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Estimation of a Time Varying Natural Interest Rate for Peru

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Abstract

Following the approach of Mésonnier and Renne (2007), we estimate a Natural Rate of Interest (NRI) using quarterly Peruvian data for the period 1996:3 - 2008:3. The model has six equations and it is estimated using the Kalman filter with output gap and NRI as unobservable variables. Estimation results indicate a more stable NRI in period 2001:3 - 2008:3 than in period 1996:3 - 2001:2 and also more stable than the observed real interest rate. Real interest rate gap (difference between real and natural rates), which measures monetary policy stance, indicates a restrictive policy for 1996-2001 and for 2003. Results also suggest a real interest rate greater than NRI for 2002 and for 2004-2008.

Keywords: Interest rate, natural interest rate, Kalman filter, output gap, unobservable components.

JEL: C32, E32, E43, E52.

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1 Introduction

The natural rate of interest (hereafter NRI) is defined as the real interest rate for macroeconomic equilibrium. It is also known as the neutral interest rate. In more formal terms, NRI is a real short-term interest rate that is consistent with potential output and with stable inflation. Historically, the concept of a *natural* real rate of interest and its use for monetary policy is associated with Wicksell (1898, 1907). In recent years, the *neo-wicksellian* framework for monetary policy analysis advocated by Woodford (2003) has emphasized its relevance for monetary authorities.

An important measure derived from the NRI is the real interest rate gap (IRG). It is calculated as the difference between the real short-term interest rate and NRI. Naturally, this indicator is a relevant candidate for assessing monetary policy stance. Actually, central banks and central banks economists pay significant attention to theoretical developments and empirical strategies for estimating the NRI and the IRG. Examples at this respect are Archibald and Hunter (2001); Christensen (2002); Williams (2003); Neiss and Nelson (2003); ECB (2004); and Crespo-Cuaresma, Gnan, and Ritzberger-Grünewald (2004).

There is an enormous literature concerning the modeling and estimation of NRI.[1] Two characteristics may guide in distinguishing models inside this vast literature. The first concerns whether the model focuses on the short-term or the medium to long-term implications of a non-zero gap. The second feature relates to the degree of economic structure built into models to obtain NRI estimates.

The first strand of literature follows the lines of Woodford (2003) and Neiss and Nelson (2003). NRI estimates are obtained from within a microfounded *new Keynesian* model, the so called dynamic stochastic general equilibrium (DSGE) model. In this framework, NRI is the equilibrium real rate of return in an economy with fully flexible prices. In other words, NRI is the real short-term interest rate that equates aggregate demand with potential output throughout time. Giammarioli and Valla (2003), and Smets and Wouters (2003) provide interesting empirical applications of this approach

[1] See Giammarioli and Valla (2004) for an excellent survey.

for the Euro area.

The second strand of literature follows Laubach and Williams (2003). In this approach, simple macroeconomic models (from the monetary policy literature) are used along the Kalman filter to estimate NRI, the natural rate of unemployment, and potential output all as unobserved variables. These type of models are known as semi-structural models. Within this context, NRI is the real short-term rate of interest consistent with output at its potential level and inflation at an stable rate in the medium run. It means the effects from demand and supply shocks upon the output gap and inflation, respectively, vanish completely. Examples of this approach can be found in Orphanides and Williams (2002); Crespo-Cuaresma, Gnan, and Ritzberger-Grünewald (2004); Basdevant, Björksten, and Karagedikli (2004); Larsen and McKeown (2004); and Garnier and Wilhelmsen (2005).

There are, of course, other simpler procedures to estimate NRI such as the application of statistical filters . Some of the more common filters are Hodrick and Prescott (1997), Baxter and King (1999), and Christiano and Fitzgerald (2003). The use of these filters, however, may be subject to critics since it lacks support from economics for its results. As Larsen and McKeown (2004) and Mésonnier and Renne (2006) suggest, the approach in the second brand of literature represents a convenient compromise between the costly DSGE modeling from the first approach and the purely statistics approach from the filters.

Another important feature in empirical studies is the stability of NRI estimates. Empirical evidence points to the plausibility of significant time variation in NRI for many economies. For instance, Rapach and Wohar (2005) find evidence of multiple structural breaks in the mean of real interest rates over the last four decades in 13 industrialized countries. Therefore, they recommend using estimation methods that allow for large persistent fluctuations in NRI.

In this paper, we follow the approach suggested by Mésonnier and Renne (2007), which in turn is derived from Laubach and Williams (2003). The approach of Mésonnier and Renne (2007) has two advantages with respect to the method proposed by Laubach and Williams (2003). First, unlike Laubach and Williams (2003), we allow for stationarity (but high persistency) in the unobservable component that drives the low-frequency common fluctuations of NRI and potential output growth. On the contrary, assuming nonstationarity in output growth and NRI, as in Laubach and Williams (2003), contradicts economic theory and intuition. Second, the real interest rate is calculated as a model-consistent ex ante real rate of interest using inflation expectations provided by the model. Other studies, including Laubach and Williams (2003), consider inflation expectations using univariate autoregressive models.

We apply the approach of Mésonnier and Renne (2007) to quarterly Peruvian data for the period 1996:3 - 2008:3. Our results are relatively sensible to the calibration of two parameters. However, in most cases, NRI estimates are very stable. The gap on the real interest rate indicates a restrictive monetary policy for the periods 1996 - 2002 and 2003. Monetary policy appears to be relatively expansionary for the period 2004-2008. The gap behavior is very stable in the second period.

This paper is organized as follow. In Section 2, the model is described. Section 3 briefly describes data and Peru's economic background. Section 4 presents and discusses the econometric results. Section 4 concludes.

2 The Model

Our specification follows closely Mésonnier and Renne (2007), which in turn is based in Laubach and Williams (2003). Both approaches follow Rudebusch and Svensson (1998). The model consists of six backward-looking linear equations. Even though they are subject to the Lucas critique, it is worth mentioning that empirical backward-looking models without explicit expectations are still widely used for monetary policy analysis and they also appear to be robust empirically. Some examples at this respect are Rudebusch and Svensson (1998, 2002), Onatski and Stock (2002), Smets (2002), Fagan, Henry, and Mestre (2001), Fabiani and Mestre (2004), Rudebusch (2005), Bernanke and Mihov (1998), Estrella and Fuhrer (1999), Leeper and Zha (2002).

The rationale for NRI dynamics follows from the basic optimal growth model. In this model, intertemporal utility maximization yields a log-linear relationship between the real interest rate r^* and the rate of labor-augmenting technological change a, which is also the per capita rate of output growth along a balanced-growth path. This relationship is expressed as $r^* = \theta a + \rho$, where θ is the constant relative risk aversion (the inverse of the intertemporal elasticity of substitution) and ρ is the time preference rate of households. It is possible to assume that the trend growth rate a_t is subject to low-frequency fluctuations. If that is indeed the case, then, it is feasible to find a link between long run fluctuations in the potential output growth rate and NRI. In this sense, this approach lies in between those of Laubach and Williams (2003) and Orphanides and Williams (2002). In the former, NRI is the sum of the trend growth rate and a second nonstationary component. The first component drives the low-frequency fluctuations of potential output growth rate. In the latter study, NRI and potential output growth are completely unrelated, which is difficult to confront with theoretical intuition.[2]

The model consists of the following six equations:

$$\begin{aligned}
\pi_t &= \alpha(L)\pi_t + \beta(L)z_t + \epsilon_t^{\pi}, &(1)\\
z_t &= \phi(L)z_t + \lambda(L)(i_t - \pi_{t+1|t} - r_t^*) + \epsilon_t^{z}, &(2)\\
r_t^* &= \mu_r + \theta a_t, &(3)\\
\Delta y_t^* &= \mu_y + a_t + \epsilon_t^{y}, &(4)\\
a_t &= \psi a_{t-1} + \epsilon_t^{a}, &(5)\\
y_t &= y_t^* + z_t, &(6)
\end{aligned}$$

where the four shocks are independently and normally distributed with variances σ_π^2, σ_z^2, σ_y^2 and σ_a^2.

The first equation may be interpreted as an aggregate supply equation or *"Phillips curve"*. It specifies that consumer price inflation relates to its own lags and output gap lags. The second equation is a reduced form of an aggregate demand equation, or *"IS curve"*, relating the output gap to its own lags and IRG (the difference between the real short-term interest rate and NRI) lags. Stable inflation is consistent with a zero output gap and zero IRG. In this sense, NRI may be named non-accelerating-inflation

[2] It may result in a non-optimal exploitation of data.

rate of interest. In this model, monetary policy affects the inflation rate through its influence on the output gap. Furthermore, the nominal short-term interest rate is assumed exogenous, which implies an implicit reaction function.

In the literature a common NRI specification is a random walk.[3] In this approach, NRI is assumed to follow a highly autoregressive process as specified by (4) and (6).[4] Even though the random walk assumption may be advantageous from some perspective[5], it hinders economic interpretation of the model. This is the case, in particular, if we assume that potential growth (Δy_t^*) shares common fluctuations with r_t^*.[6] NRI estimates (see next section) show that this process is highly persistent, which is consistent with the purpose of capturing large and low frequency fluctuations in the level of the equilibrium real rate.

Equation (6) is an autoregressive representation for a_t. It captures low-frequency variations in potential output growth assuming that these variations are common with those of NRI. Notice that equation (5) specifies the behavior of potential output growth. It states that potential output growth has another stationary component that may account for other sources of discrepancies with NRI (shocks to preferences or changes in fiscal policy, for example). A simple white noise is enough to model this second stationary component.

An acknowledged setback of the model is that it does not incorporate open-economy features. For instance, the model does not allow for an explicit influence of terms of trade variations in potential output. Thus, the effects of positive external shocks in growth would be attributable to productivity growth and, as such, would imply a larger NRI than otherwise in

[3] Nonstationarity is also specified for the potential output growth rate. Some examples are Laubach and Williams (2003), Orphanides and Williams (2002), Larsen and McKeown (2004), and Fabiani and Mestre (2001).

[4] Another exception in the stationary specification of NRI is Gerlach and Smets (1999). Furthermore, they assume that potential output is I(1).

[5] It combines persistent changes in the unobservable component with smooth accommodation of feasible but unspecified structural breaks in the actual interest rate series.

[6] A nonstationary specification for NRI and potential output growth would indeed imply that potential output is integrated of order two. In terms of the standard optimal growth model, it would mean a nonstationary path of output to the stock of capital.

an open-economy representation of NRI. Therefore, interpretation of NRI estimates should be drawn carefully over periods of external turbulence. In these cases, NRI would feasibly be considered as an upper limit to the equilibrium rate (if positive shocks were in place).

All specifications are consistent with the hypothesis that potential output is an I(1) process. Application of simple unit root tests reject the null hypothesis of an I(2) log real output.

3 Data and economic background

Peruvian quarterly data for the period 1996:3-2007:2 is used in estimations. The inflation rate is defined as the annualized quarterly growth rate of the CPI series. The ex ante real short-term rate of interest is obtained by deducting from the current level of the nominal interest rate the one-quarter-ahead expectation of the (quarterly annualized) inflation rate. The data set is complete with the log of the real GDP. All variables have been seasonally adjusted using the procedure Tramo-Seats of Gómez and Maravall (1999).

Sample size is determined in practical terms considering data availability for the interbank rate (as a measure of the short-term nominal rate) in the Peruvian financial system. More importantly, economic rationale for the estimation period responds to Peru's output and inflation dynamics. Peru suffered from hyperinflation until 1990 and the disinflation process lasted up until 1994. Business cycle fluctuations were large and highly volatile during most of the 1980's and the first half of the 1990's. A number of structural economic reforms were introduced during the first part of the 1990's, namely financial system liberalization (including a previsional pension fund reform), trade openness, reinsertion in the international financial system, tax-system reform, sound and prudent monetary and fiscal policies, investments promotion and, in general, more market-oriented policies throughout the economy. With so many structural reforms at roughly the same time, volatility of the main macroeconomic variables was wide and unstable. By the start of the second half of the 1990's, most of these first-generation reforms were well functioning and key relationships between monetary and real sector reestablished soundly. Building upon new trends in macroeconomic variables by the late 1990's (and despite holding-up effects from the international financial crises), by 2002 Peru implemented a fully-fledged inflation-targeting regime. That suggests a feasible regime change that makes worth it paying attention to two sub samples, from 1996 to 2001 and 2002 and beyond. Indeed, output, investment and other key macroeconomic variables reduced considerably their volatility in the latter period.

4 Results

The six-equations model is written in its state-space form, and the parameters are estimated by maximization of the likelihood function provided by the Kalman filter. The filter is a recursive algorithm for sequentially updating a linear projection of a dynamic system. Given a set of measurement and transition equations, the Kalman filter provides the best linear unbiased estimate of the state variables. A particular feature of this approach is its ability to quantify uncertainty around the estimated state variables. In this sense, a filtered estimate of the state variables uses information only up to time t, whereas a smoothed estimate uses information from the whole sample, that is, up to time T. The former is frequently named a one-sided estimate whereas the second is a two-sided estimated.

In the direct estimation (without restrictions) by maximum likelihood two difficulties arise. The first one is estimation of the parameter θ. Unconstrained estimation of this parameter appears to be very unstable and not statistically significant.[7] This parameter links two unobservable variables, which may render its estimation ambitious if we consider the sample size used in the estimations. The second difficulty found in the unconstrained estimation is an estimated value of zero for σ_y. In some cases estimation of the parameter σ_z also renders a zero value. It implies that idiosyncratic shocks to output are indistinguishable from transitory shocks to output. That is not surprising if we think in the output gap's high persistence.

[7] Similar difficulties have been found by Larsen and McKeown (2004) applying the methodology of Laubach and Williams (2003) to UK data. Because they interpret the problem as a dimensionality issue, they decide to reduce the number of parameter using a calibration similar to Mésonnier and Renne (2007), which is also applied here. Further instability determines to calibrate another parameter.

In order to deal with these difficulties, two calibrations are used. The first one is calibration of the ratio σ_y/σ_z. Basis to calibrate this ratio is difficult to find. Even for the US and EU economies, evidence does not suggest basis for a consensus calibration. Fabiani and Mestre (2004) find a ratio of 0.94 for their Euro area model. Peersman and Smets (1999) find a value of 0.42 for a model including five countries of the EU. For the US some estimates are due to Peersman and Smets (1999), Smets (2002) and Laubach and Williams (2003). The range of values is from 1.7 to 3.3.

The second calibration is for parameter θ. Reasonable values for this parameter should be consistent with the order of magnitude of empirical estimates of the inverse of intertemporal elasticities of substitution found in the literature. Hall (1988) finds a small parameter that is non-statistically different from zero. It corresponds to an infinite risk aversion coefficient. Other estimates of the intertemporal elasticity of substitution (ranging from 0.27 to 0.77) are due to Ogaki and Reinhart (1998). For Peru, using a stochastic dynamic equilibrium model, Castillo, Montoro and Tuesta (2006) find a value of 4.00 for the risk aversion coefficient.[8] Given the aforesaid discussion, we consider the range $[0, 20]$ as a reasonable interval for plausible values of the risk aversion parameter θ.

Equations (1) and (2) need selection of lag lengths. Based on the statistical significance, equation (1) uses three lags for inflation and one lag for output gap. In equation (2), one lag for output gap and the second lag for the IRG have been selected. Furthermore, the null hypothesis that the coefficients of inflation sum to unity is not rejected. Therefore, we impose this condition, implying that an accelerationist form of the Phillips curve is adopted. In other words, inflation depends only on nominal factors in the long run.

Table 1 reports parameter estimates under alternative estimation scenarios that differ on the values for the calibrated parameters. Last column presents the unrestricted estimates. Results for $\theta = 1$ and $\sigma_y/\sigma_z = 0.5$ are our preferred scenario.

[8] Castillo, Montoro and Tuesta (2006) consider habits in the utility function. Using a habit coefficient of 0.75, the intertemporal elasticity of substitution corresponds to 0.25. Therefore, it implies a risk aversion coefficient of 4 in quarter terms.

Figure 1 shows the output gap, the productivity measure a_t, the NRI versus the observed real short-term interest rate, and the monetary position implied by the IRG (with 90% confidence bands in all cases). Picture of the output gap indicates supply excess until 2000-2001. Meanwhile, in more recent years, the output gap approximates zero (with wide confidence regions). Graphics of NRI shows narrow bands that indicate precise estimation. In comparison with the observed real short-term interest rate, NRI appears much more stable. The IRG indicates a restrictive monetary policy for periods 1996-2001 and 2003. A slight expansive monetary policy is observed for periods 2002 and 2004-2007.

An interesting issue is the fact that NRI and IRG estimates appear to be much more stable and similar throughout all alternative estimation scenarios.

5 Conclusions

This paper uses a semi-structural model to estimate the natural rate of interest (NRI) using Peruvian data for the period 1996:3 - 2007:2. A model without restrictions has been estimated. Some scenarios with two calibrated parameters are also estimated. In comparison with the behavior of the real short-term interest rate, the NRI is very stable. It is true for all considered scenarios. The gap of the interest rate is also stable and it describes the periods of restrictive and expansive monetary policy. Other statistical procedures to estimate the NRI have been used to compare with those from our model. The results indicate strong differences between both set of estimates. It suggests that care should be taken when we use simple statistical procedures to estimate the NRI or the gap of the interest rate.

Furthermore, considering the application to Peru, the model falls short in incorporating the dollarization of the Peruvian financial system. However, effects from such an omission are not conclusive as for NRI. Further empirical work should address feasible NRI dynamics variation.

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Table 1. Parameter Estimates

	$\sigma_y/\sigma_z = 0.5$ $\theta = 4$	$\sigma_y/\sigma_z = 4$ $\theta = 4$	$\sigma_y/\sigma_z = 0.5$ $\theta = 16$	$\sigma_y/\sigma_z = 0.5$ $\theta = 1$	No Restrictions
	Value (p-value)	Value (p-value)	Value (p-value)	Value (p-value)	Value (p-value)
α_1	0.85 (0.00)	0.83 (0.00)	0.84 (0.00)	0.90 (0.00)	0.89 (0.00)
α_2	-0.02 (0.89)	-0.05 (0.77)	-0.03 (0.86)	-0.08 (0.60)	-0.08 (0.66)
α_3	0.17 (0.17)	0.21 (0.08)	0.19 (0.14)	0.19 (0.12)	0.19 (0.16)
β	0.15 (0.09)	0.48 (0.11)	0.13 (0.07)	0.80 (0.11)	0.76 (0.11)
σ_π	0.78 (0.00)	0.71 (0.00)	0.78 (0.00)	0.72 (0.00)	0.72 (0.00)
ϕ	0.73 (0.00)	0.53 (0.02)	0.67 (0.01)	0.51 (0.02)	0.51 (0.03)
λ	-0.13 (0.01)	-0.08 (0.05)	-0.11 (0.04)	-0.06 (0.08)	-0.06 (0.10)
σ_z	0.94 (0.00)	0.28 (0.00)	0.90 (0.00)	0.00 (0.99)	0.00 (0.99)
σ_a	0.25 (0.14)	0.20 (0.31)	-0.17 (0.13)	1.28 (0.00)	1.28 (0.00)
ψ	0.92 (0.00)	0.94 (0.00)	0.95 (0.00)	0.30 (0.02)	0.30 (0.07)
μ_r	5.44 (0.03)	5.05 (0.06)	6.93 (0.35)	4.72 (0.00)	4.70 (0.00)
μ_y	1.08 (0.01)	1.14 (0.05)	1.06 (0.01)	1.09 (0.00)	1.09 (0.00)
θ	4.00	4.00	16.00	1.00	0.66 (0.60)
σ_y	0.47	1.13	0.45	0.00	0.00 (0.99)

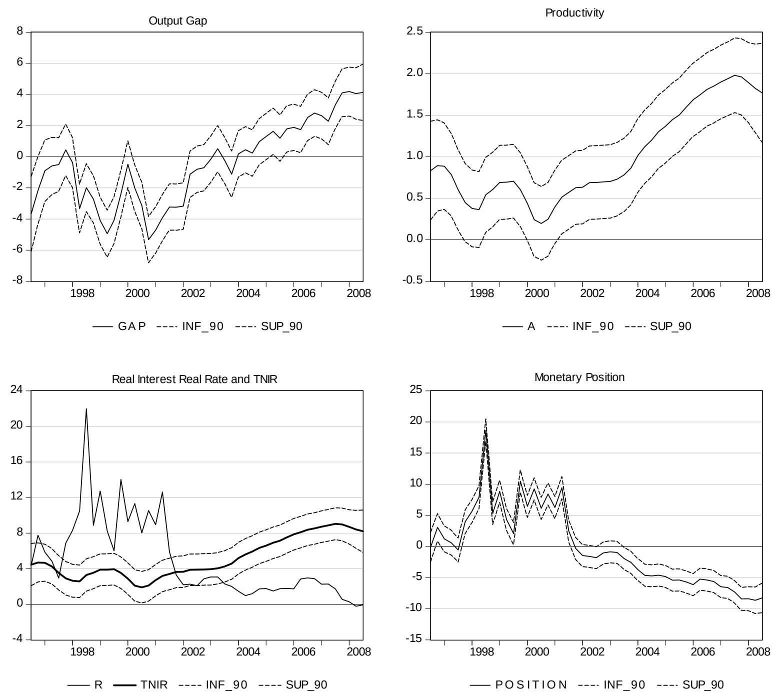

Figure 1. Estimates with $\sigma_y/\sigma_z = 0.5$ and $\theta = 4$

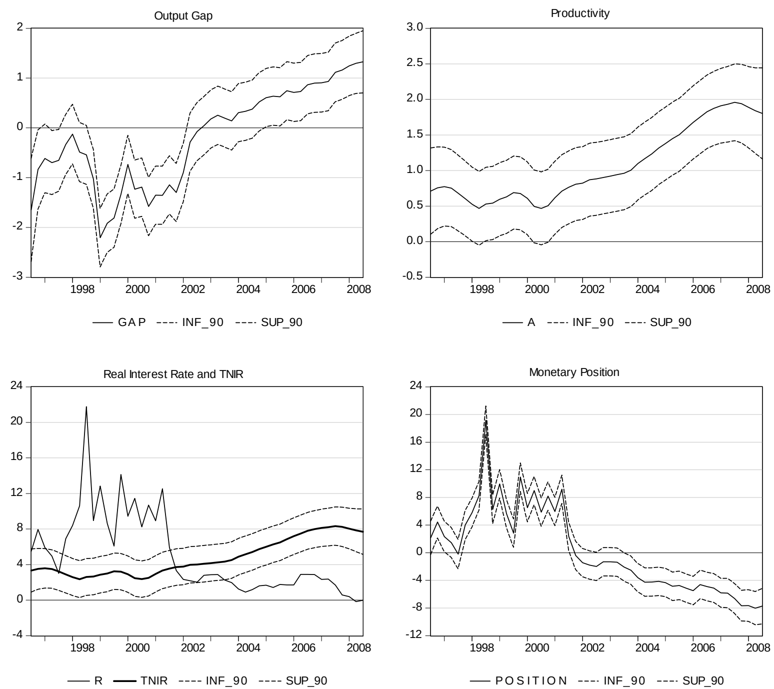

Figure 2. Estimates with $\sigma_y/\sigma_z = 4$ and $\theta = 4$

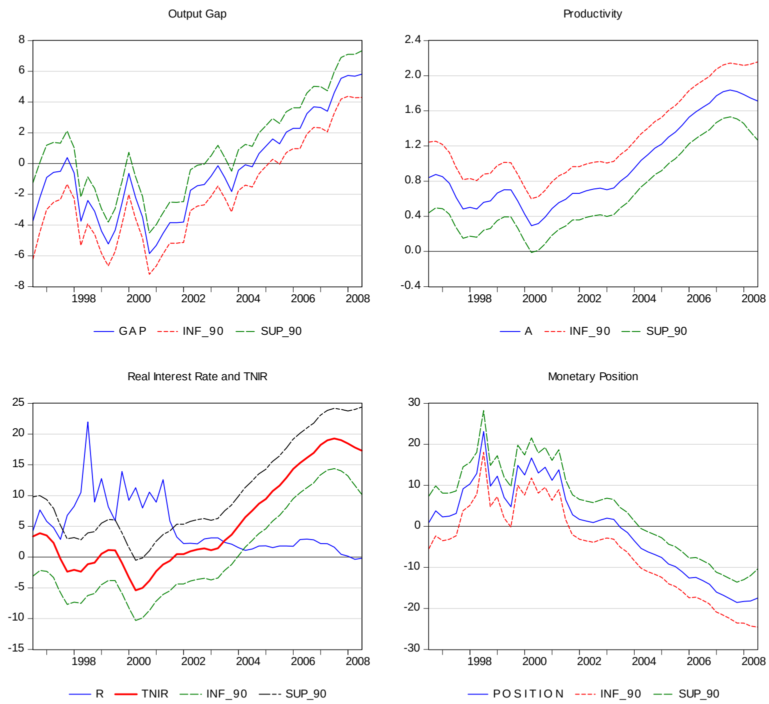

Figure 3. Estimates with $\sigma_y/\sigma_z = 0.5$ and $\theta = 16$

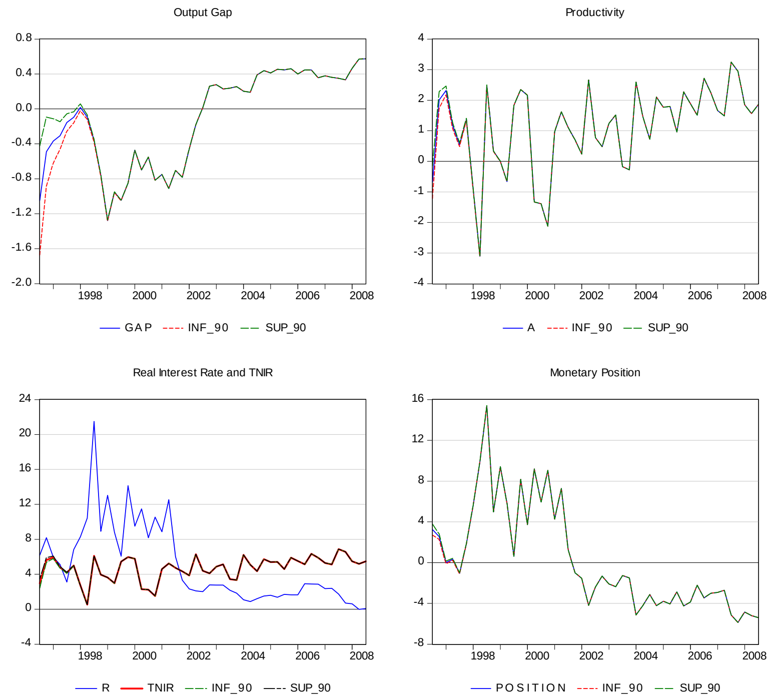

Figure 4. Estimates with $\sigma_y/\sigma_z = 0.5$ and $\theta = 1$

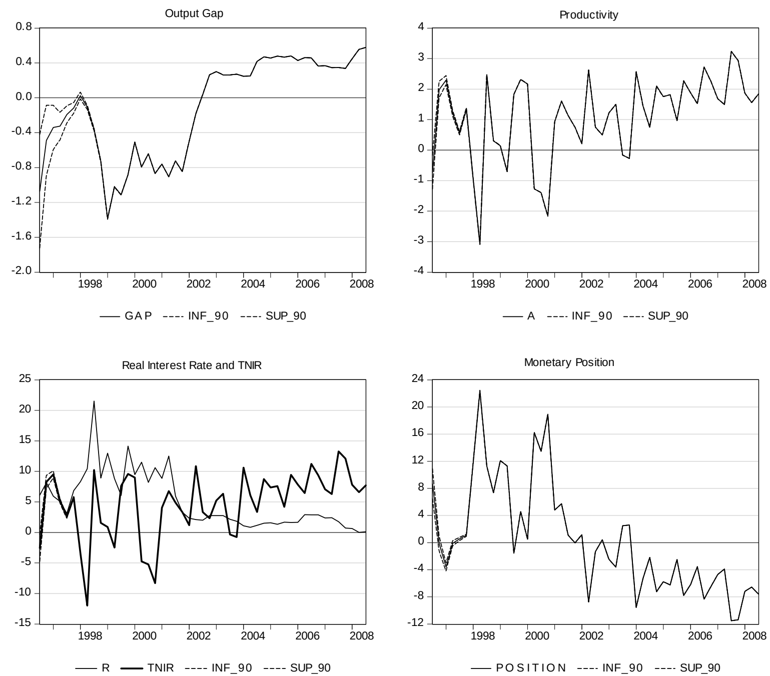

Figure 5. Estimates without restrictions