Using A Forward-Looking Phillips Curve to Estimate the Output Gap in Peru

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Abstract

This paper identifies the output gap using the theoretical definition of the gap within a Phillips curve. The results show that the output gap is large and persistent. Furthermore, the output gap is not correlated with the stochastic trend which is similar to the assumption used in the unobserved components model. The model is extended to include information coming from the unemployment rate. The results are very similar to those obtained without this variable indicating poor additional information in the unemployment rate to identify the output gap. Other estimations of the output gap are performed. I use the procedures of Hodrick and Prescott (1997), Baxter and King (1999), Beveridge and Nelson (1981), Morley, Nelson and Zivot (2003), the unobserved components model of Clark (1987) and a simple quadratic trend. The results show strong differences between our measure of output gap and the other measures. The closer measure is the one obtained using the unobserved component model and the simple quadratic trend.

Keywords: Business Cycles, Phillips Curve, Output Gap, Inflation, Unemployment, Filters.

JEL Classification: C22, C52, E31, E32.

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1 Introduction

In the seminal research of Mitchell (1927) and Burns and Mitchell (1946), the recessions are interpreted to be deviations from a full-employment level of output. These deviations are named the output gap. The literature about this topic is very extensive but it may be categorized into two groups: statistical and economic. Of course, there are sub-categories within them and interactions between them.

In the statistical approach, we may find two major sub-categories of decomposition of output into trend and cycle components. The first sub-category imposes smoothness on either the trend or the cycle. The simplest and still widely used method in this group is to fit a polynomial in time to output, the residuals being the estimated cycle. On another hand, the filter of Hodrick and Prescott (1997) imposes smoothness but not determinism on the trend. Another approach extracts an estimate of the cycle by passing the data through a filter that pre-specifies the relevant frequencies for the cycle and thus its persistence. It is the case of the filter of Baxter and King (1999) where the cycle is defined as having spectral power in the range between 6 and 32 quarters. In the same family appears the filter of Christiano and Fitzgerald (2003).

The other sub-category does not impose prior smoothness on either component. It uses a time series model and require identification of the stochastic trend component. In this sub-category we find the decomposition of Beveridge and Nelson (1981). According to it, the first differences of data is modeled as an ARMA model where the trend is identified as the long-horizon forecast, which must be a random walk. Another possibility is to use the unobserved components model based on the research of Harvey (1985), Watson (1986) and Clark (1987). In this kind of model, a zero restriction between the shocks to the cycle and trend is imposed. The trend is assumed to be a random walk with varying growth rate in some specifications. The
empirical evidence shows that the decomposition of Beveridge and Nelson (1981) yields small, less persistent cycles whereas the unobserved component decomposition yields large, more persistent cycles. In a recent paper, Morley, Nelson and Zivot (2003) show that this issue is due to the assumption that trend and cycle shocks are uncorrelated, demonstrating the strong impact of this statistical assumption on trend-cycle decompositions. In general, empirical macroeconomic results may be sensitive to which method is used; for further details, see Canova (1998).

In the side of economic approaches, one way to calculate the output gap is to use an aggregate production function. Another popular measure is calculated by the Congressional Budget Office (CBO, 1995). They use a large-scale multi-sector growth model for estimating the potential output. On another hand, Galí and Gertler (1999) recommended to use the real unit labor cost as a good approximation for the output gap. They argue that this measure provides an important empirical support for the forward-looking Phillips curve.

There also are efforts merging statistical and economic approaches. It has resulted in estimating multivariate forms of the unobserved components model. For example, Kuttner (1994) uses a bivariate model of inflation and output, assuming that the transitory component of output is the gap variable in the inflation equation. Using a similar approach for European data, Gerlach and Smets (1999) use the real interest rate as a driving variable for the cycle. It is worth to note that both works use the standard random walk trend and uncorrelated shocks assumption from the unobserved-components models to complete their model. Apel and Jansson (1999) use a bivariate model of inflation and unemployment to extract an estimate of the cyclical fluctuations in output. Other reference but applied to a multi-country study is Clark (1989). Roberts (2001) finds that the assumption of zero correlation is reasonable for the US.

A closely related literature has been on measuring the natural rate of
unemployment or the NAIRU. We may also find statistical and economic approaches in order to identify these measures. Important references are Blanchard and Katz (1997), Gordon (1997, 1998), Laubach (2001), Salemi (1999), Staiger et al. (1997b, 2001), Stiglitz (1997). From a critical perspective, Staiger et al. (1997a) point out that is very difficult to measure the NAIRU.


Estimation of the Phillips curve has not been free of critics. For instance Fuhrer (1997) provides evidence against the forward-looking price behavior. A similar conclusion is reached by Fuhrer and Moore (1995). However, Roberts (1995, 1997) provides support for the role of inflationary expectations in estimating a Phillips curve. On another hand, Galí and Gertler (1999) estimated a “hybrid” Phillips curve containing both forward-looking and backward-looking components. They show that this “hybrid” model provides a good fit. In a recent paper, Basistha and Nelson (2007) use this feature to calculate the output gap for the US. I follow this approach using Peruvian data. Furthermore, following Basistha and Nelson (2007), my approach allows the gap to differ from cycle, and relaxes the restriction that trend and cycle shocks are uncorrelated.

The document has the following sections. In Section 2, the model is presented. Section 3 discusses the estimates. Section 4 presents the augmented model introducing the unemployment rate. Section 5 discusses the results. Finally, Section 6 concludes.
2 The Model

The forward-looking New Keynesian Phillips curve may be derived based on the type of pricing model suggested by Calvo (1983). In this framework, the forward-looking New Keynesian Phillips curve is based on optimizing behavior by forward-looking and monopolistically competitive producers. See also Galí and Gertler (1999), Sbordone, Goodfriend and King (1997), Rothenberg and Woodford (1997), and Yun (1996). This curve takes the following specification:

\[ \pi_t = \beta E_t \pi_{t+1} + \delta c_t + z_t, \]

(1)

where \( \pi_t \) is the inflation rate, \( c_t \) is the output gap due to nominal rigidities, \( z_t \) is a supply shock to inflation rate, and \( E_t \pi_{t+1} \) is the (unobservable) aggregate expectation of inflation rate at the period \( t+1 \) based on information at period \( t \). In empirical research lagged inflation rate has been added to these models because its considerable explanatory power. It is named an “hybrid” Phillips curve because there are backward-looking and forward-looking behavior.

In the present framework, expectations of inflation and the output gap are considered as unobserved variables. Therefore each variable is treated as a state variable in a state-space representation of the Phillips curve. The Kalman filter is used to extract the output gap implied by the behavior of the inflation rate. The part of the actual inflation that is not related to the gap is treated as the state variable implicit in the following measurement equation:

\[ \pi_t = \bar{\pi}_t + \delta c_t. \]

(2)

The non-gap part of inflation (\( \bar{\pi}_t \)) is partially observable through its linear projection on observable variables, including survey expectations of inflation (see Roberts, 1997, 1998) and lagged actual inflation. Therefore,
the state equation is
\[ \tilde{\pi}_t = \beta_0 + \beta_1 \pi_{t-1}^{se} + \beta_2 \pi_{t-1} + \epsilon_{xt}, \quad (3) \]
where \( \pi_{t}^{se} \) denotes survey expectations of inflation and \( \epsilon_{xt} \) is a composite of both unobserved variables that play a role in expected inflation and \( z_t \), the supply shock. In order to ensure long-run neutrality, we restrict \( \beta_1 + \beta_2 = 1 \).

Concerning the decomposition of output, I follow conventional specifications. Thus, the output \( (y_t) \) consists of two unobserved components. The first one is the permanent component \( (p_t) \) which reflects the impact of permanent shocks on the equilibrium level of output. The second component is the transitory component \( (c_t) \) which is associated with nominal rigidities in the economy. The measurement equation for output is given by
\[ y_t = p_t + c_t. \quad (4) \]

In order to complete the specification of the state variables, the trend component, \( p_t \), is assumed to be a random walk with a constant drift. On the other hand, the transitory component, \( c_t \), is assumed to be an AR(2) process, which is in the tradition of Harvey (1985), Watson (1986), Clark (1987), and Harvey and Jaeger (1993). Thus, both state equations are given by
\[ p_t = \mu + p_{t-1} + \epsilon_{pt}, \quad (5) \]
\[ c_t = \phi_{1c} c_{t-1} + \phi_{2c} c_{t-2} + \epsilon_{ct}, \]
where \( \epsilon_{pt} \sim N(0, \sigma_p^2) \), and \( \epsilon_{ct} \sim N(0, \sigma_c^2) \).

Therefore, we have three shocks in the system. The generalized variance-covariance matrix to be estimated is then:
\[ \text{cov}(\epsilon_{pt}, \epsilon_{ct}, \epsilon_{xt}) = \begin{bmatrix} \sigma_p^2 & \sigma_{pc} & \sigma_{p\pi} \\ \sigma_{cp} & \sigma_c^2 & \sigma_{c\pi} \\ \sigma_{p\pi} & \sigma_{c\pi} & \sigma_\pi^2 \end{bmatrix}. \quad (6) \]

In sum, the state-space formulation of the model may be expressed as follows. Equations (2) and (4) are the measurement equations which relate
observed inflation and output respectively to state variables. The equations (3) and (5) represent the state equations which establish the behavior of the unobserved variables. The parameters are estimated using the maximum likelihood method and then I use the Kalman filter to produce filtered and smoothed estimates of the unobserved components.

3 Results I

I use quarterly data for the period 1980:1-2005:4 from the Central Bank of Peru. Output is the log of real GDP. The quarterly inflation has been computed using the seasonally adjusted CPI data and was annualized. Data for inflationary expectations is not available for the complete sample. Therefore, I use lagged inflation as an approximate measure for inflationary expectations.

Estimates of equations (2)-(5) are presented in Table 1. The estimate of the trend growth rate $\mu$ is around 2.2% percent annually. The estimated response of inflation to the gap is only 0.03 indicating a very flat-sloped Phillips curve. Compared with other estimates, it is very small; see Basistha and Nelson (2007), Rudebusch (2002). The estimates also show a negative correlation between the inflation shock and the output-gap shock ($\rho_{\pi_c}$), zero correlation between the output-gap shock and shocks to the permanent component ($\rho_{pc}$), and zero correlation between the permanent shock and the inflation shock ($\rho_{p\pi}$).

Notice that the zero correlation between the trend and cycle components ($\rho_{pc}$) obtained in the model is consistent with the assumption of the unobserved component model.

High persistence in output gap dynamics is found. The sum of the autoregressive coefficients is 0.938, whereas the estimate using the approach of Morley, Nelson and Zivot (2003) is only 0.234. Because the approach of Morley, Nelson and Zivot (2003) is univariate, the difference in the estimates suggests the important role of inflation in identifying the persistence of the
output gap. It is consistent with the findings of Kuttner (2004), Apel and Jansson (1999), and Roberts (2001).

The estimates indicate a high level of backward-looking (0.612) compared with the forward-looking side (0.388).

In order to see how different is the measure of output gap obtained from the model, I calculated other measures of output gap using some well known methods. I calculated output gap using the filter of Hodrick and Prescott (1997), the filter of Baxter and King (1999), the filter of Christiano and Fitzgerald (2003), Beveridge and Nelson (1981), the unobserved component model proposed by Clark (1987), a linear time trend, and a quadratic time trend.

The Figure 1 shows the evolution of the different output gap measures. For example, it is easy to observe that our measure is almost completely unrelated with the measure of output gap calculated using the approach of Beveridge and Nelson (1981). The simple correlation is 0.070. Unlike this measure, our measure is large and persistent. Intermediate values of correlations are obtained with the filter of Hodrick and Prescott (0.656), the filter of Baxter and King (0.691) and the unobserved components model (0.523). Our measure presents more similarity with the measures obtained using a simple linear trend (0.932) and a simple quadratic trend (0.856).

4 Adding the Unemployment Rate

Following the suggestion of Clark (1987), and in order to exploit potential useful information of the unemployment rate, I extend the previous model adding the unemployment rate. In a similar way as for the output, I define the unemployment rate to be a sum of the natural rate \(n_t\) and the unemployment gap \(c_{ut}\):

\[
 u_t = n_t + c_{ut}. \tag{7}
\]

Following the representation of the Okun’s Law used by Clark (1987),
I assume that the current and lagged output gap affect the unemployment gap:

\[ c_{ut} = \gamma_0 c_t + \gamma_1 c_{t-1}. \] (8)

Concerning the natural rate of unemployment \((n_t)\), I assume that it follows a random walk without drift which is in the same spirit as Clark (1987), Gordon (1998), and Apel and Jansson (1999):

\[ n_t = n_{t-1} + \epsilon_{nt}. \] (9)

The variance-covariance matrix of the four shocks is allowed to be completely general:

\[
\text{cov}(\epsilon_{pt}, \epsilon_{ct}, \epsilon_{\pi t}, \epsilon_{nt}) = \\
\begin{bmatrix}
\sigma^2_p & \sigma_{pc} & \sigma_{p\pi} & \sigma_{pn} \\
\sigma_{cp} & \sigma^2_c & \sigma_{c\pi} & \sigma_{cn} \\
\sigma_{\pi p} & \sigma_{\pi c} & \sigma^2_\pi & \sigma_{\pi n} \\
\sigma_{np} & \sigma_{nc} & \sigma_{n\pi} & \sigma^2_n \\
\end{bmatrix}.
\] (10)

5 Results II

The results of the extended model are presented in Table 2. The estimates of the drift and the slope of the Phillips curve are very similar as those presented in Table 1. The coefficients corresponding to the Okun’s Law are not significant. It means absence of persistence in the unemployment gap. The equation related to the inflation shows that the backward-looking component is relatively more important (0.608).

The cyclical component presents high persistence as shown by the sum of the autoregressive coefficients which is 0.955. The correlation between the shocks of the trend and cyclical components is not significant. It is the same result as in the Section 3 indicating that the assumption of the unobserved components model is not rejected.

The correlation of our measure of output gap with other measures is similar to those obtained before. Higher correlations are obtained with the
quadratic and linear methods to calculate the output gap. The correlation with the reduced model (Sections 2 and 3) is 0.968. See Figure 2.

All results indicate that unemployment rate does not contain useful information in the estimation of the output gap. Our conjecture is related on the poor quality of this variable.

6 Conclusions

This paper identifies the output gap using the theoretical definition of gap within a Phillips curve. This approach allows to differ from the cycle and relaxes the restriction that the trend and cycles are uncorrelated.

The results show that the output gap is large and persistent. Furthermore, the output gap is not correlated with the stochastic trend which is similar to the assumption used in the unobserved components model. The model has been extended to include information coming from the unemployment rate. The results are very similar to those obtained without this variable indicating poor useful additional information in the unemployment rate to identify the output gap.

For comparison, I have tried with other estimations of output gap. I used the procedures of Hodrick and Prescott (1997), Baxter and King (1999), Beveridge and Nelson (1981). I also used the unobserved components model of Clark (1987) and a simple quadratic trend to obtain the output gap. The results show strong differences between our measure of output gap and other measures. The closer measure is the one obtained using the unobserved component model and the simple quadratic trend.

References

Letters 64, 271-275.


Table 1. Results I

The trend drift, the Phillips curve slope and the autoregressive coefficients

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Std. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>0.5317</td>
<td>0.1941</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0290</td>
<td>0.0133</td>
</tr>
<tr>
<td>$\phi_{1,c}$</td>
<td>1.4642</td>
<td>0.1193</td>
</tr>
<tr>
<td>$\phi_{2,c}$</td>
<td>-0.5258</td>
<td>0.1086</td>
</tr>
</tbody>
</table>

The non-gap coefficients of the Phillips curve

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Std. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>0.0075</td>
<td>0.1022</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.3888</td>
<td>0.0871</td>
</tr>
</tbody>
</table>

The standard deviations and the correlations of the shocks

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Std. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_p$</td>
<td>1.1710</td>
<td>0.5097</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>2.3786</td>
<td>0.7097</td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>0.9364</td>
<td>0.0696</td>
</tr>
<tr>
<td>$\rho_{pc}$</td>
<td>0.2454</td>
<td>0.6150</td>
</tr>
<tr>
<td>$\rho_{px}$</td>
<td>0.2361</td>
<td>0.5946</td>
</tr>
<tr>
<td>$\rho_{\pi c}$</td>
<td>-0.8841</td>
<td>0.0918</td>
</tr>
</tbody>
</table>

Log Likelihood: -275.4021

Standard errors in parentheses.
Table 2. Results II

<table>
<thead>
<tr>
<th></th>
<th>( \mu ) 0.4168 (0.1611)</th>
<th>( \gamma_0 ) -0.0885 (0.1098)</th>
<th>( \delta ) 0.0229 (0.0106)</th>
<th>( \gamma_1 ) -0.0119 (0.0534)</th>
</tr>
</thead>
</table>

| The autoregressive coefficients and the non-gap coefficients of Phillips curve |
|---------------------------------|-----------------------------|-----------------------------|
| \( \phi_{1,c} \) 1.4563 (0.1025) | \( \beta_0 \) 0.0659 (0.0707) |
| \( \phi_{2,c} \) -0.5012 (0.1020) | \( \beta_1 \) 0.3916 (0.0829) |

| The standard deviations of the shocks |
|---------------------------------|-----------------------------|
| \( \sigma_p \) 1.1362 (0.4747) | \( \sigma_\pi \) 0.9314 (0.0673) |
| \( \sigma_c \) 2.4927 (0.6233) | \( \sigma_n \) 0.7422 (0.0726) |

| The correlations of the shocks |
|---------------------------------|-----------------------------|
| \( \rho_{pc} \) 0.1224 (0.5115) | \( \rho_{pn} \) 0.3068 (0.4573) |
| \( \rho_{pi} \) 0.3188 (0.4836) | \( \rho_{cn} \) -0.0692 (0.3098) |
| \( \rho_{pi} \) -0.8582 (0.0996) | \( \rho_{ni} \) -0.0743 (0.2969) |

Log Likelihood -295.7748

Standard errors in parentheses.
Figure 1. Estimates of Output Gap
Figure 2. Estimates of Output Gap (Augmented Model)