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Long-Run Money Demand in Latin-American countries: A Nonstationary Panel Data Approach*

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Abstract

Central banks have long been interested in obtaining precise estimations of money demand given the fact that the evolution of money demand plays a key role over several monetary variables. I use Pedroni's (2002) Fully Modified Ordinary Least Square (FMOLS) to estimate the coefficients of the long-run money demand function for 15 Latin-American countries. The FMOLS technique pools information regarding common long-run relationships while allowing the associated short-run dynamics and fixed effects to be heterogeneous across different members of the panel. For this group of countries, I find evidence of a cointegrating money demand, an income elasticity of 0.94, and an interest-rate semi-elasticity of -0.01.

Keywords: Money demand, panel cointegration, FMOLS, Latin-American.

JEL classification: C22, C23, E41.

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1 Introduction

Central banks and economists have long been interested in obtaining precise estimates of money demand for at least two reasons. First, knowing the income elasticity of long-run money demand helps in determining the rate of monetary expansion that is consistent with long-run price level stability. Second, knowing the interest rate semi-elasticity of money demand aids in calculating the welfare costs of long-run inflation. In addition, a well-identified money demand function is a key part of the IS-LM model.

Until the early 70s, the discussion between the IS–LM model of John Maynard Keynes and John Hicks and the real business cycles (RBC) paradigm of Robert Lucas and Finn Kydland and Edward Prescott seemed to be irreconcilable. RBC models assume full market clearing, whereas a central feature of IS–LM models is either wage or price rigidities. At some point, however, wage and price rigidities are introduced into those dynamic and stochastic models (now known as DSGE models), which narrowed the gap between these views. Some authors constructed DSGE models displaying some features of the IS–LM model such as [Benassy \(2007\)](#) and [Casares and McCallum \(2006\)](#). In this regard, while the IS survived most of the critiques, the LM seems to be a less important feature (the quantity of money supplied in the economy is endogenously determined for central bank’s decision over an interest rate target).

However, some authors argue that ignoring the LM curve could be troublesome. The preferences for money (money demand) motivated a series of studies such as its effects on the business cycles, correct identification of monetary policy shocks, or currency substitution. For example, [Ireland \(2004\)](#) argues that a structural model of the monetary business cycle implies that real money balances enter into a correctly-specified, forward-looking IS curve if and only if they enter into a correctly-specified, forward-looking Phillips curve. Ireland points out that empirical measures of real balances must be adjusted for shifts in money demand to accurately isolate and quantify the dynamic effects of money on output and inflation.

In this paper I follow [Ball \(2001\)](#) and [Mark and Sul \(2003\)](#) approach of long-run money demand as a cointegrating relationship. My final reduced-form specification is based on the LM set-up in which the elasticity of money with respect to income is positive and the semi-elasticity with respect to the interest rate is negative. While the original set up is heavily criticized on grounds of lack of microfoundations, most of the results can be rescued with models of Money in the Utility Function or Cash in Advance type. In this regard, [Walsh \(2010\)](#) argues that the empirical literature on money demand is vast. He presents a money in the utility function model and discusses the parameter values of the money demand function for several cases. My results are supported by those of the previous literature, and the contribution is given for the technique used in the estimation of parameters that better characterizes the money demand function.

I use Fully Modified OLS (FMOLS) proposed by [Pedroni \(1999\)](#). This panel cointegration technique allows researchers to selectively pool information regarding common long-run rela-

tionships from across members of the panel while allowing the associated short-run dynamics and fixed effects to be heterogeneous across different members of the panel. According to Pedroni, it is reasonable to think in members of a panel as sample drawing from a population that is either $I(1)$ or $I(0)$ and each member represents a sampling from its own separate population. For him, there is no theory that indicates if a member of a group that is selected would give a response (with respect to GDP, inflation, etc.) that is informative enough for all remaining members of the group. Even more, this author argues that FMOLS does a better job at estimating heterogeneous long-run relationships than a Dynamic OLS (DOLS).

Combining observations from 15 Latin-American countries I build an unbalanced panel, and employ FMOLS to estimate the coefficients of the long-run money demand function in these countries. I also test if the signs of the coefficients are consistent with the LM curve of the IS-LM model. These countries might have a cointegrating money demand with an income elasticity of 0.94, and an interest-rate semi-elasticity of -0.01.

Most studies focus on individual country cases and to the best of my knowledge there are none for this region. This is an initial effort to reveal the parameters that govern this key relationship in economics.

The remaining of the paper is organized as follows. Sections II and III introduce and discuss a theory on money demand, with special emphases in the IS-LM model. In section IV I introduce nonstationary panels, test for cointegration, and use FMOLS to estimate the money demand for Latin-American countries. Section V concludes.

2 Money demand theory

First developed by John Hicks in 1937 the IS-LM model was an attempt to portray the central ideas of Keynes' General Theory. As [Bordo and Schwartz \(2003\)](#) point out, monetarists dislike the IS-LM framework because it limits monetary influence too narrowly, essentially to the interest elasticity of money demand. The "IS-LM has survived all of its criticisms over the years [...] it is simple, elegant, and easy to manipulate [...] It is [...] the workhorse of open macroeconomics and of the IMF in its evaluation of member countries' economic balance [...] Finally it has now been endowed with the legitimacy of microfoundations based on optimizing behavior by households and firms."¹

In this section I discuss the main money-demand frameworks, in special the case of the LM of the IS-LM model. Referring to this point [Mark and Sul \(2003\)](#) comment that "in the era of dynamic general equilibrium models, Lucas (1988) shows that such a neoclassical

¹[Bordo and Schwartz \(2003\)](#), page 22.

model with cash in advance constraint generates a standard money demand function.”²

2.1 Money market equilibrium in an open economy

In order to study money demand, I consider a standard money market equilibrium model. First, I assume that the supply of money (M^s) is an exogenous policy variable decided by the monetary authority, such that:

$$M^s = \bar{M}^s \quad (1)$$

In a closed economy the return on held money is negative and is given by the inflation rate \hat{P} . The opportunity cost of holding money is what it could have earned elsewhere i.e. invested in other assets which is r . So the total opportunity cost of money is given by this nominal interest rate:

$$r - (-\hat{P}) = r + \hat{P} = i \quad (2)$$

where r is the real interest rate and i is the nominal interest rate.

Thus, the demand for money depends negatively on i and may also include Y (income) as an exogenous variable that determines the long-term demand for money.

In an open economy, assets are of two types domestic and foreign. If the uncovered interest parity condition holds, then this return is the same as at home:

$$i_{t,k} = i_{t,k}^* + \hat{S}_{t,k}^e \quad (3)$$

where $i_{t,k}$ and $i_{t,k}^*$ is the nominal interest rate in domestic and foreign, and $\hat{S}_{t,k}^e$ is the expected devaluation of the exchange rate.

If uncovered interest parity does not hold, then $i_{t,k} \neq i_{t,k}^* + \hat{S}_{t,k}^e$ so I need to consider both i and $i^* + \hat{S}^e$ as a potential opportunity costs. M^d then would depend on both i and $i^* + \hat{S}^e$:

$$M^d = M^d(\bar{i}, \bar{i}^*, \bar{\hat{S}}^e, \bar{S}) \quad (4)$$

and in equilibrium, the interest rate clear the money market i.e. an equilibrium condition that equalizes money supply and money demand.

²Mark and Sul (2003), pages 674 - 675. The authors cited Robert Lucas (1998). “Money Demand in the United States: A Quantitative Review”, Carnegie-Rochester Conference Series on Public Policy, 29, pages 137-168. For a Baumol-Tobin money demand approach, see Alvarez et al. (2003).

$$M^d = M^s \tag{5}$$

2.2 International business cycles and the Mundell - Fleming model: A Keynesian perspective of the money demand

In the short run, if prices adjust slowly (sticky prices), monetary policy can affect output. For business cycles model, the combination of real resources/loanable funds market with the money market can be represented as

$$S - I = NX \tag{6}$$

$$M^d = M^s \tag{7}$$

where S is saving, I is investment, and NX is net exports.

Equations that show equilibrium on the demand side of the economy (IS) are

$$Y = C + I + XN$$

$$Y = C + S$$

$$S - I = XN$$

where C is consumption in the typical IS-LM set-up.

To begin with, let us assume that the uncovered interest rate parity holds. Then M^d depends on the opportunity cost of holding money, which is i . But in order to relate this with the market for real resources, I express it in terms of the demand for real money balances,

$$\frac{M^d}{P} = \frac{M^d}{P}(\bar{r}) \tag{8}$$

which reflects the opportunity cost of holding money as an asset.

Since I am interested in relating the market for real loanable funds, it is important to consider the effect of changes in Y over $\frac{M^d}{P}$ i.e. demand for money because of transaction needs:³

³From Friedman's point of view, the Keynesian distinction between "active balances and idle balances" is irrelevant. "each unit of money renders a variety of services that the household or firm equates at the margin". [Bordo and Schwartz \(2003\)](#), page 7.

$$\frac{M^d}{P} = \frac{M^d}{P}(\bar{r}, \bar{Y}) \quad (9)$$

which means that real money balances depend negatively on interest rates and positively on the amount produced in an economy. Let me use L to denote the demand function for real money, so:

$$\frac{M^d}{P} = L(\bar{Y}, \bar{r}) \quad (10)$$

In an open economy, I consider that the central bank determines the money supply according to the assets that it holds both domestically and abroad. So that:

$$M^s = D + F$$

where D is the domestic component of assets (such as domestic credit, bonds, etc.) and F is the foreign component of assets (such as gold, foreign reserves, etc.)

In real terms: $\frac{M^s}{P} = \frac{D+F}{P}$ and in equilibrium:

$$\left(\frac{M^s}{P}\right) = L(Y, r) \quad (11)$$

Combining money market with the demand side (or loanable funds), I get an open economy IS-LM model (also known as the Mundell - Fleming model), where:

- The IS curve depicts the different combinations of r and Y that are consistent with $S - I = NX$ i.e. equilibrium in the loanable funds market.
- The LM curve depicts the different combinations of r and Y that are consistent with $\left(\frac{M^s}{P}\right) = L(Y, r)$ i.e. equilibrium in the money market.

3 Remarks on the IS-LM model

3.1 Recent developments on the IS-LM model

About recent developments, [Bordo and Schwartz \(2003\)](#) mention that although the IS-LM is used to evaluate and conduct monetary policy, it does not actually have money in it. The model has three equations: an IS equation where the output gap depends on the real interest rate (the nominal rate minus rationally expected inflation); a Phillip curve, which relates the inflation rate to the output gap and to both past inflation and rationally expected future inflation; a policy rule (commonly known as the Taylor rule) that relates the short-term

interest rate (central bank's policy instrument) to output.⁴

Friedman (2003) points out the work of Clarida et al. (1999) as the first and the standard new view of monetary policy, in line with the models described by Bordo and Schwartz (2003). He argues that the IS curve has survived, but the LM is gone due to changes in policymaking practice: “no central banker feels the need to be apologetic about believing the monetary policy does affect real outcomes”⁵ because monetary policy could not affect real outcomes because changes in expectations would undo the behavior that such models imply.

On the other hand, Leeper and Roush (2003) argue in favor of the role of money in monetary policy analysis. They find evidence of an essential role for money in the transmission of monetary policy. Both the money stock and the interest rate are needed to identify monetary policy effects. For a given exogenous change in the nominal interest rate, the estimated impact of monetary policy in economic activity increases monotonically with the response of the money supply; and, the path of the real interest rate is not sufficient for determining policy impacts.⁶

3.2 Caveats and other remarks

In order to be consistent with the IS-LM model, knowing the effects of income over money demand facilitates the determination of the rate of monetary expansion that is consistent with the long-run price level stability. Moreover, due to the effects of interest rate in future consumption, knowing the interest rate effects over money demand eases the calculation of the welfare costs of long-run inflation.

Since the aim of this study is the money demand (LM curve), my main interest is to show how are the effects of an expansion of money supply through the money demand characterization. At least in the short run I would have an idea of how strong is the effect over output due to an expansion of money supply. However, in order to be accurate I should identify the IS curve, too.

Another important point about the IS-LM model is that in its evolution there are a number of Friedman's critics incorporated in the model for example “inflation is always and anywhere a monetary phenomenon and can be controlled by monetary policy; that mone-

⁴“Although the model does not have an LM curve in it, one can add it on to identify the amount of money that the central bank will need to supply when it follows the policy rule, given the shocks that hit the economy. However this fourth equation is not essential for the model.” Bordo and Schwartz (2003), page 23.

⁵Friedman (2003), page 8.

⁶The authors basically estimate two models: with and without money, and analyze the outcome under a VAR approach, and find evidence that “money stock and the short term nominal interest rate jointly transmit monetary policy in the United States.” Leeper and Roush (2003), page 20.

tary policy in the short-run has important real effects because of the presence of nominal rigidities or lags in the adjustment of expected to actual inflation [...] and that policy rules are important anchors to stable monetary policy.”⁷

4 Empirical analysis

I follow [Ball \(2001\)](#) and [Mark and Sul \(2003\)](#) who approach the long-run money demand as a cointegrating relationship.⁸ However, it is important to mention that [Ball \(2001\)](#) uses time series analysis for the money demand in the U.S., [Mark and Sul \(2003\)](#) apply a panel DOLS (or pooled within-dimensions) to estimate the money demand of 19 OECD countries. My case is group mean FMOLS (or group mean between-dimensions) to estimate the money demand in 15 Latin American countries.⁹

The data I use in this paper comes from the International Financial Statistics (IFS) and covers the sample period 1948 - 2003. Money refers to M1 definition and interest rate is a short-term interest rate.

In this section I: (i) introduce nonstationary panels, (ii) test for panel unit roots, (iii) test for cointegration, (iv) discuss FMOLS, and (v) present my estimated results. I also present an exercise where the parameters of money demand can be compared between countries.

4.1 Nonstationary panels

A nonstationary panel is a time series panel with unit root components which is typical of aggregate macro panels. Some of the characteristics of a nonstationary panels are:

1. substantial time series dimension (with serial correlation),
2. substantial cross sectional dimension (with heterogeneity across members),
3. single or multiple variables, and,
4. unit root present in at least some variables.

⁷[Bordo and Schwartz \(2003\)](#), pages 23-24.

⁸[Mark and Sul \(2003\)](#) also mention the following authors whose research are in the same line: [Stock and Watson \(1993\)](#). “A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems”, *Econometrica* 61:783-820; and, [Hoffman \(1995\)](#) “The Stability of Long-Run Money Demand in Five Industrial Countries”, *Journal of Monetary Economics*, 35, pages 317-339. Regarding studies of money demand for developing countries, see [Kumar \(2011\)](#).

⁹See [Walsh \(2010\)](#), chapter 2, pages 49-52, for a review of the empirical literature on money demand.

Nonstationary panel techniques are more useful when time series dimension is relatively large (too short for reliable inference for any member alone, but long enough to treat dynamics flexibly); cross sectional dimension, or number of members, moderate (too large to be treated as a pure system); and at least some commonalities exist among the members (among parameters or hypothesis investigated).

If the time dimension is too short, it is difficult to model the underlying dynamics of the series. Typically, serial correlation properties differ across members of the panel, so sufficient time series length for each member is required to account for member specific dynamics.

[Pedroni \(2002\)](#) highlight the fact that panel methods minimally require at least some commonality across members (gain combining data from different members) so that improves upon reporting results for each member separately. The minimal types of commonalities required are either:

1. Properties of the data (parameters or moments). The data do not have any shared values across members, but often must bound differences in a probabilistic sense (for example require that the probability that they are too far apart is bounded), or,
2. Properties of the hypothesis. The hypothesis does not constrain all members to give the same answer to a hypothesis test, but different members of a panel should answer the same question, for example the answer for one member must have some bearing on a likely answer for another. Otherwise, the combination of data would not lead to better answer to the research question.

In my case I work an unbalanced nonstationary panel for the period 1948-2003, for 15 countries. About commonalities I restrain my analysis to the characteristics and parameters of the long-run money demand under a Keynesian approach for a group of countries. This group of countries are from Latin America, a particular region. In this paper, I assume that the financial systems and transactions technologies across Latin-American countries are essentially similar.

4.2 Testing for panel unit roots with heterogeneous dynamics

Panel approach involves some simplifications in order to achieve a reduction in the number of parameters to estimate. This is why exploiting commonalities must lie at the foundation of any panel time series approach.

From panel unit roots, if it is reasonable to imagine members of panel as a sample drawing of a population that is either $I(1)$ or $I(0)$ then may substantially increase power by using panel dimension which substitutes observations over $i = 1, 2, 3, \dots, N$ dimension to make

up for short T dimension when members are independent over i .

In my case, I test if Keynesian monetary approach to money demand applies equally to all members of a panel. If it is not correct, it should fail regardless of which member is considered. This theory indicates property of data generating process for the population, while individual members of the panel are treated as different realizations from this population. Inability to see this on an individual basis for all members is due to low power in finite samples of the test, so as increase number of members, hence sample realizations of $\ln\left(\frac{M_{it}}{P_{it}}\right)$, improve ability to conduct inference. Under this overview, panel test represents a direct and dramatic increase in power over individual tests.

The reduce-form equation that I estimate is:

$$\ln\left(\frac{M_{it}}{P_{it}}\right) = \alpha_i + \beta_y \ln Y_{it} + \beta_r R_{it} + u_{it} \quad (12)$$

where M_{it} is a money measure, P_{it} is a price level, Y_{it} is the real GDP, R_{it} is a short term interest rate, α_i refers to specific effects in a country, β_y is the income elasticity, and β_r is the interest rate semi-elasticity; for $i = 1, 2, \dots N$; $t = 1, 2, \dots T$; where $N = 15$ and $T = 56$.

First, I identify whether $\Delta \ln\left(\frac{M_{it}}{P_{it}}\right)$ is either $I(0)$ or $I(1)$. Then, I can infer if $\Delta \ln\left(\frac{M_{it}}{P_{it}}\right)$ has a short run serial correlation and if $\ln\left(\frac{M_{it}}{P_{it}}\right)$ may have a long run cointegration relations for all countries.

By applying unit root tests to the panel, I estimate the following relationship:

$$\Delta \ln\left(\frac{M}{P}\right)_t^* = c + \prod \ln\left(\frac{M}{P}\right)_{t-1}^* + \sum_{k=1}^k \Phi_k \Delta\left(\frac{M}{P}\right)_{t-k}^* + \varepsilon_t^* \quad (13)$$

In this case, I have stacked panel into time-series vector $\ln\left(\frac{M}{P}\right)_t^*$.

Panel unit root tests that allow heterogeneous dynamics can be classified in:¹⁰

- Pooled within dimension tests developed by [Levin et al. \(2002\)](#). They study three different tests using sequential limits:
 1. Pooled Phillips-Perron p -statistic.
 2. Pooled Phillips-Perron t -statistic.
 3. Pooled ADF t -statistic.

¹⁰For more details, see [Harris and Solis \(2003\)](#). Chapter 7, Panel Data Models and Cointegration.

These tests are distributed standard normal by sequential limit.

- Group mean test developed by [Im et al. \(2003\)](#). This test has a normal standard distribution due to the central limit theorem.

[Levin et al. \(2002\)](#) point out that their proposed panel base unit root test does have the following limitation: there are some cases in which contemporaneous correlation cannot be removed by simply subtracting the cross sectional averages (results depends crucially upon the independence assumption across individuals, and hence not applicable if cross sectional correlation is present). Also, the assumption that all individuals are identical with respect to the presence or absence of a unit root is somewhat restrictive. On the other hand, [Im et al. \(2003\)](#) worked a panel unit root test without the assumption of identical first order correlation but under a different alternative hypothesis.¹¹

¹¹“Maddala and Wu (1999) have done various simulations to compare the performance of competing tests, including IPS (Im, Pesaran, and Shin) test, LL (Levin, and Lin) test [...] Care must be taken to interpret their results. Strictly speaking, comparisons between the IPS test and LL test are not valid. Though both tests have the same null hypothesis, but the alternatives are quite different. The alternative hypothesis in this article is that all individual series are stationary with identical first order autoregressive coefficient, while the individual first order autoregressive in IPS test are allowed to vary under the alternative. If the stationary alternative with identical AR coefficients across individuals is appropriate, pooling would be more advantageous than [Im et al. \(2003\)](#) average t -statistics without pooling. Also note that the power simulations reported in Maddala and Wu (1999) are not size-corrected.” [Levin et al. \(2002\)](#), pages 15-17.

Table 1: Tests for unit roots

	$\ln \frac{M}{P}$	$\ln Y$	R
In levels			
LL ρ -statistic	0.89	1.58	-1.93
LL t -statistic	2.00	1.91	-0.24
LL ADF-statistic	2.54	0.92	-0.17
IPS ADF-statistic 1/	2.90	2.40	0.03
First differences			
LL ρ -statistic	-41.25	-42.83	-40.81
LL t -statistic	-27.21	-27.85	-18.27
LL ADF-statistic	-26.33	-49.26	-11.80
IPS ADF-statistic 1/	-48.24	-45.83	-10.73

Note: Unbalanced panel data, sample of 15 countries. LL denotes Levin and Lin, and IPS denotes Im, Pesaran, and Shin.

1/ Using large sample adjustment.

Table 1 reports the unit root test under the $H_0 : \text{Unit root}$. In all cases I fail to reject H_0 .¹²

The results from the unit root tests show that the three variables are first difference stationary.

4.3 Testing for cointegration and heterogeneity restrictions

To estimate (12), first, I test for cointegration. Then, I use the cointegrating relationship and obtain the residuals. Later on, I test those residuals for unit roots. In other words, I identify if there are any cointegrating relationships which is or might be consistent with (12).

The panel cointegrating tests that allow heterogeneity restrictions can be classified in:

¹²If the statistic is less than the critical value, -1.28, I can reject H_0 at 10 percent of confidence.

- Pooled within dimension tests developed by Pedroni (1999).¹³ He researched four different tests running individual cointegrating regression for each member, collect estimated residuals and compute pooled panel root test:¹⁴
 1. Pooled semi-parametric variance test.
 2. Pooled semi-parametric p -statistic test.
 3. Pooled semi-parametric t -statistic test.
 4. Pooled fully parametric ADF t -statistic test.
- Group mean test developed by Pedroni (2002). He researched three different tests running individual cointegrating relationships for each member, collect estimated residuals, and compute group mean unit root test:
 1. Group mean semi-parametric p -statistic test.
 2. Group mean semi-parametric t -statistic test.
 3. Group mean fully parametric ADF t -statistic test.

In each case the H_0 : *No cointegration* can be rejected if the statistic is less than the critical value.¹⁵ If it is less than -1.28 I can reject H_0 at 10 percent of confidence.

It is important to mention that the key difference between both Pooled and Group tests is that the residuals test is grouped rather than pooled. Group mean tests are preferred over the Pooled tests since they allow greater flexibility under alternative hypotheses.¹⁶

¹³“tests for the null of no cointegration in heterogeneous panels based on Pedroni (1995, 1997) have been limited to simple bivariate examples, in large part due to the lack of critical values available for more complex multivariate regressions. The purpose of this paper is to fill this gap by describing a method to implement tests for the null of no cointegration for the case with multiple regressors and to provide appropriate critical values for these cases”. Pedroni (1999), page 653. The author cited: Pedroni (1995) “Panel Cointegration: Asymptotic and Finite Sample Properties of Pooled Time Series Tests, with an Application to the PPP Hypothesis”, Indiana University Working Papers in Economics, June; and, Pedroni (1997) “Panel Cointegration; Asymptotic and Finite Sample Properties of Pooled Time Series Tests, with and Application to the PPP Hypothesis: New Results”, Indiana University Working Papers in Economics, April.

¹⁴These tests allow heterogeneous dynamics, heterogeneous cointegrating vectors, endogeneity, and normal distributed standard errors.

¹⁵“one can think of such panel cointegration test as being one in which the null hypothesis is taken to be that for each member of the panel variables of interest are not cointegrated and the alternative hypothesis is taken to be that for each member of the panel there exists a single cointegration vector, although this cointegration vector need not be the same for each member. Indeed, an important feature of these tests is that they allow the cointegration vector to differ across members under the alternative hypothesis.” Pedroni (1999), page 655.

¹⁶For details about how to build the statistics for each test, see the Technical Appendix.

Table 2: Test for Cointegration of $\ln \frac{M}{P}$, $\ln Y$, and R
(Panel Cointegration Statistics)

Pooled within - dimension tests		Group mean based tests	
v -statistic	0.7713	ρ -statistic	-0.3088
ρ -statistic	-1.2108	t -statistic (nonparametric)	-3.4881
t -statistic (nonparametric)	-1.7032	t -statistic (parametric)	2.4981
t -statistic (parametric)	0.3501		

Note: Unbalanced panel data, sample of 15 countries.

Table 2 reports the test for cointegration. I reject the H_0 of no cointegration among these series in most cases. These results suggest that there is a cointegrating relationship among these series.

4.4 Estimating a Group Mean FMOLS

If the IS-LM model is consistent with the data, and the equilibrium in the money market holds, i.e. money demand equals money supply, I can expect (10) to hold in the long run; while in equilibrium, (11) should hold.

The result should hold true regardless of any details of dollar substitution, technological change, demand side preferences, or any other characteristic, under a pure Keynesian framework. If an alternative model is correct, the expected sign of this relationship will not hold in the long run.

In my case-study, the data from any particular country is too short to reliably choose null or alternative, but data from many combined countries are sufficient to decide whether the prediction of the theory is accurate or not.

It is reasonable to picture members of a panel as a sample drawn from a population that is either $I(1)$ or $I(0)$ and each member represents a sampling from its own separate population (time series realizations from any member would represent the population). In this case, the theory does not indicate that members must all give the same answer (panel test maybe “mixed” regarding whether null or alternative is correct). Specifically, for (12) the hypothesis to test is:

$$H_0 : \beta_y = 0 \text{ and } \beta_r = 0 \text{ for all } i$$

$H_1 : \beta_y > 0$ and $\beta_r < 0$ for enough i

where “enough” in the literature is often not precisely defined.

The advantage of the panel approach is that it has broadened the class of data to which the test has been applied by exploiting commonalities. The use of the panel is not to ask if a theory is correct or not, rather, it is asking how “pervasive” this particular characterization is for the particular group of members.

4.4.1 Advantages of between-dimension group mean panel (FMOLS) over within-dimension panel (DOLS):

Pedroni (2001) discusses three important advantages of the between-dimension estimators over within-dimension estimators:¹⁷

- The form in which the data is pooled in the between-dimension estimators allow for greater flexibility in the presence of heterogeneity of the cointegrating vectors. Test statistics constructed from the within-dimension estimators are designed to test $H_0 : \beta_i = \beta_0$ for all i against $H_A : \beta_i = \beta_A \neq \beta_0$ where the value β_A is the same for all i . Test statistics constructed from the between-dimension estimators are designed to test $H_0 : \beta_i = \beta_0$ for all i against $H_A : \beta_i \neq \beta_0$ so that the values for β_i are not constrained to be the same under the alternative hypothesis. This is an important advantage for applications, because there is no reason to believe that, if the cointegrating slopes are not equal, they necessarily take on some other arbitrary common value.
- The point estimates of the between-dimension estimators have a more useful interpretation in the event that the true cointegrating vectors are heterogeneous. Specifically, point estimates for the between-dimension estimator can be interpreted as the mean value for the cointegrating vectors. This is not true for the within-dimension estimators.
- The test statistics constructed from the group mean estimators appear to have another advantage even under the null hypothesis when the cointegrating vector is homogeneous. Specifically, Pedroni (2002) shows that they appear to suffer from much lower small-sample size distortions than the within-dimension estimators.

4.4.2 Estimation:

The equation that I estimate for a money demand given by (12) is estimated by group-mean FMOLS because it has much better small sample size properties than pure time series case,

¹⁷ “[...] DOLS estimators are within-dimension estimators”. Pedroni (2001), page 728.

and has clear advantages over panel DOLS. Even in cases where those estimations are difficult for pure time series case, it does fairly well in panels with heterogeneous dynamics due to the fact that biases tend to average out over N dimension, and in addition it has usual advantage of group-mean test, where the alternative hypothesis is more flexible.¹⁸

In my case my working hypothesis using [Pedroni \(2002\)](#) in group mean tests (estimate average long run cointegrating relationship) are:¹⁹

$$H_0 : \beta_y = 0 \text{ versus } H_A : \beta_y \neq 0$$

$$H_0 : \beta_r = 0 \text{ versus } H_A : \beta_r \neq 0$$

The flexibility in this case is important since I do not have any prior value over the specific alternative hypothesis.²⁰ It has the usual advantage of group mean tests, in that estimates have more useful economic interpretation when cointegrating vectors are heterogeneous.

The test can be interpreted as the average of individual Fully Modified estimators, each individual FMOLS estimator corrects for endogeneity and for serial correlation by estimating long run covariance directly, and the average over individual Fully Modified to obtain a group mean.

Thus the group mean FMOLS estimators take the form:

¹⁸About the specification, [Ball \(2001\)](#) discuss if this is the correct money-demand function and the implicit assumption that the function does not include a time trend. He argues that “For given output and interest rates, money demand can change over time if there are changes in the economy’s transaction technology”. [Ball \(2001\)](#), page 42. However, since money measure and output have trends, it is possible to think that they cancel each other. This fact can be viewed in the estimations of [Mark and Sul \(2003\)](#) where they find small differences estimating DOLS with and without trends. Ball also mentions “a trend is highly collinear with income, so one cannot disentangle their effects”. [Ball \(2001\)](#), page 42. Another approach, like cash in advance models, is developed in [Alvarez et al. \(2003\)](#) in which the framework is the Baumol-Tobin model. Finally, I review the work of [Calza et al. \(2001\)](#) which includes in the relationship the long term interest rate in their estimation of the long-run money demand of the Euro area, obtaining the wrong sign for this variable, and deleting it.

¹⁹[Mark and Sul \(2003\)](#) estimate pooled within tests (estimate average long-run regression) $H_0 : \beta_Y = 0$ versus $H_A : \beta_Y = 1.0 \neq 0$ and $H_0 : \beta_r = 0$ versus $H_A : \beta_r = -0.05 \neq 0$

²⁰However [Mark and Sul \(2003\)](#) point out that “1.0 is a typical value of the income elasticity estimated in the literature while a common estimate of the interest rate semi-elasticity is -0.05”. [Mark and Sul \(2003\)](#), page 15. [Ball \(2001\)](#) estimate that the income elasticity money demand is 0.5 and the interest rate semi-elasticity is -0.05 for the U.S. economy.

$$\hat{\beta}_{Y,GFM}^* = N^{-1} \sum_{i=1}^N \left(\sum_{t=1}^T (\ln Y_{it} - \overline{\ln Y}_i)^2 \right)^{-1} \left(\sum_{t=1}^T (\ln Y_{it} - \overline{\ln Y}_i) (y_{Y,it}^* - T \hat{\gamma}_i) \right)$$

$$\hat{\beta}_{r,GFM}^* = N^{-1} \sum_{i=1}^N \left(\sum_{t=1}^T (r_{it} - \bar{r}_i)^2 \right)^{-1} \left(\sum_{t=1}^T (r_{it} - \bar{r}_i) (y_{r,it}^* - T \hat{\gamma}_i) \right)$$

where $[y_{Y,it}^*] = \left[\ln \left(\frac{M}{P} \right)_{it} - \overline{\ln \left(\frac{M}{P} \right)}_i \right] - \frac{\hat{\Omega}_{21i}}{\hat{\Omega}_{22i}} [\Delta \ln Y_{it}]$ is the endogeneity correction and uses $\ln Y_{it}$ as an “internal instrument.”²¹ Also $\hat{\gamma}_i \equiv \hat{\Gamma}_{21i} + \hat{\Omega}_{21i}^0 - \frac{\hat{\Omega}_{21i}}{\hat{\Omega}_{22i}} (\hat{\Gamma}_{22i} + \hat{\Omega}_{22i}^0)$ is the serial correlation correction, and Ω_i is the long run covariance.²²

Equivalently, the estimator can be expressed as: $\hat{\beta}_{Y,GFM}^* = N^{-1} \sum_{i=1}^N \hat{\beta}_{YFM,i}^*$ and likewise for the t -statistic: $t_{\hat{\beta}_{GFM}^*} = N^{-1/2} \sum_{i=1}^N t_{\hat{\beta}_{FM,i}^*}$, so that, individuals FMOLS tests are distributed $N(0, 1)$ as $T \rightarrow \infty$. Likewise, group FMOLS tests are distributed $N(0, 1)$ as $T \rightarrow \infty$ and $N \rightarrow \infty$ sequentially.

4.5 Results

In Table 3, I present the FMOLS estimations. Single equation FMOLS estimates are seen to display a cross-sectional variability. In FMOLS regressions, the income elasticities are positive, with the exception of Argentina and Uruguay,²³ ranging from 0.44 (Paraguay) to a whopping 3.27 (Brazil), but the interest semi-elasticity has the wrong sign for Brazil²⁴, and for the other countries it is ranging from -0.022 (Guatemala) to -0.001 (Peru, Bolivia, and Chile).

²¹It is similar for the case of $y_{r,it}^*$.

²²For details about the estimation of Ω and Γ see the Technical Appendix.

²³In both cases, they are statistically no significant.

²⁴This statistic is also not statistically significant.

Table 3: Long Run Money Demand
(Single-equation and Panel Fully Modified OLS estimates)

Country	β_y	t-stat	β_r	t-stat
Argentina	-1.00	-1.32	-0.002	-0.44
Bolivia	0.90	12.49	-0.001	-5.11
Brazil	3.27	11.58	0.002	1.47
Chile	1.07	16.84	-0.001	-0.39
Colombia	0.87	16.48	-0.004	-1.46
Costa Rica	1.09	12.18	-0.015	-2.50
Dominican Republic	0.75	12.86	-0.018	-4.27
Ecuador	1.06	48.26	-0.011	-4.41
Guatemala	1.66	17.49	-0.022	-5.59
Honduras	1.34	8.41	-0.008	-1.57
Mexico	0.76	4.93	-0.009	-5.71
Paraguay	0.44	1.50	-0.009	-1.74
Peru	1.05	7.94	-0.001	-0.71
Uruguay	-0.54	-1.68	-0.002	-1.65
Venezuela	1.33	26.46	-0.019	-10.19
Panel Group FMOLS	0.94	50.20	-0.008	-11.43

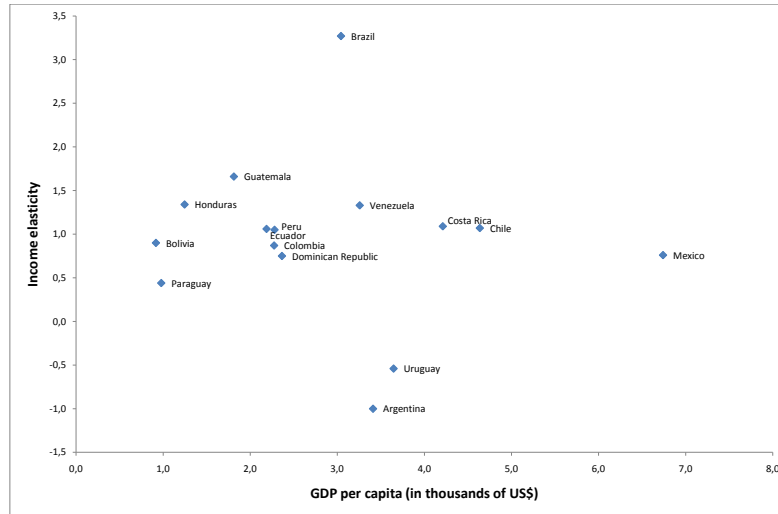
If I maintain an underlying belief that the financial systems and transactions technologies across Latin-American developing countries are essentially similar, the cross-sectional variability in these estimates must reflect the inherent difficulty of obtaining accurate estimates rather than evidence of disparate economic behavior.

Panel FMOLS estimate for income elasticity is 0.94 (with t -statistic equal to 50.2) and for the interest semi-elasticity is -0.008 (with t -statistic equal to -11.4).²⁵ There is evidence of a cointegrating money demand among Latin-American countries. This fact is important since there is the idea of a unique currency in this region. However, this is only a small piece of information that is required if a monetary union is intended (as it is the case of The Economic and Monetary European Union, EMU).²⁶

²⁵Mark and Sul (2003) use a panel dynamic OLS (DOLS) with trend among 19 developed economies (similar income levels and financial market developed) “In our analysis, single equations DOLS with trend gives such disparate income elasticity estimates as -1.23 for New Zealand and 2.42 for Canada. The corresponding interest rate semi-elasticity estimates range from 0.02 for Ireland (which has the wrong sign) to -0.09 for the UK”. Mark and Sul (2003), page 658. “The estimates in which we have the most confidence are an income elasticity near 1 and an interest rate semi-elasticity of -0.02”. Mark and Sul (2003), page 679.

²⁶The European Central Bank has a monetary aggregate operating target, which departs from other

Figure 1: Income elasticity of the money demand



As a group, the income elasticity is below 1 which implies the existence of economies of scale in money management. Across countries, Mexico, Colombia, Paraguay, Bolivia, and Dominican Republic have income elasticity below 1. This result is consistent with countries in which the process of dollarization is decreasing and the main function of the local currency (transactions) is recovering in a slow manner. In addition, there is a tendency to keep dollars for precautionary matters. (See Figure 1).

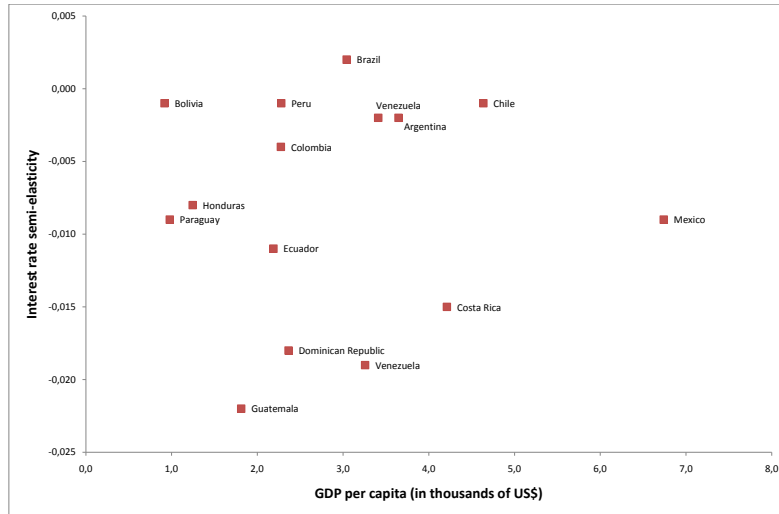
The semi-elasticity of the money demand with respect to the interest rate is low and has a low variability across countries. The reaction in the money demand to changes in the interest rate may be relatively similar within countries. Under the idea of a monetary union of Latin-American countries, target interest rate could work as it is the case in many developed countries. (See Figure 2).

As a matter of fact, [Walsh \(2010\)](#) discusses previous values found in the literature for this parameter.²⁷ My results are in line with those previously found.

inflation targeting central banks. Part of the argument is based on a stable money demand, as showed in [Funke \(2001\)](#). See [Poole \(1970\)](#) for more details.

²⁷See [Walsh \(2010\)](#), pages 50-51.

Figure 2: Interest rate semi-elasticity of the money demand



4.6 An increase in the money supply

As an exercise, I estimate the effect of the expansion of the money supply over output, based on (11). In the short run (when the monetary policy has effects over output) prices are sticky, so taking differentials,

$$\Delta M = L_Y \Delta Y + L_r \Delta r \quad (14)$$

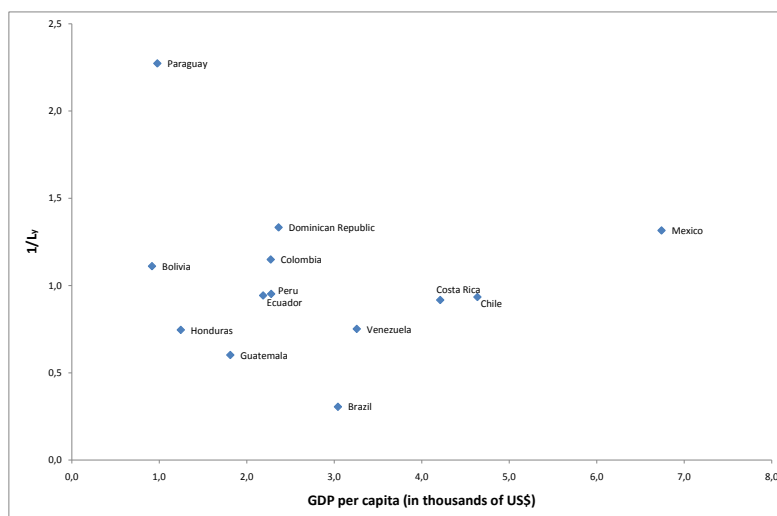
where L_Y can be approached for the elasticity of money with respect to output and L_r is the semi-elasticity of money with respect to the interest rate. Re-arranging (14), I find the following expression,

$$\frac{\Delta Y}{\Delta M} = \frac{1}{L_Y} - \frac{L_r}{L_Y} \frac{\Delta r}{\Delta M} \quad (15)$$

Since I do not know the total effect over r (I need to estimate the IS curve), I can only infer about the direct/fix effect of an increase in 1% in the money supply over output (see Figure 3) and the indirect/partial effect derived from movements in the interest rate, given the increase in money supply over output (see Figure 4).

I find that Paraguay, a country with low GDP per capita, has better opportunities to increase output with a monetary expansion. Colombia and Bolivia are also interesting cases,

Figure 3: Fix effect of a 1% increase in money supply over output - Direct effect of money



since a 1% increase in money supply increases output in more than 1%.

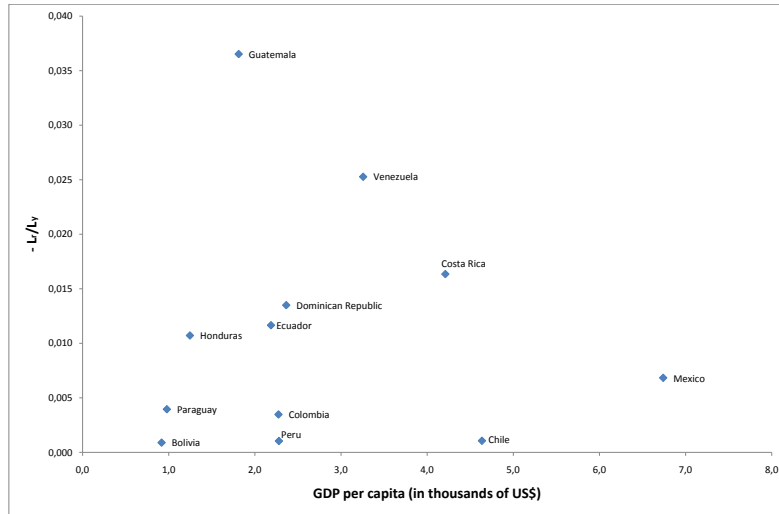
This analysis is not complete yet. In order to estimate the total change in output I also need to estimate the IS-curve. On the other hand, I have the ratio that measures the effects of the decrease in the interest rate, given the 1% expansion in the money supply, implied by the LM-curve. Even though it is the interaction between the IS and LM that would provide the final interest rate level, this is a reasonable approximation of a money growth increase effect over output. If the IS-curve is too steep, countries like Guatemala and Venezuela would have less expansionary effects.

5 Conclusions

Combining observations across countries helps obtain relatively sharp and stable estimates of money demand elasticities. In that regard, the panel cointegration approach seems a well suited technique. I applied the panel FMOLS method to estimate the long-run money demand function for 15 Latin-American countries. The estimates for this group of countries are an income elasticity of 0.94 and an interest rate semi-elasticity of -0.01.

These results are consistent with the LM approach, which expects positive values for the income elasticity money demand (for transactions) and negative values for the interest rate

Figure 4: Partial effect of a 1% increase in money supply over output - Indirect effect of interest rate



semi-elasticity money demand (for speculation/precaution). Even though some countries have the wrong sign in their estimators, those cases seem to be statistically not significant.

Regarding the results, the value slightly under 1 implies the existence of economies of scale in money management. This result is consistent with the slow process of “de-dollarization” after the successful experiences of many central banks in Latin America by controlling the high inflation processes that were a problem in the late 80s.

Another interesting result is the low variability in the money demand to changes in the interest rate across countries. Moreover, the low level of the parameter is consistent with the low opportunity cost of holding money.

Finally, I make a partial-equilibrium exercise where I measure the effects of a money-supply increase in 1% over output. In most of the cases, the direct effect is less than a 1% increase in output.

Some points remain in the agenda. Results from the exercise are not conclusive. To have accurate conclusions I should estimate the IS curve. As suggested by Kumar (2011), I should also account for structural breaks that capture the effects of financial reforms that affected the money demand.

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A Technical Appendix

A.1 Seven panel cointegration statistics

1. Panel v -statistic:

$$T^2 N^{3/2} Z_{\hat{v}_{N,T}} \equiv T^2 N^{3/2} \left(\sum_{i=1}^N \sum_{t=1}^T \hat{L}_{11i}^{-2} \hat{e}_{i,t-1}^2 \right)^{-1} \quad (16)$$

2. Panel ρ -Statistic:

$$T\sqrt{N} Z_{\hat{\rho}_{N,T-1}} \equiv T\sqrt{N} \left(\sum_{i=1}^N \sum_{t=1}^T \hat{L}_{11i}^{-2} \hat{e}_{i,t-1}^2 \right)^{-1} \sum_{i=1}^N \sum_{t=1}^T \hat{L}_{11i}^{-2} \left(\hat{e}_{i,t-1} \Delta \hat{e}_{i,t} - \hat{\lambda}_i \right) \quad (17)$$

3. Panel t -statistic:(nonparametric)

$$Z_{t_{N,T}} \equiv \left(\hat{\sigma}_{N,T}^2 \sum_{i=1}^N \sum_{t=1}^T \hat{L}_{11i}^{-2} \hat{e}_{i,t-1}^2 \right)^{-1/2} \sum_{i=1}^N \sum_{t=1}^T \hat{L}_{11i}^{-2} \left(\hat{e}_{i,t-1} \Delta \hat{e}_{i,t} - \hat{\lambda}_i \right) \quad (18)$$

4. Panel t -Statistic: (parametric)

$$Z_{tN,T}^* \equiv \left(\tilde{s}_{N,T}^{*2} \sum_{i=1}^N \sum_{t=1}^T \hat{L}_{11i}^{-2} \hat{e}_{i,t-1}^2 \right)^{-1/2} \sum_{i=1}^N \sum_{t=1}^T \hat{L}_{11i}^{-2} (\hat{e}_{i,t-1}^* \Delta \hat{e}_{i,t}^*) \quad (19)$$

5. Group ρ -Statistic:

$$TN^{1/2} \tilde{Z}_{\hat{\rho}_{N,T-1}} \equiv TN^{1/2} \sum_{i=1}^N \left(\sum_{t=1}^T \hat{e}_{i,t-1}^2 \right)^{-1} \sum_{t=1}^T (\hat{e}_{i,t-1} \Delta \hat{e}_{i,t} - \hat{\lambda}_i) \quad (20)$$

6. Group t -Statistic: (nonparametric)

$$N^{1/2} \tilde{Z}_{tN,T} \equiv N^{1/2} \sum_{i=1}^N \left(\tilde{\sigma}_i^2 \sum_{t=1}^T \hat{e}_{i,t-1}^2 \right)^{-1/2} \sum_{t=1}^T (\hat{e}_{i,t-1} \Delta \hat{e}_{i,t} - \hat{\lambda}_i) \quad (21)$$

7. Group t -Statistic: (parametric)

$$N^{-1/2} Z_{tN,T}^* \equiv N^{-1/2} \sum_{i=1}^N \left(\sum_{t=1}^T \hat{s}_i^{*2} \hat{e}_{i,t-1}^{*2} \right)^{-1/2} \sum_{t=1}^T \hat{e}_{i,t-1}^* \Delta \hat{e}_{i,t}^* \quad (22)$$

where:

$$\begin{aligned} \hat{\lambda}_i &= \frac{1}{T} \sum_{s=1}^{k_i} \left(1 - \frac{s}{k_i + 1} \right) \sum_{t=s+1}^T \hat{\mu}_{i,t} \hat{\mu}_{i,t-s}, \hat{s}_i^2 \equiv \frac{1}{T} \sum_{t=1}^T \hat{\mu}_{i,t}^2, \hat{\sigma}_i^2 = \hat{s}_i^2 + 2\hat{\lambda}_i, \\ \hat{\sigma}_{N,T}^2 &\equiv \frac{1}{N} \sum_{i=1}^N \hat{L}_{11i}^{-2} \hat{\sigma}_i^2 \\ \hat{s}_i^{*2} &\equiv \frac{1}{t} \sum_{t=1}^T \hat{\mu}_{i,t}^{*2}, \tilde{s}_{N,T}^{*2} \equiv \frac{1}{N} \sum_{t=1}^N \hat{s}_i^{*2}, \hat{L}_{11i}^2 = \frac{1}{T} \sum_{t=1}^T \hat{\eta}_{i,t}^2 + \frac{2}{T} \sum_{s=1}^{k_i} \left(1 - \frac{s}{k_i + 1} \right) \sum_{t=s+1}^T \hat{\eta}_{i,t} \hat{\eta}_{i,t-s} \end{aligned}$$

and where the residuals $\hat{\mu}_{i,t}$, $\hat{\mu}_{i,t}^*$ and $\hat{\eta}_{i,t}$ are obtained from the following regressions:

$$\begin{aligned}
\hat{e}_{i,t} &= \hat{\gamma}_i \hat{e}_{i,t-1} + \hat{\mu}_{i,t} \\
\hat{e}_{i,t} &= \hat{\gamma}_i \hat{e}_{i,t-1} + \sum_{k=1}^{K_i} \hat{\gamma}_{i,k} \Delta \hat{e}_{i,t-k} + \hat{\mu}_{i,t}^* \\
\Delta y_{i,t} &= \sum_{m=1}^M \hat{b}_{mi} \Delta x_{mi,t} + \hat{\eta}_{i,t}
\end{aligned}$$

See [Pedroni \(1999\)](#), pages 660-661, for more details.

A.2 Long-run covarianze

Taken from [Pedroni \(2001\)](#), page 728.

Let $\varepsilon_{it} = (\hat{u}_{it}, \Delta \ln Y_{it})'$ be a stationary vector consisting of the estimating residuals from the cointegrating regression and the differences in $\ln Y$.

Let $\Omega_i \equiv \lim_{T \rightarrow \infty} E \left[T^{-1} \left(\sum_{t=1}^T \varepsilon_{it} \right) \left(\sum_{t=1}^T \varepsilon'_{it} \right) \right]$ be the long-run covariance for this vector process. It can be decomposed as $\Omega_i = \Omega_i^0 + \Gamma_i + \Gamma_i'$ where Ω_i^0 is the contemporaneous covariance and Γ_i is the weighted sum of auto-covariances.

This procedure is the same for the case of r .

For more details, see [Pedroni \(2002\)](#).