BANCO CENTRAL DE RESERVA DEL PERÚ

Política Monetaria en un Entorno de dos Monedas

Guillermo Felices* y Vicente Tuesta**

* Bank of England
** Banco Central de Reserva del Perú

DT. N° 2007-006
Serie de Documentos de Trabajo
Working Paper series
Abril 2007

Monetary Policy in a Dual Currency Environment*

Guillermo Felices[†] Bank of England

Vicente Tuesta[‡] Central Reserve Bank of Perú

March 2007[§]

Abstract

We develop a small open economy general equilibrium model with sticky prices and partial dollarization - a situation where both domestic and foreign currencies coexist. We derive a tractable representation of the model in terms of domestic inflation and the output gap in which a trade-off, which depends on the degree of dollarization, arises endogenously due to the presence of foreign interest rate shocks. We use this framework to show analytically how higher degrees of dollarization induce larger volatilities of the output gap and inflation, thus hampering a central bank's effectiveness in stabilizing the economy. Our impulse-response functions show that the transmission of such shocks has a positive (negative) effect on inflation and negative (positive) effect on the output gap when money aggregates and consumption are complements (substitutes). We also show that a standard Taylor rule guarantees real determinacy of the rational expectations equilibrium. Finally, we demonstrate that a higher degree of dollarization reduces the determinacy region when the overall money aggregate and consumption are substitutes.

Keywords: Dollarization, Currency Substitution, Policy trade-off, Staggered Price Setting, Open Economy.

JEL classification: E50, E52, F00, F30, F41.

*We would like to thank Nicoletta Batini, Jess Benhabib, Pierpaolo Benigno, Paul Castillo, Luca Colantoni, Roberto Chang, Thomas Cooley, Mark Gertler, Ricardo Lagos, Gonzalo Llosa, Paul Levine, Carlos Montoro, Sydney Ludvigson, Fabrizio Perri, and seminar participants at the Central Bank of Peru and the 2002 meeting of LACEA for helpful comments. The views expressed here are those of the authors and not necessarily reflect the position of the Bank of England or the Central Reserve Bank of Perú.

[†]Financial Stability, Bank of England; e-mail: Guillermo.Felices@bankofengland.co.uk

[‡]Research Department, Central Bank of Perú, Miroquesada # 441, Lima Peru; e-mail:vicente.tuesta@bcrp.gob.pe;

[§]A previous version of this paper circulated under the title 'Monetary Policy in a Partially Dollarized Economy'.

1 Introduction

One of the central issues in emerging economies is the idea of replacing the domestic currency with the US dollar. There is not a unique position in this debate with arguments found for and against dollarization. Recent research has focused on analyzing extreme cases, either complete dollarization or an economy with only domestic currency (examples are Chang and Velasco, 2001; Cooley and Quadrini, 2001; and Schmitt-Grohe and Uribe, 2000).

In some developing countries foreign currency is legally used and it is difficult to persuade agents not to hold it. This is the case of a 'partially dollarized economy' where foreign currency can be demanded not only as a deposit of value but also as a medium of payment (commonly known as transaction dollarization or currency substitution). Some other forms of dollarization can be identified. For instance, it is common that transaction dollarization is accompanied by financial dollarization, price dollarization and vice versa, although to varying degrees.

The Peruvian economy is usually taken as a case study since it is one of the most highly dollarized economies among emerging market countries that target inflation. Armas, Batini and Tuesta (2006) discuss the difficulties of implementing an independent monetary policy aimed at price stability through IT under financial, real and transaction dollarization[1]. Figure 1 below describes the importance of different types of dollarization for the Peruvian economy.

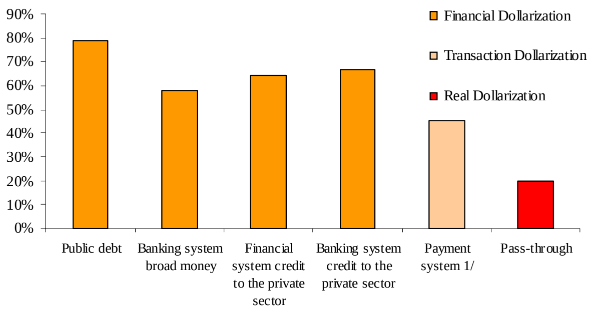

Figure 1: Degree of Transaction, Real and Financial Dollarization in Peru, 2006

(From Armas, Batini and Tuesta 2006)

Dollarization of financial assets in Peru is by far the most important form of dollarization (50-60 percent). Transaction dollarization is less strong but still very important (40 percent). Figure 1 uses the percentage of dollar-denominated private debt and the percentage of cash and

[1] See Armas, Batini and Tuesta (2006) for a description of the main channels through which partial dollarization hampers the trasnmission mechanism of monetary policy.

check payments made in dollars as a proxy measure for transaction dollarization[2]. Given the non-negligible importance of transaction dollarization, developing an analytical framework to explore its implications for monetary policy will be useful for central bankers that operate in such an environment.

Partial dollarization may not be an optimal regime but the costs of dollarizing or dedollarizing can be substantial in the short run. It is therefore important for central banks to understand the constraints they face when operating in a dual-currency environment. Policy makers are aware of some of the constraints faced by central banks when conducting monetary policy in these economies. As described in the 2000 edition of the IMF's World Economic Outlook, in a partially dollarized economy "... a significant constraint on monetary policy discretion is imposed. Not only is the monetary authority given control of a smaller share of the money supply but also this share can rapidly shrink (via currency switching) if credibility of domestic policies falters." But there is little analytical work in the literature to inform how a central bank should behave in order to overcome such limitations.

This paper seeks to provide such an analytical framework by studying monetary policy in a general equilibrium open economy model where both local and foreign currencies are demanded for transaction purposes in the home economy. We use this framework to assess qualitatively and quantitatively the extent to which higher degrees of dollarization rise the volatility of inflation and output, making the central bank less effective in stabilizing these variables. The paper also seeks to explore the transmission mechanism of shocks to foreign interest rates -a common and sometimes destabilizing shock in developing economies. The impact of foreign interest rate shocks are important in driving business cycles in the Peruvian economy as reported by Castillo, Montoro and Tuesta (2006). For example, these authors find that foreign disturbances (shocks to foreign interest rates and to the uncovered interest rate parity) account for 34 percent of output fluctuations in the Peruvian economy[3].

Hence, we develop a general equilibrium open economy framework in the spirit of Clarida, Gali and Gertler (2001 and 2002, hereafter CGG), Gali and Monacelli (2005, hereafter GM) and Benigno and Benigno (2003), allowing for partial degrees of currency substitution. We motivate partial dollatization by including a composite of both foreign and domestic currency in the utility function of the generic household in the home economy.[4] The composite is not separable from

[2] This percentage approximates the share of transactions in dollars relative to total transactions by combining data on (i) ATM dollar withdrawals; (ii) dollar checks; (iii) dollar interbank transfers; and (iv) direct debits in dollars,

[3] Castillo, Montoro and Tuesta (2006) estimate a DSGE model with partial dollarization using Bayesian techniques and Peruvian data.

[4] Partial dollarization can also be motivated using shopping-time models and cash in advance models, but at the expense of tractability that we obtain with our money-in-the-utility-function setup. For instance Castillo (2006) model currency substitution by adding transaction frictions in a cash in advance model.

consumption. We derive a tractable linearized model that embeds the extreme cases of high dollarization -defined as a very high preference for foreign currency- and low dollarization in the domestic economy.[5]

In order to obtain a tractable model we employ a two-country specification. We then explore the dynamic properties of the model in the limiting case that the size of the domestic economy is small.[6] In order to illustrate the inner-workings of the model we evaluate analytically the unconditional volatility of its key variables following a foreign interest rate shock for various degrees of dollarization. The key insight is that treating money aggregates as a composite of consumption introduces a short run trade-off between the stabilization of inflation and the output gap, thereby making the flexible price allocation no longer attainable. Moreover, such policy trade-off arises endogenously without considering an exogenous cost push shock due to the presence of exogenous shocks to the foreign interest rate.[7] Therefore, the monetary authority cannot stabilize output and inflation simultaneously. This trade-off is affected by the degree of dollarization. Furthermore, our canonical and tractable model also allows us to evaluate whether a standard Taylor rule guarantees real determinacy of the rational expectations equilibrium. We not only find that such a rule delivers real determinacy, but also that the determinacy region is reduced for higher degrees of dollarization. This is a novel result.

Our simulations and analytical results of the canonical model show that macroeconomic volatility increases for higher degrees of dollarization, thus making it more difficult for central banks to stabilize the economy. Interestingly, the transmission mechanism of a positive foreign interest rate shocks might have a positive (negative) effect on inflation and a negative (positive) effect on output when money aggregates and consumption are complements (substitutes). The intuition for the case when goods are complements is as follows: an increase in foreign interest rates reduces the demand for foreign currency, leading to a fall in the demand for the overall money aggregates. If money and consumption are complements the marginal utility of consumption is increasing in real money balances, then it follows that the marginal utility of consumption falls after the shock. Given a standard labour supply relationship, real wages and inflation rise. Finally, the output gap decreases as a result of the trade-off .

Our model results in a formulation of the marginal utility of consumption (MUC) that crucially affects the transmission mechanism of exogenous shocks in our setup. The MUC depends

[5]To perfectly characterize a fully dollarized economy an optimal price setting in foreign currency must be considered. This analysis escapes the scope of this paper since our objective is to factor in the effects of dollarization that stems from currrency substitution.

[6]Gali and Monacelli (2005) build a small open economy setting by assuming that the world is populated by a continuum of identical small open economies. Our model, differs from theirs in the sense that we are starting from a two-country general equilibrium framework.

[7]Clarida, Gali and Gertler (2001 and 2002) assume the presence of an exogenous cost push shock in order to generate the tradeoff between inflation and output gap.

not only on consumption but also on both foreign and domestic interest rates where their relative weights are functions of the ratio of foreign currency in the total money aggregates, which in turn depend on the degree of preference for foreign currency. A higher degree of preference for foreign currency reduces the effect of the interest rate on the MUC and therefore consumption and output drop further in response to any shock. This reveals the fragility of monetary policy in a partially dollarized environment.[8]

The paper is organized as follows. Section 2 outlines a general equilibrium model that allows for currency substitution in the domestic economy by introducing a composite of both domestic and foreign currency. We also describe how the optimality conditions and the international risk sharing condition change. In section 3 we present the equilibrium; simulate the loglinear version of the model; and calculate analytically the unconditional moments as well as the determinacy conditions. Finally, section 4 concludes.

2 The Model

We consider a two country open economy model with imperfect competition and nominal price rigidities along the lines of Obstfeld and Rogoff (1995), Clarida, Gali and Gertler (2002) and Benigno and Benigno (2003). In contrast, we give money a role in the model by introducing a money aggregate (composed of both local and foreign currency) as a composite of consumption for the home country[9]. We allow for tradable goods only, home bias and a complete asset market structure. In order to treat the home economy as small and open, we eliminate the effect of home variables on the foreign economy, as in Sutherland (2002).

2.1 Households

The model follows CGG (2002), Beningno and Benigno (2003) and Benigno (2004). The world size is normalized to unity. The are two countries, home and foreign. The population in the home country lies on the interval $[0, n]$, while in the foreign economy it lies on the segment $(n, 1]$. A generic agent h belonging to the home economy is a consumer of all the goods produced in both

[8]There is now lengthy literature for small open economies that show that dollarization of liabilities affect the aggregate demand through balance sheets effect (debt denominated in either domestic or foreign currency). Some prominent examples include: Gertler, Gilchrist and Natalucci (2003), Christiano, Gust and Roldos (2004) and Céspedes, Chang and Velasco (2004). In contrast with these studies, our setup we affect both aggregate demand and aggregate supply by the effect of currency substitution over the MUC.

[9]As shown in Castillo, Montoro and Tuesta (2006) this formulation is supported by the Peruvian data.

countries H and F. Preferences of the generic household h in country H are given by

$$E_t \sum_{i=0}^{\infty} \beta^i U^h \left[C_t^h, Z_t^h \left(\frac{M_t^h}{P_t}, \frac{D_t^h S_t}{P_t} \right), L_t^h \right] \tag{1}$$

$$U_{t+i}^h = \frac{1}{1-\sigma} \left\{ \left[\left(b C_{t+i}^{h\ \frac{\omega-1}{\omega}} + (1-b) Z_{t+i}^{h\ \frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}} \right]^{1-\sigma} - \frac{L_{t+i}^{h\ (1+\upsilon)}}{1+\upsilon} \right\} \tag{2}$$

where Z_{t+i}^h is a money aggregate defined as

$$Z_{t+i}^h = \left(\nu \left(\frac{M_{t+i}^h}{P_{t+i}} \right)^{\frac{\chi-1}{\chi}} + (1-\nu) \left(\frac{D_{t+i}^h S_{t+i}}{P_{t+i}} \right)^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}} \tag{3}$$

E_t denotes the expectation conditional on the information set at date t, and β is the intertemporal discount factor, and σ and $\upsilon > 0$ represent the coefficient of risk aversion and the inverse of the elasticity of labor supply, respectively. $\omega > 0$ captures the degree of complementary or substitutability between consumption and the overall money aggregate. This parameter will become important later on since it will capture the effect of money over the labor supply and consequently over the aggregate supply equation. To the extent that $\sigma = 1$, when $\omega > 1$, the marginal utility of consumption is decreasing in real money balances $U_{CZ}^h < 0$[10]. Therefore, higher interest rates along with the associated reduction in real balance holdings, increase the marginal utility of consumption, hence the overall money aggregate and consumption are substitutes. On the other hand, when $0 < \omega < 1$, the marginal utility of consumption is increasing in real money balances $U_{CZ}^h > 0$ and therefore the overall money aggregate and consumption are complements[11]. The parameter $0 < b < 1$ is the weight of consumption in the consumption-money aggregate; $\chi > 0$ represents the elasticity of substitution between domestic and foreign currency; and $0 < v < 1$ is the preference for domestic currency within the overall money aggregate. Agents get utility from consumption C_t^h and from holding both domestic and foreign real money balances, $\frac{M_t^h}{P_t}$ and $\frac{S_t D_t^h}{P_t}$, respectively. The household also supplies hours of work, L_t^h.

The novelty in this formulation is the definition of the money aggregate Z_{t+i}^h as a CES composite of real domestic and foreign money balances[12]. When the weight of domestic money, ν,

[10] Note that $U_{CZ} = \left(\frac{1-\sigma\omega}{\omega}\right) b (1-b) (ZC)^{\frac{\omega-1}{\omega}-1}$ .if $\left(\frac{1-\sigma\omega}{\omega}\right) > 0$ consumption and money aggregates are complements. For the particular case when $\sigma = 1$ the condition simplifies to $\frac{1}{\omega} - 1 > 0$. Then if $0 < \omega < 1$ C and Z are complements, on the contrary if $\omega > 1$ consumption and the overall aggregate are substitutes $U_{CZ} < 0$.

[11] See Woodford (2003) chapter 2 for a brief discussion related to the consequences of nonseparable utility function and price determination.

[12] Notice that money shows up in both the budget constraint and in the utility function. In our model the monetary distortion is taken to be non-negligible or non trivial. Morever, non-separability between money aggregates and consumption, guarantee a role for money even if monetary policy actions are defined in terms of an interest

equals 1 the model collapses to an open economy with no foreign currency used for transactions (no dollarization) in the home economy. Similarly, when $\nu = 0$, only foreign currency is used for transactions, implying full dollarization[13]. Standard models for open economies implicitly assume that there exist tight legal restrictions which prevent a country's resident from using foreign currency for domestic transactions. However, in economies which have experienced hyperinflations it is difficult for the government to persuade their citizens to use only domestic currency for day-by-day transactions[14]. Moreover, even in the presence of legal restrictions, agents often hold and use foreign currency. Obstfeld and Rogoff (1996) motivate the idea of partial dollarization by introducing a composite of foreign and domestic money in the utility function that tries to capture the existence of those legal restrictions[15]. We assume that there are no legal restrictions for holding foreign currency as it is the case in developing countries such as Peru, Bolivia and Uruguay. In these economies there are no restrictions to hold foreign currency. Moreover, foreign currency is used for day-by-day transactions. Therefore, it seems plausible to include both foreign and domestic currency as a CES aggregate in the utility function[16]. Furthermore, the advantage of considering this specification is that it allows us to endogenously pin down the dollarization ratio in the steady state.

A generic household of the foreign country obtains utility only from consumption C_t^* and supply labor L_t^*

$$E_t \sum_{i=0}^{\infty} \beta^i U^f \left[C_t^*, L_t^* \right] \tag{4}$$

$$U_{t+i}^f = \frac{{C_{t+i}^*}^{1-\sigma}}{1-\sigma} - \frac{{L_{t+i}^*}^{(1+\upsilon)}}{1+\upsilon} \tag{5}$$

We define C_t as the consumption index in the home country with partial dollarization.

rate feedback rule.

[13]Note that when $b = 1$, the model is similar to the small open economy version presented by Clarida, Galí and Gertler (2001).

[14]Even with low inflation levels, agents still demand foreign currency not only as an mean of exchange but also as deposit of value.

[15]Obstfeld and Rogoff (1996) assume a quadratic form as they consider an economy with legal restrictions in holding foreign currency. The form they consider is $a_0 \left(\frac{M_t}{P_t} \right) - \frac{a_1}{2} \left(\frac{D_t S_t}{P_t} \right)^2$ where the second term measures the evasion costs of the legal restrictions.

[16]A model with transaction technology with shopping time and real money balances in both foreign and domestic currency would be another possibility at the cost of losing tractability. See Brock (1974) for an earlier use of shopping-time model to motivate a money-in-the-utility function approach. The advantage of the way we impose currency substitution is that it delivers a more tractable model. Other approaches to give rise to a valued role for money are suggested by Kyotaki and Wright (1993) by imposing that direct exchange of commodities is assumed to be costly, but there is a fiat money that can be treated as costlessly for commodities.

We define the consumption index as

$$C_t \equiv \left[(1-\lambda)^{\frac{1}{\theta}} \left(C_{H,t}^h \right)^{\frac{\theta-1}{\theta}} + \lambda^{\frac{1}{\theta}} \left(C_{F,t}^h \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{6}$$

where θ is the elasticity of substitution between home and foreign tradable goods and $C_{H,t}$ and $C_{F,t}$ are two sub-indices that refer to the consumption of home produced and foreign goods. The parameter that determines home consumers' preferences for foreign goods, λ, is a function of the relative size of the foreign economy, $(1-n)$, and of the degree of openness, γ, That is $\lambda = (1-n)\gamma$.

The corresponding consumption index for foreign households is given by,

$$C_t^* \equiv \left[(1-\lambda^*)^{\frac{1}{\theta}} (C_{H,t}^*)^{\frac{\theta-1}{\theta}} + \lambda^{*\frac{1}{\theta}} (C_{F,t}^*)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{7}$$

where $1-\lambda^* = n\gamma$

As in Sutherland (2002), $1-\lambda$ accounts for the degree of home bias in domestic consumption Notice that $\gamma = 0$ implies a completely closed economy. The indexes $C_{H,t}$, ($C_{F,t}$), $C_{H,t}^*$($C_{F,t}^*$) are home (foreign) consumption of the differentiated products produced in countries H and F, respectively which are defined as follows

$$C_{H,t} \equiv \left\{ \left(\frac{1}{n} \right)^{\frac{1}{\epsilon}} \int_0^n [c_t(z)]^{\frac{\epsilon-1}{\epsilon}} dz \right\}^{\frac{\epsilon}{\epsilon-1}}, C_{F,t} \equiv \left\{ \left(\frac{1}{1-n} \right)^{\frac{1}{\epsilon}} \int_n^1 [c_t(z)]^{\frac{\epsilon-1}{\epsilon}} dz \right\}^{\frac{\epsilon}{\epsilon-1}} \tag{8}$$

$$C_{H,t}^* \equiv \left\{ \left(\frac{1}{n} \right)^{\frac{1}{\epsilon}} \int_0^n [c_t^*(z)]^{\frac{\epsilon-1}{\epsilon}} dz \right\}^{\frac{\epsilon}{\epsilon-1}}, C_{F,t}^* \equiv \left\{ \left(\frac{1}{1-n} \right)^{\frac{1}{\epsilon}} \int_n^1 [c_t^*(z)]^{\frac{\epsilon-1}{\epsilon}} dz \right\}^{\frac{\epsilon}{\epsilon-1}} \tag{9}$$

where $\varepsilon > 1$ is the elasticity of substitution across goods produced within a country. In this context, the general price indexes that corresponds to the previous specification are given by

$$P_t \equiv \left[(1-\lambda)(P_{H,t})^{1-\theta} + \lambda (P_{F,t})^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{10}$$

$$P_t^* \equiv \left[(1-\lambda^*)(P_{H,t}^*)^{1-\theta} + \lambda^* (P_{F,t}^*)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{11}$$

$(1-\lambda)$ and λ^* are parameters that capture the degree of home bias in preferences in each country, respectively. As in Sutherland (2002) λ corresponds to the share of foreign goods in consumption basket of home agents and it will depend on the share of foreign goods in the total measure of goods in the world $(1-n)$ and on the degree of openness γ. $\gamma = 0$ implies a completely closed economy.[17] The previous limiting case could be interpreted as if the foreign currency have complete home bias. C_H^h and C_F^h are sub-indexes of consumption across the continuum of differentiated goods produced in country H and F, and are given by

$$C_{H,t}^h \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_0^n c_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}}, C_{F,t}^h \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}} \int_n^1 c_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{12}$$

where $\varepsilon > 1$ is the elasticity of substitution across goods produced within a country. In this context, the general price indexes that corresponds to the previous specification are given by

$$P_t \equiv \left[(1-\gamma)(P_{H,t})^{1-\theta} + \gamma (P_{F,t})^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{13}$$

$$P_t^* \equiv \left[(1-\gamma^*)\left(P_{H,t}^*\right)^{1-\theta} + \gamma^* \left(P_{F,t}^*\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{14}$$

where $P_{H,t}$ ($P_{H,t}^*$) is a price sub-index for home-produced goods expressed in domestic (foreign) currency and $P_{F,t}$ $\left(P_{F,t}^*\right)$ is the price sub-index for foreign produced goods expressed in domestic (foreign) currency, respectively.

$$P_{H,t} \equiv \left\{\left(\frac{1}{n}\right) \int_0^n [p_t(z)]^{1-\epsilon} dz\right\}^{\frac{1}{1-\epsilon}}, P_{F,t} \equiv \left\{\left(\frac{1}{1-n}\right) \int_n^1 [p_t(z)]^{1-\epsilon} dz\right\}^{\frac{1}{1-\epsilon}}$$

$$P_{H,t}^* \equiv \left\{\left(\frac{1}{n}\right) \int_0^n [p_t^*(z)]^{1-\epsilon} dz\right\}^{\frac{1}{1-\epsilon}}, P_{F,t}^* \equiv \left\{\left(\frac{1}{1-n}\right) \int_n^1 [p_t^*(z)]^{1-\epsilon} dz\right\}^{\frac{1}{1-\epsilon}}$$

Prices are set in producer currency. This assumption implies that the law of one price holds, $P_{H,t} = S_t P_{H,t}^*$ and $P_{F,t} = S_t P_{F,t}^*$ where S_t denotes the nominal exchange rate (the price of foreign currency in terms of domestic currency). Note, however, that purchasing power parity (a constant real exchange rate) does not necessarily hold because of the presence of home bias in preferences. The home-bias assumption allows generating real exchange rate dynamics in a model with only tradable goods. We define the terms of trade as the price of imported goods from abroad relative to the price of the exported goods abroad, such that $T_t = P_{F,t}/S_t P_{H,t}^* = P_{F,t}/P_{H,t}$.

Given the previous definitions we can express the real exchange rate as a function of the terms

[17] Unlike GM (2005) and CGG (2002) we rely on a complete general equilibrium structure. As you will see later, in steady state, γ will represent the share of domestic consumption allocated to imported goods, so it could be interpreted as a natural index of openness. So in this sense $(1-\gamma)$ is interpreted as the degree of home bias and the larger this value (smaller γ) the closer to a closed economy counterpart.

of trade:

$$Q_t = \frac{S_t P_t^*}{P_t} = \frac{\left[(1-\lambda^*)\left(S_t P_{H,t}^*\right)^{1-\theta} + \lambda^*\left(S_t P_{F,t}^*\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\left[(1-\lambda)\left(P_{H,t}\right)^{1-\theta} + \lambda\left(P_{F,t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}} \tag{15}$$

dividing the above expression both numerator and denominator by $P_{H,t}$ and taking the limit when $n \longrightarrow 0$. Then $(1-\lambda^*)\frac{(1-n)}{n} \longrightarrow \gamma$.

$$Q_t = \left[\frac{T_t^{1-\theta}}{(1-\gamma) + \gamma T_t^{1-\theta}}\right]^{\frac{1}{1-\theta}} \tag{16}$$

2.2 Optimal Consumption Allocations and Demand

The allocation of demands across each of the goods produced within a given country are given by

$$y_t^d(h) = c_t(h) + c_t^*(h) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\epsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\theta}\left[(1-\lambda)C_t + \frac{(1-\lambda^*)(1-n)}{n}C_t^* Q_t^\theta\right] \tag{17}$$

$$y_t^d(f) = c_t(f) + c_t^*(f) = \left(\frac{p_t(f)}{P_{H,t}}\right)^{-\epsilon}\left(\frac{P_{F,t}}{P_t}\right)^{-\theta}\left[\frac{\lambda n}{1-n}C_t + \lambda^* C_t^* Q_t^\theta\right] \tag{18}$$

To portray the small open economy we use the definition of $(1-\lambda)$ and $(1-\lambda^*)$ and take the limit when $n \rightarrow 0$. We obtain

$$y_t^d(h) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\theta}\left[(1-\gamma)C_t + \gamma Q_t^\theta C_t^*\right] \tag{19}$$

and

$$y_t^d(f) = \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\varepsilon}\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\theta} C_t^* \tag{20}$$

From the above equations notice that domestic consumption and the real exchange rate do not affect the world demand.

2.3 Budget Constraint and Asset Market Structure

We assume that households have access to a complete set of state contingent nominal claims which are traded domestically and internationally[18].We represent the asset structure by assuming a

[18] Given this assumption it is not necessary to characterize the current account dynamics in order to determine the equilibrium allocations.

complete contingent one-period nominal bond denominated in home currency[19]. The household in the domestic economy with partial dollarization faces a sequence of intertemporal budget constraints of the form

$$C_t^h = \frac{W_t}{P_t}L_t^h + \Pi_t - TR_t^h - \frac{M_t^h - M_{t-1}^h}{Pt} - \frac{E_t\{\xi_{t,t+1}B_{t+1}^h\} - B_t^h}{Pt} - \frac{D_t^h S_t - D_{t-1}^h S_t}{P_t} \tag{21}$$

B_{t+1}^h is a nominal random payoff of the portfolios purchased in domestic currency at t. with $\xi_{t,t+1}$ being the stochastic discount factor of nominal payoffs[20].

The government's budget constraint is balanced every period, so that total transfers are equal to seigniorage revenues:

$$\int_0^n \left(M_t^h - M_{t-1}^h\right) dh = \int_0^n TR_t^h dh \tag{22}$$

Once we account for the optimal allocation of consumption and demands, together with the budget constraint, we can obtain the optimality condition by differentiating the objective function with respect to L_t^h, B_t^h, $\frac{M_t^h}{P_t}$, and $\frac{S_t D_t^h}{P_t}$, to obtain the following FOCs:

$$\frac{W_t}{P_t}U_{C,t} = \left(L_t^h\right)^{\upsilon} \tag{23}$$

$$\beta\frac{P_t}{P_{t+1}}\frac{U_{C,t+1}}{U_{C,t}} = \xi_{t,t+1} \tag{24}$$

$$U_{C,t} = (1+i_t)\,E_t\left\{\frac{P_t}{P_{t+1}}\beta U_{C,t+1}\right\} + U_{m,t} \tag{25}$$

$$U_{C,t} = (1+i_t^*)\,E_t\left\{\frac{P_t}{P_{t+1}}\frac{S_{t+1}}{S_t}\beta U_{C,t+1}\right\} + U_{d,t} \tag{26}$$

with marginal utility of consumption

$$U_{C,t} = \Phi_t^{\frac{1}{\omega}-\sigma}C_t^{-\frac{1}{\omega}}b \tag{27}$$

with $\Phi_t \equiv \left(bC_{t+i}^{h\,\frac{\omega-1}{\omega}} + (1-b)Z_{t+i}^{h\,\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}}$

[19] Given that markets are complete internationally, it does not matter the currency denomination of the securities.

[20] Therefore $\xi_{t,t+1}$ is the price of one unit of nominal consumption of time $t+1$, expressed in units of nominal consumption at t, contingent on the state at $t+1$ being s_{t+1}, and given any state s in t. If we define the value of the portfolio at the end of the period as A_t, the complete market assumptions implies that there exists a unique discount factor $\xi_{t,t+1}$ of a portfolio with the property that the price in period t of the portfolio with random value B_{t+1} is $A_t = E_t\left[\xi_{t,t+1}B_{t+1}\right]$

marginal utility of domestic real balances can be written as

$$U_{m,t} = \Phi_t^{\frac{1}{\omega}-\sigma}(1-b)\nu Z_t^{h\left(\frac{1}{\chi}-\frac{1}{\omega}\right)}\left(\frac{M_t^h}{P_t}\right)^{-\frac{1}{\chi}} \tag{28}$$

and marginal utility of foreign real balances

$$U_{d,t} = \Phi_t^{\frac{1}{\omega}-\sigma}(1-b)(1-\nu) Z_t^{h\left(\frac{1}{\chi}-\frac{1}{\omega}\right)}\left(\frac{D_t^h S_t}{P_t}\right)^{-\frac{1}{\chi}} \tag{29}$$

Taking conditional expectations to both sides of (24) and letting $(1+i_t)$ denote the (gross) nominal yield of a one period risk-free discount bond in domestic currency (such that $E_t\left\{\xi_{t,t+1}\right\} = (1+i_t)^{-1}$ is the price of this bond) we can derive the intertemporal home Euler equation[21]

$$U_{C,t} = (1+i_t)E_t\left\{\frac{P_t}{P_{t+1}}\beta U_{C,t+1}\right\} \tag{30}$$

2.3.1 International Risk Sharing

Given that state contingent securities are tradable internationally[22], the intertemporal efficiency condition for the foreign economy is given by

$$\beta\frac{P_t^*}{P_{t+1}^*}\frac{S_t}{S_{t+1}}\frac{U_{C^*,t+1}}{U_{C^*,t}} = \xi_{t,t+1} \tag{31}$$

then combining the above equation with equation (24) we get:

$$\frac{U_{C,t}}{P_t} = k_o\frac{U_{C^*,t}}{S_t P_t^*} \tag{32}$$

where k_o is a function of predetermined variables (see CKM (2002) for details). Using the definition of the real exchange rate the above expression can be written as

$$Q_t = k_o\frac{U_{C^*,t}}{U_{C,t}} \tag{33}$$

This equation relates consumption at home with consumption abroad via the real exchange rate. It is worthwhile mentioning that our model delivers a different risk-sharing condition than that

[21] The interest rate at home is the price of the portfolio that delivers one unit of home currency in each contingency that occurs one-period ahead.

[22] We assume complete markets for simplicity and tractability. For small open economy models with incomplete markets and stationary net foreign assets see Schmitt-Grohe and Uribe (2001) and Laxton and Pesenti (2003). See Benigno (2001) and Selaive and Tuesta (2003) for incomplete markets in two-country models.

in standard two-country models. Notice that given the assumption of non-separability between consumption and the money aggregate in the home economy, $U_{C,t}$ will be a function of domestic consumption and both domestic and foreign interest rates. In our model both interest rates play a key role in explaining the real exchange rate dynamics and consequently the cross correlation between the relative consumption and the real exchange rate across countries[23].

As in the domestic economy, we define the foreign interest rate i_t^* as the price of the portfolio that delivers one unit of foreign currency in each contingent state next period. Therefore, given the complete markets assumption we have:

$$\frac{1}{1+i_t^*} \equiv E_t\left(\xi_{t,t+1}\frac{S_{t+1}}{S_t}\right) \tag{34}$$

Combining equations (31) and (34) we can obtain the intertemporal Euler equation for the foreign country:

$$U_{C^*,t} = (1+i_t^*)E_t\left\{\frac{P_t^*}{P_{t+1}^*}\beta U_{C^*,t+1}\right\} \tag{35}$$

Notice that by combining both home and foreign Euler equations along with equation (32) we obtain a version of the uncovered interest rate parity condition (UIP) [24].

$$1 = E_t\left\{\xi_{t,t+1}\left[(1+i_t) - (1+i_t^*)\,\frac{S_{t+1}}{S_t}\right]\right\} \tag{36}$$

It is worthwhile mentioning that the UIP holds given the complete asset market structure and it does not represent an additional equilibrium condition.[25]

2.4 Relative Demand for Foreign Currency

By combining equation (30) with (25) we get an equation for the demand for domestic money balances

$$U_{m,t} = \frac{i_t}{1+i_t}U_{C,t} \tag{37}$$

[23] Backus and Smith (1993) find out that the main discrepancy between complete markets models and the data is that the complete market assumption implies a positive and high cross-correlation between the real exchange rate and relative consumption across countries, while in the data we observe the opposite. Selaive and Tuesta (2006) proposed an interesting avenue to explain this anomaly by consideing an incomplete markets model in which the net foreign position plays a crucial role in accounting for the apparent lack of risk-sharing. Instead, the risk sharing condition implied by the non-separability utility function suggests another channel through which the anomaly could be explained without relaying on the incomplete markets assumption.

[24] Relaxing this assumption would give interesting results, however, since the main goal of the paper is to derive a model for a partially dollarized economy, the assumption of completeness is reasonable to get a tractable model.

[25] The UIP holds even if we have deviations from PPP. In an incomplete markets structure without financial frictions (also known as the "bond economy") the UIP will also hold in log linear form. However, we can attain deviations from the UIP once financial frictions are taken into account, see Benigno (2001).

on the other hand by combining (35), (26) and (32) we obtain the demand for foreign currency:

$$U_{d,t} = \frac{i_t^*}{1+i_t^*} U_{C,t} \tag{38}$$

Note that we can derive a demand for foreign currency from equation (38) and a demand for local currency from (37). Using the last two equations we obtain the relative demand of foreign currency with respect to local currency

$$RF_t \equiv \frac{\frac{D_t^h S_t}{P_t}}{\frac{M_t^h}{P_t}} = \left(\frac{i_t^*}{1+i_t^*} \frac{1+i_t}{i_t} \frac{\nu}{1-\nu} \right)^{-\chi} \tag{39}$$

Notice that $\frac{\partial RF_t}{\partial i_t} > 0$, which implies that if the opportunity cost of holding domestic currency increases then the relative demand for foreign currency increases, similarly $\frac{\partial RF_t}{\partial i_t^*} < 0$ which implies that if the opportunity cost of holding foreign currency increases then the relative demand for foreign currency decreases. Also note that $\frac{\partial RF_t}{\partial v} < 0$, the higher the preference for domestic currency the lower the relative demand for foreign currency. Manipulating expression (39) we derive the ratio of dollarization, the amount of real foreign currency as a proportion of total real money aggregates (local and foreign)

$$RD_t = \left[\left(\frac{i_t^*}{1+i_t^*} \frac{1+i_t}{i_t} \frac{\nu}{1-\nu} \right)^{\chi} + 1 \right]^{-1} \tag{40}$$

Notice again that $\frac{\partial RD_t}{\partial i_t} > 0$, $\frac{\partial RD_t}{\partial i_t^*} < 0$, $\frac{\partial RD_t}{\partial v} < 0$.[26].

2.5 Price Setting

The firms' price setting decision is modelled through a Calvo-type mechanism. We assume that prices are subject to changes at random intervals. In each period a seller faces a fixed probability $(1-\alpha)$ of adjusting the price, independently of the length of the time period before the previous change. In this model suppliers behave as monopolists in selling their products. The objective of a home firm selling traded goods is to maximize the expected discounted value of profits[27]. τ is an employment subsidy that eliminates the monopolistic distortion. Since all firms resetting

[26] The derivative $\frac{\partial RF_t}{\partial \alpha}$ can be positive or negative depending on the size of $\left(\frac{i_t^*}{1+i_t^*} \frac{1+i_t}{i_t} \frac{\nu}{1-\nu} \right)$. If $\left(\frac{i_t^*}{1+i_t^*} \frac{1+i_t}{i_t} \frac{\nu}{1-\nu} \right) > 1$ then $\frac{\partial RF_t}{\partial \alpha} < 0$. If $\left(\frac{i_t^*}{1+i_t^*} \frac{1+i_t}{i_t} \frac{\nu}{1-\nu} \right) < 1$ then $\frac{\partial RF_t}{\partial \alpha} > 0$.

[27] $\xi_{t+k} = \beta^k \frac{U_C(C_{t+k})}{P_{t+k}} \frac{P_t}{U_C(C_t)}$ is the stochastic discount factor associated to the first order condition for the recursive competitive equilibrium.

prices in any given period will choose the same price, we henceforth drop the h subscript.

$$Max_{\widetilde{P}_{H,t}} E_t \sum_{k=0}^{\infty} \alpha^k \xi_{t,t+k} \{ \widetilde{P}_{H,t,t+k} \left(\widetilde{y}^d_{t,t+k} - MC^n_{t+k} \right) \} \tag{41}$$

subject to

$$y^d_{t+k}(h) = \left(\frac{\widetilde{P}_{H,t+k}}{P_{H,t+k}} \right)^{-\epsilon} \left(\frac{P_{H,t+k}}{P_{t+k}} \right)^{-\theta} \left[(1-\gamma) C_{t+k} + \gamma C^*_{t+k} Q^\theta_{t+k} \right] \tag{42}$$

where $MC^n_t \equiv (1-\tau) \frac{W_t}{A^H_t}$. Each firm produces according to a linear technology

$$y_t(h) = A^H_t L^h_t \tag{43}$$

where A^H_t is the country-specific productivity shock at time t.

The supplier maximizes (41) with respect to $\widetilde{P}_t$ given the demand function and taking as given the sequences of prices $\left\{ P^i_{H,t}, P^i_{F,t}, P^i_t, C^i_t \right\}$ for $i = H, F$.

The optimal choice of $\widetilde{P}_t$ is:

$$\widetilde{P}_{H,t} = \sum_{k=0}^{\infty} \alpha^k E_t \xi_{t,t+k} Y_{t+k} \left[\widetilde{P}_{H,t} - \frac{\varepsilon}{\varepsilon - 1} MC^n_{t+k} \right] \tag{44}$$

Finally, Calvo-price setting implies the following state equation for $P_{H,t}$

$$P^{1-\varepsilon}_{H,t} = \alpha P^{1-\varepsilon}_{H,t-1} + (1-\alpha) \widetilde{P}^{1-\varepsilon}_{H,t} \tag{45}$$

An analogous expression can be derived for the foreign economy.

2.6 Monetary Policy

For the specification of monetary policy we could consider a rule that embeds different types of rules. The general form of the interest rate rule is

$$\frac{1+i_t}{1+i} = \Psi(F, \xi^m_t) \tag{46}$$

where F is the set of target variables for the home country, and ξ^m_t is a pure monetary shock reflecting interest rate movements that do not correspond to the endogenous reaction of the monetary authority to instrumental variables. Monetary shocks can be motivated by assuming that the central bank sometimes deviates from its own rule, that it makes mistakes implementing monetary policy, or by assuming that the demand for money is itself stochastic. For simplicity we will assume a very simple Taylor type rule in the log-linear case.

2.7 The log-linear version in a limiting case: $(n \longrightarrow 0)$

In this section, we present a full log-linear version of the model. Appendices A, B, C provide details of the derivation. In what follows, a variable x_t represents the log-deviation of X_t with respect to its steady state, X .

The following equations characterize the equilibrium of the domestic small open economy:

$$-u_{c,t} = -E_t u_{c,t+1} - i_t + (1-\gamma) E_t \pi_{H,t+1} + \gamma E_t \Delta S_{t+1} \tag{47}$$

$$\sigma c_t^* = \sigma E_t c_{t+1}^* - i_t^* \tag{48}$$

$$q_t = -u_{c,t} - \sigma c_t^* \tag{49}$$

$$u_{c,t} = -\sigma c_t + \Psi \left[(1-\delta) i_t + \delta i_t^* \right] \tag{50}$$

where $\Psi \equiv \beta(\sigma\omega - 1)(1 - b_1)$ and $b_1 \equiv \frac{b}{b+(1-b)(A_2)^{\frac{\omega}{\omega-1}}}$ and $\delta = \frac{DS/P}{DS/P+M/P} = \frac{(v/(1-v))^{-\chi}}{1+(v/(1-v))^{-\chi}}$ Parameters b_1 and δ have been defined in appendix A and B.

$$\pi_{H,t} = \lambda mc_t + \beta E_t \pi_{H,t+1} \tag{51}$$

where $\lambda \equiv (1-\alpha)(1-\alpha\beta)/\alpha$

$$mc_t = \upsilon y_{H,t} + \sigma c_t - (1+\upsilon) a_t + \gamma t_t - \Psi \left[(1-\delta) i_t + \delta i_t^* \right] \tag{52}$$

$$q_t = (1-\gamma) t_t \tag{53}$$

$$t_t = t_{t-1} + \Delta S_t - \pi_{H,t} \tag{54}$$

$$y_{H,t} = \theta\gamma t_t + (1-\gamma) c_t + \gamma \left(c_t^* + \theta q_t \right) \tag{55}$$

Equations (47) -(55) along with the exogenous home-productivity process, the endogenous Taylor rule at home and an exogenous process for the foreign interest rate characterize the open economy with partial dollarization. A key equation is $u_{c,t} = -\sigma c_t + \Psi \left[(1-\delta) i_t + \delta i_t^* \right]$. See appendix B for details. Note that when $b = 1$, $b_1 = 1 \Longrightarrow \Psi = 0$, the model collapses to a standard small open economy as in CGG (2001) and GM (2005). The novelties in our setup are the second and third terms on the right hand side of the marginal utility of consumption. The *MUC* now depends on both the domestic and foreign interest rates in addition to consumption. For example, suppose that $\sigma = 1$, a positive foreign interest rate shock will reduce both the demand for real money balances denominated in foreign currency ($\downarrow D_t$) and as a consequence the overall money aggregate will fall ($\downarrow Z_t$). If $0 < \omega < 1$ (consumption and overall money aggregates are complements, $U_{CZ} > 0$), after the shock we should observe a reduction in the marginal utility of consumption. Given the first order condition for labor supply (23), which in log linear form takes the following form $w_t - p_t = vl_t - u_{c,t}$, a fall in the marginal utility of consumption induces an increase in real wages in equilibrium. This rises firms' marginal costs and inflation picks up. This in turn induces a stronger policy response of interest rates, driving down output and consumption. On the contrary, the marginal utility of consumption rises if consumption and money aggregates are substitutes, leading to a fall in inflation following an increase in the foreign interest rate.

Note also that, in this economy, the risk-sharing condition will be affected by the presence of the foreign interest rate in the MUC, therefore, real exchange rate fluctuations in this particular economy will inherit the volatility and the persistence of the foreign interest rate[28].

2.8 Interest Rate Rule

We describe monetary policy as a variation of the Taylor (1993) rule, in which the nominal interest rate responds to expected movements in inflation, reflecting the aim of the monetary authority to stabilize future inflation rates[29][30]. This rule is followed by several central banks in emerging economies. It takes the following form:[31]

$$i_t = r_t^n + \gamma_\pi E_t \pi_{H,t+1} + \gamma_x x_t \tag{56}$$

where r_t^n and x_t denote the unobservable natural real interest rate and the output gap respectively. These variables will be defined in the next section.

[28] In particular, by combining the risk-sharing condition with the log-linear form of the marginal utility of consumption we obtain the following risk-sharing condition that relates real exchange rate movements with both domestic and foreign interest rates: $q_t = \sigma (c_t - c_t^*) - \Psi [(1-\delta) i_t + \delta i_t^*]$

The above risk-sharing condition might help explain the apparent lack of risk-sharing observed in the data in developing economies. Thus, it would be interesting to test the risk-sharing condition implied by the model.

[29] Laxton and Pesenti (2003) find that Inflation-Forecast-Based rules may perform better in small open economies than conventional Taylor rules.

[30] In the Peruvian economy since the adoption of a fully-fledge IT regime in 2002 monetary monetary has been targeting the interbank lending interest rate. Before that period the central bank was implementing its monetary policy by targeting money aggregates. As a result of the change in the policy instrument, there has been a significant reduction in the mean of key nominal variables.

[31] Given that this rule has been used for a relatively short period of time (approximately five years), we decided not to estimate the parameters of the equation for the calibration exercise.

3 A Tractable Representation of the Model

3.1 A Tractable Representation of a Partially Dollarized Economy

In this section we manipulate our model to deliver a simple and tractable representation similar to that in CGG (2001, 2002) and GM (2004). The log-linearized equilibrium dynamics can be expressed in terms of the output gap and domestic inflation. This representation provides us with the basis to obtain analytical solutions and some insights regarding the endogenous trade-off.

Let $y_{H,t}^n$ be the log of the natural level of output, defined as the level of output that arises with perfectly flexible prices and no cyclical distortions in the labor market (i.e. $mc_t = 0$). Therefore $x_t = y_{H,t} - y_{H,t}^n$ is our measure of output gap.

3.1.1 Aggregate Supply in Gaps and the Endogenous Trade-off

In order to obtain the natural level of output we combine the risk-sharing equation (49) with (50) which leads to

$$c_t = c_t^* + \frac{1}{\sigma} q_t + \frac{\Psi}{\sigma} \left[(1-\delta) i_t + \delta i_t^* \right]. \tag{57}$$

Combining the above expression with (53) and (55) we obtain an expression for domestic output

$$y_{H,t} = \frac{1}{\sigma_\gamma} t_t + c_t^* + \frac{\Psi (1-\gamma)}{\sigma} \left[(1-\delta) i_t + \delta i_t^* \right] \tag{58}$$

where $\sigma_\gamma = \frac{\sigma}{[1+\gamma(2-\gamma)(\sigma\theta-1)]}$.

Combining the marginal cost equation (52) with (57) and (58) we can express the real marginal cost in terms of home productivity, foreign consumption, domestic output and both domestic and foreign interest rates, where the degree of dollarization will play a role:

$$mc_t \equiv (\upsilon + \sigma_\gamma) y_{H,t} + (\sigma - \sigma_\gamma) c_t^* - (1+\upsilon) a_t - \frac{\sigma_\gamma \Psi (1-\gamma)}{\sigma} \left[(1-\delta) i_t + \delta i_t^* \right] \tag{59}$$

the flexible and efficient level of output can be attained by making $mc_t = 0$, conditional on the policy rule $i_t^n = 0$, $i_t^{n*} = 0$ for all t as[32]

$$y_{H,t}^n = a_1 a_t + a_2 c_t^* \tag{60}$$

where $a_1 \equiv \frac{(1+\upsilon)}{\upsilon+\sigma_\gamma}, a_2 \equiv \frac{\sigma_\gamma - \sigma}{\upsilon+\sigma_\gamma}$. Notice that foreign consumption can be expressed in terms of the foreign interest rate by using equation (48). Then in our economy the natural level of output will depend in addition to productivity shocks on the foreign nominal interest rate.

[32] $i_t^n = 0$ corresponds to an interest rate peg in the flexible price allocation, not a zero nominal interest rate.

Then, combining (60) with (59) , the real marginal cost can be expressed as:

$$mc_t = (\upsilon + \sigma_\gamma) x_t - \frac{\sigma_\gamma \Psi (1-\gamma)}{\sigma} [(1-\delta) i_t + \delta i_t^*] \tag{61}$$

Finally, plugging equation (61) into the Phillips curve (51) we obtain

$$\pi_{H,t} = \kappa_x x_t + \beta E_t \pi_{H,t+1} - \kappa_i [(1-\delta) i_t + \delta i_t^*] \tag{62}$$

where $\kappa_x \equiv \lambda (\upsilon + \sigma_\gamma)$, $\kappa_i \equiv \lambda \frac{\sigma_\gamma}{\sigma} \Psi (1-\gamma)$,

Equation (62) is a short run aggregate supply (AS) curve that relates domestic inflation to the output gap. Unlike the baseline model with domestic money only, domestic inflation depends directly on deviations of the foreign interest rate. In particular, the presence of currency substitution allows the model to generate an endogenous cost-push shock in terms of the foreign interest rate. Any increases in the foreign interest rate affects directly the dynamics of inflation because there is a shift factor in the marginal cost. Note that the effectiveness of monetary policy is affected by δ. The larger the degree of dollarization (δ) the larger the effect of movements in the foreign interest rate shock on inflation. If consumption and the aggregates are substitutes $\omega > 1$, $\kappa_i > 0$, hence an increase in the foreign interest rate will decrease domestic inflation. In particular, following a positive shock in the foreign interest rate the demand for foreign currency decreases, leading to a rise in the marginal utility of consumption ($\uparrow u_{c,t}$). The increase in $u_{c,t}$ generates a reduction in real wages in equilibrium.

The higher the degree of dollarization (higher δ) the larger the effect of foreign shocks over the aggregate supply and the less effective the central bank is in stabilizing inflation. The new transmission mechanism through which the central bank can affect inflation dynamics directly stems from the composite between money and consumption in the utility function. All else equal, an increase in either the domestic or foreign interest rates causes an increase in the marginal utility of consumption (MUC). The previous mechanism arises because in equilibrium the MUC equates the disutility of work, therefore an increase in MUC implies an increase in labor supply and consequently a reduction in domestic inflation.

3.1.2 Aggregate Demand in Gaps

Combining the risk-sharing condition (57) with (58) and using the fact that $q_t = (1-\gamma) t_t$ leads to

$$c_t = a_3 y_{H,t} + (1-a_3) c_t^* + [1 - a_3 (1-\gamma)] \frac{\Psi}{\sigma} [(1-\delta) i_t + \delta i_t^*] \tag{63}$$

where $a_3 \equiv \frac{\sigma_\gamma(1-\gamma)}{\sigma}$. Plugging in the (54) and (50) into the home Euler equation (47) we obtain

$$\sigma c_t = \sigma E_t c_{t+1} - i_t + E_t \pi_{H,t+1} + \gamma E_t \Delta t_{t+1} - \Psi E_t \left[(1-\delta) \Delta i_{t+1} + \delta \Delta i^*_{t+1} \right] \tag{64}$$

From (58) we obtain

$$\Delta t_{t+1} = \sigma_\gamma \Delta y_{H,t+1} - \sigma_\gamma \Delta c^*_{t+1} - a_3 \Psi E_t \left[(1-\delta) \Delta i_{t+1} + \delta \Delta i^*_{t+1} \right] \tag{65}$$

Which combined with (64) results in

$$\begin{aligned} \sigma c_t \;\; = \;\; & \sigma E_t c_{t+1} - i_t + E_t \pi_{H,t+1} + \gamma \sigma_\gamma \Delta E_t y_{H,t+1} - \gamma \sigma_\gamma E_t \Delta c^*_{t+1} - \\ & \Psi \left(\gamma a_3 + 1 \right) E_t \left[(1-\delta) \Delta i_{t+1} + \delta \Delta i^*_{t+1} \right] \end{aligned} \tag{66}$$

Plugging (63) into the above equation we arrive at

$$y_{H,t} = E_t y_{H,t+1} - \frac{1}{\sigma_\gamma} \left(i_t - E_t \pi_{H,t+1} \right) + \frac{\sigma - \sigma_\gamma}{\sigma_\gamma} E_t \Delta c^*_{t+1} - a_3 \frac{\Psi}{\sigma_\gamma} \left[(1-\delta) \Delta i_{t+1} + \delta \Delta i^*_{t+1} \right] \tag{67}$$

Finally, using the previous equation and the natural level of output we obtain the IS equation in terms of the output gap

$$x_t = E_t x_{t+1} - \frac{1}{\sigma_\gamma} \left[i_t - E_t \pi_{H,t+1} - r^n_t \right] + s_i \left[(1-\delta) E_t \Delta i_{t+1} + \delta E_t \Delta i^*_{t+1} \right] \tag{68}$$

with $s_i \equiv -a_3 \frac{\Psi}{\sigma_\gamma}$, and where

$$r^n_t = \sigma_\gamma E_t \Delta y^n_{H,t+1} + \left(\sigma - \sigma_\gamma \right) E_t \Delta c^*_{t+1} \tag{69}$$

Equation (68) is an IS curve that relates the output gap inversely to the domestic interest rate and positively to the expected future output gap. In addition, the IS curve is also affected by the expected path of both the domestic and the foreign interest rates which corresponds to the third term of the right hand side of the IS curve. The higher the degree of dollarization (higher δ), the smaller the effect of the domestic interest rate over the aggregate demand. This reveals the fragility of monetary policy in a partially dollarized economy. It is worth mentioning that when $\gamma = 0$ the IS curve for the partially dollarized economy becomes the IS for the closed economy.

Some differences with respect to CGG (2001, 2002) are worth noting. First, there is a shift factor in the AS equation which does not allow one to stabilize both the output gap and domestic inflation following a domestic inflation targeting regime. This is in contrast with GM (2004) where the optimal policy implies full stabilization of output and inflation. On the other hand, CGG (2001) rely on the presence of an exogenous cost-push shock in order to obtain this trade-off. Second, the aggregate demand depends on expected changes in both domestic and foreign interest rates.

Finally, equations (62) and (68), together with the Euler equation for the foreign economy (48) and the exogenous process for both productivity and foreign interest rate shocks characterize a partially dollarized economy.

3.2 Analytical Solution and the Transmission Mechanism

Given the tractability of the model we can obtain analytical solutions for the endogenous variables x_t and $\pi_{H,t}$. We obtain the rational expectations solution by implementing the undetermined coefficients method. The equations we have to consider are the aggregate supply (62), the aggregate demand (68) and the policy rule (56). The only source of shocks is a foreign interest rate shock, which follows the following AR(1) process

$$i_t^* = \rho i_{t-1}^* + \varepsilon_t \tag{70}$$

We guess the following solutions for domestic inflation and the output gap

$$\pi_{H,t} = \eta_{\pi i^*} i_t^* \tag{71}$$

$$x_t = \eta_{xi^*} i_t^* \tag{72}$$

where $\eta_{\pi i^*}$ and η_{xi^*} are the partial elasticities of domestic inflation and the output gap with respect to the foreign interest rate shock . Plugging the possible solutions into the aggregate supply curve (62) we obtain

$$\pi_{H,t} = \left[\kappa_x - \kappa_i (1-\delta) \gamma_x\right] x_t + \left[\beta - \kappa_i (1-\delta) \gamma_\pi\right] E_t \pi_{H,t+1} - \kappa_i \delta i_t^* \tag{73}$$

$$\pi_{H,t} = \frac{\left[\kappa_x - \kappa_i (1-\delta) \gamma_x\right]}{1 - \left[\beta - \kappa_i (1-\delta) \gamma_\pi\right] \rho} x_t - \frac{\kappa_i \delta}{1 - \left[\beta - \kappa_i (1-\delta) \gamma_\pi\right] \rho} i_t^* \tag{74}$$

$$\pi_{H,t} = \frac{\left[\kappa_x - \kappa_i (1-\delta) \gamma_x\right] \eta_{xi^*} - \kappa_i \delta}{1 - \left[\beta - \kappa_i (1-\delta) \gamma_\pi\right] \rho} i_t^* \tag{75}$$

From the IS curve (68) we get

$$
\begin{aligned}
x_t &= E_t x_{t+1} - \frac{(\gamma_\pi - 1)\rho}{\sigma_\gamma}\eta_1 i_t^* + s_i(1-\delta)(\rho-1)i_t + \delta s_i(\rho-1)i_t^* \\
x_t &= E_t x_{t+1} - \frac{(\gamma_\pi - 1)\rho}{\sigma_\gamma}\eta_1 i_t^* + s_i(1-\delta)(\rho-1)\rho\gamma_\pi\eta_1 i_t^* \\
&\quad + s_i(1-\delta)(\rho-1)\gamma_x x_t + \delta s_i(\rho-1)i_t^* \\
(1-C-\rho)x_t &= -\frac{(\gamma_\pi - 1)\rho}{\sigma_\gamma}\eta_1 i_t^* + s_i(1-\delta)(\rho-1)\rho\gamma_\pi\eta_1 i_t^* + \delta s_i(\rho-1)i_t^* \\
x_t &= -\frac{1}{\sigma_\gamma}\frac{(\gamma_\pi - 1)\rho\eta_{\pi i^*}}{1-C-\rho}i_t^* - \frac{s_i(1-\rho)}{1-C-\rho}\left[(1-\delta)\rho\gamma_\pi\eta_{\pi i^*} + \delta\right]i_t^*
\end{aligned}
$$

where $C = s_i(1-\delta)(\rho-1)\gamma_x$

After some manipulation we can get the following two expressions

$$
\eta_{\pi i^*} = \frac{\left[\kappa_x - \kappa_i(1-\delta)\gamma_x\right]\eta_{xi^*} - \kappa_i\delta}{1 - \left[\beta - \kappa_i(1-\delta)\gamma_\pi\right]\rho} \tag{76}
$$

$$
\eta_{xi^*} = -\frac{1}{\sigma_\gamma}\frac{(\gamma_\pi - 1)\rho\eta_{\pi i^*}}{1-C-\rho} - \frac{s_i(1-\rho)}{1-C-\rho}\left[(1-\delta)\rho\gamma_\pi\eta_{\pi i^*} + \delta\right] \tag{77}
$$

the above equations represent a system of two equations and two unknowns $(\eta_{\pi i^*}, \eta_{xi^*})$ which can be solved for analytically. After further manipulation, we obtain the analytical solutions for domestic inflation and the output gap for the dual currency economy[33]

$$
\eta_{\pi i^*} = \frac{-\delta\sigma_\gamma(1-\rho)(\kappa_i + s_i\kappa_x)}{D} \tag{78}
$$

$$
\eta_{xi^*} = \frac{\delta\rho(\gamma_\pi - 1)\kappa_i - \delta s_i\sigma_\gamma(1-\rho)(1-\beta\rho)}{D} \tag{79}
$$

where $D = (1-\rho)\sigma_\gamma(1-\beta\rho)\left[1 - s_i\gamma_x(1-\delta)\right] + (\gamma_\pi - 1)\rho\left[\kappa_x - \kappa_i\gamma_x(1-\delta)\right] + (1-\delta)(1-\rho)\sigma_\gamma\gamma_\pi\rho(\kappa_i + s_i\kappa_x)$

In order to gain further intuition, let us assume that the effect of overall real money balances over the aggregate demand is zero, $s_i = 0$, and that the central bank reacts only to expected inflation $\gamma_x = 0$, then the above solutions collapse to

$$
\eta_{\pi i^*} = \frac{-\delta\sigma_\gamma(1-\rho)\kappa_i}{(1-\rho)\sigma_\gamma(1-\beta\rho) + (\gamma_\pi - 1)\rho\kappa_x + (1-\delta)(1-\rho)\sigma_\gamma\gamma_\pi\rho\kappa_i}
$$

[33] Details of the derivationa are available in an appendix upon request from the authors.

$$\eta_{xi^*} = \frac{\delta\rho\left(\gamma_\pi - 1\right)\kappa_i}{\left(1-\rho\right)\sigma_\gamma(1-\beta\rho) + \left(\gamma_\pi - 1\right)\rho\kappa_x + \left(1-\delta\right)\left(1-\rho\right)\sigma_\gamma\gamma_\pi\rho\kappa_i}$$

If $\omega > 1$, which implies that $\Psi > 0$, then $\kappa_i > 0$ and hence the denominator of the above expressions will be positive. Hence, the analytical solutions imply that after a positive foreign interest rate shock we should observe a decrease in inflation ($\downarrow \eta_{\pi i^*}$) vis-a-vis an increase in the output gap ($\uparrow \eta_{xi^*}$) when money and consumption are substitutes. This result is captured by the term $\kappa_i\left[\left(1-\delta\right)i_t + \delta i_t^*\right]$ in the aggregate supply equation. A different pattern could emerge when money and consumption are complements.

Note also that the volatility of both domestic inflation and the output gap are increasing in the implied degree of dollarization (δ). Figure 2 below confirms our analytical findings regarding volatility. As expected, the volatility of both output and domestic inflation increase monotonically with the degree of dollarization. Note further that the volatility of output rises by more than that of inflation. This result shows that the larger the implied degree of dollarization (larger δ) the larger the endogenous volatility of both domestic inflation and the output gap.

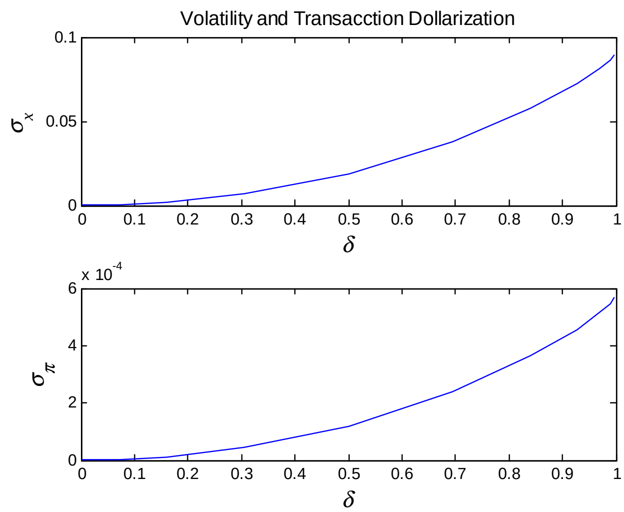

Figure 2 ($\omega = 2$)

3.3 Some Simulated Exercises

3.3.1 Parameterization

Our quantitative analysis seeks to illustrate the transmission mechanism of the model , and in particular, the role of foreign currency in total money aggregates and the non-separability of the latter from consumption. To do that we calibrate the parameters taking as a benchmark the Peruvian economy, which as shown in the introduction, is characterized by a dual currency environment. The parametrization of the model seeks to characterize the qualitative behavior of its main variables rather than to match the empirical data. The steady state equilibrium is derived in the Appendix A.

We set a quarterly discount factor, β, equal to 0.99, which implies an annualized rate of interest of 4%. In the steady state, the gross foreign interest rate, assumed to be exogenous in the model, is also equal to $\beta^{-1} = 1.01$, which implies the same annualized interest rate. The share of foreign goods in consumption, $\gamma = 0.4$, which is close to the ratio of imports over aggregate consumption for the Peruvian economy. In order to isolate the role of foreign currency in total money aggregates and the non-separability of the latter from consumption, we choose parameter values equal to 1 for the coefficient of risk aversion, σ, the inverse of the elasticity of labor supply, υ, and the elasticity of substitution between home and foreign goods, θ. We choose a degree of monopolistic competition, ε, equal to 7.66 following Rotemberg and Woodford (1997). This implies an average mark-up of 15 percent. For the monetary rule, we follow Taylor (1993) and set the coefficient on inflation, $\gamma_\pi = 1.5$. and the coefficient for output gap $\gamma_x = 0.5$. As it is common in the literature on the Calvo (1983) pricing technology, we let the probability of not adjusting prices, $\alpha = 0.75$.

Given that the main goal is to analyze the effect of foreign interest rate shocks, we calibrate this exogenous process. In order to calibrate this shock, we fit an AR(1) process to the FED funds rate (our proxy for foreign rates), by using quarterly data over the sample period 1955:01 to 2004:02. We obtain de following estimates: $\rho^* = 0.96$, and $var(\varepsilon_t^{i^*}) = (0.009)^2$. We set the elasticity of substitution between domestic and foreign money,χ , equal to 4.1 which is consistent with previous studies at the Central Bank of Perú. In our benchmark parameterization we set v equal to 0.5 which implies a steady state degree of dollarization of 70 percent (see equation A9 in appendix A). This value is roughly the average degree of dollarization from 1994-2005 for the Peruvian economy. We parameterize $b = 0.83$ which constitutes the share of consumption in the CES function.[34] Finally, ω can take two values, $\omega = 0.9$ implying that consumption and the overall aggregate are complements and, $\omega = 2$ which implies that consumption and the overall aggregate are substitutes.

[34] In particular, we use private consumption and overall money aggregate which is the sum of domestic and foreign currency liquidity.

3.3.2 Impulse Responses

Figures 3 and 4 display the impulse-responses of the simulated model for three degrees of dollarization: high ($\upsilon = 0.33 \Longrightarrow \delta = 0.96$), medium ($\upsilon = 0.5 \Longrightarrow \delta = 0.5$) and low ($\upsilon = 0.67 \Longrightarrow \delta = 0.05$). We report the impulse response functions under two possible scenarios for the parameter ω. The impulse responses confirm the analytical results. Figure 2 depicts the responses when $(\omega = 2)$. Following the foreign interest shock and provided that the money aggregate and consumption are substitutes, the demand for foreign currency decreases and the marginal utility of consumption increases. Consequently real wages decrease in equilibrium, which in turn induce a reduction in inflation, which is captured by the term $-\kappa_i \delta i_t^*$. It is useful to recall that for $\omega > 1$, $\kappa_i > 0$. Then it is clear that inflation falls on impact after the foreign interest rate shock. The limitations of the central bank under this environment are also clear. In particular, the larger the presence of foreign currency (larger δ) the larger the impact of the shock on domestic inflation. Given the reduction of inflation the central bank must react by reducing its policy rate in order to expand aggregate demand and bringing inflation to its steady state level. In figure 3 we observe that the policy rate decreases after the shock triggering an expansion of aggregate demand. Again the central bank has a stronger response the larger the degree of dollarization, highlighting the limitations of a central bank in a partial dollarized economy.

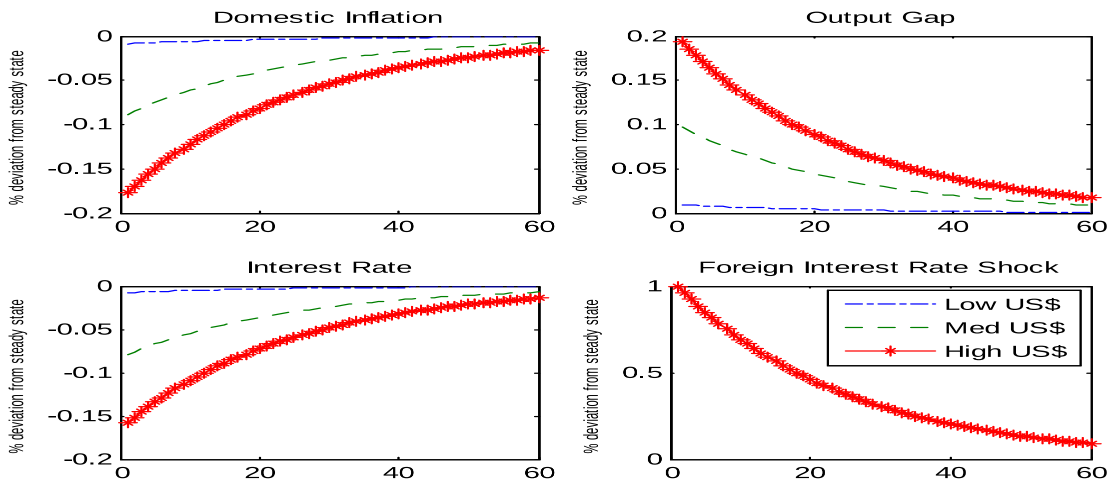

Figure 3: (High, $\delta = 0.95$, Med, $\delta = 0.5$, Low, $\delta = 0.05$) and $\omega = 2.0$

In the case where consumption and the overall aggregates are complements, $(0 < \omega < 1)$, the impulse-responses show the opposite pattern. Figure 4 depicts the results. Foreign interest rate shocks generate a persistent increase in inflation, a contractionary policy response and a fall in the output gap. The magnitude and persistence of the response of inflation to the foreign interest rate shock depends on how sensitive the aggregate supply is to the implied parameter δ.

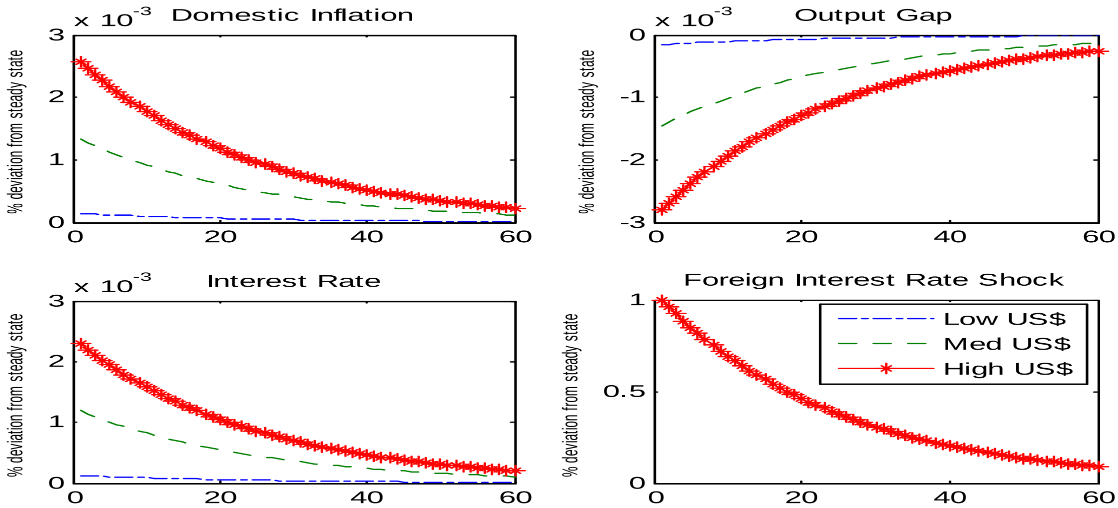

Figure 4: (High, δ=0.95, Med, δ=0.5, Low, δ=0.05) and ω=0.9.

3.3.3 Unconditional Volatilities

In this section we illustrate how dollarization could make the economy relatively unstable. We illustrate this issue in the context of our calibrated model. In particular we compute the implied standard deviation of domestic inflation, the output gap and nominal interest rate under various degrees of dollarization captured by the parameter δ as defined in the appendix. Thus given $\chi = 4.1$ we change v, the preference for domestic currency, in order to obtain the implied degree of dollarization. The analysis is made conditional on a foreign interest rate shock. In addition we also check for robustness by calculating the unconditional moments when consumption and the overall aggregate are complements ($\omega = 0.9$).

Table I reports the results. It shows how the standard deviations of domestic inflation, the output gap and the nominal interest rate, vary in response to shifts in the degree of dollarization. The larger the degree of dollarization the larger the standard deviations. The first three columns display the results for the benchmark parametrization with $\omega = 2.0$. The last three columns repeat the exercise, this time with a value of $\omega = 0.9$. In order to facilitate the analysis, we normalize to unity the standard deviations corresponding to the calibration with $\nu = 0.585$

which implies a degree of dollarization of twenty percent $(\delta = 0.20)$.

Table I: Macroeconomic Volatility and Dollarization

	Foreign interest rate shock ($\omega = 2.0$)			Foreign interest rate shock ($\omega = 0.9$)		
δ	$\sigma(\pi_H)$	$\sigma(x)$	$\sigma(i)$	$\sigma(\pi_H)$	$\sigma(x)$	$\sigma(i)$
0.20	1.00	1.00	1.00	1.000	1.000	1.000
0.30	1.56	1.56	1.56	1.530	1.529	1.529
0.40	2.10	2.10	2.10	2.018	2.018	2.018
0.50	2.58	2.60	2.60	2.529	2.528	2.529
0.70	3.87	3.78	3.76	3.553	3.554	3.554
0.95	5.42	5.43	5.49	4.866	4.865	4.865

The results show that the cyclical response of the economy to the foreign interest rate shock is quite sensitive to the degree of dollarization (δ). For example as δ decreases from 0.70 to 0.40, the macroeconomic volatility almost halves. The implied reduction in δ can be obtained by a small increase in ν, from 0.448 to 0.525. Thus, by a small increase of the parameter that captures the preference for domestic currency the model predicts a meaningful reduction in macroeconomic volatility. Finally, we note that the results we obtain in the context of the simple small open economy are robust to changes in ω which is the key parameter that generates the trade-off between stabilizing domestic inflation and the output gap. Therefore, the higher the degree of dollarization in the model economy, the higher the unconditional volatility of inflation and the output gap.

3.4 Determinacy and the Taylor Principle

In this section we evaluate under what conditions a standard Taylor rule guarantees real determinacy in our small open economy. In particular, we intend to highlight how the determinacy condition changes compared to an economy without transaction dollarization. Lets assume that the Taylor rule adopts the following standard form

$$i_t = r_t^n + \gamma_\pi \pi_{H,t} + \gamma_x x_t \tag{80}$$

where $\gamma_\pi > 0$ and $\gamma_x > 0$. We omit the exogenous shocks since for the determinacy conditions shocks are not relevant. Combining (80) and (62) we can re-express the AS equation as

$$E_t \pi_{H,t+1} = \left(\frac{1}{\beta} + \frac{\kappa_i}{\beta}(1-\delta)\gamma_\pi\right)\pi_{H,t} - \left(\frac{\kappa_x}{\beta} - \frac{\kappa_i}{\beta}(1-\delta)\gamma_x\right)x_t \tag{81}$$

Substituting (80) and (68) and after some manipulation, the IS equation becomes

$$
\begin{aligned}
\left(1+s_i\left(1-\delta\right)\gamma_x\right)E_t x_{t+1} &= \left\{\begin{array}{c} 1+\frac{1}{\sigma_\gamma}\left(\gamma_x+\frac{\kappa_x}{\beta}\right)+\left(s_i\left(1-\delta\right)+\frac{1}{\sigma_\gamma}\frac{\kappa_i}{\beta}\left(1-\delta\right)\right)\gamma_x \\ +s_i\left(1-\delta\right)\gamma_\pi\left(\frac{\kappa_x}{\beta}-\frac{\kappa_i}{\beta}\left(1-\delta\right)\gamma_x\right) \end{array}\right\} x_t \\
&+\left\{\begin{array}{c} \frac{1}{\sigma_\gamma}\left(\gamma_\pi-\frac{1}{\beta}\right)+\left(s_i\left(1-\delta\right)-\frac{1}{\sigma_\gamma}\frac{\kappa_i}{\beta}\left(1-\delta\right)\right)\gamma_\pi \\ -s_i\left(1-\delta\right)\gamma_\pi\left(\frac{1}{\beta}+\frac{\kappa_i}{\beta}\left(1-\delta\right)\gamma_\pi\right) \end{array}\right\}\pi_{H,t} \quad (82)
\end{aligned}
$$

the above system of two equations can be written in the following matrix form

$$
A \equiv \left[\begin{array}{cc} A_{11} & A_{12} \\ A_{21} & A_{22} \end{array}\right] \quad (83)
$$

where $A_{11} \equiv \left(\frac{1}{\beta}+\frac{\kappa_i}{\beta}\left(1-\delta\right)\gamma_\pi\right)$, $A_{12} \equiv -\left(\frac{\kappa_x}{\beta}-\frac{\kappa_i}{\beta}\left(1-\delta\right)\gamma_x\right)$,

$A_{21} \equiv \frac{1}{C_0}\left[\begin{array}{c} \left(\frac{1}{\sigma_\gamma}+s_i\left(1-\delta\right)\right)\gamma_\pi \\ -\left(\frac{1}{\sigma_\gamma}+s_i\left(1-\delta\right)\gamma_\pi\right)\left(\frac{1}{\beta}+\frac{\kappa_i}{\beta}\left(1-\delta\right)\gamma_\pi\right) \end{array}\right]$,

$A_{22} \equiv \frac{1}{C_0}\left[\begin{array}{c} 1+\left(\frac{1}{\sigma_\gamma}+s_i\left(1-\delta\right)\gamma_\pi\right)\frac{\kappa_x}{\beta} \\ +\left[\begin{array}{c} \frac{1}{\sigma_\gamma}+s_i\left(1-\delta\right)- \\ \left(\frac{1}{\sigma_\gamma}+s_i\left(1-\delta\right)\gamma_\pi\right)\frac{\kappa_i}{\beta}\left(1-\delta\right) \end{array}\right]\gamma_x \end{array}\right]$

and $C_0 \equiv \left(1+s_i\left(1-\delta\right)\gamma_x\right)$

Where the trace and determinant are given by

$$
\begin{aligned}
trA &= \left(\frac{1}{\beta}+\frac{\kappa_i}{\beta}\left(1-\delta\right)\gamma_\pi\right)+\frac{1}{C_0}+\frac{1}{C_0}\left(\frac{1}{\sigma_\gamma}+s_i\left(1-\delta\right)\gamma_\pi\right)\frac{\kappa_x}{\beta}+ \\
&\frac{1}{C_0}\left[\left[\begin{array}{c} \frac{1}{\sigma_\gamma}+s_i\left(1-\delta\right)- \\ \left(\frac{1}{\sigma_\gamma}+s_i\left(1-\delta\right)\gamma_\pi\right)\frac{\kappa_i}{\beta}\left(1-\delta\right) \end{array}\right]\gamma_x\right] \\
DetA &= \frac{1}{C_0}\left(\frac{1}{\beta}+\frac{\kappa_i}{\beta}\left(1-\delta\right)\gamma_\pi\right)+\frac{1}{C_0}\left(\frac{1}{\sigma_\gamma}+s_i\left(1-\delta\right)\right)\gamma_\pi\frac{\kappa_x}{\beta} \\
&\frac{1}{C_0}\frac{1}{\beta}\left(\frac{1}{\sigma_\gamma}+s_i\left(1-\delta\right)\right)\gamma_x
\end{aligned}
$$

The system is determined if the eigenvalues of A lie outside the unit circle. For the benchmark parametrization ($\omega > 1$), it is straight forward to see that both $DetA$ and trA are bigger than zero. Therefore, the only condition that the system needs to meet for real determinacy is $DetA - trA > -1$,

Solving for this inequality condition we show that the necessary and sufficient condition for real determinacy is (see Appendix C for details)

$$\kappa_x (\gamma_\pi - 1) + \gamma_x [1 - \beta + \kappa_i (1 - \delta)] > 0 \quad (84)$$

The above expression shows how dollarization, to the extent that $\kappa_i > 0$ (or when $\omega > 1$), reduces the region for a determinate equilibrium. Once non-separable preferences are considered, the smaller the value δ the larger the effect of $\kappa_i (1 - \delta)$, helping condition (84) to be met. Hence, less dollarization is desirable for determinacy. The following graph illustrates the previous result.

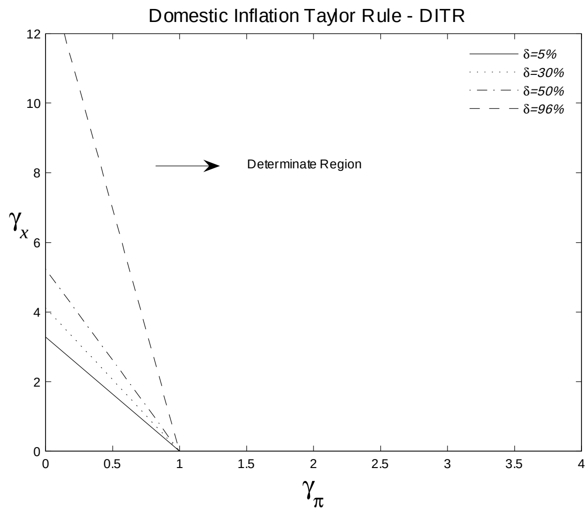

Figure 5: Regions of Determinacy in a Dual Currency Environment ($\omega = 2$)

Figure 5 depicts the determinacy regions as a function of γ_π and γ_x under different implied degrees of dollarization: 5%, 30%, 50% and 96%. In all cases the rest of the parameters are set at their baseline values.

The numerical results reveal that the constraints faced by policy makers are slightly tighter in an economy with higher degrees of dollarization. In particular, the line depicting determinacy shifts away from the origin for higher degrees of dollarization, so whenever $\gamma_\pi < 1$, more dollarized economies need greater responses to the output gap to guarantee determinacy. The explanation behind this result relies on the effects of dollarization on the term $\kappa_i (1 - \delta)$. If $\omega > 1$,then $\kappa_i > 0$ and an increase in dollarization, δ, reduces the area of determinacy through the reduction of $\kappa_i (1 - \delta)$.

The intuition behind the reduction of the determinate region stems from the effects of the foreign interest rate on inflation dynamics. Specifically, in an economy with transaction dollarization a central bank has to put more weight on output gap fluctuations because aggregate demand fluctuations have a larger impact on domestic inflation via the extra kick of both domestic and foreign interest rates.

Interestingly, the combination of non-separability and dollarization in our setup alters the condition for real determinacy. Non-separability, under our benchmark parameterization relaxes the determinacy condition.[35] However, once dollarization is considered, the condition becomes more difficult to meet. Although the determinacy condition changes with respect to the standard condition in small open economies with separable preferences, condition (84) still corresponds to the Taylor principle: in the face of inflationary pressures, the central bank increases its interest rate by more than the rise in inflation, hence raising real interest rates until inflation returns to the target.[36]

The higher chances of falling on the indeterminacy shows why a central bank that faces a high degree of transaction dollarization has to react more strongly to the output gap in order to avoid falling in an unstable region.[37]

4 Conclusions

This paper has been motivated by the experience of several developing economies, in particular de Peruvian economy, where both local and foreign currency coexist as a means of transaction and as a deposit of value. The monetary authority faces the problem of managing the domestic currency component of the money aggregate in a situation where the foreign component can change significantly over time.

We develop a model that embeds a foreign and local currency money aggregate into a simple two-country open economy model as in Clarida, Gali and Gertler (2001, 2002) and Gali and Monacelli (2005). The resulting model yields a tractable formulation for the qualitative analysis of monetary policy in economies that face currency substitution as an equilibrium outcome. The results suggest that, given shocks to the foreign interest rate, inflation and output volatility increase when dollarization is high meaning that a central bank's ability to reduce volatility is more limited in a partially dollarized economy. The transmission mechanism of these shocks can lead to lower inflation and higher output when the overall money aggregate and consumption are

[35] Indeed, in the extreme example of $\kappa_i = 0$ (separable case) the determinacy region becomes smaller.

[36] Llosa and Tuesta (2006) have analyzed conditions of determinacy and learnability for a broad set of instrument rules in a small open economy environment with separable preferences. They find that some type of managed exchange rate rules might have desirable properties in terms of determinacy, although these rules might generate large macroeconomic volatility and therefore they might not be so desirable in this dimension.

[37] Remarkably, when $\delta \longrightarrow 1$ the determinacy condition collapses to the one obtained in a cash-less economy.

substitutes.

Some novel results are worth highlighting. First, our canonical model generates an endogenous trade-off between the stabilization of inflation and the output gap. The short-run trade-off, which depends on the degree of dollarization, arises due to the presence of a foreign interest rate shock, hence it cannot be evaluated in a closed economy environment. Second, we find that a standard Taylor rule guarantees real determinacy of the rational expectations equilibrium. We find analytical results for real determinacy that show that the higher the degree of dollarization the smaller the determinacy region, meaning that more dollarized economies require a greater response to the output gap in order to guarantee determinacy. An extended version of the Taylor principle applies.

There are very interesting avenues for future research in this area. The analysis of optimal policy using a micro-founded loss function, for example, is an avenue to pursue. For instance, Batini, Levine and Pearlman (2006) have derived optimal policy based on ad-hoc quadratic loss function using our framework. The study of determinacy and learning for a broader set of instruments rules is also an interesting line of research to pursue.

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Appendix A: The Steady State

Variables without time subscript represent steady state values. For the characterization of the perfect foresight steady state equilibrium of our open economy we assume $A_t^H = A_t^F = 1$, for all t. In the steady state we also normalize $P_H = P_F$, such that $\frac{P_H}{P} = \frac{P_F}{P} = Q = 1$. We find the symmetric steady state at the limiting case when $n \longrightarrow 0$.

From firm maximizations home and abroad (equation 44) and combining with the labor supply equations (23) we get

$$\frac{\varepsilon (1-\tau)}{(\varepsilon - 1)} = \frac{L^{\upsilon}}{U_C(C, Z)} \tag{A1}$$

$$\frac{\varepsilon (1-\tau)}{(\varepsilon - 1)} = \frac{L^{*\upsilon}}{U_C(C^*)} \tag{A2}$$

From production functions home and abroad

$$Y^H = L \tag{A3}$$

$$Y^F = L^* \tag{A4}$$

From the risk-sharing condition we get:

$$U_c(C, Z) = \kappa_0 U_c(C^*) \tag{A5}$$

From the aggregate demands home and abroad

$$Y^H = (1-\gamma) C + \gamma C^* \tag{A6}$$

$$Y^F = C^* \tag{A7}$$

Combining the steady state of equations (25), (27) with (28) and (29), respectively we obtain the steady state real demand for both domestic and foreign currency,

$$\frac{M}{P} = \left(\frac{b(1-\beta) C^{-\frac{1}{\omega}}}{(1-b) v A_1^{\left(\frac{1}{\chi} - \frac{1}{\omega}\right)}} \right)^{-\omega} \tag{A8}$$

$$\frac{DS}{P} = \left(\frac{b(1-\beta) C^{-\frac{1}{\omega}}}{(1-b)(1-v)\left((\frac{v}{1-v})^{\chi} A_1 \right)^{\left(\frac{1}{\chi} - \frac{1}{\omega}\right)}} \right)^{-\omega} \tag{A9}$$

where $A_1 = \left[v + (1-v) \left(\frac{v}{1-v} \right)^{1-\chi} \right]^{\frac{\chi}{\chi-1}}$. Combining the above equations we can obtain the steady state level of the total money aggregate as a function of domestic consumption. importantly, from the above equations we can also determine the steady state degree of dollarization (δ):

$$\delta = \frac{DS/P}{DS/P + M/P} = \frac{(v/(1-v))^{-\chi}}{1 + (v/(1-v))^{-\chi}} \tag{(A9)}$$

Notice that from the above expression if the elasticity of substitution across currencies is equal to one the dollarization ratio turns out to be equal to the preference for foreign currency, $1-v$.

From expressions $(A8)$ and $(A9)$ we can further express the overall money aggregate as a function of total consumption

$$Z = A_2 C \tag{A10}$$

where $A_2 \equiv A_1^{\frac{\omega}{\chi}} \left[\frac{b(1-\beta)}{(1-b)v} \right]^{-\omega}$.

Given $(A10)$ and the risk-sharing condition $(A5)$ along with the utility function under its functional form at steady state, we can obtain a direct relationship between home and foreign consumption :

$$C^* = A_3(\kappa_0) C \tag{A11}$$

where $A_3(\kappa_0) \equiv \left[\frac{\frac{1}{1-\sigma} \left(b + (1-b) A_2^{\left(\frac{\omega}{\omega-1}\right)} \right)^{1-\sigma}}{\kappa_0} \right]^{-\frac{1}{\sigma}}$. For convenience and without loss of generality we set the initial conditions such that $A_3(\kappa_0) = 1$.Plugging $(A11)$ into the demands $(A6)$ and $(A7)$. We get that $Y^H = C = C^* = Y^F$, which implies zero net exports in the home economy at the new steady state. From the previous relation we get

$$\frac{\varepsilon(1-\tau)}{(\varepsilon-1)} = \frac{C^v}{C^{-\sigma}} \tag{A12}$$

from which we can obtain the steady state level of consumption

$$Y = C = \left\{ \frac{(\varepsilon-1)}{\varepsilon(1-\tau)} \right\}^{\frac{1}{\sigma+v}} \tag{A13}$$

Appendix B: Marginal Utility to Consume

In this appendix we will show how the marginal utility to consume at home can collapse to an equation in consumption and both domestic and foreign interest rates. First taking the log-linear approximation of equation (27) we get

$$u_{c,t} = \left[(\frac{1}{\omega} - \sigma) b_1 - \frac{1}{\omega} \right] c_t + (\frac{1}{\omega} - \sigma)(1 - b_1)\left(1 - c_1\right) z_t \tag{B1}$$

where $b_1 \equiv b_1 \equiv \frac{bC^{\frac{\omega}{\omega-1}}}{bC^{\frac{\omega}{\omega-1}} + (1-b) Z^{\frac{\omega}{\omega-1}}} = \frac{b}{b+(1-b)(A_2)^{\frac{\omega}{\omega-1}}}$ From the definition of the money aggregate, z_t, in log-linear form we get

$$z_t = (1 - \delta)\, m_t + \delta d_t \tag{B2}$$

where m_t and d_t corresponds to the log-linear approximation of the real money balances in both domestic and foreign currency, respectively. By taking log-linear approximation of equations (37) and (38) respectively we get

$$m_t = \frac{\chi}{\omega} c_t - \chi \beta i_t - (\frac{\chi}{\omega} - 1) z_t \tag{B3}$$

$$d_t = \frac{\chi}{\omega} c_t - \chi \beta i_t^* - (\frac{\chi}{\omega} - 1) z_t \tag{B4}$$

Then by combining equations $((B3)$ and $((B4)$ yields the following expression for the money aggregate

$$z_t = c_t - \delta \omega \beta i_t^* - (1 - \delta)\, \omega \beta i_t. \tag{B5}$$

Substituting expression $(B5)$ into $((B1)$ yields the expression for the marginal utility of consumption equation (50)

$$u_{c,t} = -\sigma c_t + \Psi \left[(1 - \delta)\, i_t + \delta i_t^* \right] \tag{B6}$$

with $\Psi \equiv \beta(\sigma\omega - 1)(1 - b_1)$

Appendix C: Determinacy

The system is determined if the eigenvalues of A lie outside the unit circle. For the benchmark parametrization ($\omega > 1$), it is straight forward to see that both $DetA$ and trA are bigger than zero. Therefore, the only condition that the system needs to meet for real determinacy is $DetA - trA > -1$,

Solving for this condition we get

$DetA - trA = \frac{1}{C_0}\frac{1}{\beta} + \frac{1}{C_0}\frac{\kappa_i}{\beta}(1-\delta)\gamma_\pi + \frac{1}{C_0}\frac{1}{\sigma_\gamma}\gamma_\pi\frac{\kappa_x}{\beta} + \frac{1}{C_0}s_i(1-\delta)\gamma_\pi\frac{\kappa_x}{\beta} +$
$\frac{1}{C_0}\frac{1}{\beta}\frac{1}{\sigma_\gamma}\gamma_x + \frac{1}{C_0}\frac{1}{\beta}s_i(1-\delta)\gamma_x - \left(\frac{1}{\beta} + \frac{\kappa_i}{\beta}(1-\delta)\gamma_\pi\right) - \frac{1}{C_0} - \frac{1}{C_0}\frac{1}{\sigma_\gamma}\frac{\kappa_x}{\beta} - \frac{1}{C_0}s_i(1-\delta)\gamma_\pi\frac{\kappa_x}{\beta}$
$-\frac{1}{C_0}\frac{1}{\sigma_\gamma}\gamma_x - \frac{1}{C_0}s_i(1-\delta)\gamma_x + \frac{1}{C_0}\frac{1}{\sigma_\gamma}\frac{\kappa_i}{\beta}(1-\delta)\gamma_x + \frac{1}{C_0}s_i(1-\delta)\gamma_\pi\frac{\kappa_i}{\beta}(1-\delta)\gamma_x > -1$

Since $C_0 > 1$, we can multiply both sides of the previous expression by C_0

$\frac{1}{\beta} + \frac{\kappa_i}{\beta}(1-\delta)\gamma_\pi + \frac{1}{\sigma_\gamma}\gamma_\pi\frac{\kappa_x}{\beta} + s_i(1-\delta)\gamma_\pi\frac{\kappa_x}{\beta} +$
$\frac{1}{\beta}\frac{1}{\sigma_\gamma}\gamma_x + \frac{1}{\beta}s_i(1-\delta)\gamma_x - C_0\left(\frac{1}{\beta} + \frac{\kappa_i}{\beta}(1-\delta)\gamma_\pi\right) - 1 - \frac{1}{\sigma_\gamma}\frac{\kappa_x}{\beta} - s_i(1-\delta)\gamma_\pi\frac{\kappa_x}{\beta}$
$-\frac{1}{\sigma_\gamma}\gamma_x - s_i(1-\delta)\gamma_x + \frac{1}{\sigma_\gamma}\frac{\kappa_i}{\beta}(1-\delta)\gamma_x + s_i(1-\delta)\gamma_\pi\frac{\kappa_i}{\beta}(1-\delta)\gamma_x > -C_0$

Substituting out C_0

$\frac{1}{\beta} + \frac{\kappa_i}{\beta}(1-\delta)\gamma_\pi + \frac{1}{\sigma_\gamma}\gamma_\pi\frac{\kappa_x}{\beta} + s_i(1-\delta)\gamma_\pi\frac{\kappa_x}{\beta} +$
$\frac{1}{\beta}\frac{1}{\sigma_\gamma}\gamma_x + \frac{1}{\beta}s_i(1-\delta)\gamma_x - (1 + s_i(1-\delta)\gamma_x)\left(\frac{1}{\beta} + \frac{\kappa_i}{\beta}(1-\delta)\gamma_\pi\right) - 1 - \frac{1}{\sigma_\gamma}\frac{\kappa_x}{\beta} - s_i(1-\delta)\gamma_\pi\frac{\kappa_x}{\beta}$
$-\frac{1}{\sigma_\gamma}\gamma_x - s_i(1-\delta)\gamma_x + \frac{1}{\sigma_\gamma}\frac{\kappa_i}{\beta}(1-\delta)\gamma_x + s_i(1-\delta)\gamma_\pi\frac{\kappa_i}{\beta}(1-\delta)\gamma_x > -(1 + s_i(1-\delta)\gamma_x)$

Simplifying

$\frac{1}{\sigma_\gamma}\gamma_\pi\frac{\kappa_x}{\beta} + s_i(1-\delta)\gamma_\pi\frac{\kappa_x}{\beta} + \frac{1}{\beta}\frac{1}{\sigma_\gamma}\gamma_x - \frac{1}{\sigma_\gamma}\frac{\kappa_x}{\beta} - s_i(1-\delta)\gamma_\pi\frac{\kappa_x}{\beta}$
$-\frac{1}{\sigma_\gamma}\gamma_x - s_i(1-\delta)\gamma_x + \frac{1}{\sigma_\gamma}\frac{\kappa_i}{\beta}(1-\delta)\gamma_x > -s_i(1-\delta)\gamma_x$
$\frac{1}{\sigma_\gamma}\gamma_\pi\frac{\kappa_x}{\beta} + \frac{1}{\beta}\frac{1}{\sigma_\gamma}\gamma_x - \frac{1}{\sigma_\gamma}\frac{\kappa_x}{\beta}$
$-\frac{1}{\sigma_\gamma}\gamma_x + \frac{1}{\sigma_\gamma}\frac{\kappa_i}{\beta}(1-\delta)\gamma_x > 0$

Then, the necessary and sufficient condition for real determinacy is

$$\kappa_x(\gamma_\pi - 1) + \gamma_x[1 - \beta + \kappa_i(1-\delta)] > 0.$$