BANCO CENTRAL DE RESERVA DEL PERÚ

Dedollarization and financial robustness

Rocio Gondo* Fabrizio Orrego**

* University of Maryland
** Central Bank of Peru

DT. N° 2011-022
Serie de Documentos de Trabajo
Working Paper series
Diciembre 2011

Dedollarization and financial robustness*

Rocio Gondo †
University of Maryland

Fabrizio Orrego ‡
Central Bank of Peru

December, 2011

Abstract

This paper evaluates the qualitative and quantitative implications of financial dedollarization of firms' liabilities on real aggregates in a small open economy model. We extend the standard Cespedes, Chang, and Velasco (2004) model by allowing entrepreneurs borrow in both foreign and domestic currency so as to finance firms' capital needs. A real depreciation reduces the value of firms' net worth whenever there is a currency mismatch in their balance sheets. Under flexible exchange rates, a lower degree of dollarization lessens the negative impact on output and investment, since there is a smaller increase in the cost of external borrowing. The quantitative results show that the balance sheet channel accounts for about 70 percent of the output and investment drop in Peru following the Russian Crisis, and a reduction in debt dollarization would have reduced output drop in 0.9 percentage points of GDP.

Keywords: Small open economy, balance sheet effects, dollarization.
JEL classification codes: F31, F41, G32

*We are grateful to Enrique Kawamura, Paul Castillo, Marco Vega and one anonymous referee for many useful ideas and suggestions on this paper. We also benefited from comments by seminar participants at the Central Bank of Peru. Part of this research was done when Fabrizio Orrego was affiliated to Universidad de San Andrés, Buenos Aires, Argentina. The usual disclaimer applies.
†Email address: gondomori@econ.umd.edu
‡Email address: forrego@andrew.cmu.edu

1 Introduction

In recent years, following the severe impact of the 1997-98 financial crises, emerging market countries have been implementing a set of policies aimed at improving the currency composition of their debt so as to reduce financial fragility and the likelihood of being exposed to a financial crisis.

A vast literature emerged after these crises, which provided the theoretical arguments that explain how financial shocks may have real effects, through their impact on the balance sheets of firms, due to a currency mismatch between assets and liabilities. In order to simplify the theoretical framework, a common assumption in this class of models is that agents are only allowed to borrow in foreign currency, so that all debt is denominated in dollars and is fully exposed to currency risk.[1] The main motivation is the failure of emerging markets to borrow at long maturities in their own currency.[2]

However, the ability of a country to contract debt in different currencies seems to be relevant in practice. Reinhart, Rogoff, and Savastano (2003b) show that it is not clear that emerging markets can only borrow in foreign currency as stated in the original sin hypothesis. They argue notwithstanding that the degree of debt intolerance can be explained through more underlying factors such as the quality of institutions and a history of good economic management. Debt dollarization seems to arise as a response to financial market conditions, where countries with shallow and illiquid financial markets choose dollarization as an alternative source of financing.[3] Therefore, it is relevant to analyze what the ability of debt dedollarization is in reducing the exposure of emerging countries to financial crises and how they can help to mitigate the balance sheet channel through which financial shocks affect real variables.

In this paper we evaluate the qualitative and quantitative implications of financial dedollarization of firms' liabilities on real aggregates in a small open economy. We extend the standard Cespedes, Chang, and Velasco (2004) (CCV henceforth) model by allowing entrepreneurs borrow in both foreign and domestic currency (*soles*) so as to finance firms' capital needs. To deal with the issue of borrowing in different currencies, we consider two types of entrepreneurs, investors and savers, which for the time being are catalogued exogenously. Both types are endowed each period with a certain amount of net worth, but only investors can buy productive capital. Nevertheless, savers can profit from their net worth by lending their soles to investors, who can borrow from abroad as well. The proportion of investors determines the extent of financial dollarization in this economy. External borrowing is subject to agency costs.

After characterizing the optimal contract, we show that under flexible exchange rates, a lower degree of dollarization reduces agency costs and the volatility of output in terms of home goods. Since investors face a lower risk premium, the output drop

[1]Levy-Yeyati (2006) argues that the main disadvantage of financial dollarization is related to the incidence of balance sheet effects in the event of a sharp real exchange rate depreciation, as debtors may no longer be able to service their dollar denominated debt.

[2]This is the hypothesis of original sin introduced by Eichengreen and Hausmann (1999).

[3]Countries with a bad credit history have more domestic dollarization than the ones with no default history. The sample considered in Reinhart, Rogoff, and Savastano (2003b) shows a wide variety of degrees of debt dollarization depending on their default history, with a mean dollarization of 0.3 percent for non-defaulters and 16 percent for countries that have a history of default.

following an unanticipated increase in the world interest rate is smaller the lower the degree of the currency mismatch. We provide a link between debt dollarization and financial fragility.[4] In the limit, when investors only borrow in soles, without being exposed to a currency mismatch, the economy is always in the financially robust case.

Our quantitative analysis suggests that the gains from changing the currency denomination of debt is significant compared to the effect of financial imperfections due to the costly state verification problem. Changing the currency composition of debt reduces the effect on output by 0.8 percentage points, whereas switching off the risk premium due to the agency problem reduces the effect on output by 0.1 percentage points. Similar effects are obtained in other real variables such as investment (1.9 percent due to currency mismatch versus 0.2 percent due to agency costs) and real depreciation (2.0 percent due to currency mismatch versus 1.3 percent due to agency costs).

We simulate the model for 10,000 periods and compute the variance of consumption deviations from the deterministic steady state for debt denominated in dollars and in soles as an approximate measure of the welfare component related to insurable risk. The variance of consumption is 0.007 for the benchmark case with full dollar-denominated debt, larger than the variance of consumption 0.0042 with full sol-denominated debt.

This paper contributes to several strands of the literature. The fact that the quantitative effects of financial frictions in this class of models is very mild has already been studied for closed economy models in Bernanke, Gertler, and Gilchrist (1999) and Christensen and Dib (2008). The difference in the impact of a shock to interest rates in the model with and without financial frictions is of the order of 0.1 percent of deviations from steady state.[5] Our results are also in line with Kocherlakota (2000) and Cordoba and Ripoll (2004), who find that financial frictions under standard parameter values do not create large amplifications.[6] Our model compares this result to the amplification effect of currency mismatch, which in our model is quantitatively higher than the effect of agency costs.

There are several papers on the quantitative implications of the financial accelerator mechanism, but they all consider firms with fully dollarized liabilities. Gertler, Gilchrist, and Natalucci (2007) analyze the quantitative implications of balance sheet effects in a small open economy with fixed exchange rates and calibrate the model to the case of Korea in 1997. They find that the model is able to replicate the 12 percent drop in output, where the balance sheet effect accounts for about 50 percent of output drop. Tovar (2005) presents a quantitative analysis of the real effects of the

[4] Throughout this paper, financial fragility (robustness) means the elasticity of the risk premium with respect to the real exchange rate is positive (negative).

[5] Bernanke, Gertler, and Gilchrist (1999) calibrate a closed economy model with sticky prices. The impact of a monetary shock shows an amplification on the effect on output from 1.0 percent to 1.4 percent. Christensen and Dib (2008) present a similar model with nominal contracts and monetary policy that follows a Taylor-type of interest rate rule. A positive shock to the policy interest rate amplifies the output drop from 0.4 percent to 0.5 percent.

[6] Kocherlakota (2000) finds that exogenous borrowing constraints do not create amplification, whereas with financial constraints that depend on the value of collateral, amplification is only important for certain parameter values. Cordoba and Ripoll (2004) additionally consider the general equilibrium effects on the interest rate in a closed economy model. With standard assumptions such as CES utility and Cobb Douglas production function, large amplification of exogenous shocks is a cutting edge result.

balance sheet problem in an environment with sticky prices, and the use of monetary policy as a tool to smooth these effects by including an additional expansionary effect of exchange rates on output. The quantitative results show that devaluations are expansionary despite the presence of balance sheet effects.

On the effects of financial fragility, there is empirical evidence that higher dollarization and the resulting currency mismatch in assets and liabilities increases the solvency risk of debtors and of the banking system, making the financial system more fragile and increasing the incidence of default.[7] Even though our framework abstracts from the existence of a banking system, financial fragility is higher with more dollar-denominated debt. As previously mentioned, higher dollarization means that there is a larger currency mismatch between the value of output in soles and repayment. This increases the probability of default, so that the lender has more incentives to go through the costly verification process, which reflects in an increase in the risk premium when a bad financial shock hits the economy. As borrowing costs become larger, the borrower is more financially fragile and faces lower solvency, as the incentives and capability of debt repayment are lower.

Empirical work on the importance of the currency composition of debt has been analyzed in several papers, in terms of its effect on output performance (both growth and volatility) and its effect on financial fragility and incidence of crises. Reinhart, Rogoff, and Savastano (2003b) find evidence of higher output volatility in highly dollarized economies, whereas the effect on output growth is not significantly different. Our model captures higher output volatility for economies with higher debt dollarization. A higher ratio of dollar-denominated debt increases the currency mismatch between the source of income of the domestic economy (output denominated in soles) and the repayment value (interest rate payment for foreign currency borrowing). This increases agency costs that entrepreneurs must pay to take debt, and therefore, increases the cost of borrowing, which reduces optimal debt, investment and output, amplifying the volatility of real variables in the model.

This model is also consistent with empirical studies that show that the degree of currency mismatch in highly leveraged firms is a key feature affecting the impact of exchange rates on the behavior of real aggregates, where countries with higher degrees of financial dollarization have experienced more frequent financial crises than countries with a smaller currency mismatch.[8] Firms without full hedge on the currency composition of their liabilities face a sharp drop in investment and output when facing currency depreciations during financial crises.[9] In this model, when a bad interest rate shock hits the economy, the currency mismatch amplifies the increase in the cost of borrowing due to financial frictions, with more severe drops in investment, output

[7] For instance, see De Nicolo, Honohan, and Ize (2003). Similar to Reinhart, Rogoff, and Savastano (2003a), they find evidence that dollarization arises as an alternative source of financing in countries that lack macroeconomic stability. Highly dollarized countries are more exposed to currency mismatches in debtors balance sheets that undermine the quality of their loan portfolio when large depreciations take place, increasing solvency risk as measured by the Z-index and the ratio of non-performing loans.

[8] Levy-Yeyati (2006) finds that exchange rate fluctuations have significant effects on crises propensity in the presence of financial dollarization, compared to non-dollarized economies.

[9] Aguiar (2005) shows that partially hedged Mexican firms faced a reduction in net worth and a sharp drop in investment during the Tequila crisis. Forbes (2002) presents evidence on a set of emerging markets, where after large currency depreciations, highly leveraged firms have lower net income growth.

and consumption, more severe capital account reversals and larger real exchange rate depreciations. Therefore, higher dollarization would reflect into more frequent sudden stop episodes, as defined by Calvo, Izquierdo, and Mejia (2008). The model also replicates the connection between a higher frequency of sudden stop episodes and higher liability dollarization, where balance sheet effects are more relevant when there is a higher ratio of dollar denominated liabilities.

The welfare consequences are studied in Elekdag and Tchakarov (2007), where they compare welfare under fixed and flexible exchange rate regimes for a small open economy with balance sheet effects, and find that only highly indebted countries without credible monetary policy could achieve benefits from a fixed regime. Even though we do not fully address a detailed analysis on the welfare implications of dollarization, we find that consumption is more volatile when a higher proportion of debt is denominated in dollars. If the conditions established in Woodford (2002) are satisfied, that is, when the stochastic steady state is close enough to the deterministic steady state, this would imply that welfare is higher when debt is fully denominated in soles.

The plan of this paper is as follows. Section 2 introduces the model. Section 3 discusses the qualitative results of a change in the currency composition of liabilities under flexible exchange rate regimes, and the effects under a financially robust and financially vulnerable economy. Section 4 presents a quantitative analysis of the balance sheet channel in the case of Peru during the Russian Crisis, and compare the results to the real effects of a currency depreciation if debt had been denominated only in soles. Section 5 concludes.

2 The Model

We lay out a general equilibrium framework in the spirit of recent open macroeconomics literature. There is an infinite-horizon small open economy, one single good produced domestically by firms that employ both labor and capital, and two different types of agents: households and entrepreneurs. The former supply labor force, whereas the latter finance capital on behalf of firms. A key assumption of the paper is that entrepreneurs have an initial endowment of net worth that is not enough to purchase capital. Thus, they must look for additional means of funding, and borrowing is subject to agency costs.

2.1 Firms

There is a continuum of firms indexed in the [0, 1] interval, which produce a single good in a competitive environment. They all have access to a common technology which exhibits constant returns to scale:

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

where K_t denotes capital input, L_t labor input, Y_t home output, A is a positive time-invariant parameter (also known as total factor productivity), and $0 < \alpha < 1$ is the output elasticity of capital. The market for labor is characterized by monopolistic competition, since labor services offered by workers are heterogeneous. Using a Dixit-Stiglitz operator, we let L_t be the aggregation of the services of the different workers in the economy:

$$L_t = \left[\int_0^1 L_{it}^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

where L_{it} refers to worker i's labor demand and $\sigma > 1$ is the elasticity of demand for worker i's services. Firms' profits are defined as follows:

$$P_t Y_t - R_t K_t - \int_0^1 W_{it} L_{it} di \tag{3}$$

where, as usual, P_t represents the price of the home good, R_t is what firms pay in exchange for capital usage and W_{it} is worker i's wage, all expressed in soles.

Firms maximize their profits (3) subject to equations (1) and (2). For simplicity, capital fully depreciates each period. The solution to this problem is standard. On the one hand, first order conditions show that:

$$R_t K_t = \alpha P_t Y_t \tag{4a}$$

$$W_t L_t = (1-\alpha) P_t Y_t \tag{4b}$$

where $W_t = [\int_0^1 W_{it}^{1-\sigma} di]^{\frac{1}{1-\sigma}}$ denotes the minimum cost of one unit of L_t expressed in soles. On the other hand, the demand for worker i's labor is:

$$L_{it} = \left(W_{it}/W_t\right)^{-\sigma} L_t \tag{5}$$

2.2 Households

Preferences of household i can be represented by the following expected utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_{it} - \left(\frac{\sigma-1}{\sigma} \right) \frac{L_{it}^{\tau}}{\tau} \right] \tag{6}$$

where $\tau > 1$ governs the curvature of labor supply, $0 < \beta < 1$ is the subjective discount factor and $\mathbb{E}_t(x)$ is the expected value of x conditional on information available at time t. We assume additive separability between consumption and labor supply and logarithmic utility function for convenience. As usual, C_{it} is a composite consumption index of the home good $\left(C_{it}^H\right)$ and the imported good $\left(C_{it}^F\right)$:

$$C_{it} = \frac{(C_{it}^H)^{\gamma} (C_{it}^F)^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \tag{7}$$

where $0 < \gamma < 1$ stands for the participation of home goods in the composite consumption index. The price of the imported good is normalized to one and the price of one unit of imports in soles is equal to the nominal exchange rate of S_t soles per dollar, because the Law of One Price is assumed to hold. For later reference, let $E_t \equiv S_t/P_t$ be the prevailing real exchange rate.

As in CCV, we assume that households cannot save and hence labor income is the only source of earnings. The associated budget constraint is given by:

$$W_{it}L_{it} = P_t C_{it}^H + S_t C_{it}^F \tag{8}$$

The household maximizes the expected utility (6) subject to (5), (7) and (8), which implies in a symmetric equilibrium that:

$$Q_t C_t = W_t L_t \tag{10}$$

where the minimum cost of consumption is given by:

$$Q_t = P_t^{\gamma} S_t^{1-\gamma} \tag{11}$$

Households set their wages at the beginning of each period, before main aggregate variables are observed. As wages are set in advance, it must be the case that:

$$\mathbb{E}_t L_{t+1}^{\tau} = 1 \tag{9}$$

2.3 Entrepreneurs

There is a continuum of risk-neutral entrepreneurs indexed in the [0, 1] interval, who are initially allocated a positive net worth in domestic currency. Entrepreneurs are supposed to invest in capital for the next period. However, we assume there is a threshold in the distribution of entrepreneurs so that individuals above that threshold are restricted from investing in physical capital.

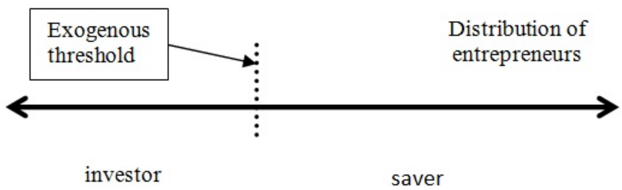

Figure 1: Distribution of entrepreneurs. There is a threshold that divides the unit interval in two segments. The position of the threshold is exogenous.

The threshold gives rise to two types of entrepreneurs, investors and savers (see Figure 1). Investors use their initial net worth to finance capital acquisitions. Should investors fall short of money, they may borrow additional resources either in foreign or domestic currency. Debt in foreign currency is provided by the world capital market at the risk-free interest rate given by ρ_{t+1}^*. On the other hand, debt in domestic currency is provided by savers, who transform costlessly their endowments into bonds and charge an interest rate in domestic currency equal to ρ_{t+1}.

Loosely speaking, we interpret the location of the threshold as the *depth* of the domestic capital market, as in Caballero and Krishnamurthy (2003). In this economy, the ratio of dollarization (i.e. the ratio of debt issued in foreign currency to total debt) is exogenously driven by the position of the threshold in the unit interval.

Investors arrange their portfolios according to the exogenously determined ratio of dollarization. Given that they are risk neutral, the optimality condition establishes that in order to have debt in two currencies in equilibrium, the expected cost of borrowing in the two different markets must be the same. If the expected cost of borrowing were strictly lower in one market, investors would choose to borrow only in that currency. Hence the uncovered interest rate parity displayed in (12) is a by-product of the entrepreneurs problem:

$$(1+\rho_{t+1}) = (1+\rho^*_{t+1})\mathbb{E}_t\left(S_{t+1}/S_t\right) \tag{12}$$

Now, if Q_t is the price of capital, investors' budget constraint can be written as:

$$P_t N^I_t + D_{t+1} + S_t D^*_{t+1} = Q_t K_{t+1} \tag{13a}$$

where D_{t+1} and D^*_{t+1} denote the amount borrowed in soles and dollars, respectively, $P_t N^I_t$ represents investors' net worth, K_{t+1} accounts for the investment in $t+1$ capital, and S_t is the nominal exchange rate. On the other hand, savers face the following budget constraint:

$$P_t N^{II}_t = D_{t+1} \tag{13b}$$

Equations (13) reflect the fact that savers' net worth has already been absorbed by investors. This can be seen more clearly in Table 1, in which we depict the entrepreneurs' balance sheets:

Investors		Savers	
Assets	Liabilities	Assets	Liabilities
$Q_t K_{t+1}$	$P_t N^I_t$ D_{t+1} $S_t D^*_{t+1}$	$B_{t+1} = D_{t+1}$	$P_t N^{II}_t$

Table 1: Economywide entrepreneurs' balance sheets. Investors and savers are endowed with $P_t N^I_t$ and $P_t N^{II}_t$ soles, respectively. The latter transform these resources costlessly into bonds B_{t+1}. Investors acquire these bonds worth D_{t+1} soles, and also purchase D^*_{t+1} dollars abroad to finance capital acquisitions.

The results from the optimal contract imply a relationship between the return on capital investment and the risk premium similar to that of CCV. Because of informational asymmetries, borrowing is subject to frictions which in turn introduce a wedge between the expected return to investment and the cost of borrowing, both in dollars and in soles:

$$\frac{\mathbb{E}_t(R_{t+1}K_{t+1}/S_{t+1})}{Q_t K_{t+1}/S_t} = (1+\rho^*_{t+1})(1+\eta_{t+1}) \tag{14a}$$

$$\frac{\mathbb{E}_t(R_{t+1}K_{t+1})}{Q_t K_{t+1}} = (1+\rho_{t+1})(1+\eta_{t+1}) \tag{14b}$$

where R_{t+1} is the rental rate of capital at $t+1$ and η_{t+1} is the so-called risk premium. The risk premium is the same for debt in both currencies because it captures the effect of the costly state verification problem, which is faced equally by both savers and foreign lenders.[10] In particular, the risk premium satisfies:

$$1+\eta_{t+1} = F\left(\frac{Q_t K_{t+1}}{P_t N_t^I}\right), F(1)=1, F'(\cdot)>0 \tag{15}$$

Finally, at the end of each period, investors collect the income from capital and repay domestic and foreign debt. They consume a portion $1-\delta$ of the remainder and only consume imports. Thus, their net worth is:

$$P_t N_t^I = \delta\left[R_t K_t - \Phi_t \alpha P_t Y_t - (1+\rho_t^*) S_t D_t^* - (1+\rho_t) D_t\right] \tag{16a}$$

where $R_t K_t$ is aggregate capital income; $\Phi_t \alpha P_t Y_t$ accounts for monitoring costs paid in period t, while terms $(1+\rho_t^*)S_t D_t^*$ and $(1+\rho_t)D_t$ refer to debt repayments in foreign and domestic currency, respectively. Similarly, savers entrepreneurs at the end of each period collect the income from the repayment of domestic debt. They consume a fraction $1-\delta$ of this income, everything in terms of imported goods. Thus, savers' net worth is:

$$P_t N_t^{II} = \delta(1+\rho_t)D_t \tag{16b}$$

Equations (16) imply that entrepreneurs die with probability $(1-\delta)$.[11]

2.4 Equilibrium

The full description of a competitive equilibrium is the content of the following definition:

Definition 1 (Competitive equilibrium) *Given $\{\rho_{t+1}^*\}_{t=0}^{\infty}$ and K_0, the competitive equilibrium in the small open economy is the set of allocations $\{L_{it}, C_{it}^H, C_{it}^F, C_{i,t}, L_t, C_t, K_{t+1}, Y_t, N_t, D_{t+1}, D_{t+1}^*\}_{t=0}^{\infty}$, together with prices $\{W_{it}, W_t, P_t, R_t, S_t, Q_t, \rho_{t+1}, \eta_{t+1}\}_{t=0}^{\infty}$ such that:*

1. *Firms: Taking $\{P_t, R_t, W_{it}\}$ as given, firms choose $\{K_t, L_{it}\}$ to maximize profits (3) subject to (1) and (2).*

2. *Households: Taking $\{P_t, S_t, W_{it}\}$ as given, each worker i chooses $\{L_{it}, C_{it}^H, C_{it}^F\}$ to maximize expected utility (6) subject to (5), (7) and (8).*

3. *Entrepreneurs: Taking $\{R_t, S_t, Q_t\}$ as given, each investor chooses $\{D_{t+1}, D_{t+1}^*\}$ so as to finance investment $\{K_{t+1}\}$, subject to savers' participation constraint, information and resource constraints.*

4. *Market clearing: All markets clear.*

   (a) $L_t = \left[\int_0^1 L_{it}^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}$,

[10] Appendix A discusses the derivation of the optimal contract.

[11] See Carlstrom and Fuerst (1997) for a related exposition.

(b) $K_{jt+1} = \frac{(K^H_{jt+1})^\gamma (K^F_{jt+1})^{1-\gamma}}{\gamma^\gamma(1-\gamma)^{1-\gamma}}$, $K_{t+1} = K_{jt+1}$,

(c) $C_{it} = \frac{(C^H_{it})^\gamma (C^F_{it})^{1-\gamma}}{\gamma^\gamma(1-\gamma)^{1-\gamma}}$, $C_t = C_{it}$,

(d) Goods market:
$Y_t = C^H_t + K^H_{t+1} + \Phi_t \alpha Y_t + X$

(e) Balance of Payments:
$S_t X - S_t C^F_t - S_t K^F_{t+1} - \Phi_t \alpha P_t Y_t + S_t D^*_{t+1} - (1+\rho^*_t) S_t D^*_t - (P_t N^I_t + P_t N^{II}_t)(1/\delta - 1) = 0$

In this small open economy, domestic production is absorbed by both residents and non-residents. The market for home goods is in equilibrium when:

$$P_t Y_t = \gamma Q_t (K_{t+1} + C_t) + S_t X \tag{17}$$

where the left hand side is the nominal outcome, C_t is aggregate consumption and X represents exports. The derivation of this equation can be found in the appendix.

In the next section, we will analyze the dynamics of the model by log-linearizing the equilibrium equations around the non-stochastic steady state.

2.5 Dynamics

Monetary policy in this model, though not formally modeled, is conducted in a very simple fashion. The monetary authority targets the home goods price index or, alternatively, operates in an environment of flexible exchange rates, as described in the following definition:

Definition 2 (Flexible exchange rate regime) *A regime of flexible exchange rates is one in which the monetary authority lets the nominal exchange rate s_t adjust to market conditions. In particular, monetary policy is conducted so as to set $p_t = \mathbb{E}_{t-1} p_t = 0$ for all t.*

Since most resulting expressions are fairly standard, we only report the evolution of the risk premium, which is the content of the following proposition:[12]

Proposition 1 (Risk premium) *The first-order difference equation that describes the dynamics of the risk premium is:*

$$\eta'_{t+1} - \phi \eta'_t = \mu \left[\left(\frac{1-\lambda}{\lambda} \right) (y_t - e_t) \right] + \mu \psi^* \delta (1+\rho^*) \left[(1-\pi)(e_t - \mathbb{E}_{t-1} e_t) - (y_t - \mathbb{E}_{t-1} y_t) \right] \tag{18}$$

where lowercase letters denote percentage deviations from the non-stochastic steady state, λ is the share of investment demand in total non-consumption demand for home goods, ψ^* is the steady state ratio of debt to net worth, μ is the elasticity of the risk premium with respect to leverage, ϕ is a coefficient that depends on the debt contract and $(1-\pi)$ represents the steady state ratio of dollarization.

[12] Appendix C shows all log-linearized expressions.

The first term in equation (18), as in CCV, is interpreted as follows. A higher level of output requires more capital input, which, given that the entrepreneur's net worth is not enough to finance capital, must be achieved by increasing debt. Higher debt increases the debt burden in terms of home production, and therefore, the lender charges a higher risk premium.

The effect of the currency denomination of debt is observed in the second term, which amplifies the response of the risk premium to shocks. An unexpected devaluation increases the entrepreneur's debt burden, as the value of repayment of the dollar-denominated debt in terms of home goods increase, whereas the repayment value of sol-denominated debt is not affected. Higher debt burden reduces the entrepreneur's net worth and therefore foreign lenders charge a higher risk premium.

The third term, related to the effect of an unexpected fall in output, is exactly the same as in CCV, as it implies that lower unexpected output reduces the reward for capitalists from previous investment regardless of the currency denomination of debt. This results in lower net worth and higher risk premium.

The first and last term show us that, regardless of the currency denomination of debt, the presence of asymmetric information and imperfect capital markets require agency costs to create the incentives that guarantee debt repayment. Therefore, even under zero dollarization, the risk premium would still be affected by changes in output, given the fact that higher output increases the entrepreneur's leverage and reduces net worth.

3 Qualitative results

We assume that, starting from the steady state, there is a shock to the world safe interest rate at t = 0, whose effects disappear from t = 1 onwards, because the economy settles again on the saddle path toward the long run.[13]

We will derive two curves in the (k_{t+1}, e_t) space, namely IS$_\pi$ and BP$_\pi$, which summarize the equilibria in the goods and loan markets, respectively. In order to derive the BP$_\pi$ curve, we use the equation for the risk premium in period 1, which is the content of the following lemma:

Lemma 1 (Risk premium in period 1) *The risk premium in period 1 is:*

$$\eta_1' = e_{0,\pi}\varepsilon_\pi \tag{19}$$

where $\varepsilon_\pi \equiv \mu\left[\psi^*\delta\left(1+\rho^*\right)\left(1-\pi\right) - \frac{1-\lambda}{\lambda}\right]$ *is the elasticity of the risk premium with respect to a change in the real exchange rate.*

The sign of the elasticity determines whether an economy is financially robust ($\varepsilon_\pi < 0$) or financially vulnerable ($\varepsilon_\pi > 0$). Notice that the ratio of dollar-denominated debt $(1-\pi)$ increases the size of this elasticity, making an economy more prone to be financially vulnerable.

With Lemma 1 at hand, we are now able to characterize the IS$_\pi$ and BP$_\pi$ curves in this economy, as the next proposition shows:

[13]Appendix D demonstrates that under the assumptions addressed here, the introduction of sol-denominated debt does not alter the saddle-path stability of the original system.

Proposition 2 (IS$_\pi$ and BP$_\pi$ curves) *Under flexible exchange rates, the IS$_\pi$ curve is:*

$$0 = \lambda k_{1,\pi} + (1 - \lambda\gamma) e_{0,\pi} \tag{20}$$

and the BP$_\pi$ curve is:

$$k_{1,\pi} = [\gamma - (1 - \zeta)\,\varepsilon_\pi]\, e_{0,\pi} \tag{21}$$

But, when we include the effect of the unanticipated increase in the foreign interest rate, the BP$_\pi$ curve becomes:

$$k_{1,\pi} = [\gamma - (1 - \zeta)\,\varepsilon_\pi]\, e_{0,\pi} - \tilde{\rho}_1^* \tag{22}$$

where $\zeta < 0$ is the saddle-path coefficient in the linear relationship $y_t - e_t = \zeta \eta_t'$.

Before the shock, we conclude from equations (20) and (21) that both curves intersect at the origin. Nevertheless, when the shock is taken into account it is straightforward to conclude from equations (20) and (22) that the values of $(e_{0,\pi}, k_{1,\pi})$ satisfy:

$$(e_{0,\pi}, k_{1,\pi}) = \left(\frac{\lambda \tilde{\rho}_1^*}{\lambda(\zeta - 1)\,\varepsilon_\pi + 1}, \frac{(\lambda\gamma - 1)\,\tilde{\rho}_1^*}{\lambda(\zeta - 1)\,\varepsilon_\pi + 1} \right) \tag{23}$$

Now we will analyze the effect of a lower degree of dollarization using the IS$_\pi$-BP$_\pi$ diagram, after an unanticipated increase in the foreign interest rate. We will study shortly what happens to the real exchange rate and capital when the economy is either robust or vulnerable.

3.1 The Financially Robust Economy

Let ε be the elasticity of the risk premium with respect to a change in the real exchange rate that would prevail in a world with full dollarization, that is when $(1 - \pi) = 1$. Let BP be the associated curve that clears the loan market in the (k_{t+1}, e_t) Cartesian plane. From equation (20), we know that IS$_\pi$ does not depend on the ratio of dollarization $(1 - \pi)$, and hence its slope is invariant to changes in π. From equations (21) or (22), we also know that the slope of BP$_\pi$ does depend on $(1 - \pi)$ through the elasticity of the risk premium with respect to the real exchange rate. Actually, BP$_\pi$ is flatter than BP as $\varepsilon > \varepsilon_\pi$.

In the case of a financially robust economy, it is clear from (22) that both BP and BP$_\pi$ shift to the left after an unanticipated increase in the foreign interest rate. The impact on both the real exchange rate and capital is less intense with partial dollarization, that is when $(1 - \pi) < 1$, as shown in Figure 2.

3.2 The Financially Vulnerable Economy

In the case of a financially vulnerable economy, we may face a downward sloping BP$_\pi$ curve under certain parameter configurations, as depicted in Figure 3. In this context, the impact of an unanticipated increase in the foreign interest rate on both the real exchange rate and investment is less sharp with partial dollarization. Of course, since

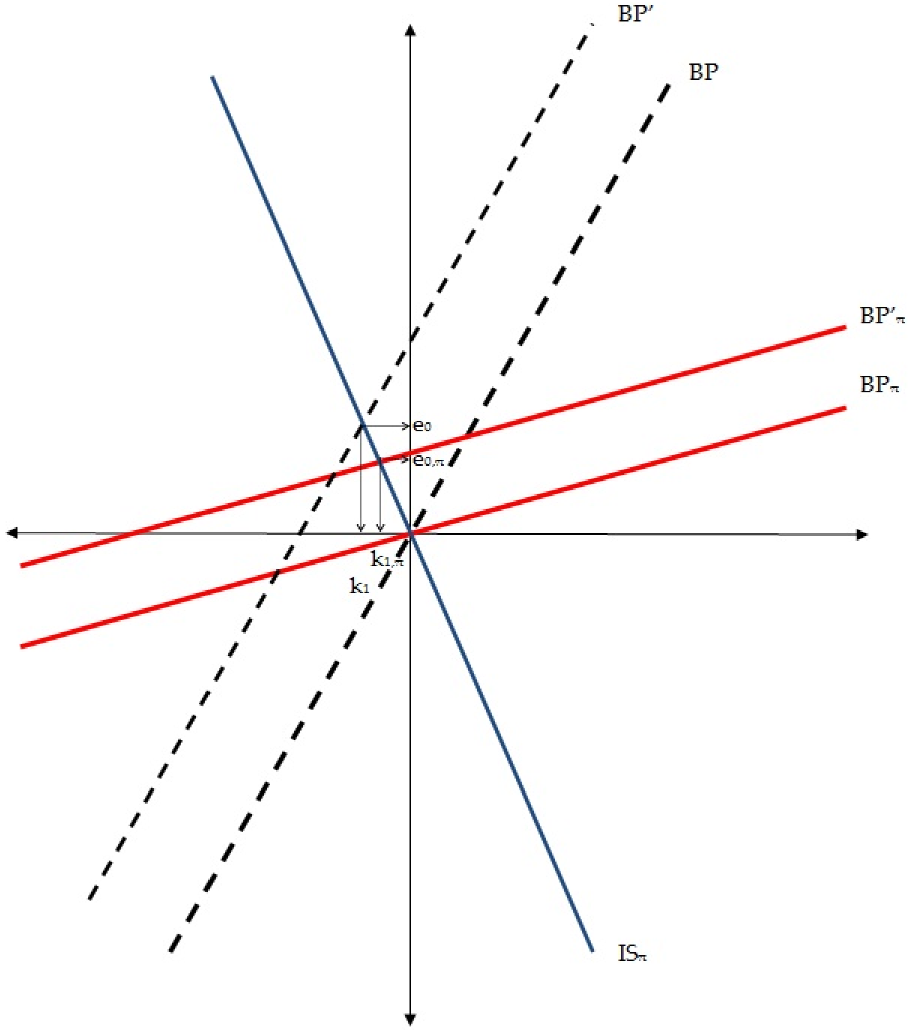

Figure 2: A financially robust economy. Initially BP_π and IS_π intersect at the origin. After an unanticipated increase in the foreign interest rate, BP_π shifts to BP'_π. The new intersection occurs at $(e_{0,\pi}, k_{1,\pi})$. When $\pi = 0$, the relevant upward sloping BP would shift to BP′ and the intersection with IS_π would occur at (e_0, k_1).

net worth effects matter, both the real exchange rate and investment fall more on impact than in the financially robust case.

This result shows that a financially vulnerable economy would find beneficial to develop the market for domestic debt, as it makes an economy less prone to financial vulnerabilities and reduces the size of the balance sheet effect. The next proposition summarizes this fact:

Proposition 3 (Optimal level of dollarization) *Let* $\overline{\pi} \equiv \frac{\lambda[\psi^*\delta(1+\rho^*)+1]-1}{\lambda\psi^*\delta(1+\rho^*)}$. *The economy is financially robust if* $(1-\pi) < (1-\overline{\pi})$. *Otherwise, the economy is financially vulnerable.*

If the economy succeeds in fostering financial dedollarization so that $(1-\pi) < (1-\overline{\pi})$, financial unsoundness is no longer a feasible outcome. In particular, Proposition 3 says that if debt is contracted only in domestic currency, then the economy will be

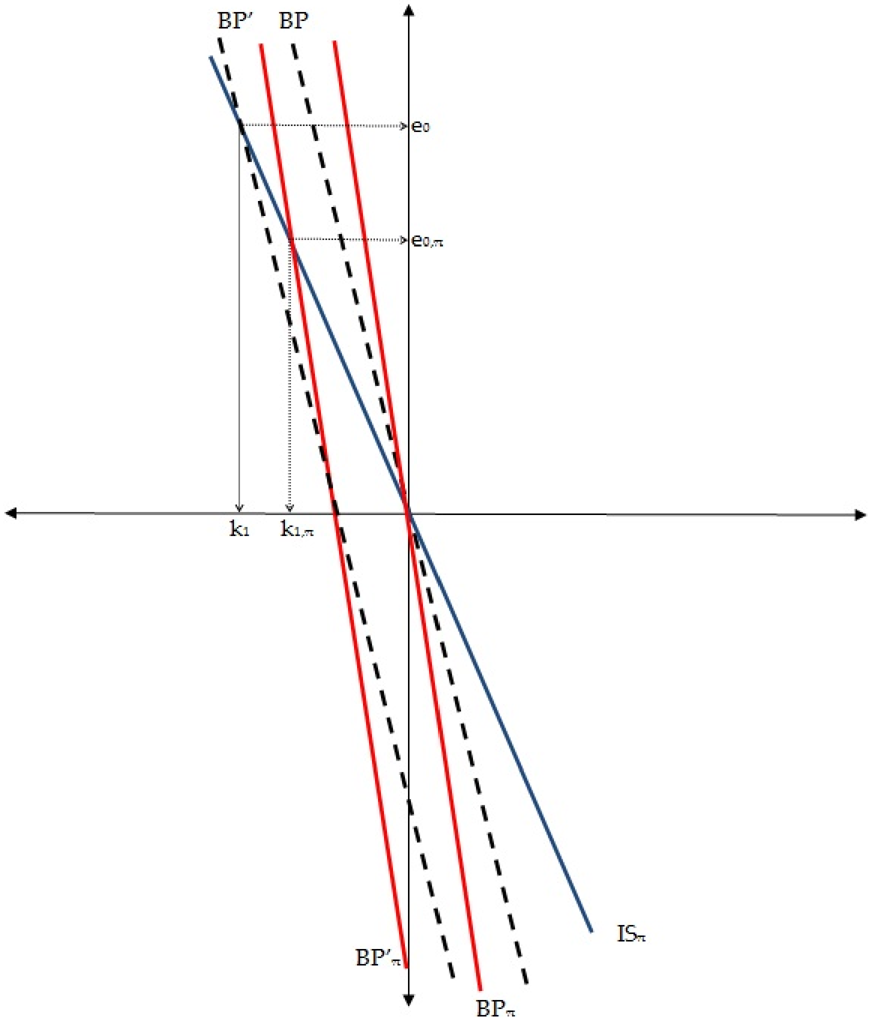

Figure 3: A financially vulnerable economy. Initially BP_π and IS_π intersect at the origin. After an unanticipated increase in the foreign interest rate, BP_π shifts to BP'_π. The new intersection occurs at $(e_{0,\pi}, k_{1,\pi})$. When $\pi = 0$, the relevant downward sloping BP would shift to BP′ and the intersection with IS_π would occur at (e_0, k_1).

robust.

Corollary 1 (Zero dollarization) *If debt is contracted only in domestic currency, which implies in the limit that* $(1 - \pi) = 0$*, the economy will always be financially robust.*

Furthermore, with the possibility of reducing currency mismatch by borrowing in domestic currency, a higher proportion of sol-denominated debt allows an economy display a larger leverage ratio without becoming financially vulnerable.

3.3 A digression on consumption and welfare

Following the previous analysis on the behavior of investment and real exchange rates, we now focus on the behavior of consumption and how it is affected by the currency composition of debt. The details on the derivation can be found in the appendix. The log-linearized behavior of consumption at each period is given by

$$c_t = \alpha k_t - (1-\gamma)e_t$$

Once again, there is a shock to the world safe interest rate at $t = 0$, whose effect disappears from $t = 1$ onwards. The behavior of consumption in period $t = 0$ is given by

$$c_{o,\pi} = -(1-\gamma)\frac{\lambda\widetilde{\rho}_1}{\lambda\left(\zeta-1\right)\varepsilon_\pi + 1}$$

As mentioned in Lemma 1, the ratio of dollar-denominated debt increases the size of the elasticity, making an economy more prone to be financially vulnerable. It is easy to show that $\frac{dc_{0,\pi}}{d\pi} < 0$. Therefore, a positive shock in $\widetilde{\rho}_1$ implies a larger fall in consumption when the economy has a higher ratio of dollar-denominated debt. This would imply a higher volatility of consumption in an economy with higher dollarization.

A first order approximation of the welfare measure depends only on the volatility, so that welfare is lower in an economy with a higher ratio of dollar-denominated debt. When the deterministic steady state is close enough to the stochastic steady state, the welfare comparison using a linear approximation gives similar results to a higher-order one, as in Woodford (2002).[14] This can also be seen from the second-order approximation of the utility-based measure of welfare $U(C, L)$.

$$\mathbb{E}(U) = \bar{U} + \mathbb{E}(c) - \left(\frac{\sigma-1}{\sigma}\right) v\bar{L}^v\mathbb{E}(l) - \frac{1}{2}\mathbb{V}(c) - \frac{1}{2}\left(\frac{\sigma-1}{\sigma}\right) v(v-1)\bar{L}^v\mathbb{V}(l) + O(\|\xi\|^3)$$

where uppercase letters with bars denote non-stochastic steady state values, $\mathbb{E}(x)$ is the unconditional expected value of the random variable x, and $\mathbb{V}(x)$ is the unconditional variance of x. Also, $\xi > 0$ and O is the standard big o notation.

Under flexible prices, this expression simplifies to

$$\mathbb{E}(U) = \bar{U} + \mathbb{E}(c) - \frac{1}{2}\mathbb{V}(c) + O(\|\xi\|^3)$$

If the conditions in Woodford (2002) are satisfied $\mathbb{E}(c|_{\pi=0}) = \mathbb{E}(c|_{\pi=1}) = 0$. Therefore, changing the currency composition of debt from dollars to soles is welfare improving if $\mathbb{V}(c|_{\pi=1}) < \mathbb{V}(c|_{\pi=0})$. We will revisit this condition in the next section.

A standard measure used in the literature to make welfare comparisons is the equivalent variation with respect to steady state, as in Lucas (1987). The welfare measure, χ, is defined as the percentage increase in consumption required to achieve the value of welfare in steady state:

$$\sum_{t=0}^{\infty}\beta^t U(\bar{C}, \bar{L}) = \sum_{t=0}^{\infty}\beta^t U((1+\chi)C_t, L_t)$$

[14] However, a complete analysis of welfare requires a second-order approximation of the model, as mentioned by Schmitt-Grohe and Uribe (2004). A first-order approximation of the policy functions may yield spurious results when the stochastic steady-state differs from the deterministic steady state, as in Kim and Kim (2003).

Given that prices are flexible in our model, aggregate labor is always chosen to be at the steady state level. Using the utility functional form for the quantitative exercise, this expression simplifies to:

$$
\begin{aligned}
\sum_{t=0}^{\infty} \beta^t \log(\bar{C}) &= \sum_{t=0}^{\infty} \beta^t \log((1+\chi)C_t) \\
1+\chi &= \exp((1-\beta)\sum_{t=0}^{\infty} \beta^t \log(\bar{C}/C_t))
\end{aligned}
$$

4 Quantitative Analysis

In the previous section, we analyzed the qualitative implications of the currency composition of debt on the impact of exchange rate depreciation on output and investment. The results show that a higher proportion of sol-denominated debt reduces the size of the balance sheet effect by making the economy more prone to be financially robust. Similarly, a lower level of debt also contributes to financial robustness.

In this section, we use the model to analyze the relative size of the balance sheet effect and its impact on output and investment during the Russian Crisis, and how quantitatively relevant is the dedollarization could the dedollarization process be in reducing the impact of currency depreciations on output and investment. We compare the effects of an unanticipated increase in the world risk free interest rate on real aggregates under the financial conditions faced during the period prior the crisis and compare them with the observed outcome.

In order to solve the model quantitatively, we compute numerically a log-linear approximation of the decision rules for the variables in the model, following Klein (2000) and Sims (2002). All variables in the system of equations are expressed as log deviations from the deterministic steady state. Using the policy rules, we calculate impulse response functions of real variables to the shock on the external interest rate under each scenario.

4.1 Calibration

We calibrate the parameter values to the Peruvian economy. For the parameters related to the real sector of the economy, we use standard values in the literature on small open economy models. We do not directly target any moments for the variables in the real sector, as these results will help us to assess the balance sheet effects of the currency mismatch mechanism. The steady state world risk free rate is set at 4 percent, a standard value is small open economy models. The home good share in consumption, γ, is set at 0.8, based on the median of the distribution estimated by Elekgad, Justiniano, and Tchakarov (2006)[15]. The capital share in the production of the home good, α, is set at 0.45, value taken from the Quarterly Forecasting Model from the Central Bank of Peru. The parameter ν comes from the optimal contract

[15] This parameter is estimated to be between 0.6 and 0.99 for advanced economies in Lewis (1999), and Elekgad, Justiniano, and Tchakarov (2006) use Bayesian methods to estimate this parameter for Korea, and find the median to be between 0.6 and 0.8

and is set at 0.3. The discount factor, β, is set at 0.96, a standard value for annual data. For the value of δ, the fraction of entrepreneur's net income that goes to net worth, we use the value from Cespedes, Chang, and Velasco (2000) of 0.92.

We calibrate the key parameters of the financial sector to match the financial environment of the Peruvian economy during the Russian Crisis. The two parameters that are calibrated are the elasticity of the risk premium with respect to the leverage ratio, μ, and the steady state value of the risk premium, η^{SS}. For the risk premium, we target a 5 percent risk premium, consistent with an average EMBI index of 500 basis points in the period prior to the crisis, as documented by Castillo and Barco (2009). The leverage ratio is targeted to a value of of 3.2, following the average leverage ratio in Loveday, Molina, and Rivas-Llosa (2004) for a sample of two thousand non-financial firms.

Parameter	Value	Definition
ρ	0.04	Risk free interest rate
γ	0.8	Home goods share in consumption
α	0.45	Capital share in home good production
β	0.96	Discount factor
δ	0.92	Entrepeneur's saving rate
ν	0.3	Parameter from optimal contract
Calibrated parameters		
η	0.05	Steady state risk premium
μ	0.03	Elasticity of risk premium with respect to leverage

Table 2: Parameter Values

4.2 Russian Crisis

In this section, we present the exercise for the two extreme cases, where debt is completely denominated in dollars, as in the benchmark CCV model, and the case where all debt is denominated in soles. Comparing these results allows us to determine the highest possible gain from dedollarization in terms of reducing output and investment volatility and experiencing sharp real exchange rate depreciations.

We analyze the response of output and investment to a one-period unanticipated increase in the world interest rate. We consider the case of a shock that is consistent with a 600 basis points increase in the interest rate spread, as the one observed during the Russian crisis. The calibration used to match the average risk premium and leverage ratio prior to the Russian crisis is consistent with a financially vulnerable economy, as the elasticity of the risk premium with respect to the exchange rate is positive.

The mechanism in the model works as follows. A financially vulnerable economy suffers a larger output fall than a financially robust economy. In both cases, the opportunity cost of savers of lending in soles goes up, and therefore they will require that investors pay a higher interest rate.

In addition, under a financially vulnerable case, the risk premium increases with unexpected devaluation. An increase in the world interest rate depreciates the exchange rate, which reduces the capitalist's net worth and increases the debt burden in terms of home output. Due to imperfect capital markets, lenders will ask for a higher risk premium to compensate for the higher debt burden. Therefore, a financially vulnerable economy shows an amplification in output and investment drops.

In contrast, under a financially robust case, the risk premium falls with unexpected devaluation. Given the higher cost of borrowing due to the unexpected increase in the foreign risk free rate, investors take less debt, reducing the size of leverage. Foreign lenders will therefore ask for a lower risk premium, as lower leverage increases the incentives for repayment.

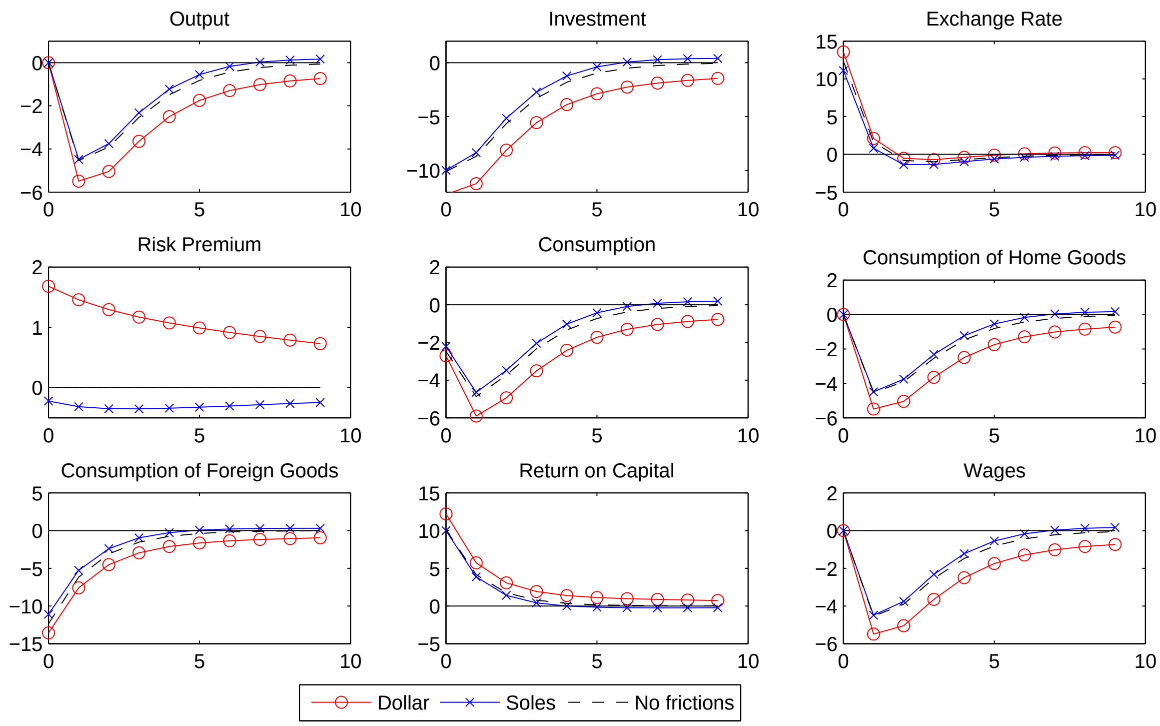

Figure 4: Impulse Response to a World Interest Rate Shock Consistent with a 600 Basis Point Increase in EMBI. Financially Vulnerable and Financially Robust Economy. Years depicted on the horizontal axis.

Figure 4 shows the response to an increase in the world interest rate consistent with a 600 basis point increase in country risk, as the one that was experienced in Peru during the Russian Crisis. For this calibration, we find that the economy economy would be classified as financially vulnerable according to the definitions in the previous section.

The sharp fall in investment reflects the increase in the cost of borrowing, through the effects of both a higher world interest rate and a higher risk premium. An increase in the world interest rate increases asymmetric information problems, so that there is a larger state space where agents would not repay their debt. Therefore, a higher risk premium is reflecting the increase in monitoring costs. Lower investment reduces the production possibilities of firms, and therefore, the sharp drop in output in the following period. This affects the payment to factors of production in $t = 1$ onwards, as a lower capital-to-labor ratio implies lower wages for workers in equilibrium. The rental rate of capital increases given the lower capital-to-labor ratio, making each unit of capital more valuable for investors, which should also be reflected in an increase in the price of capital.

From the household's perspective, under flexible prices, aggregate level of labor is always at steady state, so that aggregate income made by households falls due to lower wages. This implies that consumption falls as well. Analyzing consumption of home goods and imported goods separately we can identify two effects. On impact, at $t = 0$, there is a reduction in consumption of foreign goods, as the real exchange rate depreciates. In the next period, $t = 1$, there is an additional effect due to a reduction in the consumption of home goods. Given the functional form of aggregate consumption, households expenditure on domestic goods is a γ fraction of their income. As labor income falls, their consumption of home goods decreases as well.

We simulate the model for a baseline scenario, where we use a dollarization ratio of 100 percent, consistent with the benchmark model presented in CCV. The impulse response functions for this scenario are plotted with red lines in Figure 4 and are labeled as "Dollar". Each period corresponds to one year and the response of each variable is measured as percentage deviations with respect to their steady state.

The results for the baseline dollarization ratio predicts an output fall of 5.3 percent, whereas the output growth rate decreased from 6.9 percent in 1997 to -0.9 percent in 1998, after the Russian Crisis took place. Investment shows a similar pattern, with a predicted reduction of 11.7 percent, whereas investment growth rate fell from 16.3 percent in 1997 to -2.4 percent in 1998. The real exchange rate depreciation is predicted to be 13.1 percent, compared to the actual depreciation of 22 percent in 1998. Therefore, the model predicts that the balance sheet effect accounts for approximately 70 percent of the output drop and 60 percent of the investment drop and the real exchange rate depreciation. Notice that the model is simplified to focus on the balance sheet channel, and it ignores other aspects of the crisis, such as the effects of terms of trade shocks and exports.

Also, we must consider the fact that empirical work on the effects of financial crises in Peru show that there was a severe credit crunch during the Russian Crisis, where agents were not able to access external financing, regardless of the cost of borrowing. This effect is not considered in the model, but introducing this additional channel should help to explain larger output and investment drops.

The high level of currency mismatch in Peru during the 1990s might be related to the sharp drops in output and investment. In order to analyze the quantitative significance of dollarization in this model, we analyze the real effects under an economy with the same initial conditions as during the Russian Crisis, except for a lower level of debt dollarization. We repeat the exercise for the case where all debt is denominated in soles. As shown in the qualitative analysis, when $(1 - \pi) = 0$, the economy is always in the financially robust case.

The blue line in Figure 4 shows the impulse response functions for the extreme case where all debt is denominated in domestic currency, and is labeled as "Soles". An unanticipated increase in the world interest rate depreciates the exchange rate. If all debt is denominated in soles, the valuation of debt in terms of the home good does not change, as is the case of dollar denominated debt. Output falls by a smaller magnitude, given that it faces a higher domestic interest rate but a smaller risk premium. This second effect is explained by the elasticity of the risk premium with respect to net worth. As shown by Lemma 1, an economy with only sol-denominated debt always faces a negative elasticity of the risk premium with respect to the real exchange rate.

Therefore, even though there are still negative effects on real variables, these are quantitatively smaller than under fully dollarized debt. The intuition is as follows. Given the negative elasticity of the risk premium, an increase in the world interest rate reduces the risk premium, which partially offsets the increase in borrowing costs due to an increase in the interest rate. Therefore, a smaller increase in borrowing costs implies smaller drops in investment and output.

The payments to factors of production have milder effects as well. A reduction in the capital-to-labor ratio reduces wages and increases the rental rate of capital, but the magnitudes are smaller than in the fully-dollarized case. Consumption also falls, but the drop less intense, as explained in the digression on consumption and its relationship with the currency composition of debt.

Also, we should take into account that, even under the case with only sol-denominated debt, there is still a financial friction regarding the asymmetric information problem, and therefore, a wedge between the interest rate and the return on capital remains. However, by comparing the currency composition of debt, this wedge becomes smaller as more debt is denominated in domestic currency.

The quantitative results show that the output drop is reduced from 5.3 to 4.5 percent, whereas investment falls by 9.9 percent, compared to 11.7 percent when we considered dollar denominated debt. The economy would face a real depreciation of 11.0 percent, lower than in the benchmark case. Overall, these results show that there are some improvements in the real effects of an unanticipated foreign interest rate shock, consistent with the empirical literature where lower dollarization is associated with less frequent financial crisis and smaller balance sheet effects.

For economies that take debt in both currencies, as stated in Proposition 3, a dollarization ratio that is lower than $1-\bar{\pi}$ means that the economy is financially robust. This implies that for low values of the dollarization ratio, the amplification effect of currency mismatch are milder than for higher values. We simulate the economy for a grid of different dollarization ratios to analyze the quantitative relevance of the amplification mechanism under each scenario. Figure 5 shows the impulse response functions for different currency compositions of debt. As in Figure 4, all impulse responses are measured as percentage deviations from steady state. Each line represents different currency composition of debt, where the one labeled $\bar{\pi}$ represents the case with a constant risk premium.

Table 3 shows the one-period effect on real variables to an increase in the world interest rate for different values of the dollarization ratio. The results are consistent with the qualitative analysis, where a higher dollarization ratio increases the currency mismatch of firms' balance sheets and amplifies the negative effect on real variables. Following CCV, we establish $\bar{\pi}$ as the threshold between a financially robust and a financially vulnerable economy, that is when there is a risk premium that is inelastic to exchange rate depreciations. For the parameters in the quantitative exercise, this happens when the economy faces a dollarization ratio of about 22 percent.

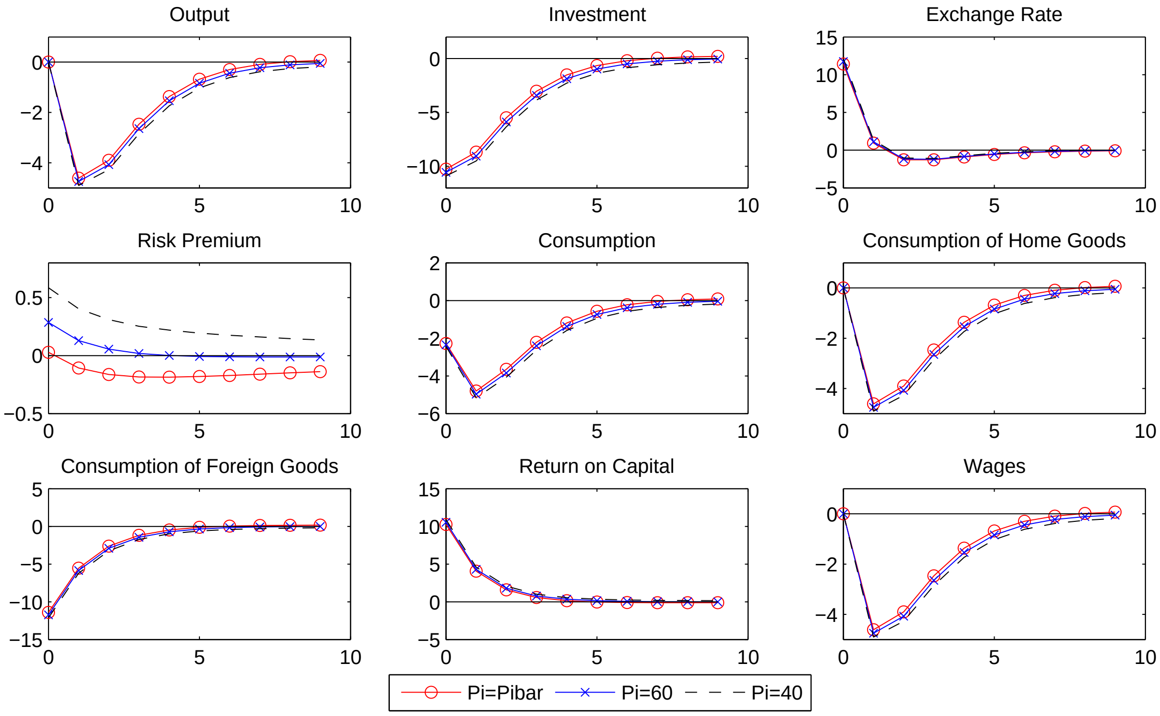

Figure 5: Impulse Response to a World Interest Rate Shock Consistent with a 600 Basis Point Increase in EMBI. Economies with debt in dollars and soles. Years depicted on the horizontal axis.

Notice that dollarization ratios that are higher than $1 - \bar{\pi}$, such as 40 percent in Peru during the Russian crisis, would imply that the economy was financially vulnerable and therefore experienced an additional amplification of the effects on real variables.

			Dollarization	ratio	(1-π)		
	0	0.2	$1-\bar{\pi}$	0.4	0.6	0.8	1
Output	-4.5	-4.6	-4.61	-4.75	-4.90	-5.07	-5.30
Investment	-9.9	-10.2	-10.3	-10.6	-10.9	-11.3	-11.7
RER	11.0	11.4	11.4	11.7	12.1	12.5	13.1
Welfare (χ)	0.010	0.015	0.015	0.020	0.025	0.030	0.036

Table 3: Welfare analysis

It is also relevant to compare these results to a model with no financial frictions, to evaluate the relative importance of the effects of dedollarizing debt. We simulate the model without financial frictions, by eliminating the wedge between the interest rate and the return on capital (and therefore a zero risk premium at all times). The results are shown in Figure 4 by the black lines labeled as “No Frictions”. An increase in the world interest rate by one standard deviation results in an output drop of 4.5 percent and an investment drop of 10.1 percent, whereas the real depreciation is 12.3 percent. These results are in line with work such as Kocherlakota (2000) and Cordoba and Ripoll (2004), where under standard parameter values, the amplification effect introduced by endogenous financial frictions is very small. Compared to this result, the introduction of a currency mismatch increases amplification by a larger magnitude, showing the potential benefits of policies oriented to dedollarization.

The model is able to capture several stylized facts obtained in the empirical literature on the effects of debt dollarization on output and financial fragility. Higher debt dollarization, as the one used in the benchmark scenario, leads to higher output volatility as in Reinhart, Rogoff, and Savastano (2003b), as the cost of borrowing becomes more volatile due to the effects of currency mismatch. A larger currency mismatch increases the volatility of output measured in dollars and results in more volatile monitoring costs. Therefore, the borrower must pay a more volatile risk premium, which results in more volatile investment and output. Higher financial fragility is also associated with higher debt dollarization as higher volatility of output in terms of debt repayment increase the probability that agents would prefer not to repay, and translate into more volatile agency costs and risk premium. Regarding the frequency of financial crises and sudden stop episodes, the results show a higher drop in foreign debt when a bad shock hits the economy under higher debt dollarization, consistent with larger capital account reversals for countries with high liability dollarization.

Finally, we simulate the model for 10,000 periods and compute the variance of consumption deviations from the deterministic steady state for debt denominated in dollars and in soles to give as an approximate measure of welfare. The variance of consumption is 0.007 for the benchmark case with full dollar denominated debt, larger than the variance of consumption 0.0042 with full sol-denominated debt. If the stochastic steady state is close enough to the deterministic steady state, as mentioned in Woodford (2002), this implies that welfare is higher in the environment with sol-denominated debt.

We also calculate the equivalent variation as an approximate measure of welfare loss compared to an economy in steady state. We follow the definition of this welfare measure, χ, as presented previously. The results of the equivalent variation, χ, is shown in Table 3 for different values of the dollarization ratio. Consistent with the previous section on welfare implications, higher debt dollarization is welfare reducing, although the effect on welfare is very small.

5 Conclusions

This work reevaluates the equilibrium properties of the model presented originally by CCV, letting entrepreneurs borrow in both domestic and foreign currency. Under the assumptions introduced here, it is observed that a lower ratio of dollarization reduces the effect of real exchange rate depreciations on investment and production decisions under a flexible exchange rate regime. The lower the degree of dollarization, the less dramatic the net worth effects are. Not trivially, Lemma 1 says that if debt is contracted only in domestic currency, the economy will always be financially robust.

The quantitative results show the effects of de-dollarization in the Peruvian economy and how financial fragility in the sense of higher debt dollarization amplified the real effects during the Russian Crisis. The model is able to account for around 70 percent of the output and investment drops in the case of the Russian Crisis, whereas it accounts for 50 percent of the exchange rate depreciation. As shown in both the qualitative and quantitative analysis, a lower level of dollarization would have reduced the impact of a financial crisis. Full sol-denominated debt in the model would imply a lower output drop of 4.8 percent, instead of the drop of 5.7 percent in the benchmark calibration.

Notice that the quantitative analysis assumes full capital depreciation as in CCV. Partial capital depreciation would reduce the size of the amplification mechanism, as entrepreneurs would face lower debt requirements and therefore a lower leverage ratio. As the cost of financing is increasing in leverage, lower leverage would result in a smaller increase in the risk premium and milder effects on output, investment and real depreciation.

Another assumption in this work is that the currency composition of debt is taken as exogenous. However, Chang and Velasco (2006) and Chamon and Hausmann (2005), among others, claim that dollarization has endogenous roots that may be dependent on the exchange rate regime expected to prevail in the economy. When individuals expect flexible exchange rates, they borrow in domestic currency, whereas the composition of debt would be indeterminate when individuals expect fully credible fixed exchange rates. This is consistent with our results. If the supply of debt in soles were higher than the demand for debt by entrepreneurs, as when $\pi = 1$, they would take all debt in soles. However, given that there is a shortage in the supply of sol-denominated debt, investors must finance the difference by taking some dollar denominated debt.

The framework presented in this work is a simplified version because the main objective is to focus on the effect of the currency composition of firms' liabilities on the magnitude of the balance sheet effect. Further work on an extension of the model should include other channels that are highly relevant in the behavior of output and investment, such as the impact of terms of trade shocks and commodity prices in the Peruvian economy. During the financial crises analyzed in this work, exports were severely affected through the worsening of terms of trade, which further reduced output and investment.

References

Aguiar, M. (2005): "Investment, devaluation and foreign currency exposure," *Journal of Development Economics*, 78(1), 95–113.

Bernanke, B., M. Gertler, and S. Gilchrist (1999): "The financial accelerator in a quantitative business cycle framework," in *Handbook of Macroeconomics*, ed. by M. Taylor, and M. Woodford, vol. 1. North-Holland.

Caballero, R., and A. Krishnamurthy (2003): "Excessive dollar debt: Financial development and underinsurance," *Journal of Finance*, 58(3), 867–893.

Calvo, G., A. Izquierdo, and L. Mejia (2008): "Systemic sudden stops: The relevance of balance-sheet effects and financial integration," Working Paper 14026, National Bureau of Economic Research.

Carlstrom, C., and T. Fuerst (1997): "Agency costs, net worth and business fluctuations: A computable general equilibrium analysis," *American Economic Review*, 87(5), 893–910.

Castillo, P., and D. Barco (2009): "Crisis financieras y manejo de reservas en el Peru," *Revista Estudios Economicos*, 17, 85–112.

Cespedes, L., R. Chang, and A. Velasco (2000): "Balance sheets and exchange rate policy," Working Paper 7840, National Bureau of Economic Research.

——— (2004): "Balance sheets and exchange rate policy," *American Economic Review*, 94(4), 1183–1193.

Chamon, M., and R. Hausmann (2005): "Why do countries borrow the way they borrow?," in *Other people's money: Debt denomination and financial stability in emerging market economies*, ed. by B. Eichengreen, and R. Hausmann. University of Chicago Press.

Chang, R., and A. Velasco (2006): "Currency mismatches and monetary policy: A tale of two equilibria," *Journal of International Economics*, 69(1), 150–175.

Christensen, I., and A. Dib (2008): "The financial accelerator in an estimated new keynesian model," *Journal of Economic Dynamics*, 11(1), 155–178.

Cordoba, J., and M. Ripoll (2004): "Credit cycles redux," *International Economic Review*, 45(4), 1011–1046.

De Nicolo, G., P. Honohan, and A. Ize (2003): "Dollarization of the banking system: Good or bad?," Working Paper 03146, International Monetary Fund.

Eichengreen, B., and R. Hausmann (1999): "Exchange rates and financial fragility," Working Paper 7418, National Bureau of Economic Research.

Elekdag, S., and I. Tchakarov (2007): "Balance sheets, exchange rate policy and welfare," *Journal of Economic Dynamics and Control*, 31(12), 3986–4015.

Elekgad, S., A. Justiniano, and I. Tchakarov (2006): "An estimated small open economy model of the financial accelerator," *IMF Staff Papers*, 53(2), 219–241.

Forbes, K. (2002): "How do large depreciations affect firm performance?," Working Paper 9095, National Bureau of Economic Research.

Gertler, M., S. Gilchrist, and F. Natalucci (2007): "External constraints on monetary policy and the financial accelerator," *Journal of Money, Credit and Banking*, 39(2), 295–330.

Kim, J., and S. Kim (2003): "Spurious welfare reversals in international business cycle models," *Journal of International Economics*, 60(2), 471–500.

Klein, P. (2000): "Using the generalized Schur form to solve a multivariate linear rational expectations model," *Journal of Economic Dynamics and Control*, 24(10), 1405–1423.

Kocherlakota, N. (2000): "Creating business cycles through credit constraints," *Quarterly Review Federal Reserve Bank of Minneapolis*, 24(3), 2–10.

Levy-Yeyati, E. (2006): "Financial dollarization: Evaluating the consequences," *Economic Policy*, 21(54), 61–118.

Lewis, K. (1999): "Trying to explain home bias in equities and consumption," *Journal of Economic Literature*, 37(2), 571–608.

Loveday, J., O. Molina, and R. Rivas-Llosa (2004): "Mecanismos de transmision de la politica monetaria y el impacto de una devaluacion en el nivel de las firmas," *Revista Estudios Economicos*, 12.

Lucas, R. (1987): *Models of business cycles*. Basil Blackwell.

Reinhart, C., K. Rogoff, and M. Savastano (2003a): "Addicted to dollars," Working Paper 10015, National Bureau of Economic Research.

——— (2003b): "Debt intolerance," Working Paper 9908, National Bureau of Economic Research.

Schmitt-Grohe, S., and M. Uribe (2004): "Solving dynamic general equilibrium models using a second-order approximation to the policy function," *Journal of Economic Dynamics and Control*, 28(4), 755–775.

Sims, C. (2002): "Solving linear rational expectations models," *Computational economics*, 20(1-2), 1–20.

Tovar, C. (2005): "The mechanics of devaluations and the output response in a DSGE model: How relevant is the balance sheet effect?," Working Paper 192, Bank of International Settlements.

Woodford, M. (2002): "Inflation stabilization and welfare," *Contributions to macroeconomics*, 2(1).

Appendix A: Sketch of the derivation of the risk premium

Consider the contracting problem between a single investor, indexed by j, and both savers and foreign lenders in any period t. At the time of contracting, j's net worth $P_t N_t^{I,j}$, the *dollar* interest rate ρ^*_{t+1}, the *sol* interest rate ρ_{t+1}, and prices in period t are known. Assume for the time being that the period $t+1$ rental rate on capital in dollars, R_{t+1}/S_{t+1}, is known.

Both types of entrepreneurs and foreign creditors are risk neutral. Their joint problem is to choose a level of investment K^j_{t+1}, a *dollar* loan D^{*j}_{t+1}, a *sol* loan D^j_{t+1}, and a repayment schedule so as to maximize the expected return to savers, so that lenders are paid at least their opportunity cost of funds, subject to resource and information constraints.

Investment in t, K^j_{t+1}, yields $\omega^j_{t+1} K^j_{t+1}(R_{t+1}/S_{t+1})$ dollars next period, where ω^j_{t+1} is a random shock. The distribution of this variable is widely known and is such that ω^j_{t+1} is i.i.d. across j and t, and its expected value is one. I assume that the realization of ω^j_{t+1} cannot be observed by lenders (both savers and foreign agents), unless they pay a proportional monitoring cost of $\zeta\omega^j_{t+1} K^j_{t+1}(R_{t+1}/S_{t+1})$; in contrast, ω^j_{t+1} is observed freely by the investor.

CCV argue that these conditions imply a standard debt contract, which stipulates a fixed repayment, say of B^{*j}_{t+1} dollars and B^j_{t+1} soles; if the investor cannot pay, lenders monitor the outcome and seize all the outcome. It is noticeable that this situation occurs as long as ω^j_{t+1} is low enough. Letting $\bar{\omega}$ be such that $B^{*j}_{t+1} + B^j_{t+1}/S_{t+1} = \bar{\omega} K^j_{t+1}(R_{t+1}/S_{t+1})$, monitoring happens whenever $\bar{\omega}$ is above ω^j_{t+1}.

Then, the problem is to provide lenders with expected returns of ρ_{t+1} and ρ^*_{t+1}. Therefore, the following condition must hold:

$$K^j_{t+1} R_{t+1}\left[\bar{\omega}(1-H(\bar{\omega})) + (1-\zeta)\int_0^{\bar{\omega}} \omega^j_{t+1} dH(\omega^j_{t+1})\right] = (1+\rho^*_{t+1})S_{t+1}D^{*j}_{t+1} + (1+\rho_{t+1})D^j_{t+1}$$

and using equation (12) in its deterministic version, the right hand side becomes:

$$\begin{aligned}
&= (1+\rho^*_{t+1})S_{t+1}D^{*j}_{t+1} + (1+\rho^*_{t+1})\frac{S_{t+1}}{S_t}D^j_{t+1} \\
&= (1+\rho^*_{t+1})\frac{S_{t+1}}{S_t}\left[S_t D^{*j}_{t+1} + D^j_{t+1}\right] \\
&= (1+\rho^*_{t+1})\frac{S_{t+1}}{S_t}\left[Q_t K^j_{t+1} - P_t N_t^{I,j}\right] \qquad \text{(A.1)}
\end{aligned}$$

where $H(\cdot)$ denotes the c.d.f. of ω^j_{t+1}, and $H(\bar{\omega})$ is the probability of bankruptcy.

The left hand side of equation (A.1) gives the expected dollar yield on investment. The first equality on the right hand side gives the opportunity cost of loans D^{*j}_{t+1} and D^j_{t+1}. The second and subsequent inequalities reflect that borrowing must equal the value of investment minus the investor's net worth.

As in the original work, the optimal contract maximizes the investor's utility:

$$\left[\int_{\bar{\omega}}^{\infty} \omega_{t+1}^{j} dH(\omega_{t+1}^{j}) - \bar{\omega}(1 - H(\bar{\omega}))\right] R_{t+1} K_{t+1}^{j} \tag{A.2}$$

subject to equation (A.1), which once simplified yields:

$$\kappa_{jt} - 1 = (1 + \eta_{t+1})\kappa_{jt} \left[\bar{\omega}(1 - H(\bar{\omega})) + (1 - \zeta) \int_{0}^{\bar{\omega}} \omega_{t+1}^{j} dH(\omega_{t+1}^{j})\right] \tag{A.3}$$

where

$$\kappa_{jt} = Q_t K_{t+1}^{j} / P_t N_t^{I,j} \tag{A.4}$$

is the ratio of the value of investment to net worth, and

$$1 + \eta_{t+1} = \frac{R_{t+1} S_t}{(1 + \rho_{t+1}^{*}) Q_t S_{t+1}} \tag{A.5}$$

is the risk premium.[16]

We can also derive the behavior of the risk premium in terms of the domestic interest rate. Using equation (A.1) and the deterministic version of (12), the participation constraint becomes:

$$K_{t+1}^{j} R_{t+1} \left[\bar{\omega}(1 - H(\bar{\omega})) + (1 - \zeta) \int_{0}^{\bar{\omega}} \omega_{t+1}^{j} dH(\omega_{t+1}^{j})\right] = (1 + \rho_{t+1}) \left[Q_t K_{t+1}^{j} - P_t N_t^{I,j}\right]$$

Once simplified, the participation constraint yields:

$$1 + \eta_{t+1} = \frac{R_{t+1}}{(1 + \rho_{t+1}) Q_t}$$

It is possible to express the entrepeneur's problem in terms of κ_{jt}, as maximizing the following objective function:

$$\left[\int_{\bar{\omega}}^{\infty} \omega_{t+1}^{j} dH(\omega_{t+1}^{j}) - \bar{\omega}(1 - H(\bar{\omega}))\right] \kappa_{jt} \tag{A.6}$$

Then, the problem reduces to choosing both κ_{jt} and $\bar{\omega}$ to maximize equation (A.6) subject to equation (A.3). It is clear that the analysis concerned here is close to that of CCV, so the ninth footnote in the main text holds.

Specifically, CCV show that under suitable conditions $\bar{\omega}$ is an increasing function of $1 + \eta_{t+1}$ (notice that the risk premium is a parameter of the investor's problem), or in its inverse form:

$$1 + \eta_{t+1} = \Delta(\bar{\omega}) \tag{A.7}$$

where $\Delta(\cdot)$ is an increasing and differentiable function in the $(0, \omega^{*})$ domain, and ω^{*} is the maximizing value of $\bar{\omega}(1 - H(\bar{\omega})) + (1 - \zeta) \int_{0}^{\bar{\omega}} \omega_{t+1}^{j} dH(\omega_{t+1}^{j})$, under certain

[16] Notice that this expression is very similar to equation (14a) in the main text. In fact, in the absence of uncertainty, both equations are the same.

conditions. Here, the optimal cutoff depends only on the risk premium, that is, it is orthogonal to j's net worth.

Then, the authors argue that the optimal investment/net worth ratio, κ_{jt} is also a function of $\bar{\omega}$:

$$\kappa_{jt} = \Psi\left(\bar{\omega}\right) \tag{A.8}$$

where $\Psi\left(\cdot\right)$ is also increasing and differentiable in the $(0, \omega^{*})$ domain. Due to the independency of $\bar{\omega}$ and j, κ_{jt} is the same for all j, so that aggregation over j is possible:

$$\frac{Q_t K_{t+1}}{P_t N_t^I} = \Psi\left(\bar{\omega}\right) \tag{A.9}$$

If equations (A.7) and (A.9) are combined, it is straightforward to get the risk premium as a function of the value of total investment relative to total net worth:

$$1 + \eta_{t+1} = \Delta\left[\Psi^{-1}\left(\frac{Q_t K_{t+1}}{P_t N_t^I}\right)\right] \equiv F\left(\frac{Q_t K_{t+1}}{P_t N_t^I}\right) \tag{A.10}$$

where F is increasing and differentiable and accounts for equation (15) in the main text.

Until now, the explanation has assumed certainty at the time of contracting. The reader can consult the original work to see what happens if R_{t+1}/S_{t+1} is not known with certainty and is replaced by its expectation at t, in order to get equation (14a) in the main text.

Appendix B: Non-stochastic steady state

To simplify expressions, we define the following parameters in terms of steady state values: $\pi = \frac{D}{D+SD^{*}}$, $\psi^{*} = \frac{D+SD^{*}}{N}$ and $\mu = \frac{F'(\cdot)}{F(\cdot)}\frac{QK}{N}$. The equations that characterize the non-stochastic steady state of the model are:

$$Y = AK^{\alpha} \tag{B.1}$$

$$Q = S^{1-\gamma} \tag{B.2}$$

$$\rho = \rho^{*} \tag{B.3}$$

$$1 + \psi^{*} = \frac{QK}{N} \tag{B.4}$$

$$\frac{\alpha Y}{QK} = (1+\rho^{*})(1+\eta) \tag{B.5}$$

$$1 = \frac{\delta \alpha Y}{N}(1-\Phi) - \delta(1+\rho^{*})\left(\frac{QK}{N} - 1\right) \tag{B.6}$$

$$Y = \gamma \left[(1-\alpha)Y + QK \right] + SX \tag{B.7}$$

Where equations (B.1), (B.2), (B.3), (B.4), (B.5), (B.6) and (B.7) are the steady state versions of equations (1), (11), (12), (13a), (14a), (16a) and (17), respectively.

These equations, together with π and the value of $\bar{\omega}$ from the optimal debt contract (see Appendix A), form a square system of nine equations in nine unknowns, namely, $Y, S, K, Q, N, D, D^*, \rho$ and η.

Appendix C: Log-linearized system of equations

The log-linearization of equations (1), (11), (12) and (14a) around the non-stochastic steady state, together with (18) and (D.2), yields, respectively:[17]

$$y_t = \alpha k_t + (1-\alpha) l_t \tag{C.1}$$

$$q_t - p_t = (1-\gamma)(s_t - p_t) \tag{C.2}$$

$$\tilde{\rho}_{t+1} = \tilde{\rho}^*_{t+1} + \mathbb{E}_t \left(s_{t+1} - s_t \right) \tag{C.3}$$

$$\tilde{\rho}^*_{t+1} = -\eta'_{t+1} + \mathbb{E}_t(y_{t+1}) - (q_t - p_t) - k_{t+1} - \mathbb{E}_t(s_{t+1} - p_{t+1}) + (s_t - p_t) \tag{C.4}$$

$$\frac{y_t}{\lambda} - \frac{1-\lambda}{\lambda}(s_t - p_t) = q_t - p_t + k_{t+1} \tag{C.5}$$

$$\eta'_{t+1} - \phi \eta'_t = \mu \left[\left(\frac{1-\lambda}{\lambda} \right) (y_t - e_t) \right] + \mu \psi^* \delta (1+\rho^*) \left[(1-\pi)\left(e_t - \mathbb{E}_{t-1} e_t \right) - \left(y_t - \mathbb{E}_{t-1} y_t \right) \right] \tag{C.6}$$

These equations and the monetary policy rule ($p_t = 0$ for flexible exchange rates) characterize the behavior of y_t, p_t, q_t, k_t, s_t, ρ_t and η_t. Additionally, we can use the log-linearization of equations (4b), (13a), (15) and (16a) around the non-stochastic steady state to get the values of d_t, d^*_t, n_t and w_t.

$$w_t - l_t = p_t - y_t \tag{C.7}$$

$$p_t + n_t = \frac{QK}{N}(q_t + k_{t+1}) + \left(1 - \frac{QK}{N} \right) (s_t + d^*_{t+1}) - \frac{D}{N}(d_{t+1} - d^*_{t+1} - s_t) \tag{C.8}$$

$$p_t + n_t = q_t + k_{t+1} - \frac{1}{\mu}\eta'_{t+1} \tag{C.9}$$

[17] Furthermore, refer to Appendix B of CCV to see that ν is the elasticity of $\int_0^{\bar{\omega}} \omega_t^j dH(\omega_t^j)$ with respect to $\bar{\omega}$ and ε_Δ is the elasticity of the Δ function in equation (A.10).

$$p_t + n_t = \frac{\delta\alpha Y}{N}(p_t + y_t) - \frac{\delta\alpha Y}{N}\Phi\left(\frac{\nu}{\varepsilon_\Delta}\eta_t' + p_t + y_t\right) +$$
$$\left[\frac{1 - \frac{\delta\alpha Y}{N}(1-\Phi)}{\frac{QK}{N} - 1}\right]\left[\left(\frac{QK}{N} - 1\right)(\tilde{\rho}_t^* + s_t + d_t^*) + \frac{D}{N}(d_t + \tilde{\rho}_t + \tilde{\rho}_t^* - s_t - d_t^*)\right] \tag{C.10}$$

Appendix D: Proofs

Proof of Proposition 1. The third term of the RHS of equation (C.10), after plugging equations (C.3), (C.4), (C.8) and (C.9) is:
$\left[1 - \frac{\delta\alpha Y}{N}(1-\Phi)\right]\left\{-\eta_t' + \mathbb{E}_{t-1}y_t - \mathbb{E}_{t-1}(s_t - p_t) + s_t + \pi\left[\mathbb{E}_{t-1}(s_t - s_{t-1}) - (s_t - s_{t-1})\right] + \frac{1}{\frac{QK}{N}-1}\frac{1}{\mu}\eta_t'\right\}$
Notice that:

$$\left[1 - \frac{\delta\alpha Y}{N}(1-\Phi)\right]\left(-\eta_t' + \frac{1}{\frac{QK}{N} - 1}\frac{1}{\mu}\eta_t'\right) = \left[1 - \frac{\delta\alpha Y}{N}(1-\Phi)\right]\left[\frac{1 + (1-\sigma)\mu}{\left(\frac{QK}{N} - 1\right)\mu}\right]\eta_t'$$
$$= -\delta(1+\rho^*)\left(\frac{1-\psi^*\mu}{\mu}\right)\eta_t'$$

Also:

$$\left[1 - \frac{\delta\alpha Y}{N}(1-\Phi)\right] = \delta(1+\rho^*)\left(\frac{QK}{N} - 1\right)$$
$$= \delta(1+\rho^*)\psi^*$$

Then, after rearranging equation (C.9), it must be the case that:

$$\eta_{t+1}' = \left[\mu\frac{\delta\alpha Y}{N}\Phi\left(\frac{\nu}{\varepsilon_\Delta}\right) + \delta(1+\rho^*)(1-\psi^*\mu)\right]\eta_t' + \mu(q_t - p_t + k_{t+1} - y_t) +$$
$$\mu\delta(1+\rho^*)\psi^*\left\{s_t - p_t - \mathbb{E}_{t-1}(s_t - p_t) - (y_t - \mathbb{E}_{t-1}y_t) + \pi\left[\mathbb{E}_{t-1}(s_t - s_{t-1}) - (s_t - s_{t-1})\right]\right\}$$

But, after using equations (B.4), (B.5) and (B.6), we have:

$$\frac{\delta\alpha Y}{N}\Phi = \frac{\delta\alpha Y}{N} - \left[1 + \delta(1+\rho^*)\psi^*\right]$$
$$= \delta(1+\psi^*)(1+\rho^*)(1+\eta) - \left[1 + \delta(1+\rho^*)\psi^*\right]$$
$$= \delta(1+\psi^*)(1+\rho^*)\eta + \delta(1+\rho^*) - 1$$

Then:

$$\eta_{t+1}' - \phi\eta_t' = \mu(q_t - p_t + k_{t+1} - y_t)$$
$$+\mu\psi^*\delta(1+\rho^*)\left\{s_t - p_t - \mathbb{E}_{t-1}(s_t - p_t) - (y_t - \mathbb{E}_{t-1}y_t) - \pi\left[(s_t - s_{t-1}) - \mathbb{E}_{t-1}(s_t - s_{t-1})\right]\right\} \tag{D.1}$$

where $\phi = \delta(1+\rho^*)(1-\psi^*\mu) + \mu\left[\delta(1+\psi^*)(1+\rho^*)\eta + \delta(1+\rho^*) - 1\right]\left(\frac{\nu}{\varepsilon_\Delta}\right)$

Plug equations (4b) and (10) into equation (17) and log-linearize the resulting expression:

$$p_t + y_t = \frac{\gamma}{1-\gamma(1-\alpha)} \frac{QK}{Y}(q_t + k_{t+1}) + \frac{s_t}{1-\gamma(1-\alpha)} \frac{SX}{Y}$$

Then, substitute equations (B.5) and (B.7) to get:

$$y_t = \frac{\alpha\gamma}{[1-\gamma(1-\alpha)]\,(1+\rho^*)(1+\eta)}(q_t + k_{t+1} - s_t) + s_t - p_t$$

Finally, after setting $\lambda = \frac{\alpha\gamma}{[1-\gamma(1-\alpha)](1+\rho^*)(1+\eta)}$, the resulting expression is:

$$\frac{y_t}{\lambda} - \frac{1-\lambda}{\lambda}(s_t - p_t) = q_t - p_t + k_{t+1} \tag{D.2}$$

After plugging this last equation (D.2) into equation (D.1), one obtains equation (18) in the main text. ■

Proof of Lemma 1. Replace time subscripts in equation (18) and recall that under flexible exchange rates it must be the case that $\eta_0' = \mathbb{E}_0 e_1 = \mathbb{E}_0 y_1 = \mathbb{E}_0 s_1 = y_1 = 0$. Therefore, the second term of the RHS of equation (18) is equal to $\mu\psi^*\delta(1+\rho^*)(e_0 - \pi s_0)$. But, since $e_0 = s_0$, it is straightforward to reach equation (19). ■

Proof of Proposition 2. For the IS curve, replace time subscripts in equations (C.1), (C.2) and (C.5), and recall that in the period of the shock, $l_0 = k_0 = y_0 = 0$ under flexible exchange rates. On the other hand, for the BP curve, replace time subscripts in equations (C.1), (C.2) and (19) under perfect foresight, and then use $z_t = \zeta\eta_t'$ (see section on saddle path coefficient). Aditionally recall that $z_1 = y_1 - e_1$ and $l_1 = 0$ under flexible exchange rates. ■

Proof of Corollary 1. Replace $\pi = 1$ in the definition of the elasticity of the risk premium with respect to a change in the real exchange rate. Since $0 < \lambda < 1$, then the elasticity is always negative and hence the economy is financially robust. ■

Saddle-path coefficient. Under perfect foresight, if $z_t = y_t - e_t$ denotes home output expressed in dollars, equation (18) reduces to:

$$\eta_{t+1}' - \phi\eta_t' = \mu\left[\left(\frac{1-\lambda}{\lambda}\right) z_t\right] \tag{D.3}$$

We rearrange equations (C.4) and (D.2); recall that there is perfect foresight and the definition of the real exchange rate. Then replace equations (C.1) and (C.2) into the former ones to get, respectively:

$$y_t = \lambda\left[(1-\gamma)\,e_t + \alpha^{-1}y_{t+1}\right] + (1-\lambda)e_t$$
$$y_{t+1} - \left[(1-\gamma)\,e_t + \alpha^{-1}y_{t+1}\right] = \tilde{\rho}_{t+1}^* + \eta_{t+1}' + e_{t+1} - e_t$$

Using these two expressions, it is possible to show that in equilibrium:

$$z_{t+1} = \lambda^{-1}z_t + \eta_{t+1}' + \tilde{\rho}_{t+1}^* \tag{D.4}$$

Then, equations (D.3) and (D.4) describe the perfect foresight dynamics of both z_t and η'_t, provided that $\tilde{\rho}^*_{t+1} = 0$. CCV state that in period t the risk premium η'_t is known, but output measured in dollars z_t is not. However, as in the original work, it is possible to find a stable saddle path, which links both terms through a negative saddle path coefficient ζ. It can be shown that there is a unique linear relationship such that $z_t = \zeta\eta'_t$, where if the risk premium rises above its steady state level, the output measured in dollars will fall below its steady state level, and vice versa.

This demonstrates that under the assumptions addressed here, the introduction of sol-denominated debt does not alter quantitatively the stability of the original system. That is, the composition of debt does not influence the saddle path but only the total amount of outstanding debt in the steady state. ■

Derivation of the Budget Constraint. Deriving the budget constraint

$$\begin{aligned} P_tY_t &= R_tK_t + W_tL_t \\ &= R_tK_t + P_tC^H_t + S_tC^F_t \end{aligned} \tag{D.5}$$

From the net worth equation

$$R_tK_t = \frac{P_tN^I_t}{\delta} + \alpha\Phi_tP_tY_t + (1+\rho^*_t)S_tD^*_t + (1+\rho_t)D_t \tag{D.6}$$

From the balance of payments equation

$$S_tX - S_tC^F_t - S_tK^F_{t+1} - \alpha P_tY_t + S_tD^*_{t+1} - (1+\rho^*_t)S_tD^*_t - \frac{(1-\delta)}{\delta}(P_tN^I_t + P_tN^{II}_t) = 0 \tag{D.7}$$

Plugging equation (D.6) in (D.5) and adding (D.7)

$$\begin{aligned} P_tY_t = &\frac{P_tN^I_t}{\delta} + \alpha\Phi_tP_tY_t + (1+\rho^*_t)S_tD^*_t + (1+\rho_t)D_t + P_tC^H_t + S_tC^F_t \\ &+ S_tX - S_tC^F_t - S_tK^F_{t+1} - \alpha\Phi_tP_tY_t + S_tD^*_{t+1} - (1+\rho^*_t)S_tD^*_t - \frac{(1-\delta)}{\delta}(P_tN^I_t + P_tN^{II}_t) \\ = &P_tN^I_t + (1+\rho_t)D_t + P_tC^H_t + S_tX - S_tK^F_{t+1} + S_tD^*_{t+1} - \frac{1-\delta}{\delta}P_tN^{II}_t \end{aligned}$$

Savers face the following budget constraint,

$$P_tN^{II}_t = D_{t+1} \tag{D.8}$$

At the end of each period, savers collect the income from the repayment of domestic debt. They consume a $1-\delta$ fraction of this income, everything in terms of imported goods. Thus, savers' net worth is:

$$P_tN^{II}_t = \delta(1+\rho_t)D_t \tag{D.9}$$

Using equations (D.8) and (D.9),

$$P_tY_t = Q_tK_{t+1} - S_tD^*_{t+1} - D_{t+1} + (1+\rho_t)D_t + P_tC^H_t + S_tX - S_tK^F_{t+1} + S_tD^*_{t+1} + D_{t+1} - \frac{1}{\delta}P_tN^{II}_t$$
$$= P_tK^H_{t+1} + S_tK^F_{t+1} + (1+\rho_t)D_t + P_tC^H_t + S_tX - S_tK^F_{t+1} - (1+\rho_t)D_t$$
$$= \gamma Q_t(C_t + K_{t+1}) + S_tX \tag{D.10}$$

■

Digression on consumption and welfare. Similar to the qualitative analysis on the behavior of investment and real exchange rates in the previous section, we analyze the response of consumption to a shock in the world interest rate and how it is affected by the currency composition of debt.

In a symmetric equilibrium, from equations (10) and (11),

$$C_t = \frac{W_tL_t}{P_t^{\gamma}S_t^{1-\gamma}}$$

Or:

$$C_t = (1-\alpha)\left(\frac{P_t}{S_t}\right)^{1-\gamma} AK_t^{\alpha}$$

Log-linearizing this equation in an economy with flexible prices ($p_t = 0$) :

$$c_t = \alpha k_t - (1-\gamma)e_t$$

Following the previous analysis, we assume a shock to the world safe interest rate at t = 0, whose effects disappear from t = 1 onwards. The behavior of consumption in period t=0 is given by

$$\begin{aligned} c_{o,\pi} &= \alpha k_{0,\pi} - (1-\gamma)e_0 \\ &= -(1-\gamma)\frac{\lambda\widetilde{\rho}_1}{\lambda\left(\zeta-1\right)\varepsilon_{\pi}^{flex}+1} \end{aligned}$$

Taking the derivative of $c_{0,\pi}$ with respect to π

$$\begin{aligned} \frac{dc_{0,\pi}}{d\pi} &= -\frac{(1-\gamma)\lambda^2(1-\zeta)\widetilde{\rho}_1}{\left[\lambda\left(\zeta-1\right)\varepsilon_{\pi}^{flex}+1\right]^2}\frac{d\varepsilon_{\pi}^{flex}}{d\pi} \\ &= \frac{(1-\gamma)\lambda^2(1-\zeta)\mu\delta(1+\rho^*)}{\left[\lambda\left(\zeta-1\right)\varepsilon_{\pi}^{flex}+1\right]^2}\widetilde{\rho}_1 \end{aligned}$$

Therefore,after a positive shock $\widetilde{\rho}_1$, there is a larger fall in consumption when the economy has a higher ratio of dollar denominated debt.

$$c_{o,\pi=0} < c_{0,\pi=1} < 0$$

■