BANCO CENTRAL DE RESERVA DEL PERÚ

Fat-Tailed Shocks and the Central Bank Reaction

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Abstract

In this paper we extend the model of Kato and Nishiyama (2005) by introducing fat-tailed shocks in a simple new Keynesian framework where the central bank explicitly considers the zero lower-bound constraint on interest rates. We find that shocks with 'excess kurtosis' make monetary policy relatively more aggressive far away from the zero lower bound region though, this difference reverts as the economy gets closer to the constrained region. From a quantitative point of view, our findings suggest that variance-preserving shifts in kurtosis, in the shape of Laplace distributed shocks, do not produce significant effects on the optimal reaction of the central bank.

Resumen

En este documento extendemos el modelo de Kato y Nishiyama (2005) al introducir choques de 'colas altas' en un marco Neo-Keynesiano simple en el que el banco central considera en forma explícita la restricción de no-negatividad de las tasas de interés. Hallamos que choques con 'curtosis excesiva' hacen que la política monetaria sea más agresiva lejos de la región restringida, sin embargo, esta diferencia se revierte a medida que la economía se aproxima a la región restringida. Desde un punto de vista cuantitativo, nuestros resultados sugieren que cambios de curtosis que preservan la varianza constante, en la forma de choques Laplacianos, no producen efectos significativos en la reacción óptima del banco central.

JEL Code: E52, E58, C63.

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"[...] we would expect policy-makers to take action when the mean and variance of forecast distributions are likely to stay the same, while the probability of some extreme bad event increases. [...] even if the variance is unchanged, an increase in the possibility of a severe economic downturn is likely to prompt action."
Cecchetti (2000).

1 Introduction

According to Mishkin (2011), one of the main lessons from the financial crisis is that key elements in the "science of monetary policy" need to be revisited. In particular those related to the non-linearities emerging in presence of the zero lower bound (ZLB), tail risk, and non-standard utility functions - such as agents' aversion to very negative outcomes. As the author points out, previous to the 2008 financial crisis, economists were aware of the presence of potential negative shocks with 'excess kurtosis' hitting the economy with a higher tail risk probability than the one implied by a Gaussian distribution.[1] In spite of acknowledging the presence of these shocks, little was done to study the importance of excess kurtosis in monetary policy design.

The presence of non-linearities is obvious when monetary policy is affected by the non-negativity constraint on nominal interest rates. If the policy rate falls below zero, agents will prefer to keep their resources in cash, which pays a zero interest rate. For this reason, the space for the policy interest rate is bounded from below, with consequences for the policy decisions. Moreover, Kato and Nishiyama (2005) and Adam and Billi (2007) show how the presence of the zero lower bound makes the (discretionary) optimal monetary policy reaction to be non-linear *outside of the constrained region* as well. In particular, central banks should become more expansionary and more aggressive as they approach the ZLB, compared to what a linear Taylor rule type of policy predicts. This result is in line with the suggestions in Blinder (2000):

"... make the response function non-linear. In particular, the coefficient a [the coefficient in the Taylor rule that controls the response of the policy rate to inflation] - *and perhaps b* [the coefficient in the Taylor rule that controls the response of the policy rate to the output gap] *as well - could be higher when inflation is low. (...) such a modification would make monetary policy looser whenever inflation was very low, thus buying more insurance against getting stuck in the liquidity trap at $i = 0$."*

Central to the non-linearities generated outside of the ZLB region is the hazard of falling in it. For this reason, when the economy faces shocks from a fat-tailed distribution or increased kurtosis, the reaction should be more aggressive. However, it is not clear how this excess kurtosis impacts optimal policy rules. The present document tackles this question by introducing fat-tailed shocks in the model of Kato and Nishiyama (2005). This is a simple Neo-Keynesian model where the central bank reacts in a "pre-emptive" manner as the probability of falling into the ZLB increases, generating non-linear responses *outside* of the zero lower bound region.[2]

[1] A fact reflecting this concern was the emergence of Financial Stability Reports as a regular publication by Central Banks where the risks that the financial system put into the economy were discussed, see Mishkin (2011).

[2] In this model the rest of the economy is characterized by a set of linear equations. Introducing agents

We perform this exercise to gauge the extent to which excess kurtosis affects the optimal behaviour of central banks outside the zero lower bound and, if this effect is significant, analyse to what extent excess kurtosis may be behind the reported change in the behaviour of central banks before and during the crisis.[3]

We focus exclusively in the role of excess kurtosis, assuming a time-invariant distribution of shocks. Our main findings are as follows: (1) under fat-tailed shocks, monetary policy becomes more aggressive further away from the zero lower bound region, compared to the model under Gaussian shocks. (2) As the economy approaches the ZLB, this pattern reverts and monetary policy is relatively *less aggressive* under shocks with excess kurtosis. (3) *Quantitatively*, these differences are not very significant as the largest differential between the optimal rates, under our baseline calibration, is lower than 10 basis points.

There is a small but growing literature related to the presence of fat-tailed shocks in macroeconomics. Fagiolo et al. (2008) pursue the hypothesis of non-normal shocks and fit via maximum likelihood the growth rate distributions for a series of OECD countries to the exponential-power (EP) family of densities, rejecting the hypothesis of normality in these series. In related work, Ascari et al. (2012) show that non-normality and fat tails characterize not only the time-series properties for GDP in the U.S, but also those for consumption, investment, employment, inflation and real wage.

By contrast, the literature on liquidity traps and the optimal policy at ZLB is extensive. The theoretical question regarding the effectiveness of monetary policy at low rates can be found in Keynes (1936). More recently, the subject received a lot of attention from policy-makers and academics as the lower inflation experienced during the early 1990s in advanced economies brought with it episodes of near-zero interest rates.[4] In October 1995, the Bank of Japan (BOJ) set its policy interest rates at 50 basis points in the midst of a deflationary crisis. A few years later, the federal funds rate in the US experienced a sharp fall, going from 6.50 percent in November 2000 to only one percent on July 2003. To date, both the Federal Reserve and the Bank of Japan maintain their policy interest rates effectively at zero.

Fuhrer and Madigan (1997) constitute one of the first efforts to analyse the dynamics of the economy in a model with forward looking agents and an explicit ZLB constraint. The authors find that after a negative shock to the economy, the recovery of the inflation rate and output takes longer when monetary policy becomes ineffective due to the ZLB. Regarding the optimal policy under the ZLB, Reifschneider and Williams (1999) find that the standard Taylor rule is suboptimal in this scenario. Orphanides and Wieland (2000) add to this result by showing that the optimal policy under the ZLB constraint will become a non-linear function of the inflation rate. The literature considers as well the idea of monetary policy being affected by the ZLB before the constraint becomes bind-

that change their behaviour in the presence of the zero lower bound increases greatly the computational costs. See Fernández-Villaverde et al. (2012).

[3] Authors such as Taylor (2007) and Calani et al. (2010) estimate Taylor rules type of policies for the pre-crisis period and simulate the paths of interests rates provided those rules would have continued during the years of turmoil, finding very large differences between the actual path of interest rates and the projected paths. They conclude that the cuts in rates represent "deviations" from the pre-crises Taylor rules.

[4] In the advanced economies, the median inflation rate fell from 7% in the 1980s to 2% in the 1990s. See Kroszner (2007).

ing (Hunt and Laxton (2004), Goodfriend (2001)). More recently, Kato and Nishiyama (2005) studied the importance of this pre-emptive motive, showing how optimal monetary policy should become more aggressive and expansionary as the economy approaches the ZLB. Nakov (2006) relaxes the assumption of perfect foresight in the Kato and Nishiyama (2005) and studies an optimal "censoredÂ´Â´ Taylor rule, which is the best lineal response conditional on the presence of the ZLB. Eggertsson and Woodford (2003) study the implications of the ZLB for monetary policy in a model that assumes a 2-state Markov chain for an exogenous disturbance. They find support for a price-level targeting type of policy, though lose the pre-emptive motive that emerges under a more general distribution for the exogenous disturbance. Finally, Fernández-Villaverde et al. (2012) adopt a fully non-linear approach in a New Keynesian model with an explicit ZLB and explore the role of fiscal policy when the economy hits the constraint. Authors relax the assumption of a time-invariant distribution and study the role of skewness and time-varying volatility for endogenous variables when the economy hits the ZLB.

The present document is structured as follows: Section 2 reviews the model of Kato and Nishiyama (2005) and explains the mechanism behind the results. In the next section, we discuss the computational strategy. Section 4 discusses the calibration of parameters and presents the results. Section 5 concludes.

2 The Model

In the current section we review the model of Kato and Nishiyama (2005). We assume the Central Bank minimizes a loss function in the spirit of Svensson (1997), Svensson (2002) and Ball (1999), namely:

$$L_t = \frac{1}{2}\{y_t^2 + \lambda(\pi_t - \pi_t^*)^2\}; \tag{1}$$

here π stands for inflation; y for the output gap and π^* the inflation target, which we assume constant. The parameter λ controls the relative importance that the central bank puts on the inflation rate deviations from the target, relative to the output gap. Following Woodford (2003), the economy is described by the following IS-AS framework:

$$y_{t+1} = \rho y_t - \delta(i_t - E_t\pi_{t+1}) + \nu_{t+1} \tag{2}$$
$$\pi_{t+1} = \pi_t + \alpha y_t + \varepsilon_{t+1} \tag{3}$$

where ν and ε are random disturbances. ρ stands for the degree of inertia over the business cycle. δ is a parameter reflecting the impact of real interest rates on the next period output - thus monetary policy affects the economy with a lag. Finally, α represents the impact of the output gap on future inflation.

Equations (2) and (3) represent the investment-savings (IS) and aggregate supply (AS) equations respectively. We substitute the expectation of inflation by a combination of the current inflation rate and the output gap, namely:

$$E_t\pi_{t+1} = \pi_t + \alpha y_t. \tag{4}$$

The inter-temporal problem of the monetary authority will be given by:

$$\min_{\{i_{t+j}\}_{j=0}^{\infty}} E_t \sum_{j=0}^{\infty} \beta^j L_{t+j}, \tag{5}$$

subject to the laws of motion for inflation and output gap given by equations (3) and (2), and an explicit zero lower constraint on the interest rate introduced through the Karush-Kuhn-Tucker approach. β reflects the time-preference of the central banker, or equivalently, the importance they assign to future losses relative to losses in the current period. This framework allows us to set up a Bellman equation with three Lagrange multipliers:

$$\begin{aligned} V(y_t,\pi_t) = \min_{i_t}[\frac{1}{2}\{y_t^2+\lambda(\pi_t-\pi^*)^2\} - E_t\phi_{t+1}\{(\rho+\alpha\delta)y_t - \delta i_t + \delta\pi_t - y_{t+1}\} \\ - E_t\mu_{t+1}(\pi_t+\alpha y_t-\pi_{t+1}) - \psi_t i_t + \beta E_t V(y_{t+1},\pi_{t+1})]. \end{aligned} \tag{6}$$

where ψ_t is the Lagrange multiplier in the non-negativity constraint for the policy interest rate. The first order condition with respect to the interest rate yields:

$$E_t\phi_{t+1}\delta = \psi_t; \tag{7}$$

which measures the "shadow cost" produced by monetary policy ineffectiveness at the zero lower bound. The first order conditions with respect to inflation and the output gap are given by the following two equations:

$$E_t\mu_{t+1} = -\beta\left[\lambda E_t(\pi_{t+1}-\pi^*) - \delta E_t\phi_{t+2} - E_t\mu_{t+2}\right] \tag{8}$$

$$E_t\phi_{t+1} = -\beta\left[E_t y_{t+1} - (\rho+\alpha\delta)E_t\phi_{t+2} - \alpha E_t\mu_{t+2}\right], \tag{9}$$

By combining equations (7), (8), and (9). It is possible to get some intuition about the restrictions for monetary policy that the ZLB imposes. In the case the ZLB is not binding we know that $\psi_t = 0$. This means, , from Equation (7), that ϕ_{t+1}, the Lagrange multiplier associated with Equation (2) - the IS equation - is zero as well. Thus, the only restriction that matters for the central bank will be the one associated with Equation (3), which represents the trade-off between stabilizing the inflation rate deviations and the output gap. In other words, the bank can fully neutralize the shocks coming from the IS equation. However, when the ZLB is binding, then $E_t\phi_{t+1} > 0$, meaning that the central bank can no longer offset the shocks coming from the IS equation. In this scenario, the central bank needs to balance the need of offsetting both the AS and IS shocks.

Kato and Nishiyama (2005) obtain an analytical derivation of the optimal interest rate.[5]

$$i^*(\pi_t,y_t) = \pi_t + \left(\alpha + \frac{\rho\theta_1+\theta_1-1}{\delta\theta_1}\right)y_t + \left(\frac{\theta_1-1}{\alpha\delta\theta_1}\right)(\pi_t-\pi^*) + \left(\frac{1}{\delta\theta_1}\right)\sum_{i=0}^{\infty}\theta_2^i E_t\psi_{t+i} \tag{10}$$

Equation (10) represents the optimal reaction function outside of the zero lower bound region. The values of θ_1 and θ_2 are combinations of the "deep parameters" α, β, and λ.

[5]We refer the reader to the paper for the derivations.

The first three terms of this expression are linear in the output gap and the inflation rate. The last term is the one generating the non-linearities, which stem from the shadow cost represented by the sequence of Lagrange multipliers associated with the non-negativity restriction $\{E_t\psi_{t+i}\}_{i=0}^{\infty}$. As we already mentioned, Equation (7) tells us that when the value of this multiplier is different from zero, the central bank is unable to offset the shocks coming from the IS equation.[6] In other words, the non-linearities are associated with the probability that the ZLB restriction becomes binding in the future. Given the difficulty of obtaining a closed-form solution for the optimal policy as a function of the inflation rate deviations and the output gap, the solution is obtained through a numerical procedure.

3 Computational strategy

The numerical strategy follows Kato and Nishiyama (2005). It is based on collocation methods.[7] The Bellman equation in (6) imposes a series of restrictions that must hold in every point of the state-space. This defines an infinite-dimensional fixed-point problem that can be discretized by approximating the value function as the sum of a finite set of basis functions. Since it is important to capture the non-linear behaviour of optimal rates, the value function is approximated through cubic splines. Obtaining the value function involves the calculation of expectations, for which we use numerical integration techniques. In particular, a Gaussian quadrature technique is used to approximate the integrals.[8] For the case of Gaussian shocks, we use the Gaussian-Hermite quadrature method, for which tables with the values of nodes and abscissas are easily found.

For the case of fat-tailed shocks we need the use of a distribution exhibiting "excess kurtosis".[9] Additionally, this distribution must exhibit finite moments (at least up to the 4th order) that are stable functions of the distribution parameters, such that we are able to control the lower moments. For this purpose, we use the Exponential Power family of distributions, attributed to Subbotin (1923). The functional form of this distribution reads:[10]

$$f(x; b, a, m) = \frac{1}{2ab^{\frac{1}{b}}\Gamma\left(1 + \frac{1}{b}\right)} e^{-\frac{1}{b}\left|\frac{x-m}{a}\right|^{b}} \tag{11}$$

where the kurtosis depends on a shape parameter (b). An interesting feature of this family of distributions is that it encompasses both the Gaussian distribution ($b = 2$) and the Laplace distribution ($b = 1$). Whenever $b < 2$ the distribution will exhibit tails fatter than the Gaussian ones (or "super-Normal" tails).[11] Due to the numerical solution

[6] It is important to mention that it is possible to express $\sum_{i=0}^{\infty} \theta_2^i E_t\psi_{t+i}$ as a function of the states (π_t, y_t). This means that we can still characterize the optimal response as a (potentially non-linear) function of these two variables.

[7] See Judd (1998).

[8] See Press et al. (1992) for a detailed description of this procedure.

[9] The "excess kurtosis" refers to the case when a distribution exhibits a kurtosis higher than 3, which is the kurtosis of the normal distribution.

[10] For instance, a problem we would faced using a t-student distribution is that the one period forward variables with t-distributed shocks will not follow a t-distribution, due to the non-zero mean. In addition, low degrees of freedom generate unbounded moments.

[11] For a detailed discussion of the properties of this family of distributions, see Fagiolo et al. (2008).

followed in the present paper, the use of Gaussian quadrature for approximating the distribution of shocks would require the calculation of quadrature weights and abscissas for each value of the shape parameter. In our case, we decided to focus on Laplace shocks, which exhibit an excess kurtosis of 3, for the following two reasons. First, Fagiolo et al. (2008) find strong support for this distribution when analysing the distribution of a set of macroeconomic series in OECD economies. Second, quadrature rules can be calculated for Laplace distribution weights through a modification of the Laguerre-Quadrature rules.

4 Calibration and Results

4.1 Calibration

Before moving forward with the numerical exercises, we need to set values for the model parameters. Table 1 shows the baseline calibration, based in Woodford (2003). From there we take values for ρ, δ and α. The parameters for the standard deviations are taken from Adam and Billi (2007), who estimate these parameters following the approach of Rotemberg and Woodford (1998). We keep the value of the time-preference parameter relatively low, at 0.6, for the baseline calibration. This value comes from Kato and Nishiyama (2005), who find that a lower value of β is needed in order to guarantee the existence of a stationary optimal policy reaction function. We set the inflation rate target at 0%. The value of λ is set at 20 which is taken from Rotemberg and Woodford (1998). We perform robustness exercises on this value since it has been documented that monetary policy becomes more *dovish* during periods of low inflation, which is the region of the state-space associated with the ZLB.[12] We perform robustness exercises for the slope of the Phillips curve (α), the real rate elasticity of output (δ), the central banker's time-preference parameter (β), and the standard deviations of the AS and IS shocks (σ_ν and σ_ε). The results are reported in Section 4.4.

Table 1: **Baseline Calibration**

Parameter	*Value*	*Description*
λ	20	Relative weight on inflation-deviations variability.
β	0.6	Central banker's time-preference parameter.
ρ	1	Persistence of output dynamics.
δ	0.5	Real rate elasticity of output.
α	-0.02	Slope of the Phillips curve (negative).
σ_ν	1.5	S.D of AS shock.
σ_ε	0.15	S.D of IS shock.
π^*	0	Target inflation rate.

4.2 Results under Gaussian shocks

We explore first the results under Gaussian shocks. As we can observe from Fig. 1, the value function under the presence of the ZLB will not be quadratic. It can be noted that

[12] Orphanides and Wilcox (2002) calls this behaviour *the opportunistic approach to monetary policy.* Martin and Milas (2007) provide empirical evidence supporting this hypothesis for the case of monetary policy in the U.S. during the 1983 to 2004 period.

when the inflation rate and output gap are negative, the loss for the central bank increases. In other words, the cost of stabilization that the central bank faces increases sharply when the economy is in this state, which the literature associates with a *deflationary spiral*. The reason can be traced back to Equation (6). When the ZLB constraint is binding, the hazard of remaining in the same region is high. Therefore, the slackness condition over the non-negativity of interest rates restriction calls for an expected positive value for ψ, the Lagrange multiplier associated with this constraint. Thus, inside the ZLB, the central bank would not be able to offset the shocks coming from the IS equation. Its ineffectiveness to stabilized the economy will be reflected in a higher variability of aggregate output and inflation rate variations, and consequently, a higher welfare loss.[13]

The optimal reaction function ceases to be linear. As we can observe from Fig. 2, the optimal reaction exhibits the pre-emptive motive. Now the interest rate outside of the ZLB region is non-linear. In Figure 3 we can compare the reaction to inflation rate deviations from the target and the output gap under a standard Taylor rule and when the ZLB restriction is taken into consideration. Panels 4a to 4c show how the optimal policy deviates from a linear policy rule as the economy approaches the ZLB region, becoming concave. As the probability of being restricted by the ZLB in future periods increases, the central bank becomes more aggressive in its response to inflation deviations. Panels 4d to 4f show how monetary policy becomes also more expansionary. As Kato and Nishiyama (2005) explain, this effect is related to the threat of a *deflationary spiral*. Under this threat, it is in the interest of the central bank to be more expansionary in comparison to the standard Taylor rule.

[13] Woodford (2003) Ch. 6, explores the problem of monetary policy under the ZLB in a model where the nonnegativity constraint is replaced by a constraint in the interest rate variability. The author results follow the same intuition. The constraint (or the additional objective) makes the stabilization of the inflation rate and the output gap harder to achieve, increasing their variability and, consequently, the welfare losses.

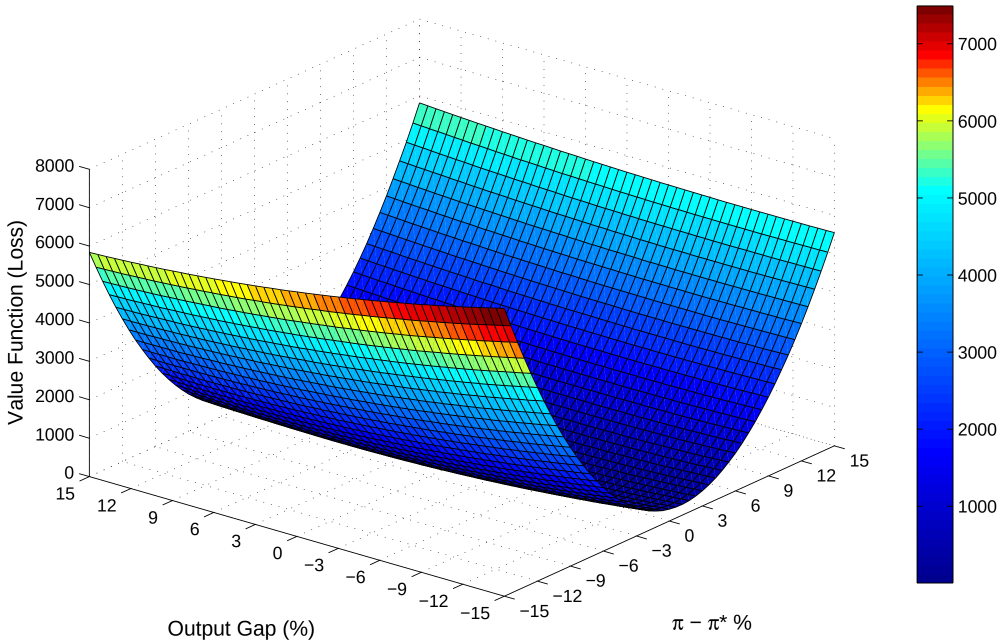


Figure 1: Value Function with Zero Lower Bound (baseline calibration)

Note: Value function for central bank under Gaussian shocks and baseline calibration. Approximation performed over 51 points for output gap and 51 points for the inflation rate deviations. Calibration follows values in Table 1.

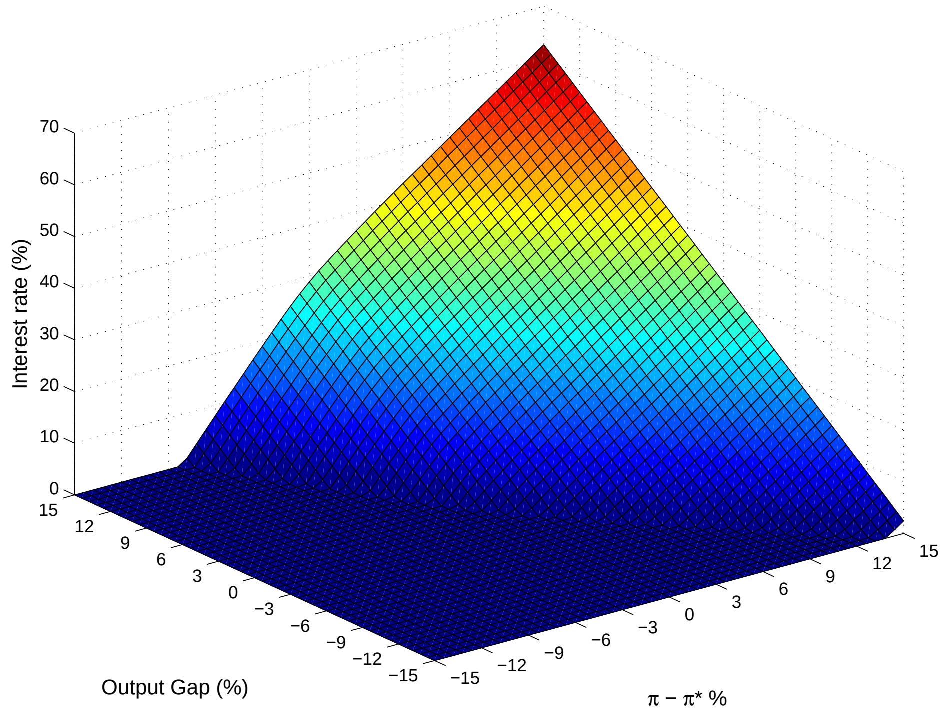
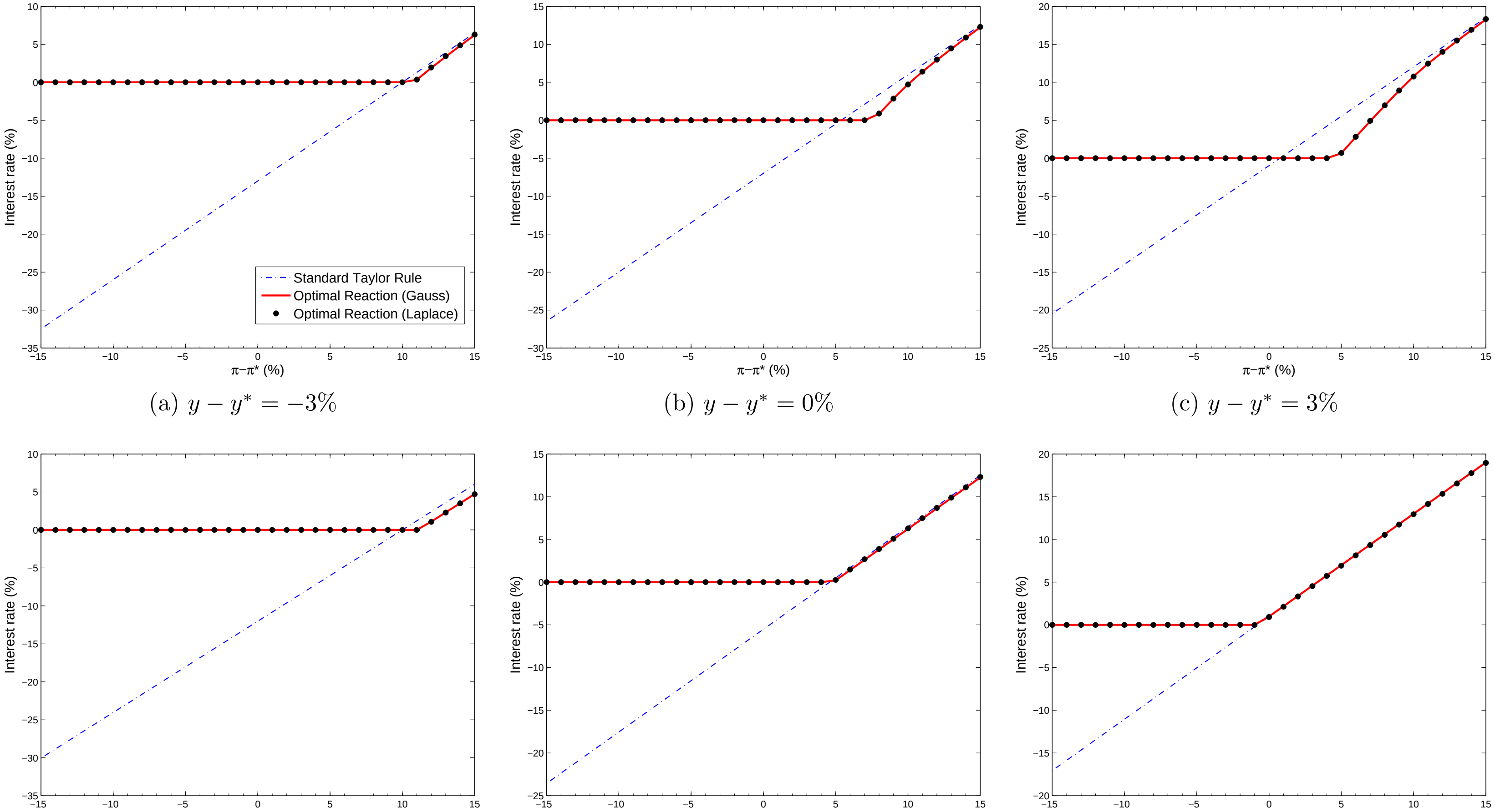


Figure 2: Optimal Policy Reaction Function (baseline calibration)

Note: Optimal reaction for central bank under Gaussian shocks and baseline calibration. Approximation performed over 51 points for output gap and 51 points for the inflation rate deviations. Calibration follows values in Table 1.



(a) $y - y^* = -3\%$ (b) $y - y^* = 0\%$ (c) $y - y^* = 3\%$

(d) $\pi - \pi^* = -3\%$ (e) $\pi - \pi^* = 0\%$ (f) $\pi - \pi^* = 3\%$

Figure 3: Optimal reaction and Taylor rule under Gaussian and Laplace Shocks

Note: Upper row shows the interest rate for different the inflation rate, leaving the output gap constant. Lower row shows the interest rate for different values of the output gap, leaving the inflation rate constant.

4.3 The role of fat-tailed shocks

Now we study how the optimal monetary policy predicted by the model changes under the presence of fat-tailed shocks. As previously discussed, we would like to assess how *excess kurtosis*, which modifies the probabilities of falling into the ZLB region, affects the optimal behaviour of central banks.

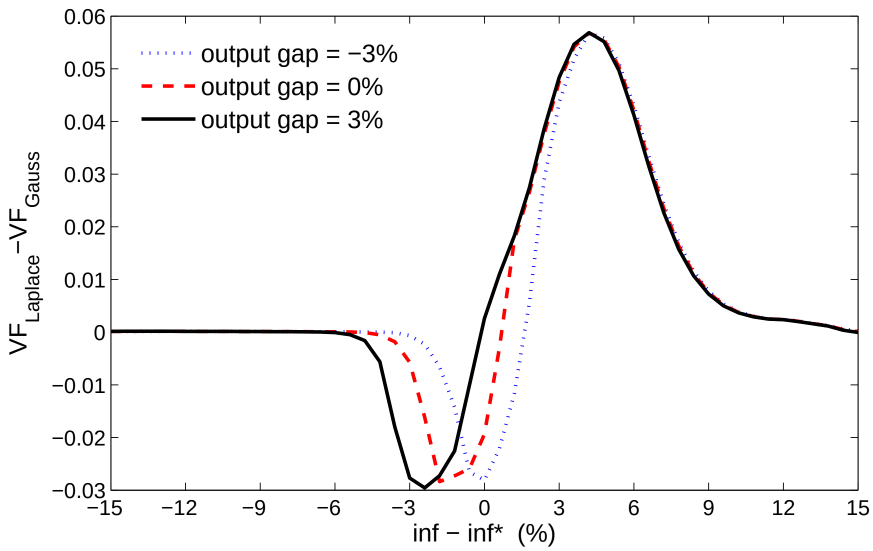
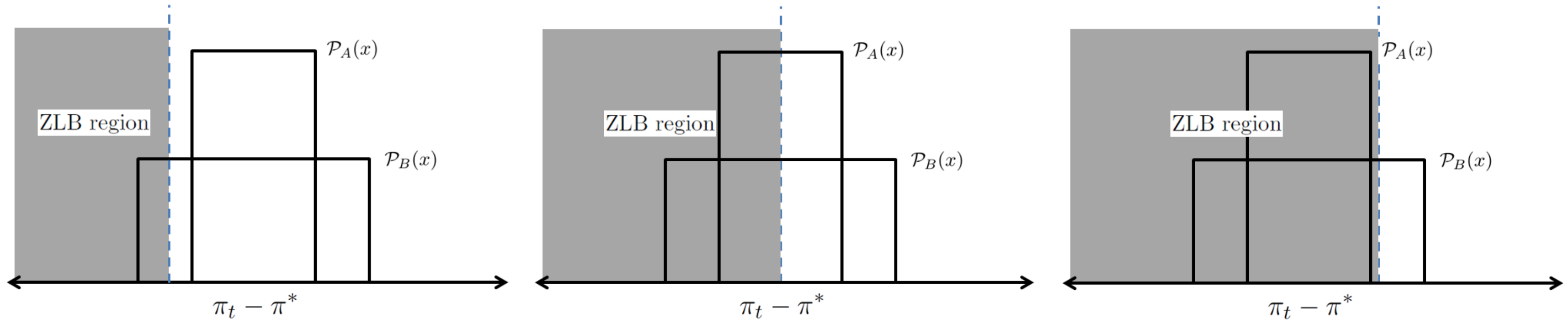


Figure 4: Central Bank's loss function, (Laplace - Gaussian)

Note: Figure shows the difference between the central bank's loss function in Eq. 5 under Laplacian and Gaussian-distributed shocks for different values of inflation deviations, keeping the output gap constant.

Since we use a global solution method we can obtain the solution to the problem for the central banker at different points of the state-space. In Figure 4 we show the differences between the loss of the central bank under both assumed distributions for different values for the inflation deviations, keeping the output gap constant. We find that, away from the zero lower bound region, the loss under fat-tailed shocks is higher. As the economy approaches the ZLB, this pattern first increases and then reverts. Inside the constrained region the difference turns negative, which means that the central bank is worse off under Gaussian shocks. Notice that when for lower values of the output gap, the difference between value functions reverts faster. In order to explain this result we make use of Figure 5, which presents a simple case of how fat tails interact with the hazard of falling or staying in the ZLB in the following period.



(a) *Case A* (b) *Case B* (c) *Case C*

Figure 5: Distribution of shocks and the ZLB

Note: Diagram shows how fat-tails affect the hazard of being in the ZLB under shocks following two different distributions. As the economy gets closer to the ZLB from high values of the inflation rate (Case A), the hazard of falling into the ZLB is higher under the relative heavy-tailed distribution ($\mathcal{P}_B$). In Case B, the probability of being in the ZLB in the next period is the same under both distributions. Finally, Case C shows that when the economy gets inside the ZLB, fat-tailed distributions might imply a higher probability of leaving the constrained region.

From the viewpoint of the central bank there is a reason to become more aggressive under Laplace shocks far away from the ZLB. As the economy approaches this region, this result reverts, as the central bank anticipates that getting near to the ZLB will be more costly under Gaussian shocks. We observe this pattern holds for the optimal interest rates, presented in Figure 6.

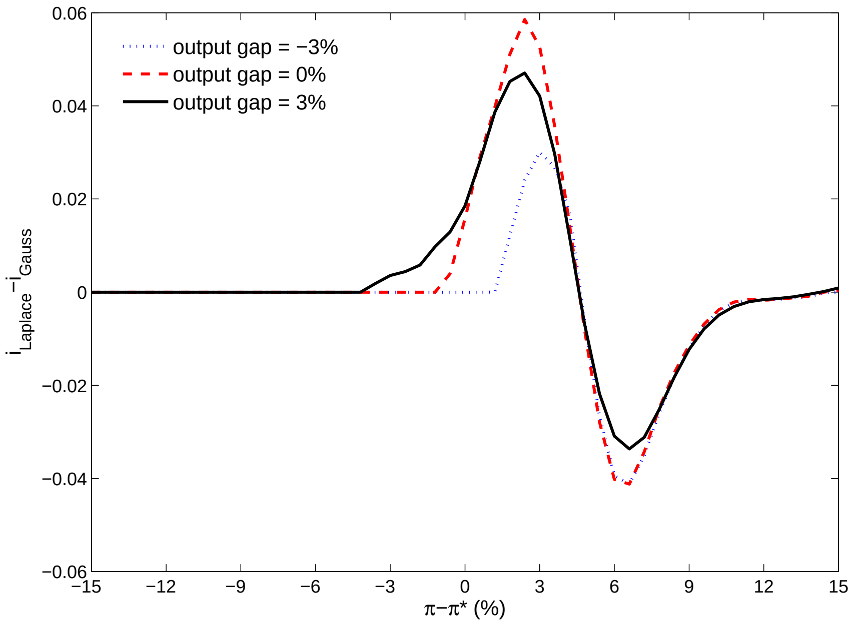


Figure 6: Difference in optimal monetary policy, (Laplace - Gaussian)

Note: Figure shows the difference between the optimal interest rates of the problem in Eq. 5 under Laplacian and Gaussian-distributed shocks for different values of inflation deviations, keeping the output gap constant.

The introduction of super-normal tails generates an interesting result as monetary policy will become relatively *less aggressive* under fat-tailed shocks near the ZLB. From a *quantitative* point of view, the difference between both cases is not significant. Figure 3, suggests that the optimal central bank's reaction is almost unaffected by the change in the assumed distribution of the shocks. Figure 6, shows that the difference between interest rates, for the cases considered, ranges between 0 and 6 basis points, far from the 25 basis point step central banks use when monetary policy changes are announced. Clearly we would require higher excess kurtosis in order to generate effects of a significant magnitude.[14]

4.4 Robustness

Alternative parameterizations are considered. Table 2 reports the maximum differences found between the optimal (discretionary) monetary policy under Gaussian and

[14]Due to the complexity in the construction of quadrature rules for distributions with higher excess kurtosis we leave these exercises for future research.

Table 2: **Robustness to alternative parameterisations**

Parameter	*Value*	*Max* $\mid i^*_{Laplace} - i^*_{Gauss} \mid$
	0.01	6.13
α	0.02 (*baseline*)	7.27
	0.03	10.54
	0.10	34.44
δ	0.25	13.08
	0.5 (*baseline*)	7.27
	5	6.29
λ	10	6.78
	20 (*baseline*)	7.27
	0.5	6.37
β	0.55	6.05
	0.6 (*baseline*)	7.27
	0.5	1.60
σ_ν	1	4.17
	1.5 (*baseline*)	7.27
	0.1	7.47
σ_ε	0.15 (*baseline*)	7.27
	0.5	6.02

Note: Table shows the maximum distance between discretionary optimal policies under Gaussian and Laplace-distributed shocks. Values are reported in basis points. In each exercise the indicated parameter value is changed, keeping the rest at the baseline calibration values in Table 1. Optimal interest rates are calculated for values of inflation and output gap in the range $[-15, 15]$ for both variables. Approximation is performed for 31 points for the output gap and 31 points for the inflation rate. α stands for slope of the Phillips curve (negative). δ is the real rate elasticity of output. λ represents the relative weight on inflation-deviations variability. β is the central banker's time-preference parameter. Finally, σ_ν and σ_ε are the standard deviations of the AS and IS shocks, respectively.

Laplace-distributed shocks. Results are not particularly sensitive to changes in most parameters. For the case of δ, which is associated with the impact monetary policy has on aggregate demand, we find a maximum difference between optimal polices of 34 basis points. When δ is low, it is harder for monetary policy to steer the economy away from the constrained region. For this reason the level of pre-emptive behaviour will be stronger and the interest rate will be more sensitive to the distribution of shocks. We confirm that loss functions follow the same pattern observed in Figure 4. Similarly, the results found in Figure 3 hold under the parameter values considered in the robustness exercises, this is, the optimal reaction is barely affected by the change in the assumed distribution of shocks.

5 Conclusions

We introduce shocks with 'super-normal tails' into the simple NK model with a monetary authority that explicitly considers the ZLB in their optimal policy design, as in Kato and Nishiyama (2005). When the central bank considers this restriction explicitly, the optimal policy ceases to be linear outside of the ZLB. These non-linearities represent a pre-emptive motive, as the central bank becomes more aggressive, in an attempt to avoid falling into a region in which monetary policy becomes ineffective. Central to this decision is the hazard of falling into the ZLB region, which is affected by the distribution of the shocks hitting the economy.

Under shocks with higher kurtosis, non-linearities in the reaction function will emerge further away from the zero interest rate region, relative to the Gaussian shocks case. However, as the economy approaches the ZLB region, this pattern reverts, as the central bank anticipates that under Gaussian shocks, it will be harder to leave the ZLB region, once the economy is inside it. This means monetary policy would actually be relatively *less aggressive* near the ZLB under fat-tailed shocks. Nonetheless, the effects of excess kurtosis are *quantitatively* very limited as the largest difference in optimal interest rates found is of 34 basis points. Changes in the baseline calibration confirm results are robust to variations in parameter values.

Our findings suggest that, in the current setup, the presence of fat-tailed shocks do not produce significant effects on the optimal monetary policy design.

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A. Numerical Algorithm

For the numerical solution we followed Kato and Nishiyama (2005), using a collocation method for solving the Bellman equation problem. The Bellman equation, given by Equation (6) follows:

$$V(\pi, y) = \min_{i}\{f(\pi, y) + \beta EV(g(\pi, y, i, \nu, \varepsilon))\} \tag{12}$$

where $f(\pi, y)$ represents the instantaneous loss of the Central Bank. The function $g(\pi, y, x, \nu, \varepsilon)$ represent the laws of motion for the state variables $\{\pi, y\}$, which are given by equations (2) and (3).

$$g(\pi, y, i, \nu, \varepsilon) = \begin{bmatrix} \rho + \alpha\delta & \delta \\ \alpha & 1 \end{bmatrix} \begin{bmatrix} y \\ \pi \end{bmatrix} + \begin{bmatrix} \delta \\ 0 \end{bmatrix} i + \begin{bmatrix} \nu \\ \varepsilon \end{bmatrix} \tag{13}$$

After setting the Bellman equation we proceed with the discretization of the state space. In this case we focus on the interval $[-15, 15]$ for both state variables and set a number interpolation nodes, which we choose to be equally distributed. We need to find approximate the form of the value function on both sides, hence we will ask the algorithm to hold the equality in equation (12) at every point of the grid. The LHS will be given by:

$$LHS_{n_y n_\pi}(c) = \sum_{i=1}^{N_\pi} \sum_{j=1}^{N_y} c_{ij} \gamma_i^\pi(\pi_{n_\pi}) \gamma_j^y(y_{n_y}) \text{ for each } (\pi_{n_\pi}, y_{n_y}) \in Node. \tag{14}$$

Here, the functions $\gamma_i^\pi(\pi_{n_\pi})$ and $\gamma_j^y(y_{n_y})$ form the basis for the splines. Hence we can form a continuous function that is a piecewise polynomial, though, smooth over the connecting points.[15]

Now, the RHS of the equation has a similar structure, however, the result is affected by the shocks ν and ε, for which we assume a known distribution. As described above, we follow two cases, in the first one we assume a Normal distribution for shocks, while in the second, we follow a Laplace or double-exponential distribution. We follow Gaussian Quadrature for the treatment of both shocks. In the first case, we use a Gaussian-Hermite quadrature, which is associated with weights that are normally distributed. In the second, we modify the Gaussian-Laguerre quadrature, used for exponential distributions. By re-weighting the quadrature weights we can approximate an exponential distribution, for an even number of abscissa. Hence, the RHS of equation (12), is given by:

$$RHS_{n_y n_\pi}(c) = \min_{i \geq 0} \left[f(\pi_{n_\pi}, y_{n_y}) + \beta \sum_{h_\nu=1}^{M_\nu} \sum_{h_\varepsilon=1}^{M_\varepsilon} \sum_{i=1}^{N_\pi} \sum_{j=1}^{N_y} w_{h_\nu h_\varepsilon} c_{ij} \gamma_{ij}(g(\pi, y, i, \nu, \varepsilon)) \right] \tag{15}$$

where the value function represented by b-splines is the same as in Equation (14), for consistency. Now however, we evaluate its value at the abscissa and nodes generated by the Gaussian quadrature. We perform a value function iteration looking for a fixed point. Convergence is attained when:

$$\max \mid V_k(\pi_{n_\pi}, y_{n_y}) - V_{k+1}(\pi_{n_\pi}, y_{n_y}) \mid < \tau, \tag{16}$$

[15] See Judd (1998), Ch 6 for a thorough description of the use of splines.

where τ is the tolerance parameter, set at $1e-4$ in our exercise. With the values of i that minimize the solution we construct a cubic spline approximation for the mapping from the states to the control. This will yield the optimal policy function.