Commodity Prices, Financial Dollarization and FX Interventions in a Small Open Economy[1]

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[1] The views expressed in this paper are our own and do not necessarily reflect those of the Central Reserve Bank of Peru.

Motivation

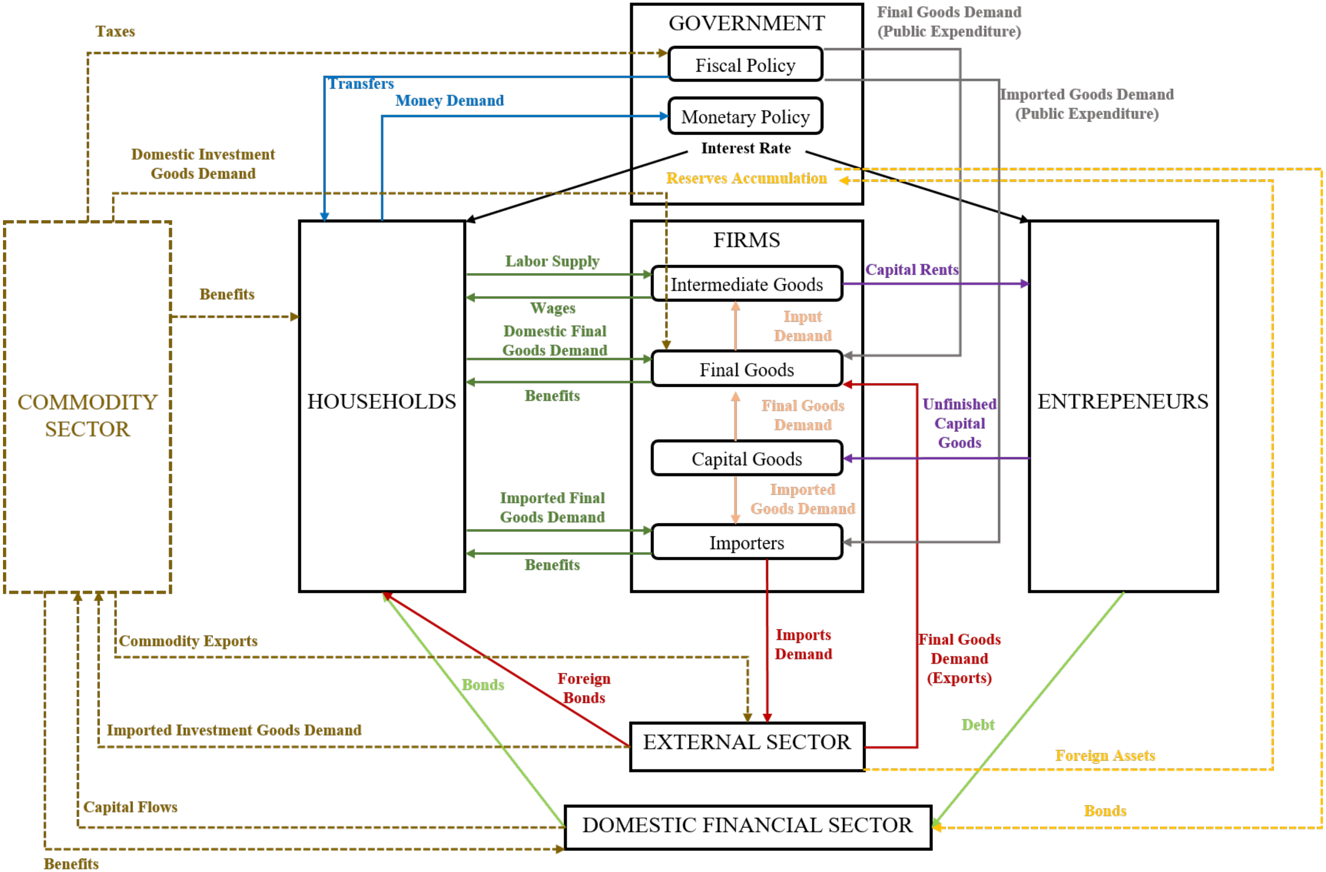
- Empirical evidence shows foreign shocks are important for small open economies.
- The 2000s were years of almost unparalleled economic growth for the Peruvian economy.
    - What role did the the foreign shocks have?
- In the same period, the Central Bank adopted an inflation targeting regime, intervened in the FX market and started a de-dollarization program.
    - The benefits of the inflation targeting regime have been widely studied.
    - What role had the FX interventions and the de-dollarization in the Peruvian economy?
- We develop a dynamic stochastic general equilibrium model for an small open economy that includes a commodity producing sector, financial dollarization, FX interventions and non-Ricardian households.

Model

- Castillo, Montoro and Tuesta (2009): MEGA-D
  - New Keynesian Small Open Economy model.
  - Financial accelerator mechanism (Bernanke, Gertler and Gilchrist 1999).
  - Financial and price dollarization.
  - Interest rate is the only monetary policy tool.
- Castillo, Florian and Toma: MEGA-ACDF
  - FX interventions as an additional tool for the Central Bank with the objective to reduce the volatility of the exchange rate.
  - A commodity producing firm that serves as the vehicle through which commodity prices fluctuations impact the rest of the model economy. Investment in this sector presents adjustment costs and time to build.
  - Non-Ricardian households to control for the fact that in emerging economies many households do not have access to financial services.

Model Scheme

Figure: Model Scheme



Financial Dollarization

- **We define δ_{FD} as the part of the interests the entrepeneurs must pay that is indexed to the foreign interest rate.**
- This is equivalent to saying entrepeneurs have debt in both domestic currency and dollars.
- With financial dollarization the arbitrage condition that determines the demand of (non-commodity sector related) stock of capital is

$$E_t\left[R^K_{t+1}\right] = E_t\left\{[1+i_t]^{1-\delta_{FD}}\left[(1+i^*_t)\frac{S_{t+1}}{S_t}\right]^{\delta_{FD}}\frac{P_t}{P_{t+1}}\right\}, \tag{1}$$

where $E_t\left[R^K_{t+1}\right]$ is the return entrepeneurs expect to obtain for investing, i_t is the nominal interest rate of the home economy, i^*_t is the foreign interest rate and P_t is the aggregate price level.

Endogenous Commodity Sector

- The production function is

$$Y_t^C = Z_t^C F^C \left(K_{t-1}^C \right), \tag{2}$$

  where Y_t^C is the commodity production, Z_t^C is the sector specific productivity, K_t^C is the specific capital this firm uses and $F^C(.)$ is the production function.

- The productivity is modelled as

$$Z_t^C = \rho_{Z^C} Z_{t-1}^C + \Upsilon P_t^C + \varepsilon_t^{Z^C}, \tag{3}$$

  where P_t^C are the commodity prices.

- For obtaining one new unit of investment, the firm must spend in investment projects X_t^C for n periods. This is the time to build mechanism and is represented by

$$INV_t^C = \sum_{j=0}^{n-1} \omega_j X_{t-j}^C, \tag{4}$$

  where $\omega_j = 1/n$.

- Commodity sector investment demands local and foreign goods.
- Commodity sector investment responds to commodity prices as well.
- The accumulation of the sector specific stock of capital is

$$K_t^C = (1 - \delta_C) K_{t-1}^C + \left[1 - \Phi \left(X_{t-n+1}^C \right) \right] X_{t-n+1}^C, \tag{5}$$

  where δ_C is the sector specific capital depreciation rate and $\Phi(.)$ is a capital adjustment cost function.

FX Interventions

- We assume the FX interventions are sterilized. In the case of a nominal appreciation, this means the monetary authority issues ΔD_t^{CB} units of Central Bank-backed bonds denominated in domestic currency in an amount equal to $S_t \Delta F_t^{CB}$.
- Thus the amount of bonds in the economy does not change and the intervention only represents a recomposition of the bond holdings of the economy.
- The balance sheet of the Central Bank is

$$S_t F_t^{*,CB} - \left(1 + i_{t-1}^*\right) \Psi_F \left(F_{t-1}^{*,CB} \frac{S_{t-1}}{P_{t-1}} \right) S_t F_{t-1}^{*,CB} = D_t^{CB} - \left(1 + i_{t-1}\right) D_{t-1}^{CB}, \tag{6}$$

where F_t^{CB} are the international reserves denominated in dollars and $\Psi_F(.)$ is the cost the Central Bank assumes for handling foreign assets. We have $\Psi_F'(.) > 0$. We also assume the international reserves earn an interest rate equal to the foreign interest rate, while the Central Bank-backed bonds denominated in domestic currency D_t^{CB} earn an interest rate equal to the domestic one.

- For the local investor, D_t^{CB} are imperfect substitutes with the private bonds in domestic currency.
- The reaction function is

$$\frac{F_t^{*,CB}}{F^{*,CB}} = \left(\frac{F_{t-1}^{*,CB}}{F^{*,CB}} \right)^{\rho_F} \left[\left(\frac{S_t - S_{t-1}}{S} \right)^{-\varphi_F} \right]^{1-\rho_F} INT_t, \tag{7}$$

where $\left(\frac{S_t - S_{t-1}}{S} \right)$ is the variation of the nominal exchange rate (also known as the depreciation rate), φ_F **represents the magnitude of the reaction of the Central Bank upon nominal exchange rate variations**, ρ_F is a persistence parameter for the international reserves and INT_t is an AR(1) process associated to the international reserves accumulation shock.

FX Interventions: Transmision Mechanism

- We take a look at the log-linearized UIP equation.

$$i_t = i_t^* + \left(\psi_B b_t^* + \psi_F f_t^{*,CB}\right) + E_t \Delta s_{t+1} \tag{8}$$

- For instance, upon a foreign interest rate shock, what would have to happen if we want to keep the nominal exchange rate constant?

$$i_t = \Uparrow i_t^* + \left(\psi_B b_t^* + \psi_F f_t^{*,CB}\right) + \overline{E_t \Delta s_{t+1}}$$

- The way the FX intervention works is by lowering the international reserves.

$$i_t = \Uparrow i_t^* + \left(\psi_B b_t^* + \psi_F \Downarrow f_t^{*,CB}\right) + \overline{E_t \Delta s_{t+1}}$$

- However, private agents may react through $\Uparrow b_t^*$ and undo what the Central Bank did.
    - The model is calibrated in such a way that the effect of the FX intervention dominates.

Preliminary Results

- Preliminary results:
  1. Bayesian Estimation
  2. Forecast Error Variance Decomposition
  3. Theoretical Moments
  4. Counterfactual Exercise: FX Intervention
  5. Counterfactual Exercise: Financial Dollarization

Estimation

- We estimate the linear version of model for the Peruvian economy during the inflation targeting (IT) period.
- We use 14 quarterly series from 2002Q1 to 2017Q4.
- All the variables are expressed in real terms and in levels.

Table: Observed Variables

Variable	Description
Y_t^{obs}	GDP
C_t^{obs}	Private Consumption
$INV_t^{TOT,obs}$	Private Total Investment
G_t^{obs}	Public Expenditure
$Y_t^{XTOT,obs}$	Total Exports
$Y_t^{MTOT,obs}$	Total Imports
RER_t^{obs}	Real Exchange Rate
i_t^{obs}	Nominal Interest Rate
π_t^{obs}	Inflation Rate (without food and energy)
$Y_t^{*,obs}$	Trade-Weighted Foreign GDP
$i_t^{*,obs}$	3-Month LIBOR Rate
$\pi_t^{*,obs}$	Trade-Weighted Foreign Inflation Rate
$Y_t^{C,obs}$	Commodity Production
$P_t^{C,obs}$	Metal Price Index

Estimation

- Measurement equations for variables in levels X_t^{obs}:

$$X_t^{obs} = x_t + x_t^{TR}, \tag{9}$$

$$x_t^{TR} = x_{t-1}^{TR} + x_t^G, \tag{10}$$

$$x_t^G = x_{t-1}^G + \varepsilon_t^x, \tag{11}$$

where x_t is the model based gap, x_t^{TR} is the trend, x_t^G is the trend growth rate and $\varepsilon_t^x \sim N(0,1)$.

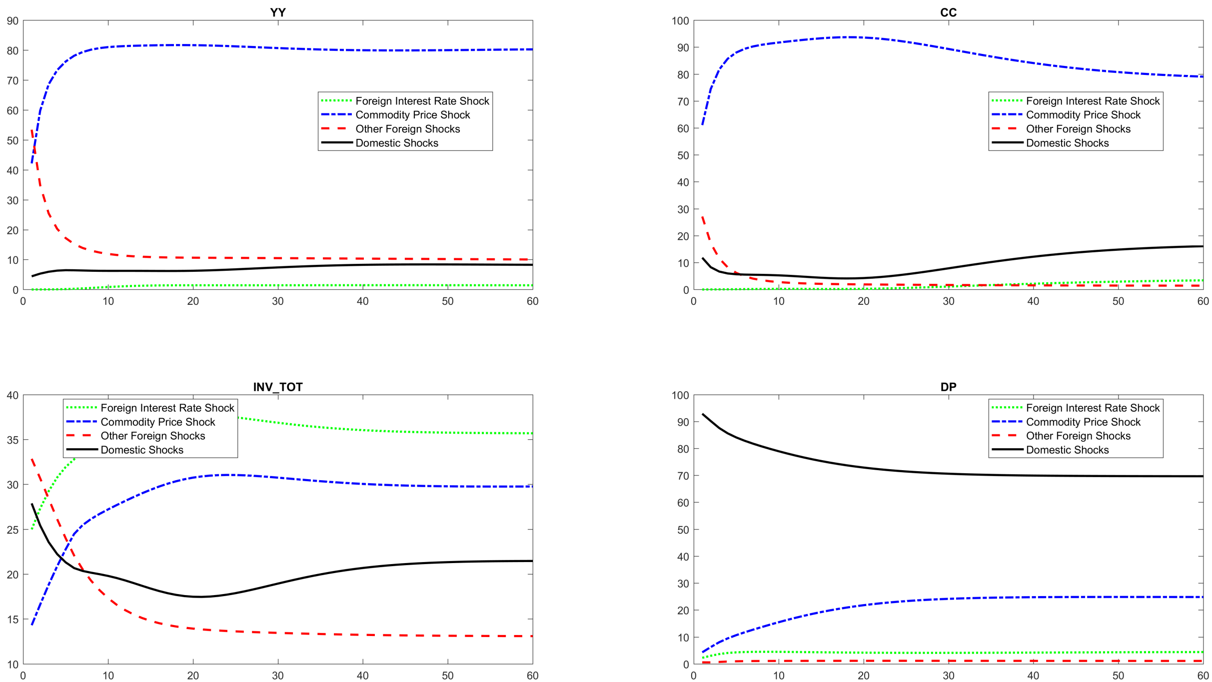
- Measurement equations for variables in rates:

$$x_t^{obs} = x_t + \overline{x} \tag{12}$$

where x_t is the model based rate and $\overline{x}$ is the mean.

Forecast Error Variance Decomposition

Figure: Forecast Error Variance Decomposition



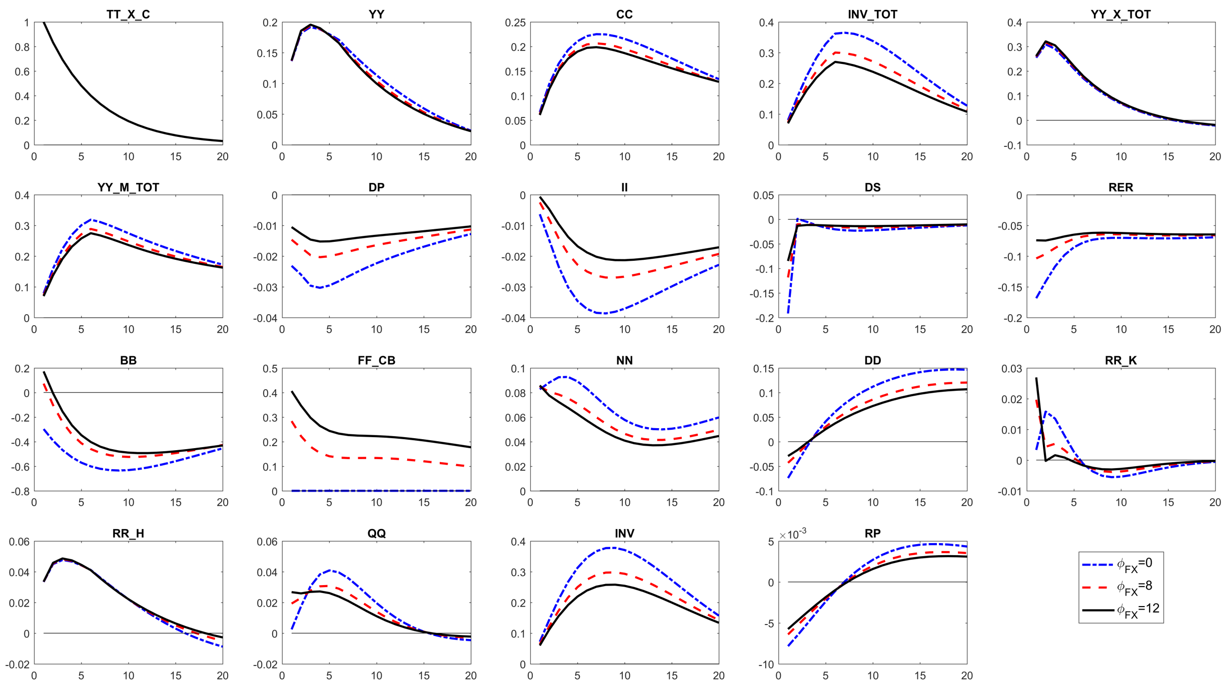
Theoretical Moments

Table: Theoretical Moments

	Correlations with GDP		Std. Dev. Ratio (GDP)	
	Observed	Model	Observed	Model
y	1.0	1.0	1.0	1.0
c	0.8	0.8	1.1	0.8
inv^{tot}	0.9	0.4	5.1	2.3
y^x	0.1	0.9	2.2	1.7
y^m	0.9	0.5	3.8	1.2
	Correlations with Inflation Rate		**Std. Dev. Ratio (Inflation Rate)**	
	Observed	Model	Observed	Model
π	1.0	1.0	1.0	1.0
$4 \times i$	0.4	0.7	2.8	4.0
Δs	0.3	0.1	6.0	4.5

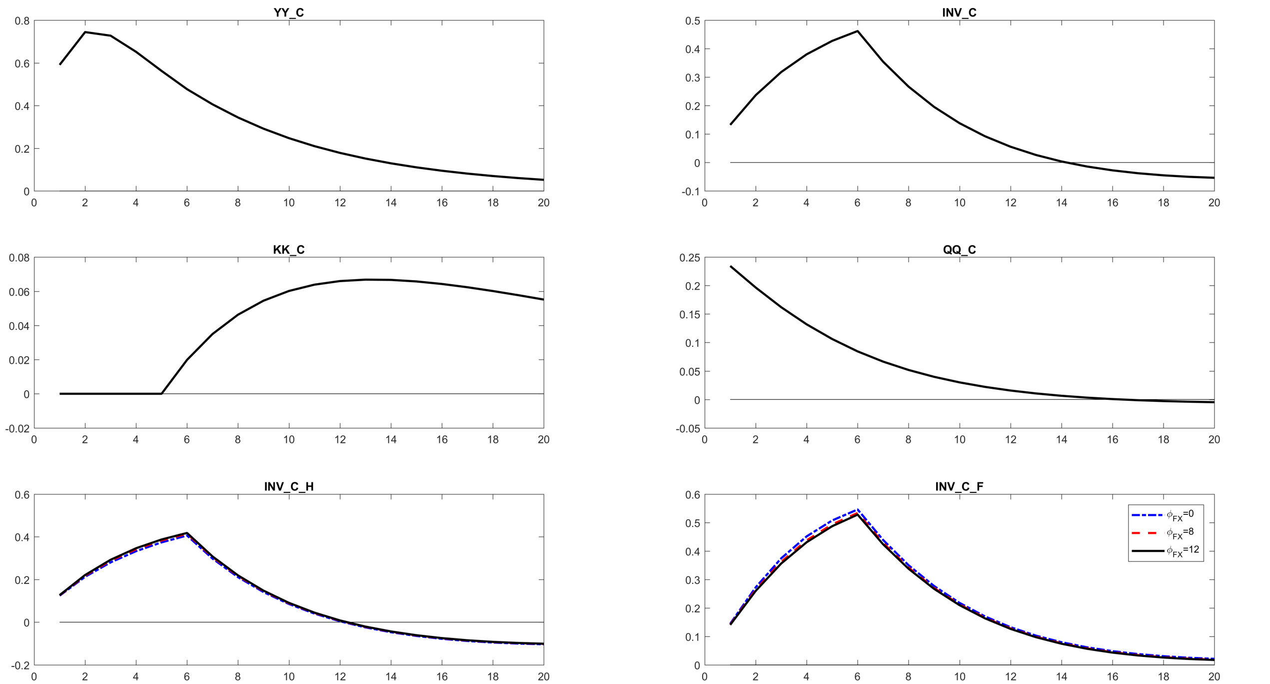
FX Intervention

Figure: Impulse Response Functions, Metal Price Index Shock (1)



FX Intervention

Figure: Impulse Response Functions, Metal Price Index Shock (2)



FX Intervention

Table: Theoretical Moments, Standard Deviations

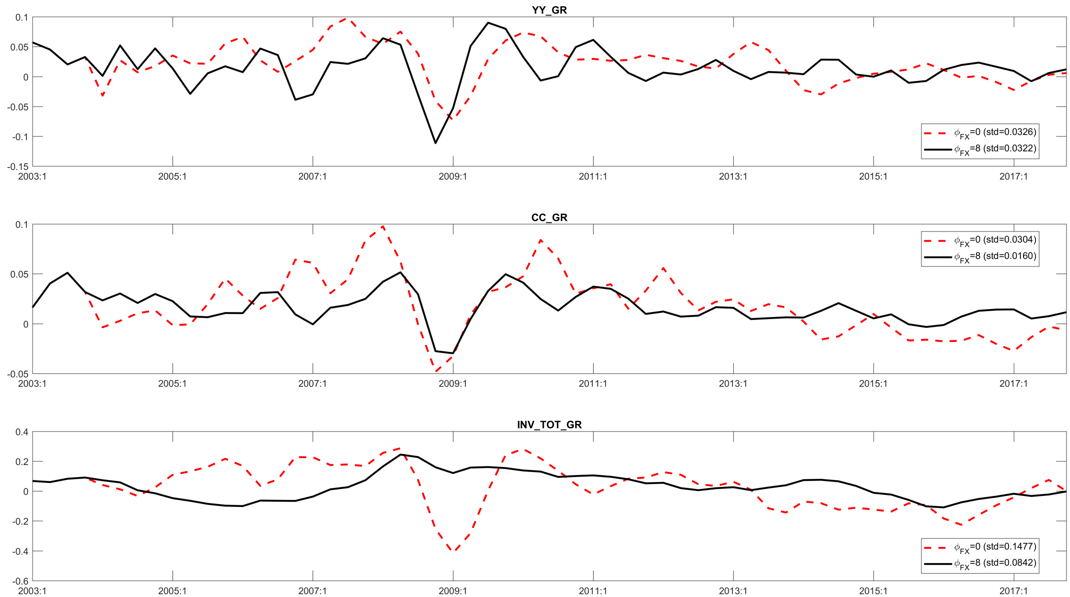
	Alternative Model ($\phi_{FX} = 0$)	Baseline Model ($\phi_{FX} = 8$)
y	0.019	0.019
c	0.016	0.014
inv^{tot}	0.051	0.043
y^x	0.031	0.032
y^m	0.026	0.022
π	0.006	0.005
$4 \times i$	0.026	0.022
Δs	0.035	0.024

Counterfactual Exercise: FX Intervention

- Based on the estimated model:
    - We feed model with observed variables for the whole sample.
    - We filter the evolution of all of the model variables.
    - We obtain the filtered shocks conditional on the model and the data.
    - We set our initial point at 2002Q1 and perform several forecasting exercises conditional on the actual realized shocks from the complete sample.
        - One scenario is that of no FX intervention ($\phi_{FX} = 0$).
        - Other case is baseline scenario ($\phi_{FX} = 8$).

Counterfactual Exercise: FX Intervention

Figure: Counterfactual Exercise (1)



Counterfactual Exercise: FX Intervention

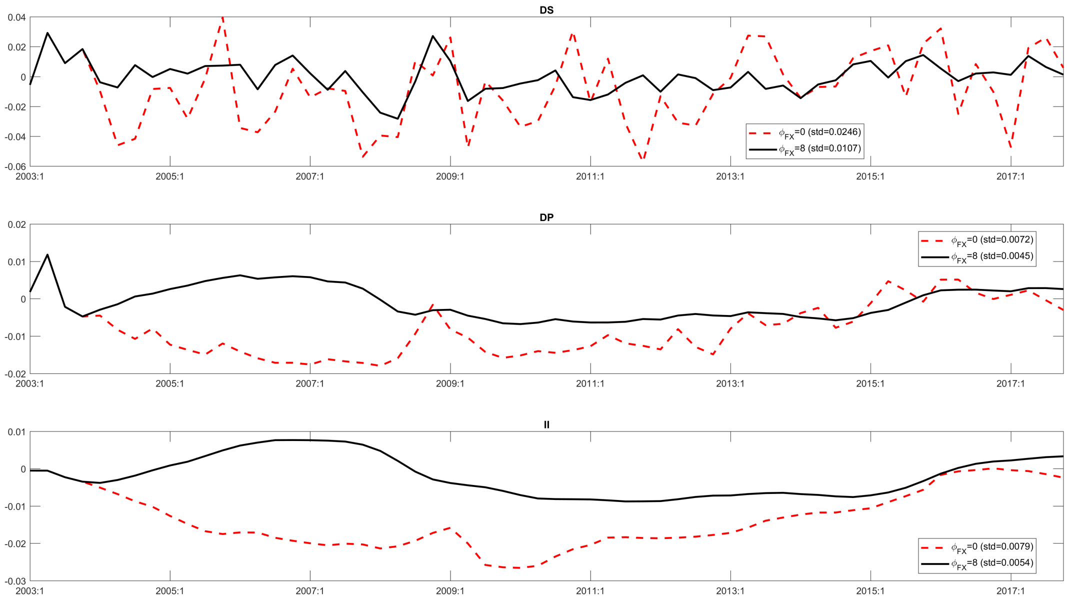
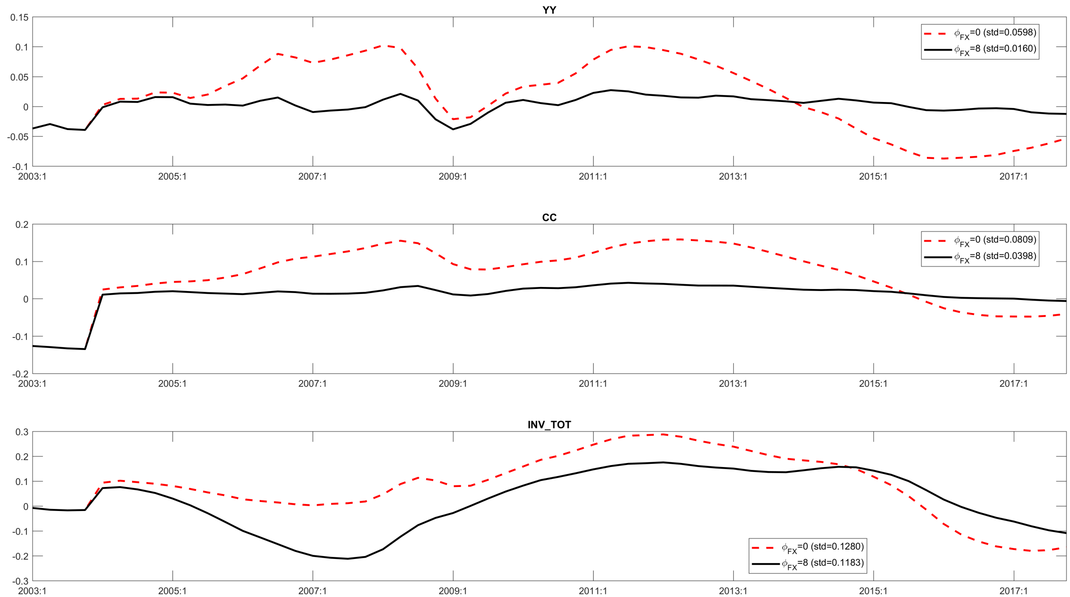


Figure: Counterfactual Exercise (2)

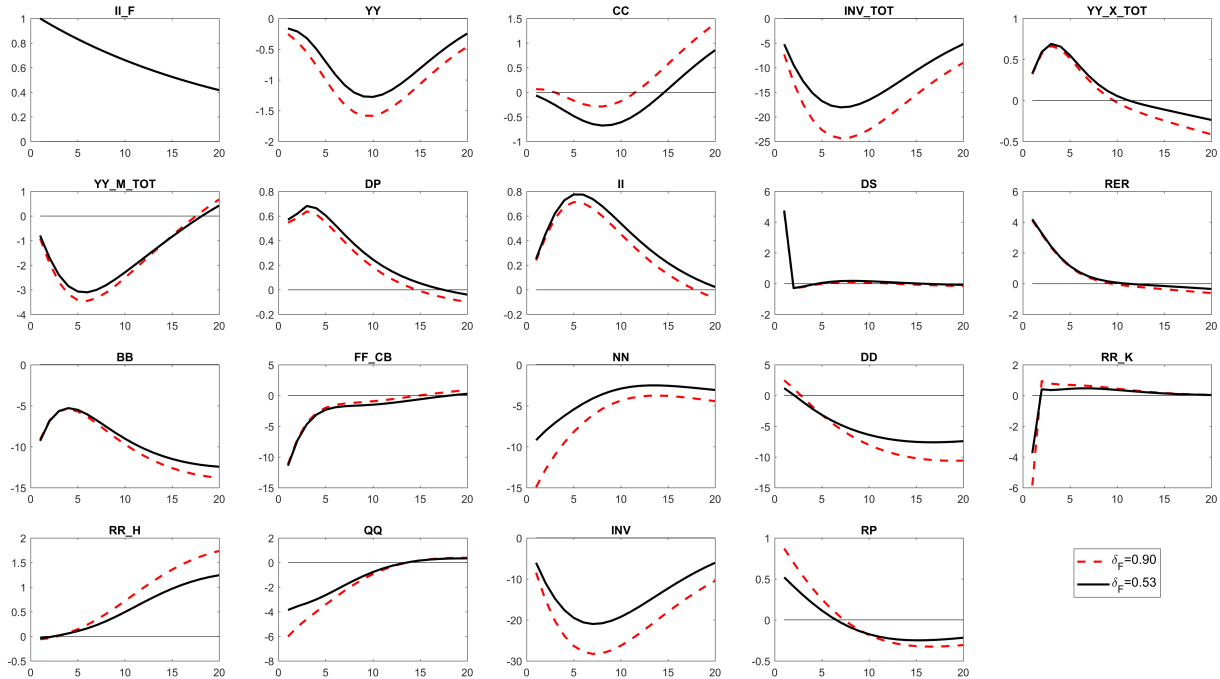
Counterfactual Exercise: FX Intervention

Figure: Counterfactual Exercise (3)



Financial Dollarization

Figure: Impulse Response Functions, Foreign Interest Rate Shock



Financial Dollarization

Table: Theoretical Moments, Standard Deviations

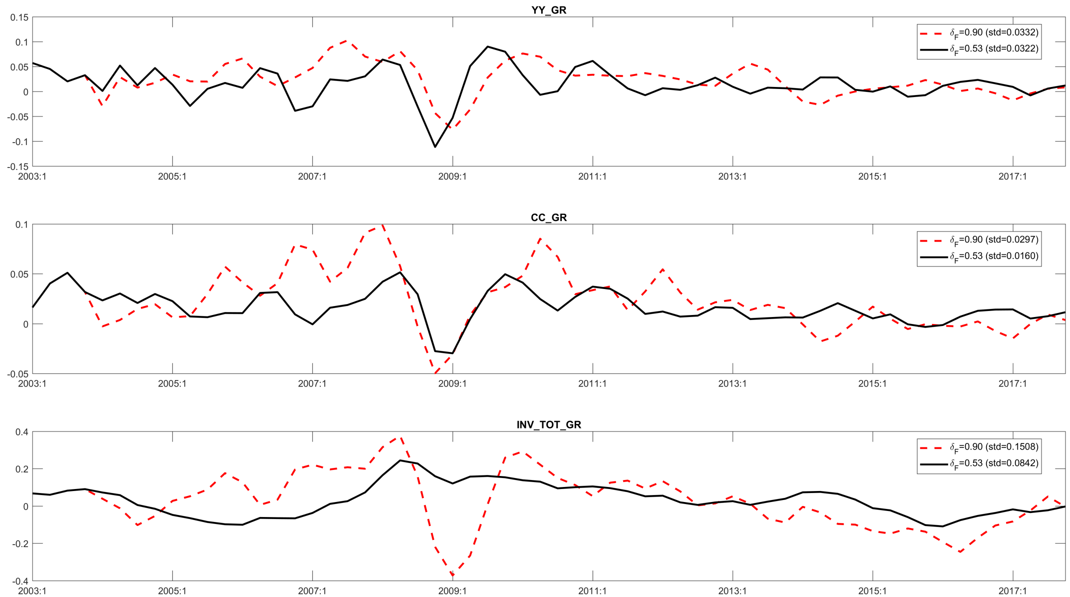
	Alternative Model ($\delta_F = 0.90$)	Baseline Model ($\delta_F = 0.53$)
y	0.019	0.019
c	0.015	0.014
inv^{tot}	0.051	0.043
y^x	0.032	0.032
y^m	0.022	0.022
π	0.006	0.005
$4 \times i$	0.022	0.022
Δs	0.024	0.024

Counterfactual Exercise: Financial Dollarization

- We analyze the effects of having a higher financial dollarization.
  - One scenario is that of high financial dollarization ($\delta_F = 0.90$).
  - Other case is baseline scenario ($\delta_F = 0.53$).

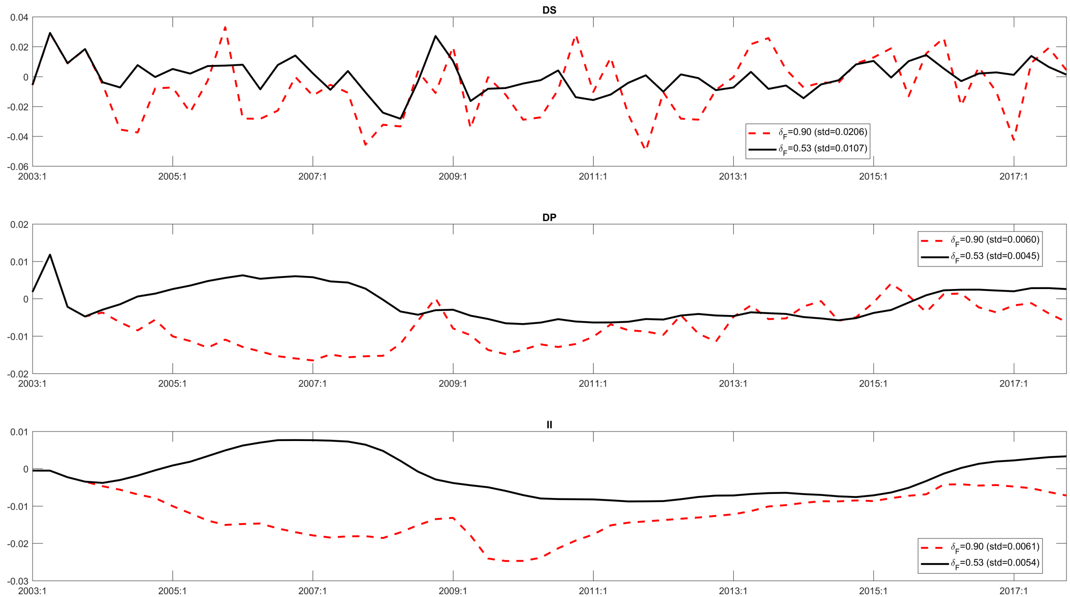
Counterfactual Exercise: Financial Dollarization

Figure: Counterfactual Exercise (1)



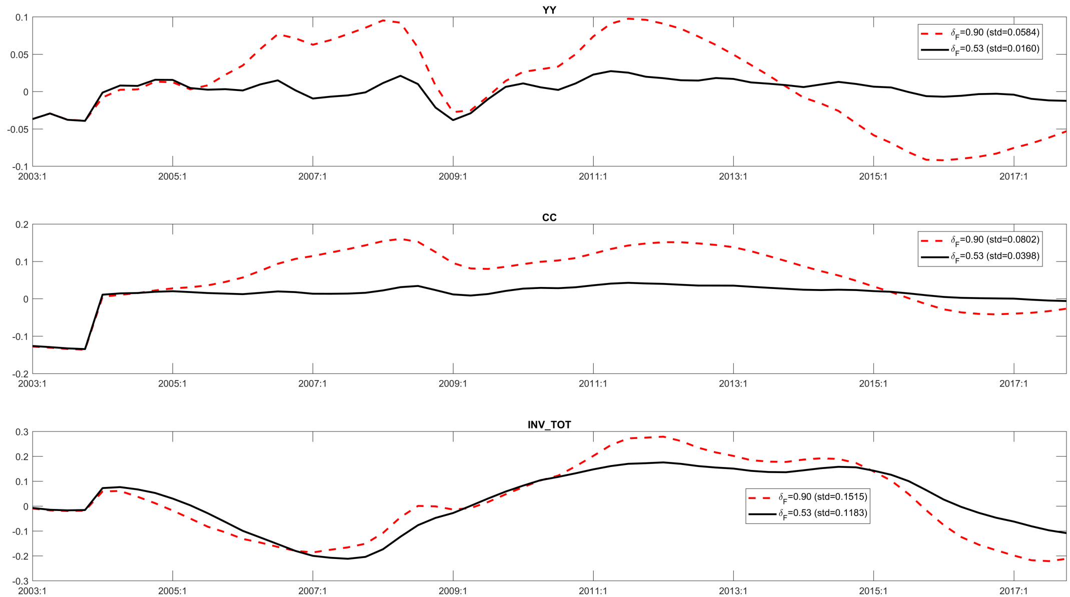
Counterfactual Exercise: Financial Dollarization

Figure: Counterfactual Exercise (2)



Counterfactual Exercise: Financial Dollarization

Figure: Counterfactual Exercise (3)



Preliminary Conclusions

- For the Peruvian economy, the commodity price index has a main role on explaining the dynamics of the macroeconomic variables.
- Counterfactual exercises show:
  - FX interventions reduce the volatility of the economy variables.
  - Lower financial dollarization levels have the same effect.

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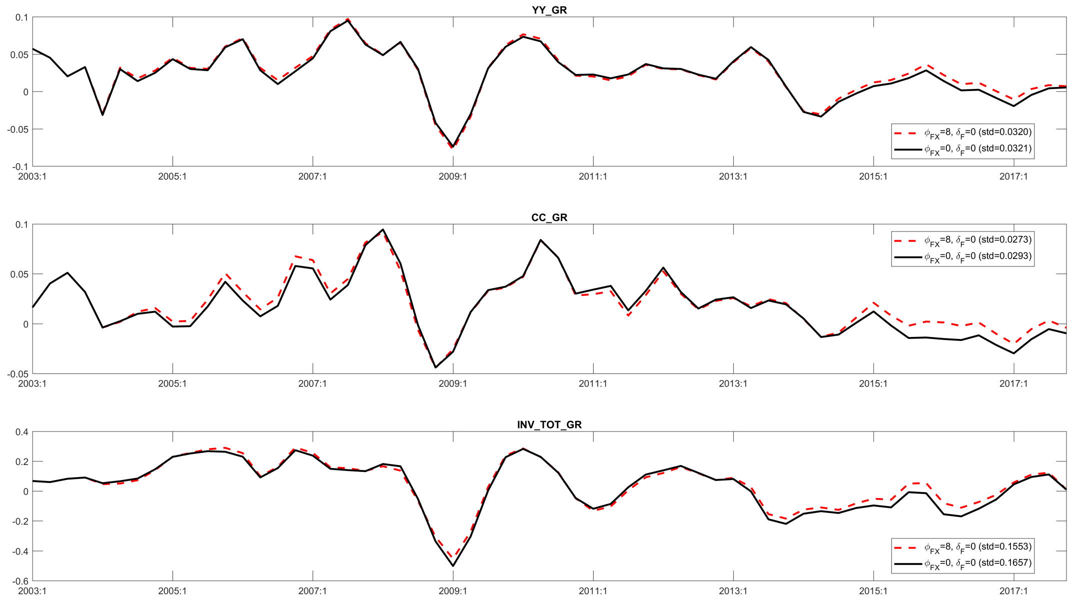
Counterfactual Exercise: FX Intervention with No Dollarization

- We analyze the effects of having FX interventions with zero financial dollarization.
  - One scenario is that of FX interventions and zero financial dollarization ($\phi_{FX} = 8, \delta_F = 0$).
  - Other scenario is that of no FX interventions and zero financial dollarization ($\phi_{FX} = 0, \delta_F = 0$).

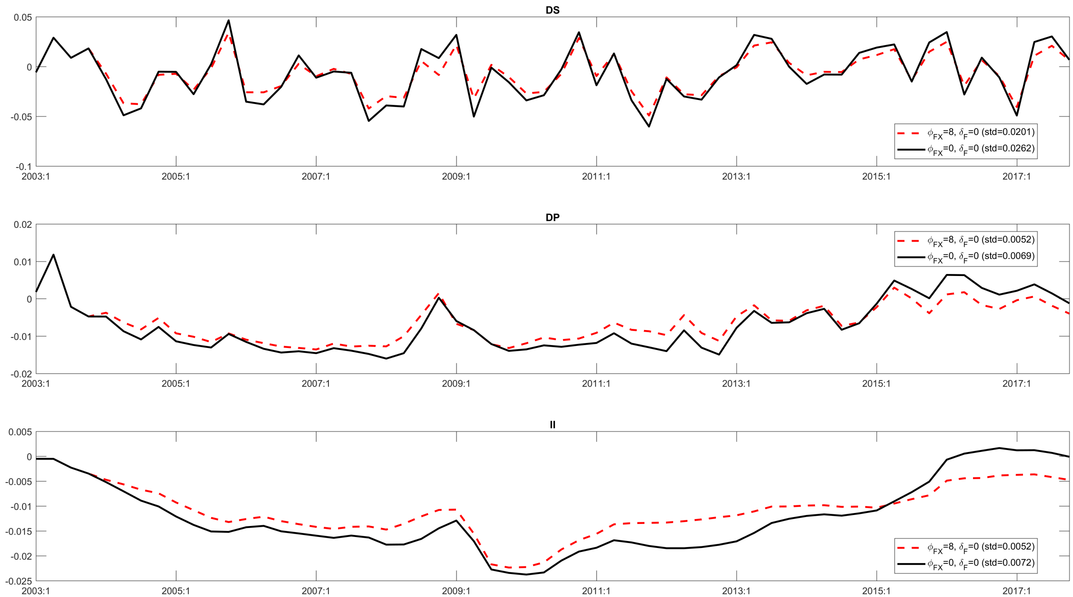
Counterfactual Exercise: FX Intervention with No Dollarization

Figure: Counterfactual Exercise (1)



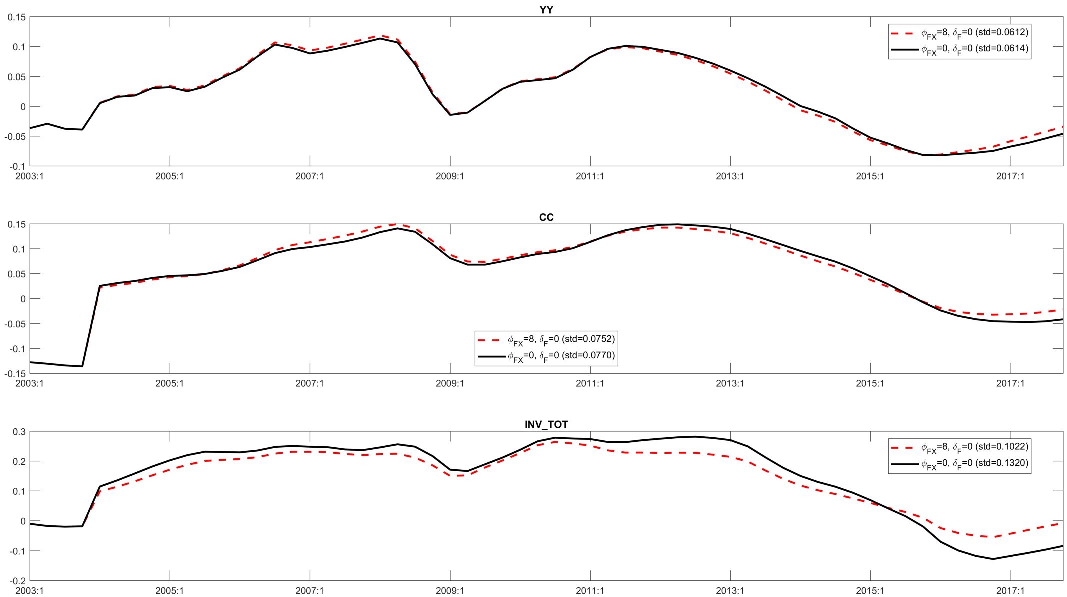
Counterfactual Exercise: FX Intervention with No Dollarization

Figure: Counterfactual Exercise (2)



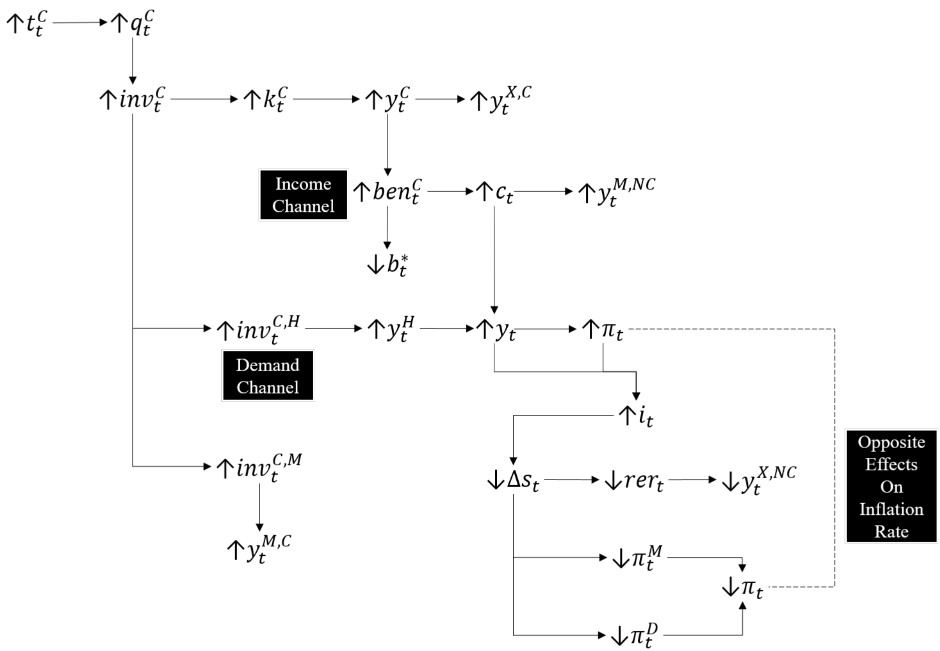
Counterfactual Exercise: FX Intervention with No Dollarization

Figure: Counterfactual Exercise (3)



Transmision Mechanisms

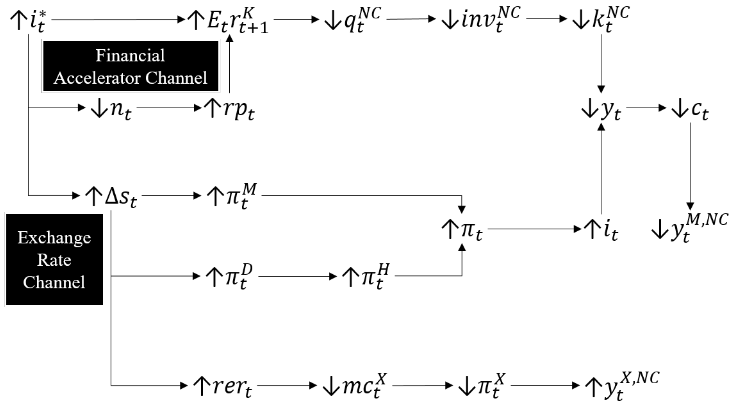
Figure: Commodity Price Shock Mechanism



Note: ben_t^C refers to the commodity producing firm benefits.

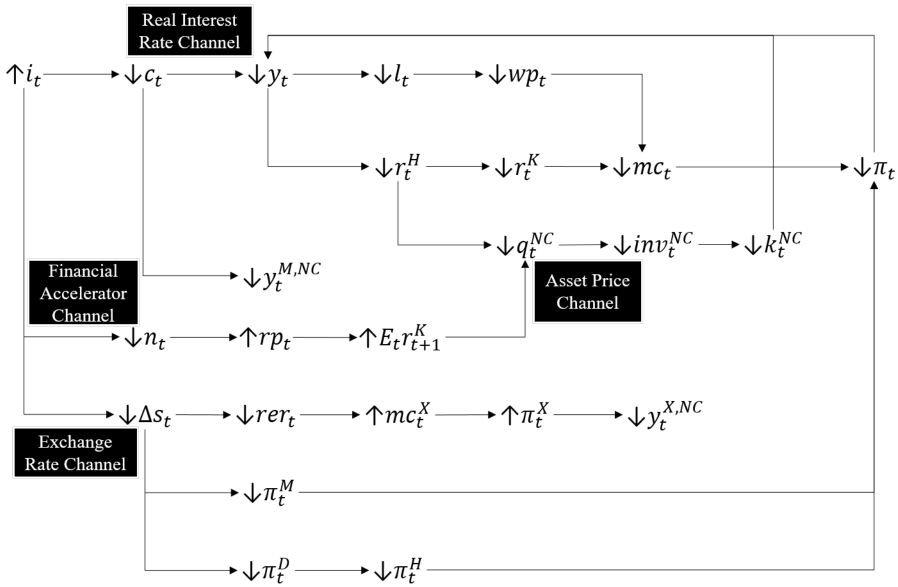
Transmision Mechanisms

Figure: Foreign Interest Rate Shock Mechanism



Transmision Mechanisms

Figure: Interest Rate Shock Mechanism



Metal Price Index

- This index is built taking into account the main 8 metals exported by the Peruvian economy: copper, gold, iron, lead, molybdenum, silver, tin and zinc.
  1. Multiply each price of the metal exports by the United States deflator, to convert the units from current dollars to constant dollars.
  2. Find the weights of the dollar exports of metal in the total amount of the 8-metal dollar export basket in each period.
  3. Take the 1994-2013 sample average of each weight to have an index with constant weights.[3]
  4. Find the 1-quarter growth rate for each of the metal price indices.
  5. Find the weighted metal price index 1-quarter growth rate using the individual weights and individual price indices.
  6. Set a metal price index with value of 100 in the first period considered in the sample.
  7. Recover the rest of the metal price index series using the growth rate.

[3] The years were chosen to match those used by Shousha (2016).

Estimation

Table: Estimated Parameters, Priors (1)

	Description	Distribution	Prior Mean	Prior Std. Dev.	Posterior Mode
h	Habit Formation	Beta	0.75	0.05	0.8741
η	Inverse of Labor Supply Elasticity	Inverse Gamma	2	0.5	1.2215
ε_H	Non-Commodity Elasticity of Substitution, Domestic and Imported Goods	Normal	0.75	0.2	1.1205
ψ_K	Non-Commodity Capital Adjustment Cost	Normal	3.3	0.1	3.2649
θ_{DC}	Calvo Probability, Domestic Currency	Beta	0.75	0.05	0.7661
θ_X	Calvo Probability, Exported Goods	Beta	0.75	0.05	0.6642
θ_M	Calvo Probability, Imported Goods	Beta	0.75	0.05	0.7549
κ_C	Commodity Investment Adjustment Cost	Normal	0.5	0.1	0.6379

Estimation

Table: Estimated Parameters, Priors (2)

	Description	Distribution	Prior Mean	Prior Std. Dev.	Posterior Mode
ρ_{PREF}	Preference Shock Persistence	Beta	0.5	0.3	0.6071
ρ_Z	Technology Shock Persistence	Beta	0.5	0.3	0.4086
ρ_{MON}	Interest Rate Shock Persistence	Beta	0.5	0.3	0.3433
ρ_{MUP}	Domestic Mark-Up Shock Persistence	Beta	0.5	0.3	0.5238
ρ_{UIP}	UIP Condition Shock Persistence	Beta	0.5	0.3	0.9220
ρ_{DP}	Inflation Shock Persistence	Beta	0.5	0.3	0.7034
ρ_{INV}	Non-Commodity Private Investment Shock Persistence	Beta	0.5	0.3	0.6280
ρ_{Z^C}	Commodity Sector Technology Shock Persistence	Beta	0.5	0.3	0.4264
ρ_g	Public Expenditure Persistence	Beta	0.7888	0.01	0.7941
ρ_{y^*}	Foreign GDP Persistence	Beta	0.8983	0.01	0.8856
ρ_{i^*}	Foreign Interest Rate Persistence	Beta	0.9817	0.01	0.9551
ρ_{π^*}	Foreign Inflation Rate Persistence	Beta	0.3651	0.01	0.3711
ρ_{p^*}	Commodity Prices Persistence	Beta	0.8339	0.01	0.8330

Estimation

Table: Estimated Standard Deviations, Priors

	Related Shock	Distribution	Prior Mean	Prior Std. Dev.	Posterior Mode
σ_{PREF}	Preferences	Inverse Gamma	0.01	0.01	0.0036
σ_{Z}	Technology	Inverse Gamma	0.01	0.01	0.0047
σ_{MON}	Interest Rate	Inverse Gamma	0.01	0.01	0.0014
σ_{MUP}	Domestic Mark-Up	Inverse Gamma	0.01	0.01	0.0132
σ_{UIP}	UIP Condition	Inverse Gamma	0.01	0.01	0.0042
σ_{INV}	Non-Commodity Private Investment	Inverse Gamma	0.01	0.01	0.0090
σ_{Z^C}	Commodity Sector Technology	Inverse Gamma	0.01	0.01	0.0400
σ_{g}	Public Expenditure	Inverse Gamma	0.0276	0.01	0.0236
σ_{y}	Foreign GDP	Inverse Gamma	0.0092	0.01	0.0027
σ_{i^*}	Foreign Interest Rate	Inverse Gamma	0.0056	0.01	0.0010
σ_{π^*}	Foreign Inflation Rate	Inverse Gamma	0.0015	0.01	0.0114
σ_{p^*}	Commodity Prices	Inverse Gamma	0.1143	0.01	0.0549

Estimation

Table: Measurement Equations Estimated Standard Deviations

	Distribution	Prior Mean	Prior Std. Dev.	Posterior Mode
$\sigma_{y^{obs}}$	Inverse Gamma	0.01	0.01	0.0086
$\sigma_{y^{x,obs}}$	Inverse Gamma	0.01	0.01	0.0308
$\sigma_{y^{m,obs}}$	Inverse Gamma	0.01	0.01	0.0310
$\sigma_{rer^{obs}}$	Inverse Gamma	0.01	0.01	0.0012
$\sigma_{y^{*,obs}}$	Inverse Gamma	0.01	0.01	0.0027
$\sigma_{c^{obs}}$	Inverse Gamma	0.01	0.01	0.0053
$\sigma_{inv^{obs}}$	Inverse Gamma	0.01	0.01	0.0220
$\sigma_{\pi^{*,obs}}$	Inverse Gamma	0.01	0.01	0.0010

Price Dollarization

- We assume a share δ^{PD} of the firms set their prices in dollars, while a share $(1-\delta_{PD})$ does so in domestic currency.
- The aggregate price level of goods produced in the home economy is

$$P_t^H = \left[(1-\delta_{PD}) \left(P_t^{DC} \right)^{\varepsilon} + \delta_{PD} \left(S_t P_t^D \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \tag{13}$$

  where P_t^H is the aggregate price level of goods produced in the home economy expressed in terms of the domestic currency, P_t^{DC} is the price level of goods produced in the home economy with prices set in domestic currency, P_t^D is the price level of goods produced in the home economy with prices set in dollars, S_t is the nominal exchange rate and ε is the elasticity of substitution.[4]
- Having firms that set prices in different currencies also means there will be two different Philips Curves, one for each currency.

[4] The nominal exchange rate is expressed as local currency for 1 dollar. This way, an increase in the nominal exchange rate means a local currency depreciation with respect to the dollar, while a decrease in the nominal exchange rate means a local currency appreciation with respect to the dollar.

Financial Dollarization

- We define δ_{FD} as the part of the interests the entrepeneurs must pay that is indexed to the foreign interest rate.
- This is equivalent to saying entrepeneurs have debt in both domestic currency and dollars.
- With financial dollarization the arbitrage condition that determines the demand of (non-commodity sector related) stock of capital is

$$E_t\left[R^K_{t+1}\right] = E_t\left\{[1+i_t]^{1-\delta_{FD}}\left[(1+i^*_t)\frac{S_{t+1}}{S_t}\right]^{\delta_{FD}}\frac{P_t}{P_{t+1}}\right\}, \tag{14}$$

where $E_t\left[R^K_{t+1}\right]$ is the return entrepeneurs expect to obtain for investing, i_t is the nominal interest rate of the home economy, i^*_t is the foreign interest rate and P_t is the aggregate price level.

Endogenous Commodity Sector

- For this extension we follow Fornero, Kirchner and Yany (2016).
- We assume the existence of a commodity goods producing firm within the modelled small open economy.
- This firm spends in sector specific investment goods to accumulate its own sector specific stock of capital. This investment has domestic and foreign shares.
- The accumulation of capital by this firm faces capital adjustment costs and time to build.
- The firm produces its commodity goods using this specific type of capital and exports the total of its production.
- Commodity producing firm does not require labor inputs to produce.
- Firm sells its output to foreign sector, it has its accounting denominated in domestic currency.
- We further assume that foreigners hold a share of the ownership of the firm equal to $(1-\chi)$.

Endogenous Commodity Sector

- The production function is

$$Y_t^C = Z_t^C F^C \left(K_{t-1}^C \right),$$

where Y_t^C is the commodity production, Z_t^C is the sector-specific productivity, K_t^C is the specific capital this firm uses and $F^C(.)$ is the production function.

- The cash flow of the firm is

$$CF_t^C = P_t^C Y_t^C - P_t^{INV^C} INV_t^C,$$

where CF_t^C is the cash flow expressed in dollars, P_t^C is the international price of the produced commodity good expressed in dollars, $P_t^{INV^C}$ is the price of the investment goods expressed in dollars and INV_t^C is the investment of the firm.

- For obtaining one new unit of investment, the firm must spend in investment projects X_t^C for n periods. This is the time to build mechanism and is represented by

$$INV_t^C = \sum_{j=0}^{n-1} \omega_j X_{t-j}^C,$$

where $\omega_j = 1/n$.

- The accumulation of the sector specific stock of capital is

$$K_t^C = (1-\delta_C) K_{t-1}^C + \left[1 - \Phi\left(X_{t-n+1}^C \right) \right] X_{t-n+1}^C,$$

where δ_C is the sector specific capital depreciation rate and $\Phi(.)$ is a capital adjustment cost function.

Cash Flow Maximization

- The first problem the commodity producing firm solves is the maximization of its cash flow by choosing optimal levels of current stock of capital and investment. The problem is

$$\max_{K_t^C, X_t^C} E_t\left[\sum_{i=0}^{\infty} \beta^t CF_{t+i}^C\right] = E_t\left[\sum_{i=0}^{\infty} \beta^t \left(P_{t+i}^C Y_{t+i}^C - P_{t+i}^{INV^C} INV_{t+i}^C\right)\right],$$

subject to

$$Y_t^C = F^C\left(K_{t-1}^C\right),$$

$$K_t^C = \left(1-\delta_C\right) K_{t-1}^C + \left[1-\Phi\left(X_{t-n+1}^C\right)\right] X_{t-n+1}^C,$$

$$INV_t^C = \sum_{j=0}^{n-1} \omega_j X_{t-j}^C,$$

where β is the discount rate.

- The first order conditions are

$$\frac{\partial L}{\partial K_t^C} = -\beta^t Q_t^C + \beta^{t+1}\left[P_{t+1}^C A_{t+1}^C \frac{\partial F^C}{\partial K_t^C}\left(K_t^C\right) + \left(1-\delta_C\right) Q_{t+1}^C\right] = 0, \quad (15)$$

$$\frac{\partial L}{\partial X_t^C} = -\beta^t P_t^{INV^C}\omega_0 - \ldots - \beta^{t+n-1} P_{t+n-1}^{INV^C}\omega_{n-1} + \beta^{t+n-1} Q_{t+n-1}^C\left[1-\Phi\left(\frac{X_t^C}{X^C}\right) - \Phi'\left(\frac{X_t^C}{X^C}\right)\frac{X_t^C}{X^C}\right] = 0, \quad (16)$$

where Q_t^C is the commodity sector Tobin's Q expressed in dollars.

Cash Flow Maximization

- The two first order conditions determine the optimal paths for the investment projects and the commodity sector Tobin's Q.
- For the rest of the model we assume

$$F^C\left(K_{t-1}^C\right) = \left(K_{t-1}^C\right)^{\alpha_C}, \tag{17}$$

$$\Phi\left(\frac{X_t^C}{X^C}\right) = \frac{\kappa_C}{2}\left(\frac{X_t^C}{X^C} - 1\right)^2, \tag{18}$$

where α_C is the parameter associated to the stock of capital within the commodity goods production function and κ_C is the parameter related to the capital adjustment cost for the commodity producing sector.

Commodity Sector Investment Bundle

- The commodity sector investment is divided among investment on domestic goods $INV_t^{C,H}$ and foreign goods $INV_t^{C,F}$. The firm must choose the optimal composition of its investment bundle by solving the problem

$$\max_{INV_t^C, INV_t^{C,H}, INV_t^{C,F}} INV_t^C = \left[(\gamma_C)^{\frac{1}{\eta_{INV^C}}} \left(INV_t^{C,H} \right)^{\frac{\eta_{INV^C}-1}{\eta_{INV^C}}} + (1-\gamma_C)^{\frac{1}{\eta_{INV^C}}} \left(INV_t^{C,F} \right)^{\frac{\eta_{INV^C}-1}{\eta_{INV^C}}} \right]^{\frac{\eta_{INV^C}}{\eta_{INV^C}-1}},$$

subject to

$$P_t^{INV^C} INV_t^C = P_t^{INV^{C,H}} INV_t^{C,H} + P_t^{INV^{C,F}} INV_t^{C,F}.$$

- Where $P_t^{INV^{C,H}}$ is the price of the domestic investment goods and $P_t^{INV^{C,F}}$ is the price of the foreign investment goods.
- Both prices are expressed in domestic currency. γ_C is the preference of the commodity sector for spending in domestic investment goods and η_{INV^C} is the degree of substitutability between domestic and foreign investment goods.
- The prices are assumed to be similar to those the domestic economy faces such that

$$P_t^{INV^{C,H}} = P_t^H, \tag{19}$$

$$P_t^{INV^{C,F}} = P_t^M, \tag{20}$$

where P_t^M is the imported goods price level.

Commodity Sector Investment Bundle

- The aggregate price index for sector specific investment goods expressed in dollars is

$$\left(P_t^{INV^C}\right)^{1-\eta_{INV^C}} = \left[\gamma_C\left(P_t^{INV^{C,H}}\right)^{1-\eta_{INV^C}} + (1-\gamma_C)\left(P_t^{INV^{C,F}}\right)^{1-\eta_{INV^C}}\right]. \tag{21}$$

- Multiplying by the nominal exchange rate to convert the price index to domestic currency and defining the price expressed in domestic currency of the commodity investment goods as $P_t^{CW} = S_t P_t^{INV^C}$ gives

$$\left(S_t P_t^{INV^C}\right)^{1-\eta_{INV^C}} = \left(P_t^{CW}\right)^{1-\eta_{INV^C}} = \left[\gamma_C\left(P_t^{H}\right)^{1-\eta_{INV^C}} + (1-\gamma_C)\left(P_t^{M}\right)^{1-\eta_{INV^C}}\right].$$

- The demand functions for the sector specific investment goods expressed in domestic currency are

$$INV_t^{C,H} = \gamma_C\left(\frac{P_t^{INV^C}}{P_t^{INV^{C,H}}}\right)^{\frac{1}{\eta_{INV^C}}} INV_t^C = \gamma_C\left(\frac{S_t P_t^{INV^C}}{P_t^H}\right)^{\frac{1}{\eta_{INV^C}}} INV_t^C = \gamma_C\left(\frac{P_t^{CW}}{P_t^H}\right)^{\frac{1}{\eta_{INV^C}}} INV_t^C, \tag{22}$$

$$INV_t^{C,F} = (1-\gamma_C)\left(\frac{P_t^{INV}}{P_t^{INV^{C,F}}}\right)^{\frac{1}{\eta_{INV^C}}} INV_t^C = (1-\gamma_C)\left(\frac{S_t P_t^{INV}}{P_t^M}\right)^{\frac{1}{\eta_{INV^C}}} INV_t^C = (1-\gamma_C)\left(\frac{P_t^{CW}}{P_t^M}\right)^{\frac{1}{\eta_{INV^C}}} INV_t^C. \tag{23}$$

Foreign Ownership

- We assume foreigners own a share equal to $(1-\chi)$ of the firm. Thus, the capital flow related to the firm is

$$S_t B_t^{*,C} - \left(1 + i_{t-1}^{*}\right) \Psi_B \left(B_{t-1}^{*} \frac{S_{t-1}}{P_{t-1}}, F_{t-1}^{*,CB} \frac{S_{t-1}}{P_{t-1}} \right) S_t B_{t-1}^{*,C} = (1-\chi)\, S_t CF_t, \tag{24}$$

  where $B_t^{*,C}$ are the dollar-denominated asset holdings of the foreign owners in the domestic economy.
- This means part of the cash flow is considered as a liability for the domestic economy. $\Psi_B(.)$ is the cost the private sector of the economy assumes for handling foreign assets. It depends on the total holdings of foreign assets by the private sector B_t^{*} and also the foreign holdings of the public sector $F_t^{*,CB}$, which we discuss in the following section. This function is increasing in the holdings, such that $\Psi_B'(.) > 0$.[5]

[5] Also, $\Psi_B(0) = 1$.

FX Interventions

- In the fashion of Benes, Berg, Portillo and Vavra (2015) and Faltermeier, Lama and Medina (2017) we consider a Central Bank that intervenes in the money market to minimize the volatility of the nominal exchange rate.
- To do so the Central Bank increases its holdings of dollar-denominated international reserves when facing a nominal appreciation (i.e. in a real economy this would mean the Central Bank buys dollars from the market) and decreases its holdings of dollar-denominated international reserves when facing a nominal depreciation (i.e. in a real economy this would mean the Central Bank sells dollars to the market).
- We assume the FX interventions are sterilized. In the case of a nominal appreciation, this means the monetary authority issues ΔD_t^{CB} units of Central Bank-backed bonds denominated in domestic currency in an amount equal to $S_t \Delta F_t^{CB}$.
- Thus the amount of bonds in the economy does not change and the intervention only represents a recomposition of the bond holdings of the economy.

Sterilized Interventions

- The balance sheet of the Central Bank is

$$S_t F_t^{*,CB} - \left(1 + i_{t-1}^*\right) \Psi_F \left(F_{t-1}^{*,CB} \frac{S_{t-1}}{P_{t-1}}\right) S_t F_{t-1}^{*,CB} = D_t^{CB} - \left(1 + i_{t-1}\right) D_{t-1}^{CB}, \tag{25}$$

where F_t^{CB} are the international reserves denominated in dollars and $\Psi_F(.)$ is the cost the Central Bank assumes for handling foreign assets. We have $\Psi_F'(.) > 0$.[6] We also assume the international reserves earn an interest rate equal to the foreign interest rate, while the Central Bank-backed bonds denominated in domestic currency earn an interest rate equal to the domestic one.

- The reaction function is

$$\frac{F_t^{*,CB}}{F^{*,CB}} = \left(\frac{F_{t-1}^{*,CB}}{F^{*,CB}}\right)^{\rho_F} \left[\left(\frac{S_t - S_{t-1}}{S}\right)^{-\varphi_F}\right]^{1-\rho_F} INT_t, \tag{26}$$

where $\left(\frac{S_t - S_{t-1}}{S}\right)$ is the variation of the nominal exchange rate (also known as the depreciation rate), φ_F represents the magnitude of the reaction of the Central Bank upon nominal exchange rate variations, ρ_F is a persistence parameter for the international reserves and INT_t is an AR(1) process associated to the international reserves accumulation shock such that

$$\ln\left(\frac{INT_t}{INT}\right) = \rho_{INT}\left(\frac{INT_{t-1}}{INT}\right) + \varepsilon_t^{INT}, \tag{27}$$

where ρ_{INT} is the persistence and ε_t^{INT} is the related shock. The variables without time subscripts denote steady state values.

[6] Also, $\Psi_F(0) = 1$.

Affecting the Nominal Exchange Rate

- Because the cost the private sector of the economy assumes for handling foreign assets $\Psi_B(.)$ now is affected by the spillovers of the foreign reserves policy, the UIP now becomes

$$\frac{(1+i_t)}{(1+i_t^*)} = \frac{\Psi_B\left(B^*_{t-1}\frac{S_{t-1}}{P_{t-1}}, F^{*,CB}_{t-1}\frac{S_{t-1}}{P_{t-1}}\right) E_t\left[U_{C,t+1}\frac{P_t}{P_{t+1}}\frac{S_{t+1}}{S_t}\right]}{E_t\left[U_{C,t+1}\frac{P_t}{P_{t+1}}\right]}, \quad (28)$$

where $U_{C,t}$ is the marginal utility of consumption. This shows how the the nominal exchange rate variation is affected by the FX intervention.

Non-Ricardian Households

- We assume a share λ_{NR} of the households are non-Ricardian.
- These households do not have access to financial markets nor they can spend on investment goods.
- They must consume all their income within the same period they receive it, without the possibility of saving for a future period.

Utility Maximization

- Non-Ricardian households solve the following maximization problem

$$\max_{C_t^{NR}, L_t^{NR}} E_t \left[\sum_{s=0}^{\infty} \beta^{t+s} U \left(C_{t+s}^{fam,NR}, L_{t+s}^{NR} \right) \right],$$

  subject to

$$P_t C_t^{fam,NR} = W_t L_t^{NR},$$

  where $C_t^{fam,NR}$ and L_t^{NR} are the consumption and the labor supplied of the non-Ricardian households. Also, these households obtain a wage W_t for their labor, which is the same wage Ricardian household receive.
- Assuming the same utility function from Castillo, Tuesta and Montoro (2009), the first order conditions of the non-Ricardian households are

$$U_{C,t}^{NR} = pref_t \left(C_t^{fam,NR} - hC_{t-1}^{fam,NR} \right)^{-1}, \tag{29}$$

$$U_{L,t}^{NR} = \left(L_t^{NR} \right)^{\eta}, \tag{30}$$

$$C_t^{fam,NR} = \frac{W_t}{P_t} L_t^{NR}, \tag{31}$$

$$MRS_t^{NR} = -\frac{U_{L,t}^{NR}}{U_{C,t}^{NR}}, \tag{32}$$

  where $U_{C,t}^{NR}$ is the marginal utility of consumption, $U_{L,t}^{NR}$ is the marginal utility of labor and MRS_t^{NR} is the marginal rate of substitution between consumption and labor, all related to the non-Ricardian households. $pref_t$ is a preferences shock, h is a habit formation parameter and η is the inverse of the labor supply elasticity.
- As Castillo, Montoro and Tuesta (2009), we assume the existence of frictions in the determination of the real wages represented by λ_{WP}, so they do not inmediately adjust to changes in the marginal rate of substitution. This is expressed as

Aggregation

- Given now the model has two types of households, in the in equilibrium their variables must be aggregated. The variables are aggregated as

$$C_t^{fam} = \left(C_t^{fam,NR}\right)^{\lambda_{NR}} \left(C_t^{fam,R}\right)^{1-\lambda_{NR}}, \tag{34}$$

$$L_t = \left(L_t^{NR}\right)^{\lambda_{NR}} \left(L_t^{R}\right)^{1-\lambda_{NR}}, \tag{35}$$

$$U_{C,t} = \left(U_{C,t}^{NR}\right)^{\lambda_{NR}} \left(U_{C,t}^{R}\right)^{1-\lambda_{NR}}, \tag{36}$$

$$U_{L,t} = \left(U_{L,t}^{NR}\right)^{\lambda_{NR}} \left(U_{L,t}^{R}\right)^{1-\lambda_{NR}}, \tag{37}$$

$$MRS_t = \left(MRS_t^{NR}\right)^{\lambda_{NR}} \left(MRS_t^{R}\right)^{1-\lambda_{NR}}, \tag{38}$$

where the superscript R denotes Ricardian households.

Goods Market

- The exports made by the commodity producing firm $Y_t^{C,X}$ are equal to it production such that

$$Y_t^{C,X} = Y_t^C. \tag{39}$$

- The imports of the commodity producing firm $Y_t^{C,M}$ include the foreign investment goods it uses such that

$$Y_t^{C,M} = INV_t^{C,F}. \tag{40}$$

- With the commodity producing firm, the absorbtion of the domestic economy becomes

$$ABS_t = C_t + INV_t + G_t + \frac{P_t^{CW}}{P_t} INV_t^C, \tag{41}$$

  where ABS_t is the domestic absorption, C_t is the consumption, INV_t is the non-commodity investment and G_t is the public expenditure. All these variables are expressed in real terms.

- The nominal GDP is

$$P_t^{def} Y_t = P_t ABS_t + S_t P_t^X Y_t^X - P_t^M Y_t^M + S_t P_t^C Y_t^{C,X} - S_t P_t^{INV^{C,F}} Y_t^{C,M}, \tag{42}$$

  where P_t^{def} is the deflator, Y_t is the real GDP, P_t^X is the export price level denominated in domestic currency, P_t^M is the import price level denominated in domestic currency, Y_t^X are the non-commodity real exports and Y_t^M are the non-commodity real imports.

Asset Market

- The equilibrium for the domestic currency-denominated bond market is

$$D_t = D_t^P + D_t^{CB} = B_t, \tag{43}$$

where D_t is the total supply of bonds denominated in domestic currency and B_t are the domestic currency-denominated bond holdings by the households.

- The equilibrium of the dollar-denominated bond market is

$$B_t^* = B_t^{*,H} - B_t^{*,C}, \tag{44}$$

where $B_t^{*,H}$ are the foreign bonds holdings by the households. We should notice the dollar-denominated asset holdings of the foreign owners in the domestic economy $B_t^{*,C}$ enter the total with a negative sign because they are a liability for the domestic economy.

Balance of Payments

- By aggregating the budget constraints and balance sheets of the households, firms, commodity producing firm and government we obtain the balance of payments equation, which is given by

$$\begin{aligned} S_t B_t^* - S_t B_{t-1}^* + S_t F_t^{*,CB} - S_t F_{t-1}^{*,CB} = \tfrac{P_t^{def}}{P_t} Y_t - ABS_t + \left(1 + i_{t-1}^*\right) \Psi_B \left(B_{t-1}^* \tfrac{S_{t-1}}{P_{t-1}}, F_{t-1}^{*,CB} \tfrac{S_{t-1}}{P_{t-1}} \right) S_t B_{t-1}^* \\ + \left(1 + i_{t-1}^*\right) \Psi_F \left(F_{t-1}^{*,CB} \tfrac{S_{t-1}}{P_{t-1}} \right) S_t F_{t-1}^{*,CB} - (1 - \chi) CF_t + REST_t \end{aligned}, \quad (45)$$

where $REST_t$ denotes the residual terms of the equation. The balance of payments is expressed in domestic currency.