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Default Externalities in Emerging Market Systemic Private Debt Crises

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Default Externalities in Emerging Market Systemic Private Debt Crises

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Abstract

This paper analyzes how default externalities lead to an excessive incidence of systemic private debt crises. An individual defaulting borrower does not internalize that her default leads to a depreciation in the exchange rate because international lenders will sell any seizable assets and flee the country. The exchange rate depreciation in turn reduces the value of non-tradable collateral and induces other borrowers to default, leading to a chain reaction of defaults. The inefficiency in default spillovers can be corrected by strengthening the enforcement of creditor rights, so that individual agents default less often, reducing the frequency of systemic default.

Keywords: Financial crisis, default, capital flows, pecuniary externalities, creditor rights, real exchange rate
JEL classification codes: D62, F32, F41

Abstract

Este trabajo analiza cómo las externalidades de default llevan a una excesiva incidencia de las crisis sistémicas de deuda del sector privado. Un prestamista individual no internaliza que su decisión de hacer default lleva a una depreciación del tipo de cambio porque los acreedores internacionales venden todo el colateral y ocasionan una salida de capitales del país. La depreciación del tipo de cambio reduce el valor del colateral denominado en bienes no transables de todos los deudores de la economía y, consecuentemente, a una reacción en cadena de default del sector privado. La ineficiencia de este efecto contagio puede ser corregido fortaleciendo el cumplimiento de los derechos de los acreedores, de tal forma que los agentes individuales hagan default menos frecuentemente y se reduzca la probabilidad de una crisis sistémica de default.

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1 Introduction

Default episodes in emerging market countries are characterized by repossession of collateral, large capital outflows and real exchange rate depreciation. When default takes place, lenders immediately seize whatever collateral they can and convert it into tradable goods. The repossession of collateral in terms of tradable goods leads to capital outflows because lenders seize collateral and repatriate it, whereas borrowers are excluded from international capital markets and cannot rollover their debt. Lenders convert their repossessed assets in domestic currency into foreign currency so an exchange rate depreciation takes place, which in turn affects the valuation of domestic currency collateral of other loans.[1]

This work analyzes the behavior of capital flows, the exchange rate and the frequency of default in a small open economy where private agents borrow from international capital markets using collateralized debt in an environment with imperfect enforceability of creditor rights. Individual default generates capital outflows because they cannot rollover their debt. Capital outflows lead to a real exchange rate depreciation, which lowers the value of non-tradable collateral of other borrowers. This creates a debt overhang problem, as the value of collateral becomes smaller relative to the value of previously taken debt, which triggers default of other agents. Default spillover effects generate an amplification mechanism, as all agents have higher incentives to default, increasing the incidence of default and the default risk premium of other borrowers and reducing the ability of private agents to borrow from abroad.

Default spillover effects create an inefficiency in the borrowing behavior of private agents because, by taking prices as given, they ignore the effect of

[1]Corsetti, Pesenti and Roubini (1999) show evidence of high collateral valuation before the 1997 East Asian crisis, followed by asset deflation and a sharp drop in the value of collateral which triggered the share of non-perfoming loans.

their decisions on the valuation of collateral and through it on the default incentives of other agents. Individual borrowers default more frequently than socially optimal. An individual defaulting borrower does not internalize that her default leads to a depreciation in the exchange rate because international lenders will sell any seizable assets and flee the country. The exchange rate depreciation in turn reduces the value of non-tradable collateral and induces other borrowers to default, leading to a chain reaction of defaults.

In order to correct this distortion, optimal policy should focus on aligning default incentives with the socially optimal default set. Stronger enforcement of creditor rights through a larger default penalty in terms of repossessed collateral increases the cost of default and reduces the frequency of default in the decentralized equilibrium. Likewise, this could also be achieved by policies aimed at lowering the cost of debt repayment, by reducing the benefit of defaulting in states where the misalignment takes place.

The inefficiency created by the distortion in default incentives is completely different to the one in models with pecuniary externalities in collateral constraints on the level of debt. To our knowledge, this is the first paper that focuses on the existence of pecuniary externalities in default incentive constraints, which generates a chain reaction in defaults, a feature that has been documented during episodes of systemic debt crises.

The model environment is a two-good endowment economy where lenders seize a fraction of the individual borrower's total income in the case of default[2]. Each individual borrower has two choices: whether to pay back or default on previously contracted debt, and if previous debt has been paid back, how much new debt to take. We derive the financial contract of collateralized debt with the possibility of default, which is reflected on an

[2]Following Uribe (2006), we need a two-good economy for a pecuniary externality to lead to a socially inefficient equilibrium, as the debt choice is socially efficient in the case of a one-good economy despite the existence of financial frictions.

interest rate schedule for each level of individual debt and analyze whether the debt and default choices in this model are socially efficient. Once we find the existence of a pecuniary externality on the default cost-benefit analysis, we introduce a default penalty to correct this distortion, by increasing the cost of default.

The qualitative results show that the model accounts for the three stylized facts previously mentioned. During an episode of default, lenders partially seize the borrower's collateral and repatriate tradable goods. This results in large capital outflows and real exchange rate depreciation due to a 'transfer problem.'[3] On the normative side, private borrowers have higher incentives to default than socially optimal. As previously mentioned, individual default has spillover effects by increasing incentives to default by lowering the recovery value of other's collateral and increasing the incentives to default for all borrowers.

The distortion in default incentives is an additional mechanism to the inefficiency created by the imperfect enforceability problem in the sovereign default literature. If the social planner could choose the degree of enforceability, she would choose one that leads to repayment in all states. In our case, even for a fixed degree of enforcement of creditor rights, there is a distortion in default incentives of private borrowers relative to the constrained efficient case. Private borrowers do not internalize the effect of their borrowing on the default risk premium of other agents through the effect on the valuation of collateral. In order to correct this distortion, decentralized agents should lower their incentives to default, which can be achieved through a stronger enforcement of creditor rights. A higher cost of default makes private agents internalize the effect of their borrowing in triggering a chain reaction of other agents' default, achieve a lower incidence of debt

[3]The transfer problem is a term used by Keynes (1929) to refer to the fact that a large transfer between two countries not only has a direct effect in terms of a capital outflow but also an indirect effect through a change in the terms of trade, especially in the case of a small open economy.

crises and improve risk sharing and consumption smoothing.

We use quantitative methods to solve for the optimal debt and default choices in the decentralized equilibrium and the constrained social planner's problem and the default penalty to correct the distortion in default incentives in the infinite horizon model. We simulate the economy to analyze the business cycle properties of the model and the behavior of real variables during default episodes. The results are consistent with the qualitative analysis, where the decentralized equilibrium shows higher incentives to default than socially efficient. This translates into a higher interest rate schedule, due to the effect of the default risk premium, and lower levels of debt compared to the case with no distortion in default incentives.

The default penalty takes positive values in the set of states where an individual borrower chooses to default whereas the social planner would not. This occurs at intermediate levels of debt, as private borrowers would efficiently choose to pay back low levels of debt and default on high ones. Higher income reduces the optimal value of the penalty, as the default penalty is proportional to tradable endowment. On the debt dimension, higher levels of debt in this set of states require a higher penalty to increase the cost of default, as the size of the default externality is increasing in debt.

Literature Review. The theoretical framework in this work is related to the literature on pecuniary externalities in incentive constraints (Greenwald and Stiglitz, 1986). In models of imperfect information and incomplete markets, the market equilibrium in economies with constraints that depend on market prices is not constrained efficient because the second order welfare loss from reducing default is smaller than the first order gain from relaxing the default incentives of other agents. In this model, the market equilibrium is not efficient because of the distortion that arises in the incentives to default due to the fact that private agents take the real exchange rate as given.

The optimal debt contract in this work is also related to the literature on optimal contract arrangements under the existence of commitment problems, such as Kehoe and Levine (1993) and Alvarez and Jermann (2000). A more restrictive version for the financial constraint is used in Zhang (1997), where the maximum level of debt is determined by the worst case scenario in terms of the exogenous shock. However, this class of contracts characterize an equilibrium which rules out default, with debt levels that satisfy an incentive compatibility constraint where paying back is strictly preferred to default. In this model, we allow for default to be preferred in a subset of states and define the participation constraint for risk neutral international lenders who are willing to engage in risky lending.

This work is also related to the vast literature on sovereign default, such as Eaton and Gersovitz (1981) and Bulow and Rogoff (1989), in the sense that we analyze an equilibrium where default does take place. Popov and Wiczer (2010) present a model with centralized default in a two good environment, where sovereign default episodes occur during periods of currency crises. A closer quantitative model of sovereign default to this work is the model with no trend shocks in Aguiar and Gopinath (2006), extended to a two good framework. However, the model differs in two important features: this paper focuses on private default, where both debt and default decisions are taken by decentralized borrowers, and debt is collateralized, whereas in the sovereign debt literature the cost of default is in terms of trade exclusion or reputation loss.

We can compare the implications of the constrained social planner's solution with models of sovereign default. The use of collateral that is subject to valuation effects create an additional amplification mechanism through the chain reaction of defaults. For a given level of debt, individual default creates a real depreciation that lowers the valuation of non-tradable collateral and triggers a chain reaction of default, which amplifies the frequency of default.

Other models of private capital flows and default include Wright (2006) and Daniel (2012). Wright(2006) presents a model of private external debt with resident default risk, where private agents choose to borrow in both domestic and foreign capital markets. Daniel (2012) uses a model with private sector risky borrowing for emerging markets where the interaction between negative productivity shocks and financial market imperfections lead to a widespread of default and triggers a severe contraction in external borrowing. Our model would be similar to a representative agent version of agents who can borrow from domestic and foreign capital markets and face resident default risk in foreign markets, as the net supply of domestic debt is zero. We expand this model to allow for an imperfect degree of collateralization, where the real exchange rate amplifies the default externality.

This work also relates to others that analyze the normative implications in models with default. Tirole (2003) presents a single good model where a negative externality arises through a government policy that is set in terms of aggregate debt, which creates a mis-alignment in default incentives of private agents that take policy as given and of a government that chooses that policy. Jeske (2006) considers a default externality in a model where private agents can engage in both domestic and foreign borrowing and can default on foreign debt but not on domestic debt, which arises from the possibility of substituting access to foreign markets with access to domestic ones. Wright(2006) presents a similar framework with domestic and external borrowing, where the optimal policy is to subsidize the repayment of capital flows.

It is also closely related to the model with pecuniary externalities and equilibrium default in Kim and Zhang (2012), where they show that decentralized borrowing and centralized default leads to the existence of a pecuniary externality through the effect of individual debt on the bond price schedule. The distortion in default incentives in our work is closely related to their bond price schedule effect, where private agents ignore that higher individual debt leads to a higher default risk premium. We extend this model by allowing for decentralized default and collateralized debt, which create an additional distortion on the default incentives of individual agents through the effect of the real exchange rate on the valuation of collateral.

The normative results in our model can be related to the literature on credit externalities and financial crises in models with endogenous borrowing constraints. When default is an off-equilibrium outcome, the pecuniary externality arises as private agents take excessive debt because they fail to consider the effect of debt on relative prices. Higher debt lowers the value of collateral and tightens the financial constraint in the following period. Therefore, the optimal policy, as shown by Bianchi (2011), Jeanne and Korinek (2010) and Korinek (2008), is one that reduces the amount of debt. A similar result is also obtained in Jeanne and Korinek (2010, 2011) and Bianchi and Mendoza (2010), where total borrowing is higher than optimal if uncontingent debt is the only financial instrument available.

The inefficiency created by the distortion in default incentives is completely different to the one in models with inefficient debt levels due to pecuniary externalities in collateral constraints. The policy recommendations to correct this distortion is also completely different from the one in models with credit constraints: the distortion in default incentives can be corrected in the period when default occurs, whereas in the model with no default distortions, optimal policy is a precautionary measure to prevent excessive leverage in periods before the economy hits the borrowing constraint.

This work highlights the externality that arises in default incentives through the effect of individual debt on relative prices. The reason why prices play a key role in this type of models is related to the link between individual and aggregate borrowing. As pointed out in Krugman (1999) for the case of currency crises, each individual's debt choice depends on the valuation of wealth. Each agent's wealth depends on aggregate borrowing, as the volume of capital inflows affects terms of trade and through it the valuation of foreign currency denominated debt, which is specially relevant in the case of a small open economy. In an episode of systemic private default, a large fraction of borrowers that default on their debt leads to a sizable real exchange rate depreciation, which exacerbates the current account reversal.

The remainder of the paper is organized as follows. Section 2 presents a model of private borrowing and default under the decentralized equilibrium and the social planner's equilibrium to show that the decentralized equilibrium is not constrained Pareto efficient. Section 3 presents the results of the quantitative analysis of the pecuniary externality and its effect on default incentives, as well as the optimal policy to correct this distortion. Section 4 concludes.

2 Model of Systemic Private Default

This section presents a small open economy model of international borrowing with collateralized debt to illustrate the interaction mechanism between debt, default and the real exchange rate in a model with infinite discrete time. There are two representative agents: a domestic private borrower and a large pool of international lenders.

Preferences. The preferences of the representative domestic borrower are given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

where $\mathbb{E}_t(.)$ is the time t expectations operator, $0 < \beta < 1$ is the discount factor, and $U(.)$ is a CRRA utility function. The consumption good, c_t, is defined as a CES aggregator:

$$c_t = [\omega c_{T,t}^{-\eta} + (1-\omega)c_{N,t}^{-\eta}]^{-1/\eta}$$

where $c_{T,t}$ and $c_{N,t}$ are the consumption of tradable and non-tradable goods, respectively, ω is the weight of tradable consumption in the aggregator and $1/(1+\eta)$ is the elasticity of substitution between tradable and non-tradable consumption.

There is a single available instrument to borrow from abroad: a one period, non-state contingent bond denominated in units of the numeraire tradable good. Every period each domestic private borrower decides whether to pay back debt contracted in the previous period, d_t, and, if she chooses to do so, she takes new debt, d_{t+1}, from a large pool of risk neutral lenders . Each domestic agent consumes two types of goods, tradable and non-tradable goods, and receives a stochastic endowment of y_t units of the tradable good with p.d.f. $f(y_t)$ and y^N units of the non-tradable good.

At the beginning of each period, borrowers decide whether to default or not. If a private borrower decides not to default, debt contracted in the previous period is paid back and new debt is taken. Each borrower chooses consumption of tradable and non-tradable goods. The budget constraint is given by:

$$c_{T,t}^R + p_t c_{N,t}^R = y_t + p_t y^N + d_{t+1} - (1+r_t)d_t \tag{1}$$

where superscript R refers to the state of repayment and p_t is the relative price of non-tradable goods, or equivalently, $1/p_t$ is a measure of the real exchange rate. The price of tradable goods is normalized to one.

If a private borrower defaults, lenders seize a fraction $0 < \lambda_1 < 1$ of the borrower's total income. There is a cost related to this process, so that lenders obtain a fraction $\lambda_2 \leq \lambda_1$ of total income, convert it into tradable goods and repatriate it.[4] Defaulters are excluded from international capital

[4] λ_1 and λ_2 are exogenous in this setup, but can be loosely related to the degree of enforcement of

markets and regain access with an exogenous probability ϕ.[5] When they regain access to capital markets, agents start with a zero debt stock. Agents choose their consumption of tradable and non-tradable goods. The budget constraint in the default state, with superscript D, is given by:

$$c_{T,t}^D + p_t c_{N,t}^D = (1 - \lambda_1)(y_t + p_t y^N) \tag{2}$$

If a private borrower defaults, there is a probability $1 - \phi$ of staying in autarky, where they consume their endowment of tradable and non-tradable goods. The budget constraint in the autarky state, with superscript A, is given by:

$$c_{T,t}^A + p_t c_{N,t}^A = y_t + p_t y^N \tag{3}$$

The key feature in this setup is the mismatch between debt and collateral, where both tradable and non-tradable goods can be used as collateral, whereas debt is denominated in terms of tradable goods. In the case of default, when lenders repossess non-tradable collateral, they would sell it against tradable goods which can be repatriated to lenders.[6]

This feature reflects in the market clearing condition for tradable goods under default, which creates the amplification mechanism in capital outflows. The demand of tradable goods by international lenders is given by the total value of seized collateral $\lambda_2[y_t + p_t y^N]$. The demand of tradable goods by domestic agents is their consumption of tradable goods in the default state, $c_{T,t}^D$. Supply is given by the endowment of tradable goods, y_t. Therefore, the market clearing condition for tradable goods in the state of

creditors' rights. See Djankov et al (2008) for references to the cost of private default and the value recovered by lenders. For the case study of a medium sized firm, they find that, on average, 48 percent of the value is lost during the debt enforcement process.

[5] We need some source of dead-weight loss due to default in order to obtain a pecuniary externality, which is obtained by setting $\lambda_2 < \lambda_1$ and/or $\phi < 1$. A detailed explanation of this is shown in the qualitative results.

[6] Tornell and Westermann (2005) provide empirical evidence on the use of non-tradable goods as collateral, where external financing fuels credit booms in the non-tradable sector. Korinek (2011) points out the use of real estate collateral during many capital inflow booms and busts.

default is:

$$c_{T,t}^{D} + \lambda_1(y_t + p_t y^N) = y_t \tag{4}$$

International lenders. There is a large pool of risk neutral international lenders. The interest schedule is derived from a participation constraint that ensures that lenders are indifferent between engaging in risky lending to domestic borrowers and their riskless outside option. Lenders receive $(1 + r)d$ if a borrower decides to pay back and they seize a fraction $\lambda_2 \leq \lambda_1$ of total income if the borrower defaults. The participation constraint is given by:

$$(1 + r_t)d_t \int_{\hat{y}_t}^{\bar{y}} f(y_t)dy_t + \lambda_2 \int_{\underline{y}}^{\hat{y}_t} (y_t + p_t y^N) f(y_t)dy_t = (1 + \rho)d_t \tag{5}$$

where ρ is the world risk free interest rate, $\bar{y}$ and $\underline{y}$ are the upper and lower bounds of the tradable endowment distribution, respectively, and $\hat{y}$ is the default threshold for the tradable endowment shock, which will be defined in the next section. For a given level of debt, borrowers repay when tradable endowment is higher than the threshold, $y_t \geq \hat{y}_t$, and default otherwise.

2.1 Decentralized equilibrium

We present the problem faced by a representative borrower in recursive form. The state at the beginning of the period is given by (d, y), where d is the stock of previously contracted debt and y is the tradable endowment shock. In states of repayment, labeled with superscript R, a private borrower chooses new debt, d', and pays back debt contracted on the previous period.

$$v^R(d, D, y) = \max_{d', c_T^R, c_N^R} U\left(c_T^R, c_N^R\right) + \beta \mathbb{E} V(d', D', y') \tag{6}$$

subject to:

$$c_T^R + p(D, y)c_N^R = y + d' - (1 + r)d + p(D, y)y^N \tag{7}$$

$$(1+r)d\int_{\hat{y}}^{\bar{y}} f(y)dy + \lambda_2 \int_{\underline{y}}^{\hat{y}} (y + py^N) f(y)dy = (1+\rho)d \tag{8}$$

$$D' = \Gamma(D, y) \tag{9}$$

where c_T^R and c_N^R are the consumption of tradable and non-tradable goods, respectively, $p(D, y)$ is the price of non-tradable goods and $r = r(d, D)$ is the interest rate schedule.[7] $\Gamma(D, y)$ is the perceived law of motion for the aggregate debt level D. $\hat{y}(d, y)$ is the threshold for the tradable endowment shock below which private borrowers default. $V(d, y)$ is the welfare value of an agent with debt stock d, in a state with aggregate debt D and tradable endowment y.

Individual borrowers repay their debt if welfare under repayment, v^R, is higher than under default, v^D.Therefore, the repayment condition is given by:

$$v^R(d, D, y) \geq v^D(D, y) \tag{10}$$

We can show that there exists a threshold for the tradable endowment shock, $\hat{y}$, such that agents choose to pay back debt for realizations of $y \geq \hat{y}$ and default otherwise. The repayment condition shows that each borrower defaults under low realizations of the tradable income shock because the cost of default is increasing in y. The threshold is defined by:

$$v^R(d, D, \hat{y}) = v^D(D, \hat{y}) \tag{11}$$

If a private agent defaults, lenders seize a fraction $0 < \lambda_1 < 1$ of total income. There is a dead-weight cost of default as lenders repatriate a fraction $0 < \lambda_2 \leq \lambda_1$ of total income in terms of tradable goods. Agents who default are banned from borrowing in international capital markets and go into autarky. They regain access to financial markets with a probability ϕ

[7]The interest rate schedule depends on both aggregate and individual debt because default risk is calculated for each individual borrower depending on her individual debt level but the recovery value of collateral depends on the price of non-tradable goods and therefore on aggregate debt.

and re-enter capital markets with no initial debt. We label this state with superscript D for default.

$$v^D(D,y) = \max_{c_T^D, c_N^D} U\left(c_T^D, c_N^D\right) + \beta(1-\phi)\mathbb{E}v^A(D',y') + \beta\phi\mathbb{E}v^R(0,D',y') \quad (12)$$

$$c_T^D + p(D,y)c_N^D = (1-\lambda_1)y + (1-\lambda_1)p(D,y)y^N \quad (13)$$

$$D' = \Gamma(D,y) \quad (14)$$

As long as private agents stay in autarky, they consume their income. We label this state A for autarky.

$$v^A(D,y) = \max_{c_T^A, c_N^A} U\left(c_T^A, c_N^A\right) + \beta(1-\phi)\mathbb{E}v^A(D',y') + \beta\phi\mathbb{E}v^R(0,D',y') \quad (15)$$

$$c_T^A + p(D,y)c_N^A = y + p(D,y)y^N \quad (16)$$

$$D' = \Gamma(D,y) \quad (17)$$

In every state, a private borrower's welfare is defined by $V(d,D,y)$, where repayment is chosen if welfare under repayment, $v^R(d,D,y)$, is higher than under default, $v^D(D,y)$, and default is chosen otherwise.

$$V(d,y) = \max\{v^R(d,y), v^D(d,y)\} \quad (18)$$

Definition 1 *A recursive decentralized competitive equilibrium for a small open economy (SOE) is a pricing function, $p(D,y)$, an interest rate schedule, $r(d,D)$, and decision rules $\{$ $d'(d,D,y)$, $c_T^R(d,D,y)$, $c_T^D(D,y)$, $c_T^A(D,y)$, $c_N^R(d,D,y)$, $c_N^D(D,y)$, $c_N^A(D,y)$, $\hat{y}(d,D,y)\}$ such that the following conditions hold:*

- *Household's problem: Taking $p(D,y)$ and $r(d,D)$ as given, decision rules $d'(d,D,y)$, $c_T^R(d,D,y)$, $c_N^R(d,D,y)$ and $\hat{y}(d,D,y)$ maximize (6) subject to (7), (8), (9) and (11); decision rules $c_T^D(D,y)$ and $c_N^D(D,y)$ maximize (12) subject to (9) and (13); and decision rules $c_T^A(D,y)$ and $c_N^A(D,y)$ maximize (15) subject to (9) and (16).*

- *Rational expectations:* $\Gamma(D,y) = d'(D,D,y)$
- *Market clearing:* $c_N^R(D,D,y) = y^N, c_N^D(D,y) = y^N, c_N^A(D,y) = y^N, c_T^R(D,D,y) = y + d'(D,y) - (1 + r(D,D))D, c_T^D(D,y) = (1-\lambda_1)y - \lambda_1 p(D,y)y^N$, $c_T^A(D,y) = y$

Assuming that the interest rate schedule is differentiable everywhere, the inter-temporal Euler equation is given by:[8]

$$U_1(c_t) = \beta \frac{1 + \rho + \left[(1 + r_{t+1})d_{t+1} - \lambda_2\left(\hat{y}_{t+1} + \hat{p}_{t+1}y^N\right)\right] f(\hat{y}_{t+1})\frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}}}{\int_{\hat{y}_{t+1}}^{\bar{y}} f(y_{t+1})dy_{t+1}} \times \int_{\hat{y}_{t+1}}^{\bar{y}} U_1(c_{t+1})f(y_{t+1})dy_{t+1} \quad (19)$$

The optimal debt choice considers the effect of debt on the risk premium, which can be decomposed in two parts. Similar to models with uncollateralized debt, the first term is given by the probability of repayment in the denominator. Higher debt reduces the probability of repayment, as it increases default incentives because more resources are needed to pay back debt. The second term is related to the marginal loss of default due to the repatriation of collateral by lenders. The square bracket represents the marginal loss of default, as lenders get the repatriation of collateral instead of the interest rate payment. Higher debt increases the value of repayment relative to collateral, which increases the probability of default, shown by the term $\partial\hat{y}_{t+1}/\partial d_{t+1}$. Lenders must receive a higher interest rate in order to be willing to engage in risky lending.

2.2 Social Planner

Consider now a benevolent social planner who faces the same financial contract with limited enforcement in international capital markets. As op-

[8] The infinite horizon problem is solved numerically using value function iteration in Section 3. We find that it is differentiable almost everywhere, except at the threshold where the probability of default becomes positive. The optimal choice of borrowing at this level is discussed in the section on the quantitative analysis, where we show the same result for the default externality.

posed to private borrowers, the constrained social planner does internalize default spillover effects, through the effect of individual default on the valuation of non-tradable collateral and through it on the default incentives of other agents in the economy. As a result, we can show that the decentralized equilibrium is not constrained Pareto efficient.

Under repayment, the planner gets a similar pay-off to the one described for private agents. The planner's welfare under repayment w^R at initial state (d, y) is given by:

$$w^R(d, y) = \max_{d', c_T^R, c_N^R} U\left(c_T^R, c_N^R\right) + \beta \mathbb{E} W(d', y') \tag{20}$$

$$c_T^R = y + d' - (1+r)d \tag{21}$$

$$c_N^R = y^N \tag{22}$$

$$(1+r)d \int_{\hat{y}}^{\bar{y}} f(y)dy + \lambda_2 \int_{\underline{y}}^{\hat{y}} (y + \frac{U_2}{U_1} y^N) f(y)dy = (1+\rho)d \tag{23}$$

$$w^R(d, \hat{y}) = w^D(\hat{y}) \tag{24}$$

where U_2/U_1 is the marginal rate of substitution between tradable and non-tradable goods.

Under default, the planner's welfare is defined as:

$$w^D(d, y) = \max_{c_T^D, c_N^D} U\left(c_T^D, c_N^D\right) + \beta(1-\phi)\mathbb{E} w^A(y') + \beta\phi \mathbb{E} w^R(0, y') \tag{25}$$

$$c_T^D = (1-\lambda_1)y - \lambda_1 \frac{U_2(c_T, y_N)}{U_1(c_T, y_N)} y^N \tag{26}$$

$$c_N^D = y_N \tag{27}$$

If the planner stays in autarky, welfare is given by:

$$w^A(y) = \max_{c_T^A, c_N^A} U\left(c_T^A, c_N^A\right) + \beta(1-\phi)\mathbb{E} w^A(y') + \beta\phi \mathbb{E} w^R(0, y') \tag{28}$$

$$c_T^A = y \tag{29}$$

$$c_N^A = y_N \tag{30}$$

In each state, the planner's welfare is defined by $W(d, y)$, where repayment is chosen if welfare under repayment, $w^R(d, y)$, is higher than under default, $w^D(d, y)$, and default is chosen otherwise.

$$W(d, y) = \max\{w^R(d, y), w^D(d, y)\} \tag{31}$$

Definition 2 *A socially efficient allocation for the small open economy (SOE) is a set of decision rules* $\{d'(d, y)$, $c_T^R(d, y)$, $c_T^D(d, y)$, $c_T^A(y)$, $c_N^R(d, y)$, $c_N^D(d, y)$, $c_N^A(y)$, $\hat{y}(d, y)\}$ *and an interest rate schedule* $r(d, y)$ *such that decision rules* $c_T^R(d, y)$, $c_N^R(d, y)$, $d'(d, y)$ *and* $\hat{y}(d, y)$ *and the interest rate schedule maximize (20) subject to (21)-(24); decision rules* $c_T^D(d, y)$ *and* $c_N^D(d, y)$ *maximize (25) subject to (26) and (27); and decision rules* $c_T^A(y)$ *and* $c_N^A(y)$ *maximize (28) subject to (29) and (30).*

The socially efficient level of debt is defined by the following Euler equation:

$$U_1(c_t) = \beta \frac{1 + \rho + \left[(1 + r_{t+1})d_{t+1} - \lambda_2\left(\hat{y}_{t+1} + \hat{p}_{t+1} y^N\right)\right] f(\hat{y}_{t+1}) \frac{d\hat{y}_{t+1}}{dd_{t+1}} + \lambda_2 y^N \int_{\underline{y}}^{\hat{y}_{t+1}} \frac{\partial p_{t+1}}{\partial d_{t+1}} f(y_{t+1}) dy_{t+1}}{\int_{\hat{y}_{t+1}}^{\bar{y}} f(y_{t+1}) dy_{t+1}} \times \int_{\hat{y}_{t+1}}^{\bar{y}} U_1(c_{t+1}) f(y_{t+1}) dy_{t+1} \tag{32}$$

where

$$\frac{d\hat{y}_{t+1}}{dd_{t+1}} = \frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}} + \frac{\partial \hat{y}_{t+1}}{\partial p_{t+1}} \frac{\partial p_{t+1}}{\partial d_{t+1}}$$

Proposition 1 *For a given level of debt, private borrowers in the decentralized equilibrium face a higher marginal effect of debt on the incentives to default than the constrained social planner, i.e.,*

$$\frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}}^{CE} > \frac{d\hat{y}_{t+1}}{dd_{t+1}}^{SP}$$

Proof. Using the results in Appendix 1, we can derive the analytical expression for the marginal effect of debt on the default threshold under the decentralized equilibrium (CE) and the social planner's solution (SP):

CE:

$$\frac{\partial \hat{y}}{\partial d} = \frac{(1+r)U_1(\hat{c}^R)}{U_1(\hat{c}^R) - (1-\lambda_1)U_1(\hat{c}^D)} \tag{33}$$

SP:

$$\frac{\partial \hat{y}}{\partial d} = \frac{(1+r)U_1(\hat{c}^R) + \lambda_1 \frac{\partial \hat{p}}{\partial \hat{c}_T} y^N U_1(\hat{c}^D)}{U_1(\hat{c}^R) - (1-\lambda_1)U_1(\hat{c}^D) + \lambda_1 \frac{\partial \hat{p}}{\partial \hat{c}_T} y^N U_1(\hat{c}^D)} \tag{34}$$

For a given d, comparing equations (33) and (34), we obtain that

$$\frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}}^{CE} > \frac{d\hat{y}_{t+1}}{dd_{t+1}}^{SP}$$

■

Let us analyze the debt and default decisions of a constrained social planner. The optimality condition for debt considers the same relationship as the one obtained by decentralized individuals, where a larger stock of debt leads to higher incentives to default, given the increase in the cost of repayment relative to the value of collateral. In addition, it considers the effect of individual debt and default on the valuation of collateral. There are two effects on the valuation of collateral: the first one related to the value of collateral for a given default set and the second effect related to the distortion in incentives to default. These effects are similar to the ones labeled 'over-borrowing' and bond price schedule effect in Kim and Zhang (2012), but extended to the case of collateralized debt and decentralized default.

The first effect is given by the additional term in the Euler equation, $\lambda_2 y^N \int \partial p_{t+1}/\partial d_{t+1} f(y_{t+1}) dy_{t+1}$, which shows that, for a given default set, higher debt leads to a lower valuation of collateral and therefore, lenders must increase the cost of borrowing to compensate for lost resources. This leads to a social planner choosing a lower level of debt than private borrowers.

The second effect is the one related to the distortion in default incentives, which is measured by the effect of individual default on others borrower's default, given by $\frac{d\hat{y}_{t+1}}{dd_{t+1}} = \frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}} + \frac{\partial \hat{y}_{t+1}}{\partial \hat{p}_{t+1}} \frac{\partial \hat{p}_{t+1}}{\partial d_{t+1}}$. Appendix 1 presents the analytical solution for the private borrower's problem in the decentralized equilibrium and the social planner's problem. A social planner that internalizes the effect of debt and default on the price of non-tradable goods has lower incentives to default because individual default leads to a chain reaction of defaults by affecting the valuation of other agents' collateral, shown by the term containing $\partial \hat{p}_{t+1}/\partial d_{t+1}$.

The result on the distortion in default incentives implies that private borrowers face a higher default risk premium, which results in a lower level of debt compared to the case where the default incentives are perfectly aligned to the socially optimal. Clearly, the total effect on the level of debt depends on the relative magnitude of the two previously mentioned effects.[9]

A key assumption to obtain a distortion in default incentives is that default creates a dead-weight loss, given by the difference between $(1+r)d$ and $\lambda_2 p y^N$. This can be achieved by lenders having to incur in a dead-weight cost to seize collateral, convert it to tradable goods and repatriate it, $\lambda_2 < \lambda_1$, and/or loss of access to international capital markets, $\phi < 1$. Appendix 2 shows that in a model with no dead-weight cost of default, private agents face the same default incentives as the socially optimal. If there is no marginal loss of default, agents would face the same interest rate schedule and choose the same optimal level of debt.

In order to correct the distortion in default incentives, it would be welfare improving to have policies that enforce debt repayment and reduce the

[9]If we consider the case where default is exogenous, we would have no distortion in default incentives, which leads to excessive borrowing in the decentralized equilibrium. For a derivation of that case, see Appendix 3: Exogenous Default. It is not just allowing for default to occur in equilibrium which results in lower borrowing but the distortion in default incentives between the socially efficient set of states and the ones chosen by decentralized agents.

frequency of default, as it would increase the benefits of risk sharing by allowing private agents to borrow more under bad states of the economy.

2.3 Optimal Policy to Correct Default Distortion

In this section, we analyze the type of policy needed to correct the distortion on private agents' incentives to default. By taking prices as given, borrowers do not internalize the effect of individual default on the valuation of other borrower's collateral. We show that the constrained efficient equilibrium can be achieved by including a default penalty which increases the cost of default and reduces incentives to default so as to match the socially optimal default set. We introduce a default penalty, $\tau(d, D, y)$, that is proportional to the value of collateral and the income from the default penalty is given back to all agents as a lump sum transfer, $T(D, y)$.[10] In this way, we are only directly affecting the default incentives for borrowers but not the amount of collateral seized by lenders. In practice, this could be enforced by taking away any additional assets to further penalize private agents who choose to default.

The policy instruments affect the budget constraint under the default state to include both the default penalty and the lump sum transfer.

$$c_T^D + pc_N^D = (1 - \lambda_1(1 + \tau))y + (1 - \lambda_1(1 + \tau))py^N + T \tag{35}$$

Definition 3 *A recursive decentralized competitive equilibrium with default penalties for the small open economy (SOE) is a pricing function $p(D, y)$, an interest rate schedule $r(d, D)$, a default penalty $\tau(d, D, y)$, a lump-sum transfer, $T(D, y)$ and decision rules $\{d'(d, D, y)$, $c_T^R(d, D, y)$, $c_T^D(D, y)$, $c_T^A(D, y)$, $c_N^R(d, D, y), c_N^D(D, y)$, $c_N^A(D, y)$, $\hat{y}(d, D, y)\}$ such that the following conditions hold:*

[10] For the full derivation of the optimal default penalty, see Appendix 1.3

- *Household's problem: Taking* $\{p(D,y), r(d,D), \tau(d,D,y), T(D,y)\}$ *as given, decision rules* $\{d'(d,D,y), c_T^R(d,D,y), c_N^R(d,D,y), \hat{y}(d,D,y)\}$ *maximize (6) subject to (7), (8), (9) and (11); decision rules* $\{c_T^D(D,y), c_N^D(D,y)\}$ *maximize (12) subject to (35) and (9); and decision rules* $\{c_T^A(D,y), c_N^A(D,y)\}$ *maximize (15) subject to (16) and (9).*
- *Rational expectations:* $\Gamma(D,y) = d'(D,D,y)$
- *Market clearing:* $c_N^R(D,D,y) = y^N, c_N^D(D,y) = y^N, c_N^A(D,y) = y^N, c_T^R(D,D,y) = y + d'(D,D,y) - (1 + r(D,D))D, c_T^D(D,y) = (1-\lambda_1)y - \lambda_1 p(D,y)y^N$ *,* $c_T^A(D,y) = y$*,* $T(D,y) = \tau(D,D,y)\lambda_1(y + p(D,y)y^N)$

The value of the default penalty, $\tau(d,D,y)$, that corrects the distortion in default incentives is characterized by:

$$\frac{(1+r)U_1(\hat{c}^R)}{U_1(\hat{c}^R) - (1-\lambda_1(1+\tau))U_1(\hat{c}^D)} = \frac{(1+r)U_1(\hat{c}^R) + \lambda_1 \frac{\partial \hat{p}}{\partial \hat{c}^T} y^N U_1(\hat{c}^D)}{U_1(\hat{c}^R) - (1-\lambda_1)U_1(\hat{c}^D) + \lambda_1 \frac{\partial \hat{p}}{\partial \hat{c}^T} y^N U_1(\hat{c}^D)} \tag{36}$$

where $\hat{c}$ and $\hat{p}$ are consumption and prices at initial state $(d, \hat{y})$.

3 Quantitative Analysis

This section presents the quantitative implications of the default spillover effects. We solve the decentralized equilibrium and the constrained social planner's problem numerically using global non-linear methods, which are described in Appendix 4. In order to analyze the quantitative properties of the model, we obtain the policy rules, default decisions and the price of debt under each state. We simulate the model to compute business cycle statistics and make an event analysis on default episodes.

3.1 Parameter Values and Functional Forms

The numerical solution takes parameters from calibrations based on data for Argentina, following other models that analyze credit externalities, such

Parameter	Value	Description	
σ	2	CRRA coefficient	
ρ	0.01	Risk-free interest rate	Aguiar and Gopinath (2006)
ω	0.3	Tradable consumption coefficient	Bianchi (2011)
$\frac{1}{1+\eta}$	0.8	Elasticity of substitution between tradable and non-tradable goods	Bianchi (2011)
ϕ	0.125	Access to capital markets	Mendoza and Yue (2011)
$\lambda_1 = \lambda_2$	0.1	Income seized under default	AG (2006)
β	0.93	Discount factor	Target: Debt-to-GDP ratio of 20 percent
ρ_y	0.9	AR(1) coefficient	AG (2006)
ξ	0.034	Standard deviation	AG (2006)

Table 1: Parameter Values

as Bianchi (2011), and on-equilibrium default, such as Aguiar and Gopinath (2006, AG hereafter). A period in our model represents a quarter. The values of the parameters used in this exercise are listed in Table (1).

Agents preferences are given by a CRRA utility in terms of the composite consumption good (c), which is a CES aggregator of the consumption of tradable (c_T) and non-tradable goods (c_N):

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

$$c = [\omega c_T^{-\eta} + (1-\omega)c_N^{-\eta}]^{-1/\eta}$$

The risk free interest rate, ρ, is set to 1 percent, which is a standard value used in the open macroeconomics literature for quarterly risk-free interest rate. The risk aversion coefficient, σ, is set at a value of 2. The probability of re-entering the credit market after default, ϕ, is set at 0.125, which implies an average exclusion period of about 10 quarters, consistent with an average exclusion of 2.5 years for Argentina. The weight on tradable consumption in the consumption aggregator, ω, is set at 0.3 and the elasticity of substitution between tradable and non-tradable consumption, $1/(1+\eta)$ is set at 0.8, following Bianchi (2011).

We should note that the elasticity of substitution is of key importance in the size of the default externality, as it drives the size of the real exchange rate depreciation when default takes place. For a given reduction in tradable consumption, a higher elasticity implies a smaller exchange rate depreciation, and therefore we should expect weaker spillover effects from the default externality.

Another key parameter in the model is the fraction of seized collateral, λ_1. This parameter differs from other models with default as it is related to a loss in terms of collateral. We use a value that is consistent with Aguiar and Gopinath's value of 2 percent loss for every period. The discount factor, β, is set to match a ratio of debt-to-GDP of 20 percent. λ_2 is set equal to λ_1 which is the case with the smallest dead-weight loss and would therefore lead to a smaller default externality case. This gives a value for the fraction of income seized by the court when the borrower defaults, λ_1, of 0.1 and a discount rate β of 0.93.

The stochastic process for tradable output follows a log-normal AR(1) process, $\log(y_t) = \rho_y \log(y_{t-1}) + \varepsilon_t^y$, where $E[\varepsilon^y] = 0$ and $E[\varepsilon^{y2}]=\xi^2$. The parameters are set to $\rho_y = 0.9$ and $\xi = 0.034$. It is necessary to create a large number of values for the discretized representation of the shock in order to get default as an equilibrium outcome, as mentioned in Aguiar and Gopinath (2006) and Arellano (2008). Therefore, the shock is discretized to a 25-state Markov chain, following the procedure proposed by Tauchen and Hussey (1991).

3.2 Results

We present the default choice of private borrowers to show that they differ from those of the social planner and then simulate it to analyze the business cycle properties and crisis dynamics of this model.

Figure (1) presents the default decisions for private individuals and the

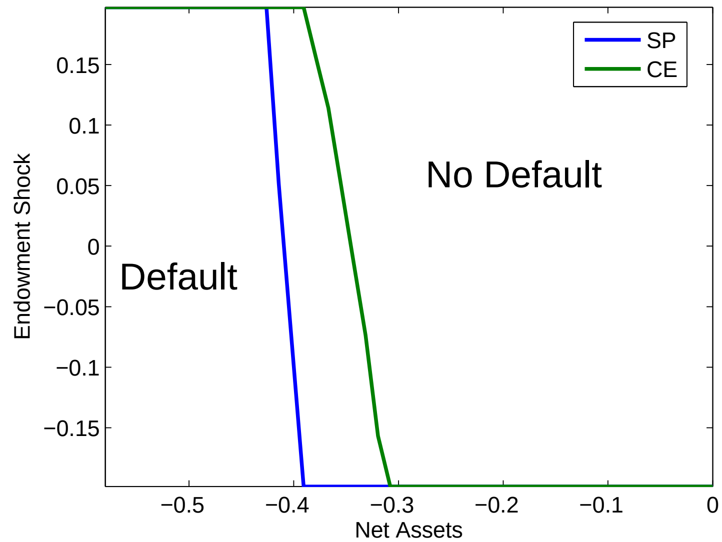

Figure 1: Default decision under the decentralized equilibrium and constrained planner's problem

social planner for a set of states. The x-axis shows different values of debt (net assets) and the y-axis shows different values of the tradable endowment shock. The blue (SP) and green (CE) lines depict the default threshold for the social planner and decentralized borrowers, respectively. The area on the left of the default threshold is the default region, where default is more likely to occur under high levels of debt (or low levels of net assets) and low levels of tradable endowment. For a given value of the endowment shock, agents have higher incentives to default under high levels of debt because higher debt increases the cost of repayment relative to default. For a given level of debt, agents have higher incentives to default under low levels of the endowment shock, as the value of seized collateral is increasing in tradable endowment.

By comparing the default sets, we observe that a distortion arises in the middle region because private agents ignore that default creates a spillover effect through the valuation of collateral. This is depicted by the region between the blue and green lines where decentralized agents default, even though it is socially optimal to repay debt. Default spillover effects are only relevant for intermediate levels of debt because, for high levels of debt, the cost of repayment is so large that it is socially optimal to default. Likewise, for very low levels of debt, the cost of repayment is so small that both private borrowers and constrained social planner choose not to default.

Figure (2) shows the price of new debt for the social planner (SP) and for individual borrowers (CE) under different states. The low state (LOW) refers to one where agents are highly indebted and get a low endowment shock. The high state (HIGH) refers to one where borrowers have low initial debt and get a high endowment shock. Consistent with default occurring less frequently under high endowment and low debt, as shown in Figure (1), the default risk premium is lower, or equivalently, the price of debt is higher in the high state.

Given the higher probability of default for decentralized borrowers, international lenders charge a higher interest rate to be willing to engage in riskier lending, which translates into a lower price of debt for private borrowers. This result is consistent with the bond price schedule effect in Kim and Zhang (2012). Notice that because private borrowers have higher incentives to default, the maximum value of debt that they would repay is lower than the socially optimal, which is shown at the debt level where there is a change in the concavity of the price of debt. At values above this maximum debt limit, there are no values of tradable income at which borrowers repay, so lenders always recover only the collateral value.

The debt choice in the private equilibrium considers two effects related to the valuation of collateral. The first effect is the one given by the recovery value of collateral on the interest rate schedule. If default incentives were perfectly aligned, such as in models with an exogenous frequency of default,

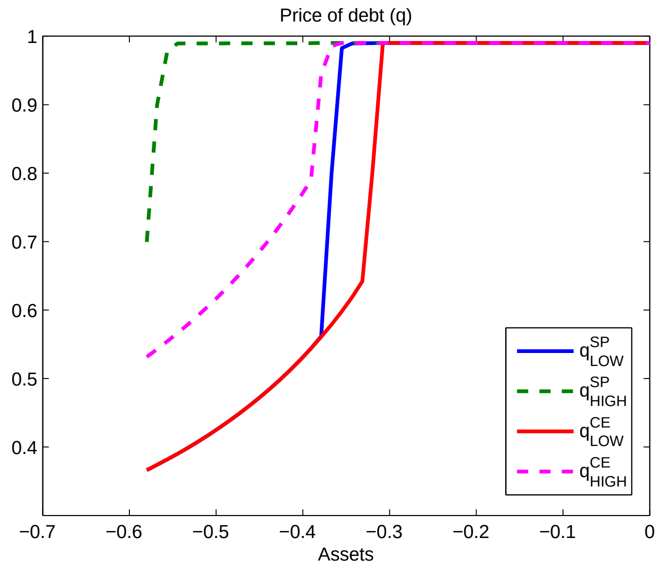

Figure 2: Price of debt

private borrowers would take more debt than the social planner.[11]The second effect is the one introduced by the distortion in default incentives. The decentralized equilibrium has higher default incentives and a lower price of debt, which reduce the marginal benefit of taking new debt. This leads to lower debt in the decentralized equilibrium than in the social planner's problem. The final effect on the level of debt depends on the relative magnitude of these two effects.

Figure (3) presents the policy rule for new debt for a tradable shock that is one standard deviation below the mean. It shows that the social planner (SP) chooses a higher level of debt than decentralized agents (CE), which is the case where the bond price schedule effect is larger than the over-borrowing effect. Note that, under high levels of debt, agents would choose to default, which is shown by the value of 0 debt for high levels of debt. Higher default incentives for private borrowers is shown by the fact

[11]A qualitative analysis of the pecuniary externality with exogenous default is presented in Appendix 3. This result is also consistent with models with endogenous borrowing constraints and no default such as Bianchi (2011) and Korinek (2010).

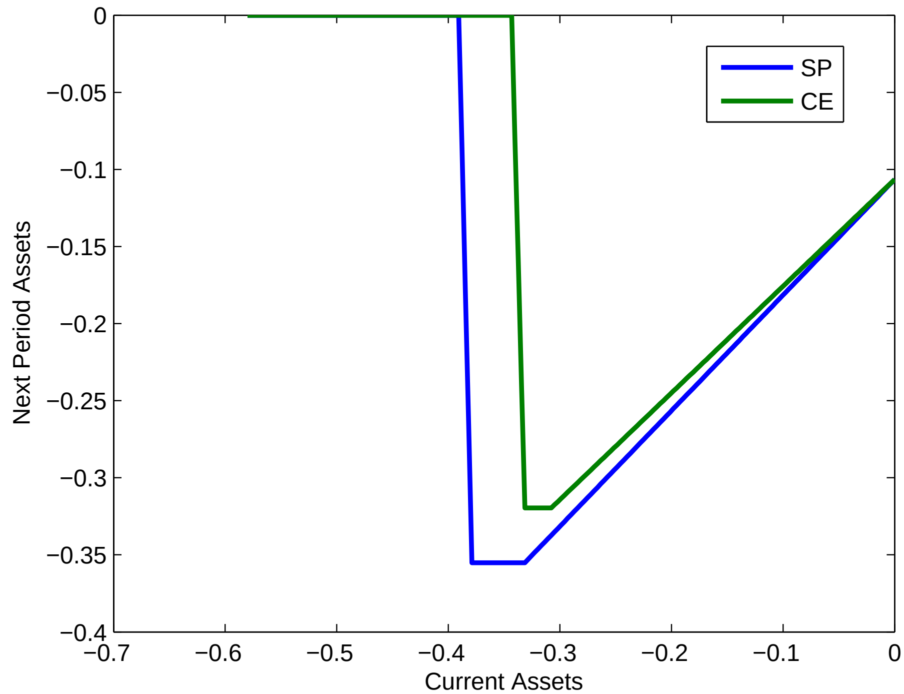

Figure 3: Policy rule for a negative 1 s.d. tradable endowment shock

that the policy rule goes back to a value of 0 under a lower level of debt than the social planner.

In contrast to an economy with no financial frictions, this economy faces an upper limit on the level of optimal debt, as there is a level above which it is always optimal to default, regardless of the value of the tradable income shock. Figure (3) shows that this limit is also lower for decentralized agents. The lack of commitment to pay back in the optimal set of states translates into a lower price and lower levels of debt than in an environment with no default distortions.

In order to analyze the behavior of real aggregates and the default externality effects during systemic private default episodes, we simulate the economy for 10,000 periods for the same path of the income shock for the decentralized equilibrium and the social planner's equilibrium. We obtain

Standard Deviation	SP	CE
Output	0.0450	0.0457
Interest Rate	0.0012	0.0013
Trade Balance	0.002	0.008
Consumption	0.0459	0.0469
Correlations with Output		
Interest Rate	0.1295	-0.0455
Trade Balance	-0.2145	-0.1662
Default Frequency	0.0706	0.1514
Debt to GDP	0.2123	0.20

Table 2: Business cycle statistics

some descriptive statistics on the behavior of real aggregates under the two settings. Table (2) summarizes the main results.

The simulation results show some standard stylized facts for business cycles in small open economies. Consumption volatility is higher than output volatility due to the financial friction, as agents face limited risk sharing due to higher default risk in the states where they need to borrow the most to smooth consumption. The volatility of the interest rate is extremely low because of the low frequency of default in the model.[12] There is a negative correlation between output and the interest rate, suggesting that default risk increases the most under low states of the tradable income shock.

Comparing the debt levels under the decentralized and the socially optimal equilibria, we get higher average debt for the social planner, consistent with the policy rules shown in Figure (3), as well as higher frequency of default in the private equilibrium. Comparing the numerical results with Aguiar and Gopinath (2006), the amplification mechanism created by the relative price of non-tradable goods allows us to obtain a higher frequency of default (0.15 compared to 0.02 percent). Compared to models with centralized default, the chain reaction of defaults creates an amplification mech-

[12] As mentioned before, proportional costs of default do not allow to sustain high levels of debt and high frequencies of default in equilibrium.

anism in the default frequency. If we compare them with our results for the social planner, we still find some amplification in the default frequency (0.07 compared to 0.02 percent), because default triggers a chain reaction in default through an exchange rate depreciation. Decentralized default amplifies this effect even more because the private cost of default is smaller than the social cost of default.

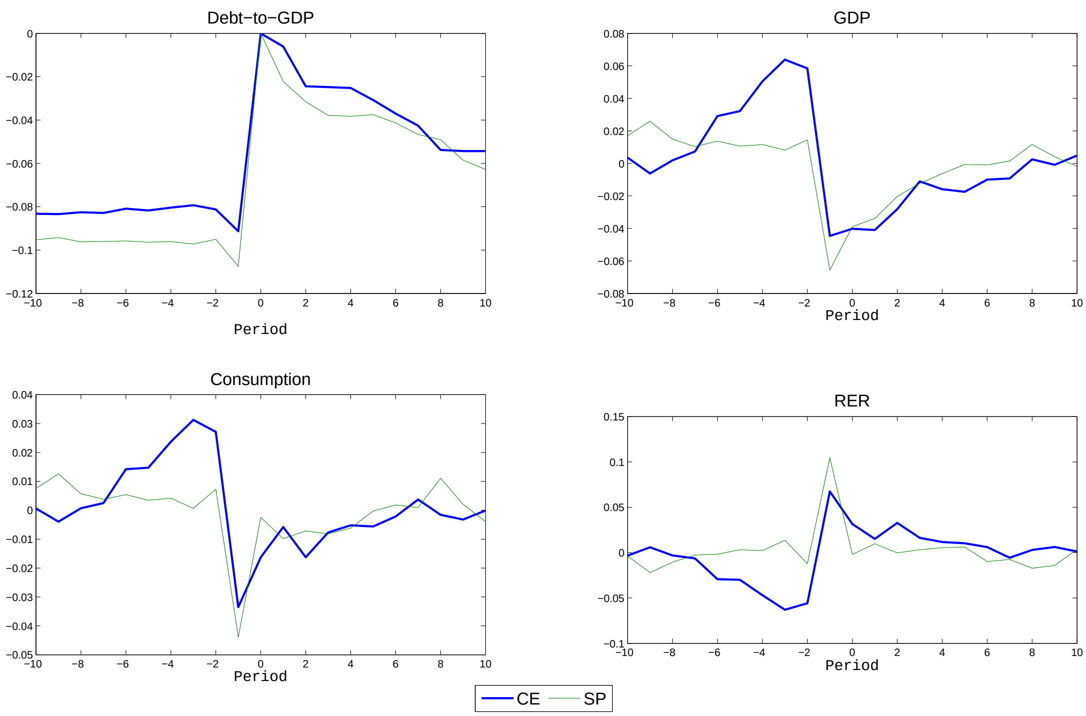

Figure 4: Default Event

Figure (4) shows the behavior of some key real aggregates under the decentralized equilibrium and social planner's problem. Each line represents the average value of all default events in the simulation. Prior to default, there is an increase in the ratio of debt-to-GDP, which triggers default due to a combination of high debt and a low endowment shock. This is also shown in the sharp fall in the figure labeled 'GDP', which is consistent with a fall in the total value of collateral. A default event leads to capital outflows and a real exchange rate depreciation, consistent with the transfer problem. Private borrowers who default lose a fraction of their collateral and access to international capital markets, which is shown as a reduction in consumption in the period when default takes place and a lower possibility of consumption smoothing in the following periods.

3.3 Sensitivity Analysis

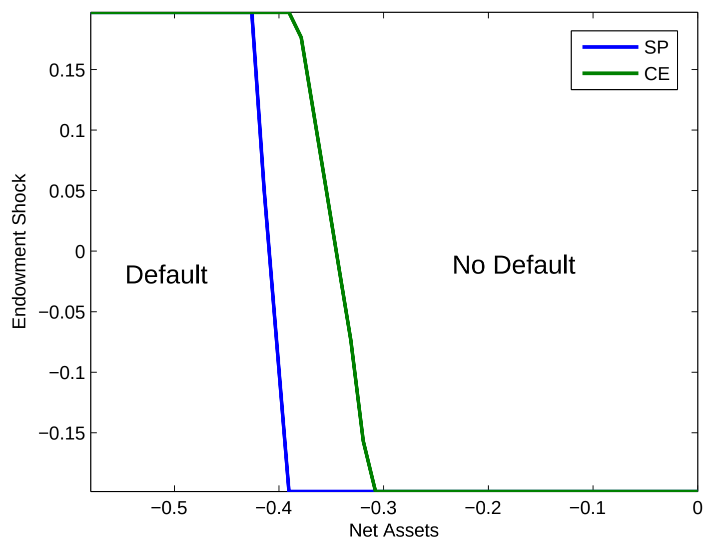

Figure 5: Default Set for $\lambda_2 = 0.9\lambda_1$

In this section, we analyze the behavior of the default externality under alternative calibrations. One of the key features of this model is to include a cost of default in terms of the value of collateral seized by lenders. Therefore, we present an analysis on different parametrizations of the value of λ_1 and λ_2. The first experiment is to increase the size of the dead-weight cost of default, by creating a wedge between λ_1 and λ_2 of 10 percent of the value of λ_1. Intuitively, an increase in the dead-weight loss reduces the amount of collateral repatriated by lenders so they would compensate it by charging a higher interest rate. An increase in the cost of borrowing has a second order effect on increasing incentives to default and on default spillover effects. However, it does have a direct effect on the interest rate faced by all agents to compensate for the dead-weight loss. Figure (5) shows a very small effect on the default set, whereas Figure (6) shows a lower price of debt to compensate for the larger dead-weight loss.

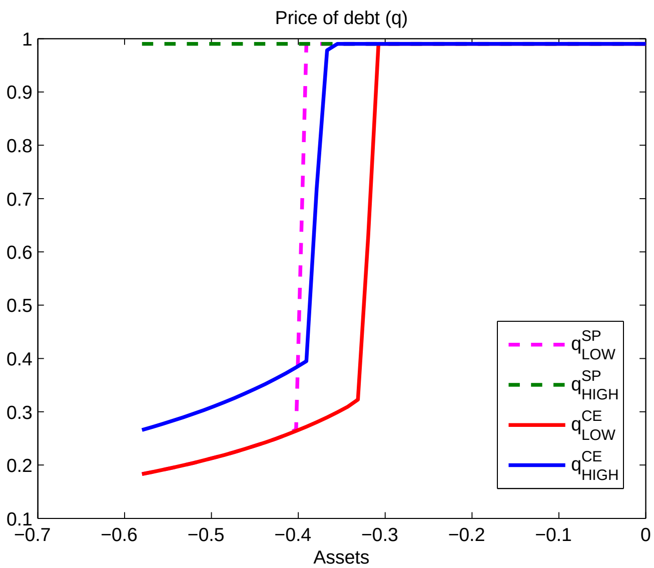

Figure 6: Price of Debt for $\lambda_2 = 0.9\lambda_1$

Figure (7) presents the results for a second experiment, where we consider a higher value of $\lambda_1 = \lambda_2 = 0.12$. A higher level of enforcement increases the cost of default for both decentralized borrowers and a constrained social planner, so that they choose to default less often than under the benchmark scenario. This is reflected in a shift in the default sets to the left, so that default is preferred only under very high levels of debt and/or very low values of tradable endowment. In terms of the default externality, the default distortion occurs under higher levels of debt and/or lower levels of tradable endowment, so that the default penalty is used less frequently.

There are several other parameters that are relevant to calculate the size of the default externality. By looking at the Euler conditions in the

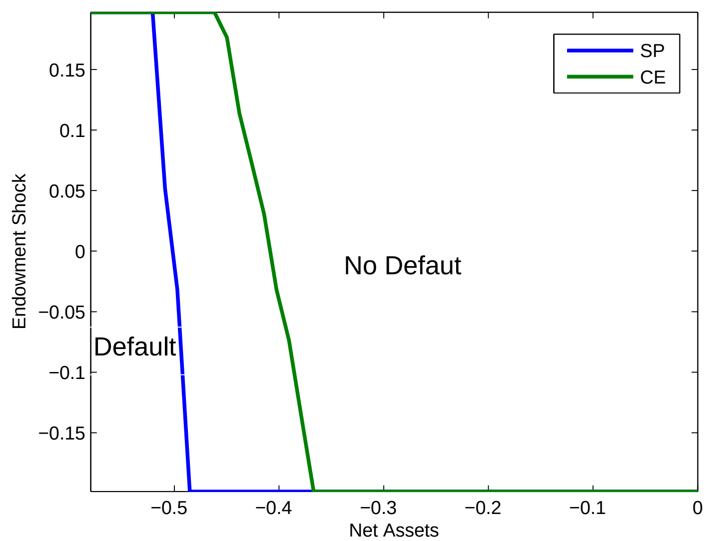

Figure 7: Default Set for $\lambda_1 = \lambda_2 = 0.12$

decentralized equilibrium and the social planner's problem, (19) and (32), the key parameters affecting the size of the default spillover effect are the ones that affect $\partial\hat{y}/\partial p\ \partial p/\partial d$. The first term measures the impact of a real exchange rate depreciation on the incentives to default. The second one is the impact of an increase in individual debt on the exchange rate.

There are three key parameters affecting the size of the spillover effect: the elasticity of substitution between tradable and non-tradable goods, $1/(1+\eta)$, the weight of non-tradable consumption in the consumption aggregator, $1-\omega$ and the discount factor, β. Our results show that the default externality is qualitatively unchanged under alternative scenarios, where internalizing the effect of default on the valuation of collateral leads to lower default incentives under the social planner's equilibrium.

The elasticity of substitution between tradable and non-tradable goods, $1/(1+\eta)$, plays a key role in determining the size of the real depreciation when default takes place. Figure (8) shows the results for an elasticity of substitution of 0.4, on the lower end of the estimates in Bianchi (2011), and

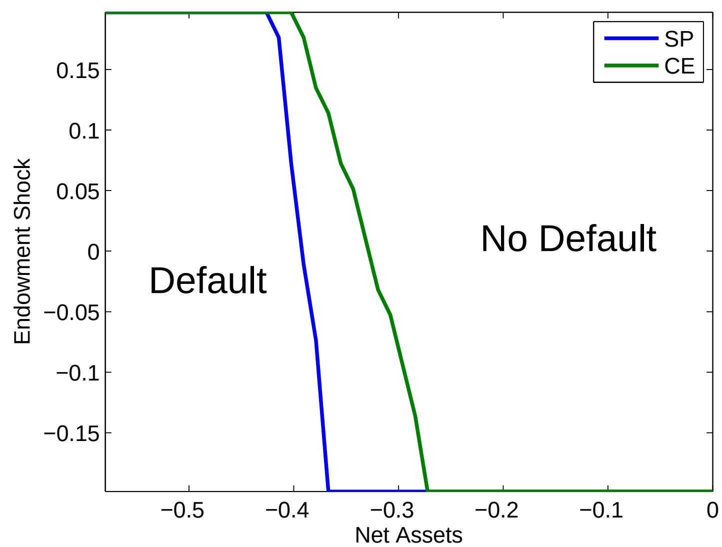

Figure 8: Default Set for $1/(1+\eta) = 0.4$

Figure (9) for a higher elasticity of substitution of 1.25, as in Benigno et al (2010), where tradable and non-tradable goods are gross substitutes. A lower elasticity increases the level of complementarity in the consumption of tradable and non-tradable goods. Therefore, a reduction in tradable consumption under default reduces the demand for non-tradable goods as well, leading to a larger real exchange rate depreciation. The size of the default externality increases for closer complements.

From a positive perspective, the size of the elasticity of substitution affects the relative cost of repayment. Default occurs more frequently as the cost of repayment is higher. This is shown by comparing the default thresholds for the social planner, depicted by the blue lines in Figures (8) and (9).

Similarly, a higher weight of tradable collateral, ω, increases the size of the default externality. A sharper exchange rate depreciation reults in more capital outflows and therefore in larger spillover effects trough a chain reaction in default. Figure (10) shows the size of the default externality for

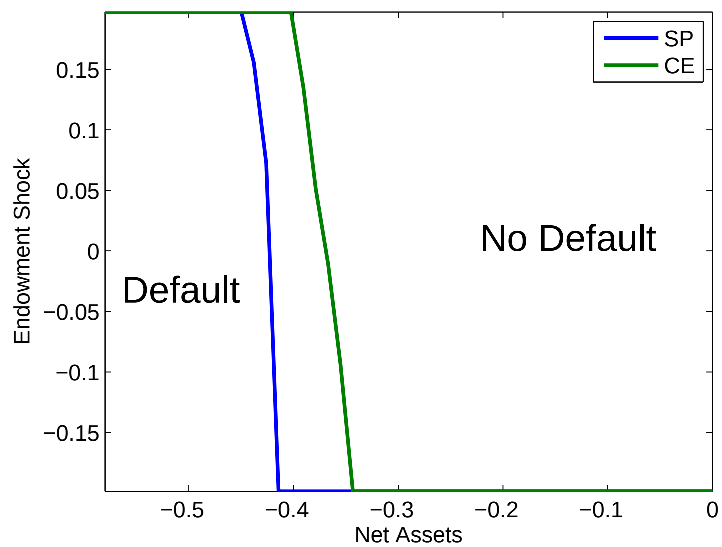

Figure 9: Default Set for $1/(1+\eta) = 1.2$

an alternative scenario with a value of $\omega = 0.5$. From a positive perspective, a higher weight on tradable consumption increases the cost of repayment in bad states, so that agents default more often, widening the set of states where default takes place.

Another key parameter is the relative size of non-tradable and tradable collateral, y^N, as it is only the fraction of non-tradable collateral which is affected by changes in valuation through the real exchange rate depreciation.[13] We repeat the numerical exercise by reducing the size of non-tradable endowment in the composition of collateral to half. [14] Figure (11) shows the reduction in the size of the default externality for a lower share of non-tradable collateral in the total composition. A lower share of non-tradable collateral decreases the magnitude of the default externality, as the real exchange rate

[13]Similar to the result obtained in Uribe (2006), if there is only tradable collateral, the pecuniary externality disappears. If only tradable goods can be used as collateral, there is no valuation effect on the default incentives and the financial contract faced by borrowers, so the decentralized equilibrium is Pareto efficient.

[14]We change the value of y^N so that the steady state value of total endowment remains the same, but with a lower ratio of y^N to y.

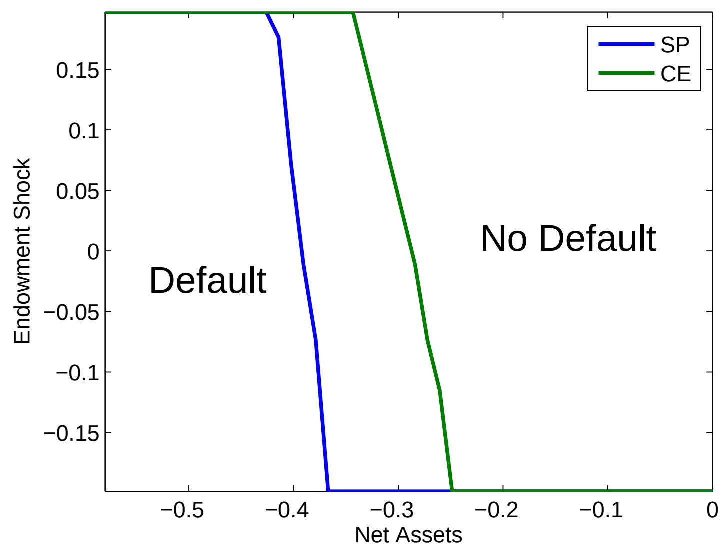

Figure 10: Default Set for $\omega = 0.5$

depreciation has a smaller impact on the total value of collateral, reducing the spillover effects on incentives to default.

The last alternative scenario considers a lower discount factor, β. On one hand, agents want to borrow more due to the impatience factor. However, on the other hand, higher debt increases incentives to default, which amplify the default spillover effect. Figure (12) shows the size of default externality with a lower value of $\beta = 0.9$. The impatience effect dominates so that agents choose to default in a smaller set of states and face a smaller default externality.

3.4 Default Penalty

We solve numerically for the default penalty in the infinite horizon model. The default penalty is the additional cost paid by individual defaulters that is proportional to the value of their own collateral. Figure (13) depicts the optimal default penalty for each state, where only the states with a distor-

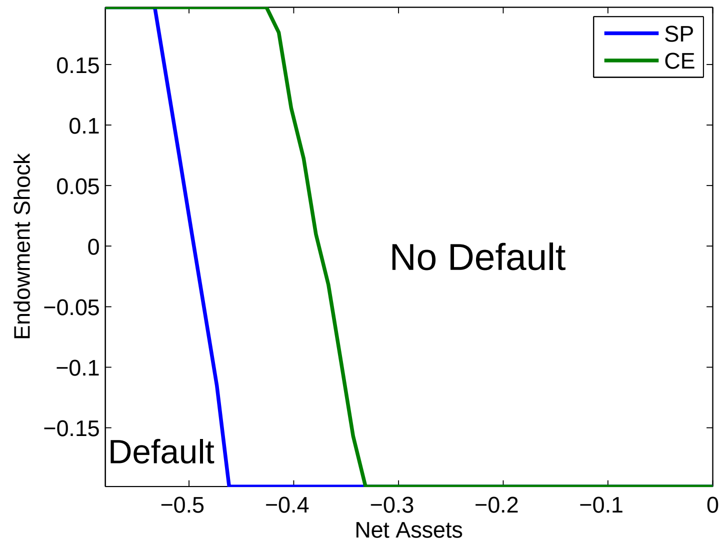

Figure 11: Default Set for $y^N = 0.5$

tion in default incentives show a positive default penalty value.[15] The x-axis shows different values of initial net assets, whereas each line corresponds to a different value of the tradable income shock. As previously mentioned, the distortion in default incentives occur in the middle range of debt values. Under high levels of debt, both the social planner and private individuals choose to repay, whereas under low levels of debt both choose to default.

On those states where a default penalty is needed to correct the externality, a higher endowment shock reduces the optimal value of the default penalty, as the cost of defaulting is increasing in tradable endowment. On the debt dimension, higher levels of debt require a higher tax to increase the cost of default and reduce incentives to default, as the size of the externality is increasing in debt.

Note that in order to correct default incentives, we could also use a policy instrument that reduces the marginal benefit of defaulting. Policy measures

[15]We could impose a positive penalty value on those states where both private agents and the social planner would choose to repay but it has no effect on the final outcome.

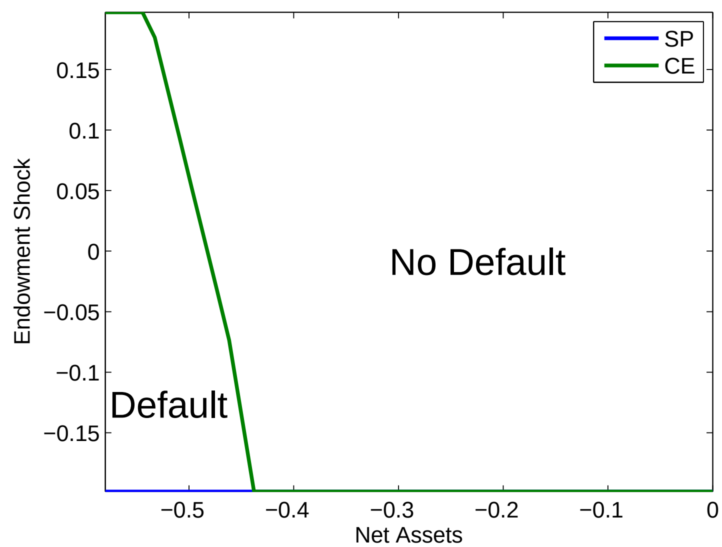

Figure 12: Default Set for $\beta = 0.9$

such as credit refinancing could also correct the externality by increasing the marginal benefit of repayment relative to defaulting. Although it would not only affect the default incentives today but will have intertemporal effects as well, another policy consistent that could potentially correct the distortion due to default spillover effects is the introduction of capital flow subsidies as in Wright (2006).

Let us compare this result to models with endogenous borrowing constraints but no default. In that case, the optimal policy measure is to impose a tax in the states where the constraint is not binding to prevent individual borrowers from taking excessive levels of debt, which would tighten the constraint through its effect on collateral prices. In periods where borrowers are already facing a binding constraint the tax does not affect their debt decision. In contrast, in the model with distortions in the default choice, the optimal policy instrument needs to address only the states where default takes place. Private borrowers can now take higher levels of debt than in models with no default, but at the expense of higher risk and lower price

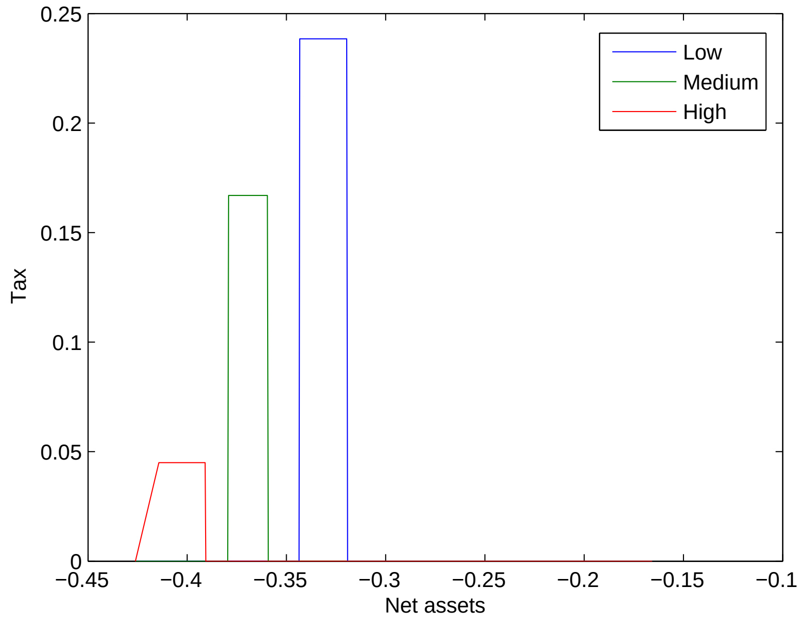

Figure 13: Optimal Policy

of debt. Therefore, the optimal policy instrument affects a different subset of states, at higher levels of debt than the ones implied by the binding borrowing constraint.

4 Conclusions

This paper analyzes the distortion in the incentives to default on collateralized debt that arises as a default spillover externality in a two-good endowment small open economy. In this model, individuals fail to internalize the effect of their default decision on the real exchange rate and through it on the incentives to default of other borrowers in the economy. An individual defaulter creates an exchange rate depreciation and capital outflows that reduce the value of non-tradable collateral and induces other borrowers to default. Therefore, by taking prices as given, private borrowers default more frequently and face a higher risk premium, which limits optimal debt taking and consumption smoothing.

Enforcement of creditor rights should be strengthened in order to align default incentives of individual agents with the socially optimal, so that individual agents default less often. This leads to decentralized borrowers to be able to take higher deb, face a lower cost of borrowing and engage in better consumption smoothing. In addition, in aggegrate terms, it also leads to a lower probability of default and a lower frequency of sharp current account reversals and real depreciations.

The model uses a simplified way of modeling the costs of the default, which has the benefit of making it easy in terms of tractability and implementation to illustrate the interaction mechanism between default and the value of collateral. However, it would be interesting to extend these results to a model that considers a more realistic approach on the punishment of the debt contract under default. Even though this paper is related to private default, similar conditions to the ones for sovereign default in Arellano (2008) and Mendoza and Yue (2011) can be applied for the costs of private default in order to obtain a more realistic default frequency and level of debt.

A further analysis of the optimal policy measures should focus on a wider variety of policies that affect the incentives to default in order to align the default choice of private borrowers to the socially optimal. This work only considers a default penalty that affects the cost of default, but other policy measures that reduce the marginal benefit of default such as credit refinancing should also be analyzed in detail.

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5 Appendix

5.1 Default externality and optimal default penalty

The intertemporal Euler equation is given by:

$$U_1(c_t) = \beta \int_{\hat{y}_{t+1}}^{\bar{y}} \frac{d[(1+r_{t+1})d_{t+1}]}{dd_{t+1}} U_1(c_{t+1}) f(y_{t+1}) dy_{t+1} \tag{37}$$

5.1.1 Intertemporal Euler equations

Decentralized Equilibrium

Total differentiation on the lenders' participation constraint:

$$\frac{\partial[(1+r_{t+1})d_{t+1}]}{\partial d_{t+1}} \int_{\hat{y}_{t+1}}^{\bar{y}} f(y_{t+1}) dy_{t+1} dd_{t+1} - \left[(1+r_{t+1})d_{t+1} - \lambda_2 \left(\hat{y}_{t+1} + \hat{p}_{t+1} y^N\right)\right] f(\hat{y}_{t+1}) \frac{d\hat{y}_{t+1}}{dd_{t+1}} = (1+\rho)$$

$$\frac{\partial[(1+r_{t+1})d_{t+1}]}{\partial d_{t+1}} = \frac{1+\rho+\left[(1+r_{t+1})d_{t+1} - \lambda_2 \left(\hat{y}_{t+1} + \hat{p}_{t+1} y^N\right)\right] f(\hat{y}_{t+1}) \frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}}}{\int_{\hat{y}_{t+1}}^{\bar{y}} f(y_{t+1}) dy_{t+1}} \tag{38}$$

The first order condition becomes:

$$U_1(c_t) = \beta \frac{1+\rho+\left[(1+r_{t+1})d_{t+1} - \lambda_2 \left(\hat{y}_{t+1} + \hat{p}_{t+1} y^N\right)\right] f(\hat{y}_{t+1}) \frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}}}{\int_{\hat{y}_{t+1}}^{\bar{y}} f(y_{t+1}) dy_{t+1}} \int_{\hat{y}_{t+1}}^{\bar{y}} U_1(c_{t+1}) f(y_{t+1}) dy_{t+1} \tag{39}$$

Social Planner

Total differentiation on the lenders' participation constraint:

$$\frac{\partial[(1+r_{t+1})d_{t+1}]}{\partial d_{t+1}} \int_{\hat{y}_{t+1}}^{\bar{y}} f(y_{t+1}) dy_{t+1} + \lambda_2 y^N \int_{\underline{y}}^{\hat{y}_{t+1}} \frac{\partial p_{t+1}}{\partial d_{t+1}} f(y_{t+1}) dy_{t+1}$$
$$- \left[(1+r_{t+1})d_{t+1} - \lambda_2 \left(\hat{y}_{t+1} + \hat{p}_{t+1} y^N\right)\right] f(\hat{y}_{t+1}) \frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}} = (1+\rho)$$

$$\frac{\partial[(1+r_{t+1})d_{t+1}]}{\partial d_{t+1}} = \frac{1+\rho+\left[(1+r_{t+1})d_{t+1} - \lambda_2 \left(\hat{y}_{t+1} + \hat{p}_{t+1} y^N\right)\right] f(\hat{y}_{t+1}) \frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}} - \lambda_2 y^N \int_{\underline{y}}^{\hat{y}_{t+1}} \frac{\partial p_{t+1}}{\partial d_{t+1}} f(y_{t+1}}{\int_{\hat{y}_{t+1}}^{\bar{y}} f(y_{t+1}) dy_{t+1}} \tag{40}$$

The first order condition becomes:

$$U_1(c_t) = \beta \frac{1+\rho+\left[(1+r_{t+1})d_{t+1} - \lambda_2\left(\hat{y}_{t+1}+\hat{p}_{t+1}y^N\right)\right] f(\hat{y}_{t+1})\frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}} - \lambda_2 y^N \int_{\underline{y}}^{\hat{y}_{t+1}} \frac{\partial p_{t+1}}{\partial d_{t+1}} f(y_{t+1})dy_{t+1}}{\int_{\hat{y}_{t+1}}^{\bar{y}} f(y_{t+1})dy_{t+1}} \times \int_{\hat{y}_{t+1}}^{\bar{y}} U_1(c_{t+1})f(y_{t+1})dy_{t+1} \tag{41}$$

5.1.2 Effect of individual debt on default incentives

Decentralized equilibrium

Total differentiation on the definition of the default threshold:

$$\frac{\partial v^R(d,D,\hat{y})}{\partial d}dd + \frac{\partial v^R(d,D,\hat{y})}{\partial \hat{y}}d\hat{y} = \frac{\partial v^D(D,\hat{y})}{\partial \hat{y}}d\hat{y} \tag{42}$$

$$-(1+r)U_1(\hat{c}^R)dd + U_1(\hat{c}^R)d\hat{y} = (1-\lambda_1)U_1(\hat{c}^D)d\hat{y}$$

$$\frac{\partial \hat{y}^{CE}}{\partial d} = \frac{(1+r)U_1(\hat{c}^R)}{U_1(\hat{c}^R)-(1-\lambda_1)U_1(\hat{c}^D)} \tag{43}$$

Social planner

Total differentiation on the definition of the default threshold:

$$\frac{\partial v^R(d,\hat{y})}{\partial d}dd + \frac{\partial v^R(d,\hat{y})}{\partial \hat{y}}d\hat{y} = \frac{\partial v^D(d,\hat{y})}{\partial d}dd + \frac{\partial v^D(d,\hat{y})}{\partial \hat{y}}d\hat{y} \tag{44}$$

$$-(1+r)U_1(\hat{c}^R)dd + U_1(\hat{c}^R)d\hat{y} = (1-\lambda_1)U_1(\hat{c}^D)d\hat{y} - \lambda_1\frac{\partial p}{\partial \hat{y}}y^N U_1(\hat{c}^D)d\hat{y} - \lambda_1\frac{\partial p}{\partial d}y^N U_1(\hat{c}^D)dd$$

$$\frac{\partial \hat{y}^{SP}}{\partial d} = \frac{(1+r)U_1(\hat{c}^R) + \lambda_1\frac{\partial \hat{p}}{\partial \hat{c}_T}y^N U_1(\hat{c}^D)}{U_1(\hat{c}^R)-(1-\lambda_1)U_1(\hat{c}^D) + \lambda_1\frac{\partial \hat{p}}{\partial \hat{c}_T}y^N U_1(\hat{c}^D)} \tag{45}$$

By comparing (43) and (45), we find that:

$$\frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}}^{CE} > \frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}}^{SP} \tag{46}$$

5.1.3 Optimal default penalty

We solve for the competitive equilibrium conditions when the default penalty instrument is available. The problem of a representative private borrower under repayment is the same, except for the new definition of welfare under the default state, v^D, which affects the default threshold, $\hat{y}$.

$$v^D(d,\hat{y}) = \max U\left(c_T^R, c_N^R\right) + \beta\phi E v^R(0,y') + \beta(1-\phi)Ev^A(y') \tag{47}$$

subject to:

$$c_T^D + pc_N^D = (1 - \lambda_1(1+\tau))(\hat{y} + py^N) + T \tag{48}$$

Taking total differentiation on the definition of the default threshold:

$$\frac{\partial v^R(d, D, \hat{y})}{\partial d} dd + \frac{\partial v^R(d, D, \hat{y})}{\partial \hat{y}} d\hat{y} = \frac{\partial v^D(D, \hat{y})}{\partial \hat{y}} d\hat{y} \tag{49}$$

$$-(1+r)U_1(\hat{c}^R)dd + U_1(\hat{c}^R)d\hat{y} = (1 - \lambda_1(1+\tau))U_1(\hat{c}^D)d\hat{y}$$

$$\frac{\partial \hat{y}}{\partial d} = \frac{(1+r)U_1(\hat{c}^R)}{U_1(\hat{c}^R) - (1 - \lambda_1(1+\tau))U_1(\hat{c}^D)} \tag{50}$$

In order to align default incentives to the social planner's equilibrium, τ must satisfy:

$$\frac{(1+r)U_1(\hat{c}^R)}{U_1(\hat{c}^R) - (1 - \lambda_1(1+\tau))U_1(\hat{c}^D)} = \frac{(1+r)U_1(\hat{c}^R) + \lambda_1 \frac{\partial \hat{p}}{\partial \hat{c}^T} y^N U_1(\hat{c}^D)}{U_1(\hat{c}^R) - (1 - \lambda_1)U_1(\hat{c}^D) + \lambda_1 \frac{\partial \hat{p}}{\partial \hat{c}^T} y^N U_1(\hat{c}^D)} \tag{51}$$

5.2 Deadweight cost of default

This section shows that if default does not create a deadweight cost, then it leads to efficient debt and default decisions. We show it in a simple two period model, where we impose that $\lambda_1 = \lambda_2$, which is the case where all seized assets under default are transfered to the lender in terms of tradable goods.

5.2.1 Decentralized equilibrium

The optimality condition for debt is given by:

$$\begin{aligned} U_1(c_1) &= \beta(1+\rho)\left\{\int_{\hat{y}}^{\bar{y}} f(y_2)dy_2 - \left[(1+r_1)d_1 - \lambda_1(\hat{y} + \hat{p}y^N)\right]\frac{1}{\lambda_1}f(\hat{y})\right\}^{-1} \\ &\quad \int_{\hat{y}}^{\bar{y}} U_1(c_2^R)f(y_2)dy_2 \end{aligned}$$

The default threshold is defined as:

$$U\left(\hat{y} - (1+r_1)d_1, y_N\right) = U(\hat{c}_T^{D,CE}, \hat{c}_N^{D,CE})$$

where $\hat{c}_T^{D,CE}, \hat{c}_N^{D,CE}$ solve

$$\begin{aligned} & \max U(\hat{c}_T^{D,CE}, \hat{c}_N^{D,CE}) \\ \text{s.t. } \hat{c}_T^{D,CE} + p\hat{c}_N^{D,CE} &= (1-\lambda_1)(\hat{y} + \hat{p}y_N) \end{aligned}$$

To simplify the exercise, assume $U(c_T, c_N) = \log c_T + \log c_N$.

$$\begin{aligned}
\hat{c}_T^{D,CE} &= \frac{1-\lambda_1}{2}(\hat{y} + \hat{p}y^N) \\
\hat{c}_N^{D,CE} &= \frac{1-\lambda_1}{2\hat{p}}(\hat{y} + \hat{p}y^N)
\end{aligned}$$

and the default threshold condition is

$$\begin{aligned}
\log(\hat{y} - (1+r_1)d_1) + \log(y_N) &= \log\left(\frac{1-\lambda_1}{2}(\hat{y} + \hat{p}y^N)\right) \\
&\quad + \log\left(\frac{1-\lambda_1}{2\hat{p}}(\hat{y} + \hat{p}y^N)\right)
\end{aligned}$$

Market clearing in the non-tradable sector implies that $\hat{c}_N^{D,CE} = y_N$.Therefore, the market clearing price becomes

$$\hat{p} = \frac{(1-\lambda_1)\hat{y}}{(1+\lambda_1)y_N}$$

Replacing the market clearing price in the default condition it becomes

$$\begin{aligned}
\log(\hat{y} - (1+r_1)d_1) + \log(y_N) &= \log\left(\frac{1-\lambda_1}{1+\lambda_1}\hat{y}\right) + \log\left(y^N\right) \\
(1+r_1)d_1 &= \frac{2\lambda_1}{1+\lambda_1}\hat{y} \\
(1+r_1)d_1 &= \lambda_1(\hat{y} + \hat{p}y_N)
\end{aligned}$$

Plugging this result in the optimality condition, it simplifies to:

$$U_1(c_1) = \frac{\beta(1+\rho)}{\int_{\hat{y}}^{\bar{y}} f(y_2)dy_2} \int_{\hat{y}}^{\bar{y}} U_1(c_2^R) f(y_2) dy_2$$

5.2.2 Social Planner

Optimal debt chosen by the social planner follows:

$$U_1(c_1) = \beta(1+\rho)\left\{\int_{\hat{y}}^{\bar{y}} f(y_2)dy_2 - \left[(1+r_1)d_1 - \lambda_1(\hat{y} + \hat{p}_2 y^N)\right]\frac{1}{\lambda_1}\left(1 - \frac{1-\lambda_1}{2}\right)f(\hat{y})\right\}^{-1} \int_{\hat{y}}^{\bar{y}} U_1(c_2^R) f(y_2) dy_2$$

The default threshold is given by:

$$\begin{aligned}
\log(\hat{y} - (1+r_1)d_1) + \log(y_N) &= \log\left((1-\lambda_1)\hat{y} - \lambda_1 \frac{\hat{U}_2}{\hat{U}_1} y^N\right) + \log(y_N) \\
(1+r_1)d_1 &= \lambda_1\left(\hat{y} + \frac{\hat{U}_2}{\hat{U}_1} y^N\right)
\end{aligned}$$

Plugging this result in the optimality condition, it simplifies to:

$$U_1(c_1) = \frac{\beta(1+\rho)}{\int_{\hat{y}}^{\bar{y}} f(y_2)dy_2} \int_{\hat{y}}^{\bar{y}} U_1(c_2^R) f(y_2)dy_2$$

5.3 Exogenous default

Agents consume only tradable goods in the first period (to simplify the problem) and tradable and non-tradable goods in the second period. The tradable endowment in the second period can take 2 values: y_L with exogenous probability π and y_H with probability $1-\pi$.

The relevant set of parameters is when default is referred in a subset of states. Agents choose to default in the low state (y_L):

$$\lambda_1(y_L + \frac{U_2}{U_1}y^N) < (1+r)d$$

and pay back in the high one (y_H):

$$\lambda_1(y_H + \frac{U_2}{U_1}y^N) \geq (1+r)d$$

5.3.1 Decentralized equilibrium

The problem faced by private agents is to maximize:

$$U(y+d) + \beta\pi U(c_{T2}^L, c_{N2}^L) + \beta(1-\pi)U(c_{T2}^H, c_{N2}^H)$$

$$c_2^L + +p_2 c_{N2}^L = (1-\lambda_1)y_L + (1-\lambda_1)p_2 y^N$$

$$c_2^H + p_2 c_{N2}^H = y_H - (1+r)d + p_2 y^N$$

$$1+\rho = (1-\pi)(1+r) + \pi\lambda_2 \frac{y_L + p_2 y^N}{d}$$

The optimal choice of debt satisfies:

$$U_1(c_1) = \beta(1+\rho)U_1(c_2^H)$$

5.3.2 Social Planner

Let us assume the following preferences $U(c_{T,}c_N) = \log(c_T) + \log(c_N)$ to simplify the problem. The social planner chooses debt to maximize:

$$U(y+d) + \beta\pi U(c_{T2}^L, c_{N2}^L) + \beta(1-\pi)U(c_{T2}^H, c_{N2}^H)$$

$$c_{T2}^L = (1-\lambda_1)y_L - \lambda_1 \frac{U_2}{U_1} y^N$$

$$c_{T2}^H = y_H - (1+r)d$$

$$c_{N2}^L = c_{N2}^H = y^N$$

$$1+\rho = (1-\pi)(1+r) + \pi\lambda_2 \frac{y_L + \frac{U_2}{U_1} y^N}{d}$$

The optimal level of debt satisfies:

$$U_1(c_1) = \beta\left[1 + r + \frac{d(1+r)}{dd} d\right](1-\pi)U_1(c_2^H)$$

$$U_1(c_1) = \beta(1+\rho)\frac{1-\pi}{1-(1+\lambda_2)\pi} U_1(c_2^H)$$

Comparing the debt choice under the decentralized equilibrium and the socially efficient level, we see that:

$$U_1(c_1^{SP}) > U_1(c_1^{CE})$$

$$d_1^{SP} < d_1^{CE}$$

Higher levels of debt increase capital outflows in the second period, which result in a real depreciation. This affects the valuation of non-tradable collateral of other agents, so that all borrowers face a higher interest rate. Higher interest rates lead to lower levels of socially optimal debt.

5.4 Algorithm for the Numerical Solution

5.4.1 Social Planner

1. Start with some guess for the price of bonds $q^0(d, y) = \frac{1}{1+\rho}$ for all d and y.

2. Given the interest rate schedule, solve for the optimal consumption $c_T(d,y), c_N(d,y)$, debt holdings $d\prime(d,y)$, and default set $\delta(d,y)$ using value function iteration. Iterate on the value function until convengence is achieved.

3. Using the default set and repayment set, compute the new price of bonds $q^1(d,y)$ that satisfies equation (23) and compare it with the one used in the previous iteration $q^0(d,y)$. If a convergence criterion is met, $\max\left|q^1(d,y) - q^0(d,y)\right| < \epsilon_r$, finish the iterative process. If not, update the bond price using a Gauss - Seidel algorithm and go back to the previous step.

5.4.2 Decentralized Equilibrium

1. Start with some guess for the relative price of non-tradable goods in each state $p^0(D,y)$. An initial guess is the price obtained from the social planner's problem.

2. Start with some guess for the price of bonds $q^0(d,D,y) = q^{SP}(d,y)$ for all D from the social planner's problem.

3. Start with some guess for the law of motion of aggregate debt $\Gamma^0(D,y) = d'^{SP}(d,y)$ from the social planner's problem.

4. Given the interest rate schedule and the law of motion for aggregate debt, solve for the optimal consumption $c_T(d,D,y), c_N(d,D,y)$, debt holdings $d\prime(d,D,y)$ and default set $\delta(d,D,y)$ using value function iteration. For every iteration, update the law of motion for aggregate debt $\Gamma^i(D,y) = d'^i(D,D,y)$. Iterate on the value function until convergence is achieved.

5. Using the default and repayment sets, compute the bond price $q^1(d,D,y)$ that satisfies equation (23) and compare it with the one used in the previous iteration $q^0(d,D,y)$.If a convergence criterion is met, $\max\left|q^1(d,D,y) - q^0(d,D,y)\right| < \epsilon_r$, go to the next step. If not, update the interest rate schedule using a Gauss - Seidel algorithm and go back to the previous step.

6. Using optimal debt holdings, compute the price of non- tradable goods $p^1(D,y)$ and compare it with the previous guess $p^0(D,y)$. If a convergence

criterion is met, $\max \left|p^1(D,y) - p^0(D,y)\right| < \epsilon_p$, finish the iterative process. If not, update the price using a Gauss - Seidel algorithm and go back to step 2.