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How Do Terms of Trade Affect Productivity? The Role of Monopolistic Output Markets

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Update

Abstract

This paper analyzes how terms of trade affect aggregate productivity using a two-country monopolistic competitive business cycle model driven by aggregate technology shocks. The inefficiency of the equilibrium implies that each country's productivity is affected by the terms of trade. This introduces a novel mechanism for business cycle synchronization. Moreover, for each country, *foreign* technology shocks have almost the same effects as *domestic* technology shocks. The paper also shows how terms of trade movements can lead to excess volatility of consumption and highly persistent productivity. On the quantitative side, the model delivers a degree of business cycle synchronization that is close to the actual comovement of the U.S. economy with the rest of the world. The model also implies that for some small open economies, specially emerging economies, foreign shocks can outperform domestic shocks in explaining their business cycles. Finally, the paper provides a quantification of the influence of the terms of trade on emerging countries' productivity and finds that it can be large.

JEL Classification: C67, E23, F12, F41, F43.

Keywords: Imperfect Competition, Input-Output Linkages, Terms of Trade, Business Cycles, Total Factor Productivity.

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1 Introduction

For some countries the data on terms of trade - defined as the ratio of import prices to export prices - and aggregate productivity are negatively correlated.[1] In other words, when the terms of trade deteriorate (i.e., increase) oftentimes productivity declines, suggesting that access to imports is crucial for productivity.[2] Moreover, the data shows signs of positive productivity spillovers across countries over the business cycle, which brings additional support to the idea that a country's low supply of exports reduces its trade patterns' productivities.[3] However, much of the standard theory on international real business cycles, initiated by Backus et. al. (1992), is silent on this evidence. First, according to this theory, a fall in a country's productivity leads to an improvement of its terms of trade. Second, the standard theory does not have any endogenous mechanism through which productivity spillovers occur. Third, as shown by Kehoe and Ruhl (2008), economic efficiency intrinsic to these models guarantees that terms of trade have no effect on productivity and thus have no role in explaining the cross-border productivity spillovers.

In this paper, I revisit the question of how terms of trade (TT) affect total factor productivity (TFP) using a two-country DSGE monopolistic competitive model. In the model, intermediate good producing firms located in each country use standard factors (capital and labor) in combination with intermediate goods (domestic or imported) and sell their output to both domestic and foreign markets. I adopt the view that output markets are monopolistic competitive as in Dixit and Stiglitz (1977). The presence of intermediate goods introduces input-output linkages as in Jones (2011). Production in each country is affected by an aggregate (Hicks-neutral) technology shock. These shocks are country-specific and independent, i.e., no spillovers. The rest of the decisions are in the hands of representative households who maximize expected discounted utilities subject to budget constraints.[4]

I find that in equilibrium *TFP in each country is not only a positive function of its technology shock, but also a negative function of its TT*, i.e., the ratio of import prices to export prices. Hence, once a country is hit by a negative technology shock, its TFP declines and its TT improve. For the other country, TT deteriorate and TFP declines. Therefore, the link between TT and TFP introduces a novel mechanism for business cycle synchronization that resembles cross-country productivity spillovers. The key behind this link is the inefficiency of the laissez-faire equilibrium. In particular, the monopolistic markup drives a positive wedge, i.e. inefficiency wedge, between the marginal product of imports and the marginal cost of imports, or TT. In equilibrium, this inefficiency wedge affects TFP.

To illustrate the above mechanism, consider the following example. Suppose that, for exogenous reasons, a country's TT deteriorate. In the model, firms respond by reducing the use of imported intermediates. The lower utilization of imports impairs production through two channels: a *direct channel*, as other factors of production are not perfect substitutes of imports, and an *indirect channel*, operating via input-output linkages. Due to the inefficiency wedge, the fall of final good production exceeds the fall of imports. As a result, the *difference* between the final good production and the real cost of imports, known

[1] See Backus et. al. (1992), Mendoza (1995), Kehoe and Ruhl (2008) and the evidence presented herein.

[2] Micro evidence supports this view, e.g., Amiti and Konings (2007) and Golberg et. al. (2010).

[3] See for example Backus et. al. (1992), Heathcote and Perri (2002) and more recently Rabanal et. al. (2011).

[4] I assume that households can only trade non-contingent risk-less bonds, i.e. asset markets are incomplete. There is also a final good producer assembling goods for consumption and investment using local intermediate goods.

as real gross domestic output, falls as well. This change in domestic output can occur even when the state of technology and the utilization of other factors of production (capital and labor) in the domestic economy remain constant. Therefore, the effects of TT spill over TFP. Explicit functional forms allow me to characterize analytically this effect. As expected, the effect is more important as the inefficiency wedge widens. When the inefficiency wedge vanishes, the effect of TT on TFP disappears. The size of the effect increases with import intensity and with the strength of input-output linkages.[5]

I also characterize the general equilibrium response of other macroeconomic variables and find that *foreign technology shocks can have almost the same effects as domestic technology shocks*. Yet, the effect of these shocks differ in three important dimensions. First, foreign and domestic shocks of the same sign imply opposite responses in TT as in the standard international real business cycle (IRBC) models. Specifically, expansionary technology shocks in one country make its exports relatively more abundant than its imports, lowering the price of exports relative to the price of imports. Second, the persistence of TFP (and other macroeconomic variables) is higher after a foreign shock than after a domestic shock, which is explained by the persistence of TT. In particular, in the model, asset market incompleteness implies that technology shocks lead to persistent wealth redistribution across countries, which induce persistent relative price movements.[6] Third, consumption tends to be more volatile than output after foreign shocks, while it is always less volatile than output after domestic shocks. For instance, after domestic disturbances, TT fluctuations attenuate the response of household's purchasing power relative to the response of output. In this case, TT provide insurance against production risk as in Cole and Obstfeld (1991). In contrast, after foreign disturbances, TT fluctuations exacerbate the response of household's purchasing power relative to the response of output.

I perform several quantitative exercises aimed at comparing the predictions of the model to the data. These quantitative exercises show that for plausible levels of markups, the effect of one country's productivity on the other country's productivity can be sizable. As a result, the model can deliver a degree of comovement that is in the order of magnitude of the actual comovement of the U.S. economy with the rest of the world. Then I explore the implications of the model for small open economies (SOE), both developed and emerging countries. A likelihood-based method, i.e., Kalman smoothing, suggests that, through the lens of the model, the observed TFP and business cycles in emerging countries were primarily the result of foreign shocks. This is because most emerging countries share the following features: highly persistent TFP, excess volatility of consumption, and negative correlation between TT and TFP.[7] This result is in stark contrast to developed SOE's, for which TFP and business cycles seem to be the result of domestic disturbances. I conclude then that TFP in emerging economies is highly influenced by TT.[8]

[5]Import intensity is determined by the share of imports on the cost of intermediate inputs. This captures the *direct effect* of imports on production. The strength of input-output linkages is approximately determined by the share of intermediate on the total cost of production. This captures the *indirect effect* of imports on production.

[6]At the same time, through general equilibrium, persistent relative movements induce persistent relative wealth dynamics. This is a very common feature in IRBC models under incomplete markets, e.g., Baxter and Crucini (1995) and Rabanal and Rubio-Ramirez (2012).

[7]The first two features are documented in Aguiar and Gopinath (2007). For the correlation between TT and TFP see the evidence presented in this paper.

[8]I also provide a quantification of the impact of TT on long-lasting TFP declines in emerging countries, which are difficult

Finally, I present micro-evidence showing that, as predicted by the model, high markups or high import intensity are associated with larger drops of TFP after a TT deterioration.

This paper bears a direct connection with large body of work trying to understand what causes TFP to move. One strand of the literature assesses the role of variable utilization of factors on measured TFP.[9] A second strand builds models in which TFP is endogenously determined, as in this paper. A common theme in these models is the existence of frictions or market imperfections that drive a wedge between market outcomes and efficiency.[10] The main difference with respect to this latter group is that I stress the role of monopolistic output markets and imported inputs in explaining the link between TT and TFP.

The mechanism highlighted in this paper, i.e., effect of TT on TFP, is reminiscent to Basu and Fernald (2002), who show that, in a closed economy context, intermediate input use affects productivity. An important difference is that, in my framework, the mere combination of intermediate inputs and monopolistic competition does not imply inefficient TFP dynamics. A necessary condition is that a fraction of inputs is imported. Furthermore, by using explicit functional forms in a fully general equilibrium model, I provide a characterization of the magnitude of the inefficiency. This allows me to set in the stage for a quantitative evaluation of the mechanism. A recent parallel and independent work by Gopinath and Neiman (2012) uses the same mechanism to explain why TFP responds to TT in a partial equilibrium model. In addition to that, these authors show how the variation in the number of imported varieties provides additional channels through which TT affect TFP. Importantly, the bulk of TFP adjustment reported in their paper comes from the mechanism stressed here. Furthermore, while these authors focus on how the micro-adjustment of trade affects TFP after an exogenous TT change, I focus on a different set of questions that involve the general equilibrium macro-adjustment after technology shocks, which endogenously generate TT variation.

Another important reference is Kehoe and Ruhl (2008), who show that, in standard macro models, exogenous TT shocks do not have first-order effects on TFP. Their result can be understood using an envelope argument. Under perfect competition, profit maximizing behavior also guarantees the maximization of aggregate domestic output and welfare. As a consequence, changes in relative prices can only have second order effects on TFP. My work indicates that the envelope argument breaks down when monopolistic firms import intermediate goods. In fact, Kehoe and Ruhl's result is nested in my framework as the knife-edge case where firms' output is perfectly substitutable.

The paper also contributes to a branch of the literature studying the transmission of shocks across countries using different variants of the IRBC model.[11] In these models, TT play a crucial role in the propagation of business cycles via a trade channel. The novelty with respect to this literature is that

to reconcile with the theory, e.g., Kehoe and Prescott (2002). I find that TT deteriorations, through the lenses of the model, contribute to up to 50 percent of the average long-lasting TFP decline.

[9]See King and Rebelo (1999) for a summary of this literature. Variable utilization can also explain why TT affects TFP, see Backus and Crucini (2000).

[10]The role of imperfect competition in productivity is studied in Hall (1990), Basu (1995), Basu and Fernald (2002), Jaimovich and Floetotto (2008), and others. The role of misallocation (due to tax distortions or regulations) in the RBC tradition is discussed in Chari et. al. (2007).

[11]An inexhaustive list includes Backus et. al. (1992), Backus et. al. (1993), Backus and Crucini (2000), Mendoza (1995), Heathcote and Perri (2002), Burstein et al. (2008) and others.

monopolistic competition introduces another channel of business cycle synchronization which operates through productivity. This channel provides an explanation for the productivity spillovers found in the data, which have been traditionally added into IRBC models as part of the exogenous characteristics of technology. In addition, in this paper I show that interdependencies in production that arise from the input-output linkages enhance business cycle comovement through the trade-channel and productivity-channel. The role of production complementarities in explaining comovement through the trade channel has been also explored in the literature, e.g., Backus et. al. (1993), Burstein et. al. (2008) and others.

Finally, the paper also offers an alternative explanation to some distinctive features of emerging economies. First, the paper shows how adverse foreign shocks can lead to highly persistent fluctuations of TFP, as it is observed in many emerging countries, see Aguiar and Gopinath (2007). Previous literature has related this feature of TFP to permanent distortions to the allocative efficiency of the economy, e.g. Chari et. al. (2007). Second, the paper shows how TT can explain the excess volatility of consumption relative to the volatility of output. Previous work highlights the role of TT in providing insurance against production risk, e.g. Cole and Obstfeld (1991). My results suggests that this mechanism is absent for some countries, especially emerging countries. This finding concurs with Berka et. al. (2012). Other literature has related the excess volatility of consumption to financial shocks, e.g., Neumeyer and Perri (2005), or to persistent income shocks, e.g., Aguiar and Gopinath (2007). Finally, the paper also contributes to a large body of literature documenting the relationship between TT and the economic performance of emerging markets.[12] This literature shows that TT are statistically important for business cycles and growth. This paper documents that TT can also be important for TFP.

The rest of the paper is organized as follows. Section 2 presents empirical patterns showing that for some countries TT and TFP are strongly negatively correlated. Section 3 outlines the model and characterizes the equilibrium. Section 4 presents the quantitative analysis for both large and small open economies. Section 5 discusses some extensions to the model and provides supporting micro evidence. Section 6 concludes.

2 Terms of trade and TFP

This section quantifies the link between TT and TFP in the data. I find that for some countries the data suggest that TT and TFP are negatively associated. This pattern is more common among emerging countries than among developed countries.

I consider a sample of countries that consists of a list of non-oil exporters, no transition, middle- or high-income countries. These countries are listed in Table B.1 in the Appendix B. I split the sample between large countries (G6), developed SOEs (other OECD), and emerging SOEs. Following Backus et. al. (1992) (hereafter BKK), TT are defined as the ratio of import prices to export prices. TFP is calculated as the SR: $TFP_t = Y_t / \left(K_{t-1}^{\alpha} L_t^{1-\alpha} \right)$, where Y_t is real GDP, K_{t-1} is the beginning of period t capital stock, and L_t is the labor input. Detailed information about these time series is given in the data appendix.

[12] See Easterly et. al (1993), Reinhart and Wickham (1994), Becker and Mauro (2005), and Izquierdo et. al. (2008). Williamsom (2011) provides an historical account of the subject.

Table B.1 reports a correlation between TT and TFP by country.[13] The correlation coefficients is reported for two sub periods, 1960-1979 and 1980-2008, and the whole sample.[14] For some countries, e.g., Switzerland, the data on TT and TFP are positively correlated (0.60). In other cases, such as Mexico, TT and TFP are strongly negatively correlated (-0.8). The rest of the countries fall in the middle. Importantly, for almost two thirds of the countries, the correlation is negative. Note also that emerging countries stand out as the group in which TT and TFP are more negatively correlated, especially after 1980.

The issue is that much of the standard IRBC models, initiated by BKK (1992), are silent on the negative correlation between TT and TFP. According to these models a fall in a country's productivity leads to an improvement of its TT, just the opposite of what we see for a large set of countries. Another possibility is that a change in one country's productivity spills over to its trade patterns' productivity. Indeed, there is strong evidence showing that aggregate productivity correlates across countries over the business cycle.[15] Yet, the standard IRBC theory does not provide a mechanism through which that productivity synchronization arises endogenously. Finally, as shown by Kehoe and Ruhl (2008), economic efficiency intrinsic to IRBC models guarantees that TT have no first order effects on TFP and hence TT have no role in explaining productivity spillovers.[16]

Section 3 outlines a two-country monopolistic competitive model. In this model, a negative foreign technology shock leads to an adverse TT for the domestic economy as in the standard IRBC framework. Importantly, the inefficiency of the laisse-faire equilibrium implies that these adverse TT reduce domestic TFP. The link between TT and TFP introduces a novel mechanism for business cycle synchronization that resembles the cross-border productivity spillovers found in the data. Section 4 explore the quantitative properties of the model. In the first part of the analysis, I show that a calibrated version of the model can deliver a degree of comovement that is in the order of magnitude of the actual comovement of the U.S. economy with the rest of the world. In the second part of the analysis, I study the implications of the calibrated model for small open economies, both developed and emerging. The results indicate that the business cycles in emerging economies are more likely to be driven by foreign shocks.

3 The model

In this section, I outline the model and characterize its equilibrium. The world consists of two countries (domestic and foreign), each of which is populated by one representative household, a final good producer, and a continuum of intermediate good producers (hereafter firms). There is a total measure $[0, 1]$ of firms in the world; a measure $[0, n]$ of them is located in the domestic country and a measure $[n, 1]$ in the foreign country. Hereafter, n indexes the size of the domestic country and $1 - n$ indexes the size of the

[13] As it common in the literature, I recover the business cycle component by applying the Hodrick-Prescott filter with a smoothing parameter of 100.

[14] This is done to avoid the influence of possible structural breaks. Backus and Crucini (2000) shows substantial changes in the cyclicality of TT in developed countries after the post-war period. Aguiar and Gopinath (2007) points out that some important stylized facts about emerging-market economies in the last decades are not present before 1980.

[15] See BKK (1992), Heathcote and Perri (2002), Crucini et. al. (2011), and Rabanal et. al. (2011).

[16] Measurement error can explain the link between TT and TFP as in Backus and Crucini (2000) or Feenstra et. al. (2009). Abstracting from measurement error, TT do not have any effect on TFP in the standard IRBC framework.

foreign country.

The representative household is standard; she consumes, invests on physical capital, supplies labor and holds a portfolio of risk-less non-contingent bonds. The final good producer is also standard; it assembles a final good using intermediate inputs produced by local firms. The final good is consumed or invested locally in new physical capital. The final good producer behaves competitively in both output and input markets. Firms produce one intermediate input using a production function that requires labor, capital, other local intermediate inputs and imported intermediate inputs. Production by firms is affected by a country-specific (Hicks-neutral) aggregate technology shock. Shocks are assumed to be independent across countries. Firms sell their output in monopolistic competitive markets as in Dixit and Stiglitz (1977). Firm's demand comes from the local final good producer, from other local firms and from firms abroad. This structure introduces input-output linkages into the model.[17]

All international trade in the model occurs at the level of intermediate goods. This is consistent with evidence showing that the bulk of international trade is concentrated in those goods, e.g., Bems et. al. (2011). Moreover, this production structure also captures the idea that even final good imports go under a series of processes that involve local factors (e.g. repackaging, transportation, retailing) before reaching their final demand, e.g., Burstein et. al. (2000).

Given the symmetry of both countries, I next provide details of the problems solved by the agents in the domestic country. Hereafter, variables for the foreign economy are indexed by the superscript $*$. Unless otherwise noted, parameters and functional forms are assumed to be the same across countries.

Representative Household Each representative household derives utility from consumption C_t and dis-utility from labor L_t. Period utility is assumed to be CRRA and quasi-linear in consumption as in Greenwood et. al. (1988),

$$U(C_t, L_t) = \frac{(C_t - \psi L_t^{\upsilon})^{1-\sigma} - 1}{1-\sigma},$$

where $\sigma \in [0, \infty)$ is the coefficient of risk aversion; $\upsilon > 1$ is a parameter controlling the Frisch elasticity of labor supply, i.e. $1/(\upsilon - 1)$; ψ is a scaling factor. The objective of the household is to maximize expected discounted utility,

$$\max_{\{C_t, L_t, B_{*t}, K_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - \psi L_t^{\upsilon})^{1-\sigma} - 1}{1-\sigma},$$

where $\beta \in (0, 1)$ is the discount factor, subject to a budget constraint and capital accumulation equation:

$$C_t + I_t + B_t + \frac{P_t^*}{P_t} B_{*t} + \frac{\kappa}{2}\left(B_t^2 + \frac{P_t^*}{P_t} B_{*t}^2\right) = \frac{w_t}{P_t} L_t + \frac{r_t}{P_t} K_{t-1} + \frac{\Pi_t}{P_t} + R_{t-1} B_{t-1} + \frac{P_t^*}{P_t} R_{t-1}^* B_{*t-1} + T_t \quad (1)$$

$$\begin{aligned} I_t &= K_t - (1-\delta) K_{t-1} + \left(\frac{\phi}{2}\right)\left(\frac{K_t}{K_{t-1}} - 1\right)^2 K_{t-1} \quad (2) \\ & K_{-1}, B_{-1} \quad \text{given.} \end{aligned}$$

[17] Specifically, in the model there are input-output linkages within a country, as in Basu (1995) and Jones (2011), and input-output linkages between countries, as in Burstein et al. (2008) and Bems et. al. (2011).

The household buys one unit of consumption C_t at a price P_t. At the same price, she invests in new physical capital I_t, or saves in one-period non-contingent risk-less bond, B_t which pays R_t (gross real interest rate). In addition, she has access to another non-contingent risk-less bond, B_{*t}, which pays R_t^* in units of the foreign final good. The price of the final good abroad is denoted by P_t^*. The household also pays portfolio quadratic costs for her bonds holdings.[18] These portfolio adjustment costs are rebated back to the household via a lump sum transfer T_t. The household receives a real wage rate w_t/P_t for every unit of labor supplied to the market and receives a real rental rent r_t/P_t for every unit of physical capital supplied to the market. She also receives a lump sum transfer of aggregate profits of local firms indexed by $i \in [0, n]$, i.e. $\Pi_t \equiv \int_0^n \Pi_t(i)\, di$. The capital accumulation includes a capital adjustment cost which modulates investment volatility.[19] Parameter $\delta \in (0, 1)$ is the depreciation rate. Parameters κ and ϕ are positive real numbers controlling the portfolio and capital adjustment costs, respectively. Optimality conditions are omitted since they are standard. See Appendix A.

Final good producer The final good is produced using local intermediate inputs $g_t(i)$ with $i \in [0, n]$. The production technology is a Dixit-Stiglitz aggregator with constant returns to scale,

$$G_t = \left(\left(\frac{1}{n} \right)^{1-\theta} \int_0^n g_t(i)^\theta \, di \right)^{\frac{1}{\theta}}, \tag{3}$$

where $\theta \in (0, 1)$ controls the elasticity of substitution among intermediate goods, i.e. $\frac{1}{1-\theta}$. As $\theta \to 1$, intermediate inputs are perfect substitutes. The final good is consumed or used in the formation of new physical capital, i.e. $G_t = C_t + I_t$. The final good producer maximizes profits,

$$\max P_t G_t - \int_0^n p_t(i)\, g_t(i)\, di,$$

subject to (3) and taking all prices of local intermediate inputs $p_t(i)$ and the final good price as given:

$$P_t = \left(\left(\frac{1}{n} \right) \int_0^n p_t(i)^{\frac{\theta}{\theta-1}} \, di \right)^{\frac{\theta-1}{\theta}}.$$

Optimality conditions are omitted since they are standard. See Appendix A.

Intermediate good producers (firms) The production function of firm $i \in [0, n]$ is,

$$q_t(i) = A_t(i) \left(k_t(i)^\alpha\, l_t(i)^{1-\alpha} \right)^{1-\mu} \left(d_t(i)^\gamma\, m_t(i)^{1-\gamma} \right)^\mu, \tag{4}$$

where $q_t(i)$ is output, $k_t(i)$ is the rented capital, $l_t(i)$ is the hired labor, $d_t(i)$ is a composite of local intermediate goods, and $m_t(i)$ is a composite of imported intermediate goods. Hereafter I refer to $q_t(i)$ as *gross* output. $A_t(i)$ represents firm i technology level which is composed of a static technology level

[18] This guarantees stationarity, see Uribe and Schmidtt-Grohe (2003).

[19] These adjustment costs are necessary because the household has access to foreign capital markets. This allows her to separate her savings decisions from her investment decisions by financing any gap between the two with external resources. As a result, investment is too volatile in the standard frictionless model. See Mendoza (1991) for a discussion.

$\bar{A}(i)$ and aggregate technology level A_t, i.e. $A_t(i) = \bar{A}(i) A_t$. The aggregate technology shock follows a logarithmic AR(1) process,

$$\log A_{t+1} = \rho_a \log A_t + \sigma_a \epsilon_{a,t+1}, \tag{5}$$

with $\epsilon_{a,t+1} \sim N(0,1)$, $0 < \rho_a < 1$ and $\sigma_a > 0$. Foreign technology shock is denoted by A_t^* and also follows a logarithmic AR(1) process with $\epsilon_{a,t+1}^* \sim N(0,1)$, $0 < \rho_a^* < 1$ and $\sigma_a^* > 0$. Innovations $\epsilon_{a,t+1}$ and $\epsilon_{a,t+1}^*$ are assumed to be independent, i.e. no spillovers.

Parameters α and μ lie in the unit interval and are the same in both countries. It is assumed that γ is a function of the relative size of the other country: $1-\gamma = (1-n)\lambda$, where parameter λ represents the degree of openness. Foreign firms' production function is analogous to (4) except that $1-\gamma^* = n\lambda$. This specification is used later when I analyze the limiting small open economy, i.e. $n \to 0$. See De Paoli (2009) for a similar application.

Domestic intermediate input $d_t(i)$ is a Dixit-Stiglitz aggregator of all local intermediate goods,

$$d_t(i) = \left(\left(\frac{1}{n}\right)^{1-\theta} \int_0^n d_t(i,j)^\theta \, dj\right)^{\frac{1}{\theta}}, \tag{6}$$

where $d_t(i,j)$ is the domestic intermediate used by firm i and produced by firm $j \in [0,n]$. Parameter $\theta \in (0,1)$ controls the elasticity of substitution among local intermediate goods, i.e. $\frac{1}{1-\theta}$. Imported intermediate input $m_t(i)$ is a Dixit-Stiglitz aggregator of all foreign intermediate goods,

$$m_t(i) = \left(\left(\frac{1}{1-n}\right)^{1-\theta} \int_n^1 m_t(i,j)^\theta \, dj\right)^{\frac{1}{\theta}}, \tag{7}$$

where $m_t(i,j)$ is the demand of firm i for the intermediate good produced by foreign firm $j \in [n,1]$. Parameter $\theta \in (0,1)$ controls the elasticity of substitution among local intermediate goods, i.e. $\frac{1}{1-\theta}$.[20]

Firm i sells its output to the final good sector, other local firms and foreign firms. The total demand faced by firm i is given by,

$$q_t(i) = g_t(i) + \int_0^n d_t(j,i) \, dj + \int_n^1 m_t^*(j,i) \, dj, \tag{8}$$

where $g_t(i)$ is the demand of firm i output used in the production of the final good, $\int_0^n d_t(j,i)\,dj$ is the aggregate demand of firm i output used by other firms in the same country, and $\int_n^1 m_t^*(j,i)\,dj$ are the exports of firm i to all firms $j \in [n,1]$ abroad.

Firm i maximizes profits,

$$\begin{aligned} \Pi_t(i) &= \max_{\varkappa_t(i)} p_t(i) q_t(i) - r_t k_t(i) - w_t l_t(i) - \int_0^n p_t(j) d_t(i,j) \, dj - \int_n^1 p_t^*(j) m_t(i,j) \, dj, \\ \text{with} &: \varkappa_t(i) \equiv \{p_t(i), q_t(i), k_t(i), l_t(i), d_t(i), d_t(i,j), m_t(i,j)\} \end{aligned} \tag{9}$$

subject to (4), (6) and (8). Intermediate producer takes the wage rate w_t, the rental rate r_t, the price of local intermediate inputs $p_t(j)$ with $j \in [0,n]$, and the price foreign intermediate inputs $p_t^*(j)$ with $j \in [n,1]$ as given.

[20] Note that θ governs the elasticity of substitution in (3), (6) and (7) is the same. This assumption simplifies the analysis.

The problem can be solved in two stages. The first stage minimizes costs, given factor prices. This provides the optimal mix of factors. The second stage is the standard pricing decision under monopolistic competition. See Appendix A for details. Before finishing this section, I emphasize that, because firms sell their products at a monopolistic price (above marginal cost), the *marginal product of all factors of production are set above their corresponding real prices*. Specifically,

$$(1-\mu)\,\alpha\frac{q_t(i)}{k_t(i)} = \frac{1}{\theta}\frac{r_t}{p_t(i)} \tag{10}$$

$$(1-\mu)(1-\alpha)\frac{q_t(i)}{l_t(i)} = \frac{1}{\theta}\frac{w_t}{p_t(i)} \tag{11}$$

$$\mu\gamma\frac{q_t(i)}{d_t(i)} = \frac{1}{\theta}\frac{P_t}{p_t(i)} \tag{12}$$

$$\mu(1-\gamma)\frac{q_t(i)}{m_t(i)} = \frac{1}{\theta}\frac{P_t^*}{p_t(i)}. \tag{13}$$

Definition of equilibrium Given $\bar{A}(i)\ \forall\, i \in [0,1]$ and the sequences of aggregate technology shocks A_t and A_t^*, the equilibrium is defined by:
(i) a sequence of allocations $\{C_t, K_t, B_t, B_{*t}, L_t\}$ and $\{C_t^*, K_t^*, B_t^*, B_{*t}^*, L_t^*\}$ for each household,
(ii) a sequence of allocations $\{G_t, g_t(i)\}$ and $\{G_t^*, g_t^*(i)\}$ for each final good producer,
(iii) a sequence of allocations $\{k_t(i), l_t(i), d_t(i,j), m_t(i,j), q_t(i)\}$, and prices $p_t(i)$ for all domestic firms $i \in [0,n]$ and $\{k_t^*(i), l_t^*(i), d_t^*(i,j), m_t^*(i,j), q_t^*(i)\}$, and prices $p_t^*(i)$ for all foreign firms $i \in [n,1]$,
(iv) a sequence of prices $\{P_t, w_t, r_t, R_t, P_t^*, w_t^*, r_t^*, R_t^*\}$
such that:
(a) given (iv), (i) solves the problem of each household,
(b) given (iv), (ii) solves the problem of each final good producer,
(c) given (iv), (iii) solves the problem of all firms $i \in [0,1]$,
(d) markets clear:

$$L_t = \int_0^n l_t(i)\,di, \qquad L_t^* = \int_n^1 l_t^*(i)\,di \tag{14}$$

$$K_{t-1} = \int_0^n k_t(i)\,di, \qquad K_{t-1}^* = \int_n^1 k_t^*(i)\,di \tag{15}$$

$$q_t(i) = g_t(i) + \int_0^n d_t(j,i)\,dj + \int_n^1 m_t^*(j,i)\,dj \qquad \forall i \in [0,n] \tag{16}$$

$$q_t^*(i) = g_t^*(i) + \int_n^1 d_t^*(j,i)\,dj + \int_0^n m_t(j,i)\,dj \qquad \forall i \in [n,1] \tag{17}$$

$$G_t = C_t + I_t, \qquad G_t^* = C_t^* + I_t^* \tag{18}$$

$$B_t + B_t^* = 0 \tag{19}$$

$$B_t^* + B_{*t}^* = 0. \tag{20}$$

Without loss of generality, I focus hereafter on a symmetric equilibrium where the firm-specific technology is equalized across firms within each country, i.e., $\bar{A}(i) = \bar{A}\ \forall\, i \in [0,n]$ and $\bar{A}^*(i) = \bar{A}^*\ \forall i \in$

$[n, 1]$.[21] In this symmetric equilibrium, $P_t = p_t(i)$, $Q_t = nq_t(i)$, $K_{t-1} = nk_t(i)$, $L_t = nl_t(i)$, $D_t = nd_t(i)$, $M_t = nm_t(i)$, $G_t = ng_t(i)$ for all domestic firms and $P_t^* = p_t^*(i)$, $Q_t^* = (1-n)\, q_t^*(i)$, $K_{t-1}^* = (1-n)\, k_t^*(i)$, $L_t^* = (1-n)\, l_t^*(i)$, $D_t^* = (1-n)\, d_t^*(i)$, $M_t^* = (1-n)\, m_t^*(i)$, $G_t^* = (1-n)\, g_t^*(i)$ for all foreign firms.

Additional definitions For each country I define the terms of trade as the ratio between the price of imports and the price of exports. For the domestic country,

$$TT_t \equiv \frac{P_t^*}{P_t}, \tag{21}$$

and for the foreign country the terms of trade are TT_t^{-1}.

For some results it is going to be useful to define a household's *real gross domestic income* (hereafter income) as the sum of labor income, capital income, and profits generated by firms in units of the final consumption good. For the domestic country, income is given by (for the foreign country the formula is analogous),

$$Z_t \equiv \frac{\Pi_t + r_t K_{t-1} + w_t L_t}{P_t}. \tag{22}$$

I denote aggregate gross output by Q_t. This measure not only includes output that satisfies final good demand, but also intermediate good demand by other local firms, see equation (16). Hence, gross output of *final goods* is gross output minus local intermediate good demand, i.e. $Q_t - D_t$. From (16) and (18), gross output of final goods equals the sum of consumption, investment and exports, which in the model is given by $G_t + M_t^*$.

Aggregate output is given by the standard real gross domestic product (GDP) formula. From the expenditure approach, nominal GDP equals the value of gross output of final goods minus the cost of imports.[22] Real GDP follows the same formulation, except that prices are constant. In the domestic country, real GDP is given by the formula,

$$Y_t = P_b G_t + P_b M_t^* - P_b^* M_t, \tag{23}$$

where b stands for "base" year, M_t^* denotes domestic exports (foreign imports), which are valued at a constant price P_b, and M_t denotes domestic imports (foreign exports), which are valued at a constant price P_b^*. The definition of real GDP in the foreign country is analogous,

$$Y_t^* = P_b^* G_t^* + P_b^* M_t - P_b M_t^*. \tag{24}$$

[21] Allocative efficiency of the model guarantees that, at the aggregate level, a symmetric equilibrium is isomorphic to an equilibrium with dispersed idiosyncratic productivities.

[22] The inclusion of imports in GDP as an offsetting entry deserves further explanation. From the point of view of the model, all imports are intermediate goods. This implies that imports are included in consumption, investment and exports. Therefore, to accurately reflect *domestic* final good production, imports of intermediates are subtracted from GDP to offset the contribution of foreign production in the final expenditures components. Standard practices for computing real GDP are detailed in "System of National Accounts 2008," published jointly by the European Commission, the IMF, the OECD, the United Nations and the World Bank, and "Concepts and Methods of the U.S. National Income and Product Accounts," published by the Bureau of Economic Analysis.

Partial characterization The goal of this section is to characterize, at least partially, some properties of the equilibrium. The main result of the paper is given in Proposition 1 which shows how TT affect TFP. In particular, this proposition implies that a fall in foreign technology, which leads to a TT deterioration for the domestic country, also depresses domestic TFP. This mechanism generates synchronization of business cycles across countries. A more general message of this section is that foreign shocks can have almost the same effects as domestic technology shocks. The differences between these two shocks are also discussed.

In the model, TT are endogenous. Following the same intuition of standard IRBC models a rise in the domestic technology deteriorates the domestic TT as domestic goods become relatively more abundant. Conversely, a rise in the foreign technology improves the domestic TT. I focus hereafter on a deterioration of TT that originates from a negative foreign technology shock while domestic technology is constant. Moreover, throughout the following discussion I neglect the general equilibrium feedback between capital, labor and TT.

From (4) , aggregate gross output at the symmetric equilibrium is,

$$Q_t = A_t \bar{A} \left(K_{t-1}^{\alpha} L_t^{1-\alpha}\right)^{1-\mu} \left(D_t^{\gamma} M_t^{1-\gamma}\right)^{\mu}. \tag{25}$$

It is relatively straightforward to show, after using the first order conditions from the firm's problem (evaluated at the symmetric equilibrium), that aggregate gross output can be rewritten as,

$$Q_t = \varpi TT_t^{-\frac{(1-\gamma)\mu}{1-\mu}} A_t^{\frac{1}{1-\mu}} K_{t-1}^{\alpha} L_t^{1-\alpha}, \tag{26}$$

where,

$$\varpi \equiv \left((1-\gamma)^{1-\gamma} \gamma^{\gamma}\right)^{\frac{\mu}{1-\mu}} (\mu\theta)^{\frac{\mu}{1-\mu}} \bar{A}^{\frac{1}{1-\mu}}. \tag{27}$$

Equation (26) shows three effects that are worth emphasizing.

First, the markup θ^{-1} depresses the level of aggregate gross output through the constant term ϖ. Intuitively, the markup generates an inefficiency by raising the marginal product of all intermediate inputs (both domestic and foreign) above their real prices. As a result, all firms underproduce.

Second, aggregate technology shocks are scaled up by $1/(1-\mu)$. This is the input multiplier arising from the input-output linkages. Specifically, this multiplier works as follows: higher technology leads to more production for all firms, which increases the demand and supply of intermediate goods, which increases production for all firms, and so on. The elasticity of gross output to intermediate inputs is μ. Hence, the overall effect is $1 + \mu + \mu^2 + ... = 1/(1-\mu)$.[23]

Third, an adverse TT deterioration (an increase of TT), reduces aggregate gross output. The economic intuition for this effect is the following: After TT deteriorate, domestic firms utilize fewer imported inputs as they become relatively more expensive. To the extent that these intermediate inputs cannot be perfectly substituted with other factors, lower imported intermediates reduce production for all firms. In the first round, the reduction in production is given by the elasticity of gross output to imported intermediates,

[23] This formula reflects the simple architecture of the input-output matrix. In the model, each firm has the same number of downstream and upstream interconnections, i.e. each firm supplies inputs to every other firm and buys inputs from every other firm. See Jones (2011) for a similar result.

i.e., $\mu(1-\gamma)$. Hereafter, I refer to this effect as the *direct effect* of import intensity. This direct effect is then amplified in further rounds by the input multiplier.[24] Hereafter, I refer to this effect as the *indirect effect* of import intensity. The following expression is the elasticity of gross output to TT that takes into account these two effects,

$$\epsilon(Q,TT) = -\frac{(1-\gamma)\mu}{(1-\mu)} < 0. \tag{28}$$

I now focus on gross domestic output and productivity. Gross domestic output equals gross output of final goods minus imports, both valued at *constant* equilibrium prices. It is straightforward to show, after using the first order conditions from the firm's problem, that the two components of output are given by,

$$G_t + M_t^* = (1-\mu\gamma\theta)\,Q_t, \tag{29}$$

$$M_t = (1-\gamma)\,\mu\theta\,TT_t^{-1}Q_t. \tag{30}$$

The above equations indicate that both gross output of final goods and imports fall after a TT deterioration. Plugging (29) and (30) into (23), delivers a formula for output that depends on technology, TT, the stock of capital and labor. Appendix A shows that output is given by,

$$Y_t = TFP_t\,K_{t-1}^{\alpha}L_t^{1-\alpha}, \tag{31}$$

where TFP_t measures the total factor productivity of the country. Proposition 1 gives the details of the TFP function.

Proposition 1 *In the decentralized equilibrium, TFP is not only a positive function of technology, but also a negative function of the terms of trade,*

$$TFP_t = TT_t^{-\frac{(1-\gamma)\mu}{1-\mu}\frac{1-\theta}{1-\mu\theta}}A_t^{\frac{1}{1-\mu}}. \tag{32}$$

See Appendix A.

Proposition 1 states that in equilibrium, TT directly affects TFP in the domestic country.[25] The intuition for this result is very simple. Suppose that there are no domestic intermediate inputs so that gross final good production equals gross output, i.e. $G_t + M_t^* = Q_t$. For the sake of the argument, suppose also that production only requires labor and imports and that labor supply and technology are fixed, i.e. $Q_t = \bar{A}\bar{L}^{1-\mu}M_t^{\mu}$. With these assumptions, it follows that up to a first order approximation,[26]

$$\frac{\partial Y_t}{\partial TT_t} \approx \left(\frac{\partial Q_t}{\partial M_t} - TT_t\right)\frac{\partial M_t}{\partial TT_t}.$$

[24]Note in the model, all local firms' production depend equally on imported inputs. This implies that the foreign country plays the role of a general-purpose technology in the language of Acemoglu et. al. (2011). In this sense, TT fluctuations that originate abroad can be interpreted as shocks to this general-purpose technology. In the domestic economy the amplification occurs downstream, as all local firms are interconnected to each other. The response of the home country will then affect the foreign economy through the supply of intermediate inputs, initiating another set of feedback loops through the TT. All these general equilibrium effects would affect the way that the relative prices and quantities respond.

[25]The formula is analogous for the foreign country, except that γ is replaced γ^* and TT are inverted.

[26]Assuming that base year relative prices and current equilibrium relative prices are arbitrarily close, i.e., $|TT_b - TT_t| < \epsilon$.

The key behind the effect of TT on TFP is the term in parenthesis, which is the difference between the *marginal product of imports and TT*. Markups drive an inefficiency wedge between these two components, see equation (13). As a result, real gross domestic product changes even if domestic technology and domestic factors are constant. It follows then that the inefficiency wedge spills over TFP. When the inefficiency wedge disappears, the marginal product of imports exactly cancels out with TT, and the term in parenthesis collapses to zero. The latter is the case of perfect competition, which is analyzed by Kehoe and Ruhl (2008). Intuitively, under perfect competition, firms' profit maximizing behavior also guarantees the maximization of aggregate output. As a consequence, changes in relative prices can only have second order effects on TFP. This envelope argument breaks down when monopolistic competitive firms import intermediate goods because output is not maximized.

Going back to the full model, Proposition 1 indicates that the effect of TT on domestic TFP is given by the elasticity,

$$\epsilon\left(TFP, TT\right) = -\frac{\left(1-\gamma\right)\mu}{\left(1-\mu\right)}\left[\frac{1-\theta}{1-\mu\theta}\right]. \tag{33}$$

The economic intuition of this elasticity is the following: The first term captures the direct and indirect effects of imports as explained above. The term in brackets captures the inefficiency wedge between the marginal contribution of imports to production and TT. This wedge arises in equilibrium due to a monopoly power, $\theta < 1$, and it is affected by $1/\left(1-\mu\theta\right)$. The latter comes from double-marginalization: firms exercising market power at successive vertical layers of the supply chain.[27] Intuitively, the elasticity contains all gains in production that arise from higher import utilization. These gains are left unexploited in equilibrium because each firm decides to underproduce. As a result of the inefficiency wedge, a change of TT affects TFP. As expected, the effect is more important as the inefficiency wedge widens. Only in the limiting case where the inefficiency wedge vanishes, $\theta \to 1$, the effect of TT on TFP disappears.[28] The size of the effect increases with import intensity, measured by $1-\gamma$, and with the strength of input-output linkages, measured by μ.

In the analysis, I have assumed that domestic capital and labor do not respond to TT. Yet, in general equilibrium domestic factors of production respond to TT and vice versa. The link between domestic factors and TT is partially characterized from the firms' first order conditions. Specifically, TT affects the real rental rate of capital and the real wage. Plugging these conditions into the household's optimality conditions, yields the following equations,[29]

$$-\frac{U_{L,t}}{U_{C,t}} = \theta\left(1-\mu\right)\left(1-\alpha\right)\frac{Q_t}{L_t} \tag{34}$$

$$1 = \beta\mathbb{E}_t\left\{\frac{U_{C,t+1}}{U_{C,t}}\left(\theta\left(1-\mu\right)\alpha\frac{Q_{t+1}}{K_t}+\left(1-\delta\right)\right)\right\}, \tag{35}$$

[27]See Rotemberg and Woodford (1993) for a similar interpretation of the monopoly inefficiency wedge with intermediate inputs.

[28]In this knife-edge case firms are unable to set prices above their marginal costs since all competing products are perfect substitutes. Hence, it is equivalent to case of perfect competition analyzed by Kehoe and Ruhl (2008).

[29]For purposes of exposition, the Euler equation omits the capital adjustment costs. See Appendix A for a complete characterization.

where $U_{L,t}$ and $U_{C,t}$ represent the marginal dis-utility of labor and the marginal utility of consumption, respectively.

The above equations show that the incentives for the domestic household to supply labor and capital to the market fall after a deterioration of TT. In other words, times of negative technology shocks abroad, are also times when the rewards of supplying capital and labor to the market at the domestic country fall. The final responses of capital and labor depend on other aspects such as the degree of risk aversion and the balance between income and substitution effects affecting the supply of labor. Regarding the latter, note that quasi-linear preferences in consumption eliminate the income effects. Hence, we should expect that domestic labor would unambiguously fall after a negative technology shock abroad.

Note that the model has two main channels for *business cycle synchronization*. The first channel operates through *trade*. In particular, when the foreign country experiences a contractionary technology shock, domestic TT deteriorate. The scarcity of imported inputs in the domestic country lead to less production, less investment and less employment. A similar force is present in standard IRBC models, e.g., Backus et. al. (1993), Heathcote and Perri (2002), and others. The second channel for business cycle synchronization operates through *productivity*. This channel is the main novelty of the model with respect to standard IRBC framework cited above.[30] Moreover, note that the productivity channel reinforces the standard trade channel. For instance, an exogenous TT deterioration not only reduces investment and employment, but also depresses productivity.

It is worth emphasizing the role of input-output linkages in explaining business cycle synchronization. In the model, input-output linkages enhance both channels via the indirect effect of import intensity. Intuitively, these indirect effects reinforce the production complementarity among firms in each country. In addition, as discussed earlier, the inefficiency wedge is amplified by double-marginalization. Input-output linkages also induce greater production complementarity among countries. In general equilibrium, greater production complementarity increases TT volatility as in the standard IRBC, e.g., Backus et. al. (1993).

To illustrate the role of the input-output linkages for TT dynamics, suppose that each household lives in financial autarky. In such a case, TT are determined by balanced trade, i.e., $P_t M_t^* = P_t^* M_t$. Plugging (25) into (30) yields an expression for imports as a function of technology, TT, capital stock and labor. Following the same steps for foreign imports (i.e., domestic exports), recalling that foreign TT are the inverse of TT_t, yields a similar expression. Plugging these expressions into the balance trade condition delivers,[31]

$$TT_t = \left(\frac{A_t}{A_t^*}\right)^{\frac{1}{1-\mu(1-\lambda)}} \left(\frac{(1-\gamma)\,\varpi}{(1-\gamma^*)\,\varpi^*}\left(\frac{K_{t-1}}{K_{t-1}^*}\right)^{\alpha}\left(\frac{L_t}{L_t^*}\right)^{1-\alpha}\right)^{\frac{1-\mu}{1-\mu(1-\lambda)}}.$$

This first element in parenthesis, i.e., ratio of technology levels, confirms the intuition given previously, i.e., a rise in a country's technology deteriorates its TT. The exponent of the ratio of technology levels captures to some extent the effects that the input-output linkages have on TT volatility. This exponent

[30]In the model, the productivity channel resembles the productivity spillovers found in the data. In contrast, in the tradition of the IRBC, these productivity spillovers have been incorporated as an exogenous characteristic of technology, see Backus et. al. (1992), Backus et. al. (1993), Heathcote and Perri (2002), Rabanal et. al. (2011), and others.

[31]Note that $1-\gamma = (1-n)\,\lambda$ and $1-\gamma^* = n\lambda$.

depends on two parameters: parameter μ, which determines the strength of the input-output linkages within a country, and parameter λ, which determines the strength of the input-output linkages between countries. Given that both μ and λ are below one, the combination of these parameters in the exponent delivers amplification. Note that input-output linkages within countries strengthen the amplification, while input-output linkages between countries weaken it. To finish this discussion, let me emphasize that, because households have access to international intertemporal trade, TT are not determined in equilibrium by the above equation.[32]

So far the characterization of the equilibrium has assumed that domestic technology is fixed. It is straightforward to show that the effects of domestic technology are quite standard. In fact, in the equations shown above, domestic technology A_t is multiplied by the inverse of TT_t. Therefore, *a negative domestic technology has almost the same effect as an adverse TT deterioration caused by a negative foreign technology shock*.

Despite their similarity, there are three important differences between domestic and foreign shocks. The *first major difference* is that, as discussed above, domestic and foreign technology shocks of the same sign imply opposite responses in TT.

The *second major difference* is the persistence of TFP. For instance, after foreign technology shock, TFP in the domestic economy is as persistent as TT. This means that domestic TFP can be highly persistent in this case because a transitory foreign shock leads to persistent wealth redistribution, which induces persistent relative price movements. This is the consequence of asset market incompleteness, see Baxter and Crucini (1993) and Rabanal and Rubio-Ramirez (2012). In contrast, after a domestic technology shock, domestic TT move in the opposite direction, and this reduces the persistence of TFP.

To uncover the relationship between the wealth distribution and TT, I look at the optimal savings choices by each household, determined by the following first order conditions (foc) for the domestic riskless bond,[33]

$$
\begin{aligned}
1+\kappa B_t &= \beta \mathbb{E}_t \left\{ \frac{U_{C\,t+1}}{U_{C,t}} R_t \right\}, \\
1+\kappa B_t^* &= \beta \mathbb{E}_t \left\{ \frac{U^*_{C\,t+1}}{U^*_{C,t}} \frac{TT_t}{TT_{t+1}} R_t \right\}.
\end{aligned}
$$

where the first equation is the foc of the domestic household and the second equation is the foc of the foreign household.

Given that the gross return of riskless bond R_t is the same for both households, the next condition expresses the optimal risk sharing across countries, log-linearized around the non-stochastic steady state,

$$
\widehat{tt}_t = \mathbb{E}_t \left[\widehat{tt}_{t+1} + (\widehat{u}_{C,t+1} - \widehat{u}_{C,t}) - \left(\widehat{u}^*_{C,t+1} - \widehat{u}^*_{C,t}\right) \right] + \kappa \left(\widehat{b}^*_t - \widehat{b}_t \right).
$$

[32] As it will discussed later, asset market incompleteness implies that TT volatility is partly affected by the wealth distribution accross countries. Yet, to some extent, the forces that affect TT volatility under financial autarky will also affect the short run TT volatility once international asset markets are open.

[33] In the model, there is a foreign riskless bond, which is denominated in units of the foreign good. In equilibrium, a no-arbitrage condition guarantees that this foreign bond provides the same risk sharing opportunities as the domestic bond.

where lower-case variables are log-deviations from steady-state values except for the bond holdings, $\hat{b}_t$ and $\hat{b}_t^*$, which denote deviations from steady state (this is because in steady state $B_t = B_t^* = 0$). Wealth distribution in period t is measured by the difference of the bond holdings, i.e., $\widehat{b}_t^* - \widehat{b}_t$. Iterating forward on the previous equation reveals,

$$\widehat{tt}_t = \mathbb{E}_t \sum_{i=0}^{\infty} \left\{ \left(\widehat{u}_{C,t+1+i} - \widehat{u}_{C,t+i} \right) - \left(\widehat{u}_{C,t+1+i}^* - \widehat{u}_{C,t+i}^* \right) + \kappa \left(\widehat{b}_{t+i}^* - \widehat{b}_{t+i} \right) \right\}.$$

Hence, TT in period t equals the sum of the expected present discounted value of the changes in marginal utilities of consumption and the expected present discounted value of the wealth distribution.[34] Consumption smoothing implies that most of the short and long run variability of TT is loaded into the wealth distribution rather than the expected changes of marginal utilities. For small values of the portfolio adjustment cost κ, changes in the wealth distribution become very persistent. Therefore, the model delivers persistent TT dynamics, which, through the productivity channel, shape the persistence of TFP.

The last major difference is the role of TT for consumption volatility. In the model, TT undo some of the domestic risk by moving in the same direction as domestic technology shocks. In this case, TT provides insurance against production risk as in Cole and Obstfeld (1991). In contrast, after foreign shocks, the response of TT exacerbates production risk. For example, after a negative foreign technology shock, TT deteriorate at the same time that output declines. This difference implies different consumption paths. To see why, I next compare how household's domestic real income, as defined by (22), responds vis-à-vis output, given by (31).

Plugging the firms' profits, equation (9), and firms' demand, equation (16), into equation (22), I have,

$$Z_t = G_t + M_t^* - TT_t M_t. \tag{36}$$

This equation is analogous to the expression for real gross domestic output, except that imports are valued at current TT. In other words, aside for the changes in quantities, income also responds directly to TT_t. Plugging (29) – (30) into equation (36), yields,

$$Z_t = (1 - \mu\theta) Q_t. \tag{37}$$

The ratio of income to output is given by

$$\frac{Z_t}{Y_t} = TT_t^{-\frac{(1-\gamma)\mu\theta}{1-\mu\theta}}.$$

Now suppose that the domestic country is hit by a negative (positive) domestic shock. In response, TT improve (deteriorate) and the ratio income to output increases (decreases). As a result, the domestic household enjoys a smoother consumption path. This insurance property is lost when shocks originate in the foreign country.

[34]Obviously this is only a partial characterization because in general equilibrium TT and the wealth distribution are jointly determined.

To summarize, in this model, monopolistic power implies that TT affect TFP. Importantly, this link between TT and TFP delivers a novel channel of business cycle synchronization that reinforces the standard trade channel. I have shown that both channels are affected by the strength of input-output linkages in the model. Overall, the model predicts that domestic technology shocks have almost the same effect as foreign technology shocks. Yet, I have highlighted three important differences between domestic and foreign technology shocks. The next section studies the numerical properties of the model.

4 Quantitative analysis

This section explores the quantitative properties of the model and confirms some of the points discussed in the previous section. I split this analysis into two parts. In the first part, I focus on the question of business cycle synchronization in large open economies. I find that the model generates a degree of international comovement that is close to the data. In the second part, I focus on small open economies (SOE) and the role of foreign shocks in their business cycles. I find that in some SOEs foreign shocks can outperform domestic shocks at explaining their business cycles.

I first briefly describe the calibration of the model and then proceed with the quantitative exercises.

Calibration

Some parameters and steady state conditions are set beforehand. The coefficient of relative risk aversion, σ, is set to 2, which is the typical value in the literature, e.g. Aguiar and Gopinath (2007). As it is standard in the RBC literature, I fix the discount factor β at 0.99 (quarterly frequency), the depreciation rate δ at 0.025, and the elasticity of output with respect to capital α at 0.36. Following Neumeyer and Perri (2005), the curvature of labor υ is set to 1.6. The preference parameter ψ is chosen to generate a labor input of $1/3$ in steady state. The steady state bond holdings are calibrated to zero. The parameter κ in the bond holding quadratic cost function is set to 10^{-5} to approximate the frictionless case and to guarantee stationarity.

A key parameter is the elasticity of substitution across competing products, $1/(1-\theta)$, which determines the markup, $1/\theta$. As a benchmark I take the estimates of the elasticity of substitution obtained for the U.S. economy, which the trade and industrial organization literatures locates between 3 and 10, e.g. Broda and Weinstein (2006) and Hendel and Nevo (2006). I consider three cases: (i) a *low* elasticity of 3 $(\theta = 2/3)$, which implies a markup of markup of 1.5, (ii) a *medium* elasticity of 5 ($\theta = 0.8$), which implies a markup of 1.25, and (iii) a *high* elasticity of 10 $(\theta = 0.9)$, which implies a markup of 1.11.

Given a value of θ, a subset of parameters is calibrated to match cross-section averages in the sample of countries listed in Table B.1. The elasticity of gross output to intermediate goods μ is chosen such that the intermediate good share (the ratio of cost of intermediate goods to the total value of gross output) in the model (denoted by $\mu\theta$) equals 0.50, which is very close to the actual cross-country average in the sample.[35] This target is useful given the relative stability of the intermediate good share, both across time

[35] Intermediate good share is approximately 0.50 for each group of countries listed in Table B.1. A similar value is used by Jones (2011).

Table 4.1: Baseline Calibration

Fixed parameters	Name	Value		
β	Discount factor	0.99		
α	Exponent of production function	0.36		
δ	Depreciation rate	0.025		
σ	Relative risk aversion	2.00		
υ	Labor curvature	1.60		
κ	Bond holding cost	1.00e-005		
Varying parameters	Name	Low	Mid	High
θ	Inverse markup	0.67	0.80	0.90
μ	Exponent of production function	0.75	0.63	0.56
ψ	Scale parameter	0.17	0.44	0.68

and across countries. Following Heathcote and Perri (2002), I set the value of γ to 0.85 to generate an import to GDP ratio in the model (denoted by $(1-\gamma)\,\mu\theta/\,(1-\mu\theta))$ of 0.15. This parameter will change when I look at small open economies.

I assume that the parameters governing the stochastic processes of technology shocks are symmetrical across countries. Following Heathcote and Perri (2002) I set ρ_a and ρ_a^* to 0.97. The standard deviation of the innovations σ_a and σ_a^* are set to 0.005. Different from the literature, I impose no cross-border technological spillovers. The parameter controlling the adjustment cost of capital is set ϕ to 0.08.

Two large economies I focus on two symmetrical economies, i.e. $n = 1/2$. I solve the model using a first-order log-linear approximation around a non-stochastic steady state. See Appendix A for the equations.

Impulse responses

Figure 1 and 2 plot the impulse response functions for all different elasticities considered in the benchmark calibration, i.e., high, mid and low. The shock is a one standard deviation, one period, unexpected positive shock to the domestic technology. All variables are presented as percentage deviations from their steady state values, except for net exports.

The responses of the model mimic those of the standard IRBC. A positive domestic technology shock provides strong incentives to increase the labor input. The complementarity between labor and capital, along with the persistence of technology, increases investment. As a consequence of higher technology and more capital and labor, output expands. The increase in technology also increases the supply of intermediate inputs, which, through the input-output linkages, amplify the initial domestic technology shock. As a result of the economic expansion, domestic TT deteriorate as domestic goods become relatively more abundant. Foreign country households buy domestic-country bonds to invest in the most productive capital.

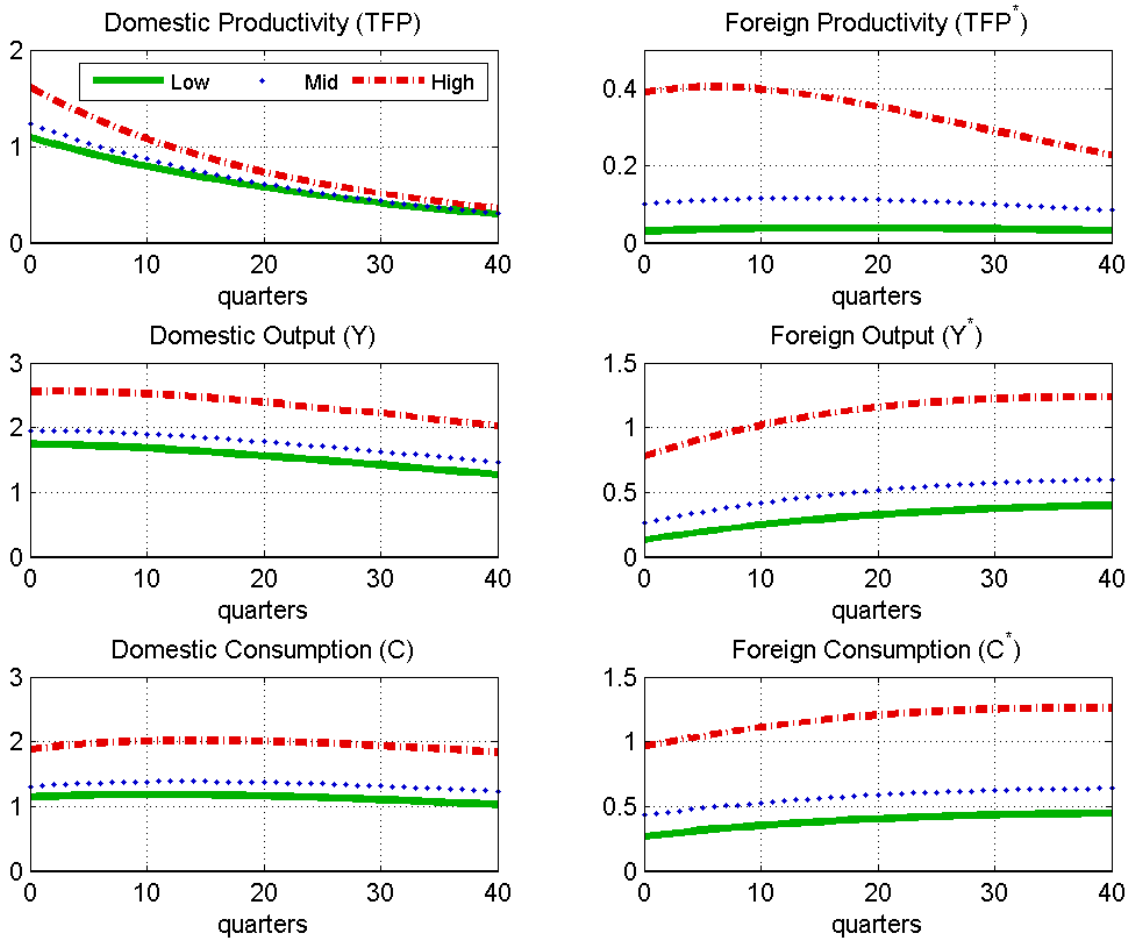

Figure 1: Impulse response functions after 1 S.D shock to domestic technology

With respect to the *international transmission of shocks*, the deterioration of domestic TT brings about an expansion in the other country. As explained earlier, this business cycle synchronization is the result of the trade and productivity channels. In the case of the trade channel, the relative abundance of imports, enable foreign firms to expand production. The complementarity of imports with other factors provide incentives for higher employment and higher investment. As for the productivity channel, access to relatively cheaper imports lifts the productivity of the foreign economy despite the fact that its technology remains fixed. The impulse responses show that this effect can be sizable, specially for low elasticities of substitution.[36] For instance, under the low elasticity, an increase of domestic TFP of 1.5 percent is accompanied by an increase of foreign TFP of about a 0.4 percent. This increase in foreign TFP represent approximately half of the increase in foreign output.

[36]This is also caused by the way the model is calibrated to match the intermediate good share in the data. In particular, the lower θ is, the larger is μ. This means that input-output linkages are stronger in the low elasticity case.

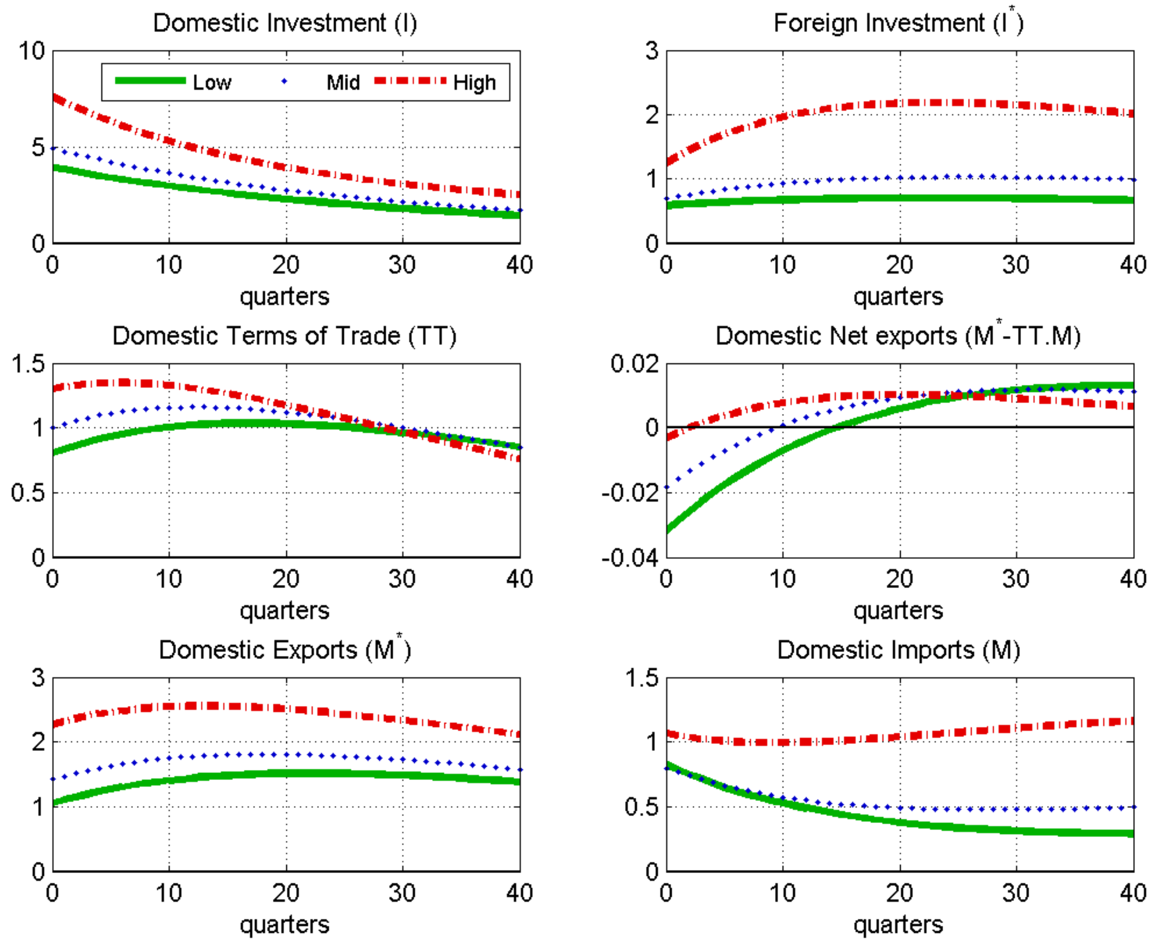

Figure 2: Impulse response functions after 1 S.D shock to domestic technology

The model delivers new implications that are worth mentioning. First, TFP is more persistent when shocks originate abroad. In this example, foreign TFP has barely changed in the first 20 quarters after the shock. In contrast, domestic TFP in the twentieth quarter is less than 50 percent of its initial level after the shock. As explained before, this quantitative implication is the result of the link between TT and wealth distribution in equilibrium. A second implication is the response of consumption relative to the response of output. In the domestic economy, consumption responds less forcefully than output. For example, on impact, consumption increases approximately 30 to 40 percent less than output. The opposite occurs in the foreign country. For example, in the low elasticity case, foreign consumption increases on impact approximately 20 percent more than output. For the high elasticity case, the response of foreign consumption is approximately twice the response of output. As discussed before, these differences are explained by the TT. Specifically, in the foreign economy, TT are insuring the household against the production risk. In contrast, in the foreign country, the response of TT exacerbates the production risk.

Table 4.2: Model versus Data

A. Volatility								
	$\sigma(Y)$	$\frac{\sigma(C)}{\sigma(Y)}$	$\frac{\sigma(I)}{\sigma(Y)}$	$\frac{\sigma(L)}{\sigma(Y)}$	$\sigma(M^*)$	$\sigma(M)$	$\sigma(NX)$	$\sigma(TT)$
U.S. Data	1.67	0.81	2.84	0.66	3.94	5.42	0.45	2.99
Standard IRBC	1.21	0.52	2.73	0.32	0.99	0.96	0.19	0.78
Model High	1.45	0.68	2.27	0.59	1.15	1.15	0.10	0.93
Model Mid	1.64	0.72	2.53	0.59	1.40	1.40	0.07	1.16
Model Low	2.23	0.83	2.90	0.61	2.12	2.13	0.03	1.50
B. Correlations with output								
	Y	C	I	L	M^*	M	NX	TT
U.S. Data	1.00	0.86	0.95	0.87	0.32	0.81	-0.49	-0.24
Standard IRBC	1.00	0.96	0.96	0.97	0.59	0.86	-0.65	0.65
Model High	1.00	0.98	0.98	0.99	0.75	0.60	-0.50	0.53
Model Mid	1.00	0.96	0.97	0.99	0.87	0.56	-0.44	0.48
Model Low	1.00	0.97	0.94	0.99	0.98	0.64	-0.15	0.36
C. Cross country correlations								
	Y	C	I	L	TFP^1			
U.S. Data	0.58	0.36	0.30	0.42	0.29			
Standard IRBC	0.17	0.68	-0.29	-0.17	0.29			
High	0.24	0.52	0.26	0.42	0.07			
Mid	0.37	0.65	0.31	0.56	0.20			
Low	0.65	0.83	0.45	0.79	0.53			

Note: Data and standard IRBC from Heathcote and Perri (2002). Statistics from the model are the averages of 100 simulations each 104 periods long. NX is the ratio of net exports to GDP (all at current prices).

[1] For data and standard IRBC, the value corresponds to the estimated correlation between technology innovations, see Heathcote and Perri (2002).

Business cycle synchronization

Table 4.2 presents the predictions of the model for volatility, correlations and international comovement. I consider the three alternative calibrations, i.e., high, mid and low elasticities. I compare these predictions to the U.S. data. As a reference, I also report the business cycles statistics predicted by a standard IRBC bond economy. Both, the data and the standard model, are taken from Heathcote and Perri (2002).

Let me start with the cross-country correlations, reported in Panel C. The standard IRBC model generates too low, and sometimes negative, cross-country correlations. This is the result of an intrinsic force in the IRBC: households reallocate their portfolio towards the most productive location. This negative comovement of factors occurs despite the fact that the IRBC is calibrated to match the cross-country correlation of productivities. In contrast to the IRBC, the model delivers plausible levels of comovement. Importantly, the model generates cross-border spillovers in productivity that are in the order of magnitude of the data. Like the standard IRBC model, the model also predicts too much comovement in consumption, relative to the correlation of output. Intuitively, there is too much risk sharing across countries.

Panel A reports the volatilities and relative volatilities and Panel B reports the correlations with output. Note that the model outperforms the standard RBC in terms of volatility of exports, imports and TT. For example, under the low elasticity, the model can explain about half of the volatility of TT, while the standard IRBC explains a quarter of the volatility. In terms of correlations, the model behaves as the standard IRBC. Like the standard framework, the model overpredicts the correlation of consumption, labor and exports. The model also fails at predicting the negative correlation between output and TT.

Small open economies In this section of the paper, I explore the implications of the model for SOE's business cycles.

Implications for SOE business cycles

Here I present an analysis suggesting that the business cycle properties of emerging SOEs are more in line with what foreign shocks predict than what domestic shocks predict. The properties of developed SOEs indicate that neither foreign nor domestic shocks are dominant.

Emerging and developed SOEs differ in many aspects. In terms of their business cycle properties, the differences are noticeable. Aguiar and Gopinath (2007) document the following differences between emerging and developed SOEs. First, at the business cycle frequency, consumption is approximately 40 percent more volatile than output in emerging markets. In developed economies the same ratio is slightly less than one on average. Second, in emerging countries, in contrast to developed countries, the trend component of growth is more important for business cycle volatility. Here I add another difference. In emerging countries, TT and TFP tend to be negatively correlated, especially after 1980, while in developed SOEs, the correlation is weakly positive, see Table B.1.

I will argue that the distinctive features of emerging countries emphasized by Aguiar and Gopinath (2007) are connected to the strong negative correlation between TT and TFP. Specifically, the model implies that a foreign technology shock have almost the same effects as a domestic technology shock of the same sign but differ in (i) the direction of change in TT, (ii) the persistence of TFP, and (iii) the relative volatility of consumption. These differences are reminiscent of the distinctive features of emerging countries highlighted above. Hence, through the lens of the model, TFP of emerging countries (and more generally their business cycles) seem to be partially driven by exogenous TT fluctuations.

In order to support the idea that emerging business cycle seems to be caused by exogenous TT movements, I show next how the predictions of the model change as the relative importance of foreign and domestic shocks varies. I set γ to 0.7 to match the average imports to GDP ratio in the sample of SOEs and let one country be infinitesimally small, i.e. $n \to 0$.[37] In this re-parameterized version I scale the relative variance of the foreign shock and the domestic shock from one extreme (only domestic shocks) to

[37] I solved the model under the limiting case where one country is a SOE. Details of this approximation are provided in the Appendix A. Specifically, I assume that the home country is the small economy, i.e. $n \to 0$. In this approximation, the exponent $1-\gamma \to \lambda$ and $1-\gamma^* \to 0$.

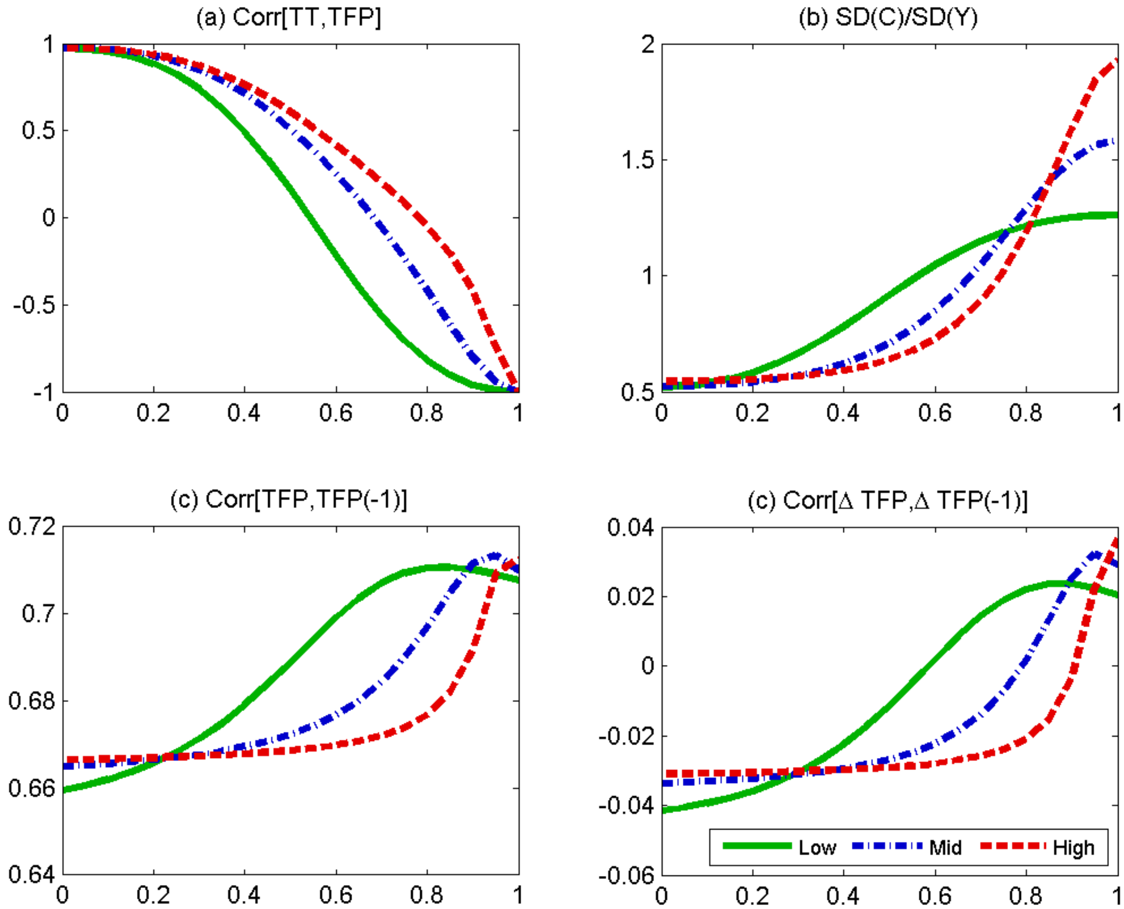

Figure 3: Moments as a function of relative volatility of foreign shocks

the other extreme (only foreign shocks). For each variation, I simulate the economy for 100 periods and compute the following statistics: (i) the correlation between TFP and TT, (ii) the ratio of standard deviation of consumption to the standard deviation of output, (iii) the first-order autocorrelation of TFP, and (iv) the first-order autocorrelation of the change in TFP. I repeat the simulation 100 times and calculate the average for each statistic. As in Aguiar and Gopinath (2007), (i)-(iii) are computed over HP-filtered data.

Figure 3 reports the results. The horizontal axis measures the relative volatility of foreign to domestic technology shocks, going from zero (no foreign shocks) to one (no domestic shocks).

Panel (a) reports the correlation between TFP and TT predicted by the model. The correlation moves from positive to negative territory as foreign shocks become relatively more important. As expected, the correlation decays faster when the inefficiency wedge widens.

Panel (b) reports the ratio of the standard deviation of consumption to the standard deviation of output. The model generates excess volatility of consumption (ratio greater than 1) when foreign shocks are predominant. Different from panel (a), increasing the inefficiency wedge imposes some limits on the excess volatility of consumption because output is also more elastic to TT in this case.

Panel (c) reports the autocorrelation of TFP (HP filtered) as a proxy of persistence. Increasing the importance of foreign shocks generates higher TFP persistence. As expected, the effect is stronger the wider the inefficiency wedge.

Finally panel (d) reports the first-order autocorrelation of the first difference in log TFP, i.e. TFP growth. When TFP is entirely driven by stationary AR(1) domestic technology shocks, this statistic is negative.[38] When TFP is a random walk, this statistic approaches to 0. As foreign shocks become more important, TFP growth displays a positive first order autocorrelation, despite the fact that all shocks are stationary.

Are shocks to TFP foreign or domestic?

Here I try to provide an answer to this question by constructing inferences about the two unobservable states A_t and A_t^* on the basis of the full history of data observed for a particular country. I construct these inferences using the Kalman smoothing algorithm, see Hamilton (1994). I use Mexico and Canada as prototypes of emerging and developed economies, respectively. Because the data is annual, I recalibrate the model to that frequency. Accordingly, the discount factor β and the depreciation rate δ are set to 0.95 and 0.06, respectively. The rest of parameters remain the same.[39]

The first step is to rewrite the model using the following state-space representation,

$$\begin{aligned}
\xi_{t+1} &= F\xi_t + H\varepsilon_{t+1}, \qquad \varepsilon_{t+1} \sim \ i.i.d.\ N\left(0, I\right) \\
x_t &= V'\xi_t + u_t, \qquad u_t \sim \ i.i.d.\ N\left(0, \Sigma\right)
\end{aligned}$$

$$\begin{aligned}
\xi_t &= \left[\widehat{k}_{t-1}^*, \widehat{b}_{t-1}, \widehat{k}_{t-1}, \widehat{a}_{t-1}, \widehat{a}_{t-1}^*, \epsilon_{a,t}, \epsilon_{a,t}^*\right]' \\
\varepsilon_t &= \left[\epsilon_{a,t}, \epsilon_{a,t}^*\right]' \\
x_t &= \left[\widehat{y}_t, \widehat{c}_t, \widehat{i}_t, \widehat{tt}_t\right]',
\end{aligned}$$

where ξ_t is the vector of states, x_t is the vector of variables observed at date t, ε_t is the vector of white noise innovations to the states and u_t are measurement errors. Matrices F, H, and V depend on parameters of the model.

I restrict the history of data to those variables that, from the point of view of the model, are informative about the unobservable states. Based on the previous analysis, I include four observables in x_t: TT, output, consumption and investment (all in logged and HP-filtered) from 1980 to 2008.[40] Kalman smoothing requires specific values for the parameters of the model. I choose the calibration under the medium elasticity of substitution, i.e. $\theta = 0.8$. Other parameters $\left(\phi,\ \rho_a,\ \sigma_a^2,\ \rho_a^*\right.$ and $\left.\sigma_a^{*2}\right)$ are estimated by Maximum Likelihood. To avoid a singularity, I include independent measurement errors for each observable, i.e., Σ is a diagonal matrix. Parameters ϕ, ρ_a, σ_a^2, ρ_a^* and σ_a^{*2} and standard errors of the measurement errors are chosen to maximize the log of the sample likelihood, and constructed via Kalman filter.[41]

The parameter estimates and their standard errors are reported in Table 4.3. Figure 4 and 5 plot the results for Mexico and Canada respectively. The top/left panel plots the observed TFP series and the TFP

[38] It is straightfoward to show that for an AR(1) process $x_t = \rho x_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim N\left(0, \sigma^2\right)$, the plim of the first autocorrelation

Table 4.3: MLE estimates

Parameter	Canada		Mexico	
	Estimate	Std. Error	Estimate	Std. Error
ρ_a	0.9414	0.0167	0.4303	0.0163
σ_a	0.0042	0.0033	0.0023	0.0046
ρ_a^*	0.7060	0.0092	0.6327	0.0227
σ_a^*	0.0071	0.0068	0.0200	0.0178
ϕ	0.7398	0.0102	0.7703	0.0552

series predicted by the model. The predicted series follows the same cyclical pattern of the observed TFP. In the case of Mexico, the model falls short at predicting the actual TFP volatility. The graph on the right shows the predicted TFP decomposed by the TT component and the domestic technology component, as in equation (32). According to Kalman smoothing, shocks that hit Mexico between 1980-2008 were predominantly exogenous to domestic technology. In contrast, the decomposition of the predicted series of TFP for Canada shows that domestic technology shocks played a predominant role. The second and third row present the predictions for the observable variables. Smoothed predictions of the model match the actual series fairly well, but fail in some occasions.[42]

5 Additional exercises

CES production function Here I discuss two extensions regarding the elasticity of substitution between factors of production. In these cases, I will depart from the Cobb-Douglas specification used in the model presented above.

Domestic and imported intermediate inputs

International business cycle models consider the case where domestic and imported intermediates are strict complements in production, see Backus et. al. (1992). Here I adopt this idea through the following nested CES production function,

$$q_t(i) = A_t(i) A_t \left(k_t(i)^\alpha l_t(i)^{1-\alpha}\right)^{1-\mu} \left[\gamma d_t(i)^\rho + (1-\gamma)(m_t(i))^\rho\right]^{\frac{\mu}{\rho}} \tag{38}$$

with $-\infty < \rho < 1$ and μ, γ and $\alpha \in (0,1)$.

of Δx_t is $(\rho - 1)/2 < 0$.

[39] Recall that for SOEs γ was set to 0.7.

[40] I do not include TFP as part of the observed variables. In that sense, I am providing less information to the Kalman filter that the one required to determine the unobservables. Results do not change dramatically if I include TFP as observable.

[41] I note that Kalman smoothing generates a perfect fit for some variables (investment and TT). Following the literature, e.g. Cole et al. (2005), I added measurement errors with pre-specified variances in the measurement equations for those variables.

[42] The model fits the data relatively well except for Mexico during 1994-1995 period (Tequila Crisis). In particular, the model underpredicts the fall in consumption.

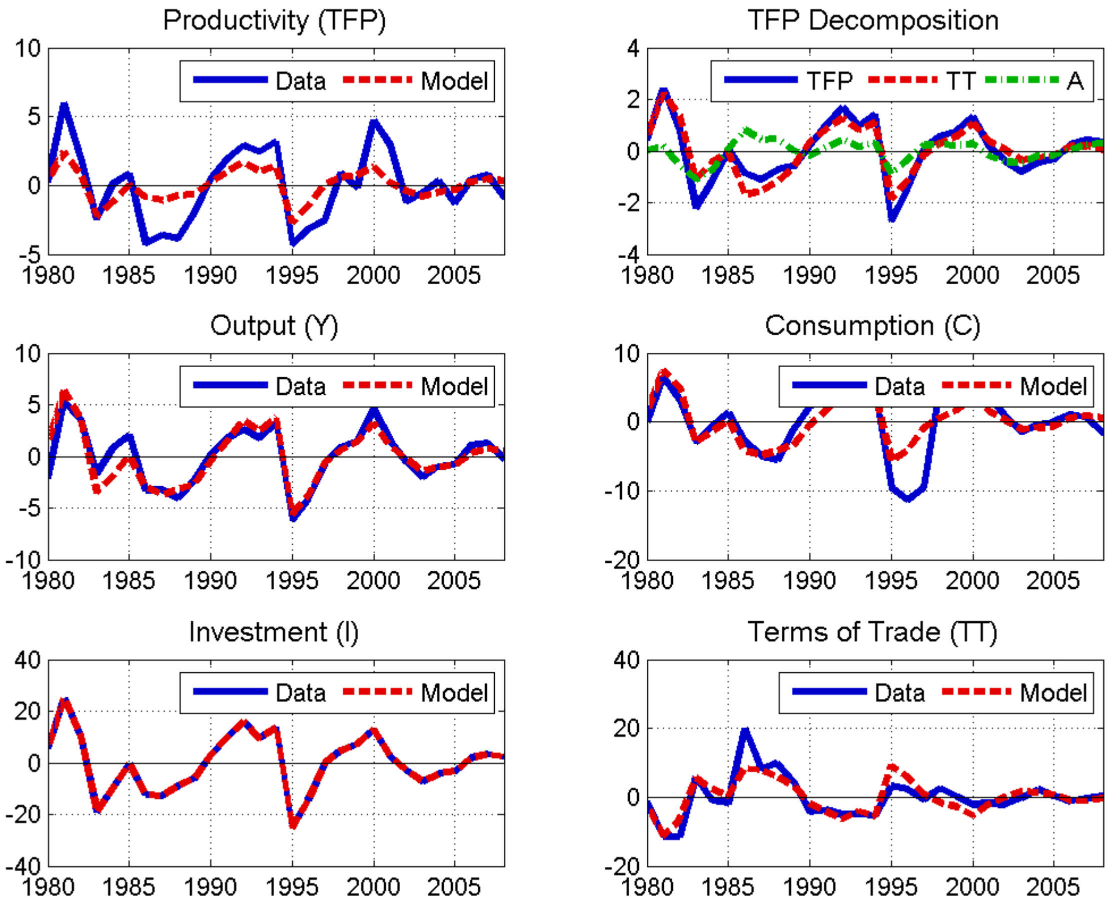

Figure 4: Results of Kalman smoothing - Mexico.

The elasticity of substitution between a domestic and imported intermediate inputs is given by $1/(1-\rho)$. It is straightforward to show that under this more general production function TFP is still given by equation (32). The main difference with the Cobb-Douglas case is the way that TT respond to shocks. Specifically, increasing the complementarity between domestic and imported intermediate inputs implies that a given change in the allocation of production across countries is achieved through larger TT movements. In the model that would also imply higher TFP volatility.

Capital-labor and intermediate inputs

There is evidence suggesting that elasticity of substitution between capital-labor and intermediate inputs is less than one (complements), e.g., Rotemberg and Woodford (1996). In this section I discuss how the link between TT and TFP is affected by the elasticity of substitution between factors of production. In particular, I consider a CES production function of the form,

$$q_t(i) = A_t(i)\left[(1-\mu)\left(k_t(i)^{\alpha} l_t(i)^{1-\alpha}\right)^{\rho} + \mu\left(d_t(i)^{\gamma} m_t(i)^{1-\gamma}\right)^{\rho}\right]^{\frac{1}{\rho}} \tag{39}$$

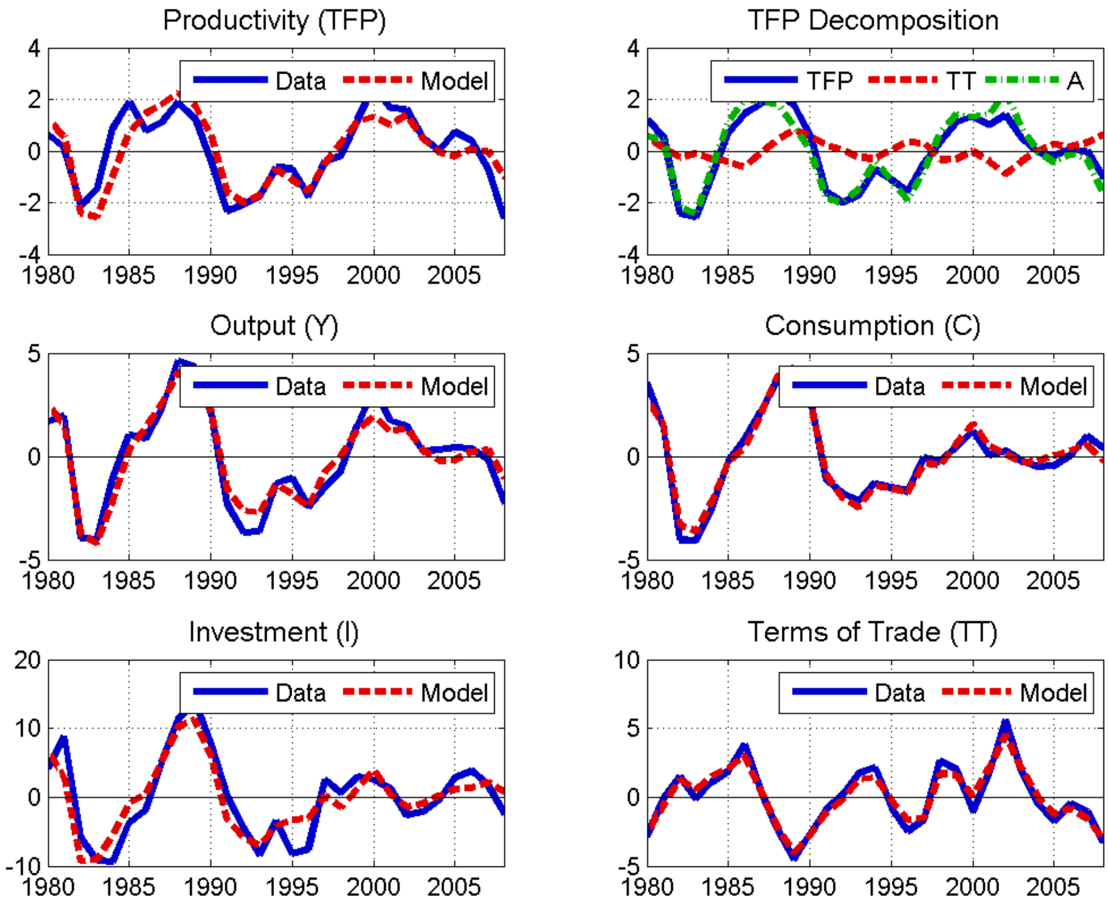

Figure 5: Results of Kalman smoothing - Canada

with $-\infty < \rho < 1$ and μ, γ and $\alpha \in (0,1)$. The elasticity of substitution between a capital-labor composite and intermediate inputs is given by $1/(1-\rho)$. This general CES production function nests the Cobb-Douglas case when $\rho \to 0$. It also nests the fixed proportions production function (i.e. Leontieff) when $\rho \to -\infty$ and the linear production function when $\rho \to 1$. The rest of the model remains the same.

Under this CES production function, aggregate gross domestic output can be also written as equation (31), where TFP (in growth rates) can be written as,

$$\frac{dTFP_t}{TFP_t} = \frac{1}{(1-\rho)}\left[\frac{1}{(1-\theta^{-1}S_t)} - \frac{\rho}{(1-S_t)}\right]\frac{dA_t}{A_t} - \frac{1}{1-\rho}\left[\frac{(1-\gamma)\,\theta^{-1}S_t}{1-\theta^{-1}S_t}\frac{1-\theta}{1-S_t}\right]\frac{dTT_t}{TT_t} \quad (40)$$

with $S_t \equiv (P_tD_t + P_t^*M_t)/P_tQ_t$ is the intermediate input revenue share.[43]

I focus on the coefficient multiplying the growth rate of TT.[44] Notice that the sign of the coefficient is the same as in the Cobb-Douglas case, i.e., an exogenous TT deterioration reduces TFP. Importantly, now TT also affect productivity through S_t. The bracketed term is the same as the Cobb-Douglas case

[43]When $\rho \to 0$, S_t collapses to $\mu\theta$ and the formula goes back to (32).

[44]Some of the effects discussed next are also present in the coefficient multiplying the growth rate of technology.

(32) except that the intermediate revenue share S_t depends on TT. The elasticity of substitution $1/(1-\rho)$ scales the bracketed term up or down depending on whether factors are strict substitutes or strict complements. The intuition for this is the following: The elasticity of substitution determines the speed at which decreasing returns set in. So, for example, when factors are strict complements, decreasing returns set in faster than the Cobb-Douglas case and thus any marginal change in imports does not affect production as much as the Cobb-Douglas case. In addition to this, the elasticity of substitution affects the coefficient via S_t. In particular, when factors are strict substitutes $S_t'(TT_t) < 0$ and when factors are strict complements $S_t'(TT_t) > 0$. Thus, as the term in brackets is strictly increasing in S_t, this additional effect counterbalances the scaling effect. Whether the scaling effect dominates this indirect effect depends itself on the elasticity of substitution. In particular, the scaling effect dominates when factors are strict complements. In the limiting case where factors are perfect complements, i.e., $\rho \to -\infty$, the effect of TT on TFP collapses to zero because decreasing returns set in quite rapidly. The opposite occurs for the strict substitute case. In the limiting case where factors are perfect substitutes, i.e., $\rho \to 1$, the effect of TT on TFP collapses to zero because S_t collapses to zero whenever TT_t increase.

Evaluating this CES case goes beyond the scope of this paper. Yet, I would like to emphasize that for plausible estimates of the elasticity of substitution, the effect of TT on TFP is still important. For instance, Rotemberg and Woodford (1996) report an elasticity of approximately 0.7. A back-of-the-envelope calculation, that only takes into account the scaling effect, indicates that for that level of complementarity the impact of TT on TFP would decline by 30 percent, at most.

Long-lasting TFP declines The previous analyses suggest that emerging countries business cycles are buffeted by exogenous TT fluctuations. Importantly, when that happens, TFP is highly persistent. Here I assess the contribution of *observed* TT on *observed* TFP persistent declines. This is related to ongoing literature that is trying to understand why TFP in emerging countries exhibit protracted recoveries, see Bergoing et. al. (2002), Cole et. al. (2005), Meza and Quintin (2007), and Benjamin and Meza (2009), among others. The analysis presented below suggests that large and persistent TT deteriorations play an important role in explaining those events.

The analysis focuses on all emerging countries in the sample from 1980-2008, see Table B.1. First I identify episodes of TFP decline. These episodes end if the pre-decline TFP level is recovered or if another TFP decline occurs. To focus on protracted recoveries, I truncate the analysis to only include those episodes with a duration of at least 5 years. Table B.2 lists all identified episodes in the sample. Long-lasting TFP declines tend to coincide with TT deteriorations of similar magnitude and persistence. The model predicts that whenever that happens, TT is causing TFP and not the other way around.

I use equation (32) to compute an alternative TFP path that is consistent with the observed path of TT, assuming that domestic technology is constant. I focus on those episodes in which TT deteriorated because in those cases, through the lenses of the model, TT is most likely exogenous to domestic technology. Figure 6 plots the average TFP (red line) decline for each year after the peak until the fifth year. TFP declines substantially in the first three years and then stagnates. The other lines denote the predictions of the model under different elasticities of substitution. For the lowest elasticity, TT contribute up to 50 percent of the average long-lasting TFP decline.

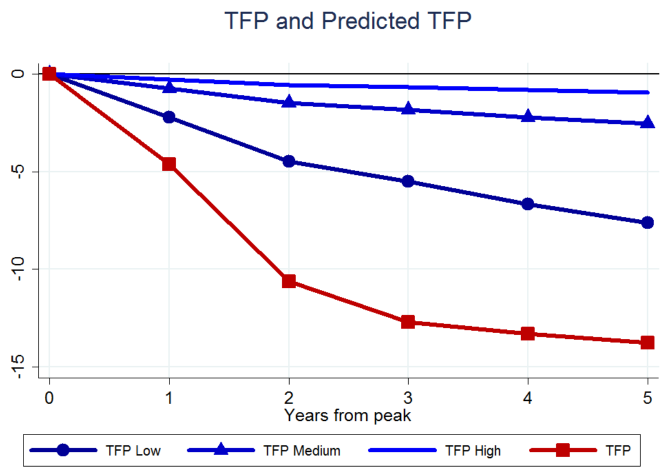

Figure 6: Long-lasting TFP declines and predicted TFP

Micro-evidence According to the model, the larger is the inefficiency wedge or the higher is the import intensity, the larger is the drop in TFP after an adverse TT deterioration. In this section, I present indirect micro-evidence supporting these implications. The micro-data comes from the annual industrial survey - *Encuesta Nacional de Industria Anual* (ENIA) - of Chile which contains establishment-level data for the years 1980 to 1995. This survey is widely used in the I-O literature, e.g., Petrin and Sivadasan (2011), and I refer to that literature for details. The ENIA is useful because the Chilean economy was hit by several shocks, including adverse TT shocks, during the first half of the 1980s, see Bergoing et. al. (2002). In fact, while aggregate TFP in Chile declined 20 percent between 1981-1982, its TT deteriorated in almost the same magnitude.

I use the micro-data to test whether industries with high markups experienced larger drops in TFP. The first step is to estimate the markups by industry. Following De Loecker and Warzynski (2012) I approximate the markup from the first order conditions (foc) for intermediate inputs. In the model, these focs can be written as:

$$\mu \frac{p_t(i)\, q_t(i)}{P_t d_t(i) + P_t^* m_t(i)} = \frac{1}{\theta}$$

In the data, I observe gross nominal output $(p_t(i)\, q_t(i))$ and the total cost of materials $(P_t d_t(i) + P_t^* m_t(i))$ by establishment. I do not know the parameters μ and θ. I proxy μ by the 3-digit sectorial elasticity of output to intermediate inputs estimated by Petrin and Sivadasan (2011). Given the estimate of μ, I back out the value θ from the foc. To reduce noise in the data, I aggregate the establishment level data by their 3-digit sector indicator. Accordingly, hereafter index i refers to a sector. I still need a value for α, which I back out from the foc of firms using the sectorial wage bill. TFP at the sectorial level is constructed using the standard formula: $TFP(i) = y(i) / k(i)^{\alpha}\, l(i)^{1-\alpha}$.

Figure 7 plots the growth rate of TFP by sector in the year 1982, when the TT deteriorated. The horizontal axis denotes the level of markups (in logs). Sectors with higher markups experienced larger TFP contractions, which is exactly what the model predicts.

I also use the micro-data to test whether industries with high import intensity experienced larger

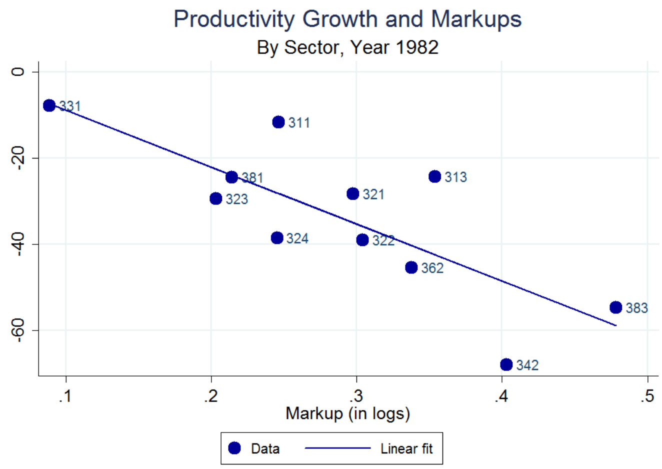

Figure 7: Micro-evidence: TFP and markups

drops in TFP. I measure import intensity by the share of imports on the cost of intermediates. In the model, the focs imply,

$$\frac{P_t^* m_t(i)}{P_t d_t(i) + P_t^* m_t(i)} = 1 - \gamma$$

In the data, I observe the cost of imported intermediate inputs $(P_t^* m_t(i))$ and the total cost of materials $(P_t d_t(i) + P_t^* m_t(i))$. I perform the analysis at the 3-digit sector level as before. Using the aggregated data, I back out γ from the previous equation. Figure 8 plots the growth rate of TFP by sector in the year 1982 sorted by the imported intensity (in logs). Sectors with higher import intensity experienced larger TFP contractions, which is exactly what the model predicts.

Measurement It is quite possible that actual national accounts use biased priced deflators.[45] The main result in Proposition 1 assumed unbiased price deflators. Here I discuss the role of these biases assuming that TT movements are exogenously driven by foreign technology. Appendix A provides the proof for all cases discussed here.

Suppose that TT at base year prices, i.e., $TT_b \equiv P_b^*/P_b$, differ from current equilibrium TT, i.e., $TT_t = P_t^*/P_t$. The implications for TFP are the following: When TT_t move towards TT_b, there is an artificial gain in efficiency because the gap between the marginal product of imports and TT_b falls. In contrast, when TT_t move away from TT_t, there is an artificial loss in efficiency. This affects the correlation between TT and TFP. For instance, if $TT_b > TT_t$ and the TT deteriorate (increase), TFP artificially rises and there is an upward bias in the correlation between TT and TFP. On the contrary, if $TT_b > TT_t$ and the TT improve (decrease), there is downward bias in the correlation between TT and TFP. The sign of the bias switches if $TT_b < TT_t$. Updating price deflators to current market condition certainly reduces the aforementioned biases, e.g., the chain-weighted Fisher index.[46]

[45] For the case of the United States, see Feenstra et. al. (2009).

[46] Assessing how the results of the numerical experiments change after considering these biases is not the main focus of the

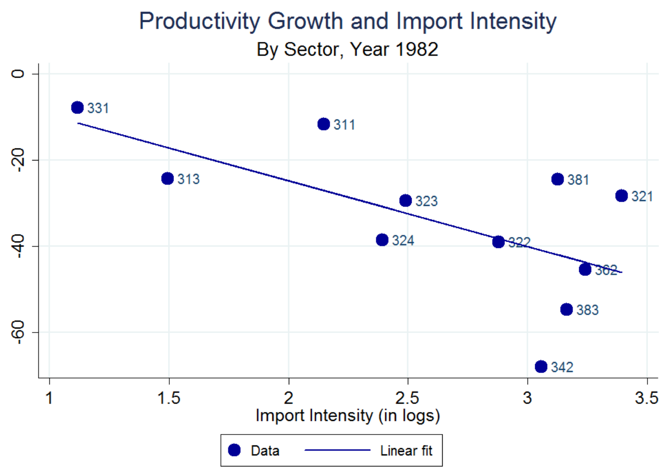

Figure 8: Micro-evidence: TFP and import intensity

6 Conclusions

In this paper, I document the correlation between the terms of trade and aggregate TFP in a group of middle/high income countries. For some countries the data on terms of trade - defined as the ratio of import prices to export prices - and TFP - measured by the Solow residual - are strongly negatively correlated. In other words, when the terms of trade deteriorate (improve) often times TFP declines (increases). This is a problem for the standard international real business cycles theory, initiated by Backus et. al. (1992).

Using a two-country monopolistic competitive model, this paper demonstrates that the terms of trade can affect productivity in the same way as in the data. The key for this result is the existence of monopolistic power in combination with imported intermediate inputs. The link between terms of trade and productivity delivers novel implications for the analysis of business cycles in open economies. First, it introduces cross-country comovement of productivities which enhance business cycle synchronization among large economies. Second, it implies that foreign technology shocks can have almost the same effects as domestic technology shocks.

Quantitatively, this paper shows that plausible levels of monopolistic power can lead to business cycle synchronization close to the one observed between the U.S. economy and the rest of the world. The paper also finds evidence suggesting that the business cycles of some small open economies, especially emerging economies, are more likely to be the result of foreign shocks. Likelihood-based methods indicate that for emerging countries, in contrast to small developed economies, TFP is mainly the result of exogenous terms of trade fluctuations.

One key point that the paper does not address is the role of financial shocks in emerging countries, which plays a central role in many business cycle models, e.g., Neumeyer and Perri (2009) and Mendoza and Yue (2011). The omission of financial shocks probably biases the results shown in the paper in favor

paper. Yet, numerical experiments suggest that in order to revert the negative correlation between TT and TFP that arises when there is monopolistic power, the bias in the base year terms of trade has to be substantial, e.g., 50 percentage points.

to the terms of trade. Hence, an interesting avenue of future research would be to add financial shocks into the model in order to disentangle their importance vis-à-vis the terms of trade. Moreover, it would also be interesting to shed light on the relationship between financial shocks and terms of trade, both in the data and in a model.

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Appendix A Main derivations

Household problem Focs,

$$
\begin{aligned}
-\frac{U_{L,t}}{U_{C,t}} &= \frac{w_t}{P_t} \\
1+\phi\left(\frac{K_t}{K_{t-1}}-1\right) &= \beta \mathbb{E}_t\left\{\frac{U_{C\,t+1}}{U_{C,t}}\left(\frac{r_{t+1}}{P_{t+1}}+(1-\delta)-\frac{\phi}{2}\left(\left(\frac{K_{t+1}}{K_t}\right)^2-1\right)\right)\right\} \\
1+\kappa B_t^* &= \beta \mathbb{E}_t\left\{\frac{U_{C\,t+1}}{U_{C,t}}\frac{TT_{t+1}^*}{TT_t}R_t^*\right\} \\
1+\kappa B_t &= \beta \mathbb{E}_t\left\{\frac{U_{C\,t+1}}{U_{C,t}}R_t\right\}
\end{aligned}
$$

where $U_{L,t}$ is the marginal dis-utility of labor and $U_{C,t}$ is the marginal utility of consumption, at period t.

Final good producer The foc,

$$
g_t(i)=\left(\frac{1}{n}\right)\left(\frac{p_t(i)}{P_t}\right)^{\frac{1}{\theta-1}} G_t,
$$

Firm's problem Given the input-output linkages in the model, it is useful to think about firm's i problem in three stages.

Cost minimization

First, firm i finds the factor mix, i.e. $k_t(i), l_t(i), d_t(i), m_t(i)$ that minimizes the total cost of production taking as given factor prices. That is,

$$
\begin{aligned}
mc_t(i)\, q_t(i) \equiv &\min_{\{k_t(i),l_t(i),d_t(i),m_t(i)\}} r_t k_t(i)-w_t l_t(i)-\widetilde{P}_t d_t(i)-\widetilde{P}_t^* m_t(i), \\
&\text{s.t.}: q_t(i)=A_t(i)\left(k_t(i)^\alpha l_t(i)^{1-\alpha}\right)^{1-\mu}\left(d_t(i)^\gamma m_t(i)^{1-\gamma}\right)^\mu
\end{aligned} \tag{41}
$$

where $\widetilde{P}_t$ and $\widetilde{P}_t^*$ are the shadow prices of the domestic intermediate input aggregator $d_t(i)$ and the imported intermediate input aggregator $m_t(i)$. First order conditions deliver,

$$
\begin{aligned}
(1-\mu)\,\alpha \frac{q_t(i)}{k_t(i)} &= \frac{r_t}{mc_t(i)} \\
(1-\mu)(1-\alpha)\frac{q_t(i)}{l_t(i)} &= \frac{w_t}{mc_t(i)} \\
\mu\gamma \frac{q_t(i)}{d_t(i)} &= \frac{\widetilde{P}_t}{mc_t(i)} \\
\mu(1-\gamma)\frac{q_t(i)}{m_t(i)} &= \frac{\widetilde{P}_t^*}{mc_t(i)}.
\end{aligned}
$$

The definition of total cost of production delivers the formula for the unitary cost of production $mc_t(i)$,

$$
\begin{aligned}
mc_t(i) &= \frac{\left(r_t^\alpha w_t^{1-\alpha}\right)^{1-\mu}\left(\widetilde{P}_t^\gamma \widetilde{P}_t^{*1-\gamma}\right)^\mu}{\varsigma A_t(i)} \\
\varsigma &\equiv \left((1-\mu)\,\alpha^\alpha(1-\alpha)^{1-\alpha}\right)^{1-\mu}\left(\mu\gamma^\gamma(1-\gamma)^{1-\gamma}\right)^\mu
\end{aligned}
$$

Intermediate input mix (cost minimization)

Given $d_t(i)$, firm i chooses the optimal mix of intermediate goods $d_t(i,j)$ taking their given prices. Firm i minimizes the cost of the domestic intermediate good composite,

$$\begin{aligned}
\widetilde{P}_t d_t(i) &\equiv \min_{\{d_t(i,j)\}_{j\in[0,1]}} \int_0^1 p_t(j)\, d_t(i,j)\, dj, \\
&\text{s.t.: } d_t(i) = \left(\left(\frac{1}{n}\right)^{1-\theta} \int_0^n d_t(i,j)^\theta\, dj\right)^{\frac{1}{\theta}}
\end{aligned}$$

The first order condition for $d_t(i,j)$,

$$d_t(i,j) = \left(\frac{1}{n}\right)\left(\frac{p_t(j)}{\widetilde{P}_t}\right)^{\frac{1}{\theta-1}} d_t(i). \tag{42}$$

Substituting (42) in $d_t(i)$ delivers the following equation,

$$\widetilde{P}_t = P_t = \left(\frac{1}{n}\int_0^n p_t(i)^{\frac{\theta}{\theta-1}}\, di\right)^{\frac{\theta-1}{\theta}}. \tag{43}$$

A similar problem for $m_t(i)$ delivers $m_t(i,j)$,

$$m_t(i,j) = \left(\frac{1}{1-n}\right)\left(\frac{p_t^*(j)}{\widetilde{P}_t^*}\right)^{\frac{1}{\theta-1}} m_t(i),$$

where,

$$\widetilde{P}_t^* = P_t^* = \left(\frac{1}{1-n}\int_n^1 p_t^*(i)^{\frac{\theta}{\theta-1}}\, di\right)^{\frac{\theta-1}{\theta}}.$$

Pricing decision

Finally, firm i sells its output to the market at a price $p_t(i)$ in a monopolistic competitive fashion. The firm maximizes profits,

$$\Pi_t(i) = \max_{p_t(i)} p_t(i)\, q_t(i) - mc_t(i)\, q_t(i),$$

subject to (8). Combining previous results, the sum of demands simplifies to a single demand schedule,

$$q_t(i) = \left(\frac{1}{n}\right)\left(\frac{p_t(i)}{P_t}\right)^{\frac{1}{\theta-1}}\left(G_t + \int_0^n d_t(j)\, dj + \int_n^1 m_t^*(j)\, dj\right),$$

The first order condition of the above problem yields the standard markup over marginal cost formula,

$$p_t(i) = \frac{1}{\theta} mc_t(i)$$

Derivation of TFP Differentiating equation (23) , keeping prices constant, yields,[47]

$$\frac{dY_t}{Y_t}=\left[\frac{P_b\left(G_t+M_t^*\right)}{Y_t}\right]\frac{d\left(G_t+M_t^*\right)}{G_t+M_t^*}-\left[\frac{P_b^*M_t}{Y_t}\right]\frac{dM_t}{M_t}. \tag{44}$$

I will focus on the case where base prices approximate current equilibrium prices, i.e. $P_b\to P_t$ and $P_b^*\to P_t^*$. From equations (29) and (30),

$$\begin{aligned}\frac{d\left(G_t+M_t^*\right)}{G_t+M_t^*}&=\frac{dQ_t}{Q_t},\\ \frac{dM_t}{M_t}&=\frac{dQ_t}{Q_t}-\frac{dTT_t}{TT_t}.\end{aligned}$$

Moreover,

$$\begin{aligned}\frac{P_t\left(G_t+M_t^*\right)}{Y_t}&=\frac{1-\mu\gamma\theta}{1-\mu\theta},\\ \frac{P_t^*M_t}{Y_t}&=\frac{(1-\gamma)\,\mu\theta}{1-\mu\theta}.\end{aligned}$$

Combining these equations with (44),

$$\frac{dY_t}{Y_t}=\frac{dQ_t}{Q_t}+\frac{(1-\gamma)\,\mu\theta}{1-\mu\theta}\frac{dTT_t}{TT_t}. \tag{45}$$

Differentiating gross output,

$$\frac{dQ_t}{Q_t}=-\frac{(1-\gamma)\,\mu}{1-\mu}\frac{dTT_t}{TT_t}+\frac{1}{1-\mu}A_t+\alpha\frac{dK_{t-1}}{K_{t-1}}+(1-\alpha)\frac{dL_t}{L_t}$$

Plugging it into (45) yields,

$$\frac{dY_t}{Y_t}=\frac{dTFP_t}{TFP_t}+\alpha\frac{dK_{t-1}}{K_{t-1}}+(1-\alpha)\frac{dL_t}{L_t}.$$

where,

$$\frac{dTFP_t}{TFP_t}=-\frac{(1-\gamma)\,\mu}{(1-\mu)}\frac{1-\theta}{1-\mu\theta}\frac{dTT_t}{TT_t}+\frac{1}{1-\mu}\frac{dA_t}{A_t}$$

Integration from time 0 to t (ignoring constants of integration) delivers equation (31) and (32) .

Log-linear model Here I present the equations of the model. All variables are log-linearized, except for the bond-holdings of the home economy which are linearized around its steady state. I assume that in the steady state bond- holdings are zero and TT are one.

For the domestic country, the equations are,

$$\begin{aligned}
\upsilon\widehat{l}_t &= \widehat{z}_t && (46)\\
\phi\left(\widehat{k}_t-\widehat{k}_{t-1}\right) &= \mathbb{E}_t\widehat{u}_{C,t+1}-\widehat{u}_{C,t}+(1-\beta(1-\delta))\left(\mathbb{E}_t\widehat{z}_{t+1}-\hat{k}_t\right)-\beta\phi\mathbb{E}_t\left(\widehat{k}_{t+1}-\widehat{k}_t\right) && (47)\\
\widehat{z}_t &= -\frac{(1-\gamma)\,\mu}{(1-\mu)}\widehat{tt}_t+\frac{1}{1-\mu}\widehat{a}_t+\alpha\widehat{k}_{t-1}+(1-\alpha)\,\widehat{l}_t && (48)\\
\kappa\widehat{b}_t &= \mathbb{E}_t\widehat{u}_{C,t+1}-\widehat{u}_{C,t}+\widehat{R}_t && (49)\\
\widehat{tt}_t+\kappa\widehat{b}_{*t} &= \mathbb{E}_t\widehat{u}_{C,t+1}-\widehat{u}_{C,t}+\mathbb{E}_t\widehat{tt}_{t+1}+\widehat{R}_t^* && (50)\\
\delta K\widehat{i}_t &= K\hat{k}_t-(1-\delta)\,K\hat{k}_{t-1} && (51)\\
C\widehat{c}_t+\delta K\widehat{i}_t+\widehat{b}_t+B_*\widehat{tt}_t+\widehat{b}_{*t} &= Z\widehat{z}_t+R\widehat{b}_{t-1}+\frac{P^*}{P}R^*\widehat{b}_{*t-1} && (52)
\end{aligned}$$

[47]This is known in the literature as the Divisia Quantity Index, see Sims (1969).

where $\widehat{x}_t \equiv \log(X_t/X)$ is the log-deviation of variable X_t from its steady state X. The bond holdings in steady state are zero. Hence, I linearize, i.e., $\widehat{b}_{*t} \approx b_{*t} - b_*$ and $\widehat{b}_t \approx b_t - b$, where b and b_* is zero in the steady state.

The log-linear approximation of the marginal utility of consumption is given by,

$$\widehat{u}_C(t) = -\frac{\sigma C}{C - \psi L^{\upsilon}}\widehat{c}_t + \frac{\sigma\psi\upsilon L^{\upsilon}}{C - \psi L^{\upsilon}}\widehat{l}_t$$

Log-linearizing the equation for output in domestic country, (31) gives ,

$$\widehat{y}_t = \widehat{tfp}_t + (1-\alpha)\,\widehat{l}_t + \alpha\widehat{k}_{t-1}, \tag{53}$$

where $\widehat{tfp}_t$ is given by,

$$\widehat{tfp}_t = -\frac{(1-\gamma)\,\mu}{(1-\mu)}\frac{1-\theta}{1-\mu\theta}\widehat{tt}_t + \frac{1}{1-\mu}\widehat{a}_t \tag{54}$$

For the foreign country, the equations are analogous except that $-\widehat{tt}_t^*$ replaces $\widehat{tt}_t$ and γ^* replaces γ.

The approximation of the bond-holding market clearing conditions,

$$\widehat{b}_t + \widehat{b}_t^* = 0 \tag{55}$$

$$\widehat{b}_{*t} + \widehat{b}_{*t}^* = 0 \tag{56}$$

Finally, to characterize the equilibrium I log-linearized equation (16) . After combining it with other conditions, I get,

$$C\widehat{c}_t + K\hat{k}_t - (1-\mu\gamma\theta)\,Q\widehat{z}_t + M^*\left(\widehat{tt}_t + \widehat{z}_t^*\right) = (1-\delta)\,K\hat{k}_{t-1} \tag{57}$$

Technology shocks,

$$\widehat{a}_t^* = \rho_a^*\widehat{a}_{t-1}^* + \sigma_a^*\epsilon_{a,t}^*,$$
$$\widehat{a}_t = \rho_a\widehat{a}_{t-1} + \sigma_a\epsilon_{a,t},$$

The aforementioned equations (including the omitted equations for the foreign country) determine a set of differential equations that can be solved via standard methods.

Small open economy To capture the dynamics of the small open economy, I follow De Paoli (2009), I implement the small open economy approximation as follows: I assume that γ (the exponent on domestic intermediate inputs in the production function) is a function of the relative size of the other country: $1-\gamma = (1-n)\,\lambda$, where parameter $\lambda \in (0,1)$ represents the degree of openness. Foreign firms' production function is analogous, except that $1-\gamma^* = n\lambda$. Assuming that the domestic country is a small economy, i.e. $n \to 0$, the exponent $1-\gamma \to \lambda$ and $1-\gamma^* \to 0$. The key difference with the set of equations shown above is that $\widehat{tt}_t^*$ disappears from foreign country equations.

In addition, I impose the following restrictions. First, the foreign bond holdings become zero: $\widehat{b}_t^*$ and $\widehat{b}_{*t}^*$ are zero. This just means that the foreign country approximates the rest of the world, which is a closed economy. Second, the domestic country is impeded from issuing domestic bonds abroad: $\widehat{b}_t$ is zero. The only financial international asset available to the small open economy is $\widehat{b}_{*t}$, which is denominated in the currency of the foreign country. This guarantees that the small open economy has no effect on the rest of the world. Moreover, the terms of trade between the small open economy and the rest of world are completely irrelevant for the latter. Because of monopoly power, the small open economy can still affect its terms of trade.

Different measurement of output Assume that production requires only labor and imported intermediate inputs, i.e. $G_t + M_t^* = Q_t(L_t, M_t)$. Moreover, to simplify the analysis further assume that the supply of labor is inelastic. First consider *output at constant base prices*:

$$Y_t = Q_t - TT_b\, M_t$$

where TT_b are the terms of trade in the base year. It follows that,

$$\frac{\partial Y_t}{\partial TT_t} = \left(\frac{\partial Q_t}{\partial M_t} - TT_b\right)\frac{\partial M_t}{\partial TT_t}, \tag{58}$$

which can be rewritten as:

$$\begin{aligned} \frac{\partial Y_t}{\partial TT_t} &= (TT_t - TT_b)\frac{\partial M_t}{\partial TT_t} + \left(\frac{\partial Q_t}{\partial M_t} - TT_t\right)\frac{\partial M_t}{\partial TT_t} \\ &= (TT_t - TT_b)\frac{\partial M_t}{\partial TT_t} + \frac{1-\theta}{\theta}TT_t\frac{\partial M_t}{\partial TT_t} \end{aligned} \tag{59}$$

Recall, $\partial M_t/\partial TT_t < 0$ and that $\partial Q_t/\partial M_t = \theta^{-1}TT_t > TT_t$. It follows that the second term is negative. The first term can be positive or negative depending on whether current equilibrium TT_t are lower or higher than base year TT_b. Suppose now that there are no markups, so that the second term drops out. In that case, if $TT_t < TT_b$, then $\partial Y_t/\partial TT_t > 0$. On the contrary, if $TT_t > TT_b$, then $\partial Y_t/\partial TT_t < 0$. These biases have obvious implications for TFP, which in this case coincides with labor productivity. The elasticity of TFP to TT:

$$\frac{d\log TFP_t}{d\log TT_t} = \frac{TT_t - TT_b}{TT_t}\left(\frac{TT_tM_t}{Y_t}\right)\left(\frac{d\log M_t}{d\log TT_t}\right) + \frac{1-\theta}{\theta}\left(\frac{TT_tM_t}{Y_t}\right)\left(\frac{d\log M_t}{d\log TT_t}\right). \tag{60}$$

Now consider the *chain weighted Fisher index* of output. Under Fisher chain-weighted output, I have:

$$Y_{t+1} = \frac{Q_{t+1} - TT_{t+1}M_{t+1}}{P_{t+1}^{Fisher}},$$

where the Fisher chain-weighted price index is the geometric average of the Paasche and Laspeyres indices between the current period and the previous period:

$$P_{t+1}^{Fisher} = \left(\frac{Q_{t+1} - TT_{t+1}M_{t+1}}{Q_{t+1} - TT_tM_{t+1}}\right)^{\frac{1}{2}}\left(\frac{Q_t - TT_{t+1}M_t}{Q_t - TT_tM_t}\right)^{\frac{1}{2}}P_t^{Fisher}.$$

This yields the Fisher chain-weighted quantity index:

$$Y_{t+1} = \left(\frac{Q_{t+1} - TT_{t+1}M_{t+1}}{Q_t - TT_{t+1}M_t}\right)^{\frac{1}{2}}\left(\frac{Q_{t+1} - TT_tM_{t+1}}{Q_t - TT_tM_t}\right)^{\frac{1}{2}}Y_t.$$

The first order change of the logarithm of chain-weighted output is approximated as:

$$\log Y_{t+1} - \log Y_t \approx \frac{dY_t}{dTT_{t+1}}(TT_{t+1} - TT_t).$$

Differentiating the natural logarithm of chain-weighted real GDP:

$$\begin{aligned} \frac{d\log Y_{t+1}}{dTT_{t+1}} &= \frac{\frac{\partial Q_{t+1}}{\partial M_{t+1}}\frac{dM_t}{dTT_t} - TT_{t+1}\frac{dM_{t+1}}{dTT_{t+1}} - M_{t+1}}{2(Q_{t+1} - TT_{t+1}M_{t+1})} \\ &\quad + \frac{M_t}{2(Q_t - TT_{t+1}M_t)} + \frac{\frac{\partial Q_{t+1}}{\partial M_{t+1}}\frac{dM_t}{dTT_t} - TT_t\frac{dM_{t+1}}{dTT_{t+1}}}{2(Q_{t+1} - TT_tM_{t+1})}. \end{aligned}$$

Since $\partial Q_t / \partial M_t = \theta^{-1} TT_t$, the above simplifies to:

$$\frac{d \log Y_{t+1}}{dTT_{t+1}} = \frac{1-\theta}{\theta} \frac{TT_{t+1} \frac{dM_{t+1}}{dTT_{t+1}}}{(Q_{t+1} - TT_{t+1} M_{t+1})} - \frac{M_{t+1}}{2\,(Q_{t+1} - TT_{t+1} M_{t+1})} + \frac{M_t}{2\,(Q_t - TT_{t+1} M_t)} + \frac{(T_{t+1} - TT_t)\, \frac{dM_t}{dTT_t}}{2\,(Q_{t+1} - TT_t M_{t+1})}. \tag{61}$$

The first term of the right hand side of (61) captures the effect imperfect competition. Evaluating the above expression at $TT_{t+1} = TT_t$, this terms remains,

$$\frac{d \log Y_t}{dTT_t} = \frac{1-\theta}{\theta} \frac{TT_t \frac{dM_t}{dTT_t}}{Q_t - TT_t M_t}.$$

It follows,

$$\frac{d \log TFP_t}{d \log TT_t} = \frac{1-\theta}{\theta} \frac{TT_t M_t}{P_t^{Fisher} Y_t} \left(\frac{d \log M_t}{d \log TT_t} \right) < 0. \tag{62}$$

Appendix B Data

Sample I consider a sample of countries that consists of a list of non-oil exporters, no transition, middle- or high-income countries.[48] The countries are listed in the Table B.1. I split the sample between large countries (G6), developed SOEs (other OECD), and emerging SOEs. The data comes from the World Bank's World Development Indicators (WDI) and Penn World Tables 7.0 (PWT). These databases contain annual data from 1960 to 2008.

TT and TFP Following BKK (1992), TT are defined as the ratio of import prices to export prices, where each price is the implicit deflator from the national accounts (nominal/real). TFP is computed as follows,

$$TFP_t = \frac{Y_t}{K_{t-1}^{\alpha} L_t^{1-\alpha}}$$

where Y_t is the period t *real GDP*, K_{t-1} is the end of period $t-1$ *stock of capital* (beginning of period t stock of capital) and L_t is a measure of labor input utilized in production. The parameter α is set at 0.36, which is the standard value used in the RBC literature. The *stock of capital* is constructed using the perpetual inventory method. This consists in constructing a time series for K_t recursively using,

$$K_t = (1-\delta)\, K_{t-1} + I_t$$

where I_t is investment in period t and parameter δ is the depreciation rate. The recursive constructed of capital is initialized using the steady state condition of capital under the balanced growth path.[49]

The *labor* input is proxied by total hours from PWT. The data are not reported directly by PWT and instead are recovered as follows (PWT's mnemonics are given in parenthesis). Total hours are the product of population (POP) with GDP Per Capita (RGDPL), divided by GDP per hour worked at 2005 constant prices (rgdpl2th). The information is available for most countries in the sample, except for Indonesia, Malaysia, The Philippines, Thailand, and Uruguay. For these countries, the labor input is approximated by total employment, which equals the POP*RGDPL, divided by GDP per employee (rgdpl2te).

[48] The classification of middle-income / high-income countries is taken from the World Bank's World Development Indicator. See IMF (2000) for a list of transition economies. See Chapter 2 of the IMF's World Economic Outlook 2006 for a list of oil exporter. This classification excludes large oil producers for which oil is not a key export, such as Canada, Ecuador, Mexico, and the United Kingdom.

[49] That is $K_0 = I_0 / (\delta + g)$, where g is growth rate of investment in a balanced growth path. The latter is estimated by the average growth rate of investment in the sample.

Table B.1: Correlation of TT with TFP

A. Large industrialized countries (G6)	1960-1979	1980-2008	1960-2008
France	-0.13	-0.51	-0.39
Germany	-0.58	0.12	-0.10
Italy	-0.29	0.16	-0.10
Japan	-0.12	-0.28	-0.20
United Kingdom	-0.04	0.56	0.22
United States	-0.21	-0.37	-0.31
Average	**-0.19**	**-0.02**	**-0.11**
B. Small developed countries (other OECD)	1960-1979	1980-2008	1960-2008
Australia	0.14	0.29	0.22
Austria	-0.03	0.13	0.01
Belgium	-0.17	-0.06	-0.11
Canada	0.03	0.18	0.10
Denmark	-0.36	-0.04	-0.19
Finland	0.20	-0.48	-0.15
Greece	0.29	0.30	0.30
Iceland	-0.54	-0.18	-0.42
Ireland	-0.09	0.17	0.04
Luxembourg	-0.60	0.25	-0.31
Netherlands	0.11	0.10	0.10
New Zealand	-0.18	0.42	0.04
Portugal	-0.55	-0.26	-0.44
Spain	0.41	-0.29	0.13
Sweden	-0.17	-0.39	-0.31
Switzerland	0.68	0.58	0.62
Average	**-0.06**	**0.04**	**-0.03**
C. Emerging countries	1960-1979	1980-2008	1960-2008
Argentina	0.02	-0.40	-0.15
Brazil	0.00	-0.56	-0.32
Chile	-0.70	-0.36	-0.52
Colombia	0.04	-0.38	-0.20
Indonesia	-0.25	-0.18	-0.17
Korea	0.35	-0.56	0.24
Malaysia	-0.56	-0.54	-0.44
Mexico	-0.06	-0.82	-0.57
Peru	0.04	-0.25	-0.16
Philippines	0.30	0.25	0.22
Thailand	-0.20	-0.27	-0.20
Uruguay	0.43	-0.13	0.09
Average	**-0.09**	**-0.35**	**-0.24**

Table B.2: Episodes of long-lasting TFP declines

Country	Start	End	Duration	TFP change	TT change
Argentina	1987	1991	5	-8.94	-10.60
Argentina	1998	2005	8	-7.13	-0.33
Brazil	1980	1986	7	-12.86	11.36
Brazil	1987	1994	8	-4.89	13.13
Brazil	1997	2005	9	-2.31	14.55
Chile	1981	1988	8	-11.72	13.89
Chile	1997	2004	8	-12.50	-5.08
Colombia	1989	2005	14	-3.52	3.27
Indonesia	1984	1988	5	-2.83	14.86
Indonesia	1995	2006	12	-9.81	-6.02
Malaysia	1980	1988	9	-4.14	11.02
Malaysia	1995	1999	5	-4.95	-0.75
Mexico	1981	1991	11	-18.58	42.71
Mexico	1992	1997	5	-4.66	4.60
Peru	1981	1986	6	-15.18	7.89
Peru	1987	1994	8	-29.54	15.03
Peru	1995	2001	6	-3.28	12.37
Philippines	1980	1989	10	-18.68	-1.80
Philippines	1990	1996	7	-5.44	-0.07
Philippines	1997	2002	5	-2.83	-15.45
Thailand	1996	2003	8	-9.24	11.38
Uruguay	1980	1987	8	-20.26	-1.30
Uruguay	1998	2004	7	-8.78	1.23