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Total factor productivity and signal noise volatility in an incomplete information setting

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Abstract

Imperfection information models where agents solve some kind of signal extraction problem are multiplying and developing fast. They have commonly been used to study the impact of imperfect information on the business cycle and the importance of news versus noise shocks. This paper attempts to apply the framework to a different, albeit related, question: that of the effect of volatility (both in news and noise) on the economy, from a long and short run perspective. An RBC model where the agent faces imperfect information regarding productivity is developed and calibrated in order to address the question, coming to the conclusion that the long run effect is insignificant while further development is required to address the short run conclusively.

1 Introduction

General equilibrium models of incomplete information are rapidly gaining prominence in the literature. They provide a useful framework to study agent responses in a context of unobservable variables complemented with signals. Thus, they have been used to analyze the impact of permanent and temporary shocks affecting a wide array of exogenous variables on agent decisions when they cannot distinguish perfectly between them. Several studies have reported on the impact of the introduction of this expectation mechanism compared to the usual full information one and perhaps the most interesting result of the comparison has been the appearance of a "hump-shaped" consumption impulse response function even with CRRA utility functions (the standard New Keynesian framework requires the assumption of habit formation in order to deliver the same result).

Other studies have used this framework to explore the relative importance of "real" versus "noise" shocks, the latter being considered as measurement error or forecast error amongst other interpretations.

Yet, we are not aware of any attempt to explore the consequences of higher volatility in either real or noise shocks. Intuitively, higher volatility of real shocks (permanent or temporary) implies a higher degree of uncertainty in future outcomes, whereas higher noise shock volatility could be interpreted as a decrease in forecast quality and/or precision.

This paper focuses on the effects of higher real or noise volatility and its impact on both long and short run dynamics. In particular, the effects of higher TFP variance will be explored, together with higher noise variance in the signal used to obtain information about the components of TFP.

Turning to related literature, the model explored here draws heavily from Blanchard et al. (2009). They present a simple consumption model where the random-walk result holds and then assume imperfect information in the form of unobservable variables coupled with signals delivering information about them. They show the consumer's signal extraction problem, solve it, and then proceed to evaluate the model empirically. Amongst other things, they demonstrate that an "econometrician" with no informational advantage to the agents cannot distinguish between news and noise shocks from the estimation of structural VAR's and that noise shocks play an important role in short-run fluctuations.

Blanchard et al. (2009) assume log productivity has a permanent (unit root) and transitory component. Productivity itself is perfectly observable but its components are not. In order to gain some information about the permanent component, a signal based on it is included in the agent's information set. This paper will employ the same setup, expanding the model to include all the pieces of a classic RBC, allowing general equilibrium analysis.

Lorenzoni (2008) presents a model of business cycles driven by shocks to consumer expectations regarding aggregate productivity. Agents are hit by heterogeneous productivity shocks, they observe their own productivity and a noisy public signal regarding aggregate productivity. The public signal gives rise to "noise shocks", which have the features of aggregate demand shocks: they increase output, employment and inflation in the short run and have no effects in the long run.

The dynamics of the economy following an aggregate productivity shock are also affected by the presence of imperfect information: after a positive productivity shock output adjusts gradually to its higher long-run level, and there is a temporary negative effect on inflation and employment.

His paper explores the idea of expectation-driven cycles, looking at a model where technology determines equilibrium output in the long run, but consumers only observe noisy signals about technology in the short run. The presence of noisy signals produces expectation errors. The role of these expectation errors in generating volatility at business cycle frequencies constitutes the main result. The author is interested in the interaction of productivity and "noise" shocks in generating the business cycle, he endows the agent with a Kalman filter that is used to "learn" about the nature of the shocks.

Lorenzoni's work differs from Blanchard et al. in that the former assumes log productivity is the result of a "permanent" (unit root) process plus an i.i.d component. Thus, his "noise" shock is mistaken initially for a real one, mechanism that drives his result. Blanchard et al. (2009) fail to include the extra i.i.d component into productivity, essentially allowing some shocks to be perfectly identifiable.

Collard et al. (2009) provide a broad review of the effects of imperfect information on the business cycle. Using a New Keynesian framework, they introduce imperfect information in several different ways, affecting productivity or monetary policy. The objective of their paper is to estimate the impact of the imperfect information assumption in business cycle fluctuations and they conclude that it is "quantitatively relevant, conceptually satisfactory and empirically plausible".

Clearly, the vast majority of work done in imperfect information models with signal extraction has not attempted to study the effects of process or signal noise volatility. Senhadji (2000) attempts to estimate TFP for several country groups over different time periods. He reports that log productivity volatility differs significantly between country groups implying there is enough cross-country variation to justify exploring its consequences both in the long (steady state) and short run (business cycle).

In order to address the question, we will construct a simple RBC model with a consumer-producer agent who faces imperfect information regarding the components of productivity and deals with the problem by applying a Kalman filter to the information at his disposition ("total" log productivity and a signal regarding its permanent component).

2 The Model

The model is largely based on a standard real business cycle structure with a representative agent in charge of consumption and production decisions. The agent's signal extraction problem will generate all expectations required to solve the optimization problem he faces. Output is produced according to:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}. \tag{1}$$

Capital accumulation follows the usual definition:

$$K_{t+1} = I_t + (1-\delta) K_t. \tag{2}$$

We will assume the only asset available in the economy is physical capital implying the following resource constraint:

$$Y_t = C_t + I_t. \tag{3}$$

The agent's objective is to maximize the (subjective) present discounted value of utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U\left(C_t\right) - \frac{L_t^{1+\eta}}{1+\eta} \right\}; \quad U\left(C_t\right) = \frac{C_t^{1-\gamma}}{1-\gamma}. \tag{4}$$

Thus, using standard solution methods it can be shown the agent must choose consumption according to the following Euler equation:

$$U'\left(C_t\right) = E_t\left[\beta U'\left(C_{t+1}\right)\left(\alpha A_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + 1 - \delta\right)\right]. \tag{5}$$

Labour supply will be the result of equating marginal disutility of labor to it's marginal product expressed in consumption units:

$$L_t^{\eta} = (1-\alpha) A_t K_t^{\alpha} L_t^{-\alpha} U'(C_t). \tag{6}$$

Following Blanchard et al. (2009), we will assume the log of TFP, $a_t = \ln A_t$, is the sum of two components:

$$a_t = x_t + z_t. \tag{7}$$

The permanent component, x_t, follows a unit root process of the form:

$$\Delta x_t = \rho_x \Delta x_{t-1} + \epsilon_t. \tag{8}$$

The transitory component, z_t, follows a stationary AR(1) process of the form:

$$z_t = \rho_z z_{t-1} + \eta_t. \tag{9}$$

Both parameters ρ_x and ρ_z are in [0,1), and ϵ_t and η_t are i.i.d. normal shocks with variance σ_ϵ^2 and σ_η^2 respectively. Agents observe productivity a_t but not the individual components. For the sake of analytical convenience, log productivity will be assumed to follow a random walk:

$$a_t = a_{t-1} + u_t, \tag{10}$$

with the variance of u_t equal to σ_u^2. Thus, certain restrictions on the parameters of its components must be imposed to guarantee consistency. In particular,

$$\rho_x = \rho_z = \rho,$$
$$\sigma_\epsilon^2 = (1-\rho)^2 \sigma_u^2, \quad \sigma_\eta^2 = \rho \sigma_u^2,$$

for some ρ in [0,1).

3 Model Solution and Calibration

The key to solving the agent's problem lies in the formulation of $E_t[A_{t+1}] = E_t[\exp(a_{t+1})]$. From (7) - (9) it can be shown that,

$$a_{t+1} = (1+\rho) x_t - \rho x_{t-1} + \rho z_t + \epsilon_{t+1} + \eta_{t+1}, \tag{11}$$

thus, given information at period t, a_{t+1} is normally distributed. This allows the application of standard log normal properties to $E_t[\exp(a_{t+1})]$:

$$E_t[\exp(a_{t+1})] = \exp\left(E_t[a_{t+1}] + \frac{1}{2} Var_t[a_{t+1}]\right). \tag{12}$$

In order to calculate the expectation and variance of next period's log productivity, the agent will have to solve a signal extraction problem by means of the Kalman filter. Following Blanchard et al. (2009) again, each period the agent observes current productivity and receives a signal, s_t, which provides information regarding the permanent component of the productivity process:

$$s_t = x_t + \nu_t, \tag{13}$$

where ν_t is i.i.d. normal with variance σ_v^2. Note that without the signal it would be impossible for the agent to decompose changes in productivity between permanent and temporary shocks. Furthermore, the agent knows the model behind productivity in detail: the particular functional forms involved and parameter values (ρ and the variance of the three shocks).

Thus, the agent enters period t with his knowledge of the model plus beliefs formed last period $\left(x_{t|t-1}, x_{t-1|t-1}, z_{t|t-1}\right)$, observes productivity and the signal (a_t, s_t) and uses all that information to update his expectations using the Kalman filter:

$$\begin{bmatrix} x_{t|t} \\ x_{t-1|t} \\ z_{t|t} \end{bmatrix} = A \begin{bmatrix} x_{t-1|t-1} \\ x_{t-2|t-1} \\ z_{t-1|t-1} \end{bmatrix} + B \begin{bmatrix} a_t \\ s_t \end{bmatrix} \tag{14}$$

where the matrices A and B depend on parameters of the model. Given the above, the Kalman filter provides the necessary ingredients to construct:

$$\begin{aligned} E_t\left[a_{t+1}\right] &= (1+\rho)\, x_{t|t} - \rho x_{t-1|t} + \rho z_{t|t} \equiv b' E_t\left[\xi_t\right]; && (15) \\ b &\equiv \begin{bmatrix} 1+\rho \\ -\rho \\ \rho \end{bmatrix}; \quad \xi_t = \begin{bmatrix} x_t \\ x_{t-1} \\ z_t \end{bmatrix} \end{aligned}$$

$$Var_t\left[a_{t+1}\right] = b' Var_t\left[\xi_t\right] b + \sigma_\epsilon^2 + \sigma_\eta^2 \tag{16}$$

where $Var_t\left[\xi_t\right] \equiv P$ is found by solving the time-invariant Ricatti equation implied by the Kalman filter and will depend on model parameters as well.

Adding these results to the Euler equation will result in:

$$\begin{aligned} U'\left(C_t\right) &= \alpha\beta \exp\left(b' E_t\left[\xi_t\right] + 0.5\left(b' P b + \sigma_\epsilon^2 + \sigma_\eta^2\right)\right) E_t\left[U'\left(C_{t+1}\right) K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha}\right] \\ &\quad + \beta\left(1-\delta\right) E_t\left[U'\left(C_{t+1}\right)\right] && (17) \end{aligned}$$

note that we have eliminated the only stochastic forward-looking variable present in the RBC model and replaced it with a process that depends exclusively on past realizations of productivity and the signal. In order to obtain this result the covariance between productivity and other variables has been ignored. This is akin to a first order approximation of the Euler equation only, log-linearization is not applied to the whole system in order to preserve the richer dynamics present in the rest of the equations.

Equations (1) - (3) and (6) together with the reformulated Euler equation and the Kalman filter result (14) form a system where all variables are driven by productivity and the signal (from the agent's point of view) which, in turn, ultimately depend on the history of the three shocks (from the researcher's point of view).

Turning to the model calibration, standard values will be used for α, β, γ, δ and η, transformed to their quarterly equivalents as shown in the following table (quarterly-adjusted values have been rounded):

Parameter	Reference value	Quarterly-adjusted value
α	0.33	0.33
β	0.96	0.99
γ	0.95	0.95
δ	0.05	0.01
η	2	2

Table 1: Calibration of RBC parameters

For the parameter governing the relative importance of permanent versus temporary shocks, ρ, a value of 0.89 will be taken, following Blanchard et al. (2009). This will imply permanent shocks that build up slowly and temporary shocks that take a long time to decay.

The only parameters left are the variances of the three shocks. Since these are the ones that will be changed in order to explore the effects of higher volatility of productivity or signal noise we will require several sets of them.

Parameter	A: Base scenario	B: Higher σ_u	C: Higher σ_ν
σ_u	0.67%	1.27%	0.67%
σ_ϵ	0.07%	0.14%	0.07%
σ_η	0.63%	1.26%	0.63%
σ_ν	0.89%	0.89%	1.78%

Table 2: Variance of shocks for different scenarios

The base scenario corresponds to Blanchard et al (2009). The other ones are based on Senhadji's (2000) estimates of the standard deviation of total factor productivity for several country groups over the 1960-1994 period. His average for "Industrial Countries" is fairly close to the value reported by Blanchard (which is for the US only). Furthermore, he finds that yearly σ_u for Middle East and North Africa is roughly 5.7% (the highest volatility he reports); that value will be used approximately for the scenario with high σ_u: for simplicity, we have constructed it as an increase in σ_u by a factor of 2. Similarly scenario C has been constructed by doubling the magnitude of σ_ν only.

4 Results

The following figures show impulse response functions for a permanent, transitory and noise shock for the base scenario. When the permanent shock takes place, consumption increases right from the outset, even at the cost of an initial fall in investment. This, together with the sustained fall in labour are the agent's responses to the wealth component of the shock. At the time of the shock, not all the observed variation in log productivity is attributed to the permanent component, in fact, the agent believes most of the shock is transitory (roughly 65% of the shock is attributed to the transitory component). As time goes by, the agent gradually realizes the true nature of the shock, thus increasing consumption slowly. This also explains the shape of investment which seems to settle at a higher level in order to sustain what will eventually become the new steady state level of consumption. Output dynamics basically follow those of productivity.

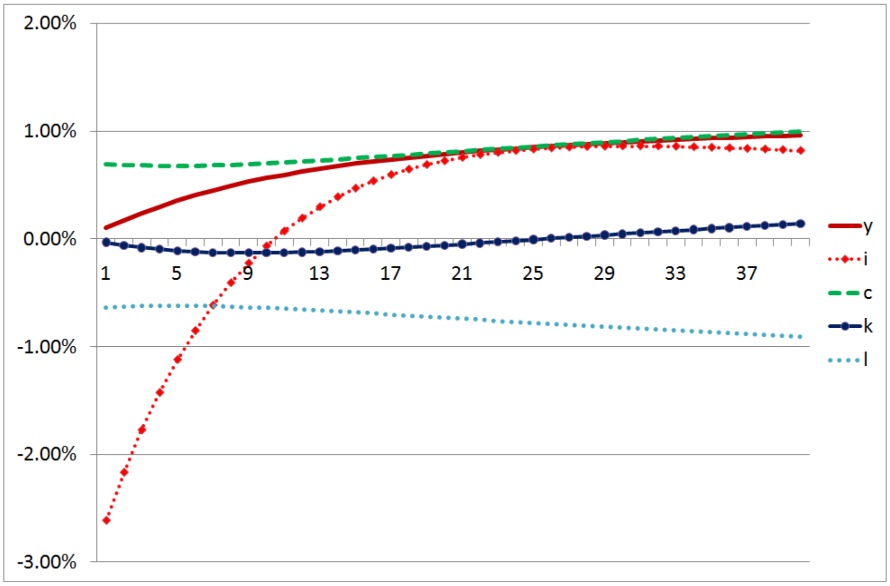

Figure 1: Impulse response function for a permanent shock

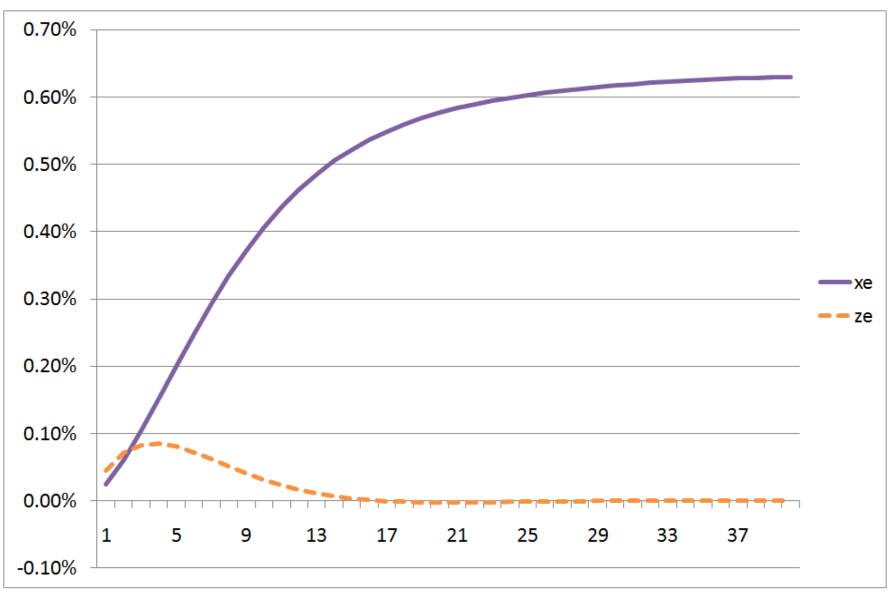

Figure 1a: Evolution of expectations after a permanent shock

The transitory shock tells a different story. The agent realizes right from the beginning the nature of the shock (95% of the variation is attributed to the transitory component). Thus, the agent proceeds to smooth out the benefits of the shock. Consumption and labour response at time zero is mild, but investment response is relatively high. The idea will be to accumulate capital in order to sustain higher output for as long as possible to allow for a longer-lasting consumption increase.

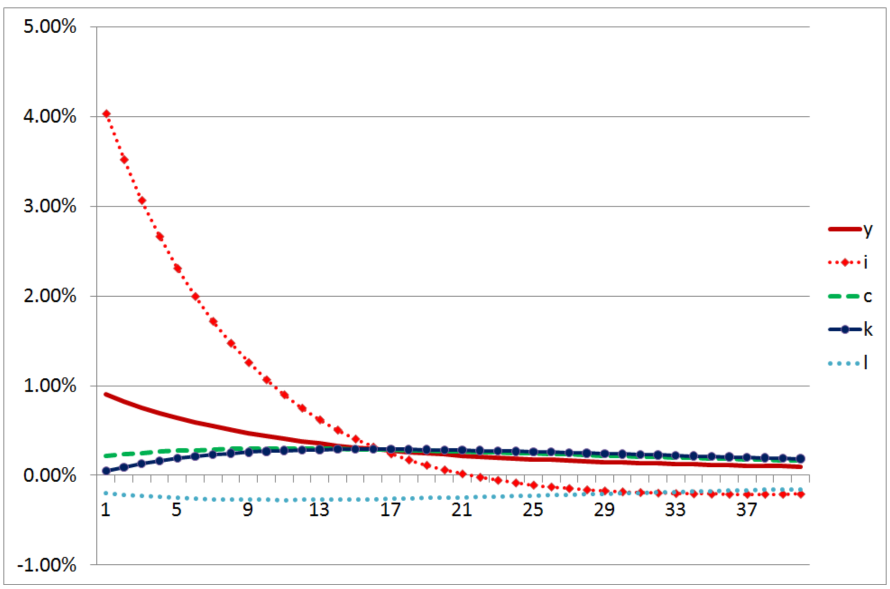

Figure 2: Impulse response function for a transitory shock

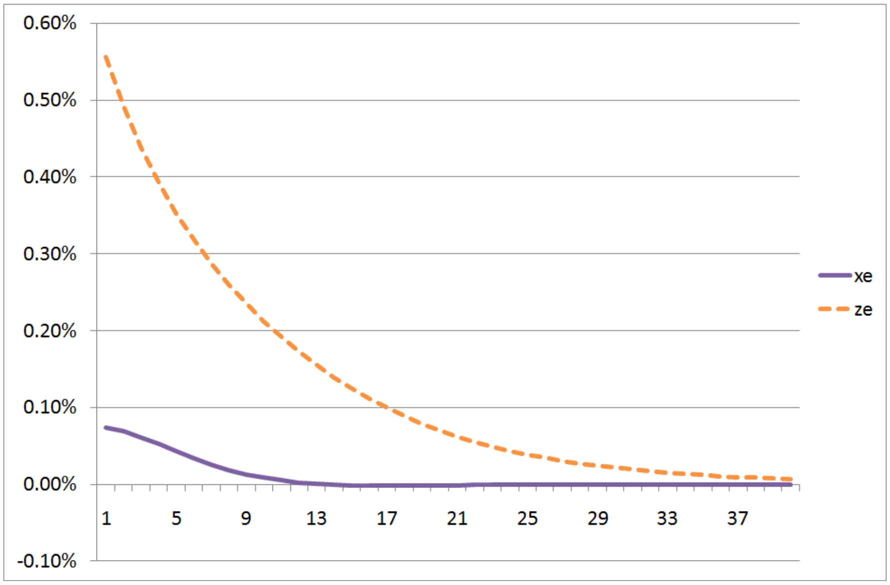

Figure 2a: Evolution of expectations after a temporary shock

The impulse response function derived from a signal noise shock delivers unexpected results.

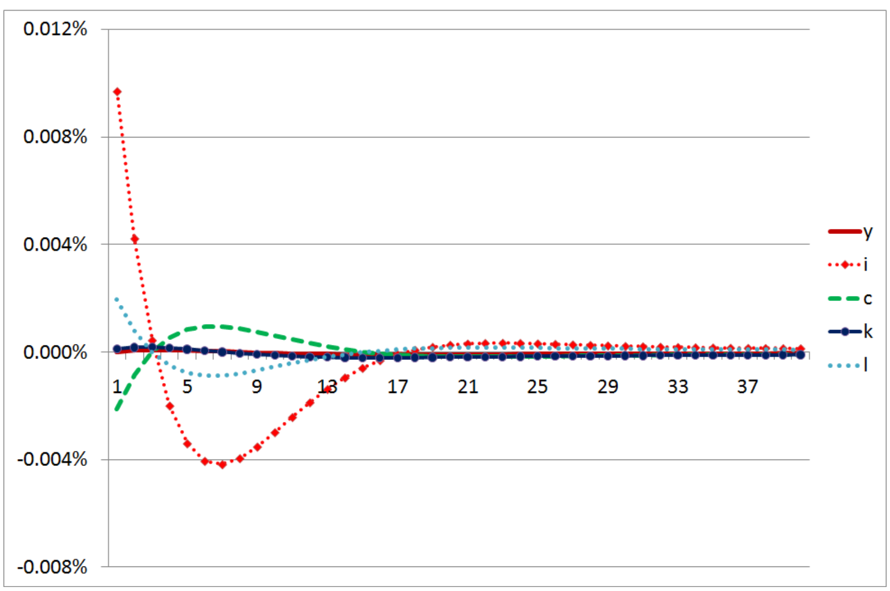

Figure 3: Impulse response function for a noise shock

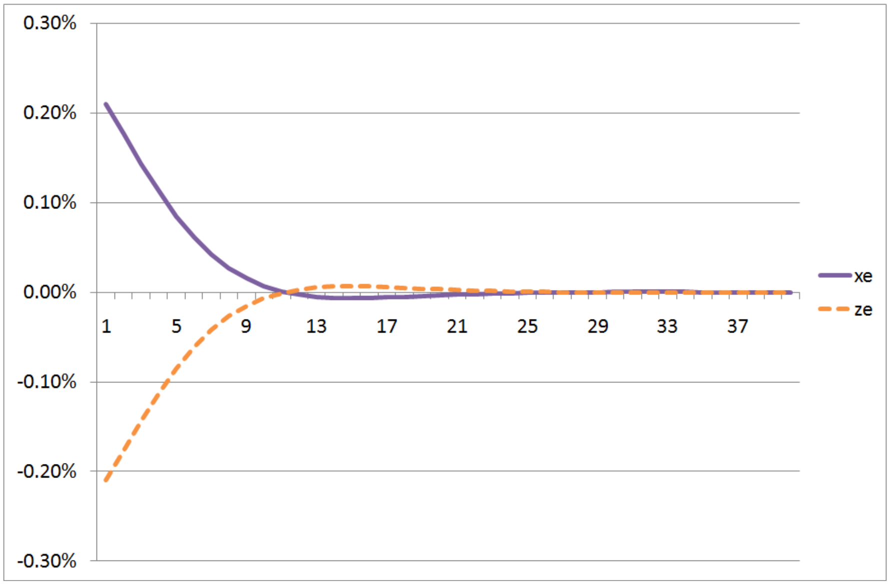

Figure 3a: Evolution of expectations after a noise shock

Differing from Blanchard et al. (2009), the response is insignificant. The biggest impulse obtained, that of investment, is below 0.01% at its highest (which occurs right at the moment the shock hits). The contradiction can be explained by comparing Euler equations. Blanchard et al. (2009) obtain the following result for consumption:

$$c_t = \lim_{j \to \infty} E_t \left[a_{t+j} \right].$$

Note that this implies consumption will depend on the expected limit value of the a_t process alone. Since any transitory shock eventually dies out, that expected value will depend solely on the current expectations of the permanent component of productivity:

$$c_t = x_{t|t} + \frac{\rho}{1-\rho} \left(x_{t|t} - x_{t-1|t} \right).$$

On the other hand, the model presented in this paper results in an Euler equation that predicts consumption depends on the entire expected present and future history of log productivity: the transitory component matters.

Thus, Blanchard et al. (2009) report consumption impulse responses that depend solely on the evolution of $x_{t|t}$ and $x_{t-1|t}$ (for the permanent shock these change obviously; for the transitory shock, recall that the agent erroneously interprets part of the transitory shock as a permanent one when the shock hits and only "learns" the truth gradually). This has no significant impact on the qualitative aspect of the response to permanent and transitory shocks but makes a big difference when it comes to signal noise shocks.

When a signal noise shock hits this system, the signal jumps but observed log productivity does not change. Given that the Kalman filter is always trying to "split" observed changes in these variables into the permanent and transitory components of productivity, the signal jump of σ_ν magnitude will be interpreted as an increase in the permanent component mirrored by a decrease in the transitory component, both with magnitude equal to roughly a quarter of σ_ν. Since Blanchard et al. (2009) have consumption depending on the permanent component only, they obtain a non-trivial impulse response, but that will not be the case for the model developed in this paper: the signal noise shock will generate insignificant responses given that the permanent and transitory components move in opposite directions and the agent learns fairly quickly that the signal jump had no "real" basis. Still, Figure 3 shows the agent's response to the noise shock is reasonable: the initial increase in investment and labour coupled with the fall in consumption would indicate an attempt to spread out the benefits of a non-existent positive shock to the future. The mistake inflates output above steady state briefly and the situation is corrected (with higher consumption and lower investment and labour) fairly quickly.

Turning to the analysis of the different scenarios presented in Table 2, it turns out that only scenario B shows an impact on the steady state worth noting. In particular, higher (double) σ_u, which implies proportionally higher σ_ϵ and σ_η, results in a 0.016% increase in the steady state values of capital, output, investment and consumption. The reason is fairly straightforward: higher log productivity variance implies higher expected productivity in level (by log normal property) which leads to a higher demand for capital in steady state. Capital then causes a proportional rise in all other model variables. Note that the model assumes no trend in log productivity (steady state exogenous growth is zero) and decreasing returns to capital (thus preventing endogenous growth from appearing). Changing the latter assumption to one of constant returns would imply the 0.016% increase would be imputed to the steady state growth rate.

Why is the impact of a change in shock volatility so small? Generally speaking, the magnitude of the change in σ_u^2 is too small to generate a significant response in steady state capital. TFP volatility apparently is too small to matter in the long run.

Changes in shock volatility also have a small impact on the short run responses of the model. In order to assess this effect it is necessary to correct impulse response functions for shock magnitude (a typical one-standard-deviation permanent shock would have different magnitudes in scenarios A and B, same reasoning applies to a one-standard-deviation signal noise shock in scenarios A and C). Once the correction is made, we can construct the changes in impulse response function (IRF) between scenarios.

Comparing scenarios A and C (the comparison between A and B is shown in the appendix) we can appreciate the following changes:

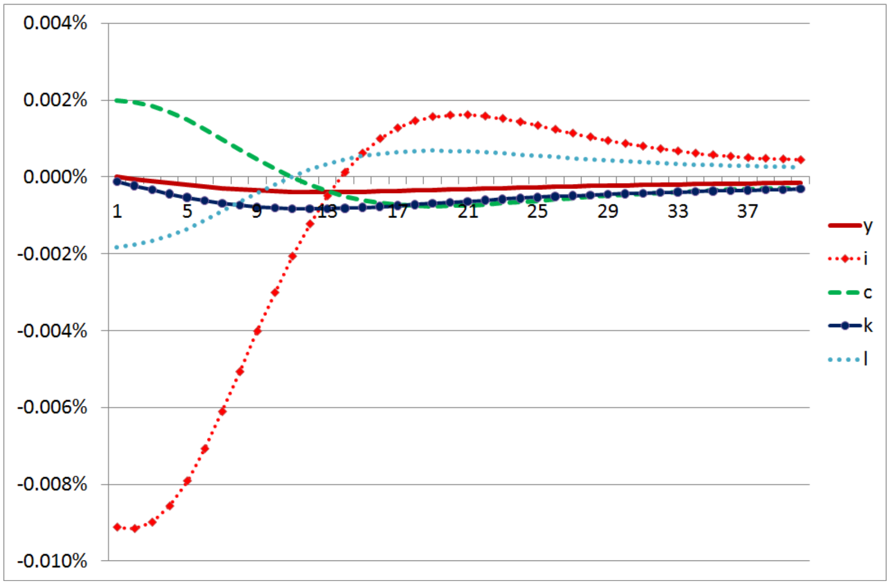

Figure 4: Permanent shock: IRF(C) - IRF(A)

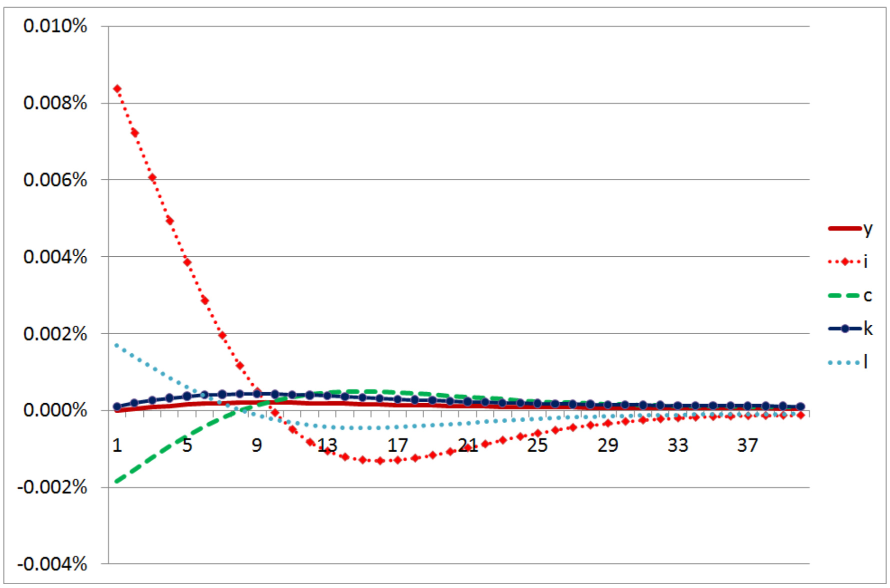

Figure 5: Temporary shock: IRF(C) - IRF(A)

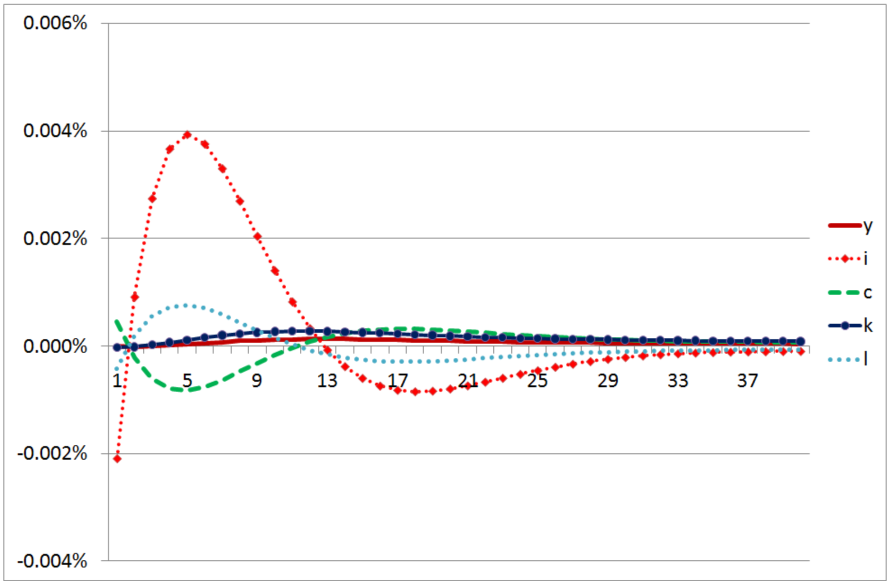

Figure 6: Noise shock: IRF(C) - IRF(A)

An increase in signal noise volatility mainly impacts consumption, investment and labour.

In the case of the permanent shock, higher noise volatility results in a bigger consumption expansion and a bigger investment contraction, this resembles, qualitatively, a negative noise shock.

In the case of the temporary shock, higher noise volatility results in a smaller consumption expansion and a bigger investment expansion, this resembles, qualitatively, a positive noise shock.

In the case of the noise shock, higher noise volatility results in a dampened response.

In general, the agent assigns a higher proportion of changes in observed productivity to the permanent component and pays less attention (weaker responses) to changes in the signal. The following figure shows how the formulation of expectations changes in response to higher volatility.

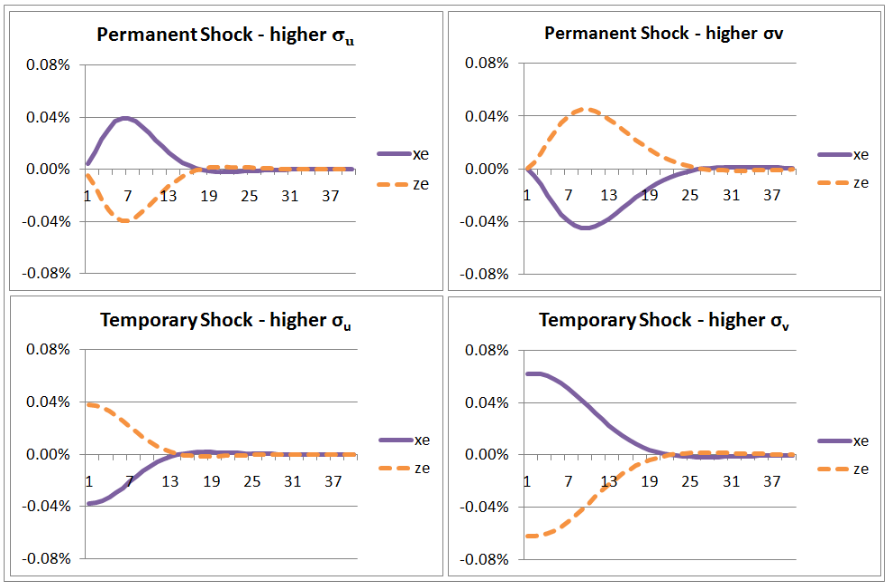

Figure 7: Changes in expectations due to higher volatility

When productivity is more volatile, the agent finds it easier to identify both permanent and temporary shocks. Implicitly, he is giving more weight to the signal and thus can discriminate better between shocks (recall the signal does not change in response to temporary shocks). On the other hand, when the extra volatility comes from signal noise his capacity to distinguish between shocks is diminished: he identifies a bigger proportion of permanent shocks as temporary and viceversa.

The model cannot address the short run impact of changes in volatility (a "volatility shock"). The reason is that productivity and signal volatility enter the model as (known) constants. Thus, the effect of any change in them is completely absorbed by adjustments to the steady state values of the model. The agent is assumed to know the magnitude of volatility in all three shocks right from the outset in every scenario. This is an assumption required to solve the Kalman filter using the method presented above, but it will have to be lifted in order to explore short run consequences of changes in volatility of the shocks (note that this would imply assuming that volatility is not constant over time and thus not necessarily observable by the agent).

Thus, the model presented seems to deliver a strong conclusion regarding the effects of TFP volatility in the long run but fails to account for short run volatility changes. Given the magnitude of these (Senhadji reports TFP growth averages for several country groups together with standard deviations: the implication is that short run values cannot be too far from those we have studied) are they worth of analysis? In order to answer this, recall the impulse response functions reported for the base scenario. They are constructed based on a one-standard-deviation change in any of the shocks: the shock magnitude is exactly the same that we'd like to apply to a short run change in volatility. Since the impulse responses are not insignificant, that could be an indication, albeit weak, that an impulse response to a "volatility shock" could have a non-trivial magnitude.

5 Conclusion

An imperfect information model with an agent facing a signal extraction problem has been developed, solved and calibrated. Then, changes in process and signal noise volatility of productivity are introduced and their impact studied. The model clearly shows that volatility of this type has no relevant long run impact but has a small effect on the economy's responses to productivity and noise shocks. Volatility shocks are not addressed, the reason being that the model studied takes volatility as a known constant from the agent's point of view, resulting in any changes to it being absorbed directly into the steady state. Analysis of short run fluctuations in volatility will require an agent that gradually learns what the true volatility behind his productivity process is or a departure from the assumption of constant volatility. In both cases, non-trivial modifications to the Kalman filter algorithm presented in this paper as the agent's method of dealing with imperfect information would be required.

6 References

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7 Appendix

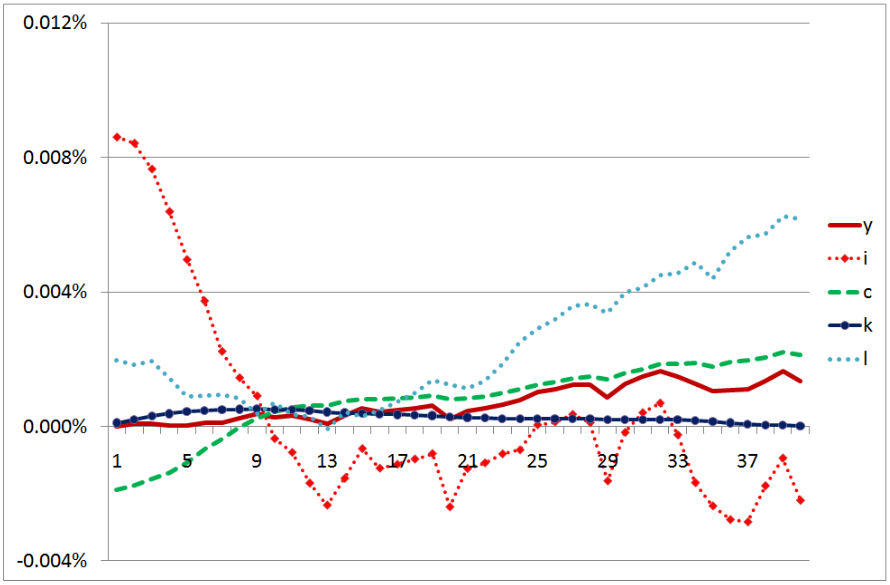

Figure A1: Permanent shock: IRF(B) - IRF(A)

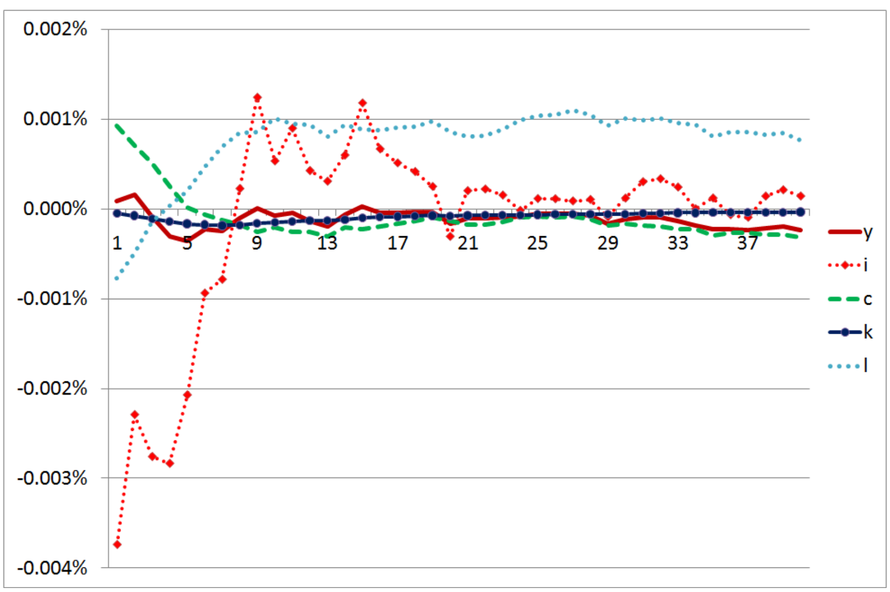

Figure A2: Temporary shock: IRF(B) - IRF(A)

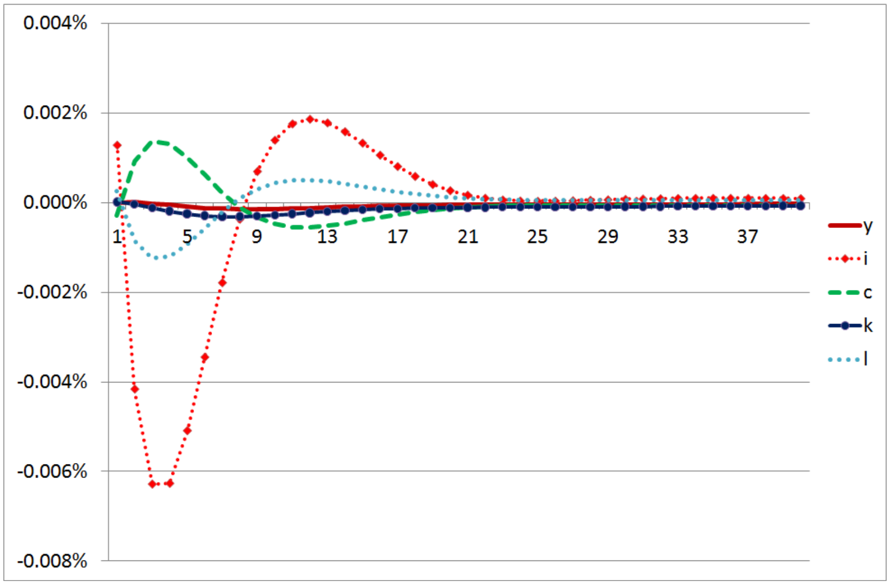

Figure A3: Noise shock: IRF(B) - IRF(A)