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Unconventional Credit Policy in an Economy with Supply and Demand Credit Frictions

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Abstract

In this paper we develop a DSGE model where we reconcile credit demand and supply frictions and evaluate the effects of an unconventional credit policy. The credit policy consists on central bank loans to firms that are directly provided by the central bank or through commercial banks and they are guaranteed by the government. Credit supply frictions allow us to mimic a more realistic dynamics of credit after a monetary policy shock. We find that the credit policy diminishes the impact of a negative shock in the economy. Since central bank loans are not subject to the moral hazard problem between bankers and depositors, credit market interventions rise aggregate credit supply. The government guarantees reduce entrepreneurs' default probability and hence increases aggregate credit demand. In periods of high uncertainty government guarantees' effects become very significant. Also, when bank loans have a higher seniority than central bank loans, the effectiveness of the credit policy on reducing real fluctuations increases.

Keywords: Credit policy, asymmetric information, moral hazard, seniority level.

JEL Classification: G21, E44, E5.

1 Introduction

The Covid-19 global shock has confronted policy makers with the limits of standard policy tools to stimulate the economy. Standard monetary and fiscal policies are not quickly enough to provide liquidity to firms experiencing a sudden fall in cash flows. Given the magnitude of the shock and their complex interaction with credit frictions,

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firms, in particular medium and small, faced in addition credit rationing which could turn the liquidity shock into a solvency shock. To alleviate the firms's liquidity shortage problem, goverments promptly adopted unconventional credit policies such as public guarantees for corporate loans or central bank liquidity facilities to fund loans backed up with government guarantees. In our view, these credit policies are named unconventional, and classified as different to conventional credit policies studied in Curdia and Woodford (2014) and Gertler and Karadi (2011a), for two reasons: 1) loans are originated by a government-guaranteed credit policy; 2) the required return of loans originated by the credit policy is not the market required return of banks loans, which is free of firm default risk premium but contains a premium due to the credit supply frictions, but the monetary policy rate. The second reason opens the door to monetary policy considerations regarding the role of central bank intermediation for accessing credit.

These unconventional credit policies have grown in importance around the world. From the 113 economies that adopted debt finance policies, 41 countries have used similar unconventional credit policies to reduce the cost of credit.[1] How should one think about the role of this type of credit programs? What mechanisms are at work? How does public credit interact with private credit? Which kinds of credit policy rules are more effective? In this paper we seek to answer these questions.

To do so we develop a DSGE model to reconcile credit demand and supply frictions and assess the effect of an unconventional credit policy. The model includes households, banks, firms (entrepreneurs), and retailers. Risk-averse households own banks and retail businesses, while entrepreneurs are risk-neutral. Households make bank deposits and banks give loans to entrepreneurs, who in turn purchase capital (which, in combination with labor, is used to produce wholesale goods). Retail firms differentiate these goods and sell them. Price stickiness faced by retail firms allows to model central bank conventional monetary interventions. In this framework unconventional credit policy plays a role due to credit frictions that hamper savings flows in financing investment opportunities and prevent banks from adequately monitoring projects. However, we are not claiming that the effectiveness of any unconventional credit policy is conditional on the existence of frictions or a socially inefficient allocation of resources. It this line, it is worth to mention that the purpose of this paper is not to look for the optimal policy that restores the socially efficient allocation, but rather to assess the implications of unconventional credit policies already implemented by several central banks.

The novelty of our framework lies in the modeling of frictions on both the credit demand and supply sides. Credit demand frictions are modeled à la Bernanke, Gertler, and Gilchrist (1999), henceforth BGG 1999. This arises by an asymmetric information problem between entrepreneurs and banks. Ex-ante identical firms face an idiosyncratic shock,

[1] Information on policies implemented around to world to face the Covid-19 shock is compiled by the World Bank and reported in the "Map of SME-Support Measures in Response to COVID-19".

which is not observable by banks, and for which a risk-premium is charged. Entrepreneurs might prefer to hold enough equity as collateral to ensure a not very high risk-premium. Credit supply frictions are modeled à la Gertler and Karadi (2011), henceforth GKa 2011. A leverage constraint arises due to a moral hazard problem between banks and depositors. In particular, the endogenous leverage constraint ensures that banks do not divert banks assets and hence can operate. As a result, firms' equity and banks' equity are crucial to determine aggregate credit demand and credit supply, respectively.

The credit policy consists of government-guaranteed loans to firms, which are provided directly by the central bank or indirectly through commercial banks. The goal of this policy is to lessen the impact of a temporal negative shock in the economy, in particular on real variables; hence, the credit policy intervention is temporal as well. Under reasonable assumptions, the indirect central bank loans are equivalent to the direct central bank loans. We mainly discuss the indirect central bank loans in order to assess the relevance of government credibility in guaranteeing them. Since (direct or indirect) central bank loans are insured by the government and their required return of central bank loans are smaller than the required return of bank loans, they are cheaper than traditional bank loans. As a result, firms first exhaust central bank loans and then resort to bank loans.

We find that adding credit supply frictions allows us to mimic a more realistic dynamics of credit after a monetary policy shock. After a contractionary policy we find that the bank credit increases if we do not let banks to bear some aggregate risk. However, if we allow for credit supply frictions and let banks to bear some risk, banks' net worth absorbe some losses, which in turn constraints the supply of credit. This is with credit supply frictions, we might observe that after a contractionary monetary policy shock, there is a credit reduction.

We find that the unconventional credit policy diminishes the impact of a negative shock on the real economy. We highlight three channels. First, as in GKa 2011, central bank loans cannot be diverted, so the credit policy increases aggregate credit supply. This occurs since less bank equity is required per unit of aggregate credit. Second, cheap central bank loans, in the sense that central bank loans have a required return smaller than traditional bank loans, reduce entrepreneurs' obligations and hence their default probability. Third, government guarantees reduce the funding costs of entrepreneurs and hence reduces their default probability and help firms to accumulate more equity overtime. A lower default probability reduces the expected monitoring costs and hence increases entrepreneurs' incentives to purchase capital and to demand credit. In normal times the first effects is more relevant; however, in high-uncertainty periods, that yields to periods of high default probability of entrepreneurs, government guarantees become also an important driver on reducing the impact of the negative shock. In other words, the positive effects of government guarantees are quantitatively significant in periods of high uncertainty.

Another important result that we find is that when bank loans have a higher seniority than central bank loans, the effectiveness of the credit policy on reducing fluctuations of real variables increases. When bank loans have higher seniority, these are paid first. Since bank loans are paid first, more resources are available to pay bank loans, which allows banks to reduces the (non-default) lending rate, and this pushes the default probability down. A lower default probability reduces expected monitoring costs and hence incentives entrepreneurs' incentives to demand capital and hence credit.

The credibility of government guarantees is also very important. We find that if commercial banks believe government guarantees on central bank loans have low credibility, the credit policy effectiveness goes down. If banks believe that the government is not going to guarantee the loans, then banks will have to put their own money to ensure central bank loans are repaid in full. In order to compensate that future losses associated with the central bank loans, banks need to claim a higher interest rates on their bank loans. As a result, a sign of no government credibility is observing higher lending rates on bank loans. However, this lower effectiveness is not quantitatively significant, unless we are in a period of high uncertainty where the impact of the government guarantees is quantitatively significant.

In addition, we find that an endogenous credit policy rule should not be automatic. This is, the rule should be flexible enough, so it can properly respond to indicators that capture the source and size of the economic deterioration. In that sense, the regulator should be capable to identify if the shock is affecting the credit supply and/or the credit demand conditions. A wrong endogenous rule might amplify the negative shock. We also find that ex-ante announcing the endogenous credit policy rule (i.e., letting entrepreneurs know that per one unit of demanded credit for sure they might get some cheap central bank loans), the one that reduces the marginal cost of external funding from entrepreneurs' perspective and increases entrepreneurs' incentives to demand credit, does not lead to significant benefits when the credit spread is small.

The remainder of this chapter is partitioned as follows. Section 2 presents the literature review. Section 3 develops de baseline model. Section 4 discusses the baseline parametrization and simulations. Section 5 presents the unconventional credit policy. Section 6 reports the simulations of the credit policy. Finally, section 7 concludes.

2 Literature Review

This work is related to the literature of demand side credit frictions as in Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999) or BGG 1999. In this literature collateral constraints limit borrowing. In particular, we follow BGG 1999 that features frictions on the credit demand side, known in the literature as the financial accelerator.

It studies the implications of the monetary policy in an economy with financial frictions. Our contribution is to complement this setup with frictions on the credit supply side to get a better picture of real and financial shocks on real and financial variables.

This paper is also related with the literature that incorporate financial intermediaries in DGSE models and develop a moral hazard problem between banks and depositors (see Gertler and Karadi 2011; Gertler and Kiyotaki 2011, Gertler and Kiyotaki 2015 and Gertler, Kiyotaki and Queralto 2011). The moral hazard problem consists in the fact that bankers can divert a fraction of bank assets and hence depositors might want bankers put some of their money (as equity) to fund bank assets to the point that the bank charter value is higher that the value of diverting bank assets. This results in a marked based capital requirement constraints or in an endogenous leverage constraint. Our contribution to this literature is that we model credit supply frictions together with credit demand frictions which provide more realism in characterizing responses to real or financial shocks.

Our paper is also related to the literature that models the interaction of both demand and supply credit constraints to study the dynamics of credit markets to allocate resources as in Elenev, Landvoigt, and Van Nieuwerburgh (2017); Justiniano, Primiceri and Tambalotti (2019). Our contribution to this literature is studying the dynamics of supply and demand frictions in a relevant model to understand monetary policy. In fact, to some extent introducing lending and borrowing frictions is important so our unconventional credit policy is not trivial.

The credit policy developed in this paper is related with the previous literature on credit policy as in Curdia and Woodford (2014) and Gertler and Karadi (2011a). In general, in this literature credit policy is developed by a central bank issuing debt to households and paying the risk-free rate to fund loans that then are issued at the market lending rate, which captures the premium due to the moral hazard problem between bankers and depositors. It is assumed that that central bank intermediation involves efficiency costs. Since the assets intermediated by the central bank do not require any collateral of bank equity, the credit policy increases the leverage ratio of total intermediated funds and hence raises aggregate credit. The key differences with this previous literature are the following: (i) we assume for simplicity that central bank intermediation does not involve any efficiency cost; (ii) central bank loans are insured by the government; and (iii) the required return of central bank loans is the risk-free interest rate and not the required return of bank loans, which is determined in the market and it is free of entrepreneurs' default risk premium but captures the premium due to the moral hazard problem between bankers and depositors. A necessary condition for (iii) is the fact that central bank loans cannot be diverted as traditional bank loans. Indeed, in this paper, we call the credit policy "unconventional" because (ii) and (iii).

Also, our work is also part of the current Covid-19 literature on policy interventions through credit markets as in Bigio, Zhang and Zilberman (2020); Drechsel and Kalemli-Ozcan (2020); Segura and Villacorta (2020); Céspedes, Chang and Velasco (2020). We seek to contribute with an additional dimension to this literature regarding the interaction of monetary policy constraints and credit policy, the former being the new element in the analysis.

3 Model

We develop a closed economy DSGE model with households, banks, firms (entrepreneurs) and retailers. Risk-averse households own banks and retail businesses, while entrepreneurs are risk-neutral. Households save only through bank deposits and supply labor. Banks issue loans to entrepreneurs. They fund loans through bank equity and deposits they issue to households. Entrepreneurs make capital purchases (which, in combination with labor, is used to produce wholesale goods) funded by entrepreneur's equity and loans from banks. Price stickiness faced by retail firms allows to model central bank monetary interventions.

The novelty of our framework is that we add credit demand and credit supply frictions. In the credit demand side, the frictions are modeled à la BGG 1999. A credit demand friction arises from a costly state verification of entrepreneurs performance and banks have to pay a monitoring cost. Ex-ante identical firms face an idiosyncratic shock, which is not observable by banks. As a result, banks charge a risk-premium, and entrepreneurs prefer to hold enough equity as collateral to ensure a lower risk-premium. Thus, entrepreneur's equity is a crucial factor in determining the demand of bank credit. Credit supply side frictions are modeled à la GKa 2011. In particular, an endogenous bank leverage constraint arises due to a moral hazard problem between banks and depositors. The endogenous leverage constraint prevents banks from diverting banks assets and also limits the amount of loans it can issue. As is the case of firms equity, bank equity is a crucial factor in determining bank credit supply.

The framework developed in this section is going to be used to study the effects of unconventional credit policies discussed in sections 5 and 6.

In the following subsections, we start describing the problem of households. Then, we describe the maximization problems of banks and entrepreneurs and present in detail the fundaments behind the credit supply and demand frictions, respectively. We continue with the problem of the capital producer firms, the entrepreneurial sector, retail sector, the market clearing conditions and finally the long-term equilibrium (or deterministic steady state).

3.1 Households

We formulate the household sector in a way that permits maintaining the tractability of the representative agent approach. In particular, there is a representative household with a continuum of members of mass unity. Within the household, there are $1-f$ "workers" and f "bankers". Workers supply labor, H_t, and return their wages, W_t, to the household. Each banker manages a financial intermediary (bank) and transfers nonnegative dividends back to the household. There is perfect consumption insurance within the family. Households do not acquire capital and they do not provide funds directly to nonfinancial firms. Rather, they supply funds to banks. It may be best to think of them as providing funds to banks other than the ones they own. Banks offer non-contingent riskless short-term real debt (one-period real deposits, D_t) to households. These deposits pay a gross real return R_t from t to $t+1$. The representative household preferences are given by,

$$\mathbb{E}_t \sum_{m=0}^{\infty} \beta^m \left[ln(C_{t+m} - hC_{t+m-1}) - \frac{\chi}{1+\varphi} H_{t+m}^{1+\varphi} \right], \tag{1}$$

where $\mathbb{E}_t$ is the expectation operator conditional on information at date t, $0<\beta<1$ is the households' discount factor, $0<h<1$ is the habit parameter, and $\varphi, \chi > 0$, φ is the inverse Frisch elasticity and χ is the utility weight of labor and C_t is real consumption. The household chooses consumption, labor supply and riskless real debt (or real bank deposits) (C_t, H_t, D_t) to maximize expected discounted utility, equation (1), subject to the flow of funds constraint,

$$C_t + D_t = W_t H_t + \Pi_t - T_t + R_{t-1} D_{t-1}, \quad \forall t.$$

Here, Π_t are the net funds from ownership of banks, capital producing firms, and retailers, and T_t are lump sum taxes. Household's first-order conditions for labor supply and consumption/saving are given respectively by,

$$u_{Ct} W_t = \chi H_t^{\varphi}, \tag{2}$$

$$\mathbb{E}_t(\Lambda_{t,t+1}) R_{t+1} = 1, \tag{3}$$

with,

$$u_{Ct} = (C_t - hC_{t-1})^{-1} - \beta h (C_{t+1} - hC_t)^{-1},$$

$$\Lambda_{t,\tau} = \beta^{\tau-t} \frac{u_{C\tau}}{u_{Ct}}, \quad \tau \geq t,$$

where u_{Ct} denotes the marginal utility of consumption and $\Lambda_{t,t+r}$ the household's stochastic discount factor.

3.2 Banks: Credit Supply Frictions

Bankers transfer funds from households (deposits) to entrepreneurs (loans). Each banker runs a bank. A Bank indexed by i gives B_t^i loans to entrepreneurs. Loans are funded by bank equity N_{bt}^j and household deposits D_t^i. A bank's balance sheet is given by,

$$B_t^i = D_t^i + N_{bt}^i, \tag{4}$$

A bank holds loans (bank assets) from t to $t+1$ to earn a gross return of R_{t+1}^l, which is going to be the required return per unit of bank loans, and pays the non-contingent real return of R_t to households deposits. In equilibrium both rates are determined endogenously.

Banks raise equity only through retained earnings. As a result, bank equity evolves as,

$$N_{bt+1}^i = R_{t+1}^l B_t^i - R_t D_t^i, \tag{5}$$

Because banks may be financially constrained, bankers will retain earnings to accumulate assets. Absent from any motive for paying dividends, they may find it optimal to accumulate to the point where any leverage constraint is no longer binding. To limit bankers' ability to save and overcome the leverage constraint, a turnover between bankers and workers is introduced. In particular, there is an i.i.d. probability $1-\sigma$ that a banker exits next period, (i.e., an average survival time $= 1/(1-\sigma)$). Upon exiting, a banker transfers retained earnings to the household and becomes a worker. Note that the expected survival time may be quite long (in our baseline calibration it is eight years). Each period, $(1-\sigma)f$ workers randomly become bankers, keeping the number in each occupation constant. Finally, because in equilibrium bankers will not be able to operate without any financial resources, each new banker receives a "startup" transfer from the family, as we describe in this section. Thus, Π_t are net funds transferred to the household; that is, funds transferred from exiting bankers minus the funds transferred to new bankers (aside from profits of capital producers and retailers).

Banks, at the end of period t, maximize the present value of future terminal dividends,

$$V_t^i = \mathbb{E}_t \left[\sum_{m=0}^{\infty} (1-\sigma)\sigma^m \Lambda_{t,t+1+m} N_{bt+1+m}^i \right], \tag{6}$$

where $\Lambda_{t,t+m}$ is the households stochastic discount factor that applies to earnings at $t+m$ since banks are owned by households.

To motivate a limit on banks ability to expand their assets indefinitely by borrowing additional funds from households, we introduce a moral hazard problem. As in GKa (2011) at the beginning of the period the banker can choose to divert some fraction λ of available funds and transfer them back to the household of which she is a member. The cost to the banker is that the depositors can force the bank into bankruptcy and recover only the remaining fraction $1-\lambda$ of assets. As a result, to ensure the existence of bank loans, the following incentive constraint must be satisfied,

$$V_t^i \geq \lambda B_t^i. \tag{7}$$

One can show that the value function is linear. i.e one can express V_t^i as follows,

$$V_t^i = \nu_t B_t^i + \eta_t N_{bt}^i,$$

with

$$\nu_t^i = \mathbb{E}_t\{(1-\sigma)\Lambda_{t,t+1}(R_{t+1}^l - R_t) + \Lambda_{t,t+1}\sigma x_{t,t+1}\nu_{t+1}^i\},$$

$$\eta_t^i = \mathbb{E}_t\{1-\sigma + \Lambda_{t,t+1}\sigma z_{t,t+1}^i \eta_{t+1}^i\},$$

where $x_{t,t+m}^i = B_{t+m}^i / B_t^i$ is the gross growth rate in assess between t and $t+m$ and $z_{t,t+m}^i = N_{bt+m}^i / N_{bt}^i$ is the gross growth rate of net worth. Then, the incentive constraint (7) becomes,

$$\nu_t^i B_t^i + \eta_t^i N_t^i, \geq \lambda B_t^i.$$

Under reasonable parameter values the constraint always binds within a local region of the steady state. In fact, we parameterize the model so the constraint is always binding. Then,

$$B_t^i = \frac{\eta_t^i}{\lambda - \nu_t^i} N_{bt}^i = \phi_t^i N_{bt}^i, \tag{8}$$

where ϕ_t^i is the ratio of bank loans to equity (or bank leverage). This constraint in equation (8) limits bank leverage ratio to the point where bank's incentives to divert funds is balanced by its cost. As a result, the moral hazard problem leads to an endogenous credit constraint on bank ability to issue loans. We rewrite the evolution of bank's net worth (5) as,

$$N_{bt+1}^i = \left[(R_{t+1}^l - R_t)\phi_t^i + R_t\right] N_{bt}^i.$$

We then rewrite $z_{t,t+1}^i$ and $x_{t,t+1}^i$ as, respectively,

$$z_{t,t+1}^i = N_{bt+1}^i / N_{bt}^i = (R_{t+1}^l - R_t)\phi_t^i + R_t,$$

$$x_{t,t+1}^i = B_{t+1}^i / B_t^i = (\phi_{t+1}^i / \phi_t^i) z_{t,t+1}^i.$$

Since ϕ_t^i does not depend on bank-specific factors, we can aggregate equation (8) to obtain a relationship between aggregate supply of bank credit B_t and aggregate bank net worth,

$$B_t = \frac{\eta_t}{\lambda - \nu_t} N_{bt} = \phi_t N_{bt}. \tag{9}$$

Equation (9) is the aggregate credit supply curve. According to this, due to the moral hazard problem, banks need to accumulate equity to be able to supply bank loans (or, equivalently, to able to capture household deposits). In particular, the higher the spread $R^l_{t+1} - R_t$, the higher bank net worth accumulation. Ceteris paribus, from equation (9) there are two ways that aggregate credit supply increases. First, that a smaller fraction of bank assets (or aggregate credit) can be diverted, i.e., a smaller λ. As we will see latter this happens with the credit policy in this paper and also in GKa 2011. And second, increments in bank equity.

Total net worth in the banking sector, N_{bt}, equal the sum of the net worth of existing banks N_{ot} (o for old) and the net worth of entering (or "new") banks N_{nt} (n for new),

$$N_{bt} = N_{ot} + N_{nt}.$$

Since a fraction σ of banks at $t-1$ will survive until t, N_{ot} is given by,

$$N_{ot} = \sigma \left[(R^l_t - R_{t-1})\phi_{t-1} + R_{t-1} \right] N_{bt-1}, \tag{10}$$

As stated before, newly entering bankers receive "startup" funds from their respective households. We suppose the household gives its new banker a transfer equal to a small fraction of the value of assets that exiting bankers had intermediated in their final operating period. Given that the exit probability is i.i.d., the total final period assets of exiting bankers at t is $(1-\sigma)B_{t-1}$. We assume that each period the household transfers the fraction $\zeta/(1-\sigma)$ of this value to its entering bankers. As a result,

$$N_{nt} = \zeta B_{t-1}. \tag{11}$$

Combining equations (10) and (11) yields the aggregate motion of bank net worth,

$$N_{bt} = \sigma \left[(R^l_t - R_{t-1})\phi_{t-1} + R_{t-1} \right] N_{bt-1} + \zeta B_{t-1}. \tag{12}$$

3.3 Entrepreneurs: Credit Demand Frictions

Entrepreneurs are modeled as in BGG 1999. Entrepreneurs are risk-neutral and ex-ante identical. Yet, they face an idiosyncratic and an aggregate shock and supply one unit of labor inelastically to the labor market. Entrepreneurs produce wholesale goods in competitive markets.

Capital acquisitions are financed with wealth (entrepreneur's equity) and borrowing (bank loans). Net worth accumulation comes from profits from previous capital investments and income from labor supply.

To avoid accumulation of equity, we assume entrepreneurs have a finite live. This is, with a constant probability γ they survive to the next period, implying an expected lifetime of $1/(1-\gamma)$. Birth rate is such that the number of entrepreneurs is constant across time.

We assume there is an asymmetric information problem between entrepreneurs and banks. Banks cannot observe idiosyncratic shock faced by each entrepreneur. Hence, banks have to pay a monitoring cost to observe the realized value of entrepreneurs' payoffs. Banks are repaid in full if entrepreneurs do not default, so banks do not have any incentive to pay a monitoring cost to verify the entrepreneurs' performance; however, when a entrepreneur defaults, banks do have incentives to pay the monitoring cost to observe the realized payoffs. Then, a higher default probability of entrepreneurs raises the agency cost of monitoring projects. Given that these costs are internalized by entrepreneurs, the higher default probability reduces entrepreneurs' incentives to demand credit. We assume only one-period loans contracts between bankers and entrepreneurs. The optimal contract is designed to minimize the expected agency costs.

In this environment, high net worth allows for increasing self-financing (or equivalently, collateralized external finance), mitigating the agency problems associated with external finance and reducing external finance premium faced by entrepreneurs in equilibrium. Hence, as will be seen later, net worth position is a key determinant for the cost of external finance. As we will see, fluctuations in net worth amplify and propagate exogenous shocks to the system.

The capital investment decisions are at entrepreneur level. Entrepreneur take the price of capital and expected return of capital as given. Firms are indexed by $j \in [0,1]$. At time t, an entrepreneur who manages firm j purchases capital, K_t^j, for use at $t+1$, and pays price Q_t per unit of capital. The return of capital is sensitive to both aggregate and idiosyncratic risk. The ex-post return of capital is $\omega^j R_{t+1}^k$, where ω^j is the idiosyncratic disturbance to firm j's return and R_{t+1}^k is the ex-post aggregate return of capital (i.e., gross return averaged across firms).[2] We set that ω^j is i.i.d. across time and firms and $\mathbb{E}\{\omega^j\} = 1$. In particular, we assume ω follows a lognormal distribution.

At the end of t (going into period $t+1$), entrepreneur j has available net worth N_t^j, and borrows B_t^j from banks,

$$B_t^j = Q_t K_t^j - N_{et}^j. \tag{13}$$

to purchase capital K_t^j. Banks pay a monitoring cost to observe entrepreneur's realized return, $\mu \omega^j R_{t+1}^k Q_t K_t^j$, with $\mu > 0$.

Assume first R_{t+1}^k is known in advance. Entrepreneur chooses, K_t^j and B_t^j, prior the realization of ω^j. The optimal contract (risky debt) is given by the gross non-default

[2]Note that ω^j is indeed ω_{t+1}^j; however, we omit time dimension for a notation simplicity.

bank loan rate Z^j_{t+1} and a threshold value of the idiosyncratic shock, $\bar{\omega}^j$, defined as,

$$\bar{\omega}^j R^k_{t+1} Q_t K^j_t = Z^j_{t+1} B^j_t. \tag{14}$$

If $\omega^j \geq \bar{\omega}^j$, entrepreneur fully repay bank loan, otherwise it defaults. In latter case, banks pay the auditing cost, seize the entrepreneur's project and obtain $(1-\mu)\omega^j R^k_{t+1} Q_t K^j_t$. A defaulting entrepreneur receives nothing.

Let say the opportunity cost of banks is R^l_{t+1}. Hence, the required return of banks loans is R^l_{t+1}. Banks can perfectly diversify the idiosyncratic risk involved in lending and indeed they do that and hence in the optimal contract banks receive a certain gross return of R^l_{t+1} per unit of bank loans. In other words, since lending risk is perfectly diversifiable, banks can ensure a certain return R^l_{t+1} for their loans. As a result, banks holds a perfectly safe portfolio (it perfectly diversify the idiosyncratic risk involved in lending). The bank loan contract ($\bar{\omega}^j$, Z^j_{t+1}) must satisfy:

$$[1 - F(\bar{\omega}^j)] Z^j_{t+1} B^j_t + (1-\mu) \int_0^{\bar{\omega}^j} \omega R^k_{t+1} Q_t K^j_t dF(\omega) = R^l_{t+1} B^j_t, \tag{15}$$

where F is the cdf of the r.v. ω^j and hence $F(\bar{\omega}^j)$ is the default probability of a j firm or equivalently the fraction of entrepreneurs that defaults at $t+1$ for a given R^k_{t+1}. The left-hand side of equation (15) is the expected return on the loan to the entrepreneurs and the right-hand side is the opportunity cost of lending. By definition, in equilibrium the bank lending rate, Z^j_{t+1}, is higher than R^l_{t+1}.

We assume again R^k_{t+1} is uncertain. Combining equations (13) and (14) with equation (15), we obtain,

$$\left[\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right] R^k_{t+1} Q_t K^j_t = R^l_{t+1}(K^j_t - N^j_{et}), \tag{16}$$

where,

$$\Gamma(\bar{\omega}^j) = \int_0^{\bar{\omega}^j} \omega dF(\omega) + (1 - F(\bar{\omega}^j))\bar{\omega}^j, \qquad G(\bar{\omega}^j) = \int_0^{\bar{\omega}^j} \omega dF(\omega). \tag{17}$$

From equation (16), $\bar{\omega}^j$ depends on the ex post realization of R^k_{t+1}. With aggregate uncertainty, the fraction of defaulting entrepreneurs is uncertain. Then, the expected default probability of an entrepreneur is given by,

$$\mathbb{E}_t\{F(\bar{\omega}^j)\},$$

where recall $F(\bar{\omega}^j)$ is the default probability given a realization of aggregate shock. The expected return (expected profits) to the entrepreneur may be expressed as:

$$\mathbb{E}_t \left\{ \int_{\bar{\omega}^j}^{\infty} (\omega R^k_{t+1} Q_t K^j_t - Z^j_{t+1} B^j_t) dF(\omega) \right\}.$$

Using (14), this is rewritten as,

$$\mathbb{E}_t \left\{ \left[1 - \Gamma(\bar{\omega}^j\right] R^k_{t+1} Q_t K^j_{t+1} \right\}. \tag{18}$$

Entrepreneurs aims to maximize (18) optimally choosing K^j_t and $\bar{\omega}^j$ schedules (as a function of the realized values of R^k_{t+1}) subject to the set of state-contingent constrains implied by the bank loan contract, equation (16), and where B^j_t is solved in bank balance sheet equation (13) taking as given R^k_{t+1}, R_{t+1} and R^l_{t+1}, which are endogenously determined in the general equilibrium. Formally, the optimal problem may be now written as:

$$\max_{K^j_t, \bar{\omega}^j} \mathbb{E}_t \left\{ \left(1 - \Gamma(\bar{\omega}^j)\right) R^k_{t+1} Q_t K^j_t + \lambda^j_{t+1} \left[\left(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right) R^k_{t+1} Q_t K^j_t - R_{t+1} B^j_t \right] \right\},$$

where λ_{t+1} is the Lagrange multiplier associated with the loan contract that requires that equation (16) holds for any realization of R^k_{t+1}. The first order conditions for $\bar{\omega}^j$:

$$-\frac{\partial \Gamma(\bar{\omega}^j)}{\partial \bar{\omega}^j} + \lambda^j_{t+1} \left(\frac{\partial \Gamma(\bar{\omega}^j)}{\partial \bar{\omega}^j} - \mu \frac{G(\bar{\omega}^j)}{\partial \bar{\omega}^j} \right) = 0. \tag{19}$$

The first order conditions for K^j_t:

$$\mathbb{E}_t \left\{ \left(1 - \Gamma(\bar{\omega}^j)\right) R^k_{t+1} + \lambda_{t+1} \left[\left(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right) R^k_{t+1} - R^l_{t+1} \right] \right\} = 0. \tag{20}$$

The first order conditions for λ_{t+1} yield the set of state-contingent constrains implied by equation (16), where,[3]

$$\frac{\partial \Gamma(\bar{\omega}^j)}{\partial \bar{\omega}^j} = 1 - F(\bar{\omega}^j), \qquad \frac{\partial G(\bar{\omega}^j)}{\partial \bar{\omega}^j} = \bar{\omega}^j f(\bar{\omega}^j).$$

Combining equations (19) and (20) yields,

$$\mathbb{E}_t \left\{ \left(1 - \Gamma(\bar{\omega}^j)\right) R^k_{t+1} + \frac{1 - F(\bar{\omega}^j)}{1 - F(\bar{\omega}^j) - \mu \bar{\omega}^j f(\bar{\omega}^j)} \left[\left(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right) R^k_{t+1} - R^l_{t+1} \right] \right\} = 0, \tag{21}$$

Note first that by construction $F(\bar{\omega}^j)$ is positive. If we assume that there is not any asymmetric problem, then $\mu = 0$, and hence equilibrium condition becomes,

$$\mathbb{E}_t \left\{ R^k_{t+1} - R^l_{t+1} \right\} = 0.$$

which is the typical equilibrium condition, where the expected marginal productivity of capital equates the expected marginal cost of capital. As a result, the asymmetric information problem distorts entrepreneur's incentives to demand capital.

[3] We assume $ln(\omega) \sim \mathcal{N}\left(-0.5\sigma^2_\omega, \sigma^2_\omega\right)$ so we have $\mathbb{E}(\omega) = 1$ and then $\Gamma(\bar{\omega}) = \Phi(z - \sigma_\omega) + \bar{\omega}[1 - \Phi(z)]$, $G(\bar{\omega}) = \Phi(z - \sigma_\omega)$, $\partial \Gamma(\bar{\omega})/\partial \bar{\omega} = 1 - \Phi(z)$ and $\partial G(\bar{\omega})/\partial \bar{\omega} = \bar{\omega} \Phi'(z)$, where $\Phi(.)$ and $\Phi'(.)$ are the c.d.f. and the p.d.f., respectively, of the standard normal and z is related to $\bar{\omega}$ through $z = (ln(\bar{\omega}) + 0.5\sigma^2_\omega)/\sigma_\omega$.

To provide more intuition of how the frictions affect the decision process of entrepreneurs, we insert equation (16) into equation (18),

$$\mathbb{E}_t \left\{ \left[1 - \mu G(\bar{\omega}^j)\right] R^k_{t+1} Q_t K^j_t - R^l_{t+1} B^j_t \right\}. \tag{22}$$

where $\mu G(\bar{\omega}^j) R^k_{t+1} Q_t K^j_t$ represents the cost of entrepreneur defaulting. As a result, if the monitoring cost is $\mu = 0$ or equivalently if the asymmetric information is overcome costlessly, we are back to a model without frictions on the credit demand. From equation (22) the interaction of entrepreneur default probability (captured by $\bar{\omega}^j$) and the monitoring cost (μ) leads to a reduction of the net marginal benefit of demanding a unit of bank loans from entrepreneur perspective. This is, ceteris paribus a higher default probability or a higher μ reduces demand of credit. And hence in that sense equation (22) shows how the distortions in the market affects entrepreneur decisions on their demand of credit (B^j_t) or equivalently their purchases of capital (K^j_t). In other words, the asymmetric information problem reduces entrepreneur capacity to demand loans and hence to invest.

Regarding the required return of bank loans, R^l_{t+1}, we assess two alternative cases. First, we can assume that R^l_{t+1} is not contingent to the aggregate risk as it is done in BGG 1999. Second, one of the contribution of this framework that also features frictions on the credit supply is that we can also study the case when aggregate risk is also beared by banks by assuming that R^l_{t+1} is contingent to the aggregate risk. As explained later, the latter is more realistic and it is aligned with the literature that suggests that credit dynamics is essentially driven by credit supply shocks. For comparison reasons, we study the two cases and presents the gains in terms of realism by introducing banks that also absorb some risk. Next, we discuss the reasoning behind these two cases and present their implications.

3.3.1 Non-State-Contingent Bank Loans Required Return

As stated in BGG 1999, since entrepreneurs are risk-neutral and households and risk-averse, in the loan contract entrepreneurs bear all aggregate risk and hence R^l_{t+1} is not contingent to the aggregate risk (or non-state-contingent). Thus, entrepreneurs are willing to guarantee the banks a return on loans that is free of any systematic risk. In other words, conditional to the ex post realization of R^k_{t+1}, the entrepreneur offers a (state-contingent) non-default payment Z^j_{t+1} that guarantees the lender a return equal in expected value to R^l_{t+1}, as suggested by equation (15), that is agreed at t. This implies that equation (15) is a set of restrictions, one for each realization of R^k_{t+1}.

As a result, a low ex post realization of R^k_{t+1} is associated with a high Z^j_{t+1} in order to compensate for the high fraction of entrepreneurs that default due to low average return on capital. This in turn, implies an increase in the cutoff value of the idiosyncratic productivity shock, $\bar{\omega}^j$. In this case, as in BGG 1999, the model implies, reasonably, that default probabilities and default premia rise when the aggregate return to capital is lower than expected. As a result, the expected spread $\mathbb{E}_t\{Z^j_{t+1} - R^l_{t+1}\}$ captures both idiosyncratic and aggregate risk premium.

Notice that since R^l_{t+1} is not contingent to the aggregate risk, any technology shock or capital quality shock is mainly absorbed by entrepreneurs and not by banks. This is, after a capital quality shock, for example, there is going to be a strong reduction in entrepreneurs' equity but not necessarily on banks' equity. As a result, the shock is expected to mainly affect the aggregate credit demand and not the aggregate credit supply.

As a result, the model with non-state-contingent bank loans required return seems qualitatively similar to a model with only credit demand frictions as in BGG 1999.

3.3.2 State-Contingent Bank Loans Required Return

Here, we assume R^l_{t+1} is contingent to the aggregate risk (or state-contingent). This means that R^l_{t+1} depends on the ex post realization of R^k_{t+1}. To our understanding this is a more realistic case. This assumption allows banks equity to absorbe gains or losses that comes from the aggregate return of capital (technology or capital quality shock) and hence the aggregate credit supply is going fluctuate more and hence bank-driven dynamics can explain more the dynamics of credit.

In this case the loan contract states that entrepreneurs are not going to bear all aggregate risk and hence R^l_{t+1} becomes contingent to aggregate risk. In other words, conditional to the ex post realization of R^k_{t+1}, the entrepreneur offers a state-contingent (non-default) payment Z^j_{t+1} that in this opportunity guarantees the lender a return equal in expected value to a state-contingent interest rate R^l_{t+1}.

In particular, we assume R^l_{t+1} is linear on R^k_{t+1}, i.e.,

$$R^l_{t+1} = \xi_t R^k_{t+1},$$

and hence ξ_t is endogenously determined in the general equilibrium. Then, equation (16) becomes,

$$\left[\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right] Q_t K^j_t = \xi_t B^j_t. \tag{23}$$

where, $\bar{\omega}^j$ is independent of the aggregate risk. In this case, the fraction of defaulting entrepreneurs does not depend on the aggregate risk. In other words,

$$\mathbb{E}_t\{F(\bar{\omega}^j)\} = F(\bar{\omega}^j).$$

In this case from equation (14), a low ex post realization of R^k_{t+1} is associated with a lower (non-default) lending rate Z^j_{t+1}. This is because after a low R^k_{t+1} ceteris paribus we might expect that a higher fraction of entrepreneurs defaulting, a lower (non-default) payment Z_{t+1} is required so $\bar{\omega}^j$ and hence the default probability keep unchanged. So, in contrast to the non-state-contingent R^j_{t+1} case, here a low ex-post realization of R^k_{t+1} is associated with a low Z^l_{t+1}. Hence, in this case it is easy to verify that the expected spread $\mathbb{E}_t\{Z^j_{t+1} - R^l_{t+1}\}$ captures only the idiosyncratic risk premium. Finally, note that the rest of equilibrium conditions still holds.

3.4 Capital producers

Competitive capital producing firms make new capital I_t and are subject to adjustment costs. They sell new capital to entrepreneurs at the price Q_t. Given that households own capital producers, the objective of a capital producer is to choose new capital I_t to solve:

$$max\mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left\{ Q_\tau I_\tau - \left[1 + f\left(\frac{I_\tau}{I_{\tau-1}} \right) \right] I_\tau \right\}, \tag{24}$$

where $f(\frac{I_\tau}{I_{\tau-1}})I_\tau$ reflects the physical adjustment costs, with $f(1) = f'(1) = 0$ and $f''(I_t/I_{t-1}) > 0$. From profit maximization, the price of the capital goods is equal to the marginal cost of investment goods as follows:

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}}\right) - \mathbb{E}_t \Lambda_{t,t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right). \tag{25}$$

Profits (which arise only outside the steady state), are redistributed to households as a lump sum. We assume that the cost of adjusting investment follows,[4]

$$f\left(\frac{I_t}{I_{t-1}}\right) = \frac{\varphi_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2.$$

3.5 Entrepreneurial Sector

Entrepreneurs purchase capital each period, which in combination with labor produces (wholesale) output. We assume that production is constant returns to scale. The aggregate production is given by,

$$Y_t = A_t(\psi_t K_{t-1})^\alpha L_t^{1-\alpha},$$

with $0 < \alpha < 1$, where Y_t is the aggregate output of wholesale goods, K_{t-1} is the aggregate of capital purchased at $t-1$, L_t is labor input, and A_t is the exogenous technology process, and ψ_t denotes the capital quality shock, so that $\psi_t K_{t-1}$ is the effective quantity of capital

[4]This function form is also used in de Groot (2014) and Akinci and Queralto (2013).

at time t. We assume the log of A_t and the log of ψ_t follow AR(1) processes. These are,

$$ln(\psi_t) = \rho_\psi ln(\psi_{t-1}) + \epsilon_{\psi,t}, \quad ln(A_t) = \rho_A ln(A_{t-1}) + \epsilon_{a,t},$$

where $\rho_\psi, \rho_a \in (0,1)$, $\epsilon_\psi \sim \mathcal{N}(0, \sigma^2_{\epsilon_\psi})$ and $\epsilon_a \sim \mathcal{N}(0, \sigma^2_{\epsilon_a})$. We assume entrepreneurs sell their output to retailers. Let X_t be the relative price of wholesale goods. Equivalently, X_t is the gross markup of retail goods over wholesale goods. The production technology implies that the ex-post gross return to holding a unit of capital from t to $t+1$ is,

$$R^k_{t+1} = \frac{\frac{1}{X_{t+1}}\frac{\alpha Y_{t+1}}{K_t} + \psi_t Q_{t+1}(1-\delta)}{Q_t}.$$

The demand curve of capital comes from aggregating the equation (21),

$$\mathbb{E}_t \left\{ (1-\Gamma(\bar{\omega}))\, R^k_{t+1} + \frac{1-F(\bar{\omega})}{1-F(\bar{\omega})-\mu\bar{\omega}f(\bar{\omega})} \left[(\Gamma(\bar{\omega}) - \mu G(\bar{\omega}))\, R^k_{t+1} - R^l_{t+1} \right] \right\} = 0\,, \quad (26)$$

which for given values of $R^k_{t+1}\,, R^l_{t+1}$ determines $\bar{\omega}$, and aggregating the bank loan contract, equation (16), which yields,

$$(\Gamma(\bar{\omega}) - \mu G(\bar{\omega}))\, R^k_{t+1} Q_t K_t = R^l_{t+1} \left(Q_t K_t - N_{et} \right)\,, \quad (27)$$

helps to determine the level of capital demand in the economy, K_t.

We assume entrepreneurs offer labor in the labor market. Total labor input L_t is obtained from the following composite of household labor and entrepreneurial labor,

$$L_t = (H_t)^\Omega (H^e_t)^{1-\Omega}.$$

We assume further that entrepreneurs supply their labor inelastically, and we normalize total entrepreneurial labor to unity. In the calibrations below we set the share of income going to entrepreneurial labor to a small value.

Let V^e_t be entrepreneurs equity, which is the accumulated wealth from operating firms, let W^e_t denote the entrepreneurial wage, and let $\bar{\omega}_t$ denote the state-contingent value of $\bar{\omega}$ set in period t. Then, aggregate entrepreneurial net worth at the end of period t, N_{et}, is given by,

$$N_{et} = \gamma V^e_t + W^e_t, \quad (28)$$

where,

$$V^e_t = R^k_t Q_{t-1} K_{t-1} - \left(R^l_t + \frac{\mu \int_0^{\bar{\omega}_t} \omega R^k_t Q_{t-1} K_{t-1} dF(\omega)}{B_{t-1}} \right) B_{t-1}.$$

where γV^e_t is the equity held by entrepreneurs at $t-1$ who are still in business at t. Entrepreneurs who do not survive at t consume the residual equity $(1-\gamma)V^e_t$, This is

$C_t^e = (1-\gamma)V_t^e$. Entrepreneurial equity equals gross earnings on holdings of equity from $t-1$ to t less repayment of borrowings. The ratio $\frac{\mu \int_0^{\bar{\omega}_t} \omega R_t^k Q_{t-1} K_{t-1} dF(\omega)}{B_{t-1}}$ reflects the premium for external finance.

The demand curves for household and entrepreneurial labor are, respectively,

$$\frac{1}{X_t}(1-\alpha)\Omega\frac{Y_t}{H_t} = W_t,$$

$$\frac{1}{X_t}(1-\alpha)(1-\Omega)\frac{Y_t}{H_t^e} = W_t^e.$$

3.6 Retail Sector

Recall that entrepreneurs produce wholesale goods in competitive markets. Retailers, who are monopolistic competitors, buy wholesale goods from entrepreneurs, differentiate them (costlessly), and then re-sell them to households. To motivate sticky prices, we allow for monopolistic competition and (implicit) costs of adjusting nominal prices at the retail level. Retailers are indexed by $z \in [0,1]$.

Let $Y_t(z)$ be the quantity of output sold by retailer z, in units of wholesale goods, and $P_t(z)$ the nominal price. The total final usable goods, Y_t^f, are the composite of individual retail goods:

$$Y_t^f = \left[\int_0^1 Y_t(z)^{(\epsilon-1)/\epsilon} dz\right]^{\epsilon/(\epsilon-1)},$$

with $\epsilon > 1$. The corresponding price index is given by,

$$P_t = \left[\int_0^1 P_t(z)^{1-\epsilon} dz\right]^{1/(1-\epsilon)}.$$

A retailer faces a demand curve given by:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\epsilon} Y_t^f. \tag{29}$$

Retailers choose the sale price $P_t(z)$, taking as given the demand curve and the price of wholesale goods P_t^w. We also assume Price inertia as is standard in the literature. Specifically, We assume that a retailer is free to change its price in a given period only with probability $1-\theta$. Let P_t^* the price set by retailers who are able to change prices at t, and let $Y_t^*(z)$ the demand given this price. They choose P_t^* to maximize expected discounted profits,

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_{t-1}\left[\Lambda_{t,k}\frac{P_t^* - P_{t+1}^w}{P_{t+k}} Y_{t+k}^*(z)\right], \tag{30}$$

where the discount rate $\Lambda_{t,k} = \beta^k \frac{u_{C_{t+k}}}{u_{C_t}}$ is the household stochastic discount factor, which retailers take as given and $P_t^w = P_t/X_t$ is the nominal price of wholesale goods. Before we take partial derivative, we write (30) as,

$$\sum_{k=0}^{\infty}\theta^k\mathbb{E}_{t-1}\left[\Lambda_{t,k}\left(\frac{P_t^*}{P_{t+k}}\left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon}Y^f - \frac{P_{t+k}^w}{P_{t+k}}\left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon}Y^f,\right)\right]. \tag{31}$$

Taking the partial derivative with respect to P_t^*, we obtain,

$$\sum_{k=0}^{\infty}\theta^k\mathbb{E}_{t-1}\left[\Lambda_{t,k}\left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon}Y_{t+k}^*(z)\left(\frac{P_t^*}{P_{t+k}} - \frac{\epsilon}{\epsilon-1}\frac{P_{t+1}^w}{P_{t+k}}\right)\right] = 0. \tag{32}$$

The aggregate price evolves according to,

$$P_t = \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{1/(1-\epsilon)}.$$

3.7 Market Clearing Condition and Monetary Policy

Final output may be either transformed into a single type of consumption good, invested, or used up in monitoring costs. In particular, the economy-wide resource constraint is given by

$$Y_t^f = C_t + C_t^e + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right]I_t + \mu\int_0^{\bar{\omega}_t}\omega R_t^k dF(\omega)Q_{t-1}K_{t-1}.$$

where the aggregate capital stock evolves according to:

$$K_t = I_t + (1-\delta)\psi_t K_{t-1}.$$

The equilibrium, the labor market clears, and labor demand equates labor supply,

$$\frac{1}{X_t}(1-\alpha)\Omega\frac{Y_t}{H_t} = \frac{1}{u_{Ct}}\chi H_t^{\varphi}.$$

The final goods market equilibrium requires,

$$\int_0^1 Y_t(z)dz = \int_0^1 Y_t^j dj = Y_t\,, \tag{33}$$

where from (29),

$$\int_0^1 Y_t(z)dz = \int_0^1\left(\frac{P_t(z)}{P_t}\right)^{-\epsilon}d(z)Y_t^f = (\ddot{p}_t)^{-\epsilon}Y_t^f. \tag{34}$$

with $\ddot{p}_t$ being the price dispersion,[5]

$$\ddot{p}_t = \left[\int_0^1 \left(\frac{P_t(z)}{P_t} \right)^{-\epsilon} d(z) \right]^{-\frac{1}{\epsilon}}.$$

As a result, combining equations (33) and (34), yields,

$$Y_t = (\ddot{p}_t)^{-\epsilon} Y_t^f.$$

We suppose monetary policy is characterized by a simple Taylor rule with interest-rate smoothing. Let i_t be the net nominal interest rate, i the steady state nominal rate, and $Y_t^{f,*}$ the natural (flexible price equilibrium) level of output. Then,

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[i_{ss} + \kappa_\pi \pi_t + \kappa_y (ln(Y_t^f) - ln(Y_t^{f,*})) \right] + \epsilon_{it},$$

where π_t is the inflation rate from $t-1$ to t, the smoothing parameter ρ lies between zero and unity, and where $\epsilon_{it} \sim \mathcal{N}(0, \sigma_{\epsilon_i}^2)$ is an exogenous shock to monetary policy and i_{ss} is the deterministic steady state value of i. The link between nominal and real interest rates is given by the following Fisher relation,

$$1 + i_t = R_{t+1} \frac{E_t P_{t+1}}{P_t}.$$

4 Baseline Simulations

In this section we present the baseline parametrization. We also assess the two cases regarding the required return on bank loans: a non-state-contingent loan return and a state-contingent loan return. In addition, we describe and solve a credit puzzle observed after a contractionary monetary policy when there are not frictions on the credit supply side.

4.1 Parametrization

Table 1 summarizes the parameter values of the model. For the discount factor β, the depreciation rate δ, the capital share α, we choose conventional values. Other conventional parameters as the habit parameter h, the relative utility weight of labor χ, and the Frisch elasticity of labor supply ψ^{-1} are set to 0.815, 3.409, and 1/0.276, respectively, following Primiceri et al. (2006).

Three parameters are specific to the financial intermediaries. The fraction of assets

[5] This becomes irrelevant when solving the model using a first-order approximation.

that can be diverted λ, the proportional transfer to entering banks ζ, and the bank survival probability σ. We set σ to 0.9687, so the average horizon of a banker is eight years. The other two parameters are set to hit the following two targets: an annualized steady state interest rate spread $(R^l - R)$ of one hundred basis points and a steady state bank leverage of four.[6] This results in $\lambda = 0.363$ and $\zeta = 0.0029$.

Table 1: Parameter

Parameters		Values
Households, technology and capital producers		
Discount factor	β	0.990
Habit parameter	h	0.815
Capital share	α	0.330
Depreciation rate	δ	0.025
Utility weight of labor	χ	3.409
Inverse Frisch elast. of labor supply	ψ	0.276
Investment adjustment costs	φ_I	1.000
Banks		
Fraction of assets that can be diverted	λ	0.363
Survival rate of bankers	σ	0.969
Transfer to entering Banks	ζ	0.003
Entrepreneurs		
Households labor share	$(1-\alpha)\Omega$	0.660
Survival probability	γ	0.982
Volatility of the log of the idiosyncratic shock	σ_ω	0.269
Monitoring costs	μ	0.286
Retail firms		
Price rigidity parameter	θ	0.750
Elasticity of substitution between goods	ϵ	4.167
Taylor rule		
Monetary policy response to inflation	κ_π	1.500
Monetary policy response to output gap	κ_y	0.125
Monetary policy rate smoothing	ρ_i	0.800
Shock processes		
Persistence of capital quality shock	ρ_ψ	0.66
Persistence of productivity shock	ρ_a	0.66

Four parameters are specific to the entrepreneurial sector. We set Ω so the household labor share is $(1-\alpha)\Omega = 0.66$, with the share of income accruing to entrepreneurs' labor is equal to 0.01. This results in $\Omega = 0.985$. The other three parameters, the "death rate" of entrepreneurs $1-\gamma$, the variance of the $\ln(\omega)$ and the fraction of realized

[6] The steady state of the economy is presented in Appendix A. Recall the deterministic steady state in the long-term equilibrium is one where the economy is not subject to aggregate shocks but only to idiosyncratic shocks.

payoffs lost in bankruptcy (or the monitoring costs) μ, are chosen to imply the following three conservative steady state outcomes: an annualized risk spread $R^k - R^l$ equal to one hundred basis points, an annualized business failure rate of three percent, and a entrepreneur leverage ratio of two. This results in $\gamma = 0.9822$, $\sigma_\omega = 0.2695$ and $\mu = 0.2862$.[7]

The price rigidity parameter θ is set to 0.75, implying that the average period between price adjustments is four quarters, and the elasticity of substitution between goods ϵ is set to 4.167. The investment adjustment parameter φ_I is set at 1 as in de Groot (2014).

For the Taylor rule, we use the conventional Taylor rule parameters of 1.5 for the κ_π and 0.125 for κ_y and 0.80 for ρ_i.[8] For simplicity, we use minus the price markup as a proxy for the output gap. We solve the model using a first order approximation with Dynare.[9]

In the following subsections, we see the economy after a negative capital quality shock and evaluate the implications of assuming a non-state-contingent loan required return R^l_{t+1} or assuming a state-contingent one. Later, we try to solve a puzzle that consists of a credit reduction observed after a contractionary monetary policy in an economy without credit supply frictions.

4.2 Who Bears The Aggregate Risk Matters

Recall that since households are risk-averse and entrepreneurs are risk-neutral, we might expect a contract with a non-state-contingent R^l_{t+1} and hence whole risk is absorbed by the entrepreneur as in BGG 1999. However, in this framework that also features frictions on the credit supply we can also study the case when risk is also absorbed by banks as observed in real life by assuming that R^l_{t+1} is state-contingent. Hence, in this subsection, we compare the effects of an (aggregate) negative capital quality shock assuming a non-state-contingent R^l_{t+1} with the effects assuming a state-contingent R^l_{t+1}

In general, we would like to discuss how this negative capital quality shock might affect the credit demand and credit supply. On the demand side, a negative capital quality shock decreases marginal return of capital, which in turn reduces the incentives to purchase capital and hence to demand bank credit. Also, a negative capital quality shock decreases firms' profits and hence firms' equity, which in turn increases firms' default probability. This increases the expected monitoring costs. Hence, through the credit demand frictions channel, the shock reduces firms' incentives to purchase capital

[7]Regarding the monitoring cost, this is a bit higher than 0.12, used in BGG 1999, and than 0.20, used in Carlstrom and Fuerst (1997).

[8]Gali 2015 and GKa 2011 set $\kappa_\pi = 1.5$ and $\kappa_y = 0.125$, while GKa 2011 set $\rho_i = 0.80$.

[9]As detailed in Appendix A we solve first for the parameters values associated with the banking sector and then for those associated with the entrepreneurial sector.

and hence to demand bank loans. On the supply side, the negative capital quality might reduce R^l_{t+1}, specially if this is state-contingent, and then bank equity, which tighten the incentive constraint for banks to operate and hence reduces bank capacity to issue loans.

Figure 1 reports the impulse response function of a five percent negative capital quality shock. In general, the directions of the real variables are the expected, consumption, capital and output decrease. However, we observe a different quantitative impact when the aggregate shock (i.e., the aggregate risk) is absorbed only by firms (non-state-contingent R^l_{t+1}) relative to when this is also absorbed by banks (a state-contingent R^l_{t+1}), not only on the financial variables as bank loans, entrepreneur equity, firm equity and spreads, but also on real aggregate variables as consumption, capital and output. First, we explain the differential impact on financial variables and then on real variables.

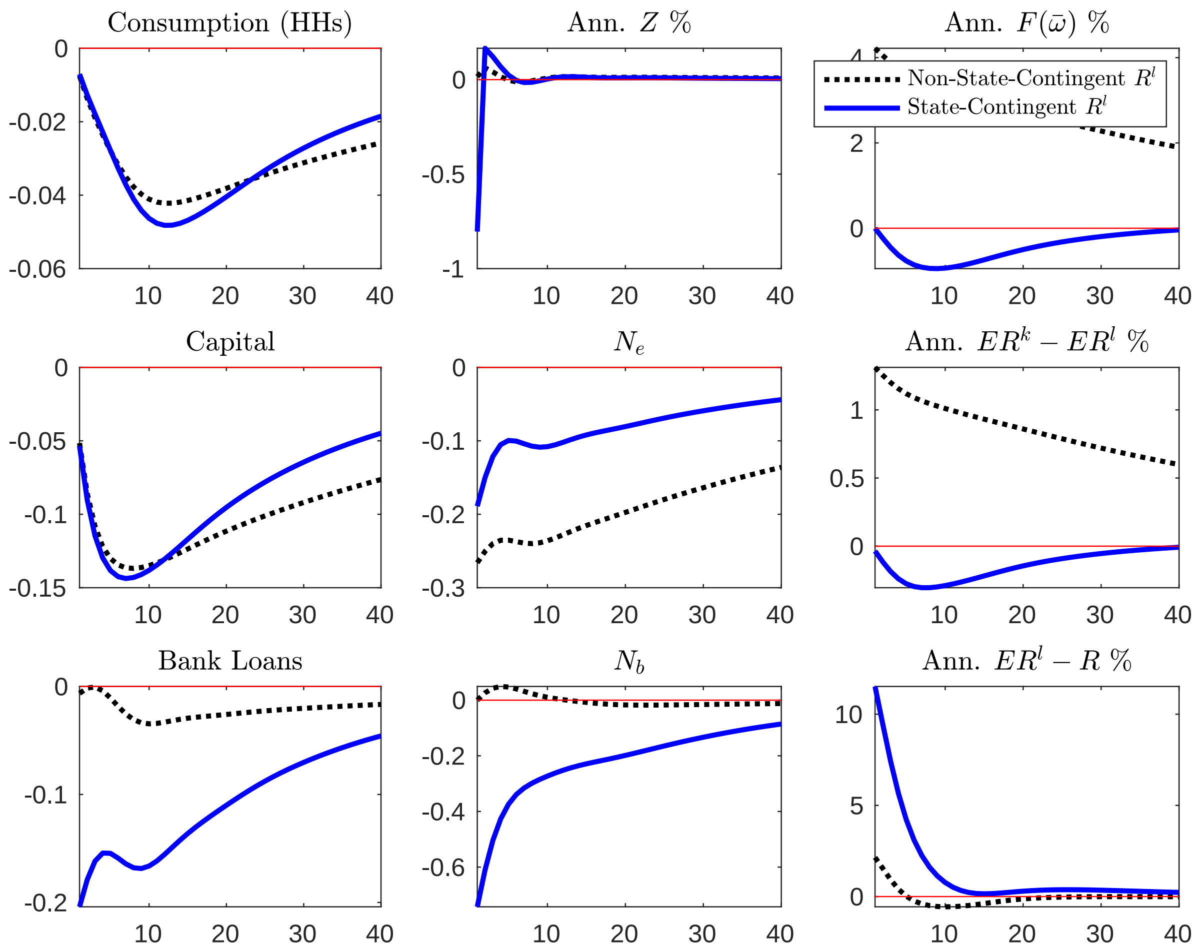
As expected with a non-state-contingent R^l_{t+1} since banks equity is not directly affected by the shock, the role of banks seems null. In other words, banks loans and bank equity are not visually affected. As a result, the drop on capital is associated by an immediate drop on entrepreneur equity. This is a consequence of entrepreneurs bearing the aggregate risk since R^l_{t+1} is not state-contingent. Note that since the entrepreneur equity takes time to recover, the higher the expected spread, $E(R^k - R^l)$, and its persistence captures the smaller capacity of entrepreneur to purchase capital.

When banks absorb some of the aggregate shock, i.e., with state-contingent R^l_{t+1}, we also observe an immediate reduction of bank's equity, and hence a smaller reduction of entrepreneur's equity. Bank's equity reduction reduces bank loan supply. As a result, the drop in aggregate capital is explained by a reduction not only of the entrepreneur's equity but also by a reduction of the aggregate supply of bank loans. In addition, a higher spread $E(R^l - R)$ captures the smaller bank capacity to supply credit, and since the impact on entrepreneur's equity is smaller, the expected spread $E(R^k - R^l)$ does not increases. Indeed, it goes down, which captures the fact that entrepreneurs are now facing higher credit supply.

In this economy the equity position of the agents that absorb the shock is crucial. In other words, the higher the leverage, the better agents are able to handle a shock. When the shock is beared only by agents that holds a relatively low leverage as entrepreneurs that corresponds to the case with a non-state-contingent R^l_{t+1}, we observe smaller fluctuations on real variables as consumption, output and capital. While if the impact of the shock is shared with agents that by definition have a higher leverage as financial intermediaries that corresponds to the case with a state-contingent R^l_{t+1}, the economy is going to suffer more and as a result we observe higher fluctuations on real variables.

These results suggest that modeling frictions on the demand and supply side allows us to have the whole story and hence to observe how the economy might response if we consider the case where banks are bearing aggregate risk or they are not. As explained latter, this characteristics of the model become very important when studying the effects of a conventional monetary policy and a unconventional credit policy.

Figure 1: A five percent negative capital quality shock



All variables are in log deviations from steady-state except spreads, failure rate and (non-deafult) lending rate, shown in level deviation from steady-state.

4.3 Credit Contraction Puzzle

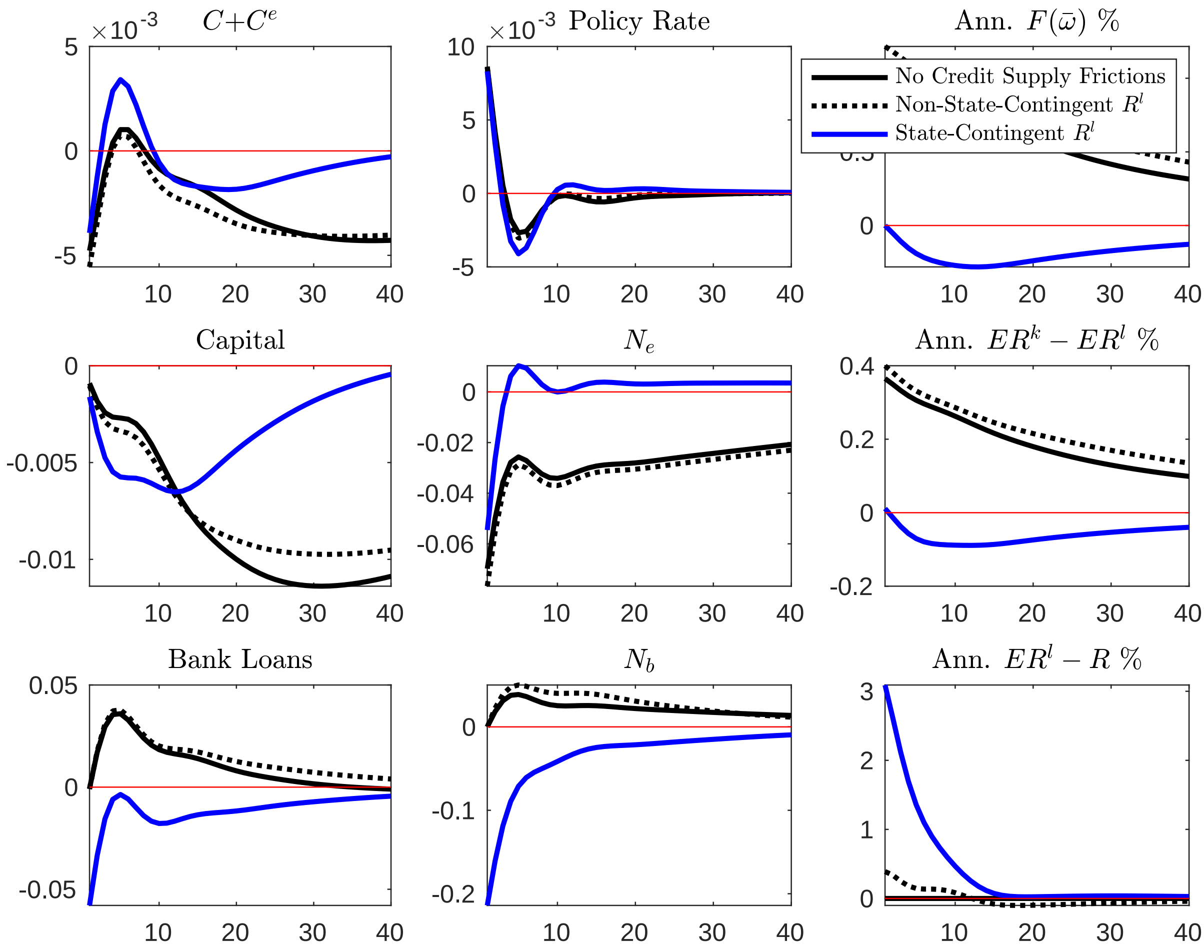
Here, we try to solve a puzzle that consists in observing a credit expansion after a contractionary monetary policy in an economy without credit supply frictions as BGG 1999.

Figure 2 shows the dynamics of the economy after a contractionary monetary policy shock of 25 basis points. In the dynamics of an economy without credit supply frictions, as in BGG 1999, we observe an increase of bank loans after the contractionary monetary policy. This is because the higher policy rate increases households' incentives to save and to hold more bank deposits, which in turn increases credit supply. Notice that this higher credit supply dominates the lower entrepreneur capacity to demand credit since the lower entrepreneur's equity. Hence, in equilibrium we observe an increase of bank loans after the contractionary monetary policy shock. In this paper, this feature is named "the credit contraction puzzle". As expected the same is observed in the model with credit supply frictions and with non-state-contingent R^l_{t+1}. As a result, it is not enough to model credit supply frictions to solve the model.

Figure 2 shows that the model with credit supply friction and with state-contingent R^l_{t+1} helps to solve puzzle. Now the contractionary monetary policy shock in the economy affects also bank's equity. The drop in bank's equity diminishes bank's capacity to supply credit and in equilibrium it dominates the higher household preference to make bank deposits.

Finally, incorporating banks and contracts between entrepreneurs and banks in a more realistic way helps us to capture what is observed after a contractionary monetary policy in reality.

Figure 2: A contractionary monetary policy shock of 25 bps



All variables are in log deviations from steady-state except spreads, failure rate and (non-deafult) lending rate, shown in level deviation from steady-state.

5 Credit Policy

Here, we define the unconventional credit policy. This policy is characterized as central bank (CB) lending facilities to entrepreneurs. We study two variations. In the first case, lending facilities are given to entrepreneurs (firms) directly by the central bank, while in a second case, these credit facilities are given to entrepreneurs indirectly through financial intermediaries (commercial banks).[10] So, in the former case we will have direct CB loans, while in the latter indirect CB loans. Notice that in both cases we consider these as CB loans, since these are going to be funded by the central bank. As we explain in subsection 5.2, under reasonable assumptions these two variations are equivalent. Thus, we mainly discuss the indirect CB loans to study the importance of government credibility regarding the government guarantees.

Independently of whether the loans are given directly by the central bank or through banks, we assume these loans are guaranteed by the government. This is, if an entrepreneur is not able to fully payback the agreed gross return on central bank loans, government transfers are enough resources so it ensures the lender receives the agreed return for these CB loans. We assume government funds their activities with lump-sum taxes to households.

The required return of central bank loans set by the central bank is the risk-free interest rate, which is the opportunity cost of the central bank as well. A necessary condition for this is that the central bank loans cannot be diverted by banks. While this strictly holds with direct central bank lending, with indirect central bank lending as explained later it is still a realistic feature. Since the required return of central bank loans is the risk-free interest rate R_t and since the government is going to bear the risk associated with these loans, the (non-default) lending interest rate of CB loans is R_t as well. Recall that since bank loans are not guaranteed by the government and are subject to the credit supply frictions, in equilibrium the bank loans (non-default) lending rate Z_{t+1}^j contains a firm default risk premium and a premium due to the moral hazard problem between bankers and depositors. Then, CB loans are going to be cheaper than bank loans. In addition, due to the premium associated with the moral hazard problem, the required return of bank loans R_{t+1}^l is higher than the required return of CB loans R_t.

We assume that entrepreneur does not internalize the effects of her capital and credit decisions on the CB loans injections. Hence, from entrepreneur's perspective the marginal cost of external funding is still given by the required return of bank loans. We believe this is a reasonable assumption in a context of unconventional credit policy, in the sense that entrepreneur cannot predict if central bank will provide lending facilities.[11]

[10]For simplicity, we assume that the central bank obtains the funds from households through lump sum taxes.

[11]However, in section 6.5 we develop the context in which CB announces its credit policy rule. So in

Then, entrepreneurs are going to demand and deplete first these CB loans and then banks loans. So, entrepreneurs aim to substitute expensive bank loans for cheap CB loans. As a result, we cannot expect a one to one multiplier effect of the credit policy on aggregate lending. With this in mind, we can preliminary suggest how this credit policy might affect aggregate credit supply and credit demand. On the aggregate credit supply side:

- First, as in GKa 2011 CB loans cannot be diverted by banks[12], the credit policy is reducing the impact moral hazard problem between banks and depositors on this economy. In other words, banks required equity per unit of aggregate credit decreases, which allows for a smaller required return of bank loans for a given aggregate credit level. As a result, CB credit policy increases the aggregate supply of credit.

- Second, since the required return of central bank loans is smaller than those of the traditional bank loans, entrepreneurs now face a limited supply of cheap CB loans in addition to the supply curve of bank loans, equation (9), which is not affected by the credit policy.

In the margin bank loan supply curve matters since the last external funding comes from bank loans. As a result, we can say that the aggregate credit supply curve changes, but the aggregate supply curve of banks loans that in the margin (together with the aggregate demand curve) determines the equilibrium level of credit is not affected. On the aggregate credit demand side:

- First, since the opportunity cost of the central bank is the risk-free interest rate R_t, banks or the central bank require a lower return per unit of these CB loans that the one required by bank loans, i.e., $\mathbb{E}_t\{\Lambda_{t,t+1}\}R_t < \mathbb{E}_t\{\Lambda_{t,t+1}R^l_{t+1}\}$.[13] This reduces entrepreneur default probability and hence reduces the defaulting costs and pushes up the aggregate demand of capital and hence of demand for credit.

- Second, the guarantee of the government avoids that the (non-default) lending interest rate associated with the CB loans reflects any risk-premium. In other words, while the (non-default) lending rate on banks loans is Z^j_{t+1}, the (non-default) lending rate on CB loans is R_t, where $Z^j_{t+1} > R_t$. Ceteris paribus for given capital and equity, CB loans reduce entrepreneur obligations, default probability and reliance on bank loans. This in turn reduces the defaulting costs and pushes up the aggregate demand of capital and hence of credit.

that case entrepreneur internalizes the effects of her decisions on CB loans.

[12]We assume it with indirect CB loans.

[13]Recall that the return required by bank loans is higher than the risk-free interest rate due to the moral hazard problem between banks and depositors and the asymmetric information problem between banks and firms.

Hence, the credit policy stimulates the aggregate credit supply and the aggregate credit demand. In other words, the credit policy is expected to produce an increase of aggregate credit. Clearly, without frictions on the credit supply side, the credit policy does not increase aggregate credit supply and the first effect on aggregate demand is null.

While these arguments might be clear for direct CB loans, as we explain later, under reasonable assumptions these also hold even if CB loans are given through banks as explained in subsection 5.2.

Next, we assess how the credit policy affects the equilibrium conditions. For a better explanation, we first focus on direct CB loans to firms and then on indirect CB loans to firms. In addition, since entrepreneurs will have two sources of external funding (CB loans and bank loans), we discuss the implications of the seniority assumption of CB loans and bank loans.

5.1 Direct Credit to Firms

Here, we introduce to the model direct CB loans to firms. As we will see only the maximization problem of entrepreneurs is affected. In this case, central bank facilitates direct lending B_t^g to firms. Since the opportunity cost of the central bank is the risk-free interest rate R_t and since CB loans are guarantee by the government, the central bank also claims R_t as the (non-default) lending rate of CB loans. As a result, at $t+1$, central bank makes zero profits.

We assume central bank is willing to provide a fraction $\psi_{CB,t}$ of total external funding for entrepreneur j, i.e.,

$$B_t^{g,j} = \psi_{CB,t}(Q_t K_t^j - N_{et}^j). \tag{35}$$

However, we assume that the entrepreneur is not aware of this credit injection rule, equation (35), and hence she cannot internalize the effects of their decisions on $B_t^{g,j}$. At the end of t (going into period $t+1$), entrepreneur j with available net worth N_t^j, borrows B_t^j from banks and $B_t^{g,j}$ from central bank to buy capital K_t^j

$$B_t^{g,j} + B_t^j = Q_t K_t^j - N_{et}^j. \tag{36}$$

Each time an entrepreneur j defaults, she needs to know the payment order to their creditors (CB and banks). There are three alternative assumptions: (1) Both CB loans and bank loans have the same seniority, (2) bank loans have higher seniority and (3) CB loans have higher seniority. In (1) both loans are paid with the same priority and hence each time entrepreneur defaults she transfers her realized capital payoffs to their creditors proportionally. In (2) if entrepreneur defaults it repays first bank loans, and then she cares on repaying CB loans. In (3) the opposite occurs.

In this subsection, we solve the model assuming that bank loans and central bank loans have the same seniority. This is, when entrepreneur defaults at $t+1$, realized revenues are use to repay CB loans and bank loans proportionally to their values at $t+1$. For example, if the debt with the CB is one quarter of the debt with the banks, twenty percent of her realized revenues goes to repay CB loans and eighty percent to repay bank loans.

Banks and government pay monitoring costs to observe entrepreneur' realized return when she defaults. These payments are proportional to what bank and central bank obtain when entrepreneur defaults, respectively. Further, we assume these auditing costs are the same for both banks and for central bank. Hence, total monitoring costs must add up $\mu\omega^j R^k_{t+1} Q_t K^j_t$. This time the threshold value of the idiosyncratic productivity, $\bar{\omega}^j$, is defined as,

$$\bar{\omega}^j R^k_{t+1} Q_t K^j_t = Z^j_{t+1} B^j_t + R_t B^{g,j}_t. \tag{37}$$

Recall that if $\omega^j \geq \bar{\omega}^j$, an entrepreneur does not default and hence is able to fully repay both bank loans and CB loans, otherwise she defaults and repay partially both loans. So, if $\omega^j < \bar{\omega}^j$, government makes sure that CB loans are fully repaid by collecting lump sum taxes. A defaulting entrepreneur receives nothing.

Combining equations (35) and (37) yields

$$\bar{\omega}^j R^k_{t+1} Q_t K^j_t = Z^j_{t+1} B^j_t + R_t B^{g,j}_t \Rightarrow \bar{\omega}^j = \frac{Z^j_{t+1}(1-\psi_{CB,t}) + R_t \psi_{CB,t}}{R^k_{t+1}} \frac{\phi_{et}-1}{\phi_{et}} < \bar{\omega}^j \Big|_{\psi_{CB,t}=0}, \tag{38}$$

where $\phi^j_{et} = K^j_t / N^j_{et}$ is the leverage of entrepreneur j. According to equation (38) ceteris paribus providing to entrepreneurs with a fraction $\psi_{CB,t}$ of cheap loans reduce $\bar{\omega}^j$ and hence entrepreneur default probability, which in turn it results in a lower expected monitoring cots. This increases the marginal benefit of capital and hence increases demand for bank loans. Clearly, the higher the $\psi_{CB,t}$, the stronger the increment of bank loan demand.

The bank loan contract $(\bar{\omega}^j,\ Z^j_{t+1})$, in equation (15), must satisfy that banks always receive a gross return R^l_{t+1} per unit of bank loans, and now becomes:

$$[1-F(\bar{\omega}^j)] Z^j_{t+1} B^j_t + (1-\mu) \int_0^{\bar{\omega}^j} \omega R^k_{t+1} Q_t K^j_t x^j_{t+1} dF(\omega) = R^l_{t+1} B^j_t, \tag{39}$$

where $x^j_{t+1} = Z^j_{t+1} B^j_t / (Z^j_{t+1} B^j_t + R_t B^{g,j}_t)$ is the proportion of the realized revenues that goes to repay bank loans when the entrepreneur defaults. For a given K^j_t the differences with a bank loan contract without credit policy, equation (15), are two: i) only a fraction $(1-\psi_{CB,t})$ of external funding comes from bank loans. This is, without credit policy $B^j_t = Q_t K^j_t - N^j_{et}$, while with credit policy $B^j_t = (1-\psi_{CB,t})(Q_t K^j_t - N^j_{et})$, and ii) only a fraction x^j_{t+1} of $\omega R^k_{t+1} Q_t K^j_t$ goes to payback bank loans each time an entrepreneur defaults.

For convenience the bank loan contract is written as,[14]

$$\left(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right) R^k_{t+1} Q_t K^j_t = R^l_{t+1}\left(Q_t K^j_t - B^{g,j}_t - N^j_t\right) + \Psi(\bar{\omega}^j) R_t B^{g,j}_t, \tag{40}$$

where,

$$\Gamma(\bar{\omega}^j) = \int_0^{\bar{\omega}^j} \omega dF(\omega) + (1 - F(\bar{\omega}^j))\bar{\omega}^j, \qquad G(\bar{\omega}^j) = \int_0^{\bar{\omega}^j} \omega dF(\omega),$$

$$\Psi(\bar{\omega}^j) = (1-\mu)\frac{1}{\bar{\omega}^j} G(\bar{\omega}^j) + (1 - F(\bar{\omega}^j)).$$

In the left-hand side of equation (40) we have the resources available to repay loans. In the right-hand side we have uses of that resources. These resources are used to fully repay the gross return required of bank loans $R^l_{t+1} B^j_t$ and to partially pay the CB loans $\Psi(\bar{\omega}^j) R_t B^{g,j}_t$. So, $\Psi(\bar{\omega}^j) R_t B^{g,j}_t$ is the effective gross return repaid to CB loans by the entrepreneur. It is composed by the amount that non default entrepreneurs transfer to repay central bank loans $(1 - F(\bar{\omega}^j)) R_t B^{g,j}_t$ and the value of seized projects from defaulting entrepreneurs $(1-\mu)\frac{1}{\bar{\omega}^j} G(\bar{\omega}^j) R_t B^{g,j}_t$, net of monitoring costs, to repay central bank loans. Note that each time a entrepreneur defaults, government honours the guarantee and hence transfers resources to ensure CB loans receive the agreed return. This implies that entrepreneurs' transfers are not enough to fully pay CB loans, i.e.,

$$\Psi(\bar{\omega}^j) < 1, \tag{41}$$

or equivalently the effective cost of CB loans from entrepreneur perspective is smaller than the risk-free interest rate, i.e.,

$$\Psi(\bar{\omega}^j) R_t < R_t.$$

This means that government transfers destined to repay CB loans are $(1 - \Psi(\bar{\omega}^j)) R_t B^{g,j}_t$.

In this case the entrepreneur aims to maximize their expected profits,

$$\mathbb{E}_t \left\{ \int_{\bar{\omega}^j}^{+\infty} \left(\omega R^k_{t+1} Q_t K^j_{t+1} - Z^j_{t+1} B^j_t - R_t B^{g,j}_t\right) dF(\omega) \right\},$$

taking as given R^k_{t+1} and $B^{g,j}_t$. Using (37) it yields,

$$\mathbb{E}_t \left\{ \left[1 - \Gamma(\bar{\omega}^j)\right] R^k_{t+1} Q_t K^j_t \right\}, \tag{42}$$

We arrive to an expression identical to the one without credit policy which is independent

[14] Proof in Appendix B.

of the loan seniority assumption. Entrepreneur chooses K_t^j and a schedule for $\bar{\omega}^j$ to maximize equation (42), subject to the state-contingent constraint implied by equation (40).[15] The aggregate credit demand curve, equation (26), becomes,[16]

$$\mathbb{E}_t \left\{ (1-\Gamma(\bar{\omega}))\, R_{t+1}^k + \frac{1-F(\bar{\omega})}{\Upsilon + 1 - F(\bar{\omega}) - \mu\bar{\omega} f(\bar{\omega})} \left[(\Gamma(\bar{\omega}) - \mu G(\bar{\omega}))\, R_{t+1}^k - R_{t+1}^l \right] \right\} = 0. \tag{43}$$

where,

$$\Upsilon = -\frac{\partial \Psi(\bar{\omega})}{\partial \bar{\omega}} \frac{R_t B_t^g}{R_{t+1}^k Q_t K_t} > 0. \tag{44}$$

Since $\Upsilon > 0$, and comparing (43) with (26), we observe that credit policy positively affects the net marginal benefit of capital and hence aggregate demand for bank loans. The intuition is that the credit policy is reducing the transfer from entrepreneur to partially repay CB loans (or equivalently is increasing the transfers from government to repay CB loans), which in turn reduces entrepreneur default probability and hence the expected defaulting costs, which in turns raises incentives to demand capital and hence bank loans.

Inserting equation (40) into equation (42) yields,

$$\mathbb{E}_t \left\{ \left[1 - \mu G(\bar{\omega}^j \right] R_{t+1}^k Q_t K_{t+1}^j - R_{t+1}^l (K_t^j - B_t^{g,j} - N_t^j) - \Psi(\bar{\omega}^j) R_t B_t^{g,j} \right\}. \tag{45}$$

Equation (45) says that the marginal cost of capital is not affected directly by the credit policy and so it continues to be R_{t+1}^l. This is because entrepreneur is not internalizing the effects of their capital decision on B_t^j since they are not aware of the credit policy rule, equation (35). Otherwise, they are aware that one unit of external funding is funded with both cheap CB loans and bank loans, reducing the marginal cost of capital from entrepreneur's perspective. This case is studied in subsection 6.5. And, V_t^e becomes,

$$\begin{aligned} V_t^e &= R_t^k Q_{t-1} K_{t-1} - R_t^l B_{t-1} - (1 - F(\bar{\omega}) + \frac{1}{\bar{\omega}} G(\bar{\omega})) R_{t-1} B_{t-1}^g \\ &- \mu \int_0^{\bar{\omega}_t} \left(\omega R_t^k Q_{t-1} K_{t-1} - \frac{\omega}{\bar{\omega}} R_{t-1} B_{t-1}^g \right) dF(\omega). \end{aligned}$$

where $(1 - F(\bar{\omega}) + \frac{1}{\bar{\omega}} G(\bar{\omega})) R_{t-1} B_{t-1}^g$ are the resources taken from entrepreneur's profits that goes to repay central bank loans, that by definition are not enough for a full repayment.

We can preliminary say that according to (38) for a given K_t credit policy reduces entrepreneur default probability and from (43) for a given $\bar{\omega}$ the net marginal benefit of capital increases. Both findings positively affect aggregate credit demand.

Next, we set an equation that allows us to understand the size of the government

[15] The first order conditions are found in Appendix B.

[16] This is obtained from aggregating equation (21) in Appendix B.

transfers (subsidies) due to the government guarantee on CB loans and the impact of no government credibility, which is explained in subsection 5.2.1. Recall these transfers are destined to make sure the central bank loans are fully repaid, which are funded with lump sum taxes. Since banks perfectly diversify the idiosyncratic risk, the following equation equates all the (entrepreneur and government) resources available to fully repay both bank loans and central bank loans (left-hand side) with the total return required by whole loans (right-hand side) for a given realization of the aggregate shock.

$$
\begin{aligned}
& [1-F(\bar{\omega})]Z_{t+1}B_t + [1-F(\bar{\omega})]R_t B_t^g + (1-\mu)\int_0^{\bar{\omega}} \omega R_{t+1}^k Q_t K_t dF(\omega) x_{t+1} \\
+ & \int_0^{\bar{\omega}} \omega R_{t+1}^k Q_t K_t dF(\omega)(1-x_{t+1}) + S_{t+1} = R_{t+1}^l B_t + R_t B_t^g \\
+ & \mu \int_0^{\bar{\omega}} \omega R_{t+1}^k Q_t K_t dF(\omega)(1-x_{t+1}). \qquad (46)
\end{aligned}
$$

In the left hand side we have, in black the entrepreneur' resources destined to fully repay bank loans, and in red entrepreneur' resources destined to partially repay CB loans, and in blue the government subsidies, S_{t+1}. So, S_{t+1} is computed as the difference between the gross return of CB loans (plus monitoring costs[17]) and entrepreneur's resources used to pay those, i.e.,

$$
S_{t+1} = R_t B_t^g - [1-F(\bar{\omega})]R_t B_t^g - (1-\mu)\int_0^{\bar{\omega}} \omega R_{t+1}^k Q_t K_t dF(\omega)(1-x_{t+1}), \qquad (47)
$$

Notice that by definition, when inserting (47) into (46) we get the bank loan contract, equation (39). And that the right-hand side of equation (47) is a different way to write $(1-\Psi(\bar{\omega}))R_t B_t^g$. For illustrative purposes, we rewrite equation (46), as,

$$
[1-F(\bar{\omega})](Z_{t+1}B_t + R_t B_t^g) + (1-\mu)\int_0^{\bar{\omega}} \omega R_{t+1}^k Q_t K_t dF(\omega) + S_{t+1} = R_{t+1}^l B_t + R_t B_t^g. \quad (48)
$$

where S_{t+1} is rewritten as,

$$
S_{t+1} = \int_0^{\bar{\omega}^j} \left[R_t B_t^g - (1-\mu)(1-x_{t+1})\omega R_{t+1}^k Q_t K_t\right] dF(\omega), \qquad (49)
$$

and hence we can clearly see that government subsidies complement entrepreneur' resources destined to repay central bank loans net of monitoring costs, $(1-\mu)(1-x_{t+1})\omega R_{t+1}^k Q_t K_t$, to make sure central bank loans are fully paid and hence receive the agreed gross return, $R_t B_t^g$.

In this case, it doesn't make sense assess the impact of no government credibility on the direct CB loans to firms. In other words, it is not realistic to say that ex-ante

[17] Recall the monitoring costs associated with entrepreneur revenues used to repay CB loans are paid by the government.

the central bank does not believe that the government ex-post wont guarantee the CB loans, since they are part of the same organization. This assumption makes more sense when the CB loans are given through banks, in this case bank might believe that the government is not going to be willing or able to guarantee the CB loans. So, we study the effects of no government credibility within the next subsection.

5.2 Indirect Credit to Firms Through Banks

Here, we study the implications of giving CB loans through banks. As it is showed next, under reasonable assumptions, this policy is equivalent to direct CB loans to firms.

In this case we assume that central bank gives funding to banks with the commitment that (1) banks give at least the same amount of loans (CB loans) to entrepreneurs and (2) charge some agreed lending interest rate to entrepreneurs for these central bank loans. This is given in three steps. Step 1: CB offers the funds in an auction. Step 2: banks demand these funds and propose a (non-default) lending rate to be charged to entrepreneurs. Step 3: CB gives the funding to those banks that offer the lowest lending rate. Since all banks are identical and perfectly compete with other banks, at the end of the day they all offer the same lending rate, which, as explained later, is going to be the risk-free interest rate. We assume that CB can costlessly enforce banks to perform (1) and (2). Recall that since the opportunity cost of the central bank is the risk-free rate, then it claims to banks a risk-free rate for funding CB loans.

We assume that there is not a moral hazard problem between banks and CB as it exists between banks and depositors. In other words, we assume that bankers cannot divert the bank assets (CB loans) that are funded by the central bank. We believe this is a realistic assumption, since the central bank might have more monitoring and enforcement power over banks than depositors.

Furthermore, we assume that banks do not incur in any administrative cost (or these are negligible) for collecting CB funding and giving these to entrepreneurs as CB loans. Hence, the cost for banks of issuing CB loans is just the interest rate claimed by the central bank, which is the risk-free interest rate.

Also, since CB loans issued by banks are guaranteed by the government, the (no-default) lending interest rate on these loans asked by banks does not contain any risk premium. This means that the (non-default) lending rate for the CB loans is going to be equal to the required return for CB loans. Thus, we implicitly assume that there is government credibility. In other words, banks ex-ante believe that government will honour the guarantee for the CB loans. In this dynamic framework, we are modeling like, the ex-ante credibility responds to the observed behavior that the central bank has fulfilled its promise in the past. Thus, we can argue that even ex-post, the government always honor the guarantee.

Finally, given these previous assumptions discussed and banks perfectly compete to obtain the CB loans, banks that get the CB loans are those who commit to charge a (no-default) lending rate for the CB loans equals to the risk-free interest rate R_t.[18]

In equilibrium, banks issue CB loans by exactly the same amount of funds received from the CB.[19] Hence, banks balance sheet becomes,

$$B_t^g + B_t = B_t^g + D_t + N_{bt}, \tag{50}$$

where B_t^g is not only the amount of CB loans but also the amount of funds received from CB to finance these loans. As a result, equation (50) collapses to the balance sheet with direct CB loans, equation (4), and hence bank loans are funded by both households' deposits and bank equity.

We assume that the government pays the monitoring costs of observing the entrepreneur's realized revenues that goes to repay CB loans. Since CB loans are guaranteed, banks do not have any incentives to pay the monitoring costs associated to observe realized revenues that goes to pay CB loans. Similarly, the central bank does not have any incentive to do so due to the government guarantee. Hence, we believe it is a reasonable assumption to say that since the government take care of her budget, she is the more interested in recover as much as it can from entrepreneur revenues and hence pays the monitoring costs.

Thus, equilibrium conditions of the banking sector are not affected. This is because banks profits are not affected given that by definition CB revenues are perfectly cancelled out with their own funding costs.

In summary, under all these assumptions, direct CB loans are equivalent to indirect CB loans. Also, since this argument does not depend on CB loans seniority, it holds for any seniority assumption.

In the case of same seniority assumption, the bank loan contract becomes as in equation (46). Hence, since we assume that banks ex-ante believe that government will honour her guarantee, and S_{t+1} takes the same form in (47) and hence the bank loan contract is indeed equivalent to the bank loans contract with direct CB loans.

Next, we discuss what happens if there is not government credibility. This is if banks ex-ante believe that government is not willing or able to guarantee CB loans.

[18]Notice that it doesn't make sense that banks propose a (non-default) lending rate bellow the risk-free interest rate.

[19]Clearly, banks are not willing to issue central bank loans funded with households deposits and/or bank equity, since the cost of collecting households deposits end up being higher than the risk-free interest rate, due to the moral hazard problem between banks and households, which is the return that they will obtain for issuing central bank loans.

5.2.1 No Government Credibility

Here, we study the case when banks ex-ante always believe that the government is not going to honour CB loans guarantees. This could be because in the past the government failed to pay the guarantees. In this dynamic framework, since the government observes that its guarantee announcement has not any (ex-ante) impact, CB has not ex-post incentives to claim CB loans and hence in equilibrium we assume that ex-post government does not honour the guarantee. We show that in this case of no government credibility, credit policy effectiveness is diminished.

We assume that the central bank charges a very high penalty to banks in case they do not fully repay the agreed return on CB funding. Thus, to make sure CB loans are fully repaid banks have to raise the (non-default) lending rate associated with bank loans so these additional revenues aim to compensate for the resources that bank believes are not going to be transferred from the government each time entrepreneurs default on CB loans. Then, the higher (non-default) lending rate of bank loans increases the entrepreneur default probability, which in turn increases the expected monitoring costs and reduces entrepreneurs' incentives to demand credit. As a result, the non government credibility diminishes the positive effect of credit policy on aggregate credit demand.

In other words, if there is zero government credibility, the credit policy should have a small effect in the economy. This is because the positive effects of having cheap CB loans in the economy are expected to be at least partially cancelled by the negative effects of having more expensive bank loans. Equivalently, the goal of the credit policy is to provide cheap credit to firms and absorb the riskiness of these loans but with zero credibility, banks do not believe government will absorb the risk and hence they have to bear the CB loans risk. So, banks have to charge a higher lending rate to compensate for the risk-taking on CB loans. Notice that government credibility does not affect aggregate credit supply, but aggregate demand.

Therefore,one policy recommendation is to monitor the lending interest rate of CB loans as an indicator of the government credibility and as an indicator of the effectiveness of central bank credit policy.

With government credibility, and under same seniority assumption, the bank loan contract is given by equation (46) at its individual level, with government subsidies defined in equation (47). Without government credibility, banks ex-ante believe that will not receive any subsidy for the government, i.e., ex-ante banks believe $S^j_{t+1} = 0$, and hence equation (46) at its individual level becomes,

$$[1 - F(\bar{\omega}^j)](Z^j_{t+1}B^j_t + R_t B^{g,j}_t) + (1-\mu)\int_0^{\bar{\omega}^j} \omega R^k_{t+1} Q_t K^j_t dF(\omega) = R^l_{t+1}B^j_t + R_t B^{g,j}_t.$$

Using (37), it yields,

$$[\Gamma(\bar{\omega}^{g,j}) - \mu G(\bar{\omega}^{g,j})]R^k_{t+1}Q_tK^j_t = R^l_{t+1}B^j_t + R_tB^{g,j}_t. \quad (51)$$

Since in equilibrium, $B^{g,j}_t = \psi_{CB,t}B^j_t$, it becomes,

$$[\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)]R^k_{t+1}Q_tK^j_t = \left(R^l_{t+1} - \frac{\psi_{CB,t}}{1+\psi_{CB,t}}(R^l_{t+1} - R_t)\right)(B^j_t + B^{g,j}_t). \quad (52)$$

Comparing equation (52) with (16) or with (23), it seems that no government credibility reduces the effectiveness of the CB credit policy, but not the whole the effect. Intuition is as follows:

- Recall that the two direct benefits of the central bank credit policy are firstly it makes that banks require a lower return per unit of the CB loans, and secondly the government guarantee avoids that the (non-default) bank lending rate associated with the CB loans reflect any risk-premium. These two benefits reduce entrepreneur default probability and hence positively affects aggregate demand of credit.
- Without government credibility, this second benefits disappear, while the first do not. As suggested by equation (52) credit policy still benefits economy by affecting the aggregate supply of credit. In other words, the credit policy still allows entrepreneurs to get on average cheap loans, where $\frac{\psi_{CB,t}}{1+\psi_{CB,t}}(R^l_{t+1} - R_t)$ captures the effects of cheap CB loans in terms of reduction on the risk premium that banks charge to the average loan to entrepreneurs.
- If we further assume that there are not credit supply frictions (i.e., $R^l_{t+1} = R_t$), the impact of the CB credit policy is null. In this case, bank loan contract, equation (51), becomes,

  $$[\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)]R^k_{t+1}Q_tK^j_t = R_t(K^j_t - N^j_t), \quad (53)$$

  which is the same loan contract under not credit policy, equation (16), with $R^l_{t+1} = R_t$. In this case, K^j_t and $\bar{\omega}^j$ are going to be the same that under not credit policy.[20] From equation (37),

  $$\bar{\omega}^j R^k_{t+1}Q_tK^j_t = Z^j_{t+1}B^j_t + R_tB^{g,j}_t, \quad (54)$$

  since the left hand side does not change, in order to keep the right hand side unchanged, bank need to increases the (no-default) interest rate of bank loans, Z^j_{t+1}, so it compensates the lower return, R_t, of the CB loans. In order words, for a given K^j_t, banks need to increases Z^j_{t+1} so the entrepreneur default probability is unchanged.

[20]Appendix F reports the first order conditions the entrepreneurs that are identical to the case of no credit policy and no credit supply frictions.

- Hence, in the model with credit supply frictions, the loan contract in equation (52), the average required gross return per unit of loans is smaller (i.e., $R^l_{t+1} - \frac{\psi_{CB,t}}{1+\psi_{CB,t}}(R^l_{t+1} - R_t) < R^l_{t+1}$). This means that entrepreneur default probability could be higher, i.e., $\bar{\omega}^j$ increases. Then, from equation (54), it requires an increase of Z^j_{t+1}, which is even higher than the one without credit supply frictions. We conclude that if banks believe the government does not fully guarantee their CB loans, banks claims a higher (non-default) lending rates for banks loans, Z^j_{t+1}, so it compensates for the government subsidies not received at $t+1$. This in turn pushes up entrepreneur default probability and hence diminishes the effects of credit policy.

5.3 Seniority implications

Here, we assess how the seniority assumption affects the impact of the credit policy.[21] As seen before, the effectiveness of the CB credit policy depends also on the level of risk that is being absorbed by the government and hence on the size of the guarantees (or subsidies) that are provided by the government in order to ensure CB loans are fully repaid. In other words, we might expect that the higher the government transfers, the stronger the impact of the credit policy.[22]

When both bank loans and CB loans have the same seniority, all loans are paid with the same priority. This is, each time an entrepreneur defaults, entrepreneur realized revenues are distributed proportionally to the size of both kinds of debts.

When bank loans have higher seniority, if an entrepreneur defaults, bank loans are paid first and hence by definition a greater fraction of realized revenues goes to repay bank loans.[23] This means that the amount recovered to repay banks loans increases and consequently the amount that goes to repay CB loans falls. Consequently, government guarantees are higher and hence government ends up giving more subsidies to repay CB loans. Since bank loans are paid first, the probability that entrepreneur defaults on bank loans decreases. It clearly pushes down the (non-default) interest rate rate Z^j_{t+1}, which in turn reduces entrepreneur default probability. This reduces expected monitoring costs and hence increases the net marginal benefits of capital, which in turn pushes up the aggregate credit demand.

When central bank loans have lower seniority, the opposite occurs. If an entrepreneur defaults, central bank loans are paid first. Since now the the amount recovered to payback CB loans are higher, this implies that government subsidies are smaller. In this case a

[21]In Appendix C we solve the maximization problem of entrepreneurs when CB loans have higher seniority and in Appendix D when CB loans have the higher seniority. As suggested in subsection 5.2, the analysis holds independently if we are talking about direct CB loans or indirect CB loans.

[22]In Appendix G we formally explore the government subsidies to repay central bank loans.

[23]There are going to be some entrepreneurs (with very low ω^j) that only are able to partially repay bank loans and repay nothing to CB loans.

smaller fraction goes to repay bank loans. Since CB loans are paid first, the probability that entrepreneur defaults on bank loans increases, it pushed up the (non-default) interest rate rate Z^j_{t+1}, which in turn increases entrepreneur default probability. This pushes down the aggregate credit demand.

6 Credit Policy Simulations

In order to quantitatively compare the effects of the credit policy, we describe the credit policy as an exogenous rule. We use an exogenous rule since under different assumption (e.g., seniority assumption, credibility assumption) an endogenous rule that responds to different general equilibrium effects might lead to different size effects of the credit policy intervention and then it becomes not comparable. In particular, we assume that the fraction of external funding that comes from the central bank resources $\psi_{CB,t}$ follows an AR(1) process,

$$\psi_{CB,t} = \rho_{CB}\psi_{CB,t-1} + \epsilon_{CB,t}, \tag{55}$$

with $\rho_{CB} = 0.95$. In the baseline simulation we set $\epsilon_{CB,1} = 10\%$ and assume future ϵ_{CB} equals to zero. This implies that immediately with the negative shock (e.g. capital quality shock) CB loans intervention will represent 10% of the credit market and then will decreases slowly. This exogenous rule is used in subsections 6.1, 6.2, 6.3 and 6.6.

And, in order to study the effectiveness of the design of a policy rule, we makes the credit policy rule endogenous. This is considered in subsections 6.4 and 6.5. In this case we assume that $\psi_{CB,t}$ follows the following dynamics,[24]

$$\psi_{CB,t} = \nu\mathbb{E}_t\{spread_{t+1} - spread_{ss}\}, \quad \nu > 0, \tag{56}$$

where $spread_{t+1} \in \{R^k_{t+1} - R^l_{t+1},\ R^l_{t+1} - R_t\}$. Equation (56) describes an endogenous credit policy rule of injecting central bank loans in the credit market. Hence, in this case credit injection depends on the deviation of some spread form its long-term value. While $R^k_{t+1} - R^l_{t+1}$ captures entrepreneur's capacity to purchases capital and hence to demand credit (external funding), $R^l_{t+1} - R_t$ captures banks' capacity to supply credit and hence to capture households' deposits. The latter is also known in the literature as the credit spread. In general, the higher spread the smaller capacity of lending. For example, higher credit spread implies that banks are facing problems to issue credit per unit of bank net worth and hence they need more equity to keep the same level of credit supply. A period where the credit spread rise sharply can be defined as a credit supply crisis. Similarly, a period where $R^k_{t+1} - R^l_{t+1}$ rises sharply can be defined as a credit demand crisis.

[24] This follows the spirit of GKa 2011.

Note that it could be the case that $\psi_{CB,t} < 0$, in this case the credit policy rule will offer firms to hold some risk-free assets in the CB (in the case of direct CB loans) or to hold bank safe deposits on banks (in the case of indirect CB loans). As a result, this rule is indeed a countercyclical. We set $\nu = 40$ as its baseline value.

Note that it is possible to assume different rules. For example, credit injection might depend on the deviations of the credit to GDP ratio from its long term, or from the percentage deviations of credit or output from its long-term value.[25] However, for illustrative purposes we set this very well used rule in several papers that study the effects of unconventional credit policies (see GKa 2011, GKi 2011 and GKQ 2012)

By definition, the unconventional credit policy rule does not affect the deterministic steady state. Next, we simulate the model to evaluate the effects of the unconventional credit policy and the implications of the seniority assumption, government credibility, credit policy design, announcing the credit rule and the unconventional feature.

Unless otherwise stated, the simulations respond to a five percent negative capital quality shock. And in our baseline model, banks bear the risk, i.e., R^l_{t+1} is state-contingent, CB loans are given through banks and all loans have the same seniority.

Note that our measure of effectiveness of the credit policy is related to how much it diminishes the impact of the negative capital quality shock on real variables. Hence, the focus on financial variable is to assess how these might affect the dynamics of real variables. For simplicity, we focus on only one variable, the aggregate capital, to measure the effectiveness of credit policy. This is, the larger the reduction of capital the smaller the effectiveness of the credit policy.

6.1 Credit Policy Effects

As figure 3 reports (baseline) the credit policy, characterized by the exogenous rule, equation (55), reduces the effects of the negative capital quality shock. Since we have already discussed, in subsection 4.2, the effects of a negative capital quality shock, here we focus on how the credit policy reduces the negative effects of the shock. On the aggregate credit supply side, as explained before the fact that the required return of CB loans is smaller than bank loans and that CB loans cannot be diverted, the credit policy increases the aggregate supply of credit. Recall that on the aggregate demand credit side the following occurs:

1. New cheap external funding reduces entrepreneurs' obligations and hence their default probability. This is because for a given K^j_t, entrepreneurs substitute bank loans with CB loans. A lower default probability decreases the costs of defaulting

[25] A study of the effectiveness of different rules of countercyclical capital buffers on macroeconomic and financial stability is presented in Pozo (2020).

(total monitoring costs) which in turn pushes upward entrepreneurs' incentives to purchase capital and hence to demand credit.

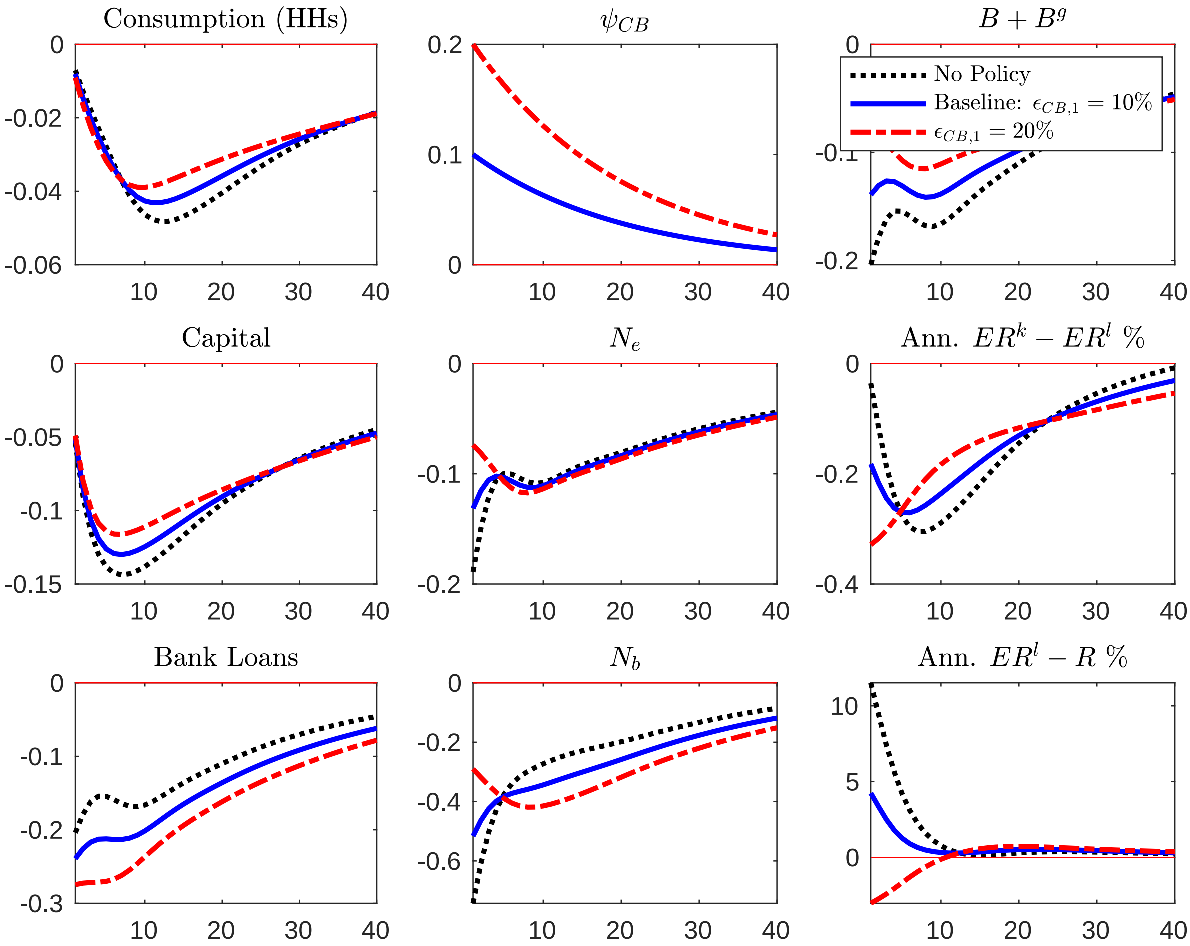
2. Government guarantees allow the central bank to extend CB loans with the (non-default) lending rate being the risk-free interest rate. This reduces the average (non-default) lending rate, which as before reduces further entrepreneur's default probability. As argued above, this increases aggregate credit demand.

As a result, the effect of credit policy is not only that the entrepreneurs substitute expensive loans for cheap ones, but maybe more importantly, it reduces entrepreneur's default probability and creates incentives to purchases more capital and take more credit, and also it reduces the frictions on the credit supply and hence increases aggregate credit supply. In Appendix E we assess the impact of the policy when there are only frictions on the credit demand side or credit supply side, respectively. In our baseline calibration we might say that the credit policy is more effective on reducing credit supply frictions than credit demand frictions. However, as we will see latter this is not necessarily true if we target a higher entrepreneur's default probability, i.e., if there is a higher uncertainty in the economy.

Quantitatively, figure 3 shows that a persistent exogenous credit policy of 10% over total aggregate credit, diminished the reduction of capital in 200 basis points. This is essentially driven by a smaller reduction on aggregate credit of around 300 basis points. Note that bank loans decreases now more because these are being substituted with CB loans. However, the magnitude of this additional reduction is smaller than the increase of CB loans. This in turn drives the higher aggregate credit in equilibrium.

In addition, figure 3 shows that if we double the intensity of credit policy, i.e. we double the CB loans participation from $\epsilon = 10\%$ to $\epsilon = 20\%$, we observe that the credit policy diminishes by more the negative effects of the negative capital quality shock. For example, with an initial CB loans participation of 10%, the capital maximum relative reduction is diminished in 200 basis points, while for a CB loans participation of 20%, this is diminished in 400 basis points.

Figure 3: A five percent negative capital quality shock



All variables are in log deviations from steady-state except spreads and CB loans share, shown in level deviation from steady-state.

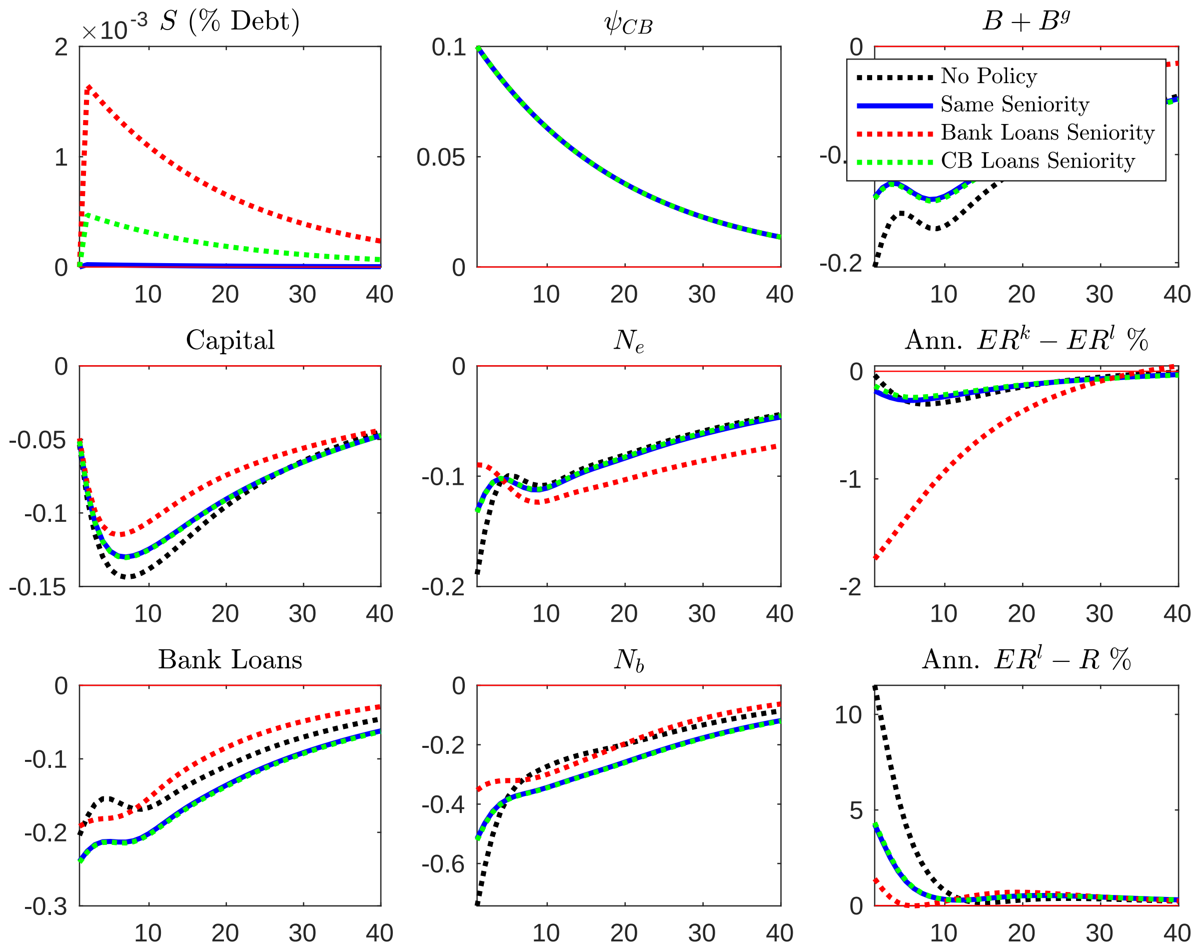
6.2 Seniority

We assess the effects of different seniority assumptions. To do so we consider the case of a state-contingent R^l_{t+1} and the exogenous credit policy rule, in equation (55).

Figure 4 suggests, by observing aggregate capital, that compared to the baseline case (same seniority), when bank loans have higher seniority, the impact of the credit policy is stronger. As discussed in subsection 5.3 this is because when bank loans have higher seniority it leads to lower default probability on bank loans, which in turn reduces the (non-default) lending rate, Z^j_{t+1}, and hence the entrepreneur's default probability. This reduces expected monitoring costs and hence pushes up entrepreneurs' incentives to purchase capital and hence expands the effectiveness of the credit policy.

However, when CB loans have higher seniority, the impact of the credit policy is similar to the same seniority case. This is because we assume the monitoring costs are paid by the government. So, on the one hand, due to higher central bank loans seniority, CB loans are paid first and it leaves less resources for bank loans, but on the other hand, more monitoring costs are being paid by the government as more entrepreneur's profits are going to pay CB loans.[26]

Figure 4: A five percent negative capital quality shock



All variables are in log deviations from steady-state except spreads and CB loans share, shown in level deviation from steady-state.

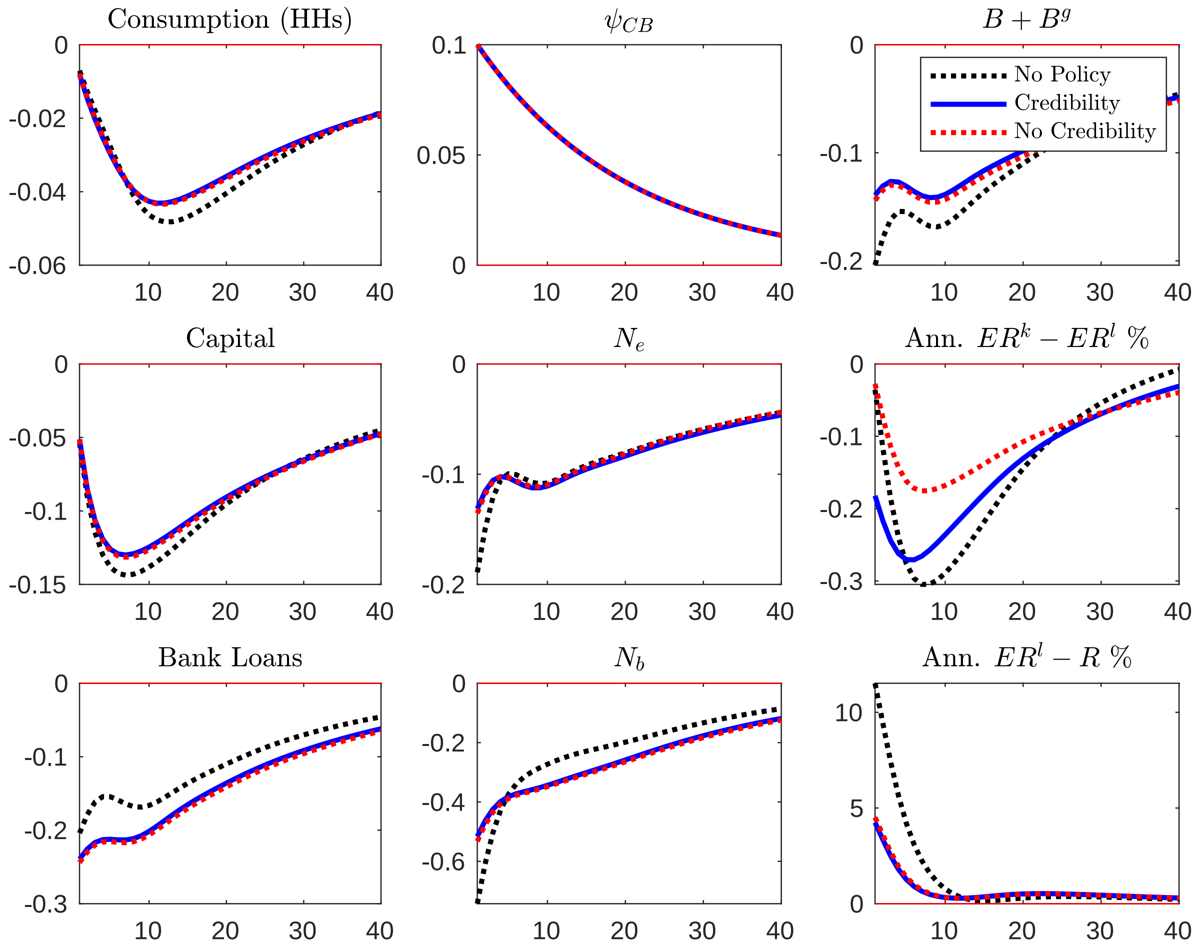
6.3 No Government Credibility

According to figure 5, by observing aggregate capital, and with the baseline calibration, no government credibility does not diminish significantly the effectiveness of the credit policy. In other words, in the baseline calibration, with state-contingent R^l_{t+1}, government guarantees do not seem to be crucial on reducing the impact of the shock. This could be because entrepreneurs are not absorbing too much risk since R^l_{t+1} is state-contingent and hence S_{t+1} is not very affected. However, according to figure 14, in

[26]These arguments also hold for the case of non-state-contingent R^l_{t+1} see figure 13 in Appendix I.

Appendix I, where R^l_{t+1} is not state-contingent, this is not the main explanation and hence government guarantees do not seem so relevant. Hence, in the baseline calibration what essentially explain the reduction on the impact of the shock is the fact that central bank loans reduces aggregate credit supply frictions. In particular, CB loans cannot be diverted by bankers and/or the fact that the required return of CB loans is smaller that the required return of bank loans. Hence, in a model with only credit demand frictions, the effect of the credit policy is very low, as suggested in figure 15, in Appendix I, under our baseline calibration. Notice also that in the case the credibility disappears completely, the impact of the credit policy is null, as suggested in subsection 5.2.1.

Figure 5: A five percent negative capital quality shock: State-Contingent R^l_{t+1}



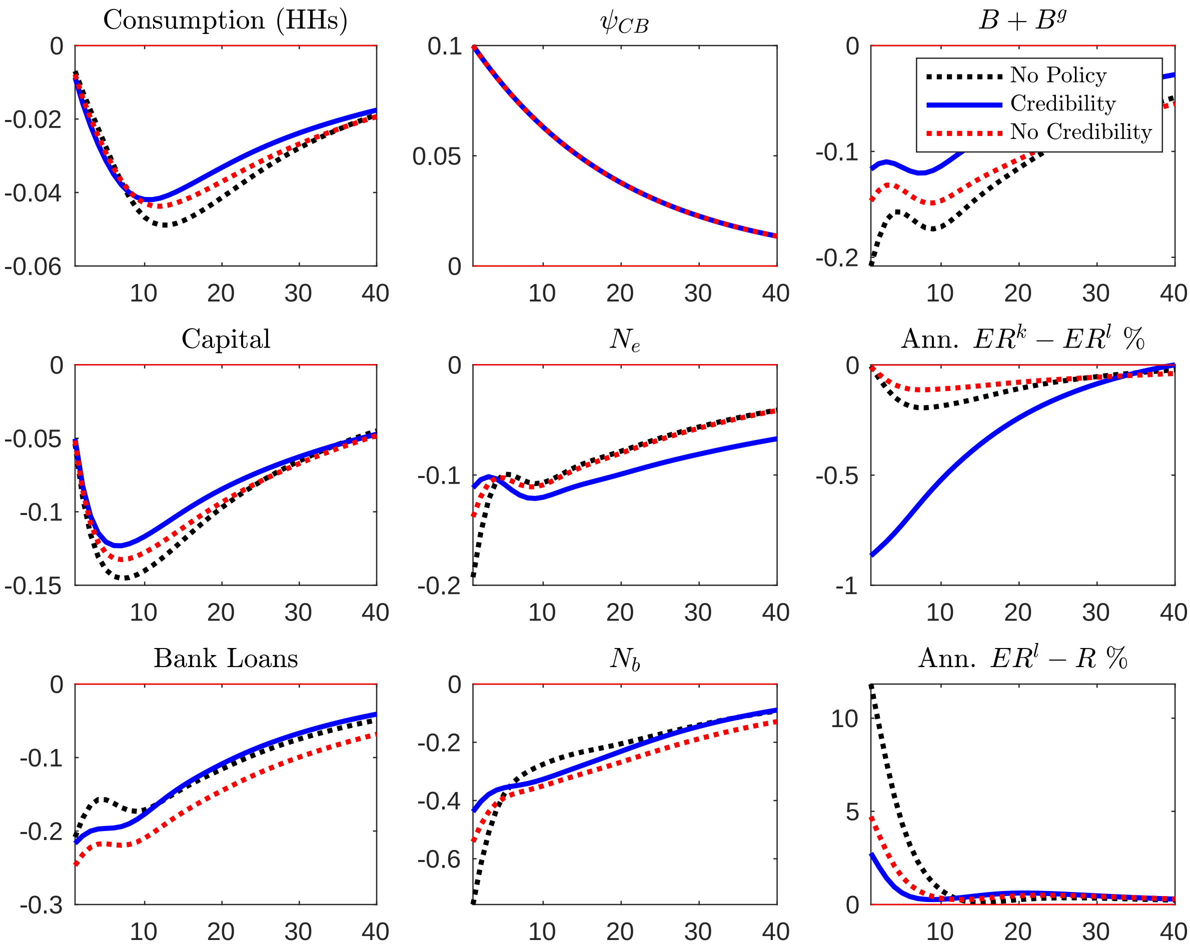
All variables are in log deviations from steady-state except spreads and CB loans share, shown in level deviation from steady-state.

Figure 6 shows the case when our target for the annualized entrepreneur default probability is 20% and keep the other targets unchanged.[27] It means that in the long-term a larger fraction of entrepreneurs is going to default. In this context of a lot

[27]This results in $\gamma = 0.983$ (0.982), $\mu = 0.0533$ (0.286) and $\sigma_\omega = 0.3689$ (0.286). Baseline calibration in parenthesis.

uncertainty, the effect of credibility seems relatively more significant. In other words, the impact of government guarantees is more significant, and then government subsidies or transfers are going to be larger.[28]

Figure 6: A five percent negative capital quality shock: State-Contingent R^l_{t+1}



All variables are in log deviations from steady-state except spreads and CB loans share, shown in level deviation from steady-state.

6.4 Endogenous Credit Policy Rule

In this section, we consider the endogenous credit policy rule, described in equation (56), and we study the effective design of an automatic rule. As we will see, a wrong endogenous rule might exacerbate the impact of the negative capital quality shock in the economy.

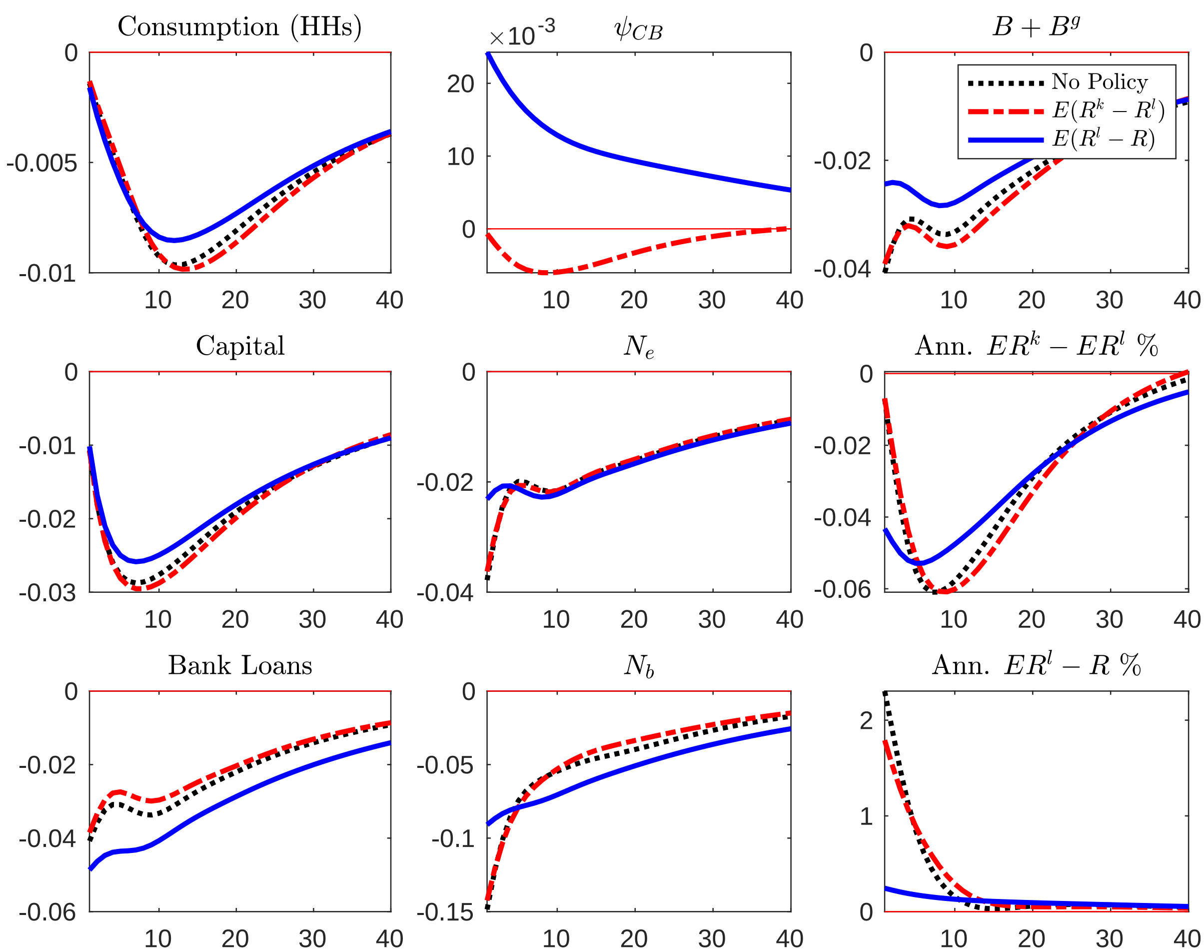
Figure 7 reports (assuming same seniority) the results for two different rules. In the first rule, credit injection responds on credit supply frictions (or credit supply conditions), i.e., $\mathbb{E}_t\{R^l_{t+1} - R_t\}$ while in the second rule, it responds on credit spread (or credit demand

[28]Figure 16 shows the results for the non-state-contingent R^l_{t+1}.

conditions), i.e., $\mathbb{E}_t\{R^k_{t+1} - R^l_{t+1}\}$.

With state-contingent R^l_{t+1} figure 7 shows that, as explained in subsection 4.2, the negative capital quality shock reduces capital level and increases the credit spread, $E(R^l - R)$, and decreases $E(R^k - R^l)$. Then, as observed in figure (7) a credit policy rule that positively responds to the credit spread $E(R^l - R)$ injects positive cheap guaranteed CB loans and hence reduces the impact of the shock. While a credit policy rule that positively responds to the problems of the credit demand side is going to inject negative CB loans, or equivalently CB requires entrepreneurs to hold deposits at the CB, and if anything it reduces entrepreneur resources to purchases capital.[29] Hence, we observe how this policy rule reduces aggregate capital and exacerbate the shock in the economy.

Figure 7: A five percent negative capital quality shock



All variables are in log deviations from steady-state except spreads and CB loans share, shown in level deviation from steady-state.

For completeness, figure 17 in Appendix I reports the case of non-state-contingent R^l_{t+1}

[29]Notice that banks might not have the incentives to male CB deposits. Hence, we assume that entrepreneurs do so otherwise they are charged a high penalty.

and verifies that in that case a policy rule that responds to the credit demand frictions, i.e., $\mathbb{E}_t\{R^k_{t+1} - R^l_{t+1}\}$, is better in mitigating the negative impact of the capital quality shock since the aggregate risk is all absorbed by the entrepreneurs and consequently, as also commented in subsection 4.2, the $\mathbb{E}_t\{R^k_{t+1} - R^l_{t+1}\}$ rise is greater than the credit spread.

These results show that, as a policy recommendation, a credit policy should not be designed as a fixed automatic rule, but it should be flexible enough so it can properly detect the source of frictions in financial markets and hence responds accordingly. In other words, credit policy effectiveness depends on regulators ability to promptly detect the source and the size of the economic deterioration. In other words, credit policy effectiveness depends on regulators ability to identify if the shock is deteriorating credit demand or credit supply conditions.

6.5 Announced Credit Policy Rule

We assume here that the CB announces ex-ante the credit policy rule, equation (35). Hence, being aware of the injection rule, entrepreneur internalizes the effects of their decisions on the size of the credit policy injection, $B^{g,j}_t$. This is, entrepreneur internalizes that the cost of capital is a weighted average of the required return of bank loans and the effective cost of central bank loans, i.e., ex-ante the entrepreneur knows that for each unit of external funding a fraction $\phi_{CB,t}$ is funded with CB loans, while a fraction $1 - \phi_{CB,t}$ is funded with bank loans. The key question is to know if the credit policy becomes more effective.

Under the same seniority assumption, substituting (35) into the bank loan contract of entrepreneur j, equation (40), it becomes,

$$\left[\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right] R^k_{t+1} Q_t K^j_t = \bar{R}^l_{t+1}\left(Q_t K^j_t - N^j_t\right). \tag{57}$$

where $\bar{R}^l_{t+1} = R^l_{t+1}(1 - \psi_{CB,t}) + \Psi(\bar{\omega}^j) R_t \psi_{CB,t}$ is the weighted average of the required return of bank loans, R^l_{t+1}, and the effective cost of CB loans $\Psi(\bar{\omega}^j)R_t$, were recall $\Psi(\bar{\omega}^j) < 1$.

With an announced credit policy rule as equation (35) from entrepreneur's perspective there are not two loan supply curves, but there is only one aggregate supply curve. In other words, entrepreneur is not going to exhaust first CB loans and then bank loans, but demand both simultaneously. This is, each unit of demanded external funding is composed by $\psi_{CB,t}$ units of CB loans and $1 - \psi_{CB,t}$ of bank loans. And the cost per unit of external funding at the margin is $\bar{R}^l_{t+1}$. Then, an announced credit policy rule has an effect on the aggregate credit supply curve, that together with the aggregate credit demand curve determines the aggregate credit in equilibrium.

Bank profits of entrepreneur, equation (45), becomes,

$$\mathbb{E}_t \left\{ \left[1 - \mu G(\bar{\omega}^j\right] R^k_{t+1} Q_t K^j_{t+1} - \bar{R}^l_{t+1}(Q_t K^j_t - N^j_t) \right\}.$$

Since $\bar{R}^l_{t+1} < R^l_{t+1}$ we see that an announced credit policy rule reduces the marginal cost of capital and hence of credit. The aggregate demand curve of credit, equation (43), becomes,[30]

$$\mathbb{E}_t \left\{ \left(1 - \Gamma(\bar{\omega}^j)\right) R^k_{t+1} + \frac{1 - F(\bar{\omega}^j)}{\Upsilon + 1 - F(\bar{\omega}^j) - \mu \bar{\omega}^j f(\bar{\omega}^j)} \left[\left(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right) R^k_{t+1} - \bar{R}^l_{t+1} \right] \right\} = 0.$$

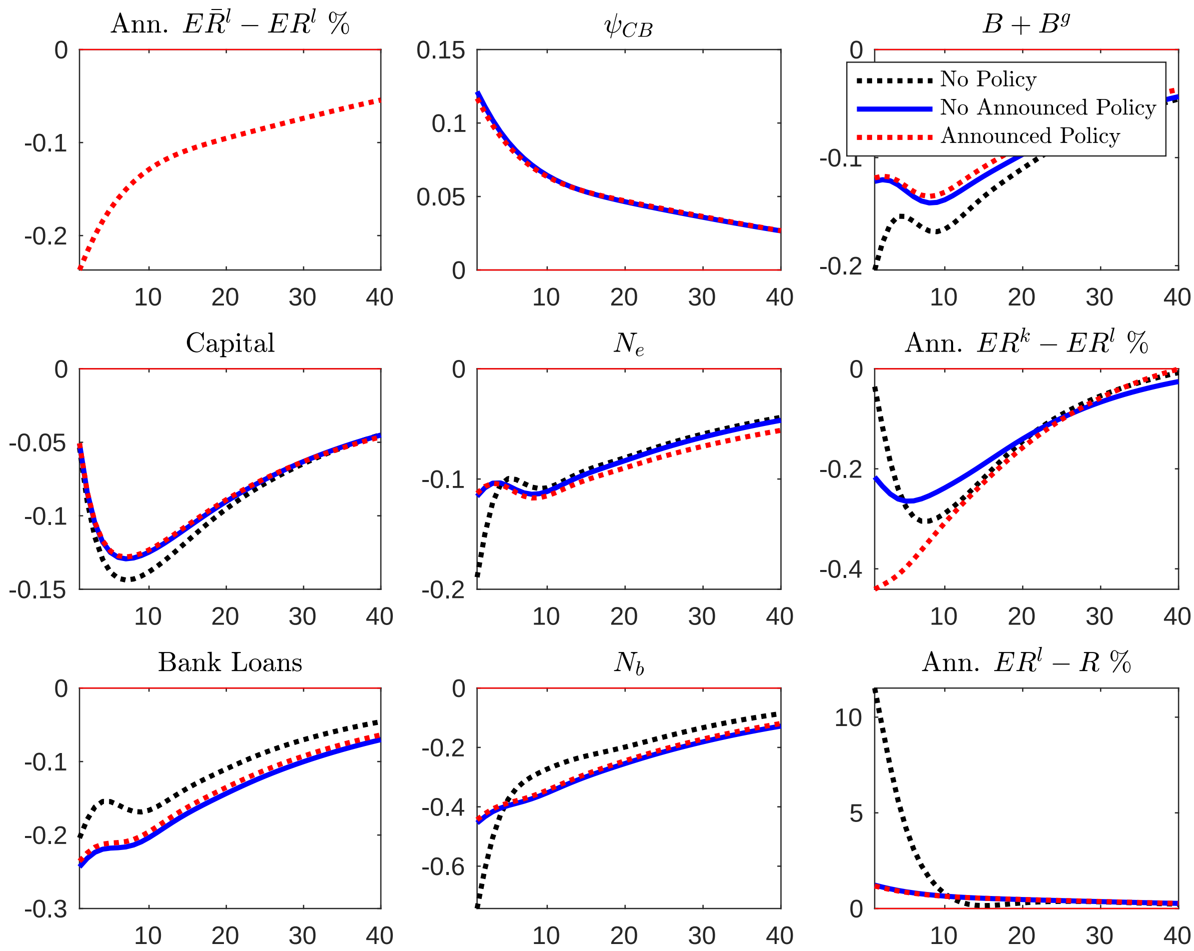
Contrasting it with equation (43), the announcement of the credit policy does not affect the aggregate demand of external funding (credit), but it positively affects the aggregate supply of credit by reducing the marginal cost of credit faced by entrepreneurs from R^l_{t+1} to $\bar{R}^l_{t+1}$.

Figure 8 reports that visually announcing a credit policy rule improves but not significantly effectiveness of the credit policy. In other words, letting entrepreneur internalizes the effects of their decisions on CB lending facilities does not significantly increase the effectiveness of the credit policy.

Figure 18 in Appendix 18 reports the results for a higher intensity of CB intervention. This is we set $\nu = 320$ (40 in baseline calibration). Since the proportion of cheap loans is now higher, this leads to a stronger reduction of the cost of a unit of external funding, $\bar{R}^l_{t+1}$, which in turns produces a more stronger recovery of capital. However, although we have a stronger CB policy, the announcement does not significantly improve the recovery of capital and hence the announcement does not still improve significantly the effectiveness of the credit policy. The small power of ex-ante announcing the policy is because the spread between the required lending rate $E_t\{R^l_{t+1}\}$ and the risk-free interest rate is only 0.25%.

[30] Proof in Appendix B.

Figure 8: A five percent negative capital quality shock



All variables are in log deviations from steady-state except spreads and CB loans share, shown in level deviation from steady-state.

6.6 Unconventional vs. Conventional Credit Policy

So far we have discussed what we have defined as unconventional credit policy in section 5. In the case of a conventional credit policy as the one proposed in GKa 2011, there are two different assumptions that depart from the unconventional credit policy: 1) the required return on the central bank loans is the market lending rate R^l_{t+1} and 2) central bank loans are not guarantee by the government. Here, we quantitatively and quantitative study the differences between the unconventional and conventional credit policy. Recall that we already know the implications of 2), the government guarantees on CB loans.

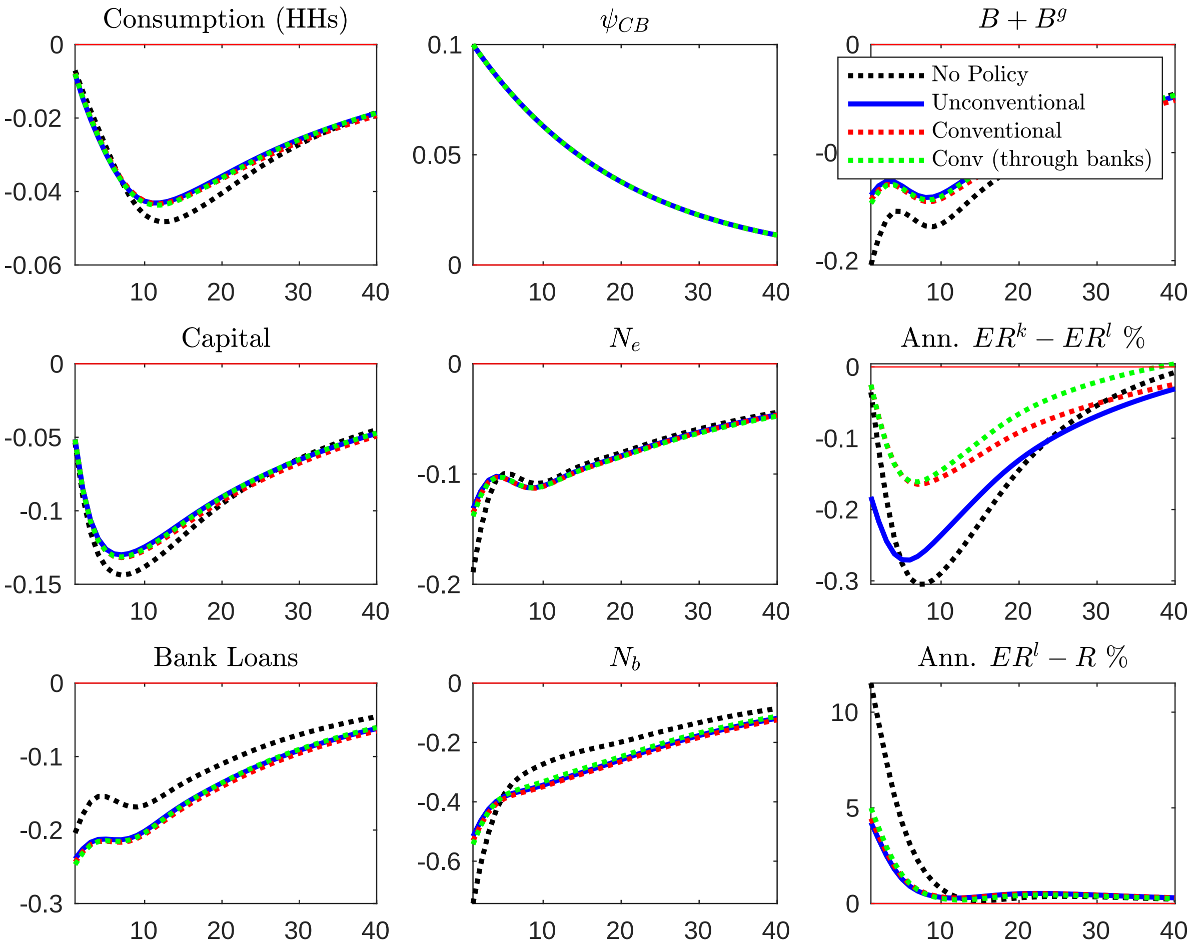
Let start assuming that CB loans are directly given by central bank. In the case of a conventional credit policy, as proved in Appendix H, given CB loans and bank loans are identical from the entrepreneurs' perspective, the aggregate credit demand is not altered by the conventional credit policy.[31] Even though the required return of CB loans is the same than bank loans, the fact that CB loans cannot be diverted increases the aggregate credit supply. In other words, as explained before, since there is not a moral hazard problem between depositors and CB, it reduces the frictions of credit supply. Indeed, this is the only transmission of the conventional policy on the credit market. Figures 9 (state-contingent R^l_{t+1}) and 10 (non-state-contingent R^l_{t+1}) reports that the real effects (effects on aggregate capital) of the conventional credit policy are quantitatively similar to the unconventional credit policy. It suggests that neither the government guarantees nor the fact that CB loans have a required return lower than bank loans have an important effect on reducing the impact of the shock. Hence, the effectiveness of the unconventional credit policy is driven by the fact that credit policy reduces the credit supply frictions since CB loans are cannot diverted. As suggested in subsection 6.3 in an economy with a higher entrepreneur default probability the government guarantees become more important and hence the impact of unconventional credit policy becomes stronger that a conventional one.

Let assume that CB loans are given through banks and that for comparison reasons (with the unconventional credit policy) we say that banks cannot divert CB loans and hence clearly credit policy is going to affect aggregate credit supply. The main difference between the conventional policy given directly and indirectly by CB is that in the latter the gains or losses are absorbed by banks' net worth.[32] Figures 9 and 10 report that the impact of banks absorbing gains or losses from CB loans is negligible.

[31] It is easy to verify that this holds for any seniority assumption.

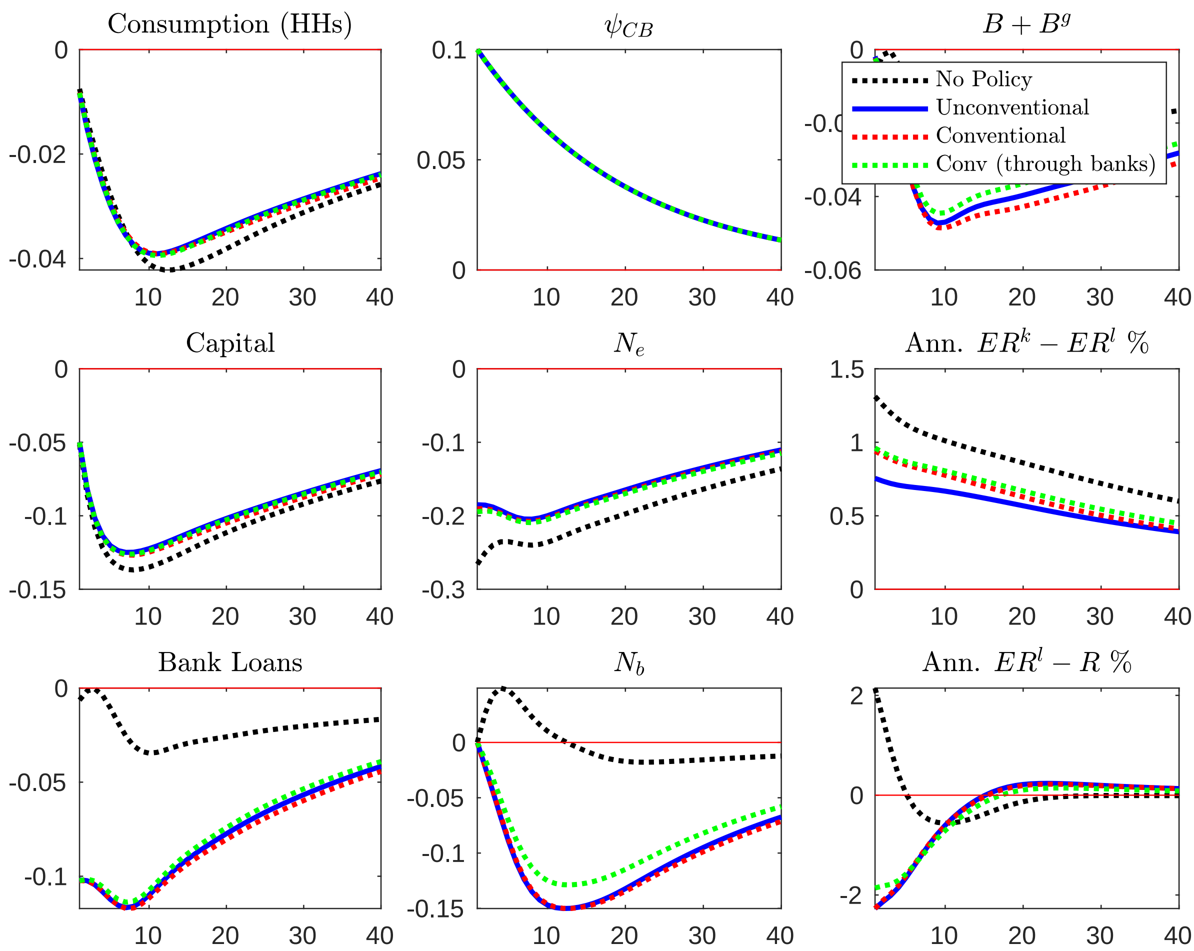
[32] Appendix H reports how the equilibrium conditions of banks are affected.

Figure 9: A five percent negative capital quality shock: State-Contingent R^l_{t+1}



All variables are in log deviations from steady-state except spreads and CB loans share, shown in level deviation from steady-state.

Figure 10: A five percent negative capital quality shock: No-State-Contingent R^l_{t+1}



All variables are in log deviations from steady-state except spreads and CB loans share, shown in level deviation from steady-state.

7 Conclusions

We develop a DSGE model with frictions on the credit demand side and credit supply side. Credit supply frictions allows us to mimic a more realistic dynamics of credit after a monetary policy shock. In particular, by considering credit supply frictions and letting banks absorbe some aggregate risk, bank's net worth absorbs some losses, and it diminishes the bank capacity to issue credit. As a result, credit supply frictions allow us to observe an aggregate credit reduction after a contractionary monetary policy. When, considering only demand side frictions we observe a puzzle: a contractionary monetary policy expands credit. With this more realistic framework we are able to address the question about the role of an "unconventional" credit policy that provides lending facilities to firms by granting central bank loans guaranteed by the government at the cost of the risk-free rate.

We find that the unconventional credit policy diminishes the impact of a negative shock on the real economy since it increases credit demand and supply: i) Since central bank loans cannot be diverted, credit policy diminishes the bank equity requirement per unit of aggregate credit, which increases aggregate credit supply; ii) entrepreneurs face a limited supply of cheap CB loans, in the sense that CB loans have a required return smaller than traditional bank loans; and iii) CB loans are guaranteed by the government. In normal times, the first effect is more relevant; however, in higher uncertainty period, the government guarantees become also an important driver on reducing the impact of the negative shock. In general, we find that the lower the seniority of the central bank loans, the higher the effectiveness of the credit policy. Since bank loans are paid first, banks can reduce the (non-default) lending rate, which pushes default probability down and increases credit demand.

In addition, we find that an endogenous credit policy rule should not be automatic. This is, the rule should be flexible enough, so it can properly respond to indicators that capture the source and size of the economic deterioration. Finally, letting entrepreneurs know that they might obtain a fraction of cheap loans which in turn reduces the marginal cost of external funding, does not lead to significant benefits when credit spread is small.

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Appendices

A Long-term Equilibrium: Deterministic Steady-State

In the deterministic steady state (SS) equilibrium $P_{ss} = P^*_{ss}$, then $X_{ss} = \frac{\epsilon}{\epsilon-1}$. From the inter-temporal condition of households, $R_{ss} = 1/\beta$, and the first order condition form capital producing firms $Q_{ss} = 1$. The nominal interest rate $i_{ss} = 1/\beta$. Inflation $\pi_{ss} = 0$. The marginal utility of consumption at the steady state is $u_{Css} = (1-\beta h)/(C_{ss}(1-h))$. From the capital and labor markets,

$$R^k_{ss} = \frac{\alpha Y_{ss}}{K_{ss}} + (1-\delta), \qquad Y_{ss} = K^{\alpha}_{ss} H^{(1-\alpha)\Omega}_{ss},$$

$$\frac{1}{X_{ss}}(1-\alpha)\Omega \frac{Y_{ss}}{H_{ss}} = \frac{1}{u_{Css}} \chi H^{\varphi}_{ss},$$

From the banks side equilibrium conditions:

$$1 = \sigma \left[(R^l_{ss} - R_{ss})\phi_{ss} + R_{ss} \right] 1 + \zeta \phi_{ss},$$

$$\phi_{ss} = \frac{\eta_{ss}}{\lambda - \nu_{ss}},$$

$$\nu_{ss} = \frac{1}{1-\beta\sigma x_{ss}}(1-\sigma)\beta(R^l_{ss} - R_{ss}),$$

$$\eta_{ss} = \frac{1}{1-\beta\sigma z_{ss}}(1-\sigma),$$

$$z_{ss} = (R^l_{ss} - R_{ss})\phi_{ss} + R_{ss},$$

$$x_{ss} = z_{ss},$$

$$B_{ss} = D_{ss} + N_{bss}.$$

$$B_{ss} = \phi_{ss} N_{bss}.$$

From entrepreneurs side equilibrium conditions:

$$\left(1 - \Gamma(\bar{\omega}_{ss})\right) R^k_{ss} + \frac{1 - F(\bar{\omega}_{ss})}{1 - F(\bar{\omega}_{ss}) - \mu \bar{\omega}_{ss} f(\bar{\omega}_{ss})} \left[\left(\Gamma(\bar{\omega}_{ss}) - \mu G(\bar{\omega}_{ss})\right) R^k_{ss} - R^l_{ss} \right] = 0,$$

$$\left(\Gamma(\bar{\omega}_{ss}) - \mu G(\bar{\omega}_{ss})\right) R^k_{ss} K_{ss} = R^l_{ss} \left(K_{ss} - N_{ess}\right),$$

$$N_{ess} = \gamma V^e_{ss} + \frac{(1-\alpha)(1-\Omega)}{X_{ss}} K^{\alpha}_{ss} H^{(1-\alpha)\Omega}_{ss}.$$

$$V^e_{ss} = R^k_{ss} K_{ss} - \left(R^l_{ss} + \frac{\mu \int_0^{\bar{\omega}_{ss}} \omega dF(\omega) R^k_{ss} K_{ss}}{B_{ss}} \right) B_{ss},$$

$$K_{ss} = B_{ss} + N_{ess}, \quad C_{ess} = (1-\gamma)V^e_{ss},$$

From market clearing of goods:

$$Y_{ss} = C_{ss} + C_{ess} + I_{ss} + \mu \int_0^{\bar{\omega}_{ss}} \omega dF(\omega) R^k_{ss} K_{ss}.$$

In the case of a state-contingent required return of bank loans, it also applies that,

$$R^l_{ss} = \xi_{ss} R^k_{ss}.$$

Procedure to find the parameter values: We first focus on the banking sector. In particular, we solve the system of equations of the first four equations from banks side. Our six variables that we want to know their values are ν_{ss} , η_{ss}, ζ and λ. In order to do so, we first set the parameter values for β and σ and set the targets $R^l_{ss} - R_{ss}$ (or R^l_{ss}) and ϕ_{ss} (and hence z_{ss} and x_{ss}). Then, with this information solve the rest of equations. Also we solve for the parameters γ, μ and σ_ω given the targets values for $R^k_{ss} - R^l_{ss}$, $F(\bar{\omega}_{ss})$ and ϕ_{ess}.

B Both central bank and bank loans have the same seniority

Recalling the bank loan contract, equation (39),

$$[1 - F(\bar{\omega}^j)]Z^j_{t+1}B^j_t + (1-\mu)\int_0^{\bar{\omega}^j} \omega R^k_{t+1} Q_t K^j_t x^j_{t+1} dF(\omega) = R^l_t B^j_t, \tag{58}$$

Recalling Z^j_{t+1} is obtained in equation (37). Then,

$$x^j_{t+1} = (\bar{\omega}^j R^k_{t+1} Q_t K^j_t - R_t B^{g,j}_t)/(\bar{\omega}^j R^k_{t+1} Q_t K^j_t) = 1 - \frac{R_t B^{g,j}_t}{\bar{\omega}^j R^k_{t+1} Q_t K^j_t}, \tag{59}$$

and so equation (58) becomes,

$$[1-F(\bar{\omega}^j)](\bar{\omega}^j R^k_{t+1} Q_t K^j_t - R_t B^{g,j}_t) + (1-\mu)\int_0^{\bar{\omega}^j} \left(\omega R^k_{t+1} Q_t K^j_t - \frac{\omega}{\bar{\omega}^j} R_t B^{g,j}_t\right) dF(\omega) = R^l_{t+1} B^j_t.$$

For convenience, this is written as,

$$-\Psi(\bar{\omega}^j) R_t B^{g,j}_t + \left(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right) R^k_{t+1} Q_t K^j_t = R^l_{t+1}\left(Q_t K^j_t - B^{g,j}_t - N^j_{et}\right), \tag{60}$$

where,

$$\Gamma(\bar{\omega}^j) = \int_0^{\bar{\omega}^j} \omega dF(\omega) + (1 - F(\bar{\omega}^j))\bar{\omega}^j, \qquad G(\bar{\omega}^j) = \int_0^{\bar{\omega}^j} \omega dF(\omega).$$

$$\Psi(\bar{\omega}^j) = (1-\mu)\frac{1}{\bar{\omega}^j} G(\bar{\omega}^j) + (1 - F(\bar{\omega}^j)).$$

The optimal contracting problem may be now written as:

$$\max_{K_t^j, \bar{\omega}^j} \mathbb{E}_t\{\left(1 - \Gamma(\bar{\omega}^j)\right) R_{t+1}^k Q_t K_t^j$$
$$+\lambda_{t+1}\left[-\Psi(\bar{\omega}^j) R_t B_t^{g,j} + \left(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right) R_{t+1}^k Q_t K_t^j - R_{t+1}^l B_t^j\right]\},$$

where $B_t^j = Q_t K_t^j - B_t^{g,j} - N_{et}^j$. The first order condition for $\bar{\omega}^j$:

$$-\frac{\partial \Gamma(\bar{\omega}^j)}{\partial \bar{\omega}^j} R_{t+1}^k Q_t K_t^j + \lambda_{t+1}\left[-\frac{\partial \Psi(\bar{\omega}^j)}{\partial \bar{\omega}^j} R_t B_t^{g,j} + \left(\frac{\partial \Gamma(\bar{\omega}^j)}{\partial \bar{\omega}^j} - \mu \frac{G(\bar{\omega}^j)}{\partial \bar{\omega}^j}\right) R_{t+1}^k Q_t K_t^j\right] = 0. \tag{61}$$

The first order condition for K_t^j:

$$\mathbb{E}_t\left\{\left(1 - \Gamma(\bar{\omega}^j)\right) R_{t+1}^k + \lambda_{t+1}\left[(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)) R_{t+1}^k - R_{t+1}^l\right]\right\} = 0. \tag{62}$$

The first order condition for λ_{t+1} yields equation (60), where,

$$\frac{\partial \Gamma(\bar{\omega}^j)}{\partial \bar{\omega}^j} = 1 - F(\bar{\omega}^j), \quad \frac{\partial G(\bar{\omega}^j)}{\partial \bar{\omega}^j} = \bar{\omega}^j f(\bar{\omega}^j).$$

$$\frac{\partial \Psi(\bar{\omega}^j)}{\partial \bar{\omega}^j} = (1-\mu)\left(-\frac{G(\bar{\omega}^j)}{(\bar{\omega}^j)^2} + \frac{1}{\bar{\omega}^j}\frac{\partial G(\bar{\omega}^j)}{\partial \bar{\omega}^j}\right) - f(\bar{\omega}^j) = -(1-\mu)\frac{G(\bar{\omega}^j)}{(\bar{\omega}^j)^2} - \mu f(\bar{\omega}^j) < 0.$$

Combining equations (61) with (62) yields,

$$\mathbb{E}_t\left\{\left(1 - \Gamma(\bar{\omega}^j)\right) R_{t+1}^k + \frac{1 - F(\bar{\omega}^j)}{\Upsilon + 1 - F(\bar{\omega}^j) - \mu \bar{\omega}^j f(\bar{\omega}^j)}\left[\left(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right) R_{t+1}^k - R_{t+1}^l\right]\right\} = 0.$$

where,

$$\Upsilon = -\frac{\partial \Psi(\bar{\omega}^j)}{\partial \bar{\omega}^j}\frac{R_t B_t^{g,j}}{R_{t+1}^k Q_t K_t^j} > 0.$$

The amount transferred back to the central bank from entrepreneur is,

$$M_{t+1} = [1 - F(\bar{\omega}^j)] R_t B_t^{g,j} + (1-\mu)\int_0^{\bar{\omega}^j} \omega R_{t+1}^k Q_t K_t^j (1 - x_{t+1}^j) dF(\omega).$$

Using (59), we obtain,

$$M_{t+1} = [1 - F(\bar{\omega}^j)] R_t B_t^{g,j} + (1-\mu)\int_0^{\bar{\omega}^j} \frac{\omega}{\bar{\omega}^j} R_t B_t^{g,j} dF(\omega).$$

$$M_{t+1} = \left([1 - F(\bar{\omega}^j)] + (1-\mu)\frac{G(\bar{\omega}^j)}{\bar{\omega}^j} \right) R_t B_t^{g,j}.$$

By construction, the entrepreneurs' revenues used to repay central bank loans (M_{t+1}) are not enough to fully repay central bank loans, and hence government collect lump sum taxes to ensure central bank loans are fully paid. In other words, It holds that $M_{t+1} < R_{t+1}B_t^g$. This implies that the government transfers $R_{t+1}B_{t+1} - M_{t+1}$ to central bank.

Announced Policy:

Here, we solve the model assuming that entrepreneur knows that $B_t^j = \psi_{CB,t}(Q_t K_t^j - N_t^j)$. Recall that the bank loan contract, equation (57), is

$$\left[\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right] R_{t+1}^k Q_t K_t^j = \bar{R}_{t+1}^l \left(Q_t K_t^j - N_{et}^j\right). \tag{63}$$

where $\bar{R}_{t+1}^l = R_{t+1}^l(1-\psi_{CB,t}) + \Psi(\bar{\omega}^j) R_t \psi_{CB,t}$. The optimal contracting problem may be now written as:

$$\max_{K_t^j, \bar{\omega}^j} \mathbb{E}_t \left\{ \left(1 - \Gamma(\bar{\omega}^j)\right) R_{t+1}^k Q_t K_t^j + \lambda_{t+1} \left[\left(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right) R_{t+1}^k Q_t K_t^j - \bar{R}_{t+1}^l (Q_t^j K_t^j - N_{et}^j) \right] \right\}.$$

The first order condition for $\bar{\omega}^j$:

$$-\frac{\partial \Gamma(\bar{\omega}^j)}{\partial \bar{\omega}^j} R_{t+1}^k Q_t K_t^j + \lambda_{t+1} \left[\left(\frac{\partial \Gamma(\bar{\omega}^j)}{\partial \bar{\omega}^j} - \mu \frac{G(\bar{\omega}^j)}{\partial \bar{\omega}^j} \right) R_{t+1}^k Q_t K_t^j - \frac{\partial \bar{R}_{t+1}^l}{\partial \bar{\omega}^j} (Q_t^j K_t^j - N_{et}^j) \right] = 0. \tag{64}$$

The first order condition for K_t^j:

$$\mathbb{E}_t \left\{ \left(1 - \Gamma(\bar{\omega}^j)\right) R_{t+1}^k + \lambda_{t+1} \left[(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)) R_{t+1}^k - \bar{R}_{t+1}^l \right] \right\} = 0. \tag{65}$$

The first order condition for λ_{t+1} yields equation (63), where,

$$\frac{\partial \bar{R}_{t+1}^l}{\partial \bar{\omega}^j} = \frac{\partial \Psi(\bar{\omega}^j)}{\partial \bar{\omega}^j} R_t \psi_{CB,t}.$$

Combining equations (64) with (65) yields,

$$\mathbb{E}_t \left\{ \left(1 - \Gamma(\bar{\omega}^j)\right) R_{t+1}^k + \frac{1 - F(\bar{\omega}^j)}{\Upsilon + 1 - F(\bar{\omega}^j) - \mu \bar{\omega}^j f(\bar{\omega}^j)} \left[\left(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right) R_{t+1}^k - \bar{R}_{t+1}^l \right] \right\} = 0.$$

where,

$$\Upsilon = -\frac{\partial \Psi(\bar{\omega}^j)}{\partial \bar{\omega}^j} \frac{R_t B_t^{g,j}}{R_{t+1}^k Q_t K_t^j} > 0.$$

C Bank loans have higher seniority

When the two external funding of the entrepreneurs have not the same seniority, we can define another threshold value of the idiosyncratic shock, $\bar{\omega}^{g,j}$,

$$\bar{\omega}^{g,j} R^k_{t+1} Q_t K^j_t = Z^j_{t+1} B^j_t, \tag{66}$$

where clearly $\bar{\omega}^j > \bar{\omega}^{g,j}$, that is associated with the lowest value of ω^j so entrepreneurs can still fully pay the external funding with higher seniority. Hence, in this case, if $\bar{\omega}^j > \omega^j \geq \bar{\omega}^{g,j}$, entrepreneur is able to fully pay bank loans but cannot fully pay CB loans, so government must intervene to ensure CB fully receive the agreed gross return. If $\bar{\omega}^{g,j} > \omega^j$, entrepreneur is not able to pay anything to the central bank, while it partially pay to banks. In this case, the government will have to pay for the whole debt of firms to CB. By definition, a defaulting entrepreneur receives nothing.

The bank loan contract ($\bar{\omega}^j$, Z^j_{t+1}), equation (39), becomes,

$$[1 - F(\bar{\omega}^{g,j})] Z^j_{t+1} B^j_t + (1-\mu) \int_0^{\bar{\omega}^{g,j}} \omega R^k_{t+1} Q_t K^j_t dF(\omega) - \mu \int_{\bar{\omega}^{g,j}}^{\bar{\omega}^j} Z^j_{t+1} B^j_t dF(\omega) = R^l_{t+1} B^j_t, \tag{67}$$

where left-hand side of equation (67) is the expected return on the loan to the entrepreneur and the right-hand side is the opportunity cost of bank lending. Clearly, in equilibrium the bank lending rate, Z^j_{t+1}, is higher than R^l_{t+1}.

The amount transferred back to the central bank from entrepreneur is,

$$M_{t+1} = [1 - F(\bar{\omega}^j)] R_{t+1} B^{g,j}_t + (1-\mu) \int_{\bar{\omega}^{g,j}}^{\bar{\omega}^j} \left(\omega R^k_{t+1} Q_t K^j_t - Z^j_{t+1} B^j_t \right) dF(\omega).$$

It is true that $M^j_{t+1} < R_{t+1} B^{g,j}_t$. This implies that the government transfers $R_{t+1} B^j_{t+1} - M^j_{t+1}$ to the central bank are such it receives the agreed gross return of R_{t+1}.

Combining equations (36) and (66) with equation (67) we obtain,

$$\left([1 - F(\bar{\omega}^{g,j})] \bar{\omega}^{g,j} + (1-\mu) \int_0^{\bar{\omega}^{g,j}} \omega dF(\omega) - \mu (F(\bar{\omega}^j) - F(\bar{\omega}^{g,j})) \bar{\omega}^{g,j} \right) R^k_{t+1} Q_t K^j_{t+1} = R^l_{t+1} B^j_t, \tag{68}$$

Note that $\bar{\omega}^j$ and $\bar{\omega}^{g,j}$ are contingent to the realization of R^k_{t+1}. Entrepreneurs aim to maximize (42). For convenience equation (68) is written as,

$$\left(\Gamma(\bar{\omega}^{g,j}) - \mu G(\bar{\omega}^{g,j}, \bar{\omega}^j) \right) R^k_{t+1} Q_t K^j_t = R^l_{t+1} B^j_t, \tag{69}$$

where,

$$\Gamma(\bar{\omega}^{g,j}) = \int_0^{\bar{\omega}^{g,j}} \omega dF(\omega) + (1 - F(\bar{\omega}^{g,j}))\bar{\omega}^{g,j},$$

$$G(\bar{\omega}^{g,j}, \bar{\omega}^j) = \int_0^{\bar{\omega}^{g,j}} \omega dF(\omega) + (F(\bar{\omega}^j) - F(\bar{\omega}^{g,j}))\bar{\omega}^{g,j}.$$

Combining (37) and (66), we obtain the following relationship or the expression for $\bar{\omega}^{g,j}$:

$$\bar{\omega}^j R^k_{t+1} Q_t K^j_t = \bar{\omega}^{g,j} R^k_{t+1} Q_t K^j_t + R_t B^{g,j}_t. \tag{70}$$

The optimal contracting problem may be now written as,

$$\max_{K^j_t, \bar{\omega}^j} \mathbb{E}_t \{ \left(1 - \Gamma(\bar{\omega}^j)\right) R^k_{t+1} Q_t K^j_t + \lambda_{t+1} \left[\left(\Gamma(\bar{\omega}^{g,j}) - \mu G(\bar{\omega}^{g,j}, \bar{\omega}^j)\right) R^k_{t+1} Q_t K^j_t - R^l_{t+1} B^j_t \right] \},$$

where $\bar{\omega}^{g,j}(\bar{\omega}^j, K^j_t)$ is obtained from (70) and $B^j_t = Q_t K^j_t - B^{g,j}_t - N^j_{et}$. The first order condition for $\bar{\omega}^j$:

$$-\frac{\partial \Gamma(\bar{\omega}^j)}{\partial \bar{\omega}^j} + \lambda_{t+1} \left(\frac{\partial \Gamma(\bar{\omega}^{g,j})}{\partial \bar{\omega}^j} - \mu \frac{G(\bar{\omega}^{g,j}, \bar{\omega}^j)}{\partial \bar{\omega}^j} \right) = 0. \tag{71}$$

The first order condition for K^j_t:

$$\begin{aligned} &\mathbb{E}_t \{ \left(1 - \Gamma(\bar{\omega}^j)\right) R^k_{t+1} + \lambda_{t+1} [\left(\Gamma(\bar{\omega}^{g,j}) - \mu G(\bar{\omega}^{g,j}, \bar{\omega}^j)\right) R^k_{t+1} \\ + &\left(\frac{\partial \Gamma(\bar{\omega}^{g,j})}{\partial K^j_t} - \mu \frac{\partial G(\bar{\omega}^{g,j}, \bar{\omega}^j)}{\partial K^j_t} \right) R^k_{t+1} K_t - R^l_{t+1}] \} = 0. \end{aligned} \tag{72}$$

The first order condition for λ_{t+1} yields equation (69), where,

$$\frac{\partial \Gamma(\bar{\omega}^j)}{\partial \bar{\omega}^j} = 1 - F(\bar{\omega}^j), \qquad \frac{\partial \Gamma(\bar{\omega}^{g,j})}{\partial \bar{\omega}^{g,j}} = 1 - F(\bar{\omega}^{g,j}),$$

$$\frac{\partial \Gamma(\bar{\omega}^{g,j})}{\partial K^j_t} = \frac{\partial \Gamma(\bar{\omega}^{g,j})}{\partial \bar{\omega}^{g,j}} \frac{\partial \bar{\omega}^{g,j}}{\partial K^j_t} = (1 - F(\bar{\omega}^{g,j})) \frac{R_t}{R^k_{t+1} Q_t} \frac{B^g_t}{(K^j_t)^2}.$$

$$\frac{\partial G(\bar{\omega}^{g,j}, \bar{\omega}^j)}{\partial \bar{\omega}^{g,j}} = F(\bar{\omega}^j) - F(\bar{\omega}^{g,j}),$$

$$\frac{\partial G(\bar{\omega}^{g,j}, \bar{\omega}^j)}{\partial K^j_t} = \frac{\partial G(\bar{\omega}^{g,j}, \bar{\omega}^j)}{\partial \bar{\omega}^{g,j}} \frac{\partial \bar{\omega}^{g,j}}{\partial K^j_t} = (F(\bar{\omega}^j) - F(\bar{\omega}^{g,j})) \frac{R_t}{R^k_{t+1} Q_t} \frac{B^g_t}{(K^j_t)^2}.$$

Since $\frac{\partial \bar{\omega}^{g,j}}{\partial \bar{\omega}^j} = 1$,

$$\frac{\partial \Gamma(\bar{\omega}^{g,j})}{\partial \bar{\omega}^j} = 1 - F(\bar{\omega}^{g,j}),$$

$$\frac{\partial G(\bar{\omega}^{g,j}, \bar{\omega}^j)}{\partial \bar{\omega}^j} = f(\bar{\omega}^j)\bar{\omega}^{g,j} + F(\bar{\omega}^j) - F(\bar{\omega}^{g,j}).$$

In this case V_t^e becomes,

$$\begin{aligned} V_t^e &= R_t^k Q_{t-1} K_{t-1} - R_t^l B_{t-1} - (1 - F(\bar{\omega})) R_{t-1} B_{t-1}^g - \int_{\bar{\omega}^g}^{\bar{\omega}} \left(\omega R_t^k Q_{t-1} K_{t-1} - Z_t B_{t-1} \right) dF(\omega) \\ &- \mu \int_0^{\bar{\omega}_t^g} \omega R_t^k Q_{t-1} K_{t-1} dF(\omega) - \mu \int_{\bar{\omega}_t^g}^{\bar{\omega}_t} Z_t B_{t-1} dF(\omega). \end{aligned}$$

where $(1 - F(\bar{\omega})) R_{t-1} B_{t-1}^g + \int_{\bar{\omega}^g}^{\bar{\omega}} \left(\omega R_t^k Q_{t-1} K_{t-1} - Z_t B_{t-1} \right) dF(\omega)$ are the resources taken from entrepreneur's profits that goes to repay central bank loans

D Central bank loans have higher seniority

When Central Bank loans have higher seniority, we redefine $\bar{\omega}^{g,j}$ as,

$$\bar{\omega}^{g,j} R_{t+1}^k Q_t K_t^j = R_t B_t^{g,j}. \tag{73}$$

If $\bar{\omega}^j > \omega \geq \bar{\omega}^{g,j}$, entrepreneur is able to fully payback central bank loans, while cannot fully pay bank loans, so government must intervene to ensure CB fully receive the agreed gross return. If $\bar{\omega}^{g,j} > \omega^j$, banks receive nothing from entrepreneurs and only pay partially to the Central Bank. In this case, the government will have to pay for the whole debt of firms to the central bank.

In this case the bank loan contract, equation (67), becomes,

$$[1 - F(\bar{\omega}^j)] Z_{t+1}^j B_t^j + (1 - \mu) \int_{\bar{\omega}^{g,j}}^{\bar{\omega}^j} (\omega R_{t+1}^k Q_t K_t^j - R_t B_t^{g,j}) dF(\omega) = R_{t+1}^l B_t^j, \tag{74}$$

Combining (37) and (73), yields,

$$(\bar{\omega}^j - \bar{\omega}^{g,j}) R_{t+1}^k Q_t K_t^j = Z_{t+1}^j B_t^j$$

Then, equation (74) yields,

$$\left([1 - F(\bar{\omega}^j)](\bar{\omega}^j - \bar{\omega}^{g,j}) + (1 - \mu) \left(\int_{\bar{\omega}^{g,j}}^{\bar{\omega}^j} \omega dF(\omega) - (F(\bar{\omega}^j) - F(\bar{\omega}^{g,j})) \bar{\omega}^{g,j} \right) \right) R_{t+1}^k Q_t K_t^j = R_{t+1}^l B_t^j.$$

For convenience this is written as,

$$\left(\Gamma_b(\bar{\omega}^{g,j}, \bar{\omega}^j) - \mu G_b(\bar{\omega}^{g,j}, \bar{\omega}^j) \right) R_{t+1}^k Q_t K_t^j = R_{t+1}^l \left(Q_t K_t^j - B_t^{g,j} - N_{et}^j \right), \tag{75}$$

where,

$$\Gamma_b(\bar{\omega}^{g,j}, \bar{\omega}^j) = \int_{\bar{\omega}^{g,j}}^{\bar{\omega}^j} \omega dF(\omega) + F(\bar{\omega}^{g,j}) \bar{\omega}^{g,j} + (1 - F(\bar{\omega}^j)) \bar{\omega}^j - \bar{\omega}^{g,j},$$

$$G_b(\bar{\omega}^{g,j}, \bar{\omega}^j) = \int_{\bar{\omega}^{g,j}}^{\bar{\omega}^j} \omega dF(\omega) - (F(\bar{\omega}^j) - F(\bar{\omega}^{g,j}))\bar{\omega}^{g,j}.$$

From (73), we obtain the expression for $\bar{\omega}^{g,j}$:

$$\bar{\omega}^{g,j} = R_t B_t^{g,j} / (R_{t+1}^k Q_t K_t^j). \tag{76}$$

Entrepreneurs aim to maximize equation (42), this time subject to equation (74). The optimal contracting problem may be now written as:

$$\max_{K_t, \bar{\omega}^j} \mathbb{E}_t \{ \left(1 - \Gamma(\bar{\omega}^j)\right) R_{t+1}^k Q_t K_t^j + \lambda_{t+1} \left[\left(\Gamma_b(\bar{\omega}^{g,j}, \bar{\omega}^j) - \mu G_b(\bar{\omega}^{g,j}, \bar{\omega}^j)\right) R_{t+1}^k Q_t K_t^j - R_{t+1}^l B_t^j \right] \},$$

where $\bar{\omega}^{g,j}(K_t^j)$ is obtained from (76) and $B_t^j = Q_t K_t^j - B_t^{g,j} - N_{et}^j$. The first order condition for $\bar{\omega}^j$:

$$-\frac{\partial \Gamma(\bar{\omega}^j)}{\partial \bar{\omega}^j} + \lambda_{t+1} \left(\frac{\partial \Gamma_b(\bar{\omega}^{g,j}, \bar{\omega}^j)}{\partial \bar{\omega}^j} - \mu \frac{G_b(\bar{\omega}^{g,j}, \bar{\omega}^j)}{\partial \bar{\omega}^j} \right) = 0.$$

The first order condition for K_t^j:

$$\begin{aligned} &\mathbb{E}_t \{ \left(1 - \Gamma(\bar{\omega}^j)\right) R_{t+1}^k + \lambda_{t+1} [\left(\Gamma_b(\bar{\omega}^{g,j}, \bar{\omega}^j) - \mu G_b(\bar{\omega}^{g,j}, \bar{\omega}^j)\right) R_{t+1}^k \\ + &\left(\frac{\partial \Gamma_b(\bar{\omega}^{g,j}, \bar{\omega}^j)}{\partial K_t^j} - \mu \frac{\partial G_b(\bar{\omega}^{g,j}, \bar{\omega}^j)}{\partial K_t^j} \right) R_{t+1}^k K_t^j - R_{t+1}^l] \} = 0. \end{aligned}$$

The first order condition for λ_{t+1} yields equation (75), where,

$$\frac{\partial \bar{\omega}^{g,j}}{\partial \bar{\omega}^j} = 0, \qquad \frac{\partial \bar{\omega}^{g,j}}{\partial K_t^j} = -R_t B_t^{g,j} / (R_{t+1}^k Q_t (K_t^j)^2).$$

$$\frac{\partial \Gamma(\bar{\omega}^j)}{\partial \bar{\omega}^j} = 1 - F(\bar{\omega}^j),$$

$$\frac{\partial \Gamma_b(\bar{\omega}^{g,j}, \bar{\omega}^j)}{\partial \bar{\omega}^j} = \bar{\omega}^j f(\bar{\omega}^j) - F(\bar{\omega}^j)\bar{\omega}^j + (1 - F(\bar{\omega}^j)) = 1 - F(\bar{\omega}^j).$$

$$\frac{\partial G_b(\bar{\omega}^{g,j}, \bar{\omega}^j)}{\partial \bar{\omega}^j} = \bar{\omega}^j f(\bar{\omega}^j) - \bar{\omega}^{g,j} f(\bar{\omega}^j) = f(\bar{\omega}^j)(\bar{\omega}^j - \bar{\omega}^{g,j})$$

$$\frac{\partial \Gamma_b(\bar{\omega}^{g,j}, \bar{\omega}^j)}{\partial K_t^j} = \left(-\bar{\omega}^{g,j} f(\bar{\omega}^{g,j}) + f(\bar{\omega}^{g,j})\bar{\omega}^{g,j} + F(\bar{\omega}^{g,j}) - 1\right) \frac{\partial \bar{\omega}^{g,j}}{\partial K_t^j} = (F(\bar{\omega}^{g,j}) - 1) \frac{\partial \bar{\omega}^{g,j}}{\partial K_t^j}$$

$$\frac{\partial G_b(\bar{\omega}^{g,j}, \bar{\omega}^j)}{\partial K_t^j} = \left[-\bar{\omega}^{g,j} f(\bar{\omega}^{g,j}) + f(\bar{\omega}^{g,j})\bar{\omega}^{g,j} - (F(\bar{\omega}^j) - F(\bar{\omega}^{g,j}))\right] \frac{\partial \bar{\omega}^{g,j}}{\partial K_t^j} = -(F(\bar{\omega}^j) - F(\bar{\omega}^{g,j})) \frac{\partial \bar{\omega}^{g,j}}{\partial K_t^j}$$

The amount transferred back to the central bank from entrepreneur is,

$$M_{t+1} = [1 - F(\bar{\omega}^{g,j})] R_t B_t^{g,j} + (1 - \mu) \int_0^{\bar{\omega}^{g,j}} \omega R_{t+1}^k Q_t K_t^j dF(\omega) - \mu (F(\bar{\omega}^j) - F(\bar{\omega}^{g,j})) R_t B_t^{g,j}.$$

In this case V_t^e becomes,

$$\begin{aligned} V_t^e &= R_t^k Q_{t-1} K_{t-1} - R_t^l B_{t-1} - (1 - F(\bar{\omega}^g)) R_{t-1} B_{t-1}^g - \int_0^{\bar{\omega}^g} \omega R_t^k Q_{t-1} K_{t-1} dF(\omega) \\ &- \mu \int_{\bar{\omega}_t^g}^{\bar{\omega}_t} \left(\omega R_t^k Q_{t-1} K_{t-1} - R_t B_{t-1}^g \right) dF(\omega). \end{aligned}$$

where $(1 - F(\bar{\omega}^g)) R_{t-1} B_{t-1}^g + \int_0^{\bar{\omega}^g} \omega R_t^k Q_{t-1} K_{t-1} dF(\omega)$ are the resources taken from entrepreneur's profits that goes to repay central bank loans

E Credit Policy Effects and Frictions

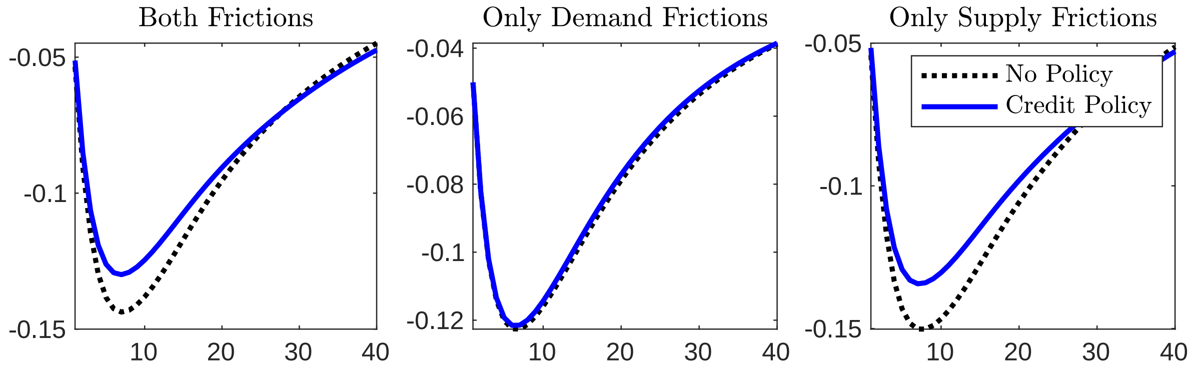
Figure 11 and Figure 12 reports the effects of the credit policy rule on the aggregate capital in an economy with either frictions only on the credit demand side or frictions only on the supply side, for both a state-contingent and non-state-contingent R_{t+1}^l, respectively.

When there are only frictions on the credit demand side, the only effect of credit policy effect is on the credit demand side due to the government guarantees. In the case of state-contingent R_{t+1}^l, according to figure 11, in the baseline calibration the government guarantees do not have a significant impact on reducing the negative effects of the capital quality shock. With non-state-contingent R_{t+1}^l, figure 12, the government guarantees should have a stronger effects since entrepreneurs are more exposed to aggregate shocks, however, this is still negligible.

When there are only frictions on the credit supply side (i.e. $\mu = 0$), the only effect of the credit policy is that it reduces the frictions of the credit supply side. In other words, since CB loans cannot be diverted, it increases the aggregate supply of credit per unit of bank net worth. According to figure 11, in the baseline calibration the fact that a fraction of aggregate credit cannot be diverted, which reduces the credit supply frictions, have a more significant impact on reducing the negative effects of the capital quality shock. With non-state-contingent R_{t+1}^l, figure 12, since shock is absorbed by entrepreneurs' net worth, which by definition does not affect credit demand, the effectiveness of the credit policy that essentially affects the credit supply is smaller.

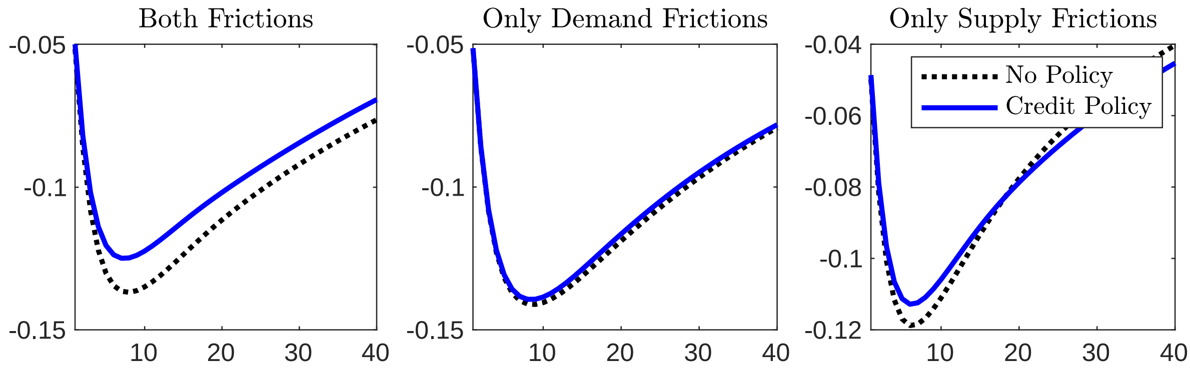
Finally, in our baseline calibration we might say that credit policy is more effective on reducing credit supply frictions than credit demand frictions. However, as we see in subsection 6.3 this is not necessarily true if we target a higher entrepreneur default probability, i.e., if there is a higher uncertainty in the economy.

Figure 11: State-contingent R^l_{t+1}: A five percent negative capital quality shock. Capital



Capital is in log deviations from steady-state.

Figure 12: Non-State-contingent R^l_{t+1}: A five percent negative capital quality shock. Capital



Capital is in log deviations from steady-state.

F No government credibility without supply credit frictions

As in the case of government credibility, entrepreneurs aim to maximize their expected profits, given by equation (42), but this time subject to the state-contingent constraints implied by equation (53). The first order conditions for $\bar{\omega}^j$, K^j_t and λ_{t+1} are respectively,

$$-\frac{\partial \Gamma(\bar{\omega}^j)}{\partial \bar{\omega}^j} + \lambda^j_{t+1}\left(\frac{\partial \Gamma(\bar{\omega}^j)}{\partial \bar{\omega}^j} - \mu \frac{G(\bar{\omega}^j)}{\partial \bar{\omega}^j}\right) = 0.$$

$$\mathbb{E}_t \left\{ \left(1 - \Gamma(\bar{\omega}^j)\right) R^k_{t+1} + \lambda_{t+1} \left[\, \left(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right) R^k_{t+1} - R_t \, \right] \right\} = 0.$$

$$\left(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right) R^k_{t+1} Q_t K^j_t = R_t \left(Q_t K^j_t - N^j_{et}\right).$$

where,

$$\frac{\partial \Gamma(\bar{\omega}^j)}{\partial \bar{\omega}^j} = 1 - F(\bar{\omega}^j), \qquad \frac{\partial G(\bar{\omega}^j)}{\partial \bar{\omega}^j} = \bar{\omega}^j f(\bar{\omega}^j).$$

Clearly, these equilibrium conditions are the same than those under no credit policy. This implies that in equilibrium K_t^j and $\bar{\omega}^j$ are also identical to those under no credit policy.

G Government transfers due to guarantees of CB loans

When bank loans have higher seniority, equation (46) becomes,

$$\begin{aligned} & [1 - F(\bar{\omega}^{g,j})] Z_{t+1}^j B_t^j + [1 - F(\bar{\omega}^j)] R_t B_t^{g,j} + (1-\mu) \int_0^{\bar{\omega}^{g,j}} \omega R_{t+1}^k Q_t K_t^j dF(\omega) \\ - & \mu \int_{\bar{\omega}^{g,j}}^{\bar{\omega}^j} Z_{t+1}^j B_t^j dF(\omega) + \int_{\bar{\omega}^{g,j}}^{\bar{\omega}^j} \left(\omega R_{t+1}^k Q_t K_t^j - Z_{t+1}^j B_t^j \right) dF(\omega) + S_{t+1}^j \\ = & R_{t+1}^l B_t^j + R_t B_t^{g,j} + \mu \int_{\bar{\omega}^{g,j}}^{\bar{\omega}^j} \left(\omega R_{t+1}^k Q_t K_t^j - Z_{t+1}^j B_t^j \right) dF(\omega), \end{aligned} \tag{77}$$

where,

$$S_{t+1}^j = R_t B_t^{g,j} - [1 - F(\bar{\omega}^j)] R_t B_t^{g,j} - (1-\mu) \int_{\bar{\omega}^{g,j}}^{\bar{\omega}^j} \left(\omega R_{t+1}^k Q_t K_t^j - Z_{t+1}^j B_t^j \right) dF(\omega).$$

It is easy to verify that equation (77) becomes as the equilibrium condition described in equation (48). However, the government subsidies, equation (49), become,

$$S_{t+1}^j = \int_{\bar{\omega}^{g,j}}^{\bar{\omega}^j} \left[R_t B_t^{g,j} - (1-\mu) \left(\omega R_{t+1}^k Q_t K_t^j - Z_{t+1}^j B_t^j \right) \right] dF(\omega) + F(\bar{\omega}^{g,j}) R_t B_t^{g,j}.$$

It says that for a fraction $(F(\bar{\omega}^j) - F(\bar{\omega}^{g,j}))$ of entrepreneurs, the government has to complement the payment to CB loans, while for a fraction $F(\bar{\omega}^{g,j})$ of entrepreneurs, which cannot repay anything of the CB loans since they already exhausted all their revenues repaying bank loans first, the government needs to fully pay the whole central bank loan debt. Contrasting with the government subsidies, when both, bank loans and CB loans, have the same seniority, equation (49), this time the government (for a given K_t^j) have to spend more, since a larger share of revenues are going to repay bank loans as these are paid first. For a given K_t we can conclude that when bank loans have higher seniority, government expends more.

Also, when central bank loans have higher seniority, equation (46) becomes,

$$[1-F(\bar{\omega}^j)]Z^j_{t+1}B^j_t + [1-F(\bar{\omega}^{g,j})]R_tB^{g,j}_t + (1-\mu)\int_{\bar{\omega}^{g,j}}^{\bar{\omega}^j}(\omega R^k_{t+1}Q_tK^j_t - R_tB^{g,j}_t)dF(\omega)$$
$$+ \int_0^{\bar{\omega}^{g,j}} \omega R^k_{t+1}Q_tK^j_t dF(\omega) + S^j_{t+1} = R^l_{t+1}B^j_t + R_tB^{g,j}_t$$
$$+ \mu\int_0^{\bar{\omega}^{g,j}} \omega R^k_{t+1}Q_tK^j_t dF(\omega) + \mu\int_{\bar{\omega}^{g,j}}^{\bar{\omega}^j} R_tB^j_t dF(\omega), \tag{78}$$

where,

$$S^j_{t+1} = R_tB^{g,j}_t - [1-F(\bar{\omega}^{g,j})]R_tB^{g,j}_t + \mu\int_{\bar{\omega}^{g,j}}^{\bar{\omega}^j} R_tB^j_t dF(\omega) - (1-\mu)\int_0^{\bar{\omega}^{g,j}} \omega R^k_{t+1}Q_tK^j_t dF(\omega),$$

It is easy to verify that equation (78) becomes as in equation (48). However, government subsidies, equation (49), become,

$$S^j_{t+1} = \int_0^{\bar{\omega}^{g,j}} \left[R_tB^{g,j}_t - (1-\mu)\omega R^k_{t+1}Q_tK^j_t\right] dF(\omega) + \mu\int_{\bar{\omega}^{g,j}}^{\bar{\omega}^j} R_tB^j_t dF(\omega),$$

This time, the government only complements payments for a fraction $F(\bar{\omega}^{g,j})$ and also pays for the monitoring costs when entrepreneur defaults, but can still fully pay central bank loans. Contrasting with (49), the fraction of entrepreneurs that default on CB loans is smaller, and consequently required transfers are smaller. For a given K_t we can conclude that when bank loans have lower seniority, government expends less.

Finally, we can immediately see that the arguments delivered regarding the effects of no government credibility in subsection 5.2.1 holds for any seniority assumption.

H Conventional Credit Policy

Equation (37) becomes,

$$\bar{\omega}^j R^k_{t+1}Q_tK^j_t = Z^j_{t+1}B^j_t + Z^{g,j}_{t+1}B^{g,j}_t, \tag{79}$$

where $Z^{g,j}_{t+1}$ is the (non-default) lending rate of CB loans since these are not longer guarantee by government. Let start assuming that CB loans are directly given by central bank. So, there is a contract for bank loans and another for CB loans. In this case, the bank loan contract, equation (39), becomes,

$$[1-F(\bar{\omega}^j)]Z^j_{t+1}B^j_t + (1-\mu)\int_0^{\bar{\omega}^j} \omega R^k_{t+1}Q_tK^j_t x^j_{t+1} dF(\omega) = R^l_{t+1}B^j_t, \tag{80}$$

and we have the CB loan contract,

$$[1 - F(\bar{\omega}^j)]Z^{g,j}_{t+1}B^{g,j}_t + (1-\mu)\int_0^{\bar{\omega}^j} \omega R^k_{t+1}Q_t K^j_t(1 - x^j_{t+1})dF(\omega) = R^l_{t+1}B^{g,j}_t. \tag{81}$$

where,

$$x^j_{t+1} = \frac{Z^j_{t+1}B^j_t}{Z^{g,j}_{t+1}B^{g,j}_t + Z^j_{t+1}B^j_t}. \tag{82}$$

From equations (80), (81) and (82),

$$Z^j_{t+1} = Z^{g,j}_{t+1}.$$

and hence CB loans and bank loans are identical from entrepreneur's perspective. Also,

$$x^j_{t+1} = \frac{B^j_t}{B^{g,j}_t + B^j_t}. \tag{83}$$

Combining the loan contracts equations (80) and (81) and using (79) yields,

$$\left(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right) R^k_{t+1}Q_t K^j_t = R^l_{t+1}\left(Q_t K^j_t - N^j_{et}\right),$$

As a result, the optimal contracting problem is identical to the maximization problem without credit policy. Hence, the first order conditions for $\bar{\omega}^j$, K^j_t and λ_{t+1} are as in the case without credit policy. Then, the aggregate demand is not altered by the credit policy.[33]

In terms of the composition of external funding, we state that in equilibrium entrepreneurs demand all CB loans available, and then $B^j_t = K^j_t - B^{g,j}_t - N^j_{et}$ in such a way that per unit of external funding a share $\psi_{CB,t}$ is demanded to the central bank while a share $1 - \psi_{CB,t}$ is demanded to banks.

Now, let assume that CB loans are given through banks. In other words, central bank gives funds to bank and charge a risk-free rate for these funds, and ask banks to issue the same amount as central bank loans. Note that if we assume that banks can also divert a fraction λ of CB loans, we are back to the case of no credit policy. This is because in that scenario CB bank loans are identical to bank loans from banks' perspective. For realism and for comparison reasons we say that banks cannot divert CB loans as they do with bank loans and hence clearly credit policy is going to affect aggregate credit supply. Formally, equation (5) becomes,

$$N^i_{bt+1} = R^l_{t+1}(B^i_t + B^{g,i}_t) - R_t(D^i_t + B^{g,i}_t),$$

[33]It is easy to verify that this holds for any seniority assumption.

and banker's incentive constraint, equation (7), becomes,

$$V_t^i \geq \lambda B_t^i + \lambda^g B_t^{g,i}, \tag{84}$$

Note that since it is more difficult to divert CB loans that bank loans, $0 < \lambda^g < \lambda$. For comparison reasons, we assume the extreme case $\lambda^g = 0$. We can express V_t^i as follows,

$$V_t^i = \nu_t(B_t^i + B_t^{g,i}) + \eta_t N_{bt}^i,$$

with

$$\nu_t^i = \mathbb{E}_t\{(1-\sigma)\Lambda_{t,t+1}(R_{t+1}^l - R_t) + \Lambda_{t,t+1}\sigma x_{t,t+1}\nu_{t+1}^i\},$$

$$\eta_t^i = \mathbb{E}_t\{1 - \sigma + \Lambda_{t,t+1}\sigma z_{t,t+1}^i \eta_{t+1}^i\},$$

where $x_{t,t+m}^i = (B_{t+m}^i + B_{t+m}^{g,i})/(B_t^i + B_t^{g,i})$. Then, the incentive constraint (84) becomes,

$$\nu_t^i(B_t^i + B_t^{g,i}) + \eta_t^i N_t^i, \geq \lambda B_t^i = \lambda(1 - \psi_{CB,t})(B_t^i + B_t^{g,i}).$$

Under reasonable parameter values the constraint always binds within a local region of the steady state. Then,

$$B_t^i + B_t^{g,i} = \frac{\eta_t^i}{\lambda(1-\psi_{CB,t}) - \nu_t^i} N_{bt}^i = \phi_t^i N_{bt}^i,$$

where $\phi_t^i = (B_t^i + B_t^{g,i})/N_{bt}^i$. We rewrite the evolution of bank's net worth (5) as,

$$N_{bt+1}^i = \left[(R_{t+1}^l - R_t)\phi_t^i + R_t\right] N_{bt}^i.$$

We then rewrite $z_{t,t+1}^i$ and $x_{t,t+1}^i$ as, respectively,

$$z_{t,t+1}^i = N_{bt+1}^i / N_{bt}^i = (R_{t+1}^l - R_t)\phi_t^i + R_t,$$

$$x_{t,t+1}^i = (B_{t+1}^i + B_{t+1}^{g,i})/(B_t^i + B_t^{g,i}) = (\phi_{t+1}^i/\phi_t^i) z_{t,t+1}^i.$$

Then, equations (11) and (12) becomes respectively,

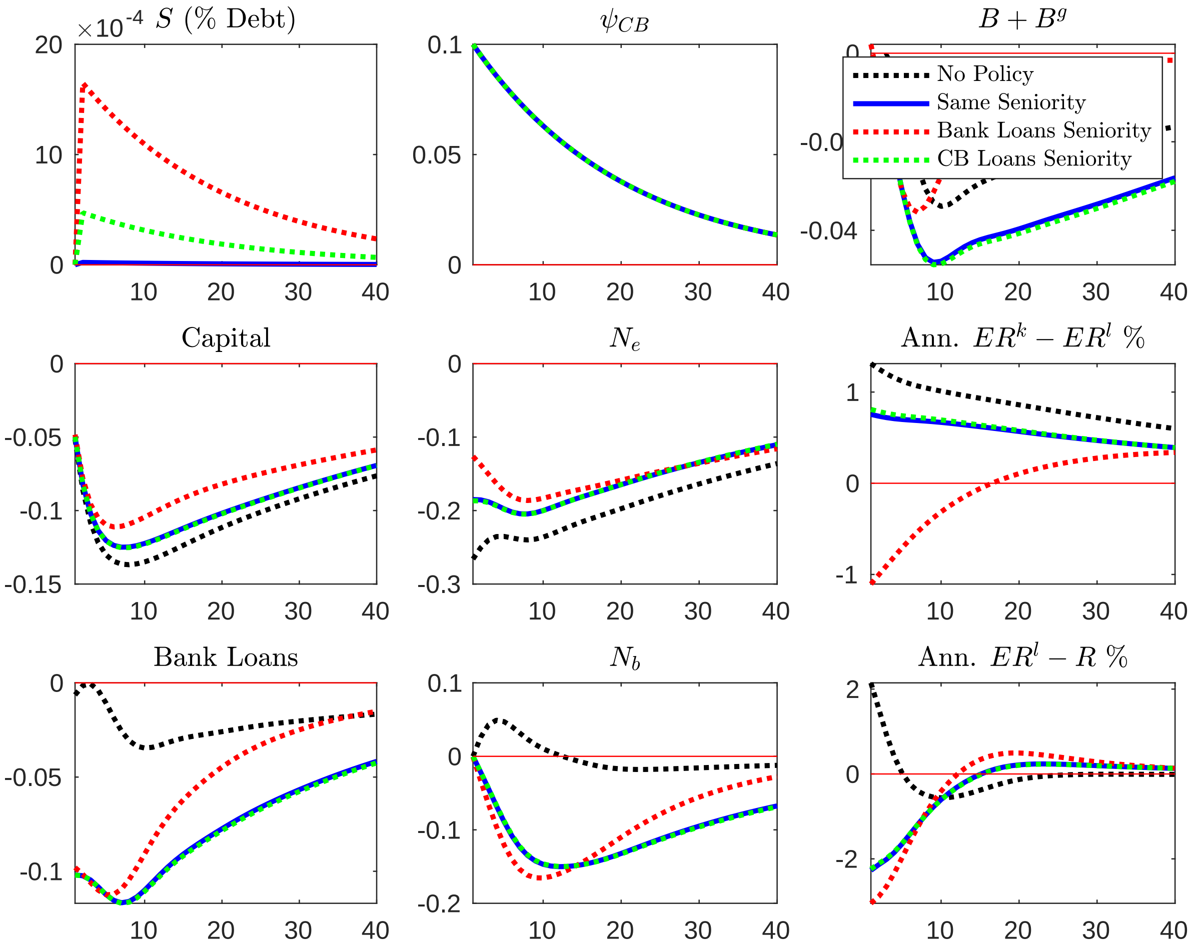
$$N_{nt} = \zeta(B_{t-1} + B_{t-1}^g).$$

Combining equations (10) and (11) yields the aggregate motion of bank net worth,

$$N_{bt} = \sigma\left[(R_t^l - R_{t-1})\phi_{t-1} + R_{t-1}\right] N_{bt-1} + \zeta(B_{t-1} + B_{t-1}^g).$$

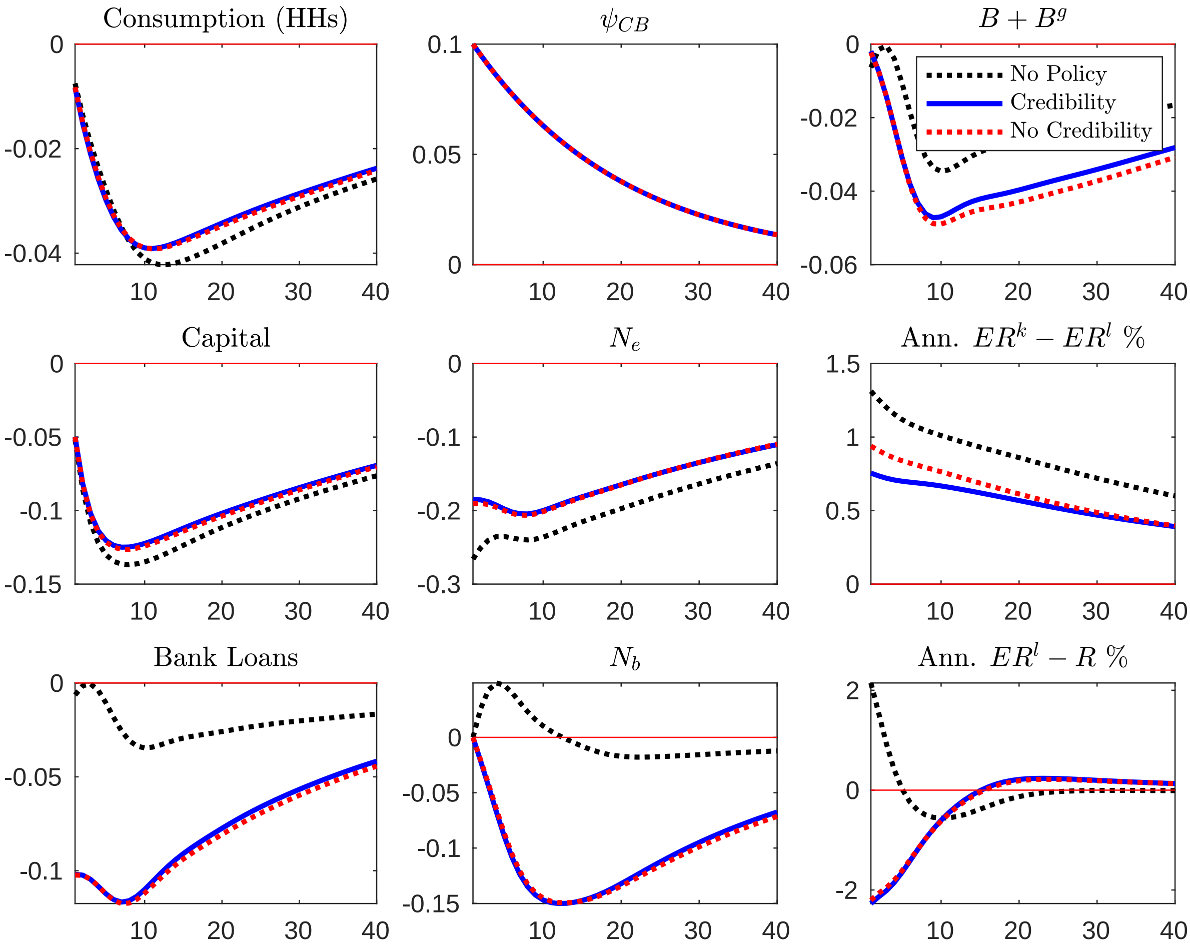
I Additional Figures

Figure 13: A five percent negative capital quality shock. Non-state-contingent. Seniority assumptions



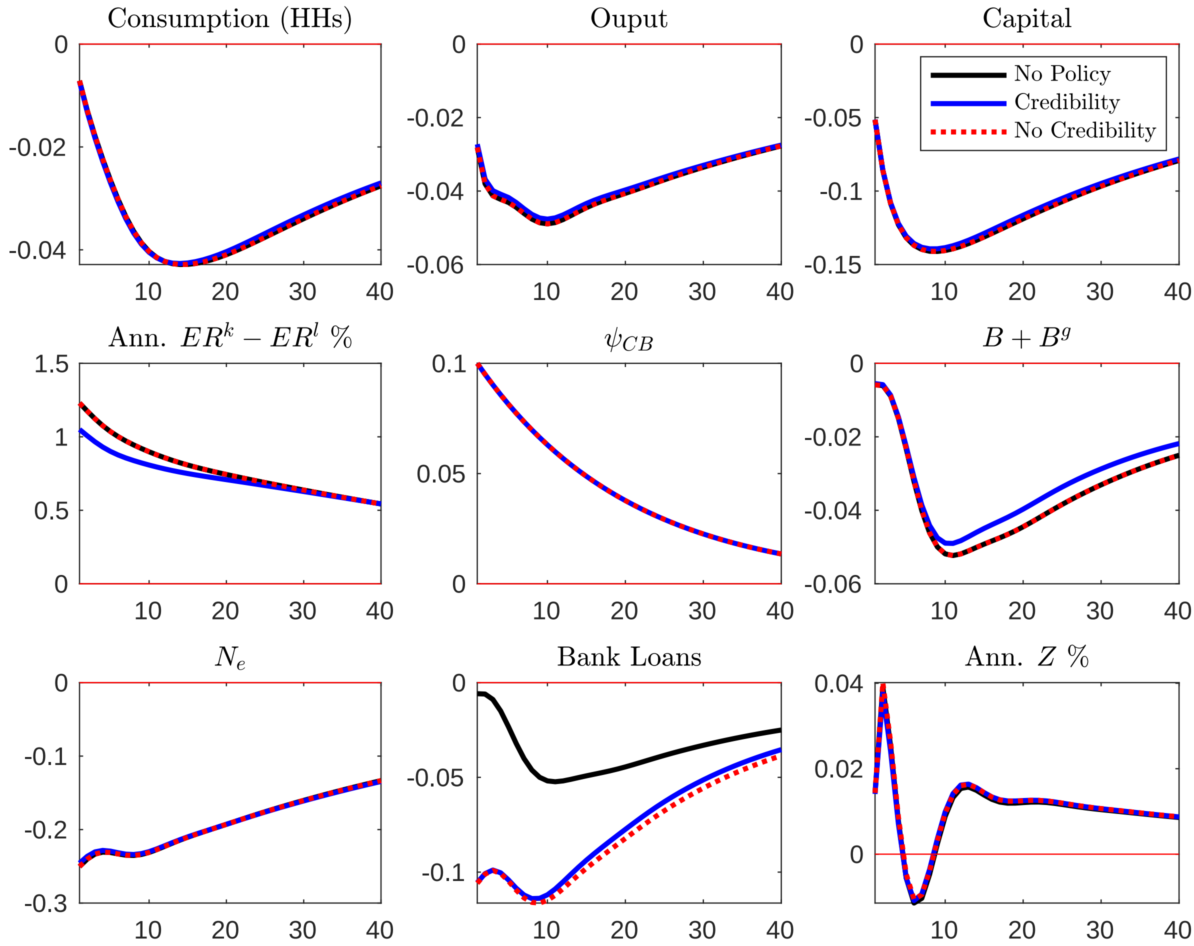
All variables are in log deviations from steady-state except spreads and CB loans share, shown in level deviation from steady-state.

Figure 14: Credibility: A five percent negative capital quality shock: Non-State-Contingent R^l_{t+1}



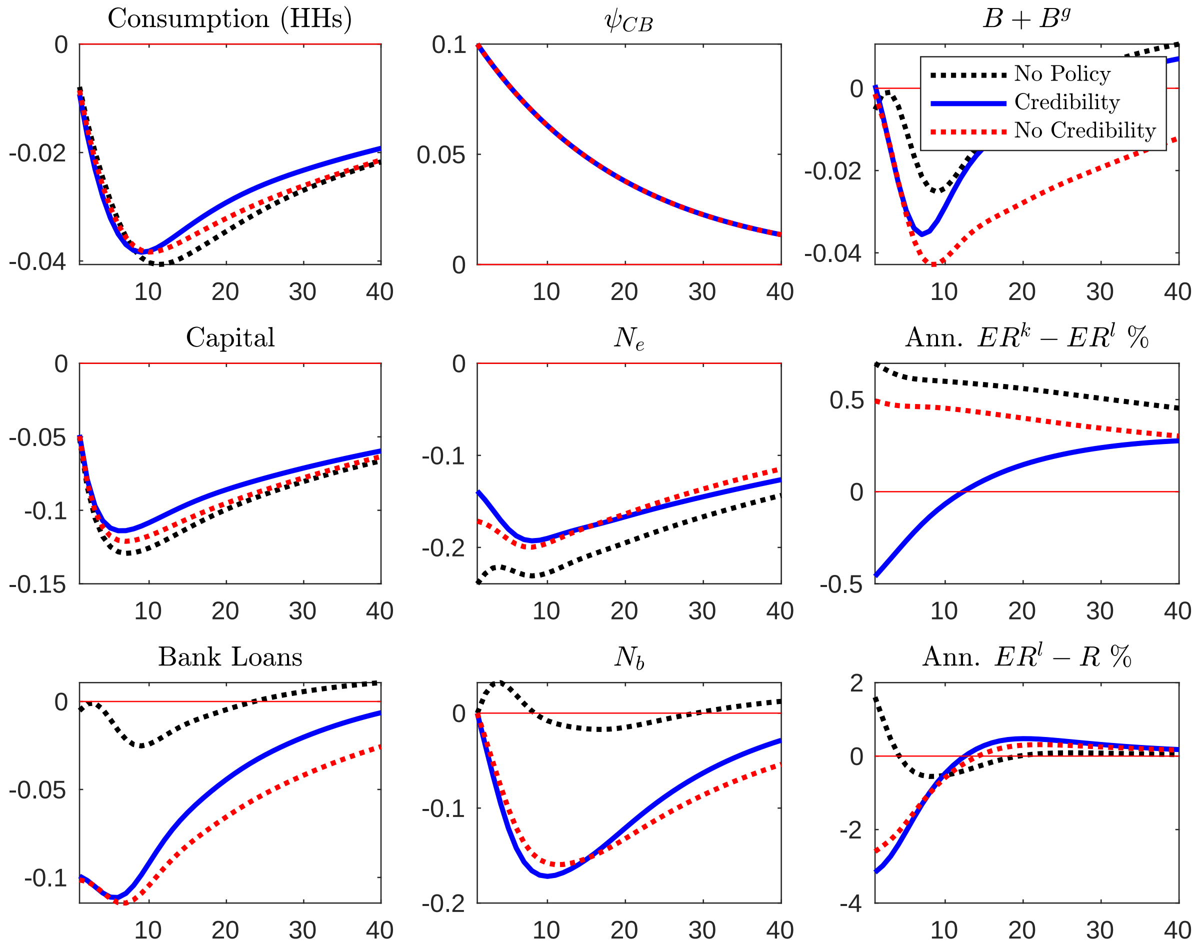
All variables are in log deviations from steady-state except spreads and CB loans share, shown in level deviation from steady-state.

Figure 15: Credibility: A five percent negative capital quality shock: No credit supply frictions



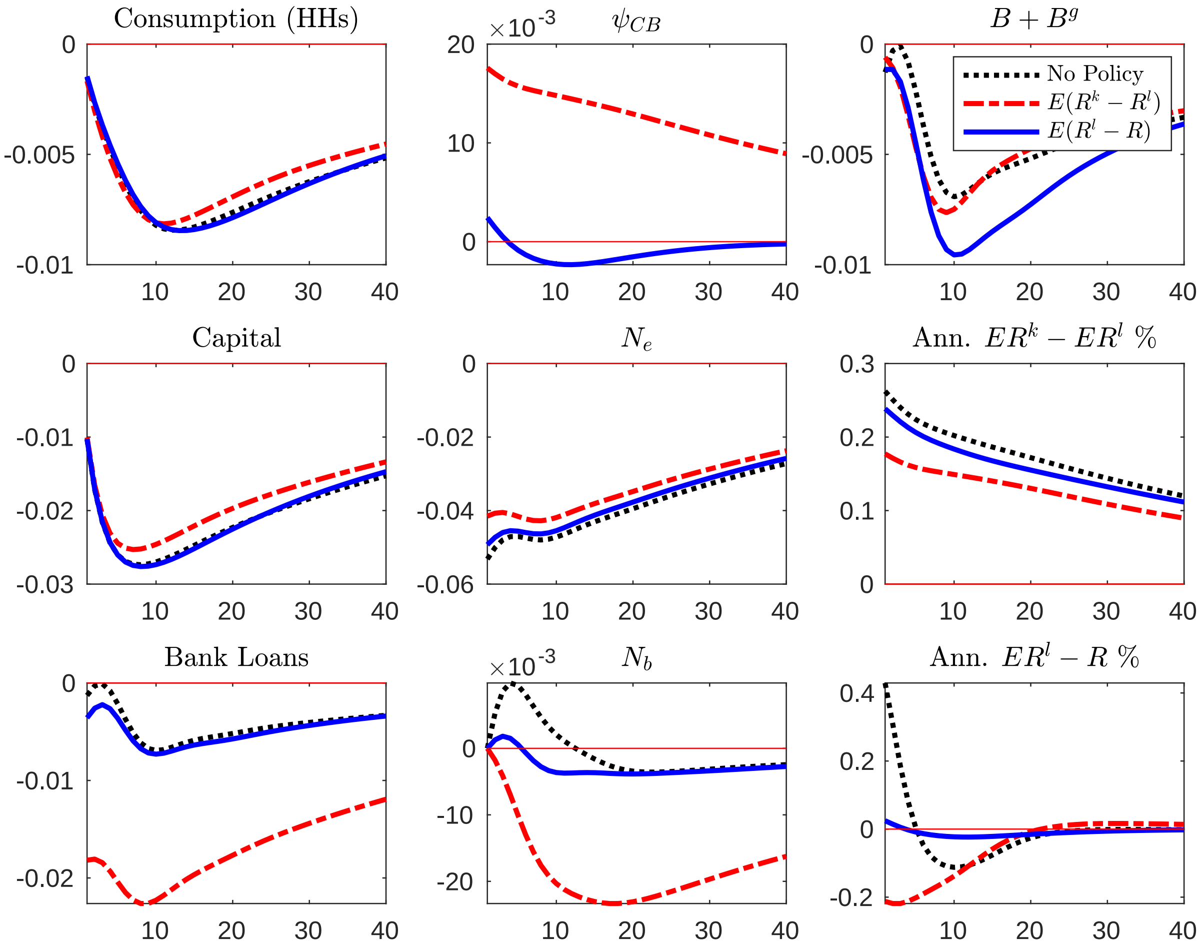
All variables are in log deviations from steady-state except spreads and CB loans share, shown in level deviation from steady-state.

Figure 16: Credibility: A five percent negative capital quality shock: Non-State-Contingent R^l_{t+1}



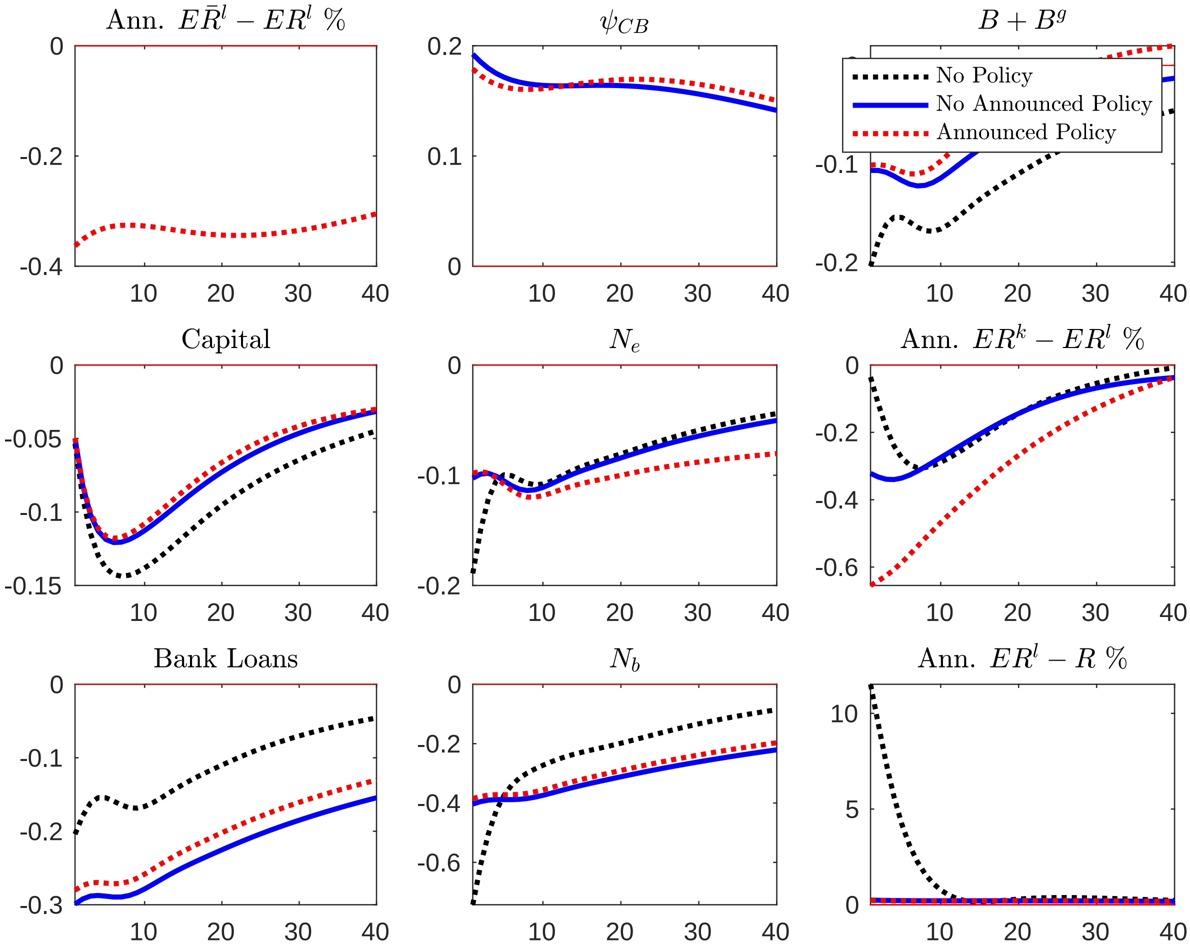
All variables are in log deviations from steady-state except spreads and CB loans share, shown in level deviation from steady-state.

Figure 17: Endogenous rule: A five percent negative capital quality shock. Non-state-contingent R^l_{t+1}



All variables are in log deviations from steady-state except spreads and CB loans share, shown in level deviation from steady-state.

Figure 18: Announcement: A five percent negative capital quality shock



All variables are in log deviations from steady-state except spreads and CB loans share, shown in level deviation from steady-state.