Financial Frictions and the Interest-Rate Differential in a Dollarized Economy

Hugo Vega*

* Banco Central de Reserva del Perú and London School of Economics

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This paper presents a partial equilibrium characterization of the credit market in an economy with partial financial dollarization. Financial frictions, in the form of costly state verification and banking regulation restrictions, are introduced and their impact on lending and deposit interest rates denominated in domestic and foreign currency studied. The analysis shows that reserve requirements act as a tax that leads banks to decrease deposit rates, while the wedge between foreign and domestic currency lending rates is decreasing in exchange rate volatility and increasing in the degree of correlation between entrepreneur’s returns and the exchange rate.

1. Introduction

The objective of this paper is to study the impact of financial frictions on lending and deposit interest rate differentials in an economy characterized by partial financial dollarization. In order to do this, I extend Bernanke, Gertler & Gilchrist’s (1999) financial accelerator mechanism to incorporate financial dollarization and banking regulation restrictions in a partial equilibrium setting.

Two types of financial frictions are incorporated in the model: first, lending banks face monitoring costs when foreclosing entrepreneurs that default on their loans. This is the standard costly state verification (CSV) mechanism of Townsend (1979) that was introduced in a DSGE framework by Carlstrom and Fuerst (1997) and Bernanke, Gertler & Gilchrist (1999) (denominated BGG from now on). The second type of financial frictions are capital and reserve requirements imposed on banks. The banking setup is a modification of the one used in Cohen-Cole and Martinez-Garcia (2009).

Financial dollarization is present because banks offer entrepreneurs loans denominated in domestic and foreign currency. The reason behind this is that banks themselves are forced to accept deposits denominated in domestic and foreign currency from households. The deposit market is assumed to be competitive.

A secondary objective of the paper is to develop a model that incorporates financial frictions in order to explore the effects of monetary policy over interest rate differentials.

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This is important because the Peruvian economy is partially dollarized and allows for deposits and loans in both domestic and foreign currency, a feature that may distort the conventional transmission mechanism.

In order to provide some intuition regarding the behaviour of lending and deposit interest rates in a dollarized economy, section 2 presents historical data showing the behaviour of interest rates and reserve requirements in Peru during the last decade. Section 3 provides the setup for the model, section 4 shows the conditions of the optimal loan contract and section 5 incorporates the solution procedure for the model. Section 6 discusses the results and section 7 concludes.

2. Data

For the Peruvian economy, there are four reference interest rates: TAMN, TIPMN, TAMEX, and TIPMEX. They are defined as follows:

- TAMN is the weighted-average lending rate in domestic currency.
- TAMEX is the weighted-average lending rate in foreign currency.
- TIPMN is the weighted-average deposit rate in domestic currency.
- TIPMEX is the weighted-average deposit rate in foreign currency.

![Figure 1: Interest rates on loans and deposits in domestic currency](image)
These rates are calculated daily, the data is public and available from the Central Bank of Peru’s website. Figures 1 and 2 show the monthly average of each rate for the period 2001 - 2010. The difference between domestic currency lending and deposits rates is 19 percentage points compared to 8 percentage points for foreign currency. As expected, lending rates are higher than deposit rates. In order to gain further insight on the factors behind this significant difference in spreads the same rates will be presented below, grouped by instrument (loans and deposits).

Figure 3 shows lending rates for both currencies. On average, the difference between the lending rate in domestic versus foreign currency is 13 percentage points. This fact explains most of the difference in spreads observed in Figures 1 and 2 and may suggest a higher external premium required for loans in domestic currency which in turn may be consistent with risk averse firms. In Figure 4, the average difference between domestic and foreign deposit rates is shown to be of just 2 percentage points. Given that households are probably risk averse as well, why isn’t the interest rate on foreign currency deposits (much) higher than its domestic currency counterpart? The only possible explanation for this would be Peru’s higher reserve requirements on foreign deposits which make this funding alternative costly for banks, pushing down the interest rate they’re willing to offer on these deposits.
Turning to reserves, the Central Bank of Peru enforces a higher reserve requirement for deposits and obligations in foreign currency compared to domestic currency as mentioned before. Figure 5 shows the evolution of the rate of reserves that banks effectively hold in order to comply with reserve requirements by type of currency (these are slightly above the required reserves imposed by the Central Bank). Reserve requirements as a monetary policy instrument became more relevant in the past three years (as a result of the crisis) and they are now actively used as a complement to the policy rate (the reference interest rate). Figure 6 illustrates this phenomenon: banks are being forced to hold more reserves when the reference interest rate increases and viceversa.
Besides the effect these reserve requirements have on the wedge between deposit rates, it is important to note that there are hints of a negative relationship between deposit rates and reserve requirements, particularly in the last 2 or 3 years.

Given the information extracted from the data presented, a good characterization of interest rate differentials in a dollarized economy such as the Peruvian one should address:

1. The wedge between domestic and foreign currency lending rates.
2. The relationship between domestic and foreign currency deposit rates and their interaction with reserve requirements.
Next section provides a highly stylized model which strives to provide a framework to study these issues.

3. The Model

This section analyzes a partial equilibrium model where an entrepreneur interacts with a bank. The entrepreneur demands loans denominated in domestic and foreign currency from the bank. The bank funds itself by taking deposits in domestic and foreign currency from households.

Given the partial equilibrium setup, some characteristics of the entrepreneur are considered exogenous. Particularly, his average return on capital and how correlated it is with the nominal exchange rate. In the bank’s case, it is assumed the deposit market is competitive and thus, deposit rates are taken as given.\(^2\)

It is assumed entrepreneurs take loans denominated in domestic \((L)\) and foreign \((L^*)\) currency in order to finance the acquisition of physical capital \(K\). The market price of physical capital is fixed at unity and the bank charges gross interest rate \(I^d\) on loans denominated in domestic currency and gross interest rate \(I^{d*}\) on loans denominated in foreign currency.

Following BGG (1999), the entrepreneur faces an idiosyncratic shock \(\omega\) to his (stochastic) return \(R^e\) over assets \(K\). On top of this, the entrepreneur also faces uncertainty with respect to next period’s nominal exchange rate \(S'\) (defined as the price in domestic currency of one unit of foreign currency). Given that interest factors are fixed before the realization of any shocks, if \(\omega\) turns out to be too small or \(S'\) too high, the entrepreneur cannot repay his debt (in domestic and foreign currency) and goes bankrupt. In this scenario, the bank pays a monitoring cost to recoup what is left of the entrepreneur’s assets.

Thus, the bankruptcy space will be:

\[
\omega R^e K < I^d L + S' I^{d*} L^*.
\] (1)

We can define the cutoff \(\overline{\omega}\) as the particular value for the idiosyncratic shock that allows the entrepreneur to pay his debt without any excess profit:

\[
\overline{\omega} R^e K = I^d L + S' I^{d*} L^*.
\] (2)

where \(S\) denotes the current nominal exchange rate.

Note that uncertainty with respect to next period’s nominal exchange rate (from here on denoted simply as "exchange rate" since the real exchange rate does not play a part in this model) and the entrepreneur’s return (which could depend on the exchange rate as well), implies the cutoff is stochastic. This is the first major departure from BGG (1999): when entrepreneurs’ income and liabilities are denominated in the same currency, the cutoff \(\overline{\omega}\) is fixed. Here, the exchange rate makes one of the liabilities stochastic, implying there will be a different cutoff for every possible realization of next period’s exchange rate.

It is assumed that entrepreneurs possess some net worth, \(N\), which is required as collateral in order to obtain loans from banks. The entrepreneur’s balance sheet links the

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\(^2\)Actually, the credit market is competitive as well, but lending rates are not considered exogenous.
entrepreneur’s net worth and outstanding loans to capital:

\[ K = L + SL^* + N. \] (3)

Banks will lend to entrepreneurs and take deposits in both currencies. Regulatory requirements will also force banks to accumulate a minimum of bank capital \( B \) which we assume is denominated in domestic currency. The bank’s balance sheet equates the loans made to entrepreneurs to bank’s liabilities (deposits) and capital\(^3\):

\[ L + SL^* = (1 - \varphi) D + (1 - \varphi^*) SD^* + B. \]

Here, \( \varphi \) and \( \varphi^* \) stand for the fractions of domestic and foreign currency deposits required as reserves by the banking regulation authority.

The entrepreneur’s expected benefit after loan repayment is,

\[
\Pi^e \equiv E \left[ \int_\omega^{\infty} (\omega R^e K - I^d L - S^d I^d L^*) \phi(\omega) \, d\omega \right],
\] (4)

where the integral comprises the stochastic return on assets and the other terms are loan repayments to the bank. Following BGG (1999), it is assumed that the idiosyncratic shock’s probability density function, \( \phi(\omega) \), corresponds to that of a lognormal distribution with \( E[\omega] = 1 \) and \( Var[\log(\omega)] = \sigma^2 \) the latter being exogenous to the model\(^4\). Using (2), the objective function can be re-written as,

\[
\Pi^e \equiv E \left[ \int_\omega^{\infty} (\omega - \overline{\omega}) \phi(\omega) \, d\omega R^e K \right]
\equiv E [f(\overline{\omega}) R^e K].
\] (5)

Function \( f(\overline{\omega}) \) has been analyzed extensively since Carlstrom and Fuerst (1997): it is the entrepreneur’s share of investment returns \( R^e K \). The marginal effect of \( \overline{\omega} \) on the entrepreneur’s share is negative and increasing.

Bank loans are paid back whenever \( \omega > \overline{\omega} \). Otherwise, the bank forecloses the entrepreneur and pays a fraction \( \mu \) of his remaining assets in order to cover monitoring costs. Thus, the bank’s objective function is,

\[
\Pi^b \equiv E \left[ \int_\omega^{\infty} \left( I^d L + S^d I^d L^* \phi(\omega) \right) \, d\omega \\
+ \left( 1 - \mu \right) \int_0^\overline{\omega} \omega R^e K \phi(\omega) \, d\omega \\
- (I - \varphi) D - S^d (I^* - \varphi^*) D^* - R^b B \right]
\] 
\[ = E \left[ \left( \overline{\omega} \int_\omega^{\infty} \phi(\omega) \, d\omega + \left( 1 - \mu \right) \int_0^\overline{\omega} \omega \phi(\omega) \, d\omega \right) R^e K \\
- (I - \varphi) D - S^d (I^* - \varphi^*) D^* - R^b B \right] 
\] 
\[ = E \left[ \left( g(\overline{\omega}) R^e K \\
- (I - \varphi) D - S^d (I^* - \varphi^*) D^* - R^b B \right) \right] (6)
\]

\(^3\)Actually, Peru’s banking system also funds its operations with foreign credit lines obtained from foreign banks and/or investment firms. Even though this type of funding is empirically relevant, this setup abstracts from it given that it would unnecessarily complicate the exposition without adding significant results.

\(^4\)It follows that \( E[\log(\omega)] = -\sigma^2/2 \).
where \( g(\varpi) \) represents the bank’s share of investment returns, \( I \) and \( I^* \) are the nominal interest factors paid on deposits in domestic and foreign currency respectively and \( R^b \) is the return paid on bank capital which will be assumed to be fixed. All three costs of bank funding, \( I, I^* \) and \( R^b \) will be considered exogenous. The marginal effect of \( \varpi \) on the bank’s share is positive and decreasing.

Besides his balance sheet, the bank faces a number of restrictions due to regulation and/or supervision. The capital requirement restriction takes the form:

\[
\frac{L+SL^*}{B} \leq v,
\]

where \( v > 1 \). Following Cohen-Cole & Martinez-Garcia (2009), the return paid on bank capital \( R^b \) is assumed to be higher than the interest factor paid on deposits denominated in domestic currency \( I \), this guarantees the capital requirement restriction is binding always.

We define the entrepreneur’s liability to net worth ratio, \( q \), as

\[
q = \frac{L+SL^*}{N},
\]

and \( d^L \), a measure of loan dollarization, as

\[
d^L = \frac{SL^*}{L}.
\]

Thus, using the entrepreneur’s balance sheet and these definitions we can re-write the cutoff as

\[
\varpi = \frac{1}{\Re^c} \frac{q}{q+1} \frac{I + I^* \left( \frac{\Re^c}{\Re^b} \right) d^L}{1 + d^L}.
\]

Similarly, the entrepreneur’s and bank’s objectives require re-writing,

\[
\Pi^e \equiv E \left[ f(\varpi) \Re^e \right] (q + 1) N,
\]

\[
\Pi^b \equiv E \left[ -\left( \left( I - \varpi^e + (1-\varpi^e) \frac{\Re^e}{\Re^b} \right) d^D \left( 1 - \frac{1}{v} \right) + \frac{R^b}{v} \right) q \right] N,
\]

where \( d^D = \frac{SD^*}{D} \) is the deposit dollarization ratio.

4. Optimal Loan Contract

The optimal contract can be obtained by maximizing (11) subject to (12) being equal to zero. The implicit assumption is that banks are competitive and they offer the best possible contract to the entrepreneur, at the cost of driving down their profits all the way to zero. Replacing the threshold with the expression shown in (10). The variables of the problem are \( q, d^L, d^D \) and the lending interest rates.

The first order conditions of the problem are:

\[
q : E \left[ \left( f(\varpi) + f'(\varpi) (q + 1) \frac{\partial \varpi}{\partial q} \right) \Re^e \right]
\]
\[ + \lambda E \left[ \left( g(\overline{w}) + g'(\overline{w})(q + 1) \frac{\hat{S}_0}{\hat{S}} \right) R^e \right] = 0, \]  
\[ d^L : E \left[ f'(\overline{w}) \frac{\partial \overline{w}}{\partial d^L} R^e \right] + \lambda E \left[ g'(\overline{w}) \frac{\partial \overline{w}}{\partial d^L} R^e \right] = 0, \]  
\[ d^P : I - \overline{\varphi} = \frac{I - \overline{\varphi}}{I - \overline{\varphi}} \lambda E \left[ \frac{S'}{S} \right], \]  
\[ I^d : E \left[ f'(\overline{w}) \frac{\partial \overline{w}}{\partial I^d} R^e \right] + \lambda E \left[ g'(\overline{w}) \frac{\partial \overline{w}}{\partial I^d} R^e \right] = 0. \]

Condition (15) is standard in the financial accelerator literature and simplifies to

\[ E [f'(\overline{w})] + \lambda E [g'(\overline{w})] = 0, \]  
which implies the Lagrangian multiplier \( \lambda \) is constant. Condition (16) is new and simplifies to

\[ E [f'(\overline{w}) S'] + \lambda E [g'(\overline{w}) S'] = 0. \]

This condition will define the relationship between interest charged in domestic and foreign currency. Given that the threshold is a function of next period's exchange rate \( S' \), the exchange rate cannot be eliminated from the expression.

The first order condition with respect to the loan "dollarization" variable, \( d^L \), simplifies to,

\[ (E [f'(\overline{w})] + \lambda E [g'(\overline{w})]) I^d = (E [f'(\overline{w}) S'] + \lambda E [g'(\overline{w}) S']) I^d. \]

It is easy to note that if the FOCs with respect to \( I \) (17) and \( I^d \) (18) hold, then (19) is guaranteed to hold as well. In the model, exchange rate risk can be compensated through the interest rates or the dollarization rate. One of the variables becomes redundant.

Condition (14) means that in order for both types of deposit (in domestic and foreign currency) to coexist, the cost of funding in each currency must be equal. Since both deposit interest factors and expected depreciation \( E_t \left[ \frac{S'}{S} \right] \) are considered exogenous, nothing in the model guarantees (14) will hold.

Thus, it will be assumed that this condition holds. There are at least two justifications for this: first, a competitive market with households offering both deposits in domestic and foreign currency should result in the deposit rates adjusting endogenously to guarantee (14); second, if (14) does not hold, banks would accept only one type of deposit implying too much or too little demand for foreign currency, and the exchange rate would have to adjust to outset this.
The remaining first order condition, with respect to \( q \), is standard as well and simplifies to:

\[
E \left[ f \left( \omega \right) R^e \right] + \lambda E \left[ g \left( \omega \right) R^e \right] = \lambda \left( \left( I - \varphi \right) + \left( I^* - \varphi^* \right) \frac{\varphi}{\varphi^*} dD \left( 1 - \frac{1}{v} \right) + \frac{R^b}{v} \right),
\]

note the bracket on the right hand side is basically the bank’s weighted cost of funding. In order to simplify it even more, an assumption tying \( R^b \) to the nominal interest factor on deposits denominated in domestic currency, \( I \), is missing:

\[
I = \varepsilon R^b. \tag{20}
\]

where \( 0 < \varepsilon < \frac{I(1 - \varphi)}{I - \varphi} \).

This guarantees the bank’s funding through equity is always more costly than through deposits. The idea is taken from Cohen-Cole & Martinez-Garcia (2009) who justify (20) arguing that households face a tax on bank dividends but not on interests gained from deposits.

Thus, condition (14) coupled with (20) allows even further simplification of the first order condition with respect to the entrepreneur’s liability-net worth ratio:

\[
E \left[ f \left( \omega \right) R^e \right] + \lambda E \left[ g \left( \omega \right) R^e \right] = \lambda \left[ \left( 1 - \frac{1}{v} \right) \frac{I - \varphi}{1 - \varphi} + \left( \frac{1}{v} \right) \frac{I}{\varepsilon} \right]. \tag{21}
\]

Note that all uncertainty has been removed from the right hand side: the bank’s average funding cost is exogenous from the bank’s point of view.

The last condition required to characterize the (partial) equilibrium of this problem is that bank’s expected profits must be zero, i.e.: there is perfect competition in banking. Simplification of that condition yields,

\[
E \left[ g \left( \omega \right) R^e \right] (q + 1) = q \left[ \left( 1 - \frac{1}{v} \right) \frac{I - \varphi}{1 - \varphi} + \left( \frac{1}{v} \right) \frac{I}{\varepsilon} \right]. \tag{22}
\]

This condition also resembles one found in BGG (1999).

5. Solution

Equations (17), (18), (21) and (22) form a system in four unknowns: \( I^l \), \( I^s \), \( q \) and \( \lambda \). All traces of the cutoff \( \omega \) can be eliminated from them using (10).

Given the functional forms and moments present in the four-equation system, in order to proceed, second order approximations will be used. In particular, a second-order approximation of (17) and (18) calculated in the vicinity of \( E \left[ \omega \right] \) and \( S \) will be used to pin down the relationship between the lending interest rates.
Combining the approximate versions of (17) and (18) it can be demonstrated that,
\[
\frac{f'( E \[ \varpi \]) + \lambda g'( E \[ \varpi \])}{f''( E \[ \varpi \]) + \lambda g''( E \[ \varpi \])} \left( E \left[ \frac{S'}{S} \right] - 1 \right) \\
= E \[ \varpi \] E \left[ \frac{S'}{S} \right] - E \left[ \varpi \frac{S'}{S} \right].
\]

(23)

This condition states that, barring expected appreciation or depreciation of the exchange rate, the cutoff and ex-ante depreciation of the exchange rate must be independent.

In order to proceed, we will assume that there is a competitive and liquid foreign currency market which guarantees that
\[
E \left[ \frac{S'}{S} \right] = 1,
\]

(24)

note this implies that deposit interest rates \( I \) and \( I^* \) must adjust to make sure (14) holds. Condition (24) also implies the nominal exchange rate follows a random walk process.

Using the latter assumption and the definition of the cutoff on (23) and simplifying implies:
\[
I^l E \left[ \frac{1}{R^e} \left( \frac{S'}{S} - 1 \right) \right] + I^{*l} E \left[ \frac{1}{R^e} \frac{S'}{S} \left( \frac{S'}{S} - 1 \right) \right] d^L = 0.
\]

(25)

This expression determines the relationship between both lending interest rates. Two particular cases are worth noting in order to gain some intuition on what (25) implies. If we assume \( R^e \) is non-stochastic (fixed) and equal to \( \bar{R} \), then (25) implies \( I^{*l} \) must be zero. The optimal contract involves no debt denominated in foreign currency. On the other hand, if \( R^e \) equals \( \frac{S'}{S} \) (which implies perfect correlation with the exchange rate) then (25) results in \( I^l \) being equal to zero, the entrepreneur is not offered any debt in domestic currency.

This reasoning leads us to the conclusion that the correlation between \( R^e \) and \( \frac{S'}{S} \) is key. This makes sense: if a well defined demand for loans is to exist in both currencies, those denominated in foreign currency must provide some additional benefit to the entrepreneur given that they expose him to exchange rate risk; when \( R^e \) and \( \frac{S'}{S} \) have some degree of correlation, foreign currency liabilities act as a form of insurance. The implication is that the BGG (1999) setup implies some degree of risk aversion, which can be traced to the presence of the cutoff on the probability of default.

In order to simplify the relationship between the lending interest rates some more, an assumption regarding the stochastic process of \( R^e \) is made:
\[
\frac{1}{R^e} = \gamma \frac{1}{\bar{R}} + (1 - \gamma) \frac{S}{S'}.
\]

(26)

Note this assumption embodies the particular cases mentioned above when \( \gamma = 1 \) (entrepreneurial return is non-stochastic) and \( \gamma = 0 \) (entrepreneurial return is perfectly

\[^5\text{See Figure 7 in the Appendix.}\]
correlated with the exchange rate). In general, \(0 < \gamma < 1\) implies there is some degree of correlation between the stochastic entrepreneurial return and the exchange rate, a condition necessary for the lending interest rates to be well defined. Using (26) on (25):

\[
I^{st} = \frac{R^e (1 - \gamma) \left( E \left[ \frac{S}{s'} \right] - 1 \right)}{\gamma \ Var \left[ \frac{S}{s'} \right] d^L} I.
\] (27)

Thus, there is a positive relationship between both lending interest rates\(^6\). Furthermore, the interest rate charged on foreign currency denominated loans is decreasing on the variance of the exchange rate and the degree of dollarization of deposits; it is increasing on the degree of correlation between the entrepreneur’s returns and the exchange rate. These results are all fairly intuitive: higher exchange rate variance implies higher exchange rate risk being taken on by the entrepreneur on foreign currency loans and he will have to be offered a lower interest rate on them in compensation. A higher degree of loan dollarization implies higher exposure to exchange rate risk as well. If the degree of correlation between the entrepreneur’s returns and the exchange rate is higher then foreign currency denominated loans become better insurance against exchange rate risk and the entrepreneur will be willing to pay a higher interest rate on loans which provide said insurance.

This result also highlights the fact that exchange rate risk faced by the entrepreneur taking foreign currency denominated credit can be compensated through a lower foreign currency lending interest rate, \(I^{st}\), or a lower credit dollarization ratio, \(d^L\). Since both options are perfect substitutes (in the sense that both can perfectly compensate the entrepreneur for exchange rate risk) then the credit dollarization ratio is not identified. This shortcoming arises from the fact that we assume (in order to simplify the exposition) that all entrepreneurs taking loans from the bank share the same \(\gamma\). A more realistic setup would involve entrepreneurs with different values of \(\gamma\) taking loans from the same bank. Given that the bank would have to offer them the same (or at least, fairly similar) interest rates, an endogenous credit dollarization ratio dependent on the distribution of \(\gamma\) should be obtainable. Such an exercise is left for further research.

The remaining equations, (17), (21) and (22) form a system in three unknowns. In order to gain some intuition, define the bank’s marginal cost of funding, \(BMCF\) as:

\[
BMCF = \left[ \left( 1 - \frac{1}{v} \right) \frac{I - \varphi}{1 - \varphi} + \left( \frac{1}{v} \right) \frac{I}{\varepsilon} \right].
\] (28)

Then, equations (21) and (22) can be expressed as:

\[
E \left[ f (\varpi) \frac{R^e}{BMFC} \right] + \lambda E \left[ g (\varpi) \frac{R^e}{BMFC} \right] = \lambda,
\] (29)

and

\[
E \left[ g (\varpi) \frac{R^e}{BMFC} \right] (q + 1) = q.
\] (30)

\(^6\)Jensen’s Inequality states that given a convex function \(g\), \(E [g (x)] > g (E [x])\). Thus, \(E [1/x] > 1/E [x]\) implying \(E [S/S'] > 1/E [S'/S] = 1\).
These two, coupled with (17) compose a system very similar to the one studied in BGG (1999). In order to ease comparison, the system analyzed by BGG (1999) is reproduced here:

\[ f' (\varpi) + \lambda g' (\varpi) = 0 \quad (31) \]
\[ f (\varpi) R^e + \lambda g (\varpi) R^e = \lambda R \quad (32) \]
\[ g (\varpi) R^e (q + 1) = q R \quad (33) \]

All variables used have similar interpretations to the ones introduced previously and \( R \) is the risk free rate: BGG (1999) assumed entrepreneurs obtained loans from a financial intermediary that funded itself from households at rate \( R \). Given that the formulation studied by BGG (1999) has a non-stochastic threshold, the solution of the system is a fixed vector \((\lambda, \varpi, q)\) satisfying (31), (32) and (33) with \( \frac{R^e}{R} \), the external finance premium, being the only "exogenous" variable, coupled with functional forms \( f \) and \( g \).

Given the non-linearity of the system, the strategy in BGG (1999) is to begin using (31) to show that \( \partial \lambda / \partial \varpi \) is positive. This result coupled with a total derivative of (32) results in \( \partial \left( \frac{R^e}{R} \right) / \partial \varpi \) being positive as well. Finally, a similar procedure on (33) is used to demonstrate that \( \partial q / \partial \varpi \) must be positive too. Thus, it must be the case that \( \partial \left( \frac{R^e}{R} \right) / \partial q \) is positive, i.e.: the external finance premium is increasing in the entrepreneur’s leverage (since the liability-net worth ratio \( q \) and leverage -the asset to net worth ratio- are positively related if net worth is fixed).

The strategy employed by BGG (1999) has to be slightly modified in order to apply it to this problem. First, the stochastic nature of the cutoff makes taking derivatives with respect to it slightly awkward. Second, the entrepreneurial return \( R^e \) is stochastic as well, so \( \partial \left( \frac{R^e}{R} \right) / \partial \varpi \) is probably not well defined.

Thus, instead of using the cutoff as the "link" variable between the external finance premium and \( q \), the lending interest rate on loans denominated in domestic currency, \( I \), will take this role.

As a first step, note that (10) implies

\[ \frac{\partial \varpi}{\partial I^l} = \frac{1}{R^e} \left( \frac{q}{q + 1} \right) \left( \frac{1}{1 + d} \right) > 0 \quad (34) \]
a result we will be relying on.

In order to show that \( \partial \lambda / \partial I^l \) is positive, note that

\[ \lambda^{-1} = - \frac{E [q' (\varpi)]}{E [f' (\varpi)]} \quad (35) \]

from (17). Furthermore,

\[ \frac{\partial \lambda^{-1}}{\partial I^l} = - \frac{1}{\lambda^2} \frac{\partial \lambda}{\partial I^l} \quad (36) \]

thus, proving \( \partial \lambda^{-1} / \partial I^l \) is negative is sufficient for our purposes. Given the (implicit) definitions of \( f \) and \( g \) given in (5) and (6), (35) can be expressed as:

\[ \lambda^{-1} = 1 - \mu \frac{E [\varpi \phi (\varpi)]}{E \left[ \int_{\varpi}^{\infty} \phi (\omega) \, d\omega \right]} \quad (37) \]
In BGG (1999), the expression\( \mathbb{E} \phi(\overline{\omega}) / (1 - \Pr[\omega < \overline{\omega}]) \) is assumed to be increasing in \( \overline{\omega} \). An analogous assumption would be sufficient for our purpose but given the nuances of the problem, further detail will be provided. In particular,

\[
\frac{\partial \lambda^{-1}}{\partial I} = -\mu \left( \frac{E \left[ (\phi(\overline{\omega}) + \overline{\omega} \phi'(\overline{\omega})) \frac{\partial \overline{\omega}}{\partial I} \right] + \frac{E [\overline{\omega} \phi(\overline{\omega})] E \left[ \phi(\overline{\omega}) \frac{\partial \overline{\omega}}{\partial I} \right]}{E \left[ \int_{\overline{\omega}}^{\infty} \phi(\omega) \, d\omega \right]^2} \right),
\]

implying we require

\[
E \left[ (\phi(\overline{\omega}) + \overline{\omega} \phi'(\overline{\omega})) \frac{\partial \overline{\omega}}{\partial I} \right] + \frac{E [\overline{\omega} \phi(\overline{\omega})] E \left[ \phi(\overline{\omega}) \frac{\partial \overline{\omega}}{\partial I} \right]}{E \left[ \int_{\overline{\omega}}^{\infty} \phi(\omega) \, d\omega \right]} > 0
\]

(39)
to guarantee our result. Given our assumption that \( \omega \) follows a lognormal distribution, it can be shown that

\[
\phi'(\overline{\omega}) = -\frac{1}{\overline{\omega}} \left( 1 + \frac{\log(\overline{\omega}) + \frac{\sigma^2}{2}}{\sigma^2} \right) \phi(\overline{\omega})
\]

(40)
which allows us to rewrite (39):

\[
E \left[ \left( \frac{E [\overline{\omega} \phi(\overline{\omega})]}{E \left[ \int_{\overline{\omega}}^{\infty} \phi(\omega) \, d\omega \right]} - \frac{\log(\overline{\omega}) + \frac{\sigma^2}{2}}{\sigma^2} \right) \phi(\overline{\omega}) \frac{\partial \overline{\omega}}{\partial I} \right] > 0.
\]

(41)

Note there are three terms being multiplied in (41). Given that the last two terms, \( \phi(\overline{\omega}) \) and \( \frac{\partial \overline{\omega}}{\partial I} \), are positive, in order for the condition to hold, the first one must be positive too. A low average cutoff is enough to guarantee that will be the case.

The next step requires demonstrating \( \frac{\partial (R_e^{BMFC})}{\partial I} \) is positive. The added difficulty comes from the fact that \( R_e \) is stochastic. Total differentiation of expression (29) with respect to \( I^l \) yields:

\[
E \left[ f(\overline{\omega}) \frac{\partial (R_e^{BMFC})}{\partial I^l} + f'(\overline{\omega}) \frac{R_e^{BMFC}}{BMFC} \frac{\partial \overline{\omega}}{\partial I^l} \right]
\]

\[
+ \lambda E \left[ g(\overline{\omega}) \frac{\partial (R_e^{BMFC})}{\partial I^l} + g'(\overline{\omega}) \frac{R_e^{BMFC}}{BMFC} \frac{\partial \overline{\omega}}{\partial I^l} \right]
\]

\[
= E \left[ 1 - g(\overline{\omega}) \frac{R_e^{BMFC}}{BMFC} \right] \frac{\partial \lambda}{\partial I^l}
\]

(42)

Using (17) and (34) on this expression and simplifying results in:

\[
E \left[ (f(\overline{\omega}) + \lambda g(\overline{\omega})) \frac{\partial (R_e^{BMFC})}{\partial I^l} \right] = \frac{\partial \lambda}{\partial I^l} \left( \frac{1}{q + 1} \right).
\]

(43)

This expression will hold if \( \frac{\partial (R_e^{BMFC})}{\partial I^l} \) is positive given that \( f(\overline{\omega}) + \lambda g(\overline{\omega}) \) is positive for every possible realization of \( \overline{\omega} \).
Finally, applying a similar procedure to (30) yields:

$$\frac{\partial q}{\partial I} = E \left[ g(\bar{w}) \frac{\partial \left( \frac{R}{BMFC} \right)}{\partial I} \right] (q + 1)^2 + E [g'(\bar{w})] \frac{q (q + 1)}{BMFC (1 + d)}.$$  

(44)

Where previous results guarantee the right hand side must be positive. Thus, the conclusion is analogous to BGG (1999):

$$\frac{\partial \left( \frac{R}{BMFC} \right)}{\partial q} > 0$$

the external finance premium must be increasing in the firm’s leverage.

6. Discussion

Results (14) and (27) are the main contributions of the model developed. The model predicts a modified interest rate parity condition to govern deposit rates and several factors affecting the wedge between domestic and foreign currency lending rates.

Result (14) requires some elaboration. The optimal contract involves not one but two dollarization decisions: that of loans and deposits. The deposit dollarization restriction results in (14). It follows that the reason behind this result is the fact that banks are basically risk neutral when it comes to the funding decision (the bank’s objective function is always linear in both types of deposit).

Turning to the implications of (14), the inverse relation between a deposit interest rate and it’s corresponding reserve requirement is evident. If we move one step further and consider $I$, the domestic currency deposit rate, to be the monetary policy instrument, then it is clear that monetary policy can influence the foreign currency deposit rate directly or through the use of reserve requirements.

There is another implication behind (14). If we abandon the (long run) assumption of $E[S^*] = S$, then short run fluctuations in the exchange rate imply short run movements in the bank’s dollarization ratios. There is evidence in Peru’s data to support this claim.7

Deposit dollarization seems to be left hanging in the air. Even though this is true in the setup, adding the household’s saving decision along the lines of Devereux & Sutherland (2007) pins down this variable, without modifying the results presented above. This is shown in the appendix.

Moving on to result (27), it provides plenty of mileage to explain the wedge between domestic and foreign currency lending rates. The first suspect is the variance of the exchange rate. Noting that $I^l$ and $I^{eL}$ are both interest factors, a small variance can help a great deal towards explaining the big difference between lending rates observed in the data. Furthermore, a small correlation between entrepreneur’s returns and the exchange rate (high $\gamma$) can also help explain the difference. Empirical analysis would be required to assess these claims, but that is left for further research.

Determining the degree of credit dollarization, $d^L$, proves troublesome. Equation (27) shows that exchange rate risk can be dealt with through the lending rate differential or by changing the degree of credit dollarization. In a more realistic context, it would be

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7See Figure 8 in the Appendix
expected that the bank has to lend to a variety of entrepreneurs with different $\gamma$. Given that the bank should offer the same interest rate differential to all of them, it follows that the degree of credit dollarization should depend on the distribution of $\gamma$.

Monetary policy can influence the variance of the exchange rate. In the extreme, a fixed exchange rate scheme should wipe the distinction between domestic and foreign currency lending rates according to (27). Regarding the correlation between entrepreneur’s returns and the exchange rate, it is unclear whether monetary policy could affect the wedge between interest rates through this channel. General equilibrium analysis would probably be required to answer that question.

6.1. Thought experiment: a monetary policy shock

What are the implications of a monetary policy shock in this (partial equilibrium) model? If we take the domestic currency deposit rate, $I$, to be the monetary policy instrument, then the analysis developed in this paper would suggest:

**Foreign currency deposit rates should increase as well.** This follows from (14). The wedge between domestic and foreign currency deposit rates remains invariant though.

**Domestic currency lending rates should increase.** An increase in the domestic currency deposit rate implies higher bank marginal funding cost ($BMFC$). Higher $BMFC$ will drive up the equilibrium cutoff $\overline{\pi}$ (this can be shown with a total derivative of (29) with respect to $BMFC$). Given the positive relationship between the cutoff and the domestic currency lending rate, the latter must increase as well.

**Foreign currency lending rates should increase.** This follows from (27) and the fact that domestic currency lending rates should increase. Again, the wedge between domestic and foreign currency lending rates should remain the same.

7. Conclusion

The model presented provides some interesting insights into the relationship between the different interest rates that arise in an economy with partial financial dollarization such as Peru. Still, there is quite a lot of work pending in order to gain more insight into these relationships.

The first point that must be made is the need for quantitative analysis. Empirical test of the propositions made and evaluation of the magnitudes involved is crucial to continue progress in this area.

Another important issue is the fact that partial equilibrium does not allow a complete analysis of the implications of this mechanism for monetary policy. Incorporating this setup into a dynamic stochastic general equilibrium model should be fairly straightforward with the added benefit of being able to quantify some of the predictions through proper calibration.

Turning to the model’s relevance, it is important to point out that even though the setup is motivated by financial dollarization in general and Peru’s characteristics in particular, the mechanism developed has other applications. The first, and most obvious one, that comes to mind are international banks operating in several countries. These institutions "lend" to financial intermediaries worldwide in several different currencies. These local financial intermediaries can be interpreted as our "entrepreneurs" with international banks incurring in country risk wherever they lend. Obviously, a similar parallel can be made
for global investment funds and other financial institutions operating worldwide. Indeed, the international macrofinance literature has recently taken great interest in this topic, encouraged by the global financial crisis.

8. References


9. Appendix

9.1. Additional figures

Figure 7: Monthly depreciation in Peru ($S_{t+1}/S_t$)

![Graph showing monthly depreciation in Peru](image)

Figure 8: Long Position of Peru’s Banking System (Net dollar assets as a percentage of total assets) and Nominal Exchange Rate

![Graph showing long position and exchange rate](image)

9.2. Household portfolio decision in a partially dollarized context

This section borrows heavily from Devereux & Sutherland (2007). Assume households have the objective:

$$\max \sum_{t=0}^{\infty} \beta^t E_t \left[ \frac{C_t^{1-\sigma^{-1}}}{1-\sigma^{-1}} \right]$$
subject to a budget constraint:

\[ P_t C_t + D_t + D_t^* = Y_t + R_{t-1}^D D_{t-1} + \frac{R_{t-1}^{D^*}}{S_{t-1}} D_{t-1}^* \]

where income \((Y_t)\) is exogenous, stochastic, and possibly correlated with the nominal exchange rate \((S_t)\). The household must divide savings between two deposit accounts, one denominated in domestic currency \((D_t)\) and the other in foreign currency \((D_t^*)\). Note that savings allocated to the foreign currency deposit account must be converted to foreign currency in order to earn interest \(R_t^{D^*}\).

In order to setup the problem properly, we introduce two transformations. Total savings \((W_t)\) are defined as:

\[ D_t + D_t^* = W_t \]

Then, deposit dollarization \((\alpha_t)\) is defined as:

\[ \alpha_t = \frac{D_t^*}{W_t} \]

Thus, the budget constraint becomes,

\[ W_t = \alpha_{t-1} \left( R_{t-1}^{D^*} \frac{S_t}{S_{t-1}} - R_t^D \right) W_{t-1} + R_{t-1}^D W_{t-1} + Y_t - P_t C_t \]

The Lagrangian of the problem would be:

\[
\max_{C_t, W_t, \alpha_t} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t E_t \left[ \frac{C_{t-\sigma}^{1-\sigma}}{1-\sigma} + \lambda_t \left( -W_t + \alpha_{t-1} \left( R_{t-1}^{D^*} \frac{S_t}{S_{t-1}} - R_t^D \right) W_{t-1} \right) \right]
\]

Taking first order conditions, we obtain the following expressions:

\[ C_t : \left( C_t^{-\frac{1}{2}} - \lambda_t P_t \right) = 0 \]

\[ W_t : \beta^t \lambda_t + \beta^{t+1} \lambda_{t+1} \left( \alpha_t \left( R_t^{D^*} \frac{S_{t+1}}{S_t} - R_t^D \right) + R_t^D \right) = 0 \]

\[ \alpha_t : \beta^t \lambda_{t+1} \left( R_t^{D^*} \frac{S_{t+1}}{S_t} - R_t^D \right) W_t = 0 \]

Combining the first and last expressions,

\[ E_t \left[ R_t^{D^*} \frac{P_t S_{t+1}}{P_{t+1} S_t} C_{t+1}^{-\frac{1}{2}} \right] - E_t \left[ R_t^D \frac{P_t S_{t+1}}{P_{t+1} S_t} C_{t+1}^{-\frac{1}{2}} \right] = 0 \]

Devereux & Sutherland (2007) take a second-order approximation of an analogous expression in order to figure out portfolio composition in their country portfolio setup. The main issue discussed by them is the particular point around which the approximation is done. They show that using the non-stochastic equilibrium as the reference point yields correct
solutions for the equilibrium portfolio composition. Furthermore, only one second-order approximation needs to be done: the budget constraint is required in order to solve for \( \alpha \) but a first-order approximation of it is sufficient.

A second-order approximation of the last expression yields:

\[
(r_t^D - r_t^D) \left( 1 - \frac{1}{\sigma} E_t [c_{t+1}] - E_t [\pi_{t+1}] \right) - \frac{1}{\sigma} E_t \left[ c_{t+1} \frac{S_{t+1}}{S_t} \right] \\
+ \frac{1}{\sigma} E_t [c_{t+1}] - E_t \left[ \frac{P_{t+1} S_{t+1}}{P_t S_t} \right] + 1 + E_t [\pi_{t+1}] \approx 0
\]

In steady state, all deviations are zero \( (r_t^D, r_t^D, c_{t+1}, \pi_{t+1}) \), but the expectations on products are not. Consumption might have some covariance with the exchange rate and prices as well (particularly if the household consumption basket includes goods priced in foreign currency).

In order to solve for the equilibrium portfolio, a first-order approximation of the budget constraint is required:

\[
c_{t+1} + R^D \frac{W}{PC} \left( \frac{P_{t+1}}{P_t} - 1 \right) = \alpha \frac{W}{PC} R^D r_t^D + (1 - \alpha) \frac{W}{PC} R^D r_t^D \\
+ \alpha R^D \frac{W}{PC} \left( \frac{S_{t+1}}{S_t} - 1 \right) + \frac{R^D}{C} \left( \frac{W_t}{P_t} - \frac{W}{P} \right) + \frac{1}{C} \left( \frac{Y_{t+1}}{P_{t+1}} - \frac{Y}{P} \right) - \frac{1}{C} \left( \frac{W_{t+1}}{P_{t+1}} - \frac{W}{P} \right)
\]

From this approximation, \( E_t \left[ c_{t+1} \frac{S_{t+1}}{S_t} \right] \) can be constructed and replaced in the previous expression. Further simplification and application of steady state values will yield:

\[
\alpha \approx \frac{(R^D \frac{W}{P} - \sigma C) \text{Cov} \left[ \frac{P_t}{P}, \frac{S_t}{S} \right] - \text{Cov} \left[ \frac{Y_t}{P}, \frac{S_t}{S} \right] + \text{Cov} \left[ \frac{W}{P}, \frac{S_t}{S} \right]}{R^D \frac{W}{P} \text{Var} \left[ \frac{S_t}{S} \right]}
\]

Which is the result we require. Deposit dollarization is increasing in the covariance between prices and the nominal exchange rate: if prices increase when the exchange rate depreciates, foreign currency deposits hedge this risk. On the other hand, higher covariance between real income \( (\frac{Y}{P}) \) and the nominal exchange rate discourages deposit dollarization since income itself would be the hedge consumers require against exchange rate risk. Higher exchange rate variance discourages deposit dollarization as well.