# Bank Liquidity Management in a Bi-Monetary Banking System\*

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#### Abstract

I propose a banking model of a Bi-Monetary Banking System. Banks face a liquidity management problem with two currencies, the local currency and the foreign currency (the U.S dollar). Banks have access to the exchange rate, FX-swap and interbank markets. The model delivers a supply of loans, reserves holdings in both currencies, a demand for deposits in both currencies and a nominal exchange rate path. Motivated by the empirical evidence on U.S monetary policy spill overs I perform a static comparative exercise simulating a change in the interest over reserves in foreign currency. Additionally, motivated by the interbank market freezing in foreign currency experienced in Europe in 2011 I present comparative statics for different values of the matching technology in the FX swap market.

 $\it keywords$ : Liquidity management, Dollarization, FX Swap Market, Interbank Market.

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# 1 Introduction

The Global Financial Crisis (GFC) brought a new agenda of research to better understand the macroeconomy, in particular the topics of monetary policy and the financial system, on a scale not seen since the great depression of the 1930s with Keynes' ideas or the 1970s with the new macroeconomy of rational expectations.

One topic of research is the functioning of the interbank market as central banks typically establish reference policy rates on those markets and adjust their monetary tools accordingly. To better understand those markets it is crucial to implement the operative procedures of monetary policy in a suitable way. Before the GFC, the study of monetary policy used to focus only on standard Neo-Keynesian models with a Taylor rule, assuming that central banks always had full control on the interbank market rates. After the GFC, it was clear that this type of model was insufficient to understand the way a central bank influences financial conditions, particularly when there is a disruption in the money market. Therefore, it is important to study the interbank market.

Another important concern for economies without a central bank that can issue international reserve currency, which is indeed the case of most central banks, is the fact that US monetary policy seems to be more important as a main driver of international financial conditions than was first realized. Rey (2016) has remarked on the importance of US monetary policy in the global financial cycle and its effects over developed and developing countries. Most of the banking systems deal with more than one currency for funding. The fact that banks residents in the United States had claims of \$ 462 billion to borrowers in Europe in 2002 and \$ 1.54 trillion in 2007, and also that the bank residents in Europe had claims of \$ 856 billion in 2002 and \$2 trillion in 2007 reflects the growing trend in cross-border claims and the importance in incorporate assets and liabilities denominated in foreign currency. Furthermore, the fact that more than half of the international banking liabilities are denominated in US dollars is a important reason to incorporate multiple currencies within a banking model. Quoting Shin (2016):

<sup>&</sup>lt;sup>1</sup>This fact is taken from Shin (2016)

 $<sup>^{2}</sup>IMF$  (2018)

"The global role of the US dollar is reflected in its pre-eminent role in the global banking system. The dollar is the unit of account in debt contracts in that borrower borrows in dollars and lenders lend in dollars, irrespective of whether the borrower or lender is located in the United States".

In this paper I provide a general equilibrium banking model in which banks use assets and liabilities denominated in local and foreign currencies. The aim of this paper is to study the liquidity management in a bi-monetary system by working with a model with two interbank markets: one without collateral and the other with US dollar reserves as collateral (FX swap market). I use the model to understand U.S monetary policy transmission through its effect on reserves in foreign currency. Finally, motivated by the foreign interbank European market freezing episode of 2011, I simulate the model when the matching becomes harder in the FX swap market.

The paper is organize as follows. Section 2 relates the model to previews literature. Section 3 presents the model. Section 4 explain how to solve the model. In section 5 I perform static comparative exercises. Section 6 concludes.

## 2 Related Literature

After the financial crisis there have been a growing interest in studying banks within macroeconomic models (see, Woodford (2010)). Two recent examples are Gertler and Karadi (2011) who propose a DSGE model with financial intermediation and balance sheet constraints in order to evaluate unconventional monetary policy in a financial crisis, and Cúrdia and Woodford (2009) who extend a standard a New Keynesian model that allows a role for Central Bank's balance sheet in equilibrium determination. This paper follows this line of literature and introduces two currencies (domestic currency and the U.S dollar) in a macroeconomic model with banks.

The aim of the paper is to study liquidity management in a bi-monetary Banking system. Bianchi and Biguio (2017) study liquidity management in a banking system with

one currency and use their model to obtain the combination of shocks that better explain the 2008 financial crisis. This paper extends the model to a bi-monetary banking model. Additionally I introduce an FX-swap market that generates a precautionary demand for reserves in foreign currency and a exchange rate market that determines the nominal exchange rate path.

This paper is also part of the dollarization literature. One of the most influential paper in this literature is Ize and Yeyati (2003). They adopt a minimum variance portfolio approach and show that financial dollarization have a strong persistence when expected inflation variance is high in comparison with the exchange rate variance. Terrones and Catao (2000) propose a bi-monetary banking model and show that deposits and loan dollarization are determined by interest rates and exchange rate risk, costly banking, credit market imperfections and the availability of tradable collateral. Basso et al. (2007) expand the existing literature by allowing different returns over loans in foreign and local currency and over deposits in foreign and local currency.

In the proposed model dollarization of assets is high when there is a large demand for loans in foreign currency or when banks want to hold large amounts of foreign reserves. The fraction of demanded loans that are in foreign currency is taken as exogenous in the model. Holdings of reserves in foreign currency are determine endogenously and these are a function of several variables, for example: the expected depreciation of the local currency, the interest rate over reserves in foreign currency (which is chosen by the U.S monetary policy) and the matching technology in the FX swap and interbank market. Deposits dollarization is also a function of several variables, for example: the reserve requirement in local and foreign currency and the matching technology in the interbank and FX swap market. Other important difference with the papers mentioned above is that in the setting of this model the inflation and exchange rate path are deterministic while in theirs at least one of these is random.

There are few papers that incorporate a foreign currency analysis in a banking model. Bruno and Shin (2015a) propose a banking model in which a global bank obtains foreign currency funding from the U.S and use it to provide funds to their branches abroad so

that these can finance loans in foreign currency. By this mechanism financial conditions in U.S can be transmitted to foreign countries. In my paper U.S monetary policy has an effect in other countries through its effect over foreign reserves holdings. My paper is also related to the literature on exchange rate determination with financial conditions. A paper that goes in this line is Gabaix and Maggiori (2015). In their model capital flows drive exchange rates by altering the balance sheets of financiers that bear the risks resulting from international imbalances in the demand for financial assets.

This paper is related to a new literature that incorporates matching frictions in interbank markets. Two papers that incorporate these into macroeconomic environments are Bianchi and Biguio (2017) and Arce et al. (2017). In Bianchi and Biguio (2017) a reserve demand and supply is obtained due to withdrawal shocks, while in Arce et al. (2017) are obtained from bank heterogeneity in their access to loan returns.

There is a large empirical literature that emphasize U.S monetary policy cross-border spillovers. Anaya et al. (2017) find that an expansionary policy shock significantly increases portfolio flows from the US, to emerging market economies for almost two quarters, accompanied by a persistent movement in real and financial variables in recipient countries. Georgiadis (2016) study the determinants of global spillovers of US monetary policy and finds that U.S monetary policy generates sizable output spillovers to the rest of the world.<sup>3</sup> Bruno and Shin (2015b) show that U.S monetary policy can be transmitted through the banking sector. Other influential papers are Miranda-Agrippino and Rey (2015) and Rey (2016) which postulate that cross-border flows and leverage of global institutions transmit monetary conditions globally even under floating exchange-rate regimes. As it will be seen later in this framework U.S monetary policy affects local financial outcomes. Nevertheless this is only the case when the Central Bank participates in the exchange rate market.

<sup>&</sup>lt;sup>3</sup>The magnitude of the spillover depends on the receiving country trade and financial integration, financial openness, exchange rate regime, financial market development, labour market rigidities, industry structure, and participation in global value chains

# 3 Model

I propose a bi-monetary banking system model based on Bianchi and Biguio (2017) framework. The model designed in this paper considers and economy in which banks have assets and liabilities in foreign currency (U.S dollars). I assume that residents in this economy exclusively save in banking deposits in local currency (D). Foreign currency arrives to the economy only through deposits of non residents in local banks  $(D^*)$ . Firms demand loans in local currency (L) and in foreign currency  $(L^*)$ . I also assume that bank owners only accept dividend payments in local currency.

Banks face a standard liquidity management problem. Each period there is a random withdrawal shock over local currency denominated deposits, a fraction  $\omega$  of deposits in local currency is withdrawn every period. Banks face reserve requirements in both currencies. In order to fulfil these constraints they interact through two markets: the local interbank market and the FX swap market. I will assume that both markets are non Walrasian. The allocation in each market is determined by a matching process and the interest rate is obtained by Nash Bargaining.

The Central Bank policy is implemented by operating in the reserve market and potentially in the currency exchange market. I will consider a Central Bank that implements an inflation targeting policy and chooses a flexible exchange rate regime. The Central Bank performs its monetary policy goal by choosing the aggregate amount of reserves in local currency. The Central Bank have the following instruments: the interest over reserves, the discount window interest rate, the capital requirement and the reserve requirement in both currencies.

#### 3.1 Bank's Problem

The owner of the bank lives forever and banks can not be bankrupt. She maximizes the stream of dividends c and has a CRRA utility function. Each period is divided in

<sup>&</sup>lt;sup>4</sup>The model can be extended to include exporters that bring in dollars from abroad.

two stages: the lending stage and the balancing stage. Banks operate in a competitive environment.

Banks start the lending stage with an amount of equity which is a function of their previous period's actions. In this stage each bank chooses its dividend policy c, supply of loans in local and foreign currency (l and  $l^*$  respectively), reserves in local and foreign currency (m and  $m^*$  respectively) and its demand of deposits in local and foreign currency (d and  $d^*$  respectively). These variables define a position in foreign assets which is translated into a net demand of foreign currency in the exchange rate market. Define this as FX. If FX > 0 the bank takes a long position in foreign currency which increases the demand for foreign currency in the exchange rate market, while if FX < 0 the bank takes a short position in foreign currency which increases the supply of foreign currency in the exchange rate market. Note that while FX can be either positive or negative,  $\{c, m, m^*, d, d^*, l, l^*\}$  can only take positive values. Also, setting a position in foreign currency determines the bank exposure to exchange rate risk.<sup>5</sup> Also banks face a leverage constraint reflected as a capital requirement constraint.

In the balancing stage each bank faces a withdraw shock in its deposits in local currency. In particular a fraction  $\omega$  of them are withdrawn. During this stage loans are illiquid and reserves are not, hence transfers of deposits in local currency between banks are settled with reserves in local currency. The distribution of  $\omega$  in known for all banks, is denoted by  $F(\omega)$ , have support  $[-1, +\infty]$  and  $E(\omega) = 0$ . This last assumption implies that deposits in local currency can not be taken outside the banking system.

Banks face a reserve requirement in foreign and local currencies during this stage. After the withdraw shock realization a distribution of reserves surpluses and reserve deficits will emerge. In order to fulfil these requirements banks participate in the interbank and FX-swap market.

In a bi-monetary banking system swap of currencies can be used to provide liquidity. I will consider a swap of currencies operation that don't deliver exchange rate risk,

<sup>&</sup>lt;sup>5</sup>In some countries the banking regulation limits the position that banks can take. I will not analyse this policy in this version of the paper but it can be easily included.

an FX-swap operation. This consist in exchanging currencies at the spot exchange rate with a repurchase agreement at the spot exchange rate at the time of the repurchase movement. This operation does not involve a change in the position of foreign currency and hence does not provide exchange rate risk. For the same reason it does not have an impact on the exchange rate.

An FX swap operation is equivalent to a swap of currencies taken as collateral the other currency. This interpretation will be used in several parts of the paper. In the framework proposed in this paper, a bank who needs liquidity during the balancing stage can swap its reserves in foreign currency for reserves in local currency to reduce its reserve deficit, inducing a precautionary demand for foreign reserves. This way the model builds over (Bianchi and Biguio, 2017) framework by adding an extra source of liquid funds, in this paper a bank with reserve deficit can get not only funds from the interbank market to fulfil its reserve requirement, additionally it can obtain FX swap founds. Interbank market funds and FX-swap funds are denoted by f and  $f^*$  respectively. I assume that first Banks get information about the withdraw shock realization, then they participate in the interbank market and finally they participate in the FX swap market<sup>6</sup>. As mentioned before allocations in those markets are determined by a matching process. Banks additionally have access to the discount window of the Central Bank. Discount window loans are denoted with w and pay the discount window interest rate  $i^{dw}$  which is set by the Central Bank. Loans in the Interbank, FX-swap and discount window loans are overnight<sup>7</sup>. Additionally interbank and FX swap markets are over the counter and in these banks with reserve surplus post lending orders and banks with reserve deficit post borrowing orders.

The lending unmatched orders are remunerated with the interest rate over reserves  $i^{ior}$  at the end of the balancing stage. This interest rate is set by the Central Bank. Reserves in foreign currency are remunerated with the interest over foreign reserves  $i^{ior*}$  which is chosen by the U.S Central Bank authority (FED).

Interbank and FX swap market interest rates are set by Nash bargaining. The first ones

 $<sup>^6{</sup>m The}$  assumption that banks first participate in the interbank market and then in the FX swap market simplifies considerable the problem.

<sup>&</sup>lt;sup>7</sup>In this framework this translates to concluding the loan contract and making the payment at the end of each period

are denoted by  $i^f$  and the second ones by  $i^{ex}$ . Since banks first meet in the interbank market and then in the FX swap market, during the bargaining for the interbank market interest rates, FX swap market outcomes will be taken into account. In the interbank market each lending order has as outside option posting the offer in the FX swap market and each borrowing order has as outside option posting a borrowing order in the FX-swap market. Consider that since reserves in foreign currency are used as collateral in the FX swap market then the amount of FX Swap funds that a bank can obtain is limited by its reserves in foreign currency holdings and hence the bargained interest rate will be a function of the foreign reserve holdings of the bank that posted the borrowing order<sup>8</sup>. In the FX swap market each lending order have as outside option keeping the reserves, do not lend them, and get the interest rate over reserve payment  $i^{ior}$ , while each borrowing order have as outside option taking a loan from the Central Bank discount window and paying the interest rate  $i^{DW}$ .

Banks operate in a competitive environment in which prices  $\{i^b, i^{b*}, i^{ior}, i^{ior*}, i^d, i^{d*}, i^{ex}, i^f, i^{dw}, P, \triangle x\}$  are taken as given,  $\{i^b, i^{b*}, i^{ior}, i^{ior*}, i^d, i^{d*}\}$  stands for nominal interest over loans in local currency, loans in foreign currency, reserves in local currency, reserves in foreign currency, deposits in local currency and deposits in foreign currency respectively. As mentioned above  $i^{ior}, i^{dw}$  and  $i^{ior*}$  are set by the local and foreign Central Bank,  $\{i^b, i^b, i^d, i^d, i^d\}$  are obtained endogenously by clearing the loans markets in both currencies and the deposits markets in both currencies. Banks know the inflation rate  $(\pi)$  and nominal exchange rate change  $(\triangle x)$  paths<sup>9</sup>, the former is obtained by finding the price level that clears the reserve market in local currency as it will be showed later and the latter is obtained from clearing the exchange rate market. Finally the interbank interest rates  $i^f$  and the FX swap interest rate  $i^{ex}$  are obtained in the interbank market and the FX swap market by a Nash Bargaining process. Remember that b stand for loans, d for deposits, m for reserves and asterisks denote variables denominated in foreign currency. Since now on for any variable h lets denote h as the variable at the beginning of the lending stage,  $\tilde{h}$ 

<sup>&</sup>lt;sup>8</sup>The model also allows to consider first the participation in the FX swap market and then in the interbank market. In this case the interbank market interest rates and the FX swap market interest rates are constant across transactions.

<sup>&</sup>lt;sup>9</sup>The only source of uncertainty is the withdraw realization,

as the variable at the end of the lending stage and  $\hat{h}$  as the variable at the end of the balancing stage.

#### 3.1.1 Lending Stage

Banks start the lending stage with equity in local currency and in foreign currency, denoted with e and  $e^*$  respectively, and are a function of last period decisions. Equity in local currency at the beginning of the period is equal to the revenues obtained by loans and reserves  $((1+i_t^b)b$  and  $(1+i_t^{ior})m$  respectively) minus deposits, taxes, interbank market loans, FX swap market loans and discount window loans  $((1+i_t^d)d, P_tT_t, (1+i_t^f)f, (1+i_t^{ex})f^*)$  and  $(1+i^{dw})w$  respectively). Interbank market and FX loans take negative values if the bank has a surplus of reserves; additionally in this case the discount window loans are equal to zero. Hence the equity in local currency is:

$$e = (1 + i_t^b)b - (1 + i_t^d)d + (1 + i_t^{ior})m - (1 + i_t^f)f - (1 + i_t^{ex})f^* - (1 + i_t^{dw})w - P_tT_t$$

Equity in foreign currency is equal to the revenues generated by loans and reserves in foreign currency  $((1+i_t^{b*})b^*)$  and  $(1+i_t^{ior*})m^*$  respectively) minus the deposits and taxes in foreign currency  $((1+i_t^{d*})d^*)$  and  $P_tT_t^*$  respectively)<sup>10</sup>, hence the equity in foreign currency is:

$$e^* = (1 + i_t^{b*})b^* + (1 + i_t^{ior*})m^* - (1 + i_t^{d*})d^* - P_tT_t^*)$$

Define  $V^l$  as the value function in the lending stage and  $V^b$  as the value in the balancing stage. The dynamic programming problem during the lending stage is:

 $<sup>^{10}</sup>$ I am considering that there are taxes payments in local and foreign currency. Assuming that taxes are collected only in local currency does not change the results of the model

$$\begin{split} V^l(b,m,d,b^*,m^*,d^*,f,f^*,w) &= \max_{\{\tilde{b},\tilde{m},\tilde{d},\tilde{b}^*,\tilde{m}^*,\tilde{d}^*,\tilde{F}X\}} + E[V^b(\tilde{b},\tilde{m},\tilde{d},\tilde{b}^*,\tilde{m}^*,\tilde{d}^*,\omega)] \\ \text{s.t.}: \\ P_tc + \tilde{b} + \tilde{m} + x_t \tilde{F}X - \tilde{d}, \\ &= (1+i_t^b)b - (1+i_t^d)d + (1+i_t^{ior})m - (1+i_t^f)f - (1+i_t^{ex})f^* - (1+i^{dw})w - P_t T_t, \\ \tilde{b}^* + \tilde{m}^* - \tilde{d}^* - \tilde{F}X, \\ &= (1+i_t^{b*})b^* + (1+i_t^{ior*})m^* - (1+i_t^{d*})d^* - P_t T_t^*, \\ \tilde{d} + x_t \tilde{d}^* < \kappa(\tilde{b} + x_t \tilde{b}^* - \tilde{d} - x_t \tilde{d}^* + \tilde{m} + x_t \tilde{m}^*). \end{split}$$

The bank has two budget constraint to fulfil, one in local currency and the other in foreign currency. These two are linked by the bank position in foreign currency. The budget constraint in local currency is:

$$P_t c + \tilde{b} + \tilde{m} + x_t \tilde{F} X - \tilde{d} =$$

$$(1 + i_t^b)b + (1 + i_t^{ior})m - (1 + i_t^d)d - (1 + i_t^f)f - (1 + i_t^{ex})f^* - (1 + i_t^{dw})w - P_t T_t .$$

Banks use their equity e and deposits in local currency (d) to issue dividends (c), supply loans and obtain reserves in local currency (l and m respectively). Banks also choose their position in foreign assets (FX). The position of a bank in foreign currency is the mirror image of its position in local currency. When FX > 0 the bank has a short position in local currency and hence it is funding foreign currency denominated assets with local denominated liabilities. The budget constraint in foreign currency is:

$$\tilde{b}^* + \tilde{m}^* - \tilde{d}^* - \tilde{FX} = (1 + i_t^{b*})b^* + (1 + i_t^{ior*})m^* - (1 + i_t^{d*})d^* - P_t T_t^*.$$

In the same token equity in foreign currency  $e^*$  and deposits in foreign currency  $(d^*)$  are used to supply loans in foreign currency  $(l^*)$ , obtain reserves in foreign currency  $m^*$  and invest in local currency denominated assets (FX < 0).

Note that the two budget constraints are equivalent to the following two equations:

$$P_t c + \tilde{b} + \tilde{m} + \tilde{b}^* x_t + \tilde{m}^* x_t = \tilde{d} + x \tilde{d}^* + e^T - P_t (T + x T^*)$$
 and 
$$FX = \tilde{b}^* + \tilde{m}^* - \tilde{d}^* - e^*.$$

Where  $e^T = e + xe^*$  is the total equity of the bank expressed in local currency using the spot exchange rate. The first equation is the budget constraint of the bank in local currency while the second equation defines the position in foreign currency of the bank. Banks also face a capital requirement constraint:

$$\tilde{d} + x_t \tilde{d}^* \le \kappa (\tilde{b} + x_t \tilde{b}^* - \tilde{d} - x_t \tilde{d}^* + \tilde{m}^* + x_t \tilde{m}^*).$$

This works as a leverage constraint for the bank since limits its capacity to obtain deposits.

#### 3.1.2 Balancing Stage:

At the beginning of the balancing stage each bank receives a withdraw shock realization, generating a distribution of banks with reserve surpluses and reserve deficits. Banks participate in two interbank markets during this stage, one without collateral (the interbank market) and the other one with collateral (the FX swap market). First they interact in the interbank market, where banks with reserve surpluses post lending orders and banks with reserves deficit post borrowing orders. Second banks participate in the FX swap market, where all the unmatched lending orders are posted and the fraction of unmatched borrowing orders that the collateral can cover are posted.<sup>11</sup> Remember that the collateral in the FX swap market is the foreign currency reserves.

By assumption banks mass of orders is equal to its reserve surplus (or deficit), additionally it is assumed that banks pay (do not pay) for the new deposits transferred (for the withdrawn deposits) hence reserves increases (decreases) in  $\frac{(1+i_{t+1}^d)}{1+i_{t+1}^{ior}}$  for each unit of new deposit that is transferred (withdrawn). The surplus in local currency  $s(\omega)$  is:

$$s(\omega) = \tilde{m} + \frac{(1+i_{t+1}^d)}{1+i_{t+1}^{ior}} \omega \tilde{d} - (1+\omega)\rho \hat{d}$$

<sup>&</sup>lt;sup>11</sup>Orders in the interbank market and FX swap market are of infinitesimal size.

while the surplus in foreign reserves  $s^*$  during the balancing stage is given by

$$s^* = \tilde{m}^* - \rho^* \hat{d}^*.$$

FX swap and Interbank market allocations are determined by a matching process. Define  $\Psi^+$  as the probability that a lending order is matched in the interbank market,  $\Psi^-$  as the probability that a borrowing order is matched in the interbank market,  $\Psi^{+*}$  as the probability that a lending order is matched in the FX swap market and  $\Psi^{-*}$  as the probability that a borrowing order is matched in the FX swap market. In the next section functional forms for these expressions are introduced, as expected these will be functions of the market tightness.

In the balancing stage expressions for f,  $f^*$  and w (interbank market loans, FX swap market loans and discount window loans) are obtained. Consider a bank with reserve surpluses after the withdraw shock realization  $(s(\omega) > 0)$ , this bank will post its surpluses of reserves in local currency as long as it gets a payment higher than  $i^{ior}$ . As mentioned above in this framework banks first post orders in the interbank market and then in the FX swap market, hence a bank a fractions  $\Psi^+$  of its posted lending orders will find a match. The remaining orders, or in other words the unmatched orders,  $(1 - \Psi^+)s(\omega)$ , will be posted in the FX swap market where a fraction  $\Psi^{+*}$  will find a match and the rest,  $(1 - \Psi^{+*})(1 - \Psi^+)s(\omega)$  will not.

A bank with reserve deficit will post borrowing orders as long as it pays less than  $i^{DW}$ , in this case first it posts borrowing orders in the interbank market where a fraction  $\psi^-$  of them is matched and  $(1-\psi^-)$  not. Ideally the bank would like to post all the remaining unmatched orders  $-(1-\psi^-)s(\omega)$  in the FX swap market, but the maximum total value of orders that can post is the value of the surpluses in foreign currency,  $x_t s(\omega)$ . If the amount needed to fulfil the local reserve requirement,  $-(1-\Psi^-)s(\omega)$ , is less than  $x_t s(\omega)$  all the unmatched orders in the interbank market are posted in the FX swap market, if this is not the case then banks post orders for a total value of  $x_t s^*$ .

Masses of lending and borrowing orders in the interbank and FX swap market are necessary to obtain the market tightness in both markets and ultimately  $\{\Psi^+, \Psi^-, \Psi^{+*}, \Psi^{-*}\}$ .

Define  $M^+$  and  $M^-$  as the lending and borrowing orders masses in the interbank market and  $M^{+*}$  and  $M^{-*}$  as the lending and borrowing orders masses in the FX swap market. These are:

$$M^{-} = -\int min(0, s(\omega))dj,$$

$$M^{+} = \int max(0, s(\omega))dj,$$

$$M^{-*} = \int 1_{s<0}min(-(1 - \psi^{-})s(\omega), x_{t}s^{*})dj \text{ and}$$

$$M^{+*} = \int max(0, (1 - \psi^{+})s(\omega))dj = (1 - \psi^{+})M^{+}.$$

Market tightness in both markets is defined as  $\theta = \frac{M^+}{M^-}$  for the interbank market and  $\theta^* = \frac{M^{+*}}{M^{-*}}$  for the FX swap market. These variables measure how much lending orders are in compare with borrowing orders. As it will be showed in the next session  $\Psi^+$  and  $\Psi^{+*}$  are weakly decreasing in  $\theta$  and  $\theta^*$  respectively, while  $\Psi^-$  and  $\Psi^{-*}$  are weakly increasing in  $\theta$  and  $\theta^*$  respectively<sup>12</sup>.

After participating in both markets the unmatched lending orders will be kept as reserves and gain the interest over reserves  $i^{ior}$ . In the other hand, in order to fulfil the reserve requirement in local currency banks with reserve deficit will take discount window loans w for the value of the unmatched borrowing orders at the interest rate  $i^{dw}$ . The balancing stage value function is given by

 $<sup>^{12}</sup>$ Note that when a banks increases its foreign currency reserves holdings generates a negative externality to other borrowers since it post more orders in the FX swap markets which reduces the FX swap market tightness and hence weakly increases  $\Psi^{+*}$  and weakly reduces  $\Psi^{-*}$ 

$$V^b(\tilde{b},\tilde{m},\tilde{d},\tilde{b}^*,\tilde{m}^*,\tilde{d}^*,\omega) = \beta V^l(\hat{b},\hat{m},\hat{d},\hat{b}^*,\hat{m}^*,\hat{d}^*,\hat{f},\hat{f}^*,\hat{w})$$

$$\hat{b} = \tilde{b}, \qquad \qquad \text{(Evolution of loans in local currency)}$$

$$\hat{d}^* = \tilde{b}^*, \qquad \qquad \text{(Evolution of loans in foreign currency)}$$

$$\hat{d}^* = \tilde{d}^*, \qquad \qquad \text{(Evolution of deposits in local currency)}$$

$$\hat{d}^* = \tilde{d}^*, \qquad \qquad \text{(Evolution of deposits in foreign currency)}$$

$$\hat{m}^* \geq \rho^* \tilde{d}^*, \qquad \qquad \text{(Reserve requirement of foreign currency)}$$

$$s = \tilde{m} + \frac{(1+ii_{t+1}^d)}{1+ii_{t+1}^{corr}} \omega \tilde{d} - (1+\omega)\rho \hat{d}, \qquad \qquad \text{Surplus of local currency)}$$

$$s = \tilde{m} + \frac{(1+ii_{t+1}^d)}{1+ii_{t+1}^{corr}} \omega \tilde{d} - (1+\omega)\rho \hat{d}, \qquad \qquad \text{Surplus of foreign reserves}$$

$$s^* = \tilde{m}^* - \rho^* \tilde{d}^*, \qquad \qquad \text{(Surplus of foreign reserves)}$$

$$\hat{f} = \begin{cases} -s\Psi_t^- & \text{if } s < 0 \\ -s\Psi_t^+ & \text{if } s \geq 0, \end{cases} \qquad \qquad \text{(Interbank loans)}$$

$$-s\Psi_t^{+*}(1-\Psi^+) & \text{if } s \geq 0, \end{cases}$$

$$\hat{w} = \begin{cases} -s(1-\Psi_t^-)(1-\min\{1,\frac{-s^*x_t}{-s(1-\Psi^-)}\}\Psi^{-*}) & \text{if } s < 0 \\ 0 & \text{if } s \geq 0, \end{cases}$$

$$\hat{m} = \tilde{m} + \frac{1+ii_{t+1}^d}{1+ii_{t+1}^d} \omega \tilde{d} + \hat{f} + \hat{f}^* + \hat{w} \text{ and} \qquad \text{(Evolution of loans in local currency)}$$

(Evolution of reserves in foreign currency)

Note that loans in both currencies are constant during the balancing stage. Additionally, since there is not withdraw shocks in foreign currency deposits, reserves in foreign currency are held constant.

 $\hat{m}^* = \tilde{m}^*$ .

#### 3.1.3 Markets clearing

There are 10 markets to be cleared: loan markets in both currencies, deposit markets in both currencies, reserves markets in both currencies, the exchange rate market, the interbank market, the FX swap market and the discount window market. Banks are

index by the  $z \in [0,1]$ . The market clearing conditions are:

$$\int_0^1 b_t^z dz + B_t^{cb} = B_t^d \qquad \qquad \text{(Loan in local currency market clearing)}$$
 
$$\int_0^1 b_t^{z*} dz = B_t^{d*} \qquad \qquad \text{(Loan in foreign currency market clearing)}$$
 
$$\int_0^1 d_t^{z*} dz = D_t^s + D_t^{cb} \qquad \text{(Deposits in local currency market clearing)}$$
 
$$\int_0^1 d_t^{z*} dz = D_t^{s*} \qquad \text{(Deposits in foreign currency market clearing)}$$
 
$$\int_0^1 m_t^z dz = M_t^{CB} \qquad \text{(Reserves in local currency market clearing)}$$
 
$$\int_0^1 FX_t^z dz + FX_t^{CB} = 0 \qquad \text{(Exchange rate market clearing)}$$
 
$$\int_0^1 f_t^z dz = 0 \qquad \text{(Exchange rate market clearing)}$$
 
$$\int_0^1 f_t^{z*} dz = 0 \qquad \text{(Exchange rate market clearing)}$$
 
$$\int_0^1 w_t^z dz = W_{t+1}^{CB} \qquad \text{(Discount window market clearing)}$$

Where  $B_t^d$ ,  $B_t^{d*}$ ,  $D_t^s$  and  $D_t^{s*}$  are the aggregate demand for loans in local, the aggregate demand for loans in foreign currency, the aggregate supply for deposits in local currency and the aggregate supply for deposits in foreign currency. By assumption the former two are downward sloping and the latter two the upward sloping. Functional form for these 4 variables are introduced later.

The Central Bank supply loans in local currency  $B^{cb}$ , supply deposits in local currency to banks  $(D^{CB})$  and choose the aggregate level of reserves in local currency  $M^{cb}$ . By assumption all reserves in foreign currency are kept in the local central bank, its total amount is  $M^{*CB}$ . Banks offers discount window loans  $W^{CB}$  and can participate in the exchange rate market choosing a position in foreign currency  $(FX^{CB})$ . More details about the local central policy are discussed later.

Note that the exchange rate market clearing condition assumes that the only participants in the exchange rate market are banks and the Central Bank. A net demand for foreign currency of agents outside the banking system can be included as in Bruno and Shin (2015a).

# 4 Model Solution

## 4.1 Solving Bank's Problem

In this section I solve the bank's problem and explain how to get tractable solutions. To this end I introduce the liquidity yield function which maps the reserve surplus in foreign currency  $s^*$  and the reserve surplus in local currency after the withdraw shock realization  $s(\omega)$  to a real value. Define  $\chi_t^{-*}$  as the mean cost of having reserve deficit when the bank has a partial access to the FX swap market and  $\chi_t^-$  as the man cost of having a reserve deficit when there is complete access to the FX swap market, the former happens when the bank does not have enough collateral, that is reserves surplus in foreign currency, to post all its remaining reserve deficit after participating in the interbank market while the later occurs when collateral is sufficient. Finally, define  $\chi_t^+$  as the mean benefit of having a reserve surplus. The expressions for these three variables are:

$$\begin{split} \chi_t^{-*} &= \Psi_t^-(i^f - i^{ior}) + \frac{s*x_t}{-s(\omega)(1 - \Psi^-)} \Psi_t^{-*}(1 - \Psi_t^-)[i^{ex} - i^{ior}] + (1 - \Psi_t^-)(1 - \frac{s*x_t}{-s(\omega)(1 - \Psi^-)} \Psi_t^{-*})[i^{dw} - i^{ior}], \\ \chi_t^{-**} &= \Psi_t^-(i^f - i^{ior}) + \Psi_t^{-*}(1 - \Psi_t^-)[i^{ex} - i^{ior}] + (1 - \Psi_t^-)(1 - \Psi_t^{-*})[i^{dw} - i^{ior}] \text{ and} \\ \chi_t^+ &= \Psi_t^+(i^f - i^{ior}) + \Psi_t^{+*}(1 - \Psi_t^+)[i^{ex} - i^{ior}]. \end{split}$$

Note that the mean cost of having full access to the FX swap market is lower than the mean cost of having partial to the FX swap market  $(\chi_t^{-**} \leq \chi_t^{-*})$  as long as  $i^{dw} > i^{ex}$  which is the case as it will be showed later, and the cost of having a reserve deficit and partial access to the FX swap market  $(\chi_t^{-*})$  is decreasing in the surplus of foreign currency reserves  $s^*$ . The later arise because of two reasons. A higher amount of reserve surplus in foreign currency allow banks to swap a higher amount of reserves in local currency for reserves in foreign currency in the FX swap market which reduces the discount window loans required to fulfil the reserve deficit in local currency, paying  $1 + i^{ex}$  instead of

 $1+i^{dw}$  for the swapped reserves. The second reason is that for a bank with reserve deficit a higher reserve surplus in foreign currency improves its outside option, improving its effective bargaining power, which reduces the interbank interest rate for all its posted borrowing orders<sup>13</sup>. The liquidity yield function is

$$\chi_t(s, s^*, \omega) = \begin{cases} s(\omega)\chi^{-*}(\omega) & \text{if } 0 \ge -s^*x_t > s(\omega)(1 - \Psi^-) \\ s(\omega)\chi^{-**} & \text{if } 0 > s(\omega)(1 - \Psi^-) \ge -s^*x_t \\ s(\omega)\chi^+ & \text{if } s(\omega) \ge 0. \end{cases}$$

The function changes is shape depending if the banks has a reserve surplus in local currency or a reserve deficit in local currency and, in the later case, if the bank has a partial access or complete access to the FX swap market. Remember that a fraction  $1 - \Psi^-$  of borrowing orders will not find a match in the interbank market. If the value of these unmatched interbank market orders  $-(1 - \Psi^-)s(\omega)$  is greater than the reserve surplus in foreign currency  $s^*x_t$  then the bank has a partial access to the FX swap market and has a liquidity cost of  $\chi^{-*}s(\omega)$ , if the contrary results,  $-(1 - \Psi^-)s(\omega) < s^*x_t$ , then the bank has a liquidity cost of  $\chi^-s(\omega)$ . Lastly if the bank has a reserve surplus it has a gain of  $s(\omega)\chi^+$ . The liquidity yield can be written in a compact way as

$$\begin{split} \chi_t^+ &= \Psi_t^+(i^f - i^{ior}) + \Psi_t^{+*}(1 - \Psi_t^+)(i^{ex} - i^{ior}) \\ \chi_t^- &= \Psi_t^-(i^f - i^{ior}) + \min[1, \frac{s*x_t}{-s(\omega)(1 - \Psi^-)}] \Psi_t^{-*}(1 - \Psi_t^-)[i^{ex} - i^{ior}] \\ &+ (1 - \Psi_t^-)(1 - \min[1, \frac{s*x_t}{-s(\omega)(1 - \Psi^-)}] \Psi_t^{-*})[i^{dw} - i^{ior}] \\ \chi(s, s^*, \omega)_t &= \begin{cases} s(\omega)\chi^- & \text{if } s(\omega) < 0 \\ s(\omega)\chi^+ & \text{if } s(\omega) \geq 0. \end{cases} \end{split}$$

The liquidity function has two kinks. The first one is in  $s(\omega) = 0$  and arrives because the average benefit of having reserve surpluses (which is also the marginal benefit of having a reserve surplus) is different from the average cost of having reserve deficits (which is also the marginal cost of having a reserve deficit). The second kink is in  $(1-\Psi^-)s(\omega) = -s*x_t$  and appears because the marginal benefit of having a reserve surpluses in foreign currency

<sup>&</sup>lt;sup>13</sup>I am not making explicit this point in the liquidity yield function to save in notation. Incorporating this implies substituting  $i^{ex}(s^*)$  for  $i^{ex}$  in  $\chi_t^{-*}$  and  $\chi_t^{-}$  and  $\mathbb{E}(i^{ex})$  for  $i^{ex}$  in  $\chi_t^{+}$ . The second substitution arrives because the lending orders are of infinitesimal size

is positive when the bank has a partial access to the FX swap market and zero when the bank has full access to the FX swap market.

During the balancing stage there is no decision making and hence the two stage problem can be simplify into a one stage problem. This problem can also be rewritten as a maximization problem with two state variables: the total real equity after tax payments and the fraction of real equity that comes from financial intermediation in foreign currency.

Redefine e as the after tax real equity in local currency and  $e^*$  as the after tax real equity in foreign currency hence the after tax total real equity is  $e^T = e + e^*$ . Assume that taxes T and  $T^*$  are proportional taxes over real equity in local currency and real equity in foreign currency respectively (that is  $T = \tau e$  and  $T^* = \tau e^*$ ), then

$$e_t^l = \frac{1-\tau}{P_t} \left[ (1+i_t^b)b_t + (1+i_t^{ior})m_t - (1+i_t^d)d_t - (1+i_t^f)f_t - (1+i_t^{ex})f_t^* - (1+i^{dw})w_t \right] \text{ and}$$

$$e_t^* = \frac{x_t(1-\tau)}{P_t} \left[ (1+i_t^{b*})b_t^* + (1+i_t^{ior*})m_t^* - (1+i_t^{d*})d_t^* \right].$$

Moving forward a period and replacing  $f, f^*$  and w, the after taxes real equity in local currency and the after taxes real equity in foreign currency are

$$e_{t+1}^{l} = \frac{1-\tau}{P_{t+1}} \left[ (1+i_{t+1}^{b}) \tilde{b_{t+1}} + (1+i_{t+1}^{ior}) \tilde{m_{t+1}} - (1+i_{t+1}^{d}) \tilde{d_{t+1}} + \chi_{t+1}(s_{t}, s_{t}^{*}, \omega) \right] \text{ and }$$

$$e_{t+1}^{*} = \frac{x_{t+1}(1-\tau)}{P_{t+1}} \left[ (1+i_{t+1}^{b*}) \tilde{b_{t+1}} + (1+i_{t+1}^{ior*}) \tilde{m_{t+1}} - (1+i_{t+1}^{d*}) \tilde{d_{t+1}} \right].$$

Define gross real return of loans, reserves and deposits in local currency as:

$$R_t^b = \frac{(1+i_{t+1}^b)}{(1+\pi_{t+1})}, R_t^m = \frac{(1+i_{t+1}^{ior})}{(1+\pi_{t+1})} \text{ and } R_t^d = \frac{(1+i_{t+1}^d)}{(1+\pi_{t+1})}$$

Where  $\pi_{t+1} = P_{t+1}/P_t$  is the inflation rate. Additionally define the gross real return of loans, reserves and deposits in foreign currency as

$$R_t^{b*} = \frac{(1 + \Delta x_{t+1})(1 + i_{t+1}^{b*})}{(1 + \pi_{t+1})}, R_t^{m*} = \frac{(1 + \Delta x_{t+1})(1 + i_{t+1}^{ior*})}{(1 + \pi_{t+1})} \text{ and } R_t^{d*} = \frac{(1 + \Delta x_{t+1})(1 + i_{t+1}^{d*})}{(1 + \pi_{t+1})}$$

Where  $\Delta x_{t+1} = x_{t+1}/x_t$  is the depreciation rate. Substituting these definitions into the one stage bank problem with two state variable it is obtained the following maximization problem:

$$\begin{split} V_{t}(e^{l},e^{*}) &= \max_{\{\tilde{b},\tilde{m},\tilde{d},\tilde{b}^{*},\tilde{m}^{*},\tilde{d}^{*},c,\tilde{F}\tilde{X}\}} u(c) \\ \text{s.t.} \\ \frac{\tilde{b}}{P_{t}} + \frac{\tilde{m}}{P_{t}} - \frac{\tilde{d}}{P_{t}} + c + \frac{x_{t}\tilde{F}\tilde{X}}{P_{t}} = e^{l}, \\ \frac{x_{t}\tilde{b}^{*}}{P_{t}} + \frac{x_{t}\tilde{m}^{*}}{P_{t}} - \frac{x_{t}\tilde{d}^{*}}{P_{t}} - \frac{x_{t}\tilde{F}\tilde{X}}{P_{t}} = e^{*}, \\ \hat{e}^{l} &= \left[ R_{t}^{b}\frac{\tilde{b}}{P_{t}} + R_{t}^{m}\frac{\tilde{m}}{P_{t}} - R_{t}^{d}\frac{\tilde{d}}{P_{t}} + \chi(\frac{\tilde{m}}{P_{t}}, \frac{\tilde{d}}{P_{t}}, \frac{x_{t}\tilde{m}^{*}}{P_{t}}, \frac{x_{t}\tilde{d}^{*}}{P_{t}}) \right] (1 - \tau), \\ \hat{e}^{*} &= \left[ R_{t}^{b*}\frac{x_{t}\tilde{b}^{*}}{P_{t}} + R_{t}^{m*}\frac{x_{t}\tilde{m}^{*}}{P_{t}} - R_{t}^{d*}\frac{x_{t}\tilde{d}^{*}}{P_{t}} \right] (1 - \tau) \text{ and} \\ \tilde{d} + x_{t}\tilde{d}^{*} &\leq \kappa(\tilde{b} + x_{t}\tilde{b}^{*} - \tilde{d} - x_{t}\tilde{d}^{*} + \tilde{m}^{*} + x_{t}\tilde{m}^{*}). \end{split}$$

Using a guess and verify approach it can be demonstrated that the value function take the following functional form

$$V(e^{l}, e^{*}) = v_{t}(e^{*} + e^{l})^{1-\gamma} - \frac{1}{(1-\beta)(1-\gamma)}.$$

Define  $\bar{c} = c/(e^l + e^*)$  as the dividend equity ratio,  $\bar{l} = \tilde{l}/((e^l + e^*)(1 - \bar{c})P_t)$  as a local currency denominated variable in real terms as a fraction of the real equity after dividend payments,  $\bar{l}^* = l^*x_t/((e^l + e^*)(1 - \bar{c})P_t)$  as a foreign currency denominated variable in real terms as a fraction of the real equity after dividend payments and  $\bar{e}^* = e^*/[(e^l + e^*)(1 - \bar{c})]$  as the fraction of total real equity after dividend payments that comes from real equity in foreign currency. Additionally define  $\bar{\chi}_t(\bar{m}, \bar{d}, \bar{m}^*, \bar{d}^*) = \frac{\chi_t(\bar{m}, \bar{d}, \bar{m}^*, \bar{d}^*)}{1 + \pi_{t+1}}$ . Replacing the value

function and using the normalizations showed above the problem can be written as

$$v_{t} = \max_{\{\bar{c}, \bar{b}, \bar{m}, \bar{d}, \bar{b}^{*}, \bar{m}^{*}, \bar{d}^{*}, \bar{F}X\}} u(\bar{c}) + \beta (1 - \bar{c})^{1 - \gamma} v_{t+1} \mathbb{E}_{\omega} [[(1 - \tau) R_{t}^{E}(\bar{b}, \bar{m}, \bar{d}, \bar{b}^{*}, \bar{m}^{*}, \bar{d}^{*})]^{1 - \gamma}],$$

$$\text{s.t.}$$

$$1 = \bar{b} + \bar{m} - \bar{d} + \bar{b}^{*} + \bar{m}^{*} - \bar{d}^{*},$$

$$F\bar{X} = \bar{b}^{*} + \bar{m}^{*} - \bar{d}^{*} - \bar{e}^{*} \text{ and}$$

$$\bar{d} + \bar{d}^{*} \le \kappa(\bar{b} + \bar{b}^{*} - \bar{d} - \bar{d}^{*} + \bar{m} + \bar{m}^{*}).$$

Where equity return  $(R_t^E)$  is

$$R_t^E = R_t^b \bar{b} + R_t^m \bar{m} - R_t^d \bar{d} - R_t^b \bar{b}^* + R_t^m \bar{m}^* - R_t^d \bar{d}^* + \bar{\chi}(\bar{m}, \bar{d}, \bar{m}^*, \bar{d}^*).$$

Dividend policy is characterized by the first order condition

$$\bar{c} = \frac{1}{1 + (\beta(1 - \gamma)v_{t+1}\Omega^{1-\gamma})^{\frac{1}{\gamma}}}.$$

The problem can be split into a dividend policy problem, the choice over  $\bar{c}$ , and a portfolio problem, the choice over  $\{\bar{b}, \bar{b}^*, \bar{m}, \bar{m}^*, \bar{d}, \bar{d}^*\}$ . The portfolio problem consist on maximizing the certainty equivalent return of total real equity subject to three constraints: i) a constraint over the sum of portfolio weights, ii) the bank position that arise from the portfolio choices and iii) a leverage constraint that emerges from the capital requirement constraint. The portfolio problem is

$$\Omega_{t} = \max_{\{\bar{c}, \bar{b}, \bar{m}, \bar{d}, \bar{b}^{*}, \bar{m}^{*}, \bar{d}^{*}, \bar{F}X\}} (1 - \tau) \left\{ \mathbb{E}_{\omega} [(R_{t}^{b}\bar{b} + R_{t}^{m}\bar{m} - R_{t}^{d}\bar{d} + R_{t}^{b}\bar{b}^{*} + R_{t}^{m}\bar{m}^{*} - R_{t}^{d}\bar{d}^{*} + \bar{\chi}^{*}(\bar{m}, \bar{d}, \bar{m}^{*}, \bar{d}^{*})]^{1 - \gamma} \right\}^{\frac{1}{1 - \gamma}}$$

$$\text{s.t.}$$

$$1 = \bar{b} + \bar{m} - \bar{d} + \bar{b}^{*} + \bar{m}^{*} - \bar{d}^{*},$$

$$F\bar{X} = \bar{b}^{*} + \bar{m}^{*} - \bar{d}^{*} - \bar{e}^{*} \text{ and}$$

$$\bar{d} + \bar{d}^{*} \leq \kappa(\bar{b} + \bar{b}^{*} - \bar{d} - \bar{d}^{*} + \bar{m} + \bar{m}^{*}).$$

Where certainty equivalent return of total equity,  $\Omega_t$  is obtained by replacing  $\Omega_t^{1-\gamma}$  by  $\mathbb{E}_{\omega}[(R^E)^{1-\gamma}]$  in the bank's maximization problem. Finally,  $v_t$  is characterized by the envelope condition

$$v_{t} = \frac{1 - \gamma}{\bar{c}^{-\gamma}} = \frac{1}{1 + \gamma} [1 + (\beta(1 - \gamma)v_{t+1}\Omega_{t}^{1-\gamma})^{\frac{1}{\gamma}}]^{\gamma}.$$

Notice that policy functions  $\{\tilde{b}_{t+1}, \tilde{b}_{t+1}^*, \tilde{m}_{t+1}, \tilde{m}_{t+1}^*, \tilde{d}_{t+1}, \tilde{d}_{t+1}^*\}$  are linear in total equity in the following way

$$\tilde{b}_{t+1} = \bar{b}[e^l + e^*](1 - \bar{c})P_t,$$

$$\tilde{m}_{t+1} = \bar{m}[e^l + e^*](1 - \bar{c})P_t,$$

$$\tilde{d}_{t+1} = \bar{d}[e^l + e^*](1 - \bar{c})P_t,$$

$$\tilde{b}_{t+1}^* = \bar{b}^*[e^l + e^*](1 - \bar{c})P_t/x_t,$$

$$\tilde{m}_{t+1}^* = \bar{m}^*[e^l + e^*](1 - \bar{c})P_t/x_t \text{ and }$$

$$\tilde{d}_{t+1}^* = \bar{d}^*[e^l + e^*](1 - \bar{c})P_t/x_t.$$

$$FX = \bar{F}X[e^l + e^*](1 - \bar{c})P_t/x_t$$

Adding up for all banks the real supply for loans denominated in local and foreign currency and the real demand for deposits denominated in local and foreign currency are obtained

$$\tilde{B}_{t+1} = \int_0^1 \tilde{b}_{t+1}/P_t dz = \int_0^1 \bar{b}[e^l + e^*](1 - \bar{c})dz,$$

$$\tilde{D}_{t+1} = \int_0^1 \tilde{d}_{t+1}/P_t dz = \int_0^1 \bar{d}[e^l + e^*](1 - \bar{c})dz,$$

$$\tilde{B}_{t+1}^* = \int_0^1 \tilde{b}_{t+1}^* x_t/P_t dz = \int_0^1 \bar{b}^*[e^l + e^*](1 - \bar{c})dz \text{ and }$$

$$\tilde{D}_{t+1}^* = \int_0^1 \tilde{d}_{t+1}^* x_t/P_t dz = \int_0^1 \bar{d}^*[e^l + e^*](1 - \bar{c})dz.$$

Real demand for loans in both currencies and real supply for deposits in both currencies

are assumed to have a constant elasticity functional form.

$$\begin{split} B^d_{t+1}/P_t &= \Theta^b \bigg[ \frac{1+i^b_{t+1}}{1+\pi} \bigg]^{-\epsilon} \ , \ \epsilon > 0, \ \Theta^b > 0 \\ D^s_{t+1}/P_t &= \Theta^d \bigg[ \frac{1+i^d_{t+1}}{1+\pi} \bigg]^{\varsigma} \ , \ \varsigma > 0, \ \Theta^d > 0 \\ B^{d*}_{t+1}x_t/P_t &= \Theta^{b*} \bigg[ \frac{(1+i^{b*}_{t+1})(1+\triangle x)}{1+\pi} \bigg]^{-\epsilon^*} \ , \ \epsilon^* > 0, \ \Theta^{b*} > 0 \\ D^{s*}_{t+1}x_t/P_t &= \Theta^{d*} \bigg[ \frac{(1+i^{d*}_{t+1})(1+\triangle x)}{1+\pi} \bigg]^{\varsigma^*} \ , \ \varsigma^* > 0, \ \Theta^{d*} > 0 \end{split}$$

These four equations pin down the returns  $\{R^{b*}, R^{b*}, R^{d*}, R^d\}$ . The Central bank chooses the aggregate level of reserves in local currency  $M^{CB}$  and doing this sets the price level.

$$M_t^{CB} = \int_0^1 \bar{m}[e^l + e^*](1 - \bar{c})P_t dz$$

The nominal exchange rate change  $(\Delta x)$  is obtained by using the first order conditions in the portfolio problem and the exchange rate market clearing condition. I will discuss details about this point in next sections. Assume that the central bank have no intervention in the exchange rate market hence since banks are the only participants in the exchange rate market then the aggregate foreign currency position must be zero, then the exchange rate market clearing condition is

$$\bar{b}^* + \bar{m}^* = \bar{d}^* + \frac{e^*}{e^* + e^l}$$

Finally, observe that the aggregate total real equity and the fraction of it that comes from foreign currency equity are the only aggregate state variables needed to solve the system of equations. Define  $ET = \int_0^1 e^{T,z}dz$  as the aggregate total real equity after taxes and  $E^* = \int_0^1 e^{*,z}dz$  as the real aggregate equity in foreign currency after taxes. These evolve according to

$$E_{t+1}^{T} = (1-\tau)[(1-c)((R_{t}^{b}\bar{b} + R_{t}^{m}\bar{m} - R_{t}^{d}\bar{d}) + (R^{b*}\bar{b}^{*} + R^{m*}\bar{m}^{*} - R^{d*}\bar{d}^{*}))(1-c)E_{t}^{T} - (i^{dw} - i^{ior})\frac{W_{t+1}}{P_{t+1}}]$$

$$E_{t+1}^{*} = (1-\tau)(R^{b*}\bar{b}^{*} + R^{m*}\bar{m}^{*} - R^{d*}\bar{d}^{*})(1-c)E_{t}^{*}P_{t}/x_{t}.$$

### 4.2 Local Central Bank

The local Central Bank chooses its monetary policy and exchange rate regimes. To maintain its monetary policy goal it issues reserves and by doing this pin downs the price level and hence the inflation rate. To keep a exchange rate regime in which the exchange rate is controlled it has to choose a foreign currency position each period. The Central Bank budget has two budget constraints. The budget constraint in local currency is:

$$B_{t+1}^{CB} + D_{t+1}^{CB} + W_{t+1}^{CB} + FX_{t+1}^{CB}x_{t+1} - M_{t+1}^{CB} =$$

$$(1+i_t^b)B_t^{CB} + (1+i_t^d)D_t^{CB} + (1+i_t^{dw})W_t^{CB} - (1+i_t^{ior})M_t^{CB} + P_tT_t \ .$$

Banks start each period with gains/looses (Right hand side of Central Bank budget constraint). The Central Bank can perform Open Market Operations by supplying  $B_{t+1}^{CB}$ . Additionally it can supply deposits  $D^{CB}$ . This is similar to issue Central Bank paper. To obtain funding for these operations the central bank issue reserves  $M^{CB}$ . Additionally the Central Bank accomplishes its Lender of last Resort role by providing discount window loans  $W^{CB}$ . I assume that Central Bank choose the taxes  $T_t$ , which are used to balance the budget constraint.

Banks can also choose the exchange rate level by changing their position in foreign currency  $FX^{CB}$ . A flexible exchange rate regime is define as one in which the Central Bank chooses  $FX^{CB} = 0$ . Since I am assuming that only banks and the Central Bank can participate in the exchange rate market if the Central Bank does not intervene in the exchange rate market the aggregate position of the banking sector is zero. In this case the change in the nominal exchange rate is obtained by using the first order conditions of the portfolio problem and clearing the exchange rate market  $(\bar{b}^* + \bar{m}^* - \bar{d}^* - e^*/(e^l + e^*) = 0)$ . In this case the exchange rate level is indeterminate however the exchange rate change is determine. In a fixed exchange rate policy a bank chooses  $FX^{CB}$  such that:

$$(\bar{b}^* + \bar{m}^* - \bar{d}^* - \frac{E^*}{E^T})(1 - c)E^T + FX^{CB}x_t/P_t = 0.$$

If the banking sector has a long position in foreign currency,  $((\bar{b}^* + \bar{m}^* - \bar{d}^* - \frac{E^*}{E^T})(1-c)E^T > 0)$  the Central Bank have to react choosing a short position in foreign currency  $FX^{CB} < 0$  such that

$$x^{CB} = \left(\frac{(\bar{b}^* + \bar{m}^* - \bar{d}^* - e^*/(e^l + e^*))}{-FX^{CB}}\right) (P_t(1 - c)E^T).$$

Where  $x^{CB}$  is the targeted level of nominal exchange rate chosen by the Central Bank. Remember that  $P_t$  is obtain clearing the local reserve market. Replacing  $P_t$ .

$$x^{CB} = (\frac{(\bar{b}^* + \bar{m}^* - \bar{d}^* - e^*/(e^l + e^*))}{-FX^{CB}})(M^{CB}/\bar{m}).$$

Note that the capacity of monetary policy to target an inflation rate now is limited. To understand why lets define the budget constraint of the Central Bank in foreign currency:

$$M_{t+1}^{*,I} - FX_{t+1}^{CB} - M_{t+1}^* = M_t^{*,I} (1 + i^{ior*_t}) - (1 + i_t^{*,ior}) M_t^* + P_t T_t^{CB*} / x_t.$$
 (1)

Where  $M^{*,I}$  are international reserves and can be considered low maturity treasure-bill bonds, deposits in foreign banks or deposits in the FED. For simplicity I am assuming that local banks put their reserve in foreign reserve holdings in the Local Central Bank and are remunerated by the interest over reserves in foreign currency  $i^{*,ior}$ . Additionally I assume that  $i^{*,ior}$  is equal to the international reserves interest rate. The local Central Bank can not determine the aggregate level of foreign currency reserves as it can with local currency reserves. In order to keep a fixed exchange rate when FX > 0 the Central Bank has to choose  $FX^{CB} < 0$ , but this later is limited by the reserves in foreign currency that the Central Bank has and its initial gain/looses. Hence Central Bank have to, additionally of choosing  $FX^{CB} < 0$ , reduce the aggregate level of reserves  $M^{CB}$  with an equal change in  $D^{CB}$  or  $B^{CB}$  to keep the budged constraint fulfilled. On the other hand, when FX < 0 the local central Bank has to choose  $FX^{CB} > 0$  which can be done by reducing  $D^{CB}$  or  $B^{CB}$ . Nevertheless when  $D^{CB} = 0$  or  $B^{CB} = 0$  there is no more space for increasing FX unless  $M^{CB}$  is increased. Since in both cases large demands of foreign currency or local currency required changes in  $M^{CB}$  the capacity of the central bank to serve its inflation targeting regime is limited.

#### 4.3 Interbank Market and FX Market

In this section Interbank and FX swap market dynamic is explained with deeper analysis and expressions for  $\{\psi^{+*}, \psi^{-*}, \psi^+, \psi^-, i^f, i^{ex}\}$  are derived.

At the beginning of the balancing stage the withdraw shock is executed producing a distribution of banks with reserve surpluses and reserve deficits. Banks during this stage exchange liquid assets (reserves) in an interbank market with no collateral (referred as the interbank market) and in an interbank market with foreign currency as collateral (the FX swap market). Banks with reserve surplus have incentives to lend their excess of reserve as long as its interest payments, on the interbank or FX swap market ( $i^f$  and  $i^{ex}$  respectively), are higher that the interest rate pay by the Central Bank over reserves  $(i^{ior})$ . On the other side, banks with reserve deficit have incentives to borrow interbank and FX swap market loans as long as their interest payments ( $i^f$  and  $i^{ex}$  respectively) are smaller than the discount window interest rate  $(i^{dw})$ . Interbank Market and FX swap markets are over the counter with orders of infinitesimal size, hence banks split their surpluses and deficits of reserves into fixed size orders  $\triangle$  and post these in the Interbank and FX swap market. Then the orders are matched following a matching process and it is assumed that the interest rate in every match is determined by Nash Bargaining. The Nash Bargaining problem will be first modelled for a fixed size orders  $\Delta > 0$  and then evaluate it when the size of the order converge to zero ( $\triangle \searrow 0$ ).

To simplify the analysis it is assumed that the interbank market opens and closes before the FX swap market opens. This is as if the period time was daily and the interbank market operated during the morning, the FX swap market during the afternoon and banks had to finish each day with a fulfilled reserve requirement.

It can be showed that under the assumptions mentioned before it is optimal for borrowers and lenders to post all theirs orders in the interbank market. After their participation in the interbank market lenders can choose to either post their unmatched lending orders in the FX swap market or keep them as reserves while borrowers can either participate in the FX swap, posting all the unmatched orders that they can, or take a discount window loan. Remember that for a borrowing bank its participation in the FX swap market

is limited by its collateral value, which is the value of its surplus in foreign currency  $(s^*x_t)^{14}$ 

The analysis that I will develop next follows Bianchi and Biguio (2017), Bianchi and Biguio (2014) and Arce et al. (2017). Bianchi and Biguio (2014) build up a one round OTC interbank market with no collateral and orders of infinitesimal size while Bianchi and Biguio (2017) use the same framework and assumes infinite rounds. Arce et al. (2017) develop a three round OTC interbank market with no collateral and use a "veto assumption" which means that if not all matched orders agree a new round is added. They also include a probability of participating in a new round. The framework in this paper considers 2 rounds in where in the first one the interbank market operates and in the second one the FX swap market operates.

I use similar expressions for the matching probabilities than the ones in Bianchi and Biguio (2017)

$$\Psi^+ = \lambda \min\{1, \theta^{-1}\}, \ \Psi^- = \lambda \min\{1, \theta\}, \ \Psi^{+*} = \lambda^* \min\{1, (\theta^*)^{-1}\} \text{ and } \Psi^{-*} = \lambda^* \min\{1, \theta^*\}.$$

Where  $\lambda$  and  $\lambda^*$  measure the efficiency in the matching process in the Interbank and FX swap market respectively and both take values between 0 and 1 ( $\{\lambda, \lambda^*\} \in (0, 1)$ ). Additionally  $\theta$  and  $\theta^*$  are the market tightness in the Interbank and FX swap markets.

The interbank and FX swap interest rates are obtained by Nash Bargaining. First the Nash bargaining problem in the FX swap market is going to be constructed and then the one in the interbank market. Define  $\xi$  as the lender market power. The outside option value of a matched lending order in the FX swap market is the value function in the lending stage with the lending order  $\Delta$  remunerated with the interest rate over reserves, this is  $V^l(e_1 + (1+i^{ior})\Delta)$  where  $e_1$  is the real equity after taxes minus the remunerated lending order, that without loss of generalization I am assuming that is the last posted

<sup>&</sup>lt;sup>14</sup>I am not introducing an interbank market in foreign currency, however this could be added and it will be in a next version of the paper. Note that borrowers that do not hold enough collateral in foreign currency have incentives to demand reserves in foreign currency. Banks with reserve surplus have incentives to lend it as long as the interest rate pay for them is higher than the interest rate over reserves in foreign currency. Hence a supply and demand for reserves in foreign currency can appear, therefore an interbank market in foreign currency can be introduced. Notice that the interbank market in foreign currency have to operate before than the FX swap market.

order. In the same token, the outside option value of a matched borrowing order in the FX swap market is the value function in the lending stage order with the borrowing order  $\triangle$  paying the discount window interest rate, this is  $V^l(e_1 - (1 + i^{dw})\triangle)$ . The expression for  $e_1^{15}$  is:

$$e_{1} = \frac{1-\tau}{P_{t}} \left[ (1+i_{t}^{b})b_{t} + (1+i_{t}^{ior})m_{t} - (1+i_{t}^{d})d_{t} - (1+i_{t}^{f})f_{t} + x_{t}((1+i_{t}^{b*})b_{t}^{*} + (1+i_{t}^{ior*})m_{t}^{*} - (1+i_{t}^{d*})d_{t}^{*}) + (1+i^{dw})(1-\Psi^{-*})s(\omega)(1-\min\{1,\frac{s^{*}x_{t}}{-(s(\omega)(1-\Psi^{-})})1_{s(\omega)<0} + \sum_{o=1}^{\eta(s,\triangle)-1}(1+i(o))\Delta \right].$$

Where  $\eta(s, \Delta)$  is the number of orders posted<sup>16</sup> and i(o) is the interest rate pay by others posted orders, for a lending order  $i(o) \in \{i^{ex}, i^{ior}\}$  and for a borrowing order  $i(o) \in \{i^{ex}, i^{dw}\}$ . Note that in the  $e_1$  expression the interbank market loans payments are included  $(1 + i_t^f)f_t$ , this is the case since the FX swap market opens after the interbank market, additionally the payments for the discount window loans carried by the unmatched interbank market orders that can not be posted in the FX swap market are also incorporated<sup>17</sup>  $((1 + i^{dw}))(1 - \Psi^{-*})s(\omega)(1 - \min\{1, \frac{s^*x_t}{-(s(\omega)(1-\Psi^{-*}))}\})$ .

The value of both agreeing in the Nash Bargaining is  $V^l(e_1 + (1+i^{ex})\Delta)$  for the lender and  $V^l(e_1 - (1+i^{ex})\Delta)$  for the borrower, these two can be written as

$$V^l(e_1 + (i^{ex} - i^{ior})\triangle + (1 + i^{ior})\triangle)$$
and  
 $V^l(e_1 - (i^{ex} - i^{dw})\triangle - (1 + i^{dw})\triangle).$ 

respectively.

To be more precise the lending (or borrowing) order  $\triangle$  should be multiply by  $\frac{1-\tau}{P_t}$  in the value function of the lending stage expression  $V^l(e_1 + \frac{1-\tau}{P_t}(1+i^{ior})\triangle)$  ( $V^l(e_1 - \frac{1-\tau}{P_t}(1+i^{dw})\triangle)$ ). I am ignoring this fact to save in notation. Moreover, since the order size will converge to zero, multiplying it by  $\frac{1-\tau}{P_t}$  will not change the results. The order size  $\triangle$  can be also interpreted as a the real order size after taxes.

<sup>&</sup>lt;sup>16</sup>I am ignoring that  $\eta(s, \Delta)$  might not be an integer. Since the order size will converged to zero making explicit this point is not relevant. To consider this point the residual  $(s(\omega) - \Delta \eta(s, \Delta))(1 + i^{ior})$   $(-(s(\omega) - \Delta \eta(s, \Delta))(1 + i^{dw}))$  can be include it in  $e_1$  if the bank has a reserve surplus (deficit).

<sup>&</sup>lt;sup>17</sup>Note that the expression considers that posted order in the interbank market have been of infinitesimal size

The interest rate that solve the Nash Bargaining problem in the FX swap market is:

$$i^{ex} = \underset{\bar{i}}{\operatorname{argmax}} \frac{1}{\Delta} [V^l(e_1 + (i^{ex} - i^{ior}) \triangle + (1 + i^{ior}) \triangle) - V^l(e_1 + (1 + i^{ior}) \triangle)]^{\xi} \times [V^l(e_1 - (i^{ex} - i^{dw}) \triangle - (1 + i^{dw}) \triangle) - V^l(e_1 - (1 + i^{dw}) \triangle)]^{(1-\xi)}$$

Where it is used that the solution of maximizing the surplus of the bargaining does not change when the objective function is divided by  $\triangle$ .

Note that the first part of the multiplication,  $V^l(e_1 + (i^{ex} - i^{ior})\triangle + (1 + i^{ior})\triangle) - V^l(e_1 + (1 + i^{ior})\triangle)/\triangle$ , can be written as:

$$\frac{V^{l}(e_{1}+(i^{ex}-i^{ior})\triangle+(1+i^{ior})\triangle)-V^{l}(e_{1})}{(1+i^{ex}-i^{ior})\triangle+(1+i^{ior})\triangle}\big(i^{ex}-i^{ior}+1+i^{ior}\big)-\frac{V^{l}(e_{1}+(1+i^{ior})\triangle)-V^{l}(e_{1})}{(1+i^{ior})\triangle}\big(1+i^{ior}\big).$$

And when the order size converge to zero  $(\triangle \searrow 0)$  it converges to  $(\partial V^l/\partial e)(i^{ex}-i^{ior})$ . The the second part of the multiplication can be written in a similar way and when the order size converge to zero its converges to  $(\partial V^l/\partial e)(i^{ex}-i^{i^{dw}})$ . Hence the bargaining problem for orders of infinitesimal size is:

$$i^{ex} = \underset{\bar{i}}{\operatorname{argmax}} [m_l(\bar{i} - i^{dw})]^{\xi} [m_d(\bar{i} - i^{ior})]^{(1-\xi)}$$

Where  $m_l$  is the lender marginal utility  $m_l = \partial V^l/\partial e = U(c)'$ , and  $m_b$  is the borrower marginal utility  $m_b = \partial V^l/\partial m = U(c)^{'18}$ . The interest rate that maximizes the bargaining problem in the FX swap market is a linear combination of the discount window rate and the interest over reserve rate  $i^{ex} = (1 - \xi)i^{dw} + \xi i^{ior}$ . Note that  $i^{ex}i^{ex} \in [i^{ior}, i^{ex}]$  and the interest rate is constant across matches.

Consider a borrowing and lending order of size  $\triangle$  that have been posted and matched in the interbank market. Both agents do a backward looking exercise during the bargaining, in particular they negotiate knowing that orders in the FX swap market are of infinitesimal size and that all negotiations in the FX swap market will be successful. Un-

 $<sup>^{18}</sup>$ These functions are obtained by using the envelope theorem

der these assumptions the opportunity cost of a lending order matched in the interbank market is to not accept the deal and post the orders in the FX swap market in where with a probability of  $\Psi^{+*}$  it will find a match (and accept the deal) ,and be remunerated with the FX swap market interest rate, and with probability  $1-\Psi^{+*}$  will not find a match, keep the reserves and be remunerated with the interest over reserves. In the same token, under these assumptions the opportunity cost of a matched borrowing order in the interbank market is to not accept the deal and post a fraction of the order size (the one covered with the collateral) in the FX swap market in where with a probability  $\Psi^{-*}$  will find a match (and accept the deal), and pay the FX swap interest rate, and with probability  $1-\Psi^{-*}$  will not find a match and take a discount window loan for the value of the order and pay the discount window interest rate. Additionally the discount window loan will also have to include the value of the order size that could not be posted in the FX swap market. For a lending and borrowing order that have been matched in the interbank market their outside options are:

$$V^{l}(e_0 + (\Psi^{+*}(1+i^{ex}) + (1-\Psi^{+*})(1+i^{ior}))\triangle).$$

and

$$V^{l}(e_{0} + [\Psi^{-*} \min\{1, s^{*}/(\sum_{o=1}^{\eta(s, \triangle)-1} 1_{matched} \triangle + \triangle)\}(1 + i^{ex}) + (1 - \Psi^{-*} \times min\{1, s^{*}/(\sum_{o=1}^{\eta(s, \triangle)-1} 1_{matched} \triangle + \triangle)\})(1 + i^{dw})]\triangle).$$

respectively. Where  $\min\{1, s^*/(\sum_{o=1}^{\eta(s,\triangle)-1} 1_{matched} \text{ is the fraction of the reserve deficit that is cover by the collateral}^{19}, 1_{matched} \text{ is an indicator function that take the value of one if the borrowing order was matched and zero if not and } e_0 \text{ is:}$ 

$$e_{0} = \frac{1 - \tau}{P_{t}} \left[ (1 + i_{t}^{b})b_{t} + (1 + i_{t}^{ior})m_{t} - (1 + i_{t}^{d})d_{t} + x_{t}((1 + i_{t}^{b*})b_{t}^{*} + (1 + i_{t}^{ior*})m_{t}^{*} - (1 + i_{t}^{d*})d_{t}^{*}) + \sum_{o=1}^{\eta(s,\triangle)-1} (1 + i(o))\Delta \right].$$

<sup>&</sup>lt;sup>19</sup>I am assuming that the same fraction of coverage applies to all orders. Assuming a different distribution for the coverage does not change the results since orders are of infinitesimal size. Note that the infinitesimal size assumption ensures that all the collateral is used independently of the coverage distribution. I choose to develop the model with a constant fraction of coverage across orders for simplicity.

Applying the same steps as before and using that  $\sum_{o=1}^{\eta(s,\triangle)-1} 1_{matched} \triangle + \triangle \to (1-\Psi^-)s(\omega)$  when  $\triangle \searrow 0^{20}$  the bargaining problem in the interbank market with infinitesimal size orders is:

$$i^{f} = \underset{\bar{i}}{\operatorname{argmax}} [m_{l}(\bar{i} - \min\{1, \frac{s^{*}x_{t}}{-(s(\omega)(1 - \Psi^{-})}\}\Psi^{-*}i^{ex} + (1 - \min\{1, \frac{s^{*}x_{t}}{-(s(\omega)(1 - \Psi^{-}))}\}\Psi^{-*})i^{dw})]^{\xi} \times [m_{d}(\bar{i} - \Psi^{+*}i^{ex} - (1 - \Psi^{+*})i^{ior})]^{(1-\xi)}$$

and the interbank interest rate that solve the problem is:

$$i^{f} = (1 - \xi)(\min\{1, \frac{s^{*}x_{t}}{-(s(\omega)(1 - \Psi^{-})}\}\Psi^{-*}i^{ex} + (1 - \min\{1, \frac{s^{*}x_{t}}{-(s(\omega)(1 - \Psi^{-})}\}\Psi^{-*})i^{dw}) + \xi(\Psi^{+*}i^{ex} + (1 - \Psi^{+*})i^{ior}).$$

Note that the interbank market interest rate depends in the coverage ratio and this in the reserve deficit amount. Since the withdraw shock generates a distribution of reserve deficit it also produces a distribution of interbank market interest rates. Moreover, interbank market interest rate can be mapped to the withdraw shock. Also observe that the interbank market weakly decrease in the foreign currency reserve surplus. This happens because an increase in the foreign currency surplus increases the effective market power of the borrower. Furthermore note that since orders are of infinitesimal size banks with reserve surplus will pay the average interbank market interest rate. Define  $\Psi^{aux} = \int 1_{s<0} \min\{1, s^*x_t/-s(1-\Psi^-)\}$  as the mean coverage ratio. The average interbank market is:

$$i^{f,average} = (1 - \xi)(\Psi^{aux}\Psi^{-*}i^{ex} + (1 - \Psi^{aux}\Psi^{-*})i^{dw}) + \xi(\Psi^{+*}i^{ex} + (1 - \Psi^{+*})i^{ior}).$$

 $<sup>^{20}\</sup>mathrm{I}$  am assuming that when both agents bargain for the interbank interest rate they belief that in the other matches both agents accept the deal regardless of the result in this bargain. Another possibility, not included in this paper, is to assume that if this bargain is not accepted for the borrower order then in the other matches borrower orders will also not accept the bargain, in this case when  $\triangle \searrow 0$   $\sum_{o=1}^{\eta(s,\triangle)-1} 1_{matched}\triangle + \triangle \to s(\omega)$ . Under this assumption and extra kink is added in the liquidity yield.

## 4.4 Portfolio Analysis

Banks portfolio decisions establish the aggregate level of loans, deposits and reserves in both currencies. Remember that for a variable h,  $\bar{h}$  is defined as that variable in real terms divided by the equity after dividend payments. The portfolio problem consists on choosing  $\{\bar{b}, \bar{m}, \bar{d}, \bar{b}^*, \bar{d}^*, \bar{m}^*, \bar{F}X\}$ . This portfolio problem is not standard since returns in reserves and deposits are not linear, these non-linearities are included in the liquidity yield function<sup>21</sup>. Replacing  $\bar{b}$  for the constraint over the sum of portfolio weights the portfolio problem is:

$$\max_{\bar{m}, \bar{d}, \bar{b}^*, \bar{d}^*, \bar{m}^*, \bar{F}X} \left\{ \mathbb{E}_{\omega} ([(R_t^b - (R_t^b - R_t^m)\bar{m} + (R_t^b - R_t^d)\bar{d} + (R_t^{b*} - R^b)\bar{b}^* - (R_t^b - R_t^{m*})\bar{m}^* + (R_t^b - R_t^{d*})\bar{d}^* + \bar{\chi}(\bar{m}, \bar{d}, \bar{m}^*, \bar{d}^*)]^{1-\gamma}) \right\}^{\frac{1}{1-\gamma}}$$

$$.s.t.$$

$$F\bar{X} = \bar{b}^* + \bar{m}^* - \bar{d}^* + \bar{e}^* \text{ and}$$

$$\bar{d} + \bar{d}^* \leq \kappa.$$

First lets study the decision over loans in foreign and local currency. The choice over loans in foreign currency ( $\bar{b}^*$ ) is characterize by its first order condition for a positive  $\bar{b}^*$  and  $\bar{b}$ :

$$R^{b*} = R^b$$

Hence the uncovered interest parity holds for loans in an equilibrium where loans in both currencies are provided. The portfolio problem does not determine the amount of loans in both currencies but instead the solution delivers the total amount of supplied loans<sup>22</sup>.

Now lets study how the reserves and deposits are determine. If a bank only uses equity to fund its supply of loans it will not be exposed to the withdraw shock and

<sup>&</sup>lt;sup>21</sup>Remember that the liquidity yield function is kinked in two points of the support

<sup>&</sup>lt;sup>22</sup>Even though the portfolio solution do not establish the fraction of loans that are denominated in each currency these will be determine in the general equilibrium by the demand of loans in local currency and the demand of loans in foreign currency

hence the certainty equivalent return over equity is a linear combination of the return over loans in local and foreign currency using as weights the share of loans in local and foreign currency,  $R^{b*}\bar{b^*} + R^b(1-\bar{b^*})$ , under the assumption that the return over reserves in both currencies  $(R^{m*})$  and  $R^{m}$  are lower than the minimum return over loans in both currencies  $(\min\{R^b, R^{b*}\})$ . Assume for the sake of simplicity that the bank supplies loans in both currencies, hence we are in an scenario where  $R^{b*}=R^b$ . A bank can choose to increase even more its supply of loans by holding deposits in local or foreign currency. In an equilibrium where deposits in local currency are not demanded by the bank an increase in deposits in foreign currency improve the revenues of the bank by  $\max\{R^b,R^{b*}\}(1-\rho^*)\}+R^{m*}\rho^*-R^{d*}$  for each unit of deposit in foreign currency. On the other hand, when the bank holds deposits in local currency it is exposed to withdraw shocks, in this case an increase of deposits in foreign currency allow the bank to do banking intermediation but also reduce the liquidity yield since reduces the collateral value in the FX swap market and hence increases the cost of being in a reserve deficit scenario after the withdraw shock realization<sup>23</sup>. The first order condition of deposits in foreign currency is:

$$R_t^b - R_t^{d*} + \frac{\mathbb{E}_{\omega}[(R^E)^{-\gamma} \frac{\partial \chi(\bar{m}, \bar{d}, \bar{m}^*, \bar{d})}{\partial \bar{d}^*})]}{\mathbb{E}_{\omega}(R^E)^{-\gamma}} \ge 0 \text{ with equality if } \bar{d} + \bar{d}^* < \kappa.$$

The bank can also do banking intermediation with residents deposits, this way increases for each unit of deposits in local currency the revenue increases in the loan-deposit in local currency spread  $R^b - R^d$ , but also reduces the expected liquidity yield since increase the likelihood of being in a reserve deficit situation. The first order condition of deposits in local currency is:

$$R_t^b - R_t^d + \frac{\mathbb{E}_{\omega}[(R^E)^{-\gamma} \frac{\partial \chi(\bar{m}, \bar{d}, \bar{m}^*, \bar{d})}{\partial \bar{d}})]}{\mathbb{E}_{\omega}(R^E)^{-\gamma}} \ge 0 \text{ with equality if } \bar{d} + \bar{d}^* < \kappa.$$

The costs associated to negative withdraw shock realizations can be reduced by increasing the holdings of reserves in local and foreign currency. The former reduces the range of withdraw shock realization in which there is a deficit of reserves while both rise the

 $<sup>^{23}</sup>$ In this version of the paper I am not providing necessary conditions for obtaining positive reserves and deposits in both currencies. These would be included in future versions

effective bargaining power in the interbank market and the FX-swap access in states of nature where the bank has a reserve deficit in local currency after the withdraw shock. The first order conditions over reserves in local currency is:

$$R_t^b - R_t^m - \frac{\mathbb{E}_{\omega}[(R^E)^{-\gamma} \frac{\partial \chi(\bar{m}, \bar{d}, \bar{m}^*, \bar{d}^*)}{\partial \bar{m}}]}{\mathbb{E}_{\omega}((R^E)^{-\gamma})} = 0.$$

Since in the balancing stage reserves in local currency are liquid and loans are not the difference between  $R_t^b - R_t^m$  is the liquidity premium of reserves. The last term can be decompose in the expected effect that an increase in reserves have over the return in both interbank markets (the one with collateral and the one with no collateral) and the risk premium component that appears because of the risk aversion of the bank owner.

$$\frac{\mathbb{E}_{\omega}[(R^{E})^{-\gamma}\frac{\partial\chi(\bar{m},\bar{d},\bar{m}^{*},\bar{d}^{*})}{\partial\bar{m}}]}{\mathbb{E}_{\omega}((R^{E})^{-\gamma})} = \underbrace{\mathbb{E}_{\omega}[\frac{\partial\chi(\bar{m},\bar{d},\bar{m}^{*},\bar{d}^{*})}{\partial\bar{m}}]}_{\text{Interbank Markets Return}} + \underbrace{\frac{COV_{\omega}[(R^{E})^{-\gamma},\frac{\partial\bar{\chi}_{t}(\bar{m},\bar{d},\bar{m}^{*},\bar{d}^{*},\omega)}{\partial\bar{m}}]}{\mathbb{E}_{\omega}[(R^{E})^{-\gamma}]}}_{\text{Risk Premiun}}.$$

The first order condition over reserves in foreign currency is is:

$$R_t^{b*} - R_t^{m*} - \frac{\mathbb{E}_{\omega}[(R^E)^{-\gamma} \frac{\partial \chi(\bar{m}, \bar{d}, \bar{m}^*, \bar{d}^*)}{\partial \bar{m}^*}]}{\mathbb{E}_{\omega}(R^E)^{-\gamma}} = 0.$$

In the same token the last term can be break down in:

$$\frac{\mathbb{E}_{\omega}[(R^E)^{-\gamma}\frac{\partial\chi(\bar{m},\bar{d},\bar{m}^*,\bar{d}^*)}{\partial\bar{m}^*}]}{\mathbb{E}_{\omega}(R^E)^{-\gamma}} = \underbrace{\mathbb{E}_{\omega}[\frac{\partial\chi(\bar{m},\bar{d},\bar{m}^*,\bar{d}^*)}{\partial\bar{m}^*}]}_{\text{Interbank Markets Return}} + \underbrace{\frac{COV_{\omega}[(R^E)^{-\gamma},\frac{\partial\bar{\chi}_t(\bar{m},\bar{d},\bar{m}^*,\bar{d}^*,\omega)}{\partial\bar{m}^*}]}{\mathbb{E}_{\omega}[(R^E)^{-\gamma}]}}_{\text{Risk Premiun}}.$$

Remember that reserves in foreign currency are also liquid during the balancing stage and can be used to obtain reserves in local currency in the FX swap market hence  $R_t^{b*} - R_t^{m*}$  is the liquidity premium of reserves in foreign currency and is the sum of two elements: the effect that it has over the returns in both interbank markets and the risk premium.

Lets disentangle the expected liquidity yield to obtain expressions for the expected value of the derivative of the liquidity yield with respect to reserves in local currency and reserves in foreign currency. Along the support of withdraw shocks there are two

points where the marginal effect of an extra unit of reserves changes its value, one is the point where the bank has neither a reserve deficit nor a reserve surplus in local currency and the point where the bank collateral coverage in the FX swap market is equal to one. These are two points that generate kinks in the liquidity yield function. Named  $\omega^*(\bar{m},\bar{d})$  the withdraw shock realization that satisfies  $s(\omega^*)=0$  and  $\omega^{**}(\bar{m},\bar{d},\bar{m}^*,\bar{d}^*)$  the withdraw shock realization that satisfies  $s(\omega^{**})=-s^*x_t/(1-\Psi^-)$ . Note that since  $s(\omega)$  is increasing in  $\omega$ ,  $\omega^{**}<\omega^*$  as long as  $s^*>0$ . The expressions for these two withdraw realizations values are:

$$\omega^* = \left(\rho - \bar{m}/\bar{d}\right)/(R^d_{t+1}/R^m_{t+1} - \rho) \text{ and }$$

$$\omega^{**} = -\left(\frac{m^* - \rho^* d^*}{1 - \Psi^{-*}} + \bar{m} - \rho \bar{d}\right)/\bar{d}(\frac{R^d_{t+1}}{R^m_{t+1}} - \rho).$$

Using these definitions the expected liquidity yield is given by:

$$\mathbb{E}_{\omega}[\chi(\bar{d}, \bar{m}, \bar{d}^*, \bar{m}^*)] = \int_{-1}^{\omega^{**}} \chi^{-*}(\bar{s}, \bar{s}^*, \omega) s(\omega) dF(\omega) + \int_{\omega^{**}}^{\omega^{**}} \chi^{-}\bar{s}(\omega) dF(\omega) + \int_{\omega^{*}}^{+\infty} \chi^{+}\bar{s}(\omega) dF(\omega).$$

Remember that for values of the withdraw shock that produce a reserve deficit in local currency an increase in the reserve surplus in foreign currency affects the liquidity yield function by two channels, i) the increase in the effective bargaining power in the interbank market and ii) the increase in the access to the FX swap market. Concerning the former channel remember that the interbank market interest rate varies with the level of access to the FX swap market of the borrowing part which is measured by its collateral coverage ratio  $\frac{\bar{s}^*}{-(\bar{s}(\omega)(1-\Psi^-))}$ , the interbank market interest rate can be written as:

$$i^{f}(\bar{s}^{*}, \bar{s}) = i^{f}(0, \bar{s}) - (1 - \xi)\Psi^{-*}(i^{dw} - i^{ex})\min\{1, \frac{\bar{s}^{*}}{\bar{s}(\omega)(1 - \Psi^{-})}\}.$$

where the first term is the interbank market interest rate when the borrowing part has no reserves surplus in foreign currency i.e. when the bank has no collateral to use in the FX swap market, the expression of  $i^f(0,\bar{s})$  is:

$$i^f(0,\bar{s}) = (1-\xi)i^{dw} + \xi(\Psi^{+*}i^{ex} + (1-\Psi^{+*})i^{ior}).$$

The second term is the reduction in the interbank market interest rate due to the increase in the access to the FX swap market and hence the improvement in the outside option value of the borrower. The average cost of having a reserve deficit can be written in a way that it is made explicit the two channels mentioned before:

$$\chi_{t}^{-}(\bar{s}^{*},\bar{s})\} = \chi_{t}^{-}(0,\bar{s}) \underbrace{-(1-\Psi^{-})\Psi^{-*}\min\{1,\frac{\bar{s}^{*}}{-\bar{s}(\omega)(1-\Psi^{-})}\}(i^{dw}-i^{ex})}_{\text{Access to the FX swap market}} \underbrace{-\Psi^{-}(1-\xi)\Psi^{-*}\min\{1,\frac{\bar{s}^{*}}{\bar{s}(\omega)(1-\Psi^{-})}\}(i^{dw}-i^{ex})}_{\text{Effective beautiging a way in the interbral market}}.$$

Where  $\chi_t^-(0,\bar{s})$  is the average cost of having a reserve deficit if the bank does not have a reserve surplus in foreign currency:

$$\chi_t^-(0,\bar{s}) = \Psi^-(i^f(0,\bar{s}) - i^{ior}) + (1 - \Psi^-)(i^{dw} - i^{ior})$$

Note the reduction in the average cost due to the access to the FX swap market is greater when it is harder to find a match in the interbank market for a borrowing order  $(1-\Psi^-)$ , while the reduction in the average cost cause by the increase in the effective bargaining power is greater when it is more likely to find a match in the interbank market for a borrowing order  $(\Psi^-)$  and when the bargaining power of the borrower is larger  $(1-\xi)$ . Consider the average liquidity cost when there is a partial access to the FX swap market, that is when  $\omega < \omega^{**}$ , that is:

$$\chi_t^{-*}(\bar{s}^*, \bar{s}) = \chi_t^{-}(0, \bar{s}) - ((1 - \Psi^{-}) + \Psi^{-}(1 - \xi))\Psi^{-*} \frac{\bar{s}^*}{-\bar{s}(\omega)(1 - \Psi^{-})} (i^{dw} - i^{ex}).$$

Define the reduction in the average liquidity cost due to collateral coverage ratio divided by the fraction of borrowed orders not matched in the interbank market as  $\chi^{-**,s*}$ :

$$\chi^{-**,s^*} = \frac{((1-\Psi^-) + \Psi^-(1-\xi))\Psi^{-*}(i^{dw} - i^{ex})}{1-\Psi^-}.$$

then the average liquidity cost when there is a partial access to the FX swap market can

be written as:

$$\chi_t^{-*}(\bar{s}^*, \bar{s}) = \chi_t^{-}(0, \bar{s}) - \chi^{-**, s^*} \frac{\bar{s}^*}{-\bar{s}(\omega)}.$$

Replacing these expression in the expected liquidity yield function:

$$\mathbb{E}_{\omega}[\bar{\chi}(\bar{d}, \bar{m}, \bar{d}^*, \bar{m}^*)] = \int_{-\infty}^{\omega^{**}} \chi_t^-(0, \bar{s})\bar{s}(\omega)dF(\omega) + \chi^{-**,s^*}F(\omega^{**})\bar{s}^* + \int_{\omega^{**}}^{\omega^*} \chi^{-**}s(\omega)dF(\omega) + \int_{\omega^{*}}^1 \chi^+s(\omega)dF(\omega).$$

Taking first order conditions with respect to  $\bar{m}$  and  $\bar{m}$  deliver the interbank market and FX swap market return<sup>24</sup>:

$$\mathbb{E}_{\omega}\left[\frac{\partial \chi(\bar{d}, \bar{m}, \bar{d}^*, \bar{m}^*)}{\partial \bar{m}}\right] = F(w^{**})\chi_t^-(0, \bar{s}) + (F(w^*) - F(w^{**}))\chi^{-**} + (1 - F(w^*))\chi^+ \text{ and }$$

$$\mathbb{E}_{\omega}\left[\frac{\partial \chi(\bar{d}, \bar{m}, \bar{d}^*, \bar{m}^*)}{\partial \bar{m}^*}\right] = F(\omega^{**})\chi^{-**, s^*}.$$

From these expressions it can be seen that the liquidity premium of reserves in local currency is different from the liquidity premium of reserves in foreign currency and hence there is no uncovered interest parity in reserves. The reason behind this results is that, even though both assets are liquid during the balancing stage they are different in nature, while reserves in local currency holdings reduce possible deficits in reserves the reserves in foreign currency allow the bank to make less costly a reserve deficit situation. Noting that  $\mathbb{E}_{\omega}\left[\frac{\partial \chi(\bar{d},\bar{m},\bar{d}^*,\bar{m}^*)}{\partial \bar{d}}\right] = \mathbb{E}_{\omega}\left[\frac{\partial \chi(\bar{d},\bar{m},\bar{d}^*,\bar{m}^*)}{\partial \bar{m}}\right] * (-\rho) \text{ and } \mathbb{E}_{\omega}\left[\frac{\partial \chi(\bar{d},\bar{m},\bar{d}^*,\bar{m}^*)}{\partial \bar{d}^*}\right] = \mathbb{E}_{\omega}\left[\frac{\partial \chi(\bar{d},\bar{m},\bar{d}^*,\bar{m}^*)}{\partial \bar{m}^*}\right] * (-\rho^*) \text{ it can be seen that there is no uncovered interest parity over deposits. Finally, the change in the nominal exchange rate, <math>\Delta x$ , it obtained by using the first order conditions introduced before and the foreign exchange market clearing condition introduced in the previews section.

The expectation operator does not consider the zero-measure points where  $\frac{\partial \chi(\bar{d},\bar{m},\bar{d}^*,\bar{m}^*)}{\partial \bar{m}}$ ,  $\frac{\partial \chi(\bar{d},\bar{m},\bar{d}^*,\bar{m}^*)}{\partial \bar{m}^*}$  are not defined which are  $(\omega=\omega^*)$  and  $(\omega=\omega^{**})$ .

## 5 Simulations

In this section I perform two comparative static exercises. In the first one the model is evaluated for different values of the interest rate over reserves in foreign currency ( $i^{ior*}$ ). In this framework changes in  $i^{ior*}$  is the channel by which U.S monetary policy can have cross-border spillovers. In the second exercise the model is evaluated for different levels of matching efficiency in the FX swap market ( $\lambda^*$ ), this way it is analysed the effect that a reduction in the liquidity of an interbank market which is not the main provider of liquidity has over the perform of the economy. This exercise is motivated by the 2011 freezing in the interbank market in foreign currency episode in Europe. In the analysis I will assume an inactive role for the Central Bank. The section is organize in the following order: first it is displayed the values of the parameters used for the simulation, second the steps to compute the equilibrium and finally the static comparative results. T

### 5.1 Preliminary Calibration

The model has 22 parameters: 2 preference parameters  $\{\beta, \lambda\}$ , 3 technology parameters  $\{\lambda, \lambda^*, \xi\}$ , 7 policy variables  $\{\kappa, \rho, \rho^*, i^{ior}, i^{ior*}, i^{dw}, \pi\}$ , 6 market parameters  $\{\Theta^b, \Theta^d, \Theta^{d*}, \epsilon, \varsigma, \varsigma^*\}$  and 3 parameters of the withdraw distribution  $\{a, b, \sigma\}$ . Additionally a withdraw distribution F have to be chosen. In this first version of the paper parameters values are not calibrated. In the calibration table I feature possible targets for each parameter, most of the moments targeted are taken from Bianchi and Biguio (2017). Depending of the targeted country (or monetary union) the targeted moments can change depending on the availability of the data.

Preference and technology parameters							
Parameters	Definition		Value	(Possible) Source/Target			
β	Discount Factor		0.993	Dividend Ratio			
λ	Interbank Market matching efficiency		0.5	DW/Reserves in local currency			
$\lambda^*$	FX swap matching efficiency		0.5	DW/Reserves in foreign currency			
ξ	Bargaining power		0.5	Baseline Value height			
$\gamma$	Risk Aversion		1	Constant divided-Equity ratio			
	Policy Variables	3					
Parameters	Definition		Value	Value (Possible) Source/Target			
$\kappa$	Capital Requirement		10	Regulator Parameter			
$\rho$	Reserve Requirement	0.05		Regulator Parameter			
$ ho^*$	Reserve Requirement in foreign currency	0		Regulator Parameter			
$i^{ior}$	Nominal interest rate over reserves		0.01	Regulator Parameter			
$i^{ior*}$	Nominal interest rate over reserves in foreign curr	ency	0.02	Regulator Parameter			
$i^{dw}$	Discount Window interest rate		0.6	Regulator Parameter			
pi	Inflation Targeting		0	Regulator Parameter			
	Market paramete	ers					
Parameters	Definition	Valu	ıe	(Possible) Source/Target			
$\Theta^b$	Loan demand intercept	7.4	1	Unit steady state Equity			
$\Theta^d$	Deposit supply intercept	3.6	5	Dep. in local currency rate			
$\Theta^{d*}$	Deposit supply in foreign currency intercept	1.4	1	Dep. in foreign currency rate			
$\epsilon$	Loan Demand elasticity	24	Ba	Bank credit response to policy rate			
ς	Deposit supply elasticity	24	:	Equal to loan elasticity			
$\varsigma^*$	Deposit supply in foreign currency elasticity	23		Equal to loan elasticity			
Withdraw shock (Truncated Normal Distribution)							
Parameters	Definition	Valı	1e	(Possible) Source/Target			
$\overline{a}$	Lower bound of withdraw shock	-1		Baseline Value			
b	Upper bound of withdraw shock	2		Baseline Value			
$\sigma$	Volatility	0.8	3	Reserve-balance distribution			
F	Reserve Distribution	0.8	3   I	Fit Excess Reserves distribution			

# 5.2 Computation

In this section I explain how to compute the equilibrium.  $^{25}\,$ 

Remember that in this model U.S monetary policy only affects local variables as long as the Central Bank choose a fixed exchange rate regime, hence in the first static compar-

<sup>&</sup>lt;sup>25</sup>This equilibrium is not necessarily the steady state equilibrium. Computation of steady state requires only a couple more of steps that the ones that I enumerate bellow.

ative exercise it will be assumed that the exchange rate is fixed. Additionally it will be assumed that the Central Bank performs an inflation targeting policy.

The computation consist on guessing a set of interbank market and FX swap market variables that delivers all the inputs to construct the liquidity yield function, then using the first order conditions and the market clearing conditions compute  $\bar{b}, \bar{m}, \bar{m}^*, \bar{d}, \bar{d}^*, R^b, R^d, R^{d*}$ . Finally it is obtained the guessed variables and compare them with their original guess. The steps for the computation are the followings:

- Assume that E=1
- Guess market tightness in both markets  $\{\theta, \theta^*\}$  and average collateral in FX swap market  $(\Psi^{*aux})$ .
- Compute liquidity yield function
- Using the first order conditions and market clearing conditions compute  $\{\bar{b}, \bar{m}, \bar{m}^*, \bar{d}, \bar{d}^*\}$  and  $\{R^b, R^d, R^{d*}\}$ .
- Compute  $\{\theta, \theta^*, \Psi^{*aux}\}$ . If they are not arbitrarily close to the guess value, obtain new values by bisection.

Expectations are computed by using integral function in Matlab. Also it has been assumed that capital requirement constraint is binding.

In the second simulation exercise the matching efficiency in the FX swap market ( $\lambda^*$ ) is changed. In this case the Central Bank has no participation in the exchange rate market and sets an inflation targeting regime. In order to clearing the exchange rate market remember that the exchange rate clearing condition requires information over two statistics: the fraction of loans in foreign currency and the fraction of total equity that comes from equity in foreign currency. I assume that the former is 15% and the later 10%. The computation is similar with the difference that in the fourth step  $\bar{b}, \bar{m}, \bar{m}^*, \bar{d}, \bar{d}^*, R^b, R^d, R^{d*}, \triangle e$  are computed.

 $<sup>^{26} \</sup>rm{Alternatively}$  it can be chosen values for the intercepts of the loan demand in local and foreign currency that delivers a loan dollarization of 15%

#### 5.3 Statistics

Here I display some statistics for the first Benchmark Model.

Variable	Value			
Portfolio problem results				
Reserves in local currency (% of assets)	5.31 %			
Reserves in foreign currency (% of assets)	2.14~%			
Loans (% of assets)	92.55~%			
Reserves dolarization (% of assets)	28.76~%			
Deposits dolarization ( $\%$ of assets)	27.27~%			
Real Returns				
Loan Returns	1.0306			
Deposits in local currency Returns	1.03			
Deposits in foreign currency Returns	1.0297			
Interbank Market and FX Swap Market outcomes				
FX SWAP / transactions	8.73 %			
Average interbank market rate	3.46~%			

In the benchmark model assets are mostly loans (92.55%). Reserves stand for a little portion of assets. While reserves in local currency mean the 5.31% of assets, reserves in foreign currency are 2.14 % of assets. Deposits dollarization is 28.76% while reserves dollarization is 28.76 %. Loans returns are 1.0306 and hence there is a positive liquidity premium for reserves in local currency and foreign currency (their returns are 1.001 and 1.002 respectively). The deposits return in local and foreign currency are different and hence there is no uncovered interest parity in deposits (the former is 1.03 and the later 1.0297). From the Interbank and the FX swap market it is displayed two statistics, the ratio FX swap market matches to Interbank Market matches which is 8.73 % and the average interbank market which is 3.46%. Remember that the interest rate over local currency reserves is 1% and the discount window interest rate is 6%, since bargaining power is equal for the lender and the borrower in the interbank market it is reasonable that the average interbank market is close to the middle point.

Here I display some statistics of the second benchmark market, this considers no participation of the Central Bank in the exchange market clearing.

Variable	Value				
Portfolio problem results					
Reserves in local currency (% of assets)	0.00 %				
Reserves in foreign currency (% of assets)	8.1 %				
Loans (% of assets)	91.89 %				
Reserves dolarization (% of assets)	1.00 %				
Deposits dolarization (% of assets)	23.09 %				
Real Returns					
Loan Returns	1.0679				
Deposits in local currency Returns	1.03				
Deposits in foreign currency Returns	1.022				
Interbank Market and FX Swap Market outcomes					
FX SWAP / transactions	34.6 %				
Average interbank market rate	3.5~%				
Exchange rate market outcomes					
Nominal exchange rate depreciation	3.8 %				

The model delivers a depreciation of the nominal exchange rate of 3.8%. The return over foreign reserves increase (comparing with the Central Bank fixed exchange rate regime scenario) which generates an increase in reserves in foreign currency holdings (8.1% in contrast with the 2.14 % of the assets showed before).

Since the liquidity cost has decreased because of the increase in foreign reserves holdings the demand for local reserves is reduced, and under the chosen parameters values is approximately zero<sup>27</sup>. Return over loans increases considerably (1.0679 instead in comparison with 1.03) because to obtain an equilibrium with reserves in local currency very close to zero the liquidity premium over local denominated reserves must be very high.

<sup>&</sup>lt;sup>27</sup>In future versions it will studied reserves in local currency satiation conditions. It is important to remark that I find that with certain combinations of parameters values reserves in local currency are not close to zero.

#### 5.4 Simulation Results

In this section I show results for different values of the interest rate over foreign reserves (from 2% to 3.5%).

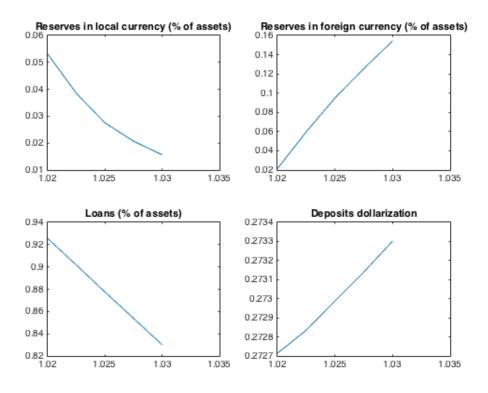


Figure 1: Interest rate over foreign reserves (Static Comparative)

An increase in the interest over foreign reserves increases its return augmenting the foreign reserves holdings. This generates a reduction in the supply of credits which increases the return of loans. Additionally banks reduce their holdings in local reserves. Since reducing reserves in local currency increases the states of nature in which a bank can have a reserve deficit and incur in a liquidity cost the bank reacts by reducing its holdings in deposits in local currency, increasing deposit dollarization. This dynamic in deposits produce an increase in the return of deposits in foreign currency and a reduction in the return of deposits in local currency.<sup>28</sup>

<sup>&</sup>lt;sup>28</sup>This change is very small and hence is not showed in the figures. Increasing the elasticity of the supply of deposits in local and foreign currency should increase the reactions of deposit returns.

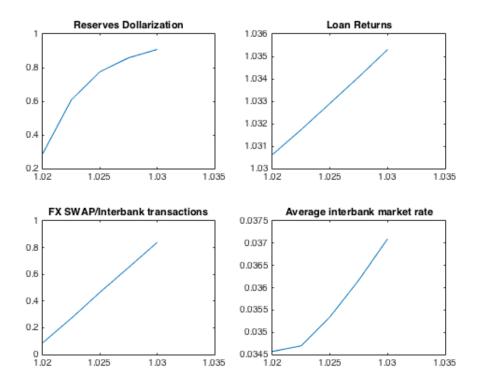


Figure 2: Interest rate over foreign reserves (Static Comparative)

The increase in foreign reserves holdings amplifies difference between the number of matches in the FX swap market and the number matches in the interbank market. Lastly the increase in reserves in foreign currency reduces the market tightness in the FX swap market which increases the effective market power of lenders in the interbank market and hence increases the average interbank market rate.

Next I present the statistics for different values of the matching technology in the FX-swap market ( $\lambda^*$ ), from 0.2 to 0.6. Remember that in this case there is no intervention of the Central Bank in the exchange rate market.

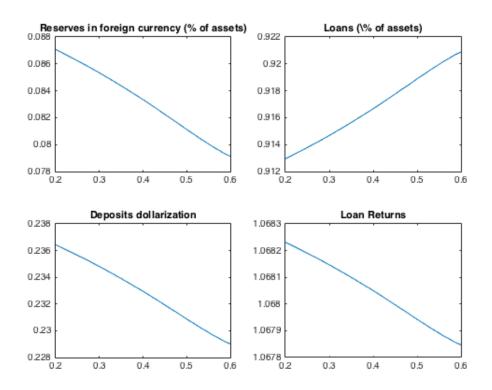


Figure 3: FX swap matching efficiency (Static Comparative)

When the efficiency in the FX swap market is reduced it is harder for a borrowing order in the FX swap market to find a match. Banks react by increasing the fraction of reserve deficit in local currency that can be posted in the FX swap market by increasing their foreign reserves holdings. Loan supply is reduced because of substitution effect and this increases the return of loans. In order to reduce the liquidity cost generated by the reduction on  $\lambda^*$  banks also reduce their holdings of deposits in local currency which increases deposit dollarization. The reduction in the demand of local currency deposits reduce its return while the increase in the demand of foreign currency deposits increase its return.

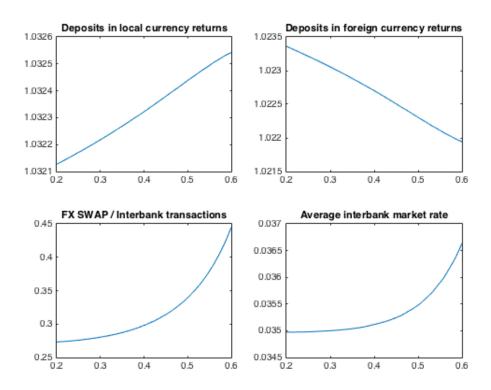


Figure 4: FX swap matching efficiency (Static Comparative)

Since it is more difficult to find a match in the FX swap market, the ratio FX - swap market matches over the Interbank market matches decreases. Finally the average interbank market is reduced since the increase in foreign reserve holdings reduce the market tightness in the FX swap market which augments the outside option of the lending orders in the interbank market.

## 6 Conclusions

I propose a general equilibrium banking model with assets and liabilities in local and foreign currencies. Banks face a standard liquidity management problem. The model contain an interbank and FX swap market. Dynamics in these markets produce a demand for reserves in local and foreign currency. Additionally the model adds an exchange rate market in where local banks and Central Bank participate.

One classical result obtained from the model is that U.S monetary policy only affects local variables as long as Central Bank does not participate in the exchange rate market. Another result of the model is the lack of uncovered interest parity for deposits and reserves. Dollarization in the model is driven by the desire of banks to have reserves in foreign currency and the effect that deposits in foreign currency have over the liquidity cost.

In the proposed model the mechanism by which U.S monetary policy affect other countries is by changing the interest rate of foreign reserves, which increases foreign reserves holding and ultimately reduces loan supply. The model also predicts that an increase in the difficulty of finding a match in the FX swap market increases foreign reserve holdings and reduce loan supply.

The paper have room for improvements and these will be developed in future versions of the paper.

# References

- Hélène Rey. International channels of transmission of monetary policy and the mundellian trilemma. *IMF Economic Review*, 64(1):6-35, May 2016. ISSN 2041-417X. doi: 10. 1057/imfer.2016.4. URL https://doi.org/10.1057/imfer.2016.4.
- Hyun Song Shin. Global liquidity and procyclicality. In speech at the World Bank conference  $\ddot{i}_{\dot{c}}\frac{1}{2}$  The state of economics, the state of the world $\ddot{i}_{\dot{c}}\frac{1}{2}$ , Washington DC, volume 8, 2016.
- IMF. CONSIDERATIONS ON THE ROLE OF THE SDR. Imf staff report, International Monetary Fund, March 2018. URL https://www.imf.org/en/Publications/Policy-Papers/Issues/2018/04/11/pp030618consideration-of-the-role-the-sdr.
- Michael Woodford. Financial intermediation and macroeconomic analysis. *Journal of Economic Perspectives*, 24(4):21–44, 2010.
- Mark Gertler and Peter Karadi. A model of unconventional monetary policy. *Journal of monetary Economics*, 58(1):17–34, 2011.
- Vasco Cúrdia and Michael Woodford. Conventional and unconventional monetary policy. 2009.
- Javier Bianchi and Saki Biguio. Banks, Liquidity Management and Monetary Policy. NBER Working Papers 20490, National Bureau of Economic Research, Inc, September 2017. URL https://ideas.repec.org/p/nbr/nberwo/20490.html.
- Alain Ize and Eduardo Levy Yeyati. Financial dollarization. *Journal of International Economics*, 59(2):323-347, March 2003. URL https://ideas.repec.org/a/eee/inecon/v59y2003i2p323-347.html.
- Marco Terrones and Luis Catao. Determinants of Dollarization; The Banking Side. IMF Working Papers 00/146, International Monetary Fund, August 2000. URL https://ideas.repec.org/p/imf/imfwpa/00-146.html.

- Henrique S Basso, Oscar Calvo-Gonzalez, and Marius Jurgilas. Financial dollarization: the role of banks and interest rates. 2007.
- Valentina Bruno and Hyun Song Shin. Cross-Border Banking and Global Liquidity. Review of Economic Studies, 82(2):535-564, 2015a. URL https://ideas.repec.org/a/oup/restud/v82y2015i2p535-564.html.
- Xavier Gabaix and Matteo Maggiori. International liquidity and exchange rate dynamics.

  The Quarterly Journal of Economics, 130(3):1369–1420, 2015.
- Óscar Arce, Galo Nuño, Dominik Thaler, and Carlos Thomas. A large central bank balance sheet? the role of interbank market frictions. 2017.
- Pablo Anaya, Michael Hachula, and Christian J. Offermanns. Spillovers of u.s. unconventional monetary policy to emerging markets: The role of capital flows. *Journal of International Money and Finance*, 73:275 295, 2017. ISSN 0261-5606. doi: https://doi.org/10.1016/j.jimonfin.2017.02.008. URL http://www.sciencedirect.com/science/article/pii/S0261560617300256. Global Economy: Future Financial and Macro Challenges.
- Georgios Georgiadis. Determinants of global spillovers from us monetary policy. *Journal of International Money and Finance*, 67:41 61, 2016. ISSN 0261-5606. doi: https://doi.org/10.1016/j.jimonfin.2015.06.010. URL http://www.sciencedirect.com/science/article/pii/S0261560615001102.
- Valentina Bruno and Hyun Song Shin. Capital flows and the risk-taking channel of monetary policy. *Journal of Monetary Economics*, 71:119 132, 2015b. ISSN 0304-3932. doi: https://doi.org/10.1016/j.jmoneco.2014.11.011. URL http://www.sciencedirect.com/science/article/pii/S0304393214001688.
- Silvia Miranda-Agrippino and H�1�ne Rey. World asset markets and the global financial cycle. Working Paper 21722, National Bureau of Economic Research, November 2015. URL http://www.nber.org/papers/w21722.