

## Informality and wealth distribution: an heterogeneous agent model

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# Outline

- 1 Motivation
- 2 Strategy
- 3 The model
- 4 Results
- 5 Conclusions

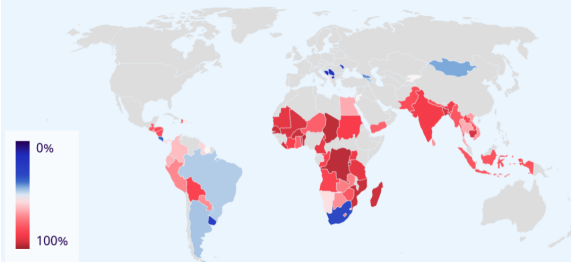
# Motivation

An overview of informality

Informality in labor markets is a prominent feature of developing countries.

## ► High shares of informality in developing countries

Share of informal employment by country (in percent), latest year



*The boundaries shown on this map do not imply endorsement or acceptance by the ILO.*

Last updated on 19 July 2021.

Source: [ILOSTAT](#)

# Motivation

## Informality in Numbers

Informality in labor markets is a prominent feature of developing countries.

	Africa	Americas	Arb States	Asia and the Pacific	Europe and Central Asia	Total
World	85.8	40	68.6	68.2	25.1	61.2
EM and developing countries	85.8	53.8	68.6	71.4	36.8	69.6
Developed countries	-	19.4	-	21.7	15.6	18.3

**Source:** International Labour Organization, statistic of the informal economy (2018).

# Informal employment

## Definition

The **International Labor Organization (2018)** defines informal employment as:

Employer-employee relationship that it is not subject to:

- national labor legislation
- income taxation
- social protection or
- entitlement to employment benefits  
(advance notice of dismissal, sick leave, etc)

# Literature

Given the relevance of informality, the literature has focused on...

Understanding the causes and consequences of informality

- **Theoretical studies** have focused on the **average effect of informality on aggregate variables such as (i) trade, (ii) tax collection and productivity, and (iii) economic development** (e.g., [Leal Ordóñez, 2014](#); [Almeida and Poole, 2017](#); [Castillo and Montoro, 2010](#))
- **Empirical studies** have shown heterogeneity among formal and informal workers
  - IW have **less income** (the formal-informal wage gap) (e.g., [Maya and Pereira, 2020](#))
  - IW' income is **more volatile** (e.g., [Gomes, 2021](#))
  - IW pay **higher interest rate** for borrowing (e.g., [Horvath, 2018](#))
  - IW do **not pay taxes**
  - IW are **more risk averse** (e.g., [Bennett et al., 2012](#))

**However**, our knowledge is limited w.r.t. the effects of informality on *wealth* and *consumption distribution*.

## Questions and contribution

- What are the effects of informality on the distribution of wealth and consumption?
- Are there aggregated effects derived from the impact of informality on distribution of wealth and consumption?

By addressing this questions, we will contribute to the literature in the following dimensions:

- Building a framework to study the effects of informality on wealth/consumption distribution withing a structural model
- Take into account explicitly the heterogeneity between formal and informal workers
- Shed a light on the macro and policy implications originated by a sizable informality sector

## What we do, what we do not do

- We answer this question theoretically for a calibration relevant to an EM
  - No structural estimation is performed
- Informality is exogenously given
  - We calibrate the degree of informality and it is not state dependent
- We abstract from other standard macro sources of variation to obtain an equilibrium in which informality and market incompleteness are the only determining factors
  - There are no macro shocks but ...
  - households move from formality to informality in a non-deterministic way



## How we do it

We build a tractable and simplified model which is calibrated as follows:

- **Benchmark economy:** low informality size, but formal/informal agents
  - pay taxes
  - same borrowing interest rate
  - same risk-aversion parameter
- **Informal economy:** high informality size with
  - informal agent does not pay taxes
  - informal agent pays a higher borrowing interest rate (risk premium)
  - informal agent is more risk-averse

We progressively add each of these informality features to the benchmark model and study how the equilibrium changes

## what we get

- Compared to the benchmark, an economy with a sizable informal sector exhibits wealth and consumption distributions with lower median and higher dispersion
  - The informal population experiences a more substantial reduction in median wealth
  - The formal population exhibits higher dispersion than their informal counterparts.
- Each informality feature has distinct effects, both in magnitude and direction, on the wealth and consumption distribution.
  - The spread in wealth's dispersion between the informal and the benchmark economy is primarily explained by the size of informality and high-risk aversion
  - On the other hand, the interest rate premium reduces this spread
  - while the absence of tax payment has minimal effects
- These informality characteristics also have heterogeneous effects on the wealth and consumption distribution of each specific group
- U-shaped interest rate with the degree of informality

# The Model

We extend the continuous-time version of the Huggett model developed by [Achdou et al. \(2021\)](#) to capture the four features of informality:

1. **Income level:** Informal agents have **lower** income ( $y_I$ ) than formal agents ( $y_F$ )

$$y_I < y_F$$

2. **Taxes:** Informal agents do **not pay** income taxes

$$\tau_F > \tau_I = 0$$

3. **Interest rate:** Informal agent pay a **premium**  $\theta$  when they borrow

$$\text{Informal: } R = r + \theta \text{ vs Formal: } R = r$$

4. **Risk aversion:** Informal agents are **more risk averse** than formal agents

$$\gamma_I > \gamma_F$$

# The Model

## Agents' Optimization Problem

$$\max_{\{c_t\}} E_t \left[ \int_0^{\infty} \exp(-\rho t) \left( \frac{c_t^{1-\gamma_t}}{1-\gamma_t} + gov_t \right) dt \right]$$

subject to

Agent wealth dynamic :  $\dot{a} = (1 - \tau_t)y_t + R_t a_t - c_t$

Borrowing constraint :  $a_t \geq \underline{a}$

Income process :  $y \in \{y_I, y_F\}$  with  $\lambda_I, \lambda_F$  and  $y_I < y_F$

Risk aversion :  $\gamma \in \{\gamma_I, \gamma_F\}$  with  $\gamma_I > \gamma_F$

Taxes :  $\tau \in \{\tau_I = 0, \tau_F > 0\}$

Interest rate : Informal : if  $a < 0 \rightarrow R = r + \theta$

Formal : if  $a < 0 \rightarrow R = r$

$gov_t$  are public goods provided by the Gov. which has the following budget constraint:

$$\int \tau_I y_I * g_I(a) da + \int \tau_F y_F * g_F(a) da = \int gov * g(a) d(a)$$

## Calibration

Parameter <sup>1</sup>		Formal Agent	Informal Agent
Subj. discount rate <sup>2</sup>	$\rho$	0.05	0.05
Relative risk aversion <sup>2</sup>	$\gamma$	0.15	0.3
Income level <sup>3</sup>	$y$	1	0.33
Borrowing limit <sup>2</sup>	$\underline{a}$	30% $\times y_1$	30% $\times y_1$
Intensity to jump <sup>4</sup>	$\lambda$	1.69	2.25
Tax rate <sup>5</sup>	$\tau$	0.18	0
Interest rate premium <sup>6</sup>	$\theta$	0	20%
Informal sector size <sup>7</sup>	$\eta$		0.64

1. In the benchmark (formal) economy, the calibration is identical but  $\eta = 0.2$ ,  $\tau_I = \tau_F = 0.18$ ,  $\gamma = 0.15$  and  $\theta = 0$

2. Standard in literature

3. Peruvian income data of from 2007Q1 to 2022Q2: informal income is approximately 1/3 of formal income

4. Estimated with annual data of proportion of formal and informal Peruvian workers

5. Average income tax rate in Peru for formal workers (4th and 5th labor categories) between 2016 and 2023

6. Average spread between interest rate of consumer loans from banks and from *cajas de ahorros* in Perú since 2015 to 2019

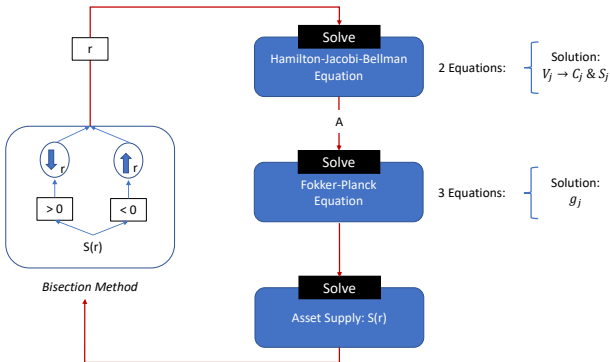
7. Average percentage of Peruvian informal workers between from 2011 and 2020

# The PDEs System

1. **HJB:**  $\rho V_j(a) = \max_{\{c\}} \{U(c, gov) + V'_j(a)S_j(a) + \lambda_j(V_{-j}(a) - V_j(a))\}$
2. **FP:**  $0 = \partial_a[S_j(a)g_j(a, t)] - \lambda_j g(a, t) + \lambda_{-j} g_{-j}(a, t)$ 
  - **FOC from HJB:**  $c_j(a) = (U')^{-1}(V'_j(a))$
  - **FOC from solving:**  $S_j(a) = y_j + Ra - c_j(a)$
3. **The state constraint boundary condition:**  $V'_j(\underline{a}) \geq U'(y_j + Ra)$
4. **Interest rate:**
  - Informal: if  $a < 0 \rightarrow R = r + \theta$
  - Formal: if  $a < 0 \rightarrow R = r$
5. **The Market clearing condition:**  $S(r) \equiv \int_{\underline{a}}^{\infty} adG_1(a) + \int_{\underline{a}}^{\infty} adG_2(a) = B$
6. **The aggregation of distributions:**  $\int_{\underline{a}}^{\infty} g_1(a)da + \int_{\underline{a}}^{\infty} g_2(a)da = 1$

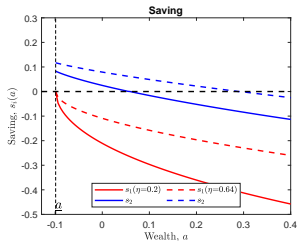
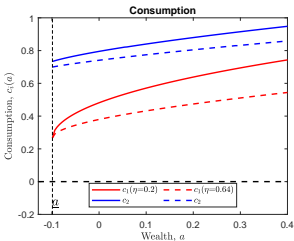
# Numerical Solution

1. Approx. of HJB and Fokker-Planck Eq.:  
**Finite Difference Method**
2. Update VF: **implicit method**
3. Forward or backward approximation: **Upwind scheme**

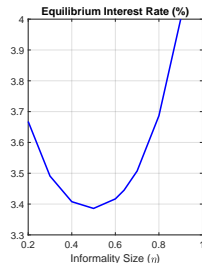
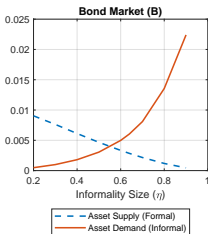
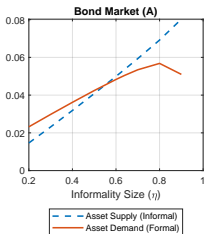


# Increase in the size of informality $\eta$

## Policy functions



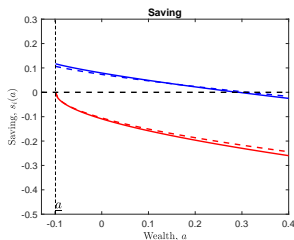
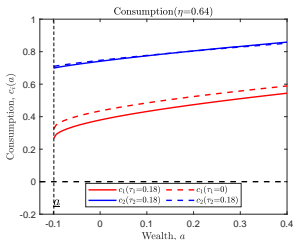
## Asset supply and demand



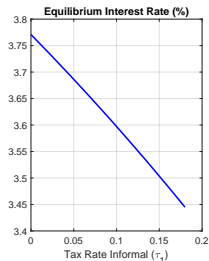
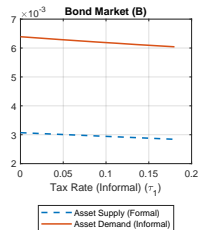
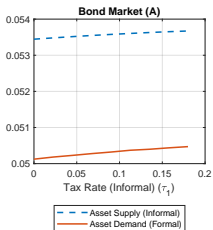


$(\eta = 64\%) +$  no tax payment for informal agents ( $\tau_I = 0$ )

### Policy functions

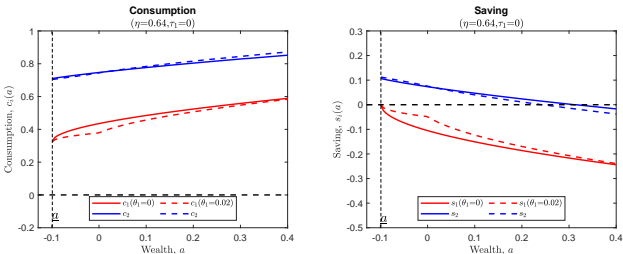


### Asset supply and demand

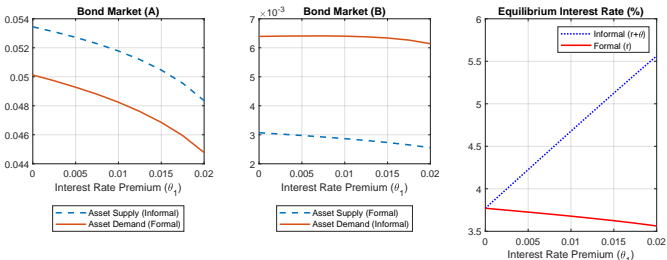


$(\eta = 64\%) + (\tau_I = 0) + \text{premium faced by informal agents } (\theta = 20\%)$

### Policy functions

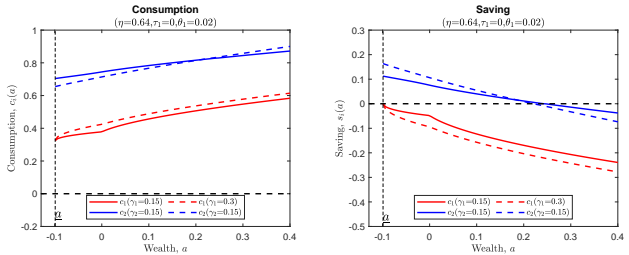


### Asset supply and demand

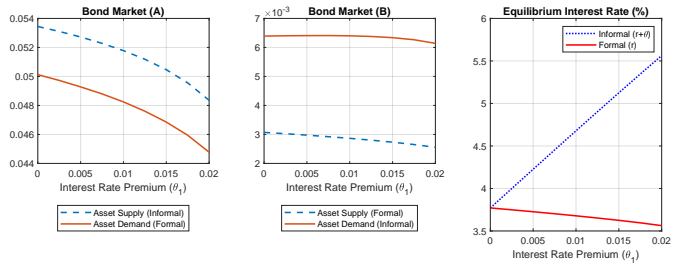


$(\eta = 64\%) + (\tau_I = 0) + (\theta = 20\%) + \text{informal higher RRA } (\gamma_I = 2\gamma_F)$

### Policy functions



### Asset supply and demand



Distribution:  $(\eta = 64\%) + (\tau_I = 0) + (\theta = 20\%) + (\gamma_I = 2\gamma_F)$

		Median			St Deviation			
	$\eta$	Informal	Formal	Total	Informal	Formal	Total	
<b>(B) Consumption Distribution</b>								
Benchmark	$0.2^a$	5.371	5.991	47.429	0.088	0.022	5.559	
High $\eta$	0.64	25.598	0.756	5.051	0.052	0.036	12.518	
+ No taxes	0.64	$\tau_I = 0$	31.067	0.726	3.518	0.051	0.033	15.290
+ Premium	0.64	$\theta = 0.02$	24.995	0.934	3.172	0.040	0.035	11.801
+ $\neq$ RRA	0.64	$\gamma_I = 0.3$	30.284	1.083	38.062	0.049	0.046	14.447
<b>(A) Wealth Distribution</b>								
Benchmark	$0.2^a$	-1.992	0.249	0.106	0.041	0.042	0.055	
High $\eta$	0.64	-9.484	0.140	-0.267	0.063	0.112	0.130	
+ No taxes	0.64	$\tau_I = 0$	-9.427	0.130	-0.273	0.065	0.117	0.131
+ Premium	0.64	$\theta = 0.02$	-7.626	0.159	-0.123	0.062	0.096	0.116
+ $\neq$ RRA	0.64	$\gamma_I = 0.3$	-9.186	0.219	-0.196	0.059	0.090	0.122

<sup>a</sup>benchmark economy (low informality and taxes paid by formal/informal)

## Conclusions

- Within a simplified HAM we can study the distributional and aggregated implications of labor informality
- In average, informal agents are net borrowers (negative wealth) and formal agents are net lenders (positive wealth)
- Different features of informality have differentiated effects
  - Higher informality size: more wealth average of formal people • more inequality and less consumption in both populations • more informal people at the constraint ( $\underline{a}$ )
  - Tax evasion has mild effects • informal agents are still net borrowers but fewer are at the constraint
  - Higher risk premium reduces consumption and wealth dispersion
  - Higher RRA increases consumption but also its dispersion
- Aggregated prices (such are interest rate) are affected by informality through its distributional implications
- Future work: expand the model by incorporating nominal or financial frictions to study policy or other relevant aspects as, for instance, the role of the informality in the determinations of the term structure