to engage in full risk sharing. This work shows a similar environment where we limit access to only one state contingent asset, which is closer in nature to a GDP-linked bond. Bai and Zhang (2010) present a model of limited enforcement and limited spanning for a production economy, where the financial constraint comes from an incentive compatibility constraint that ensures repayment even under the worst case scenario, and agents have access to non-state contingent bonds only. This work extends the results to a case where an additional state contingent bonds is available.

A closely related strand of the literature analyzes the quantitative implications of contingent financial instruments as a substitute to large accumulation of precautionary savings. Caballero and

Panageas (2008) present a quantitative model to analyze the gains of using contingent financial instruments to reduce the probability of sudden stops. A similar quantitative analysis is presented in Borensztein et al (2013), where borrowers use precautionary savings jointly with commodity futures as contingent hedging instruments for commodity exporter countries and calculate the welfare gains.

This work is also related to models where a pecuniary externality arises through the effect of valuation of collateral in credit constraints. Bianchi (2011) and Korinek (2010) analyze the presence of an externality effect and the over-borrowing result associated with it. We compare the size of the pecuniary externality and find that spillover effects are smaller in an environment with access to state contingent assets. This is because they provide partial hedging against shocks, reducing the need for precautionary savings and have an asymmetric effect on the collateral constraint, by loosening it in bad states. However, it is not possible to achieve the second best solution as in the case of using Pigouvian taxes.

The chapter is organized as follows. Section 2 presents a description of the model. Section 3 presents the analytical results on the benefits of having access to contingent financial instruments with pro-cyclical debt service. Section 4 presents an analysis of the quantitative results. Section 5 concludes.

2 Model

We present an infinite horizon model of a continuum of domestic private agents who have limited access to international capital markets from a large pool of risk neutral lenders. Domestic agents' preferences are defined on the consumption of a composite good, which depend on the consumption of tradable and non-tradable goods. They can borrow using two types of financial instruments: a non-state contingent bond, with a fixed interest payment, and a bond with state contingent interest rate payments. International borrowing is subject to limited enforcement, so that private agents can only borrow up to a fraction of the value of their total income.

2.1 Consumers

We make reference to the two-good model presented in Mendoza (2006) of a small open economy inhabited by a continuum of identical private agents that consume a composite good c. The composite good consists of tradable (c^T) and non-tradable goods (c^N).

$$c(c_t^T, c_t^N) = \left[a(c_t^T)^{-\mu} + (1-a)(c_t^N)^{-\mu}\right]^{-\frac{1}{\mu}} \tag{1}$$

where a is the CES weighting factor for tradable goods ($0 \leq a \leq 1$) and $\frac{1}{1+\mu}$ is the elasticity of substitution between tradable and non-tradable goods ($\mu \geq -1$).

Each agent's preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \tag{2}$$

where γ is the relative risk aversion coefficient.

Agents have access to two types of financial instruments that are traded in international capital markets: a non-state contingent bond (b_{t+1}), with a gross interest rate of R, and a state contingent financial instrument ($\hat{b}_{t+1}$), where the repayment of the principal is fixed but the interest rate payment (R_t^y) is contingent on the realization of the endowment shock. Agents choose consumption of tradable (c_t^T) and non-tradable goods (c_t^N) and foreign asset holdings (b_{t+1} and $\hat{b}_{t+1}$).

There is a large pool of risk neutral international lenders, where the participation constraint that allows for both types of contracts in equilibrium is given by:

$$\mathbb{E}_t R_{t+1}^y = R \tag{3}$$

2.2 Collateral Constraint

Due to imperfect enforceability of contracts, agents face a borrowing constraint in international credit markets. We can think of this as the requirement of collateral in order to obtain credit, based on the value of the current income of domestic agents. Only collateralized debt is offered by international creditors up to a fraction κ of the domestic agent's current income. A key feature in order to obtain an inefficiency in this type of models is the effect of relative prices on the collateral constraint. The use of non-tradable goods as collateral is related to the empirical evidence on credit booms in the non-tradable sector with external credit, as shown in Tornell and Westermann (2005). The other important feature is that collateral is calculated in terms of current income, consistent with the evidence found in Japelli (1990) where current income is a key factor affecting the probability of having access to credit markets.

The collateral constraint is given by:[5]

$$b_{t+1} + \hat{b}_{t+1} \geq -\kappa(y_t^T + p_t^N y^N) \tag{4}$$

which is more binding than the natural debt limit. y_t^T is the tradable endowment shock, y^N is non-tradable endowment and p_t is the relative price of non-tradable goods, which is also an indicator of the inverse of the real exchange rate.

In comparison to a model with no borrowing constraints, domestic agents reduce consumption and borrowing and accumulate precautionary savings to hedge against negative shocks that make the borrowing constraint binding.

2.3 Model Equilibrium

We solve for the allocations in the constrained social planner's problem and compare them to the ones in the competitive equilibrium in order to measure the size of the pecuniary externality, the price

[5] Notice that positive values of non-state contingent and state contingent assets can also be used as collateral. Going long on one asset and short on the other one by the same amount does not affect the collateral constraint.

distortion and the frequency of financial crises.

2.3.1 Competitive Equilibrium

Each private agent in the domestic economy solves the following recursive problem:

$$V(b,\hat{b},B,\hat{B},y^T) = \max_{b',\hat{b}'} \left\{ \frac{\left[\left[a(c^T)^{-\mu} + \left(1-a\right)(c^N)^{-\mu}\right)\right]^{-1/\mu}\right]^{1-\gamma}}{1-\gamma} + \beta \mathbb{E} V(b',\hat{b}',B',\hat{B}',y^{T\prime}) \right\} \quad (5)$$

s.t.

$$b' + \hat{b}' + c^T + p^N c^N \quad = \quad y^T + bR + \hat{b}R^y + p^N y^N \quad (6)$$

$$b' + \hat{b}' \quad \geq \quad -\kappa[y^T + p^N y^N] \quad (7)$$

$$B' = \Gamma(B,\hat{B},y^T) \quad (8)$$

$$\hat{B}' = \Theta(B,\hat{B},y^T) \quad (9)$$

where the relative price of non-tradable goods, $p^N = p^N(B,\hat{B},y^T)$, is a function of aggregate borrowing and the tradable endowment shock.

Definition 1 *The recursive competitive equilibrium is defined by the policy functions* $\{c^T(b,\hat{b},B,\hat{B},y^T)$, $c^N(b,\hat{b},B,\hat{B},y^T), b'(b,\hat{b},B,\hat{B},y^T), \hat{b}'(b,\hat{b},B,\hat{B},y^T)\}$, *prices* $\{p^N(B,\hat{B},y^T)\}$, *laws of motion* $\{B'(B,\hat{B},y^T),\hat{B}'(B,\hat{B},y^T)\}$ *such that:*

1. *Borrowers: Taking* $p^N(B,\hat{B},y^T)$ *as given, each agent solves the borrower's problem to obtain the policy functions.*

2. *Rational expectations:*

$$\Gamma(B,\hat{B},y^T) = b'(B,\hat{B},B,\hat{B},y^T) \quad (10)$$

$$\Theta(B,\hat{B},y^T) = \hat{b}'(B,\hat{B},B,\hat{B},y^T) \quad (11)$$

3. Market clearing: Markets for tradable and non-tradable goods clear:

$$c^N(B,\hat{B},B,\hat{B},y^T) = y^N \tag{12}$$

$$c^T(B,\hat{B},B,\hat{B},y^T) + b'(B,\hat{B},B,\hat{B},y^T) + \hat{b}'(B,\hat{B},B,\hat{B},y^T) = y^T + BR + \hat{B}R^y \tag{13}$$

2.3.2 Constrained Social Planner Solution

Definition 2 *The constrained social planner's equilibrium is defined by policy functions* $\{c^T(b,\hat{b},y^T)$, $c^N(b,\hat{b},y^T), b'(b,\hat{b},y^T), \hat{b}'(b,\hat{b},y^T)\}$ *that solve:*

$$W(b,\hat{b},y^T) = \max_{b',\hat{b}'}\left\{\frac{\left[\left[a(c^T)^{-\mu} + \left(1-a\right)(c^N)^{-\mu}\right)\right]^{-1/\mu}\right]^{1-\gamma}}{1-\gamma} + \beta\mathbb{E}W(b',\hat{b}',y^{T\prime})\right\} \tag{14}$$

s.t.

$$\begin{aligned} c^T &= y^T - b' - \hat{b}' + bR + \hat{b}R^y && (15)\\ c^N &= y^N && (16)\\ b' + \hat{b}' &\geq -\kappa[y^T + \frac{(1-a)}{a}\left(\frac{c^T}{c^N}\right)^{1+\mu} y^N] && (17) \end{aligned}$$

3 Analytical Results

In this section, we present a qualitative analysis of the implications of state contingent assets in terms of risk sharing and the size of the pecuniary externality. With pro-cyclical interest payments, where borrowers pay a higher interest rate in good states and lower interest rate in bad ones, state contingent assets allow borrowers to partially hedge against the borrowing constraint in bad states, which are exactly the states where they need to borrow more in order to smooth consumption. By being less credit constrained, collateral prices drop less than in the case with only non-state contingent bonds and, therefore, the spillover effects on the valuation of collateral of other agents.

3.1 Two bonds with i.i.d. tradable income shocks

A first step is to analyze an environment with i.i.d. shocks, where state contingent interest payments can be used to offset the fluctuations in tradable income. Debt with pro-cyclical interest payments allows agents to improve the transfer of resources from periods of high GDP to periods of low GDP, hence smoothing consumption across states.

When the tradable income shock is i.i.d., agents can perfectly hedge against uncertainty in the income shock across states with state contingent debt and use the non-state contingent bond to engage in risk sharing between periods.[6]

Proposition 3 *If the tradable income shock is i.i.d., state contingent bonds are used to perfectly hedge against uncertainty in the income shock, whereas the non-state contingent bond is used for inter-temporal risk sharing. If $\beta R < 1$, the optimal choice for the non-state contingent bond is given by the binding collateral constraint:*

$$b' = -\kappa[y^T + p^N y^N] - \hat{b} \tag{18}$$

Proof. For any two realizations of the tradable endowment shock, y and y', it is possible to choose a level of the state contingent debt, $\hat{b}$, so that consumption is the same across states.

$$\begin{aligned} c(y) &= y - b' - \hat{b}' + bR + \hat{b}R^y \\ c(y') &= y' - b' - \hat{b}' + bR + \hat{b}R^{y'} \\ \hat{b} &= -\frac{y - y'}{R^y - R^{y'}} = -\frac{1}{\alpha} \end{aligned}$$

where α is the degree of indexation of the interest rate payment for state contingent debt.[7]

If $\beta R < 1$, it is optimal to borrow up to the binding borrowing constraint. The constrained social

[6] The derivation is presented in Appendix 1.

[7] We define the gross interest rate $R^y = R + \alpha(y^T - \mathbb{E}\{y^T\})$.

planner and decentralized agents choose the same amount of debt:

$$b' = -\kappa[y^T + p^N y^N] - \hat{b}$$

Let us analyze the case of a $-\epsilon$ shock to tradable endowment in period 0. Agents borrow up to the collateral constraint:

$$b_1 = \kappa[\epsilon + p^N(\epsilon)y^N] + \frac{1}{\alpha} \tag{19}$$

and from t=2 onwards:

$$b_{t+1} = \frac{1}{\alpha} \tag{20}$$

$$c_0 = \epsilon(1-\kappa) - \kappa p^N(\epsilon)y^N \tag{21}$$

$$c_1 = R\kappa[\epsilon + p^N(\epsilon)y^N] \tag{22}$$

and $c_t = 0$ for $t \geq 2$.

This result is optimal and satisfies the inter-temporal budget constraint:

$$\sum_{t=0}^{\infty} \frac{c_t}{R^t} = \epsilon(1-\kappa) - \kappa p^N(\epsilon)y^N + \epsilon[\kappa + p^N(\epsilon)y^N] = \epsilon \tag{23}$$

$$\sum_{t=0}^{\infty} \frac{y_t}{R^t} = \epsilon \tag{24}$$

■

Figure (1) shows the ability to smooth consumption across different states for an i.i.d. tradable endowment shock. The horizontal axis represents different values of the tradable endowment shock, centered at zero. Agents would optimally choose to trade the amount of the state contingent asset that allows for perfect hedging. The optimal amount of $\hat{b}$ is such that, every period, agents would trade their income realization, y_t, for a fixed income, $E_{t-1}[y_t] = E[y_t]$, and achieve full consumption smoothing across states, shown by the constant consumption value at c^*. In order to achieve this

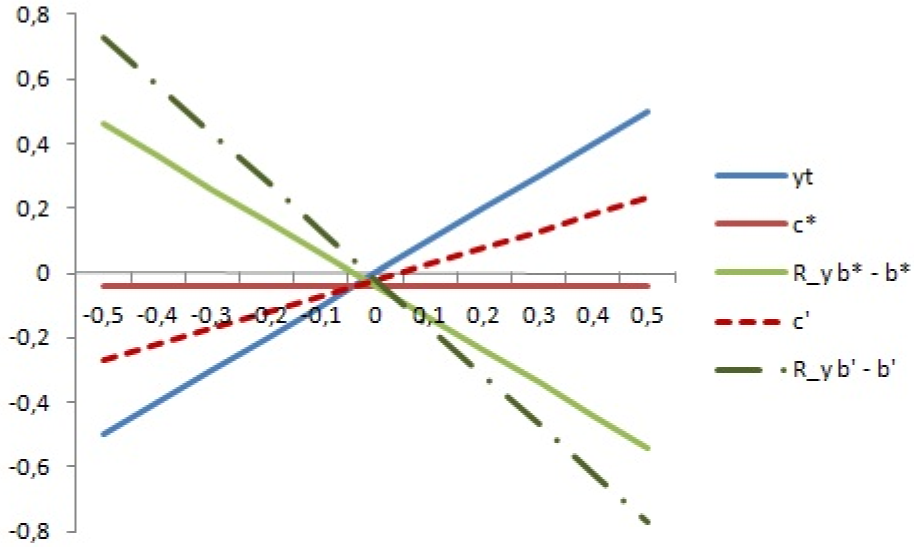

Figure 1: Optimal insurance across states using state contingent assets. The horizontal axis depicts different values of the tradable endowment shock. Full consumption smoothing c^* is achieved under the optimal amount of heding b^*. If lower heding is taken$b\prime > b^*$, only partial insurance is provided and consumption $c\prime$ is suboptimal and positively correlated with the shock.

result, agents need to borrow $\hat{b} = -\frac{1}{\alpha}$, so that the net flow in state contingent debt, $R_t^y \hat{b}_t - \hat{b}_{t+1}$, perfectly offsets the effect of the tradable shock. If agents choose a lower amount of state contingent assets, like for example $\hat{b} = -\frac{1}{2\alpha}$, they would not be able to completely insure against the tradable income shock and would face a positive correlation between the shock and tradable consumption. This results in a positively correlated tradable consumption path, c'.

In terms of the effect on the pecuniary externality, both decentralized borrowers and a constrained social planner would find it optimal to borrow up to the binding collateral constraint, so that there is no pecuniary externality.[8]

3.2 Model with two bonds and persistent shocks

When shocks are persistent, state contingent debt provides partial hedging against income shocks across states, even though it is not possible to achieve full risk sharing. Partial exposure to tradable income requires agents to accumulate some precautionary savings, which makes it suboptimal to borrow up

[8]A pecuniary externality would arise in a special case where the coefficient of the equilibrium path is equal to one, so that the equilibrium oscillates between the constrained and unconstrained cases.

to the binding collateral constraint. State contingent debt provides two benefits: it provides partial hedging to reduce consumption fluctuations and it reduces the tightness of collateral constraints in bad states.

From a normative perspective, we find that a constrained social planner accumulates more precautionary savings (or equivalently takes lower debt) as she internalizes that higher debt leads to a fall in the value of collateral of other agents as well, consistent with the results in an environment with bonds only. The pecuniary externality arises because agents fail to internalize the effect of their decisions in the valuation of collateral, and hence the whole economy faces financial crises more frequently.

However, in terms of the composition of total debt, a constrained social planner also internalizes that state contingent debt allows for partial hedging against the tightening of the collateral constraint, as it dampens the exchange rate depreciation in bad states. Lower interest payments allow for more debt rollover, which reduces the fall in tradable consumption and hence the size of the real depreciation. Therefore, it is socially optimal to take more state contingent debt than the one observed in the private equilibrium.

Let us analyze why the collateral constraint does not allow for perfect hedging against persistent tradable income shocks. Under a scenario with no borrowing constraint and no impatience factor, $\beta R = 1$, we show that agents can perfectly hedge against risk across states and time. We present the results for the effect of a one time $-\epsilon$ shock on tradable endowment in period 0 on consumption and debt.

Tradable endowment follows an AR(1) process:

$$y_t = \rho y_{t-1} + \epsilon_t \tag{25}$$

For a one time shock in period 0, this becomes:

$$y_t = -\rho^t \epsilon \tag{26}$$

Given that $\beta R = 1$, the inter-temporal Euler equation establishes that agents would optimally choose

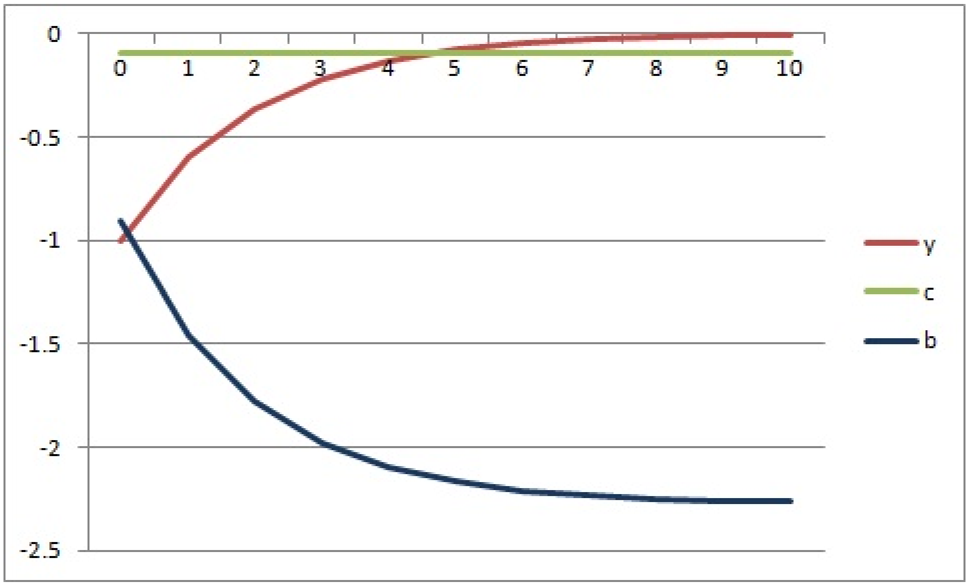

Figure 2: Effect of -ϵ shock on tradable endowment

to engage in full consumption smoothing and accumulate debt to finance this consumption path. From the inter-temporal budget constraint:

$$c_t = -\frac{R}{R-\rho}\epsilon \tag{27}$$

$$b_{t+1} = -\frac{1-\rho^t}{R-\rho}\epsilon \tag{28}$$

Figure (2) shows the time path for tradable endowment (y), tradable consumption (c) and assets (b) in the case with no borrowing constraint. Agents can optimally choose to accumulate debt until they get a positive endowment shock that allows them to pay it back.

Let us compare this result to the case with the borrowing constraint and $\beta R < 1$. The following proposition shows that it is not possible to achieve full risk sharing as in the i.i.d. case.

Proposition 4 *In an environment with persistent tradable endowment shocks, if $\beta R < 1$, agents borrow less than the amount given by the collateral constraint, $-\kappa[y^T + p^N y^N]$.*

Proof. By contradiction, let us assume that agents borrow up to the collateral constraint.

$$b_{t+1} = -\kappa y_t + \frac{1}{\alpha} \tag{29}$$

For a one time $-\epsilon$ shock in period 0, this becomes:

$$b_{t+1} = -\kappa \epsilon \rho^t + \frac{1}{\alpha} \tag{30}$$

Tradable consumption must satisfy the budget constraint:

$$c_t = -(1+\kappa)\rho^t \epsilon + R\kappa\epsilon\rho^{t-1} \tag{31}$$

The inter-temporal budget constraint does not hold:

$$\sum_{t=0}^{\infty} \frac{c_t}{R^t} = -\frac{R}{R-\rho}\epsilon[\frac{R\kappa}{\rho} - \rho(1+\kappa)] \tag{32}$$

$$\sum_{t=0}^{\infty} \frac{y_t}{R^t} = -\frac{R}{R-\rho}\epsilon \tag{33}$$

Therefore, b_{t+1} is less than the value given by the collateral constraint. ■

3.2.1 Decentralized Equilibrium

Optimal bond holdings in the decentralized equilibrium are given by:

$$U_1(c_t) = \beta R E_t \left\{ U_1(c_{t+1}) \right\} + \xi_t \tag{34}$$

where $c_t = (c_t^T, c_t^N)$ and ξ_t is the shadow price of the collateral constraint. Compared to a standard model with no financial frictions, the Euler equation shows that agents borrow less if they face a borrowing constraint.

The inter-temporal Euler equation for the bond with state contingent interest payment, $\hat{b}$, is given by:

$$\begin{aligned} U_1(c_t) &= \beta E_t \left\{ R^y_{t+1} U_1(c_{t+1}) \right\} + \xi_t \\ &= \beta R E_t\{U_1(c_{t+1})\} + \beta cov(R^y_{t+1}, U_1(c_{t+1})) + \xi_t \end{aligned} \tag{35}$$

Pro-cyclical interest payments reduce the need to accumulate precautionary savings. There is a negative covariance between the marginal utility of consumption of tradable goods and the interest rate, so that debt provides partial hedging against income fluctuations. By partially hedging against the tradable income shock, agents do not need to borrow as much in bad states, so that they hit the collateral constraint less frequently, which allows for better consumption smoothing and risk transfer. This is consistent with the results in Caballero and Panageas and the idea behind the rationality of introducing GDP indexed bonds to stabilize the debt service ratio in Borensztein and Mauro (2004).

Private agents are unable to engage in perfect risk sharing across states because of the collateral constraint. There is a trade-off between the two margins: across states and across time, where higher risk sharing across states tightens the collateral constraint, so that there is less space for risk sharing across time using the non-state contingent bond. In equilibrium, the marginal benefit of increasing risk sharing across time, given by the right hand side of equation (34) must be equal to the marginal benefit of increasing risk sharing across states, given by the right hand side of equation (35).

If agents choose to borrow with bonds only ($\hat{b}_t = 0$), $cov(R^y, U_1(c_{t+1})) < 0$. By increasing debt holdings in the state contingent asset, agents are able to optimally reduce the correlation between interest rate fluctuations and marginal utility of consumption. This result is similar to the argument in Figure (1), for the case of partial consumption smoothing. The analytical result is obtained by combining equations (34) and (35):

$$cov(R^y_{t+1}, U_1(c_{t+1})) = 0 \tag{36}$$

3.2.2 Constrained social planner

Consider now a benevolent social planner who faces the same financial contract as private borrowers, but does internalize the spillover effects of one agent's debt on collateral prices. As a result, we show that the decentralized equilibrium is not constrained Pareto optimal, but the magnitude of the spillover effect is dampened when there is access to state contingent bonds.

The inter-temporal Euler equation for uncontingent bonds is given by:

$$U_1(c_t) = \beta R E_t \left\{ U_1(c_{t+1}) + \xi_{t+1}\kappa \frac{\partial p^N_{t+1}}{\partial c_{T,t+1}} y^N \right\} + \xi_t[1 - \kappa \frac{\partial p^N_t}{\partial c_{T,t}}] \tag{37}$$

By comparing equations (34) and (37), a constrained social planner chooses lower debt, $-b_{t+1} - \hat{b}_{t+1}$, or equivalently, accumulates more precautionary savings to insure against the fact that a higher level of debt lowers the price of non-tradable collateral, and therefore, further tightens the borrowing constraint.

The inter-temporal Euler equation for state contingent assets is given by:

$$\begin{aligned} U_1(c_t) = \beta E_t \left\{ R^y_{t+1}[U_1(c_{t+1}) + \xi_{t+1}\kappa \frac{\partial p^N_{t+1}}{\partial c_{T,t+1}} y^N] \right\} + \xi_t(1 - \kappa \frac{\partial p^N_t}{\partial c_{T,t}} y^N) \\ = \beta R E_t\{U_1(c_{t+1}) + \xi_{t+1}\kappa \frac{\partial p^N_{t+1}}{\partial c_{T,t+1}} y^N\} + \xi_t(1 - \kappa \frac{\partial p^N_t}{\partial c_{T,t}} y^N) \\ + \beta cov(R^y_{t+1}, U_1(c_{t+1}) + \xi_{t+1}\kappa \frac{\partial p^N_{t+1}}{\partial c_{T,t+1}} y^N) \end{aligned} \tag{38}$$

By comparing equations (37) and (38), we get that:

$$cov(R^y_{t+1}, U_1(c_{t+1})) + cov(R^y_{t+1}, \kappa \xi_{t+1} \frac{\partial p^N_{t+1}}{\partial c_{T,t+1}} y^N) = 0 \tag{39}$$

Similar to the decentralized equilibrium, increasing state contingent debt provides partial hedging against income fluctuations, given by the first term in equation (39). In addition, it also provides partial hedging against the collateral constraint, by making it less tight in bad states. Comparing equations (36) and (39), we can show that private agents take lower state contingent debt than the constrained social planner, $-\hat{b}^{CE}_{t+1} < -\hat{b}^{SP}_{t+1}$.

The intuition behind this result is that the constrained social planner does internalize that state contingent debt provides the additional benefit of relaxing the collateral constraint through lower interest rate payments, which lead to a lower drop in the price of non-tradable goods.

We analyze the size of spillover effects on total debt holdings for a private agent who is currently unconstrained ($\xi_t = 0$) and compare it to the case with bonds only. The pecuniary externality is smaller because state contingent debt provides partial hedging against hitting the collateral constraint in bad times, so that the amplification mechanism given by the spillover effects of debt through collateral prices is dampened, as shown by the term $\beta RE\left\{\xi_{t+1}\kappa\frac{\partial p^N_{t+1}}{\partial c_{T,t+1}}y^N\right\}$.

This result is consistent with the result in Bianchi (2011) and Korinek (2010) that a constrained social planner that internalizes the effect of debt on relative prices chooses lower levels of debt than decentralized private borrowers.

By comparing the two Euler equations for state contingent debt, we find that the pecuniary externality term for a borrower that is currently unconstrained ($\xi_t = 0$) is given by:

$$\beta RE\left\{\xi_{t+1}\kappa\frac{\partial p^N_{t+1}}{\partial c_{T,t+1}}y^N\right\} + \beta cov(R^y_{t+1}, \xi_{t+1}\kappa\frac{\partial p^N_{t+1}}{\partial c_{T,t+1}}y^N)$$

Using the result in equation (39), agents who have access to borrowing with state contingent interest rate payments get a smaller pecuniary externality effect because

$$cov\left(R^y_{t+1}, \xi_{t+1}\kappa\frac{\partial p^N_{t+1}}{\partial c_{T,t+1}}y^N\right) < 0$$

The pecuniary externality is smaller because agents are allowed to borrow more in bad states of the economy, which is precisely when they need to borrow more to smooth consumption at the expense of facing tighter constraints in good states, where they do not need to borrow as much.

Given that agents can borrow more in bad states of nature, they face a milder drop in relative

Parameter	Value	Parameter	Value
a	0.31	y^N	1
β	0.945	κ	0.32
γ	2	α	0.2
μ	-0.205	R	1.045

Table 1: Parameter Values

prices compared to the case where there is only access to non-state contingent bonds. The smaller drop in relative prices and smaller amplification mechanism in the tightening of the collateral constraint reduces the size of the pecuniary externality. This results in lower need for precautionary savings and the constraint is binding in a lower frequency of states as well.

4 Quantitative Results

This section presents the solution to the infinite horizon problem using value function iteration. We analyze the distribution of debt, the frequency of financial crises and the size of the pecuniary externality in an environment with access to non-state and state contingent bonds. The results are consistent with the qualitative results in the previous section.

In order to analyze the quantitative properties of the model, we obtain the policy rules, the distribution of debt levels, the size of the pecuniary externality and the price of non-tradable goods under each state. Then, we simulate the model for 10,000 periods, where each period represents a year, in order to get the distribution of real variables and the frequency of financial crises.

4.1 Parameter Values

The values of the parameters are shown in Table (3.1). We use the parameter values for the quantitative exercise in Bianchi (2011), which is calibrated to the economy of Argentina at an annual frequency. For the endowment shock process, we match a standard deviation of 0.059 and an autocorrelation value of 0.54. We use the discretization method proposed by Tauchen (1987) to approximate the first order autocorrelation process with a five grid point first-order Markov process.

4.2 Policy functions

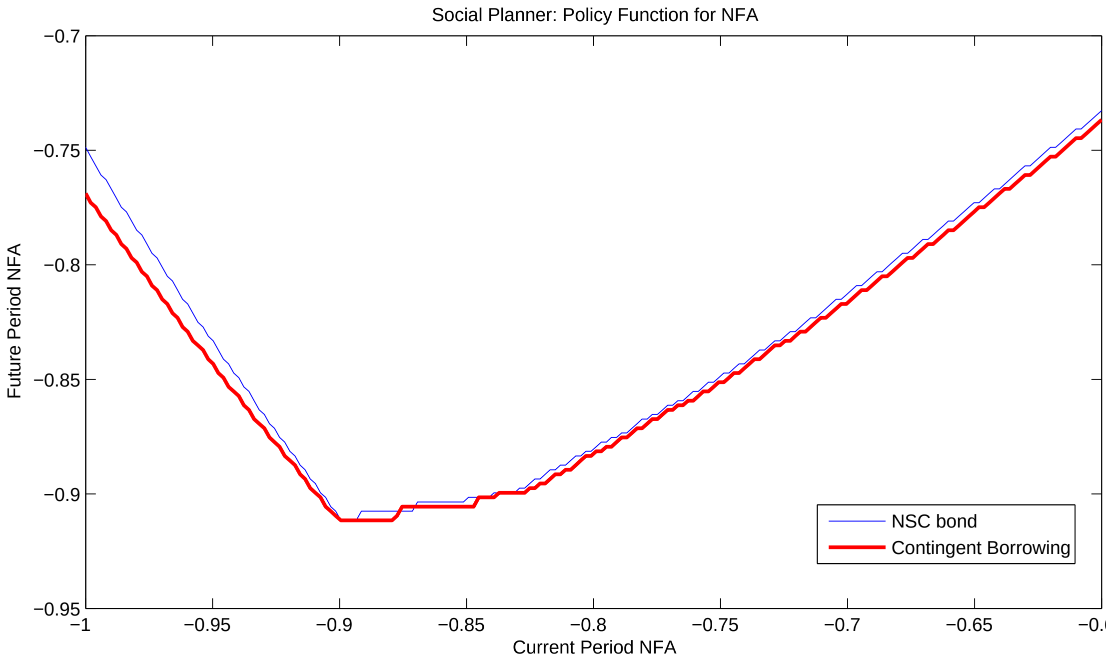

Figure 3: Social Planner: Policy Functions for NFA

Figure (3) shows the policy functions for net foreign assets (NFA, defined as $b' + \hat{b}'$), chosen by a social planner who faces a tradable endowment shock that is one standard deviation below the mean, with and without access to contingent bonds. The line labeled 'NSC bond' shows the total assetposition chosen by a constrained social planner with access to non-state contingent bonds only and the line labeled 'Contingent Borrowing' is the total asset position when they have access to both types of instruments. Pro-cyclical interest payments provide partial hedging against the shock and allows the social planner to borrow more in states with a low endowment shock, as they need to accumulate lower precautionary savings.

The largest gain in terms of the ability to borrow in bad states is seen in periods where the agent is already highly indebted, due to the insurance benefit of state contingent debt which allows to relax the binding collateral constraint. A smaller drop in consumption of tradable goods dampens the effect of the real exchange rate depreciation on the valuation of collateral and therefore on the tightness of the collateral constraint.

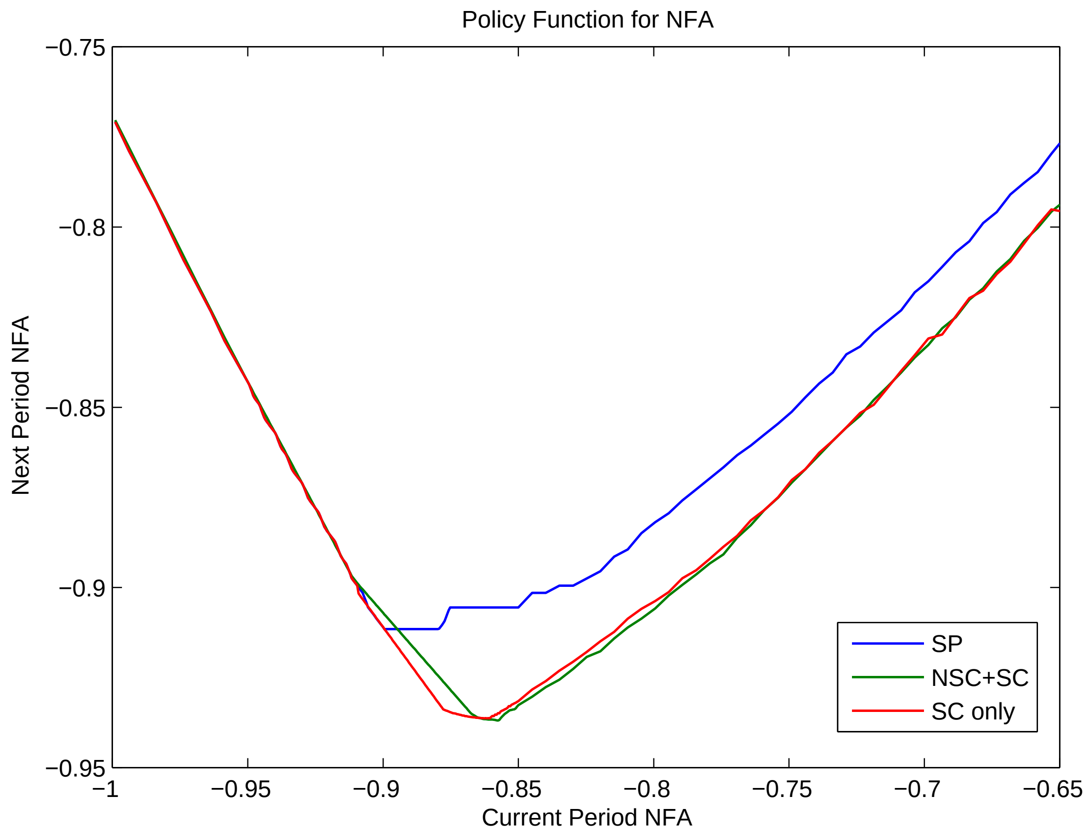

Figure 4: Policy Functions for NFA with and without access to non-state contingent bonds

Now, let us compare the optimal debt choice in an environment with access to state contingent repayment assets only. Figure (4) presents the results for an environment with state contingent assets only, labeled 'SC only', and both types of assets, labeled 'NSC+SC'. State contingent repayment allows for better insurance against shocks to the tradable endowment, reducing the volatility of net income. If all debt has state contingent repayment, then borrowers need to repay less in bad states of nature, making the constraint less tight than if total debt has a combination of state contingent and non-state contingent repayment. As shown by the policy functions, this effect is especially important in intermediate levels of debt, where the pecuniary externality is the largest.

Figure (5) presents the policy functions for NFA in an environment with access to both types of

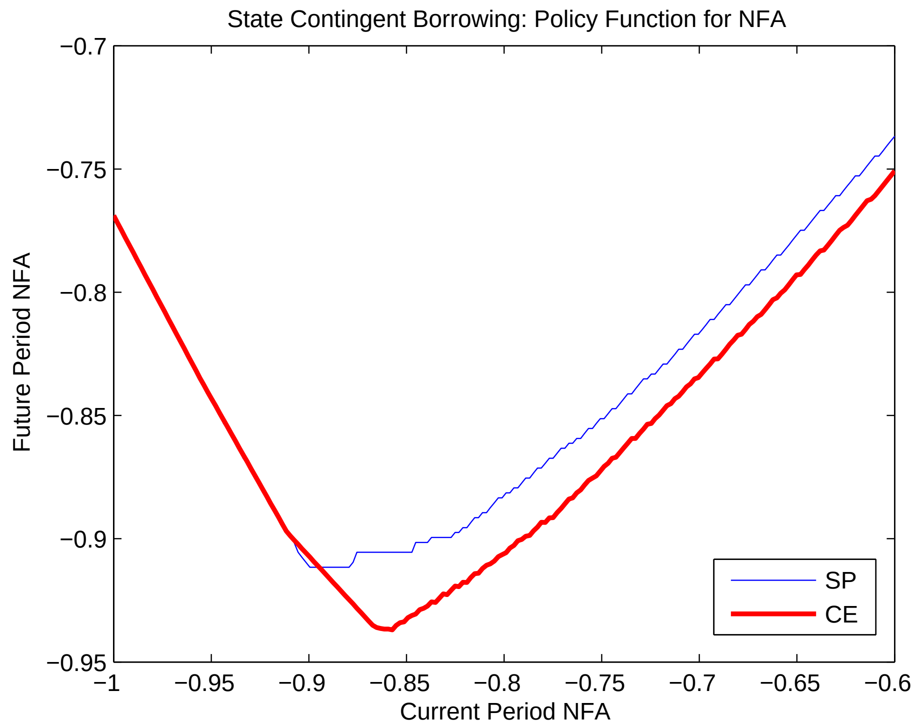

Figure 5: Two Bonds: Policy Functions for NFA

bonds. (SP) represents the policy function for a constrained social planner, whereas (CE) represents the policy function for decentralized agents who do not internalize the effect of their decisions on the exchange rate. The social planner internalizes the effect of n the valuation of other agents' collateral. In addition, with two types of bonds, she internalizes a differentiated effect, where state contingent debt provides additional insurance against hitting the collateral constraint by dampening the effect of individual debt on the real exchange rate.

Regarding total debt levels, there exists a pecuniary externality because agents are not aware of the spillover effects through collateral valuation. Highly leveraged agents are limited by the borrowing constraint, so they must lower borrowing in bad states of nature. A fall in consumption translates into a real exchange rate depreciation due to capital outflows that occur during sudden stop episodes.

However, with contingent assets, agents face the binding borrowing constraint less often, as the value of repayment is much lower in bad states of nature. Given that agents do not need to reduce their tradable consumption as much, the effect of the current account reversal on the exchange rate is dampened. Decentralized agents face a smaller real exchange rate depreciation and a smaller drop in the nominal value of collateral. Therefore, the collateral constraint allows for higher amounts of debt by relaxing the borrowing constraint in low states, which are exactly the ones where agents want to borrow more.

4.3 Distribution of NFA

We compute the stationary distribution of net foreign asset holdings for the decentralized equilibrium and the constrained socially optimal solution with non-state contingent bonds only and with both types of bonds. The stationary distribution of net foreign assets shows that there is a higher probability of reaching higher debt levels under a private equilibrium with a single non-state contingent bond than under the constrained efficient case. This result is consistent with the policy functions, where agents in the decentralized economy borrow more than the constrained efficient level of debt. The result for the case with a regular bond only is similar to the one obtained by Bianchi (2011), where the average debt in terms of tradable income is 87 percent (27,5 percent of GDP) for the constrained efficient case, while the average debt is 90 percent (28,6 percent of GDP) for the decentralized economy. Moreover, 49 percent of the highest debt levels for the decentralized equilibrium are not achieved under the constrained efficient equilibrium.

Figure (6) shows the distribution of NFA for the constrained efficient case (SP) and for the decentralized equilibrium (CE) in an environment with the two types of bonds. The results are consistent with the qualitative analysis in that there is a smaller difference in the distribution of total debt, and hence a smaller pecuniary externality. The average debt level under the decentralized equilibrium rises to 90,2 percent of tradable income (28,7 percent of GDP), whereas a constrained social planner who internalizes the effect of higher debt on the valuation of collateral has an average debt level of 88 percent of tradable income (27,7 percent of GDP).

If we compare these results with an environment with access to state contingent repayment only, shown in Figure (7), we find that 44,9 percent of the highest debt levels for the decentralized equilibrium

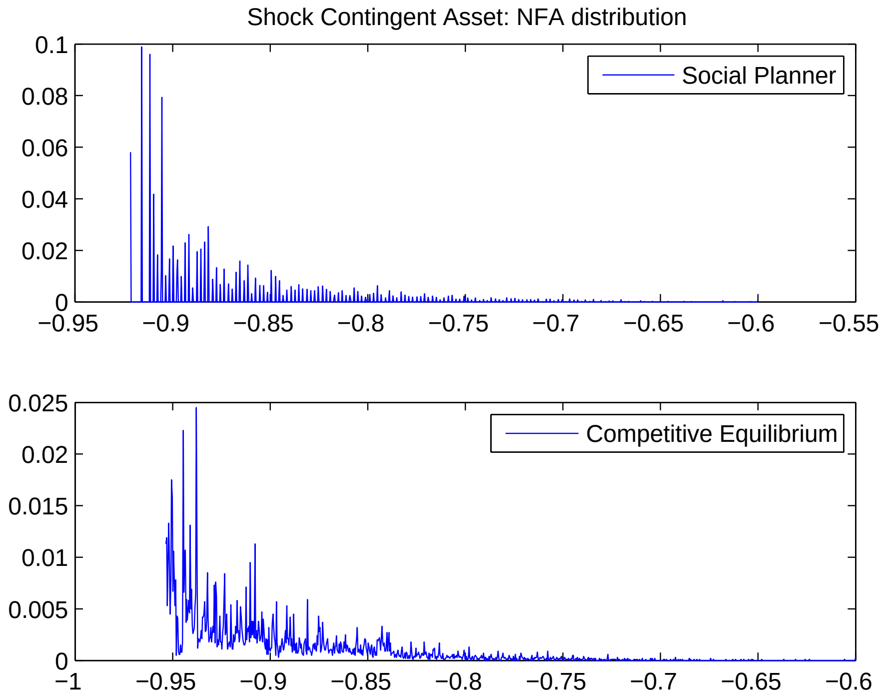

Figure 6: NFA distribution with Contingent Instruments

are not achieved under the constrained efficient equilibrium, compared to 46,5 percent in the case with access to both types of assets. This result shows that there is a smaller pecuniary externality in an environment where agents have access only to bonds with state contingent repayment. The intuition behind this result is that, with state contingent repayment only, the pecuniary externality only affects the total level of debt, while benefiting from the insurance properties of the state contingent repayment. In contrast, in an environment with both types of bonds, the pecuniary externality affects the total amount and the composition of debt, creating a larger distortion from the second best solution.

4.4 Crisis Probability

A financial crisis is defined in our setup as a state where the economy is constrained by the collateral requirement and where the current account suffers a reversal with a magnitude larger than one standard deviation. There are two channels that create a wedge between the frequency of financial crises under the constrained social planner's problem and the decentralized equilibrium. The first one is related to

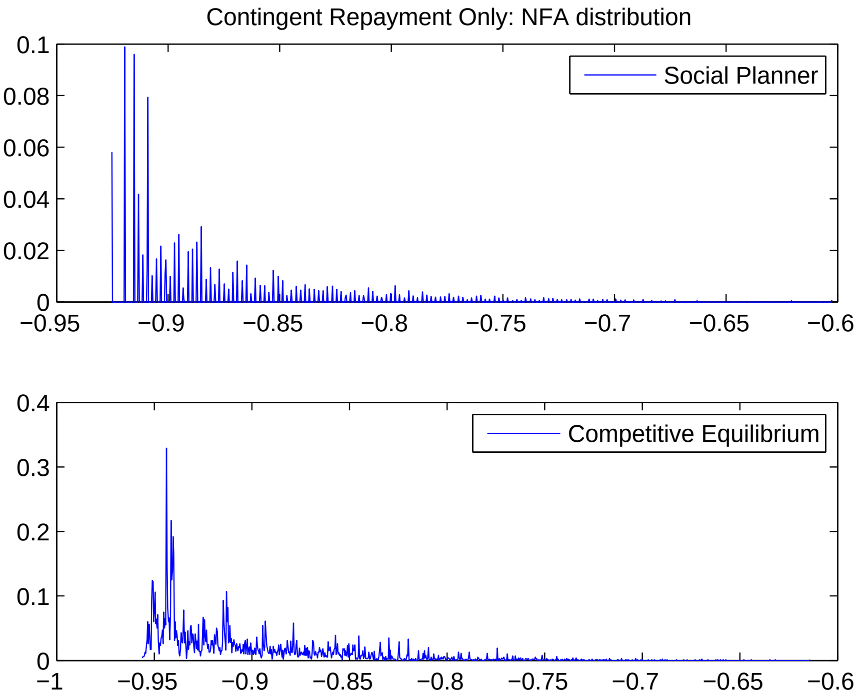

Figure 7: NFA distribution with Contingent Instruments Only

the higher proportion of state contingent debt chosen by a constrained social planner. Higher state contingent debt provides insurance by reducing the tightness of the collateral constraint in bad times so that binding collateral constraints are less frequent. The second channel is related to the fact that the social planner chooses a lower amount of total debt, as they internalize that higher debt tightens the collateral constraint through the effect on the valuation of collateral. Lower borrowing translates into lower probability of facing a binding collateral constraint as well.

In the environment with bonds only, the probability of a financial crisis is 4,6 percent, compared to 0,8 percent for the constrained social planner. However, if agents have access to the two types of bonds, the probability of a financial crisis in the decentralized equilibrium falls to 4,2 percent.

5 Conclusions

In this paper we analyze the effect of using alternative hedging instruments with contingent interest payments on risk sharing, the probability of a financial crisis and the size of the price externality in a two-good endowment economy subject to an endogenous collateral constraint.

Access to state contingent bonds allows agents to obtain partial hedging against income fluctuations and therefore engage in better consumption smoothing. In addition, state contingent debt creates an asymmetric effect on the collateral constraint, where pro-cyclical interest payments relaxes the collateral constraint in bad states, when agents need to borrow more, at the cost of tightening it in good states, when they do not need to borrow as much. Lower volatility of consumption dampens the fall in the price of non-tradable collateral in bad states, as agents would be facing a less tight collateral constraint and would not need to reduce their borrowing and consumption as much.

Having access to state contingent financial instruments reduce the probability of experiencing a financial crisis and the dampens the amplification effect created by the spillover effect of individual debt on the valuation of collateral of other agents. However, it is not possible to fully correct the pecuniary externality. As shown in Bianchi (2011) and Korinek (2010), the use of a Pigouvian tax that depends on the level of individual debt is able to align the private equilibrium to the one obtained by a constrained social planner. The optimal tax level is higher in states of higher debt levels, as there is a higher probability that the economy could face a binding collateral constraint in future periods. A financial instrument that is only contingent on the income shock does not allow to differentiate its interest payments between agents with high and low leverage, who face different marginal benefits of hedging against the income shock.

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6 Appendix

6.1 Pecuniary externalities in an environment with i.i.d. shocks and with bonds with state contingent interest payments

When shocks are i.i.d., there is no pecuniary externality, as both the constrained social planner and private agents are able to perfectly hedge against the income shock using the state contingent asset and take non-state contingent debt up to the limit given by the borrowing constraint given the impatience factor.

For any two states, y and y', tradable consumption is given by:

$$c_{T,t} = y + Rb_t + R_t^y \hat{b}_t - b_{t+1} - \hat{b}_{t+1} \tag{40}$$

$$c'_{T,t} = y' + Rb_t + R_t^{y'} \hat{b}_t - b_{t+1} - \hat{b}_{t+1} \tag{41}$$

The interest rate is given by

$$R_t^y = R + \alpha(y_t^T - \mathbb{E}_{t-1}\left\{y_t^T\right\}) \tag{42}$$

It is possible to choose $\hat{b}$ to make consumption constant across states

$$\hat{b}_t = -\frac{y' - y}{R^{y'} - R^{y}} = -1/\alpha \tag{43}$$

In a more general setup, we can think about $\hat{b}$ as a synthetic of a non-state contingent asset and a swap instrument where borrowers exchange their tradable income endowment and get paid the expected value of the tradable income distribution. For a borrower who goes short on $\hat{b}_t$ units of the synthetic instrument in period t-1, he pays $\alpha y_t + R$ and gets $\alpha \mathbb{E}_{t-1}\{y_t\}$ per unit in period t. Therefore, the budget constraint is the same as before:

$$c_{T,t} + p_t c_{N,t} = y_t + Rb_t - b_{t+1} - \hat{b}_{t+1} + [R + \alpha(y_t - \mathbb{E}_{t-1}y_t)]\hat{b}_t \tag{44}$$

6.2 Pecuniary externalities with and without access to debt with state contingent interest payments

6.2.1 Competitive equilibrium

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_{Tt}, c_{Nt})$$

s.t.

$$c_{Tt} + p_t^N c_{Nt} = y_t^T - b_{t+1} + Rb_t - \hat{b}_{t+1} + R_t^y \hat{b}_t + p_t^N y^N \tag{45}$$

$$b_{t+1} + \hat{b}_{t+1} \geq -\kappa \left[y_t^T + p_t^N y^N \right] \qquad \text{with multiplier } \xi_t \tag{46}$$

Euler equation for regular debt (b_{t+1})

$$U_1(c_t) = \beta R \mathbb{E}_t \left\{ U_1(c_{t+1}) \right\} + \xi_t \tag{47}$$

Euler equation for debt with state contingent interest payments ($\hat{b}_{t+1}$)

$$\begin{aligned} U_1(c_t) &= \beta \mathbb{E}_t \left\{ R^y_{t+1} U_1(c_{t+1}) \right\} + \xi_t \\ &= \beta R \mathbb{E}_t \left\{ U_1(c_{t+1}) \right\} + \beta cov \left(R^y_{t+1}, U_1(c_{t+1}) \right) + \xi_t \end{aligned} \tag{48}$$

The interest payment for $\hat{b}$ is given by:

$$R^y_t = R + \alpha(y_t - \mathbb{E}_{t-1} y_t) \tag{49}$$

Plugging it into the previous equation, we get:

$$U_1(c_t) = \beta R \mathbb{E}_t \left\{ U_1(c_{t+1}) \right\} + \alpha \beta cov \left(y_{t+1}, U_1(c_{t+1}) \right) + \xi_t \tag{50}$$

Lower precautionary savings with access to $\hat{\mathrm{b}}$ are shown by the fact that:

$$cov \left(y_{t+1}, U_1(c_{t+1}) \right) < 0 \tag{51}$$

Procyclical interest payments makes it less likely to hit the collateral constraint, so agents need to accumulate less precautionary savings to insure against this risk. Higher individual debt increases the probability of being financially constrained next period if borrowers get a low income shock, but due to lower interest payments in the procyclical case, the tightening of the constraint is less severe and it allows for more debt rollover compared to the case with regular bonds only.

6.2.2 Social Planner

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_{Tt}, c_{Nt})$$

s.t.

$$c_{Tt} = y^T_t - b_{t+1} + R b_t - \hat{b}_{t+1} + R^y_t \hat{b}_t \tag{52}$$

$$c_{Nt} = y^N \tag{53}$$

$$b_{t+1} + \hat{b}_{t+1} \geq -\kappa \left[y_t^T + \frac{U_2(c_t)}{U_2(c_t)} y^N \right] \qquad \text{with multiplier } \xi_t \tag{54}$$

Euler equation for regular debt (b_{t+1})

$$U_1(c_t) = \beta R \mathbb{E}_t \left\{ U_1(c_{t+1}) \right\} + \xi_t \left[1 - \kappa \frac{\partial p_t^N}{\partial c_{Tt}} y^N \right] + \beta R \mathbb{E}_t \left\{ \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right\} \tag{55}$$

Euler equation for debt with state contingent interest payments ($\hat{b}_{t+1}$)

$$\begin{aligned} U_1(c_t) &= \beta \mathbb{E}_t \left\{ R_{t+1}^y U_1(c_{t+1}) \right\} + \xi_t \left[1 - \kappa \frac{\partial p_t^N}{\partial c_{Tt}} y^N \right] + \beta \mathbb{E}_t \left\{ R_{t+1}^y \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right\} \\ &= \beta R \mathbb{E}_t \left\{ U_1(c_{t+1}) \right\} + \beta cov \left(R_{t+1}^y, U_1(c_{t+1}) \right) + \xi_t \left[1 - \kappa \frac{\partial p_t^N}{\partial c_{Tt}} y^N \right] \\ &\quad + \beta R \mathbb{E}_t \left\{ \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right\} + \beta cov \left(R_{t+1}^y, \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right) \end{aligned} \tag{56}$$

Using (49), we can simplify this to:

$$\begin{aligned} U_1(c_t) &= \beta R \mathbb{E}_t \left\{ U_1(c_{t+1}) \right\} + \xi_t \left[1 - \kappa \frac{\partial p_t^N}{\partial c_{Tt}} y^N \right] + \beta R \mathbb{E}_t \left\{ \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right\} \\ &\quad + \beta \alpha cov \left(y_{t+1}, U_1(c_{t+1}) \right) + \beta \alpha cov \left(y_{t+1}, \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right) \end{aligned} \tag{57}$$

By comparing equations (55) and (57), we can show that access to $\hat{\text{b}}$ lowers precautionary savings because:

$$cov \left(y_{t+1}, U_1(c_{t+1}) \right) + cov \left(y_{t+1}, \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right) < 0 \tag{58}$$

Procyclical interest payments leads to higher interest payments in good times with higher consumption (and lower marginal utility of consumption). The second term is also negative because it is more likely for agents to be financially constrained in bad times, as they need to borrow more to smooth consumption, hence the shadow price of the collateral constraint is higher is periods of low income.

Comparing equations (47) and (55), we observe that when agents only have access to non-state contingent bonds, the pecuniary externality is given by the term:

$$\beta R \mathbb{E}_t \left\{ \xi_{t+1} \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right\} > 0$$

Similar to the results in Bianchi (2011) and Korinek (2010), in an environment with a single non-state contingent bond, private agents who ignore the effect of individual borrowing on the valuation of collateral of other agents borrow more than socially optimal and therefore face financial crises more frequently.

$$U_1(c_t^{CE}) < U_1(c_t^{SP})$$

$$c_t^{CE} > c_t^{SP} \rightarrow -b_{t+1}^{CE} > -b_{t+1}^{SP} \tag{59}$$

When we add access to debt with state contingent interest payments, the pecuniary externality term becomes

$$\beta \mathbb{E}_t \left\{ R_{t+1}^y \xi_{t+1} \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right\} > 0 \tag{60}$$

Similarly, private borrowers ignore the effect of their individual borrowing on the valuation of collateral of other agents. Higher individual debt leads to a higher probability of being financially constrained next period, which results in inability to rollover debt and leads to capital outflows and real depreciation. This in turn lowers the value of collateral for other agents in the economy and hence leads to larger capital outflows.

However, by comparing the two pecuniary externality terms, we can see that the feedback effect through the valuation of collateral is milder when there is access to $\hat{\mathrm{b}}$. Higher individual debt leads to a higher probability of being financially constrained next period if borrowers get a low income shock. However, it is less likely to be financially constrained if the interest payment is procyclical, as a low income shock is partially offset by the lower debt service, so that there is more space for debt rollover.

This in turn makes the capital ouflows and the real depreciation milder, which is given by the term:

$$cov\left(y_{t+1}, \xi_{t+1}\kappa\frac{\partial p^N_{t+1}}{\partial c_{Tt+1}}y^N\right) < 0 \tag{61}$$