BANCO CENTRAL DE RESERVA DEL PERÚ

Self-Fulfilling Risk Panics

Philippe Bacchetta*, Cédric Tille** y Eric van Wincoop***

* University of Lausanne, CEPR
* Graduate Institute, Geneva, CEPR
* University of Virginia, NBER

Self-Fulfilling Risk Panics[1]

Philippe Bacchetta
University of Lausanne
CEPR

Cédric Tille
Graduate Institute, Geneva
CEPR

Eric van Wincoop
University of Virginia
NBER

December 1, 2010

Abstract

Recent crises have seen large spikes in asset price risk without dramatic shifts in fundamentals. We propose an explanation for these risk panics, based on self-fulfilling shifts in beliefs about risk, that are driven by a negative link between the current asset price and risk about the future asset price. This link implies that risk about the future asset price depends on uncertainty about future risk. This dynamic mapping of risk into itself gives rise to the possibility of multiple equilibria and can generate risk panics. In a panic, risk beliefs are coordinated around a macro fundamental that becomes a sudden focal point of the market. The magnitude of the panic is larger the weaker this macro fundamental. The sharp increase in risk leads to a large drop in the asset price, decreased leverage and reduced market liquidity. While the model is not aimed at modeling the specifics of any particular financial crisis, we show that its implications are broadly consistent with what happened during the 2007-2008 crisis.

1   Introduction

Sharp surges in risk are a prominent feature of financial panics, such as the turmoil in the Fall of 2008 or the 2010 Eurozone debt crisis. Volatility, as measured by the VIX index, more than quadrupled in the wake of the Lehman Brothers failure, and tripled during the debt crisis. Explaining such huge and sudden spikes in risk is an important challenge for economic theory and the literature has not yet offered a formal explanation. In this paper we propose a theory for such spikes in risk, which we refer to as "risk panics". These involve a large and sudden self-fulfilling increase in risk, coordinated around a macro fundamental that becomes a sudden focal point of the market.

The main contribution of this paper is to show how such risk panics can develop in a model where agents have simple mean-variance preferences. In that case portfolio demand depends on risk associated with the future asset price.[1] In equilibrium the asset price today depends negatively on risk associated with the asset price tomorrow. There is then a dynamic degree of freedom in the model. Risk is defined in terms of uncertainty about the asset price tomorrow. The asset price tomorrow in turn depends on risk perceptions tomorrow. Therefore risk today depends on uncertainty about risk tomorrow. The dependence of risk on uncertainty about future risk opens up the possibility of dynamic multiple equilibria associated with the perceived stochastic process of risk.

We find that beyond a regular fundamental equilibrium where risk is constant over time, there are equilibria in which the perceived process of risk is time-varying. In the latter case the perceived risk is tied to the stochastic process of a variable that can be extrinsic to the model or be a macro fundamental that is part of the model. We refer to these as respectively sunspot and sunspot-like equilibria.[2] In a sunspot-like equilibrium the fundamental variable plays a dual role. It affects

---

[1]Since we take a macroeconomic perspective, we refer to "the asset price" as the price of a market portfolio of risky stocks rather than the equity price of a particular firm.

[2]The term "sunspot-like" equilibria was first coined by Manuelli and Peck (1992). They write: "There are two ways that random fundamentals can influence economic outcomes. First, randomness affects resources which intrinsically affects prices and allocation. Second, the randomness can endogenously affect expectations or market psychology, thereby leading to excessive volatility." In the limiting case where fundamental uncertainty goes to zero, sunspot-like equilibria converge to pure sunspot equilibria.

the asset price both through its regular role as fundamental (e.g., through asset payoffs or wealth) and as a coordination device for beliefs about risk. The latter role is entirely separate from its fundamental role.

Risk panics are closely related to the presence of sunspot-like equilibria. In addition to the pure fundamental equilibrium and sunspot-like equilibrium described above, we consider switching equilibria between a low-risk and a high-risk state, with shifts driven by a Markov process. A panic is a switch from the low to the high-risk state. During a panic, a macro variable suddenly becomes a focal point for shifts in beliefs about risk. The panic is therefore not triggered by a change in the variable, but by the sudden self-fulfilling shift in beliefs about risk that is coordinated around the fundamental.[3] Moreover, the panic is larger when this variable is weak at the time of the shift (e.g., the net worth of leveraged institutions is low or the Greek debt is high).

The paper is related to the broader literature on multiple equilibria with self-fulfilling shifts in beliefs. In this literature there is a coordination of beliefs among agents, such that a common shift in beliefs leads to actions of all agents that make the change in expectations self-fulfilling. In terms of asset prices, there are many applications of this phenomenon for both stock prices and exchange rates. In particular, there is a large literature with self-fulfilling speculative attacks on currencies.[4] A key distinction here is that the self-fulfilling shift in beliefs is not about the level of the asset price but about the level, and more generally the process, of risk. This is critical as we wish to explain large spikes in risk.

There is also a small literature in which self-fulfilling shifts in beliefs about risk can occur due to an interaction between risk and liquidity. This occurs in limited participation models such as Pagano (1989), Allen and Gale (1994) and Jeanne and Rose (2002). When agents believe that risk is high, market participation is low. This implies low market liquidity, which leads to a large price response to asset demand shocks and therefore high risk.[5] In this paper multiple equilibria are not associated with an interaction between risk and liquidity, but rather are the

---

[3]Our mechanism is thus distinct from a financial accelerator where a fundamental shock is amplified through financial frictions.

[4]E.g., see Obstfeld (1986), Aghion et al. (2004), or Burnside et al. (2004).

[5]This phenomenon is not limited to limited participation models of asset prices. For other applications see Bacchetta and van Wincoop (2006) and Walker and Whiteman (2007).

result of the dynamic nature of the model where risk today depends on uncertainty about risk tomorrow.[6] In contrast to the static limited participation models, in the model here risk is time-varying in the high risk equilibrium and its process is closely connected to that of a macro fundamental. This connects more closely to what we have seen during recent crises.[7]

We derive our results under the assumption that agents have simple mean-variance preferences. The mean-variance portfolio model has a long history in academics and remains extensively used today. It is also widely used in the financial industry and can therefore be considered as a reasonable description of actual behavior.[8] An alternative avenue would be to introduce micro founded risk-based portfolio constraints, such as value-at-risk constraints or margin constraints, so that asset demand would depend explicitly on uncertainty about the future asset price.[9] This would, however, make the model significantly more complicated. The mean-variance portfolio assumption in this paper should then be considered as an approximation of more complex behavior.

As an illustration of the theory, we consider an application to the financial crisis of 2007-2008. We show that an extension of the model, where we introduce shocks to the wealth of leveraged investors, delivers results that are qualitatively consistent with what happened during the crisis to stock prices, market liquidity, stock price risk, the volatility of risk and financial leverage. Of particular interest in the context of this paper is that the model can generate a very large sudden spike in risk.

We want to emphasize though that while our model can account for the broad patterns observed during the 2008 panic, it is not aimed at modeling the specifics of any particular financial crisis. We do not encompass important aspects of the

---

[6]The dynamic nature of the multiplicity only arises when the variable around which expectations are coordinated displays persistence.

[7]The spike in risk during recent crises was accompanied by a similar spike in the volatility of risk. Moreover, after the spike in risk the VIX fluctuated closely with the variable of concern, for example any information about a possible bailout package during the Greek debt crisis.

[8]See Basak and Chabakauri (2009) for further motivation.

[9]A substantial literature introducing such constraints has developed in recent years. Examples are Brunnermeier and Pedersen (2009), Danielsson, Shin and Zigrand (2010) and Gromb and Vayanos (2002). For the same reason of analytic tractability as in this paper, these constraints are often introduced in a reduced-form way rather than based on explicit micro foundations.

recent crisis, such as a run on wholesale bank deposits (e.g. through repo contracts) that most observers would consider to be a major aspect of the crisis. Moreover, every crisis is different. During the Greek debt crisis in the Spring of 2010, the VIX more than tripled while there was no bank run at all, and no security complexity issues and adverse selection problems that characterized the breakdown in liquidity during the 2008 panic. In our framework there just needs to be some variable, or set of variables, that becomes a natural focal point of the market, instilling a significant level of "fear" that implies a self-fulfilling increase in perceived risk. In both of these two recent crises there was clearly such a variable: the health of leveraged financial institutions in 2008 and the magnitude of Greek debt in 2010.

The remainder of the paper is organized as follows. In Section 2 we develop the possibility of sunspot and sunspot-like equilibria in a simple mean-variance portfolio model with stochastic asset payoffs (dividends). We consider both a simple model with a closed form solution and a more general one. In Section 3 we show that the model can generate risk panics. Section 4 presents an application to the 2007-2008 crisis relying on wealth shocks. Section 5 concludes.

2 A Simple Mean-Variance Portfolio Choice Model

Consider a simple mean-variance portfolio choice model where agents buy a stock and a risk-free bond. We first consider the case where the return on the bond is exogenous as this allows us to derive closed-form solutions. We then endogenize the interest rate in a full general equilibrium setup.

2.1 Model Description

The model complexity is kept to a strict minimum. We consider an overlapping generation (OLG) setup where investors are born with wealth W_I. They invest in equity and bonds and consume the return on their investment when old. The bond pays an exogenous constant gross return R. This assumption, which is often made in the finance literature, allows us to derive a closed form solution. It implicitly assumes that there is a risk-free technology with a constant real return R that is in infinite supply. This assumption is not crucial to our results and is relaxed in Section 2.5.

Equity consists of a claim on a tree with stochastic payoff. There are K trees, each producing an exogenous stochastic output (dividend) A_t. Denoting the equity price by Q_t, the equity return from t to $t+1$ is:

$$R_{K,t+1} = \frac{A_{t+1} + Q_{t+1}}{Q_t} \tag{1}$$

Agents face uncertainty both about the dividend and the future equity price. The dividend is equal to $\bar{A}(1 + mS_t)$, where S_t follows an AR process:

$$S_{t+1} = \rho S_t + \epsilon_{t+1}, \tag{2}$$

$\rho \in \langle 0, 1 \rangle$ and the innovation ϵ_{t+1} has a symmetric distribution with mean zero and variance σ^2. We denote the variance of ϵ_{t+1}^2 by ω^2. S_t is the only state variable in the model. When $m = 0$ the dividend is a constant ($A_t = \bar{A}$) and S_t becomes a pure sunspot. When $m > 0$, S_t has a fundamental impact on the equity payoff.

Investors born at time t maximize a mean-variance utility over their portfolio return:

$$E_t R_{t+1}^p - 0.5 \gamma var_t(R_{t+1}^p) \tag{3}$$

where γ measures risk aversion and the portfolio return is:

$$R_{t+1}^p = \alpha_t R_{K,t+1} + (1 - \alpha_t)R$$

α_t denotes the portfolio share invested in equity. The gross returns on equity and bonds are $R_{K,t+1}$ and R respectively. The equity market clearing condition is

$$\alpha_t W_I = Q_t K \tag{4}$$

The OLG assumption is not critical to the results but simplifies the analysis in two ways. First, it avoids the well-known dynamic hedge term in the optimal portfolio that arises in multi-period portfolio problems. Second, the wealth level would be an additional state variable (in addition to S_t) if agents had infinite lives. We would then be unable to solve the model analytically or even represent the equilibria graphically. While we cannot get a closed form solution when the bond interest rate is endogenous, we can still represent the equilibria graphically as there is only one state variable. A shortcoming of the OLG assumption is that it prevents movements in asset prices from feeding back into the wealth of investors, a channel that can be important in a crisis. We introduce such a feedback effect in the Technical Appendix through a simple extension of the OLG setting, with a brief discussion in Section 4.4.

2.2 Equilibrium Condition for Equity Price

The maximization of (3) with respect to α_t gives the optimal portfolio share, which reflects the expected excess return on equity scaled by the variance of the equity return:

$$\alpha_t = \frac{E_t R_{K,t+1} - R}{\gamma var_t(R_{K,t+1})} \tag{5}$$

The portfolio share of equity can exceed one when the equity return is not very risky, or when investors put little weight on risk. In that case investors are *leveraged*, with long positions in equity and short positions in bonds.

Using (5), the market clearing condition (4) becomes:

$$E_t(A_{t+1} + Q_{t+1} - RQ_t) = \frac{\gamma K}{W_I} var_t(Q_{t+1} + A_{t+1}) \tag{6}$$

Equation (6) equates the equilibrium expected excess payoff on equity to a risk premium that depends on the variance of the payoff $Q_{t+1} + A_{t+1}$. We use it to solve for the equilibrium asset price Q_t as a function of the single state variable S_t.

2.3 Sunspot Equilibria

First consider the case where $m = 0$, so that S_t is a pure sunspot. In that case (6) can be written as

$$Q_t = \lambda_0 + \lambda_1 Risk_t + \lambda_2 E_t Q_{t+1} \tag{7}$$

where $Risk_t = var_t(Q_{t+1})$ measures risk associated with the future asset price and the parameters are $\lambda_0 = \bar{A}/R$, $\lambda_1 = -\gamma K/(RW_I)$ and $\lambda_2 = 1/R$.

There are two equilibria.[10] The first is a fundamental equilibrium where the asset price is constant:

$$Q_t = \frac{\bar{A}}{R - 1} \tag{8}$$

The second is the sunspot equilibrium $Q_t = \tilde{Q} - VS_t^2$ where:[11]

$$V = \frac{W_I}{K\gamma} \frac{R - \rho^2}{4\rho^2 \sigma^2} \tag{9}$$

---

[10]We only consider stationary equilibria, ruling out rational explosive bubbles.

[11]An additional restriction to make sure that the asset price is always positive is that the distribution of ϵ_t is bounded. In that case S_t is bounded as well. Then a sufficient condition for the asset price to always be positive is that $\bar{A}$ is sufficiently large, since $(R-1)\tilde{Q} = \bar{A} - V\sigma^2 - (\gamma K/W_I)V^2\omega^2$.

$$\tilde{Q} = \frac{1}{R-1}\left(\bar{A} - \frac{\gamma K}{W_I}V^2\omega^2 - V\sigma^2\right) \tag{10}$$

In the sunspot equilibrium the asset price fluctuates with the sunspot S_t. Risk is time-varying with the sunspot as well:

$$Risk_t = var_t(Q_{t+1}) = 4V^2\rho^2\sigma^2 S_t^2 + V^2\omega^2 \tag{11}$$

The perceived equilibrium process for risk is therefore either a constant or time-varying and tied to the process for the sunspot S_t. In order to provide some intuition for this result, it is useful to consider setting $\lambda_2 = 0$ in (7). This suppresses, just for the purpose of intuition, the standard dynamic link between the asset price today and the expected future asset price, a link that is not central to the mechanism we focus on. We then have

$$Q_t = \lambda_0 + \lambda_1 Risk_t \tag{12}$$

The same equation forwarded by one period shows that the future asset price depends on future risk:

$$Q_{t+1} = \lambda_0 + \lambda_1 Risk_{t+1}$$

Taking the variance on both sides, current risk is linked to uncertainty about future risk:

$$Risk_t = \lambda_1^2 var_t(Risk_{t+1}) \tag{13}$$

Risk therefore depends on uncertainty about future risk itself. It is this dynamic mapping of risk into itself that opens up the possibility for multiple equilibria. Clearly, zero risk is an equilibrium. But any process for $Risk_t$ is an equilibrium as long as it satisfies (13). This process must clearly lead to joint shifts in risk and uncertainty about risk as they are proportional in (13). One process that satisfies (13) is where $Risk_t$ is linear in S_t^2. Uncertainty about future risk will then depend on S_t^2 as well because $var_t(S_{t+1}^2) = 4\rho^2\sigma^2 S_t^2 + \omega^2$.

An interesting point is that the impact of the sunspot on the equity price is larger when investors have a low risk aversion γ or a large wealth W_I. As can be seen from (6), low risk aversion or large wealth reduce the risk premium and make it less sensitive to changes in risk. It is precisely because agents respond less to risk (i.e. are less risk averse) that it is possible to have an equilibrium with large time-variation in perceived risk.

2.4 Sunspot-Like Equilibria

Next consider the case where $m > 0$, so that shocks to S_t are fundamental shocks to the asset payoff. We conjecture that the asset price is linear-quadratic in S_t:

$$Q_t = \tilde{Q} + vS_t - VS_t^2 \tag{14}$$

There are again two equilibria: a fundamental one and a sunspot-like one. In the fundamental equilibrium we have $V = 0$, $v = m\rho/(R - \rho)$, and $\tilde{Q} = (\bar{A} + (\gamma K/W_I)v^2\sigma^2)/(R-1)$. The asset price is then:

$$Q_t = \tilde{Q} + \frac{m\bar{A}\rho}{R - \rho}S_t \tag{15}$$

Shocks have a bigger impact on the asset price when they are persistent. Asset price risk is constant.

In the sunspot equilibrium we have:

$$V = \frac{W_I}{K\gamma}\frac{R - \rho^2}{4\rho^2\sigma^2} \tag{16}$$

$$v = -\frac{m\bar{A}}{1 - \rho} \tag{17}$$

$$\tilde{Q} = \frac{1}{R - 1}\left(\bar{A} + \frac{1}{R}v^2\sigma^2 - \frac{\gamma K}{W_I}V^2\omega^2 + V\sigma^2\right) \tag{18}$$

The key parameter here is V. When V is non-zero, as is clearly the case, then perceived risk in this equilibrium is time-varying and depends on S_t:

$$var_t(Q_{t+1}) = 4V^2\rho^2\sigma^2S_t^2 + V^2\omega^2 \tag{19}$$

The role of S_t in coordinating beliefs about risk is entirely separate from its role as a fundamental. Specifically, V does not depend on m and is thus independent of the fundamental role of S_t. The role of S_t in driving time-varying perceptions of risk is therefore unrelated to its fundamental role.

We call this equilibrium a sunspot-like equilibrium, and the variable S_t a sunspot-like variable, because S_t has a role similar to that of a sunspot. S_t clearly is not a pure sunspot as it affects the Home dividend when $m > 0$, but its role in coordinating beliefs about risk is exactly the same as that of a sunspot variable. Although in a very different context, not involving time-varying shifts in risk,

8

Manuelli and Peck (1992) and Spear, Srivastava and Woodford (1990) also present models with sunspot-like equilibria. Spears, Srivastava and Woodford (1990) point out that "...a sharp distinction between "sunspot equilibria" and "non-sunspot equilibria" is of little interest in the case of economies subject to stochastic shocks to fundamentals." Indeed, as we raise m slightly above 0, the sunspot-like equilibrium is technically no longer a pure sunspot equilibrium, but it is effectively indistinguishable.

2.5   Full General Equilibrium

We now move to a full general equilibrium approach by relaxing our assumption of an exogenous risk-free return R. We introduce an interest-rate elastic supply of bonds so that investors can reallocate between stocks and bonds in equilibrium.[12] We do so by introducing another set of agents, which we call households, who invest in bonds and in a household technology detailed below.

There are overlapping generations of households born with wealth W_H. Households invest their endowment in bonds and a risk-free household technology, and consume the proceeds when old. Investing $K_{H,t+1}$ in the household technology at time t yields an output $f(K_{H,t+1})$ at $t+1$. The technology exhibits decreasing returns to scale, $f'(.) > 0$ and $f''(.) < 0$. Households born at time t maximize consumption at time $t+1$, which is equal to $f(K_{H,t+1}) + R_{t+1}(W_H - K_{H,t+1})$ where R_{t+1} is the interest rate on the bond. This choice equalizes the marginal return on the technology to the bond yield: $f'(K_{H,t+1}) = R_{t+1}$.

For convenience we assume a simple quadratic form for household technology. The capital demand is then linear in the interest rate:[13] $K_{H,t+1} = \nu - \eta R_{t+1}$, and the demand for bonds by households is:

$$W_H - K_{H,t+1} = W_H - \nu + \eta R_{t+1} \tag{20}$$

Equation (20) can be positive, in which case households lend bonds to investors, or negative, in which case they borrow from investors.

---

[12]With a constant supply of bonds, the equity price would be entirely pinned down by investors' wealth and there would be no sunspot or sunspot-like equilibria. There are many ways to introduce an interest-rate elastic supply or demand schedule of bonds, for example by introducing interest elastic consumption/savings or investment decisions.

[13]Specifically, we assume that $f(K_{H,t+1}) = \left[\nu K_{H,t+1} - 0.5 K_{H,t+1}^2\right]/\eta$.

9

The bond market clearing condition shows that the demands for bonds by households and investors add up to zero, as bonds are in zero net supply:

$$(1 - \alpha_t)W_I + W_H - \nu + \eta R_{t+1} = 0$$

Using the equity market clearing condition (4), we rewrite this as

$$Q_t K + \nu - \eta R_{t+1} = W \tag{21}$$

where $W = W_I + W_H$ is the aggregate initial wealth. (21) gives a linear positive relationship between the equity price and the interest rate. A higher equity price raises the supply of equity. Clearing the equity market then requires investors to shift their portfolio towards equity and reduce their purchase of bonds (or borrow more from households). Bond market clearing then requires households to lower their borrowing (or increase their bond purchase), which they are induced to do through a higher interest rate.[14]

Using (21), the equity market clearing condition (6) becomes:

$$E_t \left(A_{t+1} + Q_{t+1} - \frac{\nu - W}{\eta} Q_t - \frac{K}{\eta} Q_t^2 \right) = \frac{\gamma K}{W_I} var_t(Q_{t+1} + A_{t+1}) \tag{22}$$

The equilibrium condition (22) only involves the equity price, which we again solve with the method of undetermined coefficients. As (22) is non linear in the asset price, we no longer have an analytical solution. We therefore adopt a numerical approximation method along the following lines (details are given in Appendix A). As is standard in the literature, we consider an approximation of the equilibrium asset price in logs:

$$q_t = \tilde{q} + vS_t - VS_t^2 \tag{23}$$

We then take a quadratic approximation of Q_t and Q_{t+1} around $S_t = S_{t+1} = 0$, and use the result to compute the expectation and variance of $Q_{t+1} + A_{t+1}$. We substitute the resulting expressions into (22). We finally take a quadratic approximation around $S_t = 0$, which gives a linear-quadratic expression in S_t:

$$Z_0 + Z_1 S_t + Z_2 S_t^2 = 0 \tag{24}$$

---

[14]There is a third market clearing condition, for goods, but we can drop it thanks to Walras' Law.

where Z_0, Z_1, and Z_2 are functions of $\tilde{Q} = e^{\tilde{q}}$, v, and V. We solve for the value of these parameters by setting $Z_0 = 0$, $Z_1 = 0$, and $Z_2 = 0$.

While we are solving for three parameters, $\tilde{Q}$, v and V, we can represent the equilibria graphically in a $(\tilde{Q}, v)$ space. Define $\tilde{V} = \tilde{Q}V$. In Appendix A we show that $Z_0 = 0$ implies

$$\tilde{V} = \alpha_1 + \alpha_2 v^2 \tag{25}$$

where α_1 and α_2 are functions of $\tilde{Q}$. Substituting this into the expressions associated with $Z_1 = 0$ and $Z_2 = 0$ we obtain

$$h_1 + h_2 v + h_3 v^2 + h_4 v^3 = 0 \tag{26}$$

$$g_1 + g_2 v + g_3 v^2 + g_4 v^3 + g_5 v^4 = 0 \tag{27}$$

where h_i and g_i are functions of $\tilde{Q}$.

We solve numerically for the roots of the third and fourth order polynomials (26) and (27). The polynomials represent two schedules that map a given $\tilde{Q}$ into v, with possibly multiple solutions. We plot these two schedules in a $(\tilde{Q}, v)$ space with each intersection representing an equilibrium combination of $\tilde{Q}$ and v. $\tilde{V}$, and therefore V, then follow from (25).

For a given process for S_t a typical parameterization gives 4 equilibria. This is illustrated in Figures 1 and 2 for respectively $m = 0$ and $m = 1$. Schedule (26) is represented by the solid line and (27) by the broken line. When $m = 0$ the variable S_t is a pure sunspot. Figure 1 shows that there is one fundamental equilibrium where $v = V = 0$. The other three equilibria are all sunspot equilibria. The fact that for a given process for S_t there are now three sunspot equilibria rather than the single sunspot equilibrium we found before is a result of the non-linearity generated by the time-varying interest rate.

In Figure 2, where $m = 1$, S_t is a fundamental that drives the asset payoffs. There are again 4 equilibria. Equilibrium 1 is a pure fundamental equilibrium. As we let $m \to 0$, it converges to Equilibrium 1 in Figure 1 where $v = V = 0$. The other three equilibria are all sunspot-like equilibria. As we let $m \to 0$, they converge to the corresponding sunspot equilibria in Figure 1. Figure 3 illustrates the convergence of the sunspot-like Equilibrium 2 of Figure 2 to the sunspot Equilibrium 2 of Figure 1 when m goes to zero. It is remarkable that even when we get far away from $m = 0$, $\tilde{Q}$, v and V change very little, especially in

11

comparison to the near-zero levels of v and V in the fundamental equilibrium. This suggests that even when the fundamental role of S_t is important, the impact of S_t on the asset price is dominated by shifts in beliefs about risk that are unrelated to this fundamental role.

3   Risk Panics

3.1   Switching across states

Risk panics can happen in equilibria that allow for a switch between low and high risk states. In the previous section the economy was either in a fundamental or a sunspot-like equilibrium. We now consider an equilibrium that allows for switches between a low risk state (indexed by 1, akin to the fundamental equilibrium) and a high risk state (indexed by 2, akin to the sunspot equilibrium). Switching occurs through an exogenous Markov process. The probability that we remain in a low risk state next period when we are in a low risk state today is $p_1 > 0.5$. Similarly, the probability that we remain in a high risk state next period when we are in a high risk state today is $p_2 > 0.5$.[15]

Equilibria 1 and 2 in Figure 2 are the points to which the low and high risk states converge, respectively, in the limit where switching is not possible ($p_1 = p_2 \to 1$). When switching is possible, the low risk state becomes riskier than the pure fundamental Equilibrium 1 in Figure 2. This is because there is now a possibility of switching to the high risk state, a switch that implies a significant drop in the equity price. Even when the probability of switching is low, the main source of uncertainty in the low risk state becomes the possibility of a jump to the high risk state rather than the pure fundamental uncertainty in Equilibrium 1 of Figure 2.[16] Agents take the possibility of switching into account when forming their expectations.

---

[15]It may be realistic to allow these probabilities to depend on the state variable itself. For example, a switch to the high risk state may be more likely when the fundamental is weak. Here we abstract from that possibility and only consider the simpler case of constant probabilities of switching.

[16]This is similar to what is found in the "rare disaster" literature (e.g., Barro, 2006, Gabaix, 2008) where a small probability of a large disaster affects what happens in the no disaster periods.

We conjecture that the log equity price in state i is

$$q_{i,t} = \tilde{q}_i + v_i S_t - V_i S_t^2 \tag{28}$$

As there are two such equations we solve for 6 unknown parameters (3 for each state). This is done by imposing equity market equilibrium as before, but separately for both states. We compute the expectation and variance of Q_{t+1} taking into account that a switch to a different state is possible. The algebra is presented in Appendix B.

As an illustration, Figure 4 shows the values of $\tilde{Q}_i$, v_i and V_i in the low and high risk states for the case where $p_1 = p_2$. As pointed out above, the two states correspond exactly to Equilibria 1 and 2 of Figure 2 when $p_1 = p_2 = 1$. Switching equilibria only exist when the probability of remaining in the same state is high enough. But when $p_1 = p_2$ is higher than this cutoff (sufficiently low probability of switching), the difference between the two states quickly becomes very big. A lower probability of switching particularly reduces risk in the low risk state (lower values of v and V).

A risk panic is a switch from the low to the high risk state. It involves a self-fulfilling shift in beliefs about the process of risk. For example, when $p_1 = p_2$ are close to 1, there is a self-fulfilling shift in beliefs about risk from the low and constant risk in the fundamental equilibrium to the high and time-varying risk in the sunspot-like equilibrium. Apart from the spike in risk, the panic also entails an increase in the volatility of risk, a sharp drop in the equity price and a shift out of equity (i.e. deleveraging when investors initially hold leveraged portfolios). We graphically illustrate these effects in Section 4 in an application to the 2007-2008 financial crisis.

3.2   Panics and fundamentals

It is important to be clear both about the role that fundamentals *do* and *do not* play in a risk panic. First, a panic is not caused by a change in fundamentals. It happens for a given level of S_t. Second, the magnitude of the panic is larger the weaker the fundamental (the more negative S_t). Finally, once a panic occurs the asset price becomes much more sensitive to subsequent fluctuations in the fundamental. The market becomes on edge regarding any news about S_t.

Consider the first point: a panic does not result from a change in the fundamental. As can be seen from Figure 4, during the switch to the high risk state the coefficients v and V increase, generally by a large magnitude. This affects risk and the asset price for a *given* level of S_t. What changes is not S_t itself but rather the role it plays. As we switch to the high risk state, S_t suddenly becomes a key variable around which agents coordinate their perceptions of risk. There is a sudden self-fulfilling increase in beliefs about risk, with the variable S_t being the focal point for the change in risk perceptions.[17]

Notice that a risk panic is therefore conceptually distinct from financial accelerator models where the impact of shocks is magnified through financial constraints. While small shocks have a large effect in such models, the mechanism at work is a purely fundamental mechanism. Our framework instead puts the coordination of beliefs about risk center stage. During the panic, asset prices and risk move sharply even though the state variable does not change.

Next consider the second point: the magnitude of the panic is larger the weaker the fundamental. To illustrate this point, consider the change in the equity price from the low to the high risk state. From (28) it follows that the change in the log equity price is

$$q_{2,t} - q_{1,t} = \tilde{q}_2 - \tilde{q}_1 + (v_2 - v_1)S_t - (V_2 - V_1)S_t^2 < 0$$

Since $v_2 - v_1$ and $V_2 - V_1$ are both positive (see Figure 4), the drop in the equity price is larger the more negative is S_t (i.e. the weaker the fundamental). Consider for instance $p_1 = p_2 = 0.65$. In that case a panic lowers the equity price by only 13% when $S_t = 0$, but by 65% when S_t is two standard deviations below its unconditional mean of 0.

In this light a large risk panic can also be viewed as a delayed amplification effect. Consider a deterioration of the fundamental (a drop in S_t) when the economy is in the low risk state. The shock lowers the equity price through the standard fundamental mechanism, but this impact is relatively small. The delayed amplification effect occurs if at some later date there is a switch to the high risk equilibrium. At that point, the sunspot role of S_t suddenly surges. The impact of

_____

[17]Even in the low risk state S_t plays to some extent a sunspot role if $p_1 < 1$. But this role is generally much stronger in the high risk state. In the low-risk state this role only reflects the possibility of switching to the high risk state.

14

the panic on the asset price is much larger than the fundamental impact of S_t in the first stage. We will further illustrate this point in Section 4 in the context of the recent financial crisis.

Finally consider the last role of the fundamental in a panic: once a panic occurs the asset price becomes much more sensitive to subsequent fluctuations in the fundamental. Once we switch to the high risk state, the fundamental S_t becomes the focal point around which investors coordinate their beliefs about risk. This causes them to react strongly to any change in the variable. A further deterioration can lead to a significant further drop in the equity price. Conversely, an improvement in the fundamental becomes a significant stabilizing force. In the example above with $p_1 = p_2 = 0.65$, the equity price drops from 100 to 35 during a panic when the fundamental is two standard deviations below its mean. But when the fundamental reverts to it mean, the equity price goes all the way back to 87, even though we are still in the high risk state.

4   Application to 2007-2008 Financial Crisis

This section uses our setting to shed light on what happened in the equity market during the 2007-2008 financial crisis. We first discuss some basic financial data associated with the dynamics of risk, leverage, liquidity and equity prices. After that we slightly alter the model to make it more relevant to the crisis by introducing shocks to the wealth of leveraged investors (think of mortgage market losses). We then present a simulation and show that the outcome is qualitatively similar to what happened during the crisis. We finish by briefly discussing a variety of sensitivity analyses.

As emphasized in the introduction, there are many aspects of the recent crisis that are well beyond the scope of this paper. To the extent that our model is applicable in shedding light on the crisis, it is primarily in the context of the self-fulfilling shifts in risk perceptions that are the focus of this paper. This key mechanism applies beyond the particular episode of the 2007-2008 crisis, so this section is only an illustration.

4.1 Dynamics of Risk, Leverage, Liquidity, and Asset Prices

The crisis came in the form of a one-two punch. The first part is the relatively calm period from the beginning of 2007 until September 2008. The second part is the financial panic that started in September 2008. The panic peaked by the end of 2008 and it took several quarters for the situation to return to a more normal state. Using data for the United States, we focus on the following variables: (1) stock prices, (2) T-bill rate, (3) equity price risk, (4) volatility of risk, (5) net worth of leveraged institutions, (6) leverage, and (7) market liquidity. A description of the data and data sources can be found in Appendix C.

The dynamics of these variables during the crisis are illustrated in Figure 5. The vertical line represents the collapse of Lehman Brothers on September 15, 2008, which we consider to be the start of the financial panic. After a modest decline in stock prices and a small increase in risk during the tranquil period of the crisis, stock prices suddenly crashed and risk spiked in September 2008. The volatility of risk also shot up, while it showed no trend in the first stage. A flight to quality lowered the T-bill rate to near zero. Net worth gradually declined after mid 2007 until the third quarter of 2008, to quickly recover after the crisis. Financial leverage first rose significantly during the tranquil period, and then fell sharply during the panic stage. Finally, liquidity fell modestly during the tranquil part of the crisis, followed by a sharp drop in liquidity during the panic and then a return back to normal by mid-2009.

4.2 Model with Financial Shocks

In applying the model to the recent crisis we introduce financial shocks that redistribute wealth between households and investors. These shocks fit more closely the storyline of the 2007-2008 financial crisis where financial institutions experienced large negative shocks to their wealth (net worth) connected to mortgage market losses. We introduce shocks to the wealth of investors as follows:

$$W_{I,t} = e^{-m\theta_t - 0.5m^2\theta_t^2} \bar{W}_I \tag{29}$$

where

$$\theta_{t+1} = \rho_\theta \theta_t + \epsilon_{t+1}^\theta \tag{30}$$

and ϵ_{t+1}^θ is a shock with mean zero and variance σ_θ^2.

Equation (29) ensures that investors' wealth is linear in θ_t up to a quadratic approximation: $W_{I,t} = \bar{W}_I(1 - m\theta_t)$. A rise in θ_t implies a drop in the relative wealth of investors. Financial shocks redistribute wealth and leave aggregate wealth unchanged ($W_{I,t} + W_{H,t} = W$), although in sensitivity analysis we find that results are similar when there are shocks to the wealth of investors only. We assume that financial shocks ϵ^{θ}_{t+1} and asset payoffs A_{t+1} are uncorrelated. Finally, we assume that asset payoff shocks have no persistence ($\rho = 0$). This simplifies as the wealth of investors is then the only state variable. The solution method is analogous to that for the asset payoff shocks and can be found in the Technical Appendix. There are again 4 equilibria, similar to those in Figure 2.

These wealth shocks also introduce links in the model between wealth, financial leverage, risk and market liquidity. For example, lower wealth of investors reduces market liquidity, which implies increased risk. Similarly, higher risk reduces financial leverage, which lowers market liquidity and increases risk further. We refer to a previous version of this paper for a more detailed discussion of these links,[18] which have been extensively discussed in the literature.[19]

While we have changed the nature of the state variable in the model in order to make it more applicable to what happened in 2007-2008, we should emphasize that the exact nature of the fundamental is not so important. The key point is that a large risk panic can develop when there is a self-fulfilling shift in beliefs coordinated around a very weak fundamental. The exact nature of the fundamental does not matter much in this regard.

4.3   Model Simulation

We illustrate the dynamics of the variables in the model, and relate them to the recent crisis, using the two-state switching equilibrium as described in Section 3. The key point of this section is that large movements in asset prices and risk require both a risk panic and a weak fundamental, with either having little impact on its own. The parameters are shown at the bottom of Figure 6. The main results are robust to the precise parameter values chosen, as discussed below. We

---

[18]See NBER working paper 16159.

[19]See Adrian and Shin (2008), Brunnermeier and Pedersen (2009), Brunnermeier and Sannikov (2010), Gromb and Vayanos (2008), He and Krishnamurthy (2008a, b), Kyle and Xiong (2001) and Xiong (2001).

set $p_1 = 0.95$ and $p_2 = 0.7$. This ensures that the high risk state occurs much less frequently than the low risk state, as the economy spends only 14% of the time in the high risk state. Panics of a large magnitude are even less frequent because they require not only a switch to the high risk state but also a very weak fundamental.

The parameterization is chosen to make sure that investors are substantially leveraged. Investors' initial equity holdings are four times their net worth (wealth), and are financed by borrowing from households through bonds. High leverage is characteristic of most financial institutions. We therefore also refer to the investors as leveraged financial institutions.

We simulate the model over 16 periods, which we interpret as quarters. We do no make any attempt to match the process of financial losses in the data, but instead illustrate the drivers of the model through a simple step function for θ_t, along with a simple switching between low and high risk states. The dynamics of θ_t are illustrated through the wealth of investors, which follows the same path, in the upper left chart of Figure 6. The economy is initially in a low risk state with θ_t at its unconditional mean of zero. θ_t rises from 0 to 0.3 in period 2, which we can think of as Q1 2007 when the losses of leveraged institutions on mortgage securities became apparent, leading to a reduction of their wealth. This situation lasts until period 8, which we can think of as Q3 2008, where the economy switches to the high risk state. It stays in that situation until period 11 (Q2 2009) when θ_t falls back to zero thanks, for example, to a recapitalization of leveraged institutions. The economy reverts back to the low risk state in period 14 (Q1 2010).

These dates are not meant to match the exact length of the panic or the period of financial weakness of leveraged financial institutions. Our focus is instead to highlight the separate roles of the financial health of leveraged institutions and the specific risk state. This is done by considering all possible combinations of financial health (normal versus bad) and the state (low risk, high risk) in order to evaluate the specific contribution of both elements.

The simulation is presented in Figure 6. Periods during which θ_t changes are marked by vertical dotted lines, while the shaded area denotes the time spent in the high risk state. The wealth of investors follows the overall pattern seen in the data for brokers and dealers in Figure 5, although the deterioration of the net worth of financial institutions was obviously more gradual in the data. The other panels show the paths of the equity price, risk, the volatility of risk, interest rate, leverage

18

and illiquidity. The stock price (normalized at 100 initially) and gross interest rate are Q_t and R_{t+1}. Risk is measured as the standard deviation of Q_{t+1}/Q_t, taking into account the possibility of switching to another state. The volatility of risk is the standard deviation at time t of our risk measure at $t + 1$.[20] Leverage is equal to the share of equity in investors' portfolio, α_t. Finally, illiquidity is measured as the absolute value of the derivative of the log equity price with respect to θ_t.[21]

During the tranquil part of the crisis the shift in wealth away from leveraged financial institutions reduces demand for equity and therefore its price. It also leads to a decline in liquidity as less money is on the line in the equity market. The drop in liquidity increases risk somewhat. Nonetheless Figure 6 shows that these effects are all modest. The only large change is leverage, which almost doubles. While the small increase in risk reduces leverage, this is more than offset by an increase in the expected excess return due to the lower equity price.[22]

The second stage of the crisis, when the economy shifts into the high-risk stage, is characterized by a surge in risk and its volatility. This prompts a sharp reduction in the equity price and leverage. The drop in leverage in turn dries up liquidity in the equity market as investors reduces their exposure to equity. The switch to bonds leads to a sharp drop in the interest rate.

The key message to take away from Figure 6 is that a large surge in risk requires two ingredients, either one of which alone is not sufficient. First, there needs to be a self-fulfilling risk panic (switch to the high risk state). Second, the fundamental around which the market perceptions of risk coalesce (in this case the net worth of leveraged institutions) must be weak. A deterioration of the macro fundamental alone is not enough to generate a surge in risk. Even though the net worth of leveraged institutions drops by more than 50% during the first stage of the crisis, risk remains relatively modest. A switch to the high risk state by itself is not enough either. Risk is restored slightly below its pre-panic level in period 11, when we are still in the high risk state but the leveraged institutions

---

[20]In computing the volatility of risk, we assume that we remain in the same state the next period. This makes it more consistent with the data, where it is measured as the volatility of risk over the past 30 days, which usually captures volatility within the same state.

[21]This connects well to the illiquidity measure used in the data, which is also meant to capture the price impact of asset demand shocks. See Appendix C.

[22]The model does not account for the drop in the interest rate prior to the panic as that is largely related to monetary policy.

are recapitalized.

While the simple exercise we have conducted here is not meant to match precise data, the overall pattern in these variables is broadly in line with the data in Figure 5. During the pre-panic state of the crisis the impact on the equity price, risk and liquidity is quite modest in both the data and the model. The substantial increase in financial leverage during this period is also consistent with the model. Then, during the switch to the panic state the model accounts for the sharp drop in the equity price, financial leverage, and market liquidity and the sharp increase in risk.

The volatility of risk also behaves similarly to that in the data. It surges together with risk during the panic and later on declines with the fall in risk itself. This joint behavior of risk and the volatility of risk is a critical element of the model, as discussed in Section 2.1. Risk spikes in the model only because future risk becomes more uncertain.

4.4   Sensitivity Analysis

Self-fulfilling shifts in risk can occur as long as the asset price is negatively affected by risk about the future asset price. One might therefore expect the findings in the simulation above to apply much more broadly than for the particular model assumptions and parameterization underlying Figure 6. We confirm this through a variety of sensitivity analysis that we summarize here, with the details given in the Technical Appendix.

We first check that the results in the simulation exercise presented in Figure 6 are robust to alternative parameter values. This is done by halving and doubling most parameters. The results remain qualitatively intact for all alternative parametrizations. In particular, a risk panic leads to a sharp increase in risk and the volatility of risk, and a large decrease in the equity price, market liquidity and leverage. The precise magnitudes are certainly sensitive to parameterization. In particular, the size of the risk panic is larger the smaller η, ρ_θ, γ, and m and the larger $\nu - W$.

Second, we assess how the specifics of the model affect the results. We have already seen that the nature of the fundamental around which risk panics are coordinated is not critical to the results, as shocks to asset payoffs also lead to multiple equilibria and risk panics. Another modeling aspect is the assumption

that financial shocks redistribute wealth between investors and households, with no aggregate loss. We consider an alternative where the wealth loss for investors is not offset by a gain for households and find that the results remain very similar. Lastly, we abstracted from any feedback of the asset price to wealth. We include this aspect in our OLG setting by assuming that some of the endowment when born consists of trees. This amplifies the risk panic. For example, when 29% of the wealth is subject to asset price shocks (in the low risk state at $\theta_t = 0$), we find that the feedback effect from the asset price to wealth increases the magnitude of risk panics, with risk spiking from 26% during the tranquil part of the crisis all the way to 129% at the height of the panic.

Finally, we check the robustness with respect to the approximation in the solution method. This is done by considering a cubic approximation of the market clearing condition instead of a quadratic one. The simulation results are not substantially affected, providing confidence that the precision of the approximation method is not critical to the results.

5   Conclusion

Motivated by several recent crises that have shown very large spikes in risk without correspondingly large shifts in fundamentals, we develop a theory for self-fulfilling shifts in risk. These shifts can occur when the asset price depends negatively on perceived risk about the future asset price. Risk associated with tomorrow's asset price then depends on uncertainty about risk tomorrow. This dynamic mapping of risk into itself gives rise to the possibility of self-fulfilling shifts in risk.

Although a risk panic occurs without any change in fundamentals, it has a larger impact the weaker the macro fundamental on which agents coordinate their perceptions of risk at the time of the panic. The sharp increase in risk and accompanying volatility of risk in turn give rise to a large drop in the asset price, decreased leverage and reduced market liquidity. While the model is not intended to capture the events of any particular crisis episode, we have shown that the implications of the model are nonetheless broadly consistent with what happened during the 2007-2008 financial crisis.

Our findings open up several directions for future research. First, the equilibria that we have identified can be found in any model where the actions of agents

depend on the risk of an endogenous variable. While we have focused on asset markets, the same may be the case for example in goods and labor markets. The issue is also not limited to prices. We could replace Q with any other variable that depends on risk associated with its future level. This could for example be output. It is well-known that reduced uncertainty about the future economic environment is good for business today (e.g. see Bloom, 2009).

Another direction for future research is to consider multiple assets. In our entire analysis there is only one risky asset. This should therefore be interpreted as the market portfolio of risky assets, which could be a country-wide or even a global equity index. A natural question is what the implications are for stocks of individual firms. Closely related, in an open economy context one would like to know whether all countries will be affected by a risk panic or whether it could be contained to a limited number of countries. This question relates to the widely discussed issue of financial contagion and is analyzed in Bacchetta and van Wincoop (2010).

A final direction for further research pertains to financial crises. We have kept the model as simple as possible to focus on the role of self-fulfilling shifts in perceived risk. A natural question is how this interacts with other elements that we have ignored. A non-exhaustive list includes financial constraints on leveraged institutions (borrowing constraints, value at risk constraints), bank runs, and the interaction between the financial crisis and real economic activity. Moreover, a crucial issue is the policy recommendation that arises from our analysis. In Bacchetta et al. (2010) we examine the role of leveraged institutions in the context of our model. We find that, despite their stabilizing role in normal times, less risk averse leveraged institutions increase the magnitude of risk panics. We conclude that a policy making financial institutions more risk averse, or more prudent, could substantially reduce volatility.

Appendix

A   Numerical Solution of Model in Section 2.5

In this Appendix we describe the solution of the equilibria in the version of the model in Section 2.5. We take a quadratic approximation of the market clearing condition around $S_t = 0$. Before doing so, we first need to compute the expectation and variance of $Q_{t+1} + A_{t+1}$. From the conjecture (23) we have

$$Q_t = \tilde{Q}e^{vS_t - VS_t^2} \tag{31}$$

where $\tilde{Q} = e^{\tilde{q}}$. A quadratic approximation around $S_t = 0$ gives

$$Q_t = \tilde{Q}(1 + vS_t + (-V + 0.5v^2)S_t^2) \tag{32}$$

For consistency we now also model the asset payoff in logs: $ln(A_t) = ln(\bar{A}) + mS_t - 0.5m^2S_t^2$. This specification implies that a quadratic approximation of A_t around $S_t = 0$ is $A_t = \bar{A}(1 + mS_t)$. Using these quadratic approximations of Q_t and A_t at $t + 1$ and then substituting $S_{t+1} = \rho S_t + \epsilon_{t+1}$ gives

$$
\begin{aligned}
E_t(Q_{t+1} + A_{t+1}) &= \tilde{Q}\left(1 + v\rho S_t + (-V + 0.5v^2)(\rho^2 S_t^2 + \sigma^2)\right) + \\
&\quad \bar{A} + m\bar{A}\rho S_t \tag{33} \\
var(Q_{t+1} + A_{t+1}) &= \tilde{Q}^2\left(v + (-V + 0.5v^2)2\rho S_t\right)^2 \sigma^2 + \\
&\quad m^2\bar{A}^2\sigma^2 + 2\tilde{Q}\bar{A}\left(v + (-V + 0.5v^2)2\rho S_t\right)\sigma^2 m \tag{34}
\end{aligned}
$$

Here we have simplified slightly by adopting approximation $\epsilon_{t+1}^2 = \sigma^2$ or $var(\epsilon_{t+1}^2) = 0$. This holds exactly in a simple distribution where ϵ_t can only take on the values $-\sigma$ and $+\sigma$. More generally, it is frequently adopted as a continuous time approximation. Under a normal distribution the variance of ϵ_{t+1}^2 is $2\sigma^4$, which is a small fourth-order term. Dropping this small term makes it easier to represent the equilibria graphically.

Substituting these results into the market clearing condition (22) and taking a quadratic approximation around $S_t = 0$ gives an equation of the form (24). Setting the coefficients Z_0, Z_1 and Z_2 equal to zero, we obtain respectively

$$
\begin{aligned}
\bar{W}\left(\bar{A} + \tilde{Q} + \tilde{Q}(-V + 0.5v^2)\sigma^2 - \frac{1}{\eta}(\nu - W)\tilde{Q} - \frac{1}{\eta}K\tilde{Q}^2\right) &= \\
Km^2\bar{A}^2\sigma^2 + \tilde{Q}^2Kv^2\sigma^2 + K2\tilde{Q}\bar{A}v\sigma^2 m \tag{35}
\end{aligned}
$$

$$\bar{W}\tilde{Q}v(\rho - \frac{1}{\eta}(\nu - W) - \frac{1}{\eta}2K\tilde{Q}) + m\bar{W}\bar{A}\rho = 4K\tilde{Q}^2v(-V + 0.5v^2)\rho\sigma^2 +$$
$$4K\tilde{Q}\bar{A}(-V + 0.5v^2)\rho\sigma^2 m \tag{36}$$

$$\bar{W}\left[(-V + 0.5v^2)\rho^2 - \frac{1}{\eta}(\nu - W)(-V + 0.5v^2) - \frac{1}{\eta}2K\tilde{Q}(-V + v^2)\right] =$$
$$4K\tilde{Q}(-V + 0.5v^2)^2\rho^2\sigma^2 \tag{37}$$

Here we define $\bar{W} = W_I/\gamma$.

The strategy is as follows. For a given value of $\tilde{Q}$ we first solve $\tilde{Q}V$ from (35) as a quadratic function of v. We substitute the result in (36) and (37). This gives respectively a third and fourth order polynomial in v that needs to be solved numerically. This leads to two schedules that map $\tilde{Q}$ into v (possibly multiple values of v) that can be graphed. Equilibria are the points where these schedules intersect.

From (35) we can solve

$$\tilde{Q}V = \alpha_1 + \alpha_2 v + \alpha_3 v^2 \tag{38}$$

where

$$\alpha_1 = \frac{1}{\sigma^2}\left(\bar{A} + \tilde{Q} - \frac{1}{\eta}(\nu - W)\tilde{Q} - \frac{1}{\eta}K\tilde{Q}^2\right) - \frac{K\bar{A}^2m^2}{\bar{W}} \tag{39}$$

$$\alpha_2 = -\frac{2K\tilde{Q}\bar{A}m}{\bar{W}} \tag{40}$$

$$\alpha_3 = 0.5\tilde{Q} - \frac{\tilde{Q}^2K}{\bar{W}} \tag{41}$$

From (36) we have

$$\beta_1 + \beta_2 v + \beta_3 v^2 + \beta_4 v^3 + \beta_5[\tilde{Q}V] + \beta_6[\tilde{Q}V]v = 0 \tag{42}$$

where

$$\beta_1 = \bar{W}\bar{A}\rho m \tag{43}$$

$$\beta_2 = \bar{W}\tilde{Q}\left(\rho - \frac{1}{\eta}(\nu - W) - \frac{1}{\eta}2\tilde{Q}K\right) \tag{44}$$

$$\beta_3 = -2K\tilde{Q}\bar{A}\rho\sigma^2 m \tag{45}$$

$$\beta_4 = -2K\tilde{Q}^2\rho\sigma^2 \tag{46}$$

$$\beta_5 = 4K\bar{A}\rho\sigma^2 m \tag{47}$$

$$\beta_6 = 4K\tilde{Q}\rho\sigma^2 \tag{48}$$

Finally, (37) can be written as

$$\lambda_1 v + \lambda_2 v^2 + \lambda_3 v^4 + \lambda_4[\tilde{Q}V] + \lambda_5[\tilde{Q}V]^2 + \lambda_6[\tilde{Q}V]v^2 = 0 \tag{49}$$

where

$$\lambda_1 = 0 \tag{50}$$
$$\lambda_2 = 0.5\bar{W}\tilde{Q}\rho^2 - \frac{1}{\eta}0.5\bar{W}\tilde{Q}(\nu - W) - \frac{1}{\eta}2\bar{W}K\tilde{Q}^2 \tag{51}$$
$$\lambda_3 = -K\tilde{Q}^2\rho^2\sigma^2 \tag{52}$$
$$\lambda_4 = -\bar{W}\left(\rho^2 - \frac{1}{\eta}(\nu - W) - \frac{1}{\eta}2\tilde{Q}K\right) \tag{53}$$
$$\lambda_5 = -4K\rho^2\sigma^2 \tag{54}$$
$$\lambda_6 = 4K\tilde{Q}\rho^2\sigma^2 \tag{55}$$

Substituting (38) into (42), we have

$$h_1 + h_2 v + h_3 v^2 + h_4 v^3 = 0 \tag{56}$$

where

$$h_1 = \beta_1 + \beta_5\alpha_1 \tag{57}$$
$$h_2 = \beta_2 + \beta_6\alpha_1 + \beta_5\alpha_2 \tag{58}$$
$$h_3 = \beta_3 + \beta_5\alpha_3 + \beta_6\alpha_2 \tag{59}$$
$$h_4 = \beta_4 + \beta_6\alpha_3 \tag{60}$$

Substituting (38) into (49), we have

$$g_1 + g_2 v + g_3 v^2 + g_4 v^3 + g_5 v^4 = 0 \tag{61}$$

where

$$g_1 = \lambda_4\alpha_1 + \lambda_5\alpha_1^2 \tag{62}$$
$$g_2 = \lambda_1 + \lambda_4\alpha_2 + 2\lambda_5\alpha_1\alpha_2 \tag{63}$$
$$g_3 = \lambda_2 + \lambda_4\alpha_3 + 2\lambda_5\alpha_1\alpha_3 + \lambda_6\alpha_1 + \lambda_5\alpha_2^2 \tag{64}$$
$$g_4 = 2\lambda_5\alpha_2\alpha_3 + \lambda_6\alpha_2 \tag{65}$$
$$g_5 = \lambda_3 + \lambda_5\alpha_3^2 + \lambda_6\alpha_3 \tag{66}$$

25

Equations (56) and (61) are third and fourth order polynomials that we solve numerically. The solutions map $\tilde{Q}$ into v. There may be multiple solutions (multiple v for a given $\tilde{Q}$). We then plot these two schedules in a space with v on the vertical axis and $\tilde{Q}$ on the horizontal axis, as in Figures 1-2. There is an equilibrium when the two schedules intersect. The precise equilibria can be found by solving (35)-(37) numerically in Gauss as a fixed point problem in v, V and $\tilde{Q}$. We choose starting values that are close to the equilibria found through visual inspection of where the two schedules intersect. Visual inspection gives approximate values for $\tilde{Q}$ and v. The corresponding value for V follows from (38).

B   Solving the Switching Equilibria

We now consider the equilibria in Section 3.1 of the paper where we allow for a switch between a low and high risk state. p_1 (p_2) is the probability that next period we will be in the low (high) risk state when this period we are in the low (high) risk state. The log equity prices in the low and high risk states are

$$q_t^{\text{low risk}} = \tilde{q}_1 + v_1 S_t - V_1 S_t^2 \tag{67}$$

$$q_t^{\text{high risk}} = \tilde{q}_2 + v_2 S_t - V_2 S_t^2 \tag{68}$$

Assume that currently we are in the low risk state at time t. Analogous to (33), the expectation of $Q_{t+1} + A_{t+1}$, conditional on being in a low risk state in $t+1$, is

$$E_{t+1}(Q_{t+1} + A_{t+1}|t+1 \text{ is low}) = a_{1,low} + a_{2,low}S_t + a_{3,low}S_t^2$$

where $a_{1,low} = \tilde{Q}_1(1 + \omega_1\sigma^2) + \bar{A}$, $a_{2,low} = \tilde{Q}_1 v_1 \rho + m\bar{A}\rho$, $a_{3,low} = \tilde{Q}_1 \omega_1 \rho^2$ and $\omega_1 = -V_1 + 0.5v_1^2$. Similarly, the expectation of $Q_{t+1} + A_{t+1}$ conditional on being in the high risk state at $t+1$ is

$$E_{t+1}(Q_{t+1} + A_{t+1}|t+1 \text{ is high}) = a_{1,high} + a_{2,high}S_t + a_{3,high}S_t^2$$

where $a_{1,high} = \tilde{Q}_2(1 + \omega_2\sigma^2) + \bar{A}$, $a_{2,high} = \tilde{Q}_2 v_2 \rho + m\bar{A}\rho$, $a_{3,high} = \tilde{Q}_2 \omega_2 \rho^2$ and $\omega_2 = -V_2 + 0.5v_2^2$.

The expectation of $Q_{t+1} + A_{t+1}$ is then

$$
\begin{aligned}
&E_t(Q_{t+1} + A_{t+1}) \\
=\ & p_1 E_{t+1}(Q_{t+1} + A_{t+1}|t+1 \text{ is low}) + (1 - p_1)E_{t+1}(Q_{t+1} + A_{t+1}|t+1 \text{ is high}) \\
=\ & d_{1,low} + d_{2,low}S_t + d_{3,low}S_t^2
\end{aligned}
\tag{69}
$$

where $d_{i,low} = p_1 a_{i,low} + (1 - p_1) a_{i,high}$, $i = 1, 2, 3$.

The variance of $Q_{t+1} + A_{t+1}$ is

$$var(Q_{t+1} + A_{t+1}) = E_t(Q_{t+1} + A_{t+1})^2 - (E_t(Q_{t+1} + A_{t+1}))^2 \qquad (70)$$

Dropping terms in S_t that are third and higher order, (69) gives

$$(E_t(Q_{t+1} + A_{t+1}))^2 = d_{1,low}^2 + 2d_{1,low}d_{2,low}S_t + (d_{2,low}^2 + 2d_{1,low}d_{3,low})S_t^2 \qquad (71)$$

Next consider $E_t(Q_{t+1} + A_{t+1})^2$. Conditional on being in a low risk state at $t + 1$, we have

$$Q_{t+1} + A_{t+1} = a_{1,low} + a_{2,low}S_t + a_{3,low}S_t^2 + a_{4,low}\epsilon_{t+1} \qquad (72)$$

where $a_{4,low} = \tilde{Q}_1(v_1 + \omega_1 2\rho S_t) + m\bar{A}$. Using the definition of $a_{4,low}$, we then have

$$E_t((Q_{t+1} + A_{t+1})^2 | t + 1 \text{ is low}) = b_{1,low} + b_2 S_{t,low} + b_3 S_{t,low}^2 \qquad (73)$$

where $b_{1,low} = a_{1,low}^2 + (\tilde{Q}_1 v_1 + m\bar{A})^2 \sigma^2$, $b_{2,low} = 2a_{1,low}a_{2,low} + 4\tilde{Q}_1(\tilde{Q}_1 v_1 + m\bar{A})\omega_1 \rho \sigma^2$, and $b_{3,low} = a_{2,low}^2 + 2a_{1,low}a_{3,low} + 4\tilde{Q}_1^2 \omega_1^2 \rho^2 \sigma^2$. Similarly, conditional on being in a high risk state at $t + 1$ we have

$$E_t((Q_{t+1} + A_{t+1})^2 | t + 1 \text{ is high}) = b_{1,high} + b_{2,high}S_t + b_{3,high}S_t^2 \qquad (74)$$

Here $b_{i,high}$ $(i = 1, 2, 3)$ is defined analogously to $b_{i,low}$ with subscripts low replaced by $high$ and subscripts 1 for $\tilde{Q}$, v and ω replaced by 2. This implies that in the low risk state at t:

$$E_t(Q_{t+1} + A_{t+1})^2 = c_{1,low} + c_{2,low}S_t + c_{3,low}S_t^2 \qquad (75)$$

where $c_{i,low} = p_1 b_{i,low} + (1 - p_1)b_{i,high}$, $i = 1, 2, 3$.

It follows that

$$var(Q_{t+1} + A_{t+1}) = \left(c_{1,low} - d_{1,low}^2\right) + (c_{2,low} - 2d_{1,low}d_{2,low}) S_t$$
$$+ \left(c_{3,low} - d_{2,low}^2 - 2d_{1,low}d_{3,low}\right) S_t^2 \qquad (76)$$

Finally, a quadratic approximation around $S_t = 0$ of $Q_t R_{t+1}$ gives

$$Q_t R_{t+1} = e_{1,low} + e_{2,low}S_t + e_{3,low}S_t^2 \qquad (77)$$

27

where $e_{1,low} = \frac{1}{\eta}\left[(\nu - W) + K\tilde{Q}_1\right]\tilde{Q}_1$, $e_{2,low} = \frac{1}{\eta}\left[(\nu - W) + 2K\tilde{Q}_1\right]\tilde{Q}_1 v_1$ and $e_{3,low} = \frac{1}{\eta}\left[(\nu - W)\omega_1 + 2K\tilde{Q}_1(-V_1 + v_1^2)\right]\tilde{Q}_1$.

Substituting these results into the market equilibrium condition (22), and taking a second order approximation around $S_t = 0$, again gives (24). Setting $Z_0 = 0$, $Z_1 = 0$ and $Z_2 = 0$ gives respectively

$$\bar{W}(d_{1,low} - e_{1,low}) = K(c_{1,low} - d_{1,low}^2) \tag{78}$$

$$\bar{W}(d_{2,low} - e_{2,low}) = K(c_{2,low} - 2d_{1,low}d_{2,low}) \tag{79}$$

$$\bar{W}(d_{3,low} - e_{3,low}) = K(c_{3,low} - d_{2,low}^2 - 2d_{1,low}d_{3,low}) \tag{80}$$

All of this is conditional on being in the low risk state at t. We can similarly impose market equilibrium conditional on being in the high risk state at t. Define $c_{i,high}$ and $d_{i,high}$ $(i = 1, 2, 3)$ the same as $c_{i,low}$ and $d_{i,low}$, with p_1 replaced by $1 - p_2$. Also define $e_{i,high}$ $(i = 1, 2, 3)$ the same as $e_{i,low}$, with the subscripts 1 for $\tilde{Q}$, v, V and ω replaced by subscripts 2. Then imposing market clearing we get three equations analogous to (78)-(80) with the subscripts *low* replaced by *high*. Solving these six equations jointly gives the solutions for $\tilde{Q}_1$, $\tilde{Q}_2$, v_1, v_2, V_1 and V_2. This is done numerically in Gauss, using as starting values the solutions for equilibria 1 and 2 without switching.

C   Data Sources for Figure 5

The data presented in Figure 5 are constructed in the following way. Stock prices are measured by the DJ U.S. total stock market index. Risk is measured as the CBOE SPX volatility VIX index. Volatility of risk is the standard deviation of the VIX index over the past 30 days. Net worth and leverage are based on U.S. brokers and dealers as reported by the Federal Reserve Flow of Funds. For market liquidity we construct a measure similar to Amihud (2002) which, of different market liquidity measures, correlates the most with estimates of price impact computed using very high-frequency data (see Goyenko et al., 2009). Starting with individual stocks, we compute the average absolute daily stock price change over a month per dollar of daily trading volume. This is then averaged over 100 stocks from the S&P index. We are grateful to Giorgio Valente for providing us with the updated measure. Holding period returns and volumes are from Reuters Datastream. To

deal with stationarity, in the spirit of Acharya and Pedersen (2005) the illiquidity measure is multiplied by the ratio of the aggregate volume for all stocks in the sample at the end of a month to the same aggregate volume at the beginning of the sample. A high value of our measure indicates low market liquidity. It is therefore a measure of illiquidity.
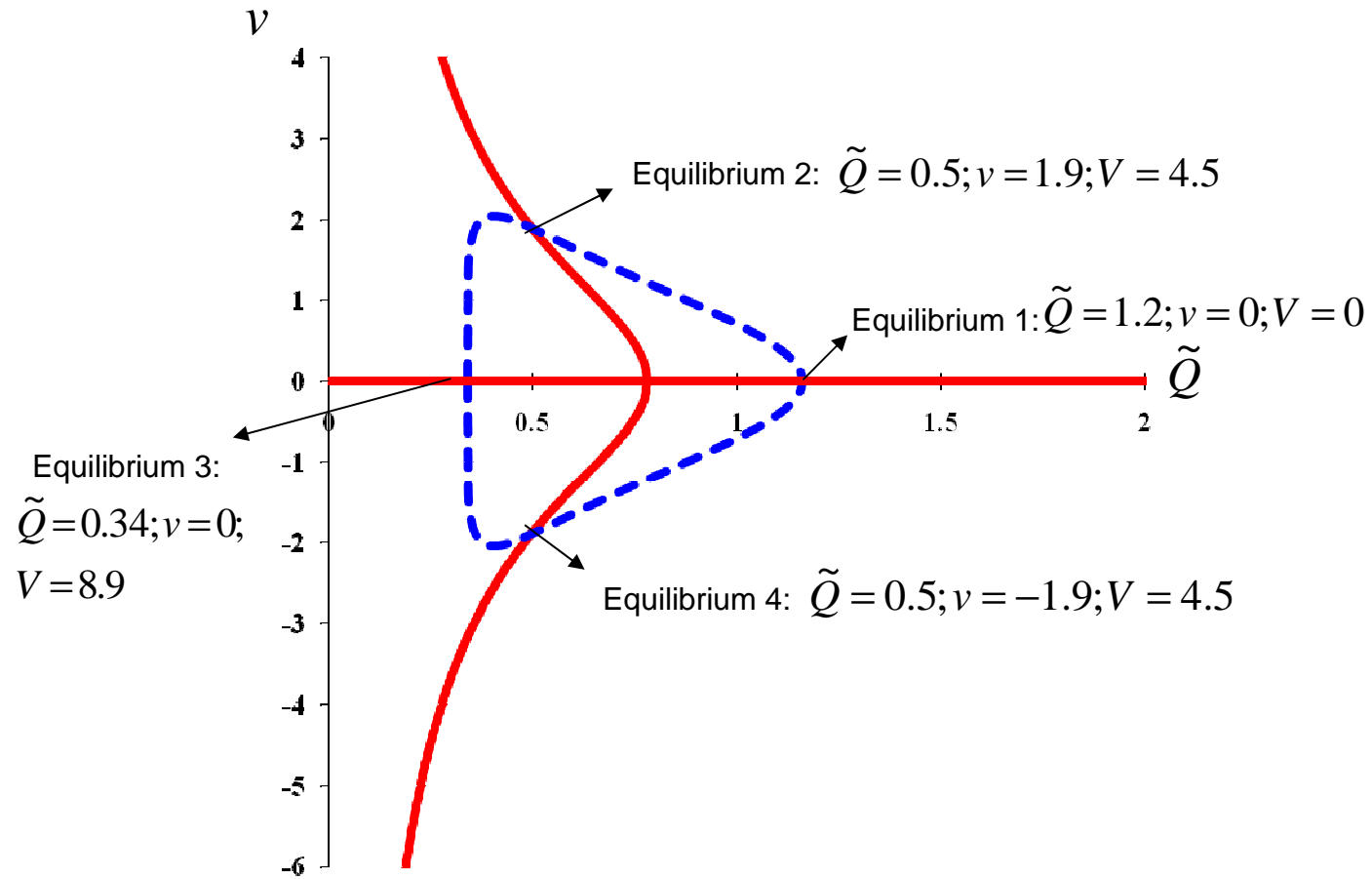
References

[1] Aghion, Philippe, Philippe Bacchetta, and Abhijit Banerjee (2004), "A Corporate Balance-Sheet Approach to Currency Crises," *Journal of Economic Theory* 119, 6-30.

[2] Acharya, Viral V. and Lasse H. Pedersen (2005), "Asset Pricing and Liquidity Risk," *Journal of Financial Economics* 77(2), 375-410.

[3] Adrian, Tobias and Hyun S. Shin (2008), "Financial Intermediary Leverage and Value-at-Risk," Federal Reserve Bank of New York Staff Reports no. 338.

[4] Allen, Franklin and Douglas Gale (1994), "Limited Market Participation and Volatility of Asset Prices," *American Economic Review* 84, 933-955.

[5] Amihud, Yakov, Haim Mendelson and Lasse H. Pedersen (2005), *Liquidity and Asset Prices*, Foundations and Trends in Finance, now Publishers.

[6] Bacchetta, Philippe and Eric van Wincoop (2006), "Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?," *American Economic Review* 96, 552-576.

[7] Bacchetta, Philippe and Eric van Wincoop (2010), "Explaining Sudden Spikes in Global Risk," mimeo.

[8] Bacchetta, Philippe, Cédric Tille, and Eric van Wincoop (2010), "Leverage and Risk Panics," mimeo.

[9] Barro, Robert J. (2006), "Rare Disasters and Asset Markets in the Twentieth Century," *Quarterly Journal of Economics* 121(3), 823-866.

[10] Basak, Suleyman and Georgy Chabakauri (2010), "Dynamic Mean-Variance Asset Allocation " *Review of Financial Studies* 23(8), 2970-3016.

[11] Bloom, Nicholas (2009), "The Impact of Uncertainty Shocks," *Econometrica* 77, 623-685.

[12] Brunnermeier, Markus K. and Lasse H. Pedersen (2009), "Market Liquidity and Funding Liquidity," *Review of Financial Studies* 22, 2201-2238.

[13] Brunnermeier, Markus K. and Yuliy Sannikov (2010), "A Macroeconomic Model with a Financial Sector," mimeo.

[14] Burnside, Craig, Eichenbaum, Martin, and Sergio Rebelo (2004), "Government Guarantees and Self-fulfilling Speculative Attacks," *Journal of Economic Theory* 119, 31–63.

[15] Danielsson, J., H.S. Shin, and J.-P. Zigrand (2010), "Risk Appetite and Endogenous Risk," Financial Market Group Discussion Papers dp647.

[16] Gabaix, Xavier (2008), "Variable Rare Disasters: A Tractable Theory of Ten Puzzles in Macro-finance," *American Economic Review* 98(2), 64-67.

[17] Goyenko, Ruslan, Craig Holden and Charles Trzcinka (2009), "Do Liquidity Measures Measure Liquidity?," *Journal of Financial Economics* 92, 153-181.

[18] Gromb, Denis and Dimitri Vayanos (2002), "Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs," *Journal of Financial Economics* 66, 361-407.

[19] Gromb, Denis and Dimitri Vayanos (2008), "Leverage and Liquidity Dry-Ups: A Framework and Policy Implications," mimeo.

[20] He, Zhiguo. and Arvind Krishnamurthy (2008a), "Intermediary Asset Pricing," NBER WP No. 14517.

[21] He, Zhiguo and Arvind Krishnamurthy (2008b), "A Model of Capital and Crises," NBER WP No. 14366.

[22] Jeanne, Olivier and Andrew K. Rose (2002), "Noise Trading and Exchange Rate Regimes," *Quarterly Journal of Economics* 77, 537-569.

[23] Kyle, Albert S. and Wei Xiong (2001), "Contagion as a Wealth Effect," *The Journal of Finance* 56(4), 1401-1440.

[24] Manuelli, Rodolfo E. and James Peck (1992), "Sunspot-like Effects of Random Endowments," *Journal of Economic Dynamics and Control* 16, 193-206.

[25] Obstfeld, Maurice (1986), "Rational and self-fulfilling balance-of-payments crises," *American Economic Review* 76, 72-81.
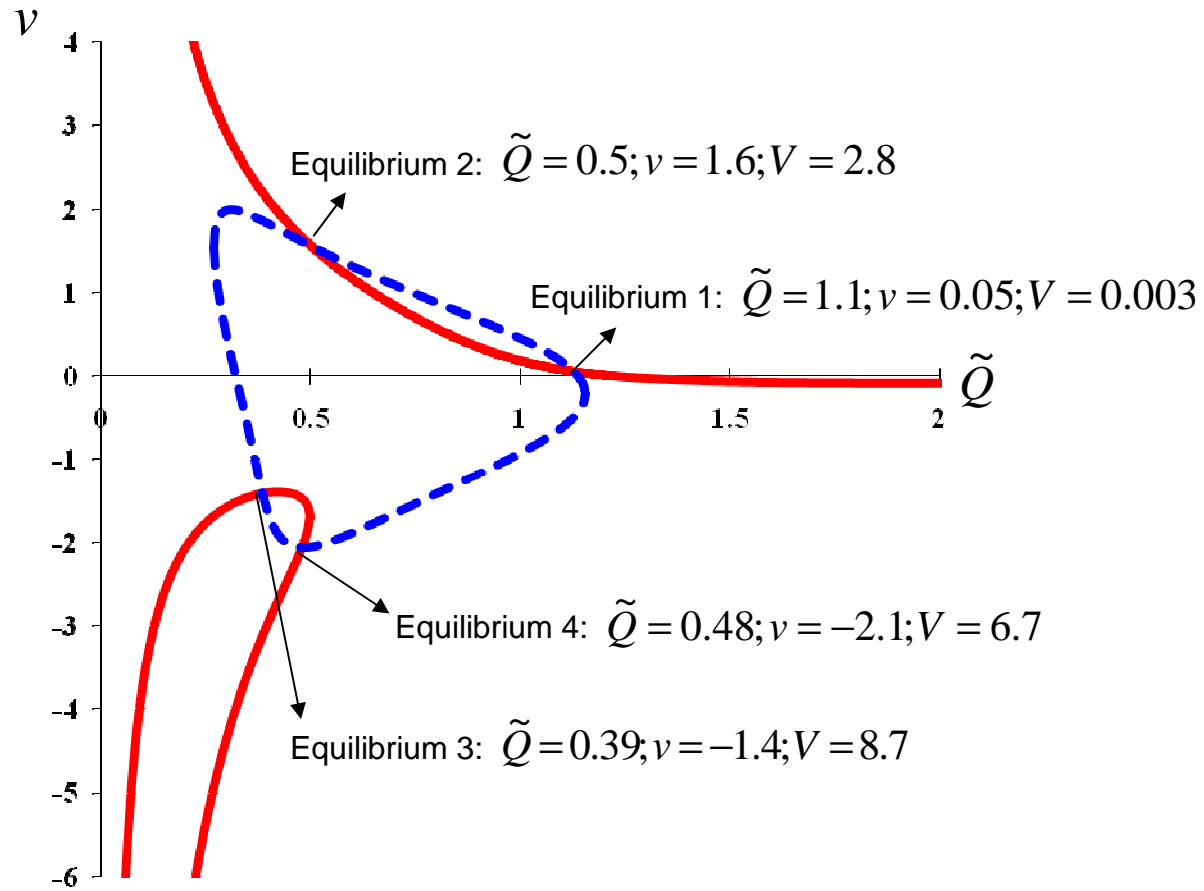
[26] Pagano, Marco (1989), "Endogenous Market Thinness and Stock Price Volatility," *Review of Economic Studies* 56, 269-288.

[27] Spear, Stephen E., Sanjay Srivastava, and Michael Woodford (1990), "Indeterminacy of Stationary Equilibrium in Stochastic Overlapping Generations Models," *Journal of Economic Theory* 50, 265-84.

[28] Walker, Todd B. and Charles H. Whiteman (2007), "Multiple Equilibria in a Simple Asset Pricing Model, " *Economic Letters* 97(3), 191-196.

[29] Xiong, Wei (2001), "Convergence Trading with Wealth Effects: An Amplification Mechanism in Financial Markets," *Journal of Financial Economics* 62, 247-292.

Figure 1 Sunspot Equilibria*

Equilibrium 2: $\tilde{Q}=0.5; v=1.9; V=4.5$

Equilibrium 1: $\tilde{Q}=1.2; v=0; V=0$

Equilibrium 3:
$\tilde{Q}=0.34; v=0;$
$V=8.9$

Equilibrium 4: $\tilde{Q}=0.5; v=-1.9; V=4.5$

*$\bar{A}=0.3; \upsilon-W=0.1; \eta=1; \sigma=0.4; \rho=0.4; \gamma=4; W_I=2; m=0; K=1$

Figure 2 Sunspot-Like Equilibria*

Equilibrium 2: $\tilde{Q} = 0.5; v = 1.6; V = 2.8$

Equilibrium 1: $\tilde{Q} = 1.1; v = 0.05; V = 0.003$

Equilibrium 4: $\tilde{Q} = 0.48; v = -2.1; V = 6.7$

Equilibrium 3: $\tilde{Q} = 0.39; v = -1.4; V = 8.7$

* Parameters are as in Figure 1 except that m=1.

Figure 3 Solution as Function of m (Equilibrium 2)*

$\tilde{Q}$

v

V

m

m

m
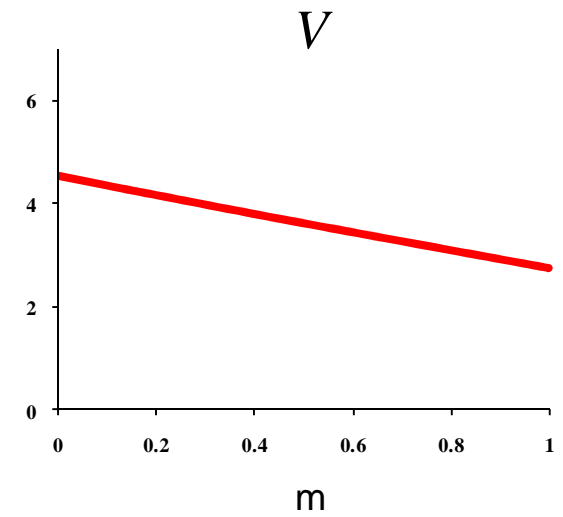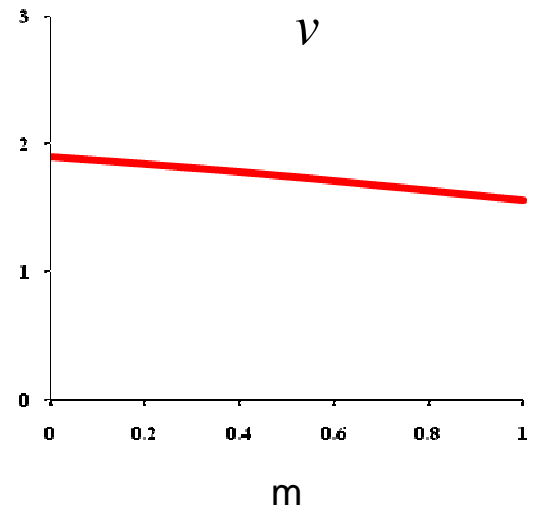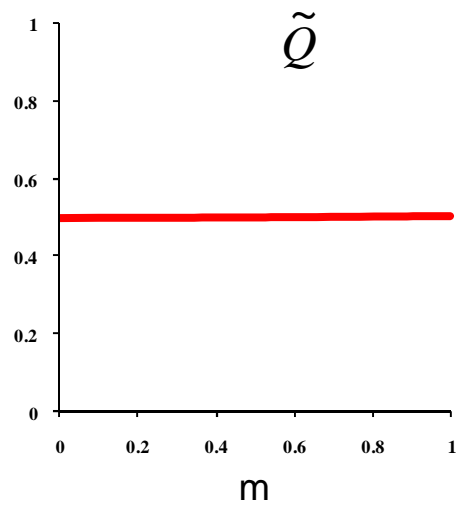
* Other than the value of m, parameters are identical to those in Figure 1.
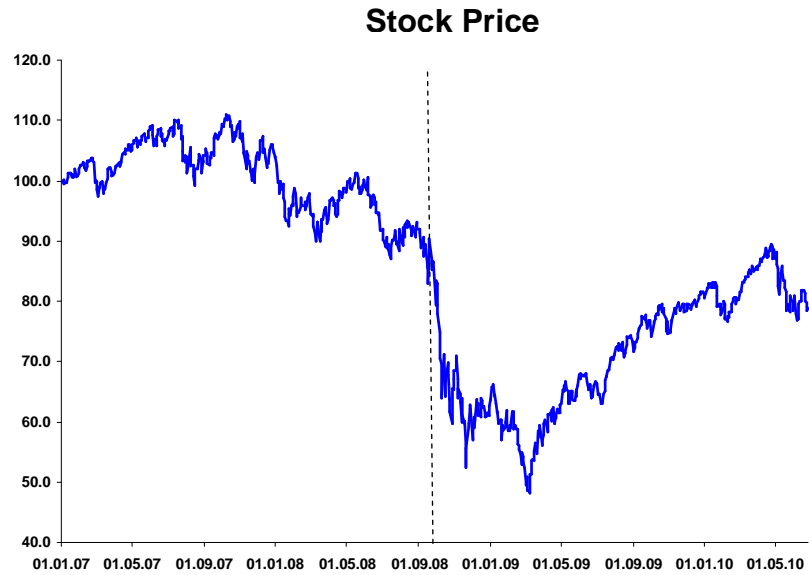
Figure 4 Switching Equilibria*

solid=low risk state; broken=high risk state
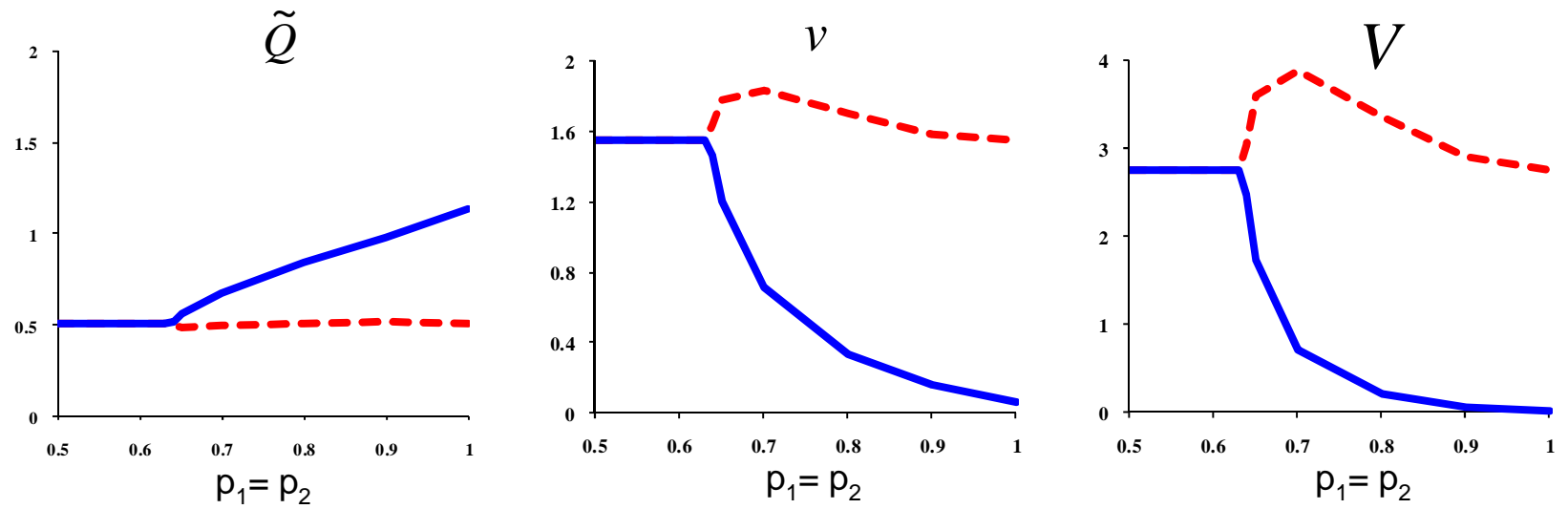
* This is based on the parameters of Figure 2. When $p_1=p_2=1$, the high and low risk states correspond exactly to equilibria 1 and 2 in Figure 2.

Figure 5a Stock prices, interest rate, and risk
vertical lines = Lehman Brothers bankruptcy (Sept. 15, 2008)

Stock Price

Interest Rate

Risk
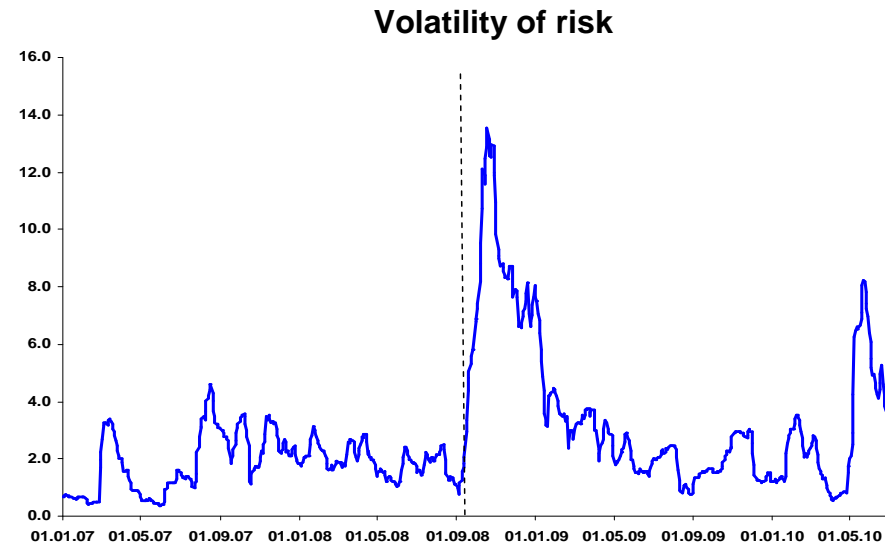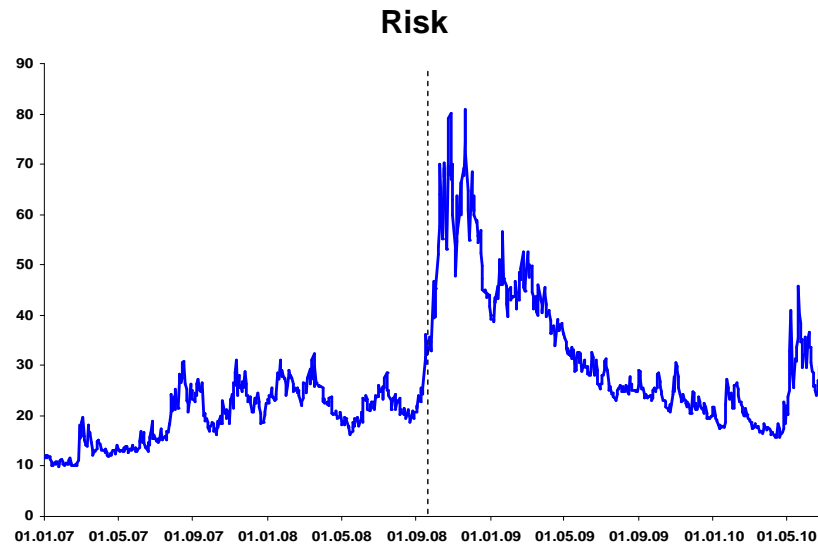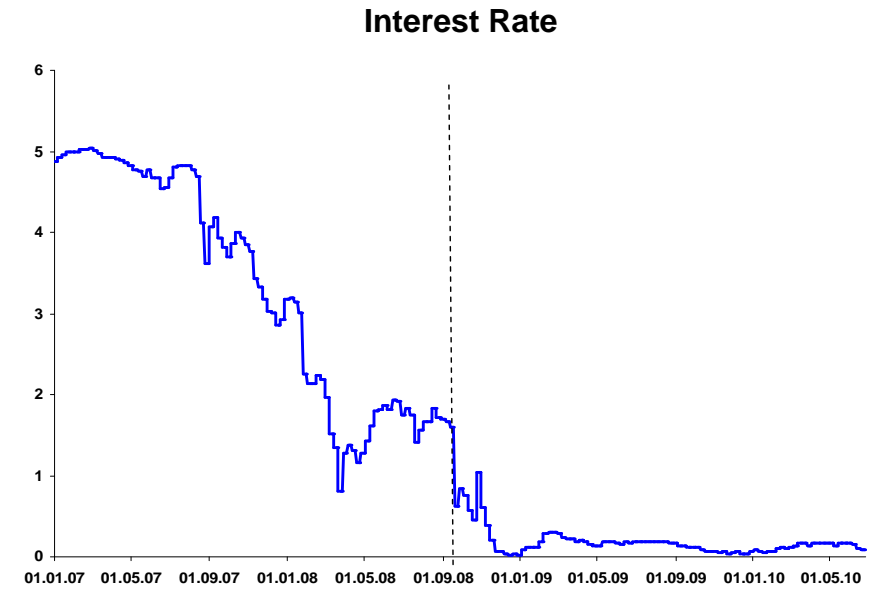
Volatility of risk

Source: Datastream, daily data. Stock prices are the DJ U.S. total market price index (January 1, 2007 = 100). The interest rate is the U.S. 3 month Treasury bill. The risk measure is the CBOE SPX volatility VIX index. The volatility of risk is the 30 days standard deviation of the VIX index.
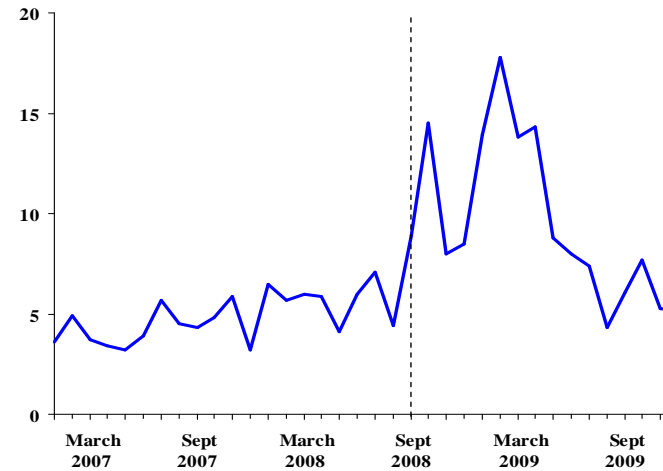
Figure 5b Net worth, leverage and illiquidity

Net worth brokers and dealers
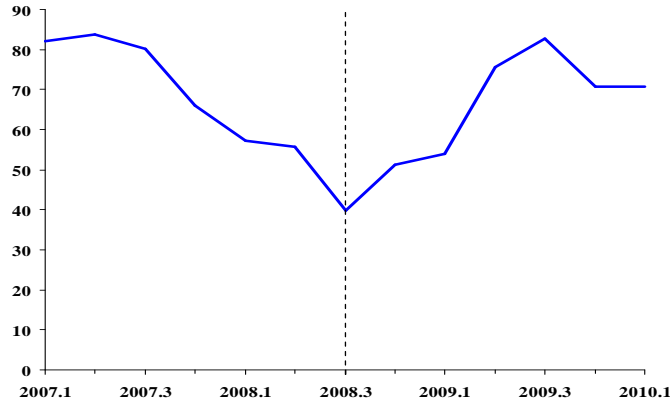
illiquidity

Leverage brokers and dealers
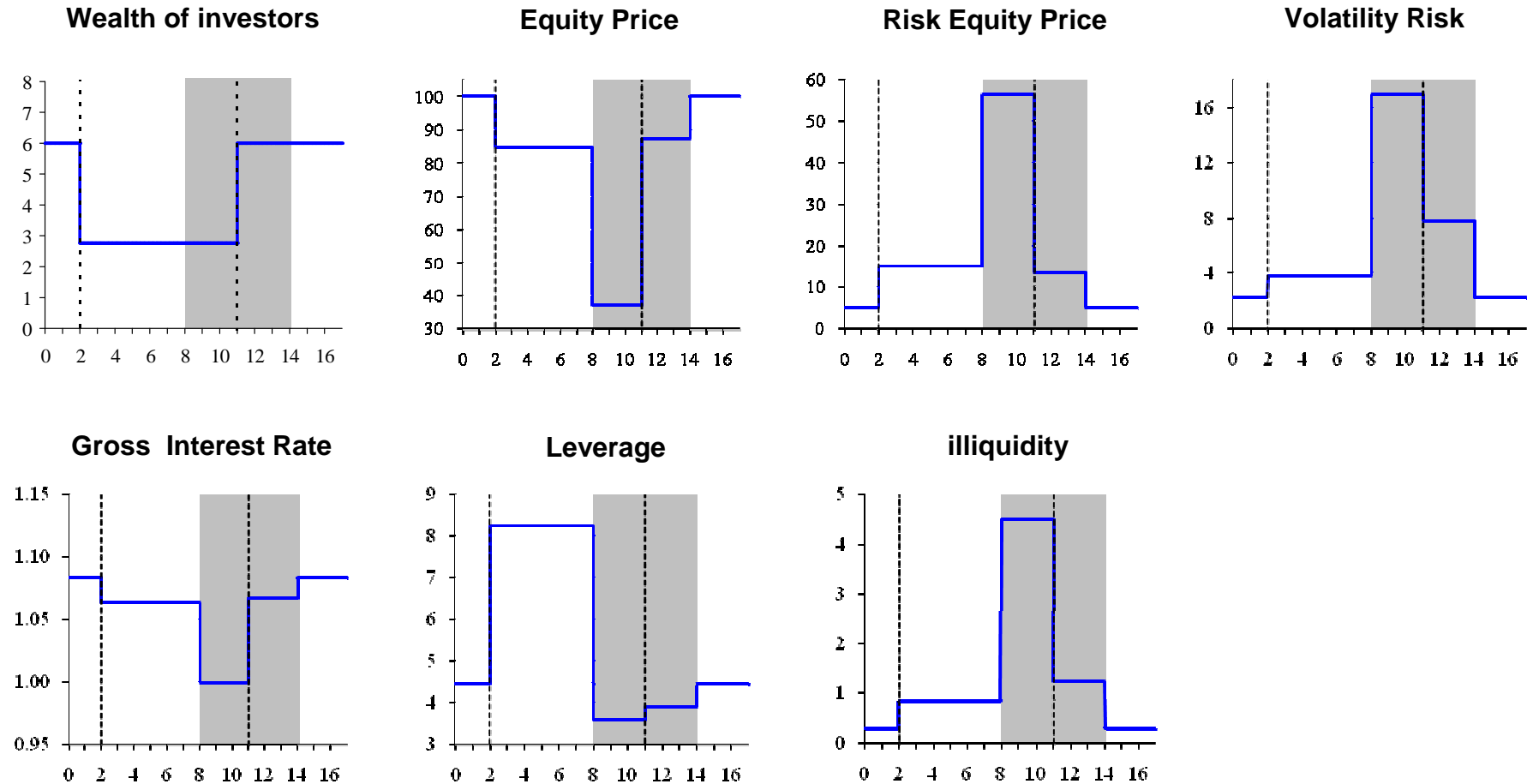

Source: Data on brokers and dealers from the Fed's Flow of Funds (L.129); net worth is assets minus liabilities, billion US $; leverage is net worth divided by assets. The illiquidity measure is an updated measure of Amihud (2002). The vertical lines represent Q3 2008 for the quarterly net worth and leverage series and September 2008 for the monthly illiquidity series.

Figure 6 Model Simulation*
shaded area = high risk equilibrium; vertical lines = endowment shock

Wealth of investors   **Equity Price**   **Risk Equity Price**   **Volatility Risk**

Gross Interest Rate   **Leverage**   **illiquidity**

* The economy starts in the low risk equilibrium. At time 2 the endowment of investors falls from 6 to 2.8. The economy stays in the low risk equilibrium until time 8, at which point is shifts to the high risk equilibrium. At time 11 endowments shift back towards the initial allocation. The economy remains in the high risk equilibrium until time 14, at which points it shifts back to the low risk equilibrium.

$$\overline{A} = 0.15; \upsilon - W = 190; \eta = 200; \sigma_a = 0.1; \sigma_\theta = 0.1; \rho_\theta = 0.7; \gamma = 1; \overline{W}_I = 6; m = 2; K = 20; p_1 = 0.95; p_2 = 0.7$$