Cointegrated TFP Processes and International Business Cycles

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- High persistence of TFP shocks: high volatility of RER, low volatility of output.
- High spillovers of TFP shocks: low volatility of RER, high volatility of output.
We also show that in the period know as "The Great Moderation" the relative volatility of the RER w.r.t output has increased.
The Great Moderation and the Real Exchange Rate

Figure: Standard Deviation of HP-Filtered Data. USA and UK.
The Great Moderation and the Real Exchange Rate

Figure: Standard Deviation of HP-Filtered Data. Canada and Australia.
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- We derive results that relate RER volatility with the parameters of the VECM.

- We show that the volatility increase can be related to changes in the parameter estimates of the VECM.
Related Literature


- Explaining RER volatility in DSGE models.
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The Model

- Standard IRBC model. Two countries, two final goods, two intermediate goods.
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- Incomplete markets.
- Firms in the intermediate and final goods sectors operate under perfect competition.
- Departure from the literature: TFP processes are C(1,1) and can be characterized with a VECM.
The Model: Households

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left\{ C(s^t)^\tau [1 - L(s^t)]^{1-\tau} \right\}^{1-\sigma}}{1 - \sigma}
\]

s.t.

\[
P(s^t) \left[ C(s^t) + X(s^t) \right] + P_H(s^t) Q(s^t) D(s^t) \leq P_H(s^t) D(s^{t-1}) + P(s^t) \left[ W(s^t) L(s^t) + R(s^t) K(s^{t-1}) \right] - P_H(s^t) \Phi(D(s^t)),
\]

and

\[
K(s^t) = (1 - \delta) K(s^{t-1}) + X(s^t),
\]
The Model: Final Goods Producers

\[ \max P (s^t) Y (s^t) - P_H (s^t) Y_H (s^t) - P_F (s^t) Y_F (s^t) \]

s.t.

\[ Y (s^t) = \left[ \omega \frac{1}{\theta} Y_H (s^t) \frac{\theta - 1}{\theta} + (1 - \omega) \frac{1}{\theta} Y_F (s^t) \frac{\theta - 1}{\theta} \right]^{\frac{\theta}{\theta - 1}} \]
The Model: Intermediate Goods Producers

\[
\begin{align*}
\text{Max} & \left\{ \begin{array}{c}
P_H(s^t) [Y_H(s^t) + Y^*_H(s^t)] - \\
P(s^t) [W(s^t) L(s^t) + R(s^t) K(s^{t-1})]
\end{array} \right\} \\
\text{s.t.} & \\
Y_H(s^t) + Y^*_H(s^t) = A(s^t)^{1-\alpha} K(s^{t-1})^\alpha L(s^t)^{1-\alpha}
\end{align*}
\]
The Model: TFP

\[
\begin{pmatrix}
\Delta \log A(s^t) \\
\Delta \log A^*(s^t)
\end{pmatrix}
= \begin{pmatrix}
c \\
c^*
\end{pmatrix} + \rho(L) \begin{pmatrix}
\Delta \log A(s^{t-1}) \\
\Delta \log A^*(s^{t-1})
\end{pmatrix}
+ \begin{pmatrix}
\kappa \\
\kappa^*
\end{pmatrix} \left[ \log A(s^{t-1}) - \gamma \log A^*(s^{t-1}) - \log \xi \right] + \begin{pmatrix}
\varepsilon^a(s^t) \\
\varepsilon^{a,*}(s^t)
\end{pmatrix}
\]

- Implies that:
  - \(\Delta \log A(s^t)\)
  - \(\Delta \log A^*(s^t)\), and
  - \(\log A(s^{t-1}) - \gamma \log A^*(s^{t-1})\) are stationary processes.
The Model: Equilibrium Conditions

\[ U_C (s^t) = \lambda (s^t), \]

\[ U_L (s^t) \left( \frac{U_C (s^t)}{U_C (s^t)} \right) = W (s^t), \]

\[ \lambda (s^t) = \beta E_t \left\{ \lambda (s^{t+1}) [R (s^{t+1}) + (1 - \delta)] \right\}, \]

\[ K (s^t) = (1 - \delta) K (s^{t-1}) + X (s^t), \]
The Model: Equilibrium Conditions

\[ \bar{Q}(s^t) = \beta E_t \left[ \frac{\lambda(s^{t+1})}{\lambda(s^t)} \frac{\tilde{P}_H(s^{t+1})}{\tilde{P}_H(s^t)} \right] - \frac{\Phi'[D(s^t)]}{\beta} \]

\[ \tilde{P}_H(s^t) \bar{Q}(s^t) D(s^t) \quad = \quad \tilde{P}_H(s^t) Y^*_H(s^t) - \tilde{P}_F^*(s^t) RER(s^t) Y_F(s^t) 
+ \tilde{P}_H(s^t) D(s^{t-1}) - \tilde{P}_H(s^t) \Phi [D(s^t)] \]

\[ E_t \left[ \frac{\lambda^*(s^{t+1})}{\lambda^*(s^t)} \frac{\tilde{P}_H(s^{t+1})}{\tilde{P}_H(s^t)} \right] \frac{RER(s^t)}{RER(s^{t+1})} - \frac{\lambda(s^{t+1})}{\lambda(s^t)} \frac{\tilde{P}_H(s^{t+1})}{\tilde{P}_H(s^t)} \] 
\[ = - \frac{\Phi'[D(s^t)]}{\beta} \]
The Model: Equilibrium Conditions

\[ W(s^t) = (1 - \alpha) \tilde{P}_H(s^t) A(s^t)^{1-\alpha} K(s^{t-1})^\alpha L(s^t)^{-\alpha}, \]

\[ R(s^t) = \alpha \tilde{P}_H(s^t) A(s^t)^{1-\alpha} K(s^{t-1})^{\alpha-1} L(s^t)^{1-\alpha}, \]

\[ Y_H(s^t) = \omega \tilde{P}_H(s^t)^{-\theta} Y(s^t), \]

\[ Y_F(s^t) = (1 - \omega) \left( \tilde{P}_F^*(s^t) RER(s^t) \right)^{-\theta} Y(s^t), \]
The Model: Equilibrium Conditions

\[ C \left( s^t \right) + X \left( s^t \right) = Y \left( s^t \right), \]

\[ Y \left( s^t \right) = \left[ \omega^{\frac{1}{\theta}} Y_H \left( s^t \right)^{\frac{\theta-1}{\theta}} + \left( 1 - \omega \right)^{\frac{1}{\theta}} Y_F \left( s^t \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \]

\[ Y_H \left( s^t \right) + Y_H^* \left( s^t \right) = A \left( s^t \right)^{1-\alpha} K \left( s^{t-1} \right)^{\alpha} L \left( s^t \right)^{1-\alpha}, \]

\[ Y_F \left( s^t \right) + Y_F^* \left( s^t \right) = A^* \left( s^t \right)^{1-\alpha} K^* \left( s^{t-1} \right)^{\alpha} L^* \left( s^t \right)^{1-\alpha}, \]

and

\[ D \left( s^t \right) + D^* \left( s^t \right) = 0. \]
The Model: Balanced Growth

- Preferences and technology satisfy King, Plosser, and Rebelo (1988) restrictions for the existence of a balanced growth path in the closed economy.
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But in the open economy we need an additional restriction

\[
\hat{Y}_F (s^t) = (1 - \omega) \left[ \tilde{P}_F^* (s^t) \text{RER} (s^t) \right]^{-\theta} \hat{Y} (s^t) \frac{A(s^{t-1})}{A^*(s^{t-1})}
\]

where \( \hat{Y}_F (s^t) = Y_F (s^t) / A^* (s^{t-1}) \), \( \hat{Y} (s^t) = Y (s^t) / A (s^{t-1}) \).
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- \( \frac{A (s^{t-1})}{A^* (s^{t-1})} \) is stationary if \( \gamma = 1 \).
Estimation of the VECM for TFP

- Take U.S. data for real GDP (BEA) and employment (Payroll Survey).
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- Rest of the world: Euro Area, Canada, Japan, Australia, and the UK. Also Mexico and South Korea.

- Aggregate GDPs using PPP-adjusted exchange rates. We aggregate number of employees.

- Follow Heathcote and Perri (2002)

\[
\log A(s^t) = \left[ \log Y(s^t) - (1 - \alpha) \log L(s^t) \right] / (1 - \alpha)
\]

\[
\log A^*(s^t) = \left[ \log Y^*(s^t) - (1 - \alpha) \log L^*(s^t) \right] / (1 - \alpha)
\]
Estimation of the VECM for TFP
Unit root tests: we cannot reject a unit root for the level of (log) TFP processes. We can reject a unit root for their first difference. TFP’s are I(1).
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- We estimate the VECM with 2 lags and cannot reject that $\gamma = 1$.
- We run several likelihood ratio tests to test for symmetry.
Estimation of the VECM for TFP

\[
\begin{pmatrix}
\Delta \log A(s^t) \\
\Delta \log A^*(s^t)
\end{pmatrix}
= \begin{pmatrix}
c \\
c^*
\end{pmatrix} + \rho_1 \begin{pmatrix}
\Delta \log A(s^{t-1}) \\
\Delta \log A^*(s^{t-1})
\end{pmatrix} \\
+ \rho_2 \begin{pmatrix}
\Delta \log A(s^{t-2}) \\
\Delta \log A^*(s^{t-2})
\end{pmatrix} \\
+ \begin{pmatrix}
\kappa \\
\kappa^*
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\varepsilon^a(s^t) \\
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\]
### Table 4: Likelihood ratio tests.

<table>
<thead>
<tr>
<th>Restriction</th>
<th>Likelihood value</th>
<th>Degrees of freedom</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>744.18</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma = 1$</td>
<td>743.33</td>
<td>1</td>
<td>0.19</td>
</tr>
<tr>
<td>$\kappa = -\kappa^*$</td>
<td>741.71</td>
<td>2</td>
<td>0.09</td>
</tr>
<tr>
<td>$c = c^*$</td>
<td>740.43</td>
<td>3</td>
<td>0.06</td>
</tr>
<tr>
<td>Symmetry across VAR coefficients</td>
<td>736.51</td>
<td>7</td>
<td>0.032</td>
</tr>
</tbody>
</table>
### Table 5: Parameter Estimates, VECM model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate 1980 – 2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>0.0071* (5.83)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>$-0.0045^*$ (2.65)</td>
</tr>
<tr>
<td>$\rho_{11}^{1}$</td>
<td>0.2041* (2.97)</td>
</tr>
<tr>
<td>$\rho_{11}^{2}$</td>
<td>0.1026 (1.54)</td>
</tr>
<tr>
<td>$\rho_{12}^{1}$</td>
<td>0.1035 (1.55)</td>
</tr>
<tr>
<td>$\rho_{12}^{2}$</td>
<td>$-0.1497^*$ (2.40)</td>
</tr>
</tbody>
</table>

T-statistics in parenthesis.

* means significance at the 5 percent level.
## Table 6: Calibration

<table>
<thead>
<tr>
<th>Preferences</th>
<th>$\beta = 0.99$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu = 0.34$</td>
</tr>
<tr>
<td></td>
<td>$\sigma = 2$</td>
</tr>
<tr>
<td></td>
<td>$\phi = 0.01$</td>
</tr>
<tr>
<td>Technology</td>
<td>$\alpha = 0.36$</td>
</tr>
<tr>
<td></td>
<td>$\delta = 0.025$</td>
</tr>
<tr>
<td></td>
<td>$\omega = 0.9$</td>
</tr>
<tr>
<td></td>
<td>$\theta = [0.85, 0.62]$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_Y$</td>
</tr>
<tr>
<td>------------------</td>
<td>------------</td>
</tr>
<tr>
<td><strong>Data</strong></td>
<td>1.25</td>
</tr>
<tr>
<td><strong>Coint. TFP, $\theta = 0.85$</strong></td>
<td>0.81</td>
</tr>
<tr>
<td><strong>Coint. TFP, $\theta = 0.62$</strong></td>
<td>0.70</td>
</tr>
<tr>
<td><strong>Stat. TFP, $\theta = 0.85$</strong></td>
<td>1.19</td>
</tr>
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<td>1.12</td>
</tr>
</tbody>
</table>

$^+$ denotes relative to output.
<table>
<thead>
<tr>
<th>Full Sample</th>
<th>CORR(Y,N)</th>
<th>CORR(Y,C)</th>
<th>CORR(Y,X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.79</td>
<td>0.81</td>
<td>0.91</td>
</tr>
<tr>
<td>Coint. TFP, $\theta = 0.85$</td>
<td>0.94</td>
<td>0.95</td>
<td>0.97</td>
</tr>
<tr>
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<td>0.92</td>
<td>0.93</td>
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</tr>
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<td>0.97</td>
<td>0.93</td>
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Explaining the Mechanism

- Estimated stationary TFP shocks imply somewhat high persistence and fast spillovers (Heathcote and Perri, 2002).
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- First, we discuss the role of persistence in a stationary model. Then we discuss the role of spillovers in a non-stationary model.
Explaining the Mechanism

- When persistence increases at home, there is a stronger income effect at home. Hence:
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  - Lower production of home intermediate good and higher production of foreign intermediate good lead to larger RER and TOT depreciation.
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  - Lower production of home intermediate good and higher production of foreign intermediate good lead to larger RER and TOT depreciation.

- Hence higher persistence leads to higher RER volatility.
Figure: Impulse Response to a Home-Country TFP shock. Model with stationary TFP shocks.
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Explaining the Mechanism

- Now we switch to the model with VECM shocks:

\[
\begin{align*}
\Delta a_t &= -\kappa(a_{t-1} - a^*_{t-1}) + \varepsilon^a_t \\
\Delta a^*_t &= \kappa(a_{t-1} - a^*_{t-1}) + \varepsilon^{a^*}_t
\end{align*}
\]

Increased \( \kappa \) implies a stronger “news channel” in the foreign country:
- Labor supply and investment decreases, and output in the foreign country decreases on impact.
- Consumption increases, leading to more demand of the home intermediate good.
- Lower production of foreign intermediate good and higher production of home intermediate good lead to RER and TOT appreciation.

Hence higher speed of convergence leads to lower RER volatility.
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\[
\Delta a_t = -\kappa (a_{t-1} - a_{t-1}^*) + \varepsilon_t^a
\]

\[
\Delta a_t^* = \kappa (a_{t-1} - a_{t-1}^*) + \varepsilon_t^{a^*}
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\Delta a_t = -\kappa (a_{t-1} - a_{t-1}^*) + \varepsilon_t^a
\]

\[
\Delta a_t^* = \kappa (a_{t-1} - a_{t-1}^*) + \varepsilon_t^a^*
\]

- Increased \( \kappa \) implies a stronger “news channel” in the foreign country:
  - Labor supply and investment decreases, and output in the foreign country decreases on impact.
  - Consumption increases, leading to more demand of the home intermediate good.
  - Lower production of foreign intermediate good and higher production of home intermediate good lead to RER and TOT appreciation.

P. Rabanal, J. Rubio-Ramírez and V. Tuesta

Cointegrated TFP Processes.

October 15, 2008
Explaining the Mechanism

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Hence **higher speed of convergence leads to lower RER volatility.**
Explaining the Mechanism

Figure: Impulse Response to a Home-Country TFP shock. Model with cointegrated TFP shocks.
Explaining the Mechanism

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## Explaining the Mechanism

Table 8: Changing $\rho_a$ and $\kappa$

<table>
<thead>
<tr>
<th>$\rho_a$</th>
<th>$SD(RER)$</th>
<th>$SD(Y)$</th>
<th>$SD(RER)^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>1.43</td>
<td>1.33</td>
<td>1.07</td>
</tr>
<tr>
<td>0.95</td>
<td>1.96</td>
<td>1.2</td>
<td>1.64</td>
</tr>
<tr>
<td>0.975</td>
<td>2.47</td>
<td>1.06</td>
<td>2.33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>$SD(RER)$</th>
<th>$SD(Y)$</th>
<th>$SD(RER)^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>1.98</td>
<td>0.64</td>
<td>3.1</td>
</tr>
<tr>
<td>0.05</td>
<td>1.02</td>
<td>0.82</td>
<td>1.25</td>
</tr>
<tr>
<td>0.25</td>
<td>0.71</td>
<td>0.86</td>
<td>0.82</td>
</tr>
</tbody>
</table>
Therefore, one unit root in the joint process of TFP across countries is not enough. We need the second root to be very close to one.

\[
\begin{pmatrix}
    a_t \\
    a_t^*
\end{pmatrix} = \begin{pmatrix}
    1 - \kappa & \kappa \\
    \kappa & 1 - \kappa
\end{pmatrix} \begin{pmatrix}
    a_{t-1} \\
    a_{t-1}^*
\end{pmatrix} + \begin{pmatrix}
    \varepsilon_t^a \\
    \varepsilon_t^{a,*}
\end{pmatrix}.
\]
Explaining the Mechanism

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a_t^*
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1 - \kappa & \kappa \\
\kappa & 1 - \kappa
\end{pmatrix} \begin{pmatrix}
a_{t-1} \\
a_{t-1}^*
\end{pmatrix} + \begin{pmatrix}
\varepsilon_t^a \\
\varepsilon_t^{a,*}
\end{pmatrix}.
\]

With \( \kappa = 0.0045 \) we have that \( \lambda_1 = 1, \lambda_2 = 1 - 2\kappa = 0.991 \).
Explaining the Mechanism

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\begin{pmatrix}
  a_t \\
  a^*_t
\end{pmatrix} = 
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  \kappa & 1 - \kappa
\end{pmatrix}
\begin{pmatrix}
  a_{t-1} \\
  a^*_{t-1}
\end{pmatrix} + 
\begin{pmatrix}
  \varepsilon^a_t \\
  \varepsilon^{a*,t}
\end{pmatrix}.
\]

- With \( \kappa = 0.0045 \) we have that \( \lambda_1 = 1, \lambda_2 = 1 - 2\kappa = 0.991 \).

- BKK implies \( \lambda_1 = 0.994, \lambda_2 = 0.812, \) and correlation between innovations of 0.26. Rel RER volatility: 0.65.
Explaining the Mechanism

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  a_t \\
  a_t^*
\end{pmatrix}
= \begin{pmatrix}
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  \kappa & 1 - \kappa
\end{pmatrix}
\begin{pmatrix}
  a_{t-1} \\
  a_{t-1}^*
\end{pmatrix}
+ \begin{pmatrix}
  \varepsilon_t^a \\
  \varepsilon_t^{a,*}
\end{pmatrix}.
\]

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Heathcote and Perri (2002) \(\lambda_1 = 0.995, \lambda_2 = 1 - 2\kappa = 0.945\), and innovations have correlation of 0.29.
Therefore, one unit root in the joint process of TFP across countries is not enough. We need the second root to be very close to one.

\[
\begin{pmatrix}
    a_t \\
    a^*_t
\end{pmatrix} =
\begin{pmatrix}
    1 - \kappa & \kappa \\
    \kappa & 1 - \kappa
\end{pmatrix}
\begin{pmatrix}
    a_{t-1} \\
    a^*_{t-1}
\end{pmatrix} +
\begin{pmatrix}
    \varepsilon^a_t \\
    \varepsilon^{a,*}_t
\end{pmatrix}.
\]

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- Heathcote and Perri (2002) \( \lambda_1 = 0.995, \lambda_2 = 1 - 2\kappa = 0.945 \), and innovations have correlation of 0.29.
- Heathcote and Perri (2008) \( \lambda_1, \lambda_2 = 0.91 \). Rel RER volatility: 1.05.
A VAR in levels or a VECM?

- In principle we could have estimated a VAR in levels instead of a VECM.
A VAR in levels or a VECM?

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- Engle and Granger (1987): small sample improvements from estimating a VECM, estimating a VAR in levels leads to ignoring important constraints that are only satisfied asymptotically.
The Great Moderation and the Real Exchange Rate

Figure: Standard Deviation of HP-Filtered Data. USA and UK.
Figure: Standard Deviation of HP-Filtered Data. Canada and Australia.
## Estimation of the VECM for TFP

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>0.007*</td>
<td>0.008*</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>$-0.008*$</td>
<td>$-0.003$</td>
</tr>
<tr>
<td>$\rho_{11}$</td>
<td>0.22*</td>
<td>0.13</td>
</tr>
<tr>
<td>$\rho_{11}$</td>
<td>0.07</td>
<td>0.12</td>
</tr>
<tr>
<td>$\rho_{11}$</td>
<td>0.07</td>
<td>0.13</td>
</tr>
<tr>
<td>$\rho_{12}$</td>
<td>0.01</td>
<td>$-0.36*$</td>
</tr>
</tbody>
</table>

* means significance at the 5 percent level
### Table 7a: Results

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_Y$</th>
<th>$\sigma_C^+$</th>
<th>$\sigma_X^+$</th>
<th>$\sigma_N^+$</th>
<th>$\sigma_{RER}^+$</th>
<th>$\rho(RER)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1980-1993</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>1.57</td>
<td>0.80</td>
<td>3.08</td>
<td>0.89</td>
<td>3.97</td>
<td>0.85</td>
</tr>
<tr>
<td>Coint. TFP, $\theta = 0.85$</td>
<td>1.12</td>
<td>0.63</td>
<td>2.17</td>
<td>0.25</td>
<td>1.33</td>
<td>0.72</td>
</tr>
<tr>
<td>Coint. TFP, $\theta = 0.62$</td>
<td>0.95</td>
<td>0.65</td>
<td>2.15</td>
<td>0.25</td>
<td>3.17</td>
<td>0.71</td>
</tr>
<tr>
<td><strong>1994-2007</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>0.83</td>
<td>0.76</td>
<td>4.20</td>
<td>0.96</td>
<td>5.17</td>
<td>0.81</td>
</tr>
<tr>
<td>Coint. TFP, $\theta = 0.85$</td>
<td>0.64</td>
<td>0.55</td>
<td>2.74</td>
<td>0.38</td>
<td>2.04</td>
<td>0.71</td>
</tr>
<tr>
<td>Coint. TFP, $\theta = 0.62$</td>
<td>0.62</td>
<td>0.43</td>
<td>3.01</td>
<td>0.42</td>
<td>5.06</td>
<td>0.69</td>
</tr>
</tbody>
</table>

$^+$ denotes relative to output.
<table>
<thead>
<tr>
<th></th>
<th>CORR(Y,N)</th>
<th>CORR(Y,C)</th>
<th>CORR(Y,X)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1980-1993</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>0.82</td>
<td>0.82</td>
<td>0.93</td>
</tr>
<tr>
<td>Coint. TFP, $\theta = 0.85$</td>
<td>0.93</td>
<td>0.96</td>
<td>0.97</td>
</tr>
<tr>
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<td>0.96</td>
</tr>
<tr>
<td><strong>1994-2007</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
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<td>0.76</td>
<td>0.90</td>
</tr>
<tr>
<td>Coint. TFP, $\theta = 0.85$</td>
<td>0.89</td>
<td>0.82</td>
<td>0.94</td>
</tr>
<tr>
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<td>0.78</td>
<td>0.97</td>
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Concluding Remarks

- In this paper, we document two empirical facts:
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Concluding Remarks

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  - TFP processes of the U.S. and the “rest of the world” are cointegrated with cointegrating vector $(1, -1)$ and
  - The relative volatility of the real exchange rate with respect to output has increased in the United States, the United Kingdom, Canada, and Australia during the last 20 years.
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We have shown that introducing cointegrated TFP processes in an otherwise standard IRBC model increases the ability of the model to explain real exchange rate volatility.
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  - The relative volatility of the real exchange rate with respect to output has increased in the United States, the United Kingdom, Canada, and Australia during the last 20 years.

- We have shown that introducing cointegrated TFP processes in an otherwise standard IRBC model increases the ability of the model to explain real exchange rate volatility.

- If we allow the speed of convergence to the cointegrating vector to change as it does in the data, the model can also explain the observed increase in the relative volatility of the real exchange rate.
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Cointegration of TFP processes should be introduced in larger-scale models (Adolfson et al., 2007)