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Demography, stock prices and interest rates: The Easterlin hypothesis revisited

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Abstract

During the twentieth century, the U.S. witnessed a cyclical birth rate. This in turn shaped the evolution of the ratio of middle-age to young adults, or MY ratio, which captures the stance of the population pyramid at any given time. In this paper, I study the effects of demographic change, as measured by the MY ratio, on stock prices and interest rates.

I construct an equilibrium model in the spirit of Geanakoplos et al. (2004). The model relates the economic fortune of a cohort to its relative size (Easterlin hypothesis) and matches qualitatively the long-run trends in real interest rates and stock prices in the U.S. postwar era. The first prediction of the model is that the price-earnings ratio and stock prices should be in phase with the MY ratio. The second prediction is that real interest rates should move inversely with the MY ratio, except after the peak in the MY ratio. Unlike Geanakoplos et al. (2004), this model does not predict that stock prices should move inversely with real interest rates. On the contrary, this model shows that in a stationary cyclic equilibrium there may be independent movements in stock and bond prices, which are necessary to prevent arbitrage opportunities.

Keywords: Overlapping generations, age structure, habits, consumption socialization

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1 Introduction

During the twentieth century, the U.S. witnessed a cyclical birth rate: 52 million people were born between 1925 to 1944 (i.e. Depression babies), 79 million from 1945 to 1964 (i.e. baby boomers), and 69 million in the baby bust from 1965 to 1984. This in turn shaped the evolution of the ratio of middle-age to young adults, or MY ratio, which captures the stance of the population pyramid at any given time. In this paper, I study the effects of demographic change, as measured by the MY ratio, on stock prices and interest rates.

I build an equilibrium model in the spirit of Geanakoplos et al. (2004). Basically I embed a cyclic age structure that resembles the Great Depression and baby-boom generations, which repeats itself every 40 years, into an otherwise standard deterministic six-period overlapping generations model. Two predictions are derived from the model. The first is that the price-earnings ratio and stock prices should be in phase with the MY ratio. The second prediction is that real interest rates should move inversely with the MY ratio, except after the peak in the MY ratio.

The rationale behind these results has to do with the fact that people's financial needs change at different periods in life. In terms of the life-cycle model of Modigliani and Brumberg (1954), a higher proportion of people in the prime working years, who save a larger fraction of their current income than at any other time in life, will result in higher stock and bond prices.

The model's predictions are qualitatively consistent with U.S. postwar data. To begin with, the MY ratio is calculated as the size of the cohort aged 40-59 to the size of the cohort aged 20-39 from 1950 to 2008, based on estimates and information provided by the U.S. Census Bureau. As expected, the MY ratio is cyclic (starting from 1950, it has two peaks in 1966 and 2006, and one trough in 1986), and coincides with the long-run trend of the price-earnings ratio and stock prices over the same time span with a phase shift of four years[1]. Moreover, U.S. data show that there has been a negative relation between changes in the MY ratio and changes in the trend of real bond yields since the mid 1950s, except for the 1970s when both the MY ratio and interest rates fell[2].

This is not the end of the story. In equilibrium, the baby boomers are worse off than the Depression babies, presumably because the former experience more adverse economic conditions than the latter (the boomers face higher asset prices in the prime saving years, but lower asset prices in the retirement years). Thus the model relates the economic fortune of a cohort to its relative size, which is nothing but the first-order effect of the Easterlin hypothesis (see Macunovich and Easterlin, 2008).

I pursue this hypothesis further by considering second-order effects, as defined by Macunovich and Easterlin (2008). Specifically, I study the role of parent's standards of living in setting their children's material aspirations. Boomers in this model foresee a deterioration in their living level

[1] That is, the model accounts for the bull market of 1945-1965, the bear market of the 1970s and early 1980s, the subsequent bull market of 1983-1999, and the bear market of the 2000s.

[2] Note that these results are also in line with Barsky (1989), who find that the 1970s were associated with decreasing real interest rates and large drops in the price-earning ratio.

in relation to that of their parents, the Depression babies, and therefore have fewer children as an attempt to maintain the status quo. The model not only readily accommodates exogenous changes in cohort fertility, but also assumes that aspirations have long-lasting effects on later adult economic behavior (aspirations eventually interact with consumption habits).

Young parents are altruistic, in the sense that they care about their children's well-being. Parents determine their children's consumption in the first and second period of life, as children do not make any economic decisions, and thus transmit intergenerationally some sort of aspirations or living standards. The introduction of altruism may tend to weaken the demand for stocks, as parents now need more resources to bring up their children. Inherited tastes work in the same direction, as young parents struggle to cope up with the standards of living determined in childhood by their own parents. However, the presence of consumption persistence works in the opposite way, because parents may save more in the early stages of life in order to keep up with previous consumption levels and the habit stock. In the calibration, the latter channel receives relatively more weight. Because the complementarity of dated goods supposedly increases, now large swings in stock prices are required to clear the markets.

The full-blown version of the model predicts that the peak-to-trough ratio of stock prices is 4.0, halfway between the value of 2.5 attained in the simple version of the model and the historical value of 5 or 6 observed in the postwar period. Furthermore, the model predicts that the price-earnings ratio varies from 7.7 to 30.4, and according to U.S. data, the price-earnings ratio increases from a low of 7.5 in 1950 to around 20.0 in the mid 1960s, and then decreases in the following two decades to 7.8, after which it increases to around 33.9 in 2002. Finally, the model predicts that 10-year real interest rates vary between -4.1 and 9.9 percent, and U.S. postwar real interest rates vary between -3.8 and 9.5 percent.

It is worth noting that Macunovich and Easterlin (2008) argue that impacts of demographic change on interest rates and stock prices should be viewed as third-order effects of the Easterlin hypothesis. Consequently, this paper presents an equilibrium model that fully embraces the hypothesis, or at least a highly stylized version of it.

Related literature. This paper relates to several strands of the literature on the economic implications of a changing U.S. age distribution. Jaimovich and Siu (2009) investigate the consequences of demographic change for business cycle analysis since World War II in the G7 countries. They show that workforce age composition has had a large and significant effect on cyclical volatility, accounting for one-fifth to one-third of the U.S. recent macroeconomic moderation. In the same vein, Fair and Dominguez (1991) use reduced-form equations to examine the effects of the changes in the U.S. population age distribution on the behavior of aggregate consumption, housing-investment, money demand and labor force participation.

Using U.S. data from the last century, Poterba (2001, 2004) studies the relationship between population age structure and real returns on Treasury bills, long-term government bonds, and corporate stock. They find weak support for the effects of a changing age composition on asset returns. However, they find a stronger historical correlation between asset levels and summary measures of the population age structure. He argues that his econometric tests may have limited power, because there are not enough effective degrees of freedom in the historical record of age structure. His empirical findings provide modest support, at best, for the view that asset prices could decline as the share of households over the age of 65 increases.

The inclusion of material aspirations in models of overlapping generations is adapted from Higgins and Williamson (1997) and Brooks (2002), in which parents provide for the consumption of their children. The fact that aspirations have long lasting effects on later adult economic behavior borrows from the vast literature on habit formation in macroeconomics and finance (e.g. see Abel, 1990). This modelling assumption is also motivated by Malmendier and Nagel (2010), who argue that experiences at young age -perhaps conveyed by parents- might be particularly formative and have a relatively strong influence on individuals' decisions today.

This paper is structured as follows. Section 2 introduces the layout of the model, discusses the calibration and presents initial results. Section 3 deals with the full-blown version of the model. Section 4 contrasts the theoretical predictions of the model with the data. The last section concludes.

2 The model

This section follows closely Geanakoplos et al. (2004). The model considers an overlapping generations exchange economy with a single consumption good, in which the economic life of an agent lasts for six periods of ten years each. The economy is closed and time is discrete, with periods indexed by $t = 1, 2, \ldots$.

2.1 Demographic structure

Let Δ_i stand for the age distribution -i.e. pyramid- in the economy when a cohort of size n_i enters the economic scene as young. The population cycle repeats itself every forty years, which is a simplification of the actual birth rate in the U.S. in the last century, and therefore the number of people entering the economy in period $t+5$ is the same as in t. Thus -and this is the key modelling assumption- there are four pyramids Δ_i in this economy, which are composed as follows:

	Pyramids			
Age	Δ_1	Δ_2	Δ_3	Δ_4
20-30	n_1	n_2	n_3	n_4
30-40	n_4	n_1	n_2	n_3
40-50	n_3	n_4	n_1	n_2
50-60	n_2	n_3	n_4	n_1
60-70	n_1	n_2	n_3	n_4
70-80	n_4	n_1	n_2	n_3

Table 1. Cyclic age structure

where Δ_i^j represents the j-th cohort size in pyramid i (e.g. $\Delta_1^2 = \Delta_4^5 = n_4$). On the other hand, the MY ratio in each pyramid is defined as $MY_i = (\Delta_i^3 + \Delta_i^4)/(\Delta_i^1 + \Delta_i^2)$.

2.2 Preferences

Preferences over lifetime consumption streams are represented by a standard discounted sum of utilities:

$$U\left[c_i\right] = \sum_{t=1}^{6} \beta^{t-1} u\left(c_i^t\right)$$

where $c_i = (c_i^1, c_i^2, c_i^3, c_i^4, c_i^5, c_i^6)$ denotes the consumption stream of an agent born in pyramid i, for $i = 1, 2, 3, 4$, and β is the intertemporal discount factor. In other words, the consumption profiles differ by the date at which agents *enter* the economic scene.

For the calibration, u will be taken to be a power utility function

$$u(c_i^t) = \frac{\left(c_i^t\right)^{1-\alpha}}{1-\alpha}, \ \alpha > 0$$

where α stands for the inverse of the intertemporal elasticity of substitution.

2.3 Market arrangements

Agents can trade two financial instruments (a riskless bond b and an equity contract e) to redistribute their income over time. The bond pays one unit of income next period and is in zero net supply; the equity contract is an infinitely-lived security in positive supply (normalized to one), which pays a dividend D each period.

Let $q_t^b = (1 + r_t)^{-1}$ be the price of the bond at time t, where r_t is the interest rate from period t to period $t+1$. Similarly, let q_t^e be the price of equity at time t. In the deterministic economy, the bond and the equity contract are perfect substitutes in each period. Then, from the no-arbitrage condition, the interest rate on bonds and the rate of return on the equity contract must be the same:

$$\frac{D+q^e_{t+1}}{q^e_t}=\frac{1}{q^b_t}=1+r_t$$

For later use, let $\tilde{r}_t$ be the annualized interest rate for a ten-year period, which satisfies $(1+r_t)^{\frac{1}{10}}=1+\tilde{r}_t$.

2.4 The household problem

The problem that the household solves is:

$$\max_{\left\{c^t_i,b^t_i,e^t_i\right\}^6_{t=1}}\sum_{t=1}^{6}\beta^{t-1}u\left(c^t_i\right)$$

$$\begin{aligned}
\text{s.t.:}\qquad & c^1_i=\omega_1-q^b_t b^1_i-q^e_t e^1_i\\
& c^2_i=\omega_2-q^b_{t+1}b^2_i-q^e_{t+1}e^2_i+b^1_i+(D+q^e_{t+1})e^1_i\\
& c^3_i=\omega_3-q^b_{t+2}b^3_i-q^e_{t+2}e^3_i+b^2_i+(D+q^e_{t+2})e^2_i\\
& c^4_i=\omega_4-q^b_{t+3}b^4_i-q^e_{t+3}e^4_i+b^3_i+(D+q^e_{t+3})e^3_i\\
& c^5_i=-q^b_{t+4}b^5_i-q^e_{t+4}e^5_i+b^4_i+(D+q^e_{t+4})e^4_i\\
& c^6_i=b^5_i+(D+q^e_{t+5})e^5_i
\end{aligned}$$

Notice that each agent has an endowment $\omega=(\omega_1,\omega_2,\omega_3,\omega_4,0,0)$, with $\omega_i>0$ for $i=1,2,3,4$, which can be interpreted as the agent's labor income. Of course, income in the last two periods of life (the so-called retirement) is zero.

In a stationary cyclic equilibrium, the sequences of equilibrium stock and bond prices are arithmetic modulo 4, which implies for example that q^b_{t+4} is congruent to q^b_t. Put it differently, stock and bond prices are cyclic:

$$\begin{aligned}
\left(q^b_t,q^b_{t+1},q^b_{t+2},q^b_{t+3},q^b_{t+4},\ldots\right) &= (q_i,q_{i+1},q_{i+2},q_{i+3},q_i,q_{i+1},\ldots)\\
\left(q^e_t,q^e_{t+1},q^e_{t+2},q^e_{t+3},q^e_{t+4},\ldots\right) &= \left(q^e_i,q^e_{i+1},q^e_{i+2},q^e_{i+3},q^e_i,q^e_{i+1},\ldots\right)
\end{aligned}$$

With complete markets, it is possible to reduce the sequential budget constraints into the following Arrow-Debreu budget constraint, which is easier to deal with:

$$
\begin{aligned}
c_i^1 + q_i c_i^2 + q_i q_{i+1} c_i^3 + q_i q_{i+1} q_{i+2} c_i^4 + q_i q_{i+1} q_{i+2} q_{i+3} c_i^5 + q_i q_{i+1} q_{i+2} q_{i+3} q_i c_i^6 &= \\
\omega_1 + q_i \omega_2 + q_i q_{i+1} \omega_3 + q_i q_{i+1} q_{i+2} \omega_4 &
\end{aligned}
$$

plus the usual non-negativity constraints on consumption. Since the objective function is continuous and the budget set is compact, this problem has a solution $c_i = f(q)$, where $q = (q_1, q_2, q_3, q_4)$ and c_i is a 6×1 vector.

2.5 Equilibrium

To fully characterize a stationary equilibrium, it is mandatory to include the market-clearing conditions. In this model, the number of market-clearing conditions is the same as the number of population pyramids in the economy. Because of Walras law, it is enough to clear the goods market:

$$
\begin{aligned}
n_1\left(c_1^1-\omega_1\right)+n_4\left(c_4^2-\omega_2\right)+n_3\left(c_3^3-\omega_3\right)+n_2\left(c_2^4-\omega_4\right)+n_1 c_1^5+n_4 c_4^6-D &= 0 \\
n_2\left(c_2^1-\omega_1\right)+n_1\left(c_1^2-\omega_2\right)+n_4\left(c_4^3-\omega_3\right)+n_3\left(c_3^4-\omega_4\right)+n_2 c_2^5+n_1 c_1^6-D &= 0 \\
n_3\left(c_3^1-\omega_1\right)+n_2\left(c_2^2-\omega_2\right)+n_1\left(c_1^3-\omega_3\right)+n_4\left(c_4^4-\omega_4\right)+n_3 c_3^5+n_2 c_2^6-D &= 0 \\
n_4\left(c_4^1-\omega_1\right)+n_3\left(c_3^2-\omega_2\right)+n_2\left(c_2^3-\omega_3\right)+n_1\left(c_1^4-\omega_4\right)+n_4 c_4^5+n_3 c_3^6-D &= 0
\end{aligned}
$$

Formally, a stationary equilibrium for this economy is (i) a collection of consumption bundles $c = (c_1, c_2, c_3, c_4)$, where c is a 6×4 matrix, bond prices q and stock prices $q^e = (q_1^e, q_2^e, q_3^e, q_4^e)$ that satisfy the no-arbitrage condition, such that, (ii) given these prices, agents in each pyramid solve the constrained utility maximization problem, and (iii) markets clear.

Since this definition departs from the usual textbook treatment of steady states, in which prices and allocations remain constant, it remains to show that there exists such a cyclic stationary equilibrium.

Proposition 1 *There is at least one cyclic stationary equilibrium in this economy.*

Proof. To begin with, let $\mathbb{R}_+^4$ be the space of prices q, and let $\mathbb{R}^4$ be the space of excess demand functions $\mathbf{Z}(q) = [Z_1(q), Z_2(q), Z_3(q), Z_4(q)]'$, such that the i-th row of $\mathbf{Z}(q)$ corresponds to the left hand side of the i-th market-clearing condition as defined above. Now I use standard results from general equilibrium theory to argue that there exists a sufficiently large $\bar{q}$ such that $Z_i(q) < 0$, when $q \geq \bar{q}$. Similarly, there exists a sufficiently small $\varepsilon > 0$, such that $Z_i(q) > 0$ when $q \leq \varepsilon\iota$, where $\iota = [1, 1, 1, 1]'$. Then it is possible to restrict the domain of q to $S = \left\{ q \in \mathbb{R}_+^4 \middle| \varepsilon \leq q_i \leq \bar{q}_i \right\}$. Finally, let $f(q) = \pi[q - \mathbf{Z}(q)]$, where π is a projection operator onto S. Note that this map $f : S \rightarrow S$ is a solution to the programming problem $\min_f \frac{1}{2} \|f - [q - \mathbf{Z}(q)]\|^2$ subject to $f \in S$. Since f is continuous and S is compact, the Brouwer theorem allows me to conclude that there exists a fixed point $q^* \in S$, which is interior by construction. It can be verified that $q^* = f(q^*)$ implies $\mathbf{Z}(q^*) = 0$, so this must correspond to a stationary competitive equilibrium. ■

It important to note that cycles do not arise endogenously as in Azariadis and Guesnerie (1986); on the contrary, 4-period cycles originate from the inclusion of the same number of market-clearing conditions. In general, it could be argued that n-period cycles could exist together with n market-clearing conditions, but the study of cycles of higher periodicity is beyond the scope of this paper.

Finally, there is nothing in this model that prevents net real interest rates from being negative, even in the presence of an infinitely-lived secutiry (or land). Since gross interest rates satisfy:

$$1 + r_i = \frac{1}{q_i} = \frac{D + q_{i+1}^e}{q_i^e}$$

then net real interest rates may become negative if q_{i+1}^e is low enough relative to q_i^e.

2.6 Calibration

Cohort sizes. To begin with, the choice of cohort sizes $(n_1, n_2, n_3, n_4) = (26.07, 26.4, 36.76, 41.96)$ mimics the Great Depression and the baby-boom generations. During the first two periods the small cohorts -i.e. the Depression babies- enter, with $n_1 + n_2 = 52$, and in the next two periods the large cohorts -i.e. the boomers- enter, with $n_3 + n_4 = 79$. The cycle then repeats itself. These numbers imply that the values of the MY ratio at each pyramid are $(MY_1, MY_2, MY_3, MY_4) = (0.92, 1.50, 1.08, 0.67)$.

Discount factor and elasticity of substitution. Since a *period* in the model represents ten years in the lifetime of an agent, I take the discount factor to be $\beta = 0.75$ (corresponding to an annual intertemporal discount factor of 0.975). The inverse of the intertemporal elasticity of substitution is equal to $\alpha = 4$.

Wage income. I use the information from the U.S. Census Bureau (Table H-10, Age of household: All races by median and mean income: 1967 to 2008) in order to calculate the wedges among the average annual real incomes of agents in the age groups 15-24, 25-34, 35-44 and 45-54. Because preferences are homothetic, only relative wages matter. The maximum ratio of the average annual incomes of agents in the age groups 45-54 and 25-34 is 1.54, and the corresponding ratios between age groups 35-44 and 25-34, and 25-34 and 15-24 are 1.32 and 1.63, respectively. If $\omega_2 = 2$ serves as a numeraire, the endowment profile is $\omega = (1.23, 2, 2.64, 3, 0, 0)$.

Dividends. Following Geanakoplos et al. (2004), I assume that the ratio of generalized dividends to generalized wages is 0.19 on average. Total wages in Δ_i are equal to $\omega_{\Delta_i} = \Delta_i^1\omega_1 + \Delta_i^2\omega_2 + \Delta_i^3\omega_3 + \Delta_i^4\omega_4 + \Delta_i^5 \times 0 + \Delta_i^6 \times 0$. Thus, in this economy $(\omega_{\Delta_1}, \omega_{\Delta_2}, \omega_{\Delta_3}, \omega_{\Delta_4}) = (292, 306, 293, 273)$. Finally, $D = 0.19[(292 + 306 + 293 + 273)/4] \approx 55$.

For the sake of exposition, I report again the parameter values in the following table:

α	inverse elast. substitution	4.00
β	intertemp. discount factor	0.75
n_1	initial size of cohort in pyramid 1	26.07
n_2	initial size of cohort in pyramid 2	26.40
n_3	initial size of cohort in pyramid 3	36.76
n_4	initial size of cohort in pyramid 4	41.96
ω_1	endowment 20-30 ys old	1.23
ω_2	endowment 30-40 ys old	2.00
ω_3	endowment 40-50 ys old	2.64
ω_4	endowment 50-60 ys old	3.00
ω_5	endowment 60-70 ys old	0.00
ω_6	endowment 70-80 ys old	0.00
D	dividends	55.27

Table 2. Parameter configuration

2.7 Results

In order to compute an equilibrium price vector, first I write the 6×4 matrix $c(q)$ in closed form. Then I feed the nonlinear system of market-clearing equations laid out above, and use the Levenberg-Marquardt algorithm to solve for $q^* \in \mathbb{R}_+^4$. With q^* at hand, I obtain the associated interest rates and the prices of the equity contract, as well as the matrix c. The price-earnings ratio is constructed assuming that corporate firms distribute half their earnings as dividends, as in Geanakoplos et al. (2004). In other words, the price-earnings ratio is $PE_i = [q_i^e/(D/10)]/2$.

The equilibrium prices and interest rates, together with the values of the MY ratio, are presented in Table 3:

	Pyramids			
	Δ_1	Δ_2	Δ_3	Δ_4
q_i	0.46	0.96	1.04	0.44
q_i^e	108.24	180.21	133.22	72.35
PE_i	9.79	16.30	12.05	6.55
$\tilde{r}_i$	8.08	0.45	-0.43	8.50
MY_i	0.93	1.50	1.08	0.67

Table 3. Prices in a stationary equilibrium

These results can be better understood with the help of Figure 1, which uses the property that the equilibrium sequences are arithmetic modulo 4. In this model with 4 pyramids, stock prices and the price-earnings ratio are in phase with the MY ratio (see Figure 1, quadrants 1.1 and 1.2), but bond prices and the MY ratio do not comove and consequently real interest rates are no longer in reverse phase with the MY ratio (see Figure 1, quadrant 1.3). It is also clear that real interest rates move inversely with the price-earnings ratio, except after the peak in the latter (see Figure 1, quadrant 1.4).

In a stationary equilibrium, the peak-to-trough ratio of stock prices is 2.5. The model also predicts that the price-earnings ratio varies from 6.6 to 16.3. Finally, the model predicts that 10-year real interest rates should vary between -0.4 and 8.5 percent.

What is the story behind these results? Households are more likely to save as they approach retirement, because of their hump-shaped income profile. With perfect foresight, they are indifferent between the two assets and without loss of generality, only one security -the equity contract- is needed in this economy. Note that when the number of households in their prime saving years is high relative to the number of young households, stock prices rise in response to excess demand, as stocks are in fixed net supply. On the other hand, when the number of households in their prime saving years is low relative to the number of young households, stock prices decrease.

Real interest rates (or bond prices) adjust accordingly to prevent arbitrage opportunities: interest rates are high when stock prices are expected to increase and viceversa. In particular, interest rates are low in pyramids 2 and 3, because agents anticipate drops in stock prices in pyramids 3 and 4, as households born in large cohorts begin to enter the job market (or, alternatively, because households born in large cohorts begin to retire). Because interest rates decrease after the peak in the MY ratio, it is not always possible to observe a negative relation between changes in the MY ratio (and therefore stock prices) and changes in the long-run trend of interest rates.

The previous paragraphs heavily rely on the number of pyramids though. Geanakoplos et al. (2004) consider an economy with 2 pyramids and 20-year periods, in which the MY ratio is either high or low. When the MY is high, the price of the equity contract is high and the real interest rate is low, and viceversa. They further study the case with 10 pyramids and 4-year periods, and predict that: (i) long-term (20-year) real interest rates are in reverse phase with equity prices and the MY ratio, and (ii) long-term rates are below short-term (4-year) rates on the ascending phase of equity prices and above them on the descending phase. Unfortunately, as I will show in section 4, both predictions are difficult to obtain from the data.

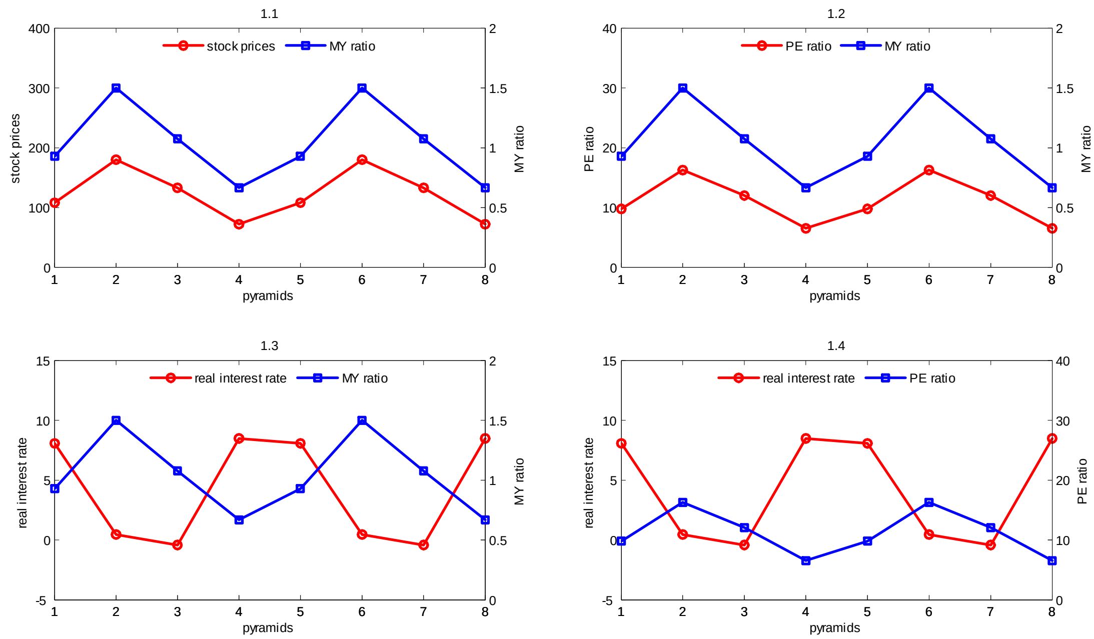

Figure 1. Stock prices, price-earning ratio, long-term real interest rates and MY ratio in a stationary equilibrium.

This is not the end of the story, however. In Table 4, I calculate the utility level $U[c_i]$ of the household born at the beginning of pyramid Δ_i. It is evident that the baby boomers (i.e. people born in large cohorts n_3 and n_4) are worse off than the Depression babies (i.e. people born in small cohorts n_1 and n_2), because the former experience more adverse lifetime economic conditions than the latter (the boomers face higher asset prices in the prime saving years, but lower asset prices in the retirement years). Thus the model relates the economic fortune of a cohort to its relative size, which is nothing but the first-order effect of the Easterlin hypothesis (see Macunovich and

Easterlin, 2008)[3].

	Cohort i			
	n_1	n_2	n_3	n_4
Utility level $U[c_i]$	-0.19	-0.16	-0.23	-0.30

Table 4. Economic fortune and cohort size

3 Aspirations, family size and habit formation

In the previous section, the dynamics were largely driven by age distribution effects. Yet the model could not capture either the peak-to-trough ratio of stock prices of 5 or 6 observed in the postwar era, or the range of long-term real interest rates over the same time span (real ex-post 10-year interest rates vary between -3.5 and 8.1 percent, and real ex-ante 10-year interest rates vary between -3.8 and 9.5 percent). The model also fails to account for the peak in the price-earnings ratio of 33.9 in the early 2000s.

In order to improve the quantitative predictions of the model, I pursue the Easterlin hypothesis further by considering second-order effects, as defined by Macunovich and Easterlin (2008). Specifically, I study the role of parent's standards of living in setting their children's material aspirations. Boomers in this model foresee a deterioration in their living level in relation to that of their parents, the Depression babies, and therefore have fewer children as an attempt to maintain the status quo. For simplicity, the model studies the effects of parent's standards of living on children's savings plans, *after* accommodating exogenous changes in cohort fertility. To enrich the model, I assume that aspirations eventually interact with consumption habits and therefore have long-lasting effects on later adult economic behavior.

3.1 Cohort fertility

Parents in each pyramid between 20 and 30 years of age have children. Once children enter the job market at the age of 20, they become young parents automatically as well as active decision makers. The implicit demographic structure is as in Table 5:

[3]Is worth saying that the original channel operates mainly through the labor market, because an expansion in the relative supply of younger workers deteriorates their relative wage rates, unemployment conditions and upward job mobility (see Welch, 1979).

	Pyramids			
Age	Δ_1	Δ_2	Δ_3	Δ_4
0-10	n_3	n_4	n_1	n_2
10-20	n_2	n_3	n_4	n_1
20-30	n_1	n_2	n_3	n_4
30-40	n_4	n_1	n_2	n_3
40-50	n_3	n_4	n_1	n_2
50-60	n_2	n_3	n_4	n_1
60-70	n_1	n_2	n_3	n_4
70-80	n_4	n_1	n_2	n_3

Table 5. Augmented pyramids

In each pyramid the number of children per parent is given by $(v_1, v_2, v_3, v_4) = \left(\frac{n_3}{n_1}, \frac{n_4}{n_2}, \frac{n_4}{n_2}, \frac{n_2}{n_4}\right)$.

3.2 Preferences

Preferences over lifetime consumption streams are represented by a discounted sum of utilities:

$$U\left[\mathbf{c}_i, c_{i+2}^0\right] = \lambda v_i^{1-\varepsilon}\left[u\left(c_i^{-1}\right) + \beta u\left(c_i^0\right)\right] + \Sigma_{t=1}^6 \beta^{t-1} w\left(c_i^t, s_i^t\right)$$

where $\mathbf{c}_i = \left(c_i^{-1}, c_i^0, c_i^1, c_i^2, c_i^3, c_i^4, c_i^5, c_i^6\right)$ is an 8×1 vector, $u(\cdot)$ is the power utility function and:

$$w\left(c_i^t, s_i^t\right) = \frac{\left(c_i^t - \rho s_i^t\right)^{1-\alpha}}{1-\alpha}, \ \alpha > 0$$

where:

$$s_i^t = \begin{cases} (1-\theta)\, s_i^{t-1} + \theta c_i^{t-1}, & t > 1 \\ (1-\theta)\, c_{i+2}^0, & t = 1 \end{cases}$$

Young parents are altruistic, in the sense that they care about their children's well-being. Parents allocate c_i^{-1} and c_i^0 to their children in the first and second period of life, respectively, as children do not make any economic decisions. The weight attached to the children's utility is given by $\lambda v_i > 0$, where λ is a parameter related to the child-equivalent consumption and v_i stands for the number of children. On the other hand, $c_i = (c_i^1, c_i^2, c_i^3, c_i^4, c_i^5, c_i^6)$ now denotes the consumption stream of a young parent in pyramid i, for $i = 1, 2, 3, 4$, and β is the intertemporal discount factor.

Similarly, c_{i+2}^0 stands for the consumption received by the young parent in pyramid i from his own parents just before entering the job market. Under this formulation, c_{i+2}^0 is a consumption externality that generates an intergenerational correlation in consumption. In a sense, it allows for the transmission of living standards or aspirations. Finally notice that whenever $\rho > 0$, the $w(\cdot)$ function is consistent with *external* habit formation or *keeping up with the Joneses if* $\theta = 0$, and $\theta = 1$ corresponds to *internal* habit formation.

3.3 The household problem

The problem that the household solves is:

$$\max_{\{c_i^t\}_{t=-1}^{6}} \lambda v_i^{1-\varepsilon}\left[u\left(c_i^{-1}\right)+\beta u\left(c_i^{0}\right)\right]+\Sigma_{t=1}^{6}\beta^{t-1}w\left(c_i^t, s_i^t\right)$$

$$\begin{aligned} \text{s.t. } v_i c_i^{-1} + v_i q_i c_i^0 + c_i^1 + q_i c_i^2 + q_i q_{i+1} c_i^3 + q_i q_{i+1} q_{i+2} c_i^4 + q_i q_{i+1} q_{i+2} q_{i+3} c_i^5 + q_i q_{i+1} q_{i+2} q_{i+3} q_i c_i^6 &= \\ \omega_1 + q_i\omega_2 + q_i q_{i+1}\omega_3 + q_i q_{i+1} q_{i+2}\omega_4 & \end{aligned}$$

plus the usual non-negativity constraints on consumption, taking s_i^1 (or c_{i+2}^0) as given. The right hand side of the Arrow-Debreu budget constraint looks exactly the same as in section 2, and the left hand side takes into account the children's consumption of the unique good in this economy.

I also include additional restrictions that prevent the slope of indifference curves from being positive (see Lahiri and Puhakka, 1998 for details), and restrict the arguments inside $w(\cdot)$ to be non-negative.

3.4 Equilibrium

As in the previous section, it is mandatory to include the market-clearing conditions to fully characterize a stationary equilibrium. Because of Walras law, it is enough to clear the goods market (notice that the children's consumption is also accounted for):

$$\begin{aligned}
n_1\left(c_1^1+v_1c_1^{-1}-\omega_1\right)+n_4\left(c_4^2+v_4c_4^0-\omega_2\right)+n_3\left(c_3^3-\omega_3\right)+n_2\left(c_2^4-\omega_4\right)+n_1c_1^5+n_4c_4^6-D &= 0\\
n_2\left(c_2^1+v_2c_2^{-1}-\omega_1\right)+n_1\left(c_1^2+v_1c_1^0-\omega_2\right)+n_4\left(c_4^3-\omega_3\right)+n_3\left(c_3^4-\omega_4\right)+n_2c_2^5+n_1c_1^6-D &= 0\\
n_3\left(c_3^1+v_3c_3^{-1}-\omega_1\right)+n_2\left(c_2^2+v_2c_2^0-\omega_2\right)+n_1\left(c_1^3-\omega_3\right)+n_4\left(c_4^4-\omega_4\right)+n_3c_3^5+n_2c_2^6-D &= 0\\
n_4\left(c_4^1+v_4c_4^{-1}-\omega_1\right)+n_3\left(c_3^2+v_3c_3^0-\omega_2\right)+n_2\left(c_2^3-\omega_3\right)+n_1\left(c_1^4-\omega_4\right)+n_4c_4^5+n_3c_3^6-D &= 0
\end{aligned}$$

Formally, a stationary equilibrium for this economy is (i) a collection of consumption bundles $\mathbf{c} = (\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3, \mathbf{c}_4)$, where $\mathbf{c}$ is an 8×4 matrix, bond prices q and stock prices $q^e = (q_1^e, q_2^e, q_3^e, q_4^e)$ that satisfy the no-arbitrage condition, such that, (ii) given these prices, agents in each pyramid solve the constrained utility maximization problem, and (iii) markets clear.

3.5 Calibration

Fertility. Changes in cohort fertility are exogenous in this model. Because $(n_1, n_2, n_3, n_4) = (26.07, 26.4, 36.76, 41.96)$, it is straightforward to calculate $(v_1, v_2, v_3, v_4) = (1.41, 1.58, 0.70, 0.63)$. These numbers suggest that parents in small cohorts have more children than those in large cohorts, which has been historically the case in the U.S. during the twentieth century.

Preferences. As in Geanakoplos et al. (2004), I consider $\lambda = 0.0625$. Moreover, I assume $\varepsilon = 0.5$ as in Brooks (2002).

Aspirations and habit formation. The parameter that captures the habit stock is set to $\rho = 0.5$, although there is a wide range in the literature that goes from 0.174 (Kocherlakota, 1996) to 0.87 (Campbell and Cochrane, 1999). The parameter that measures persistence is set to $\theta = 0.95$.

The rest of parameter values remain the same as before. For the sake of exposition, I report again the parameter values in Table 6:

λ	child equivalent consumption	0.06
ε	weight per child	0.50
v_1	number of children of young agent in pyramid 1	1.41
v_2	number of children of young agent in pyramid 2	1.58
v_3	number of children of young agent in pyramid 3	0.70
v_4	number of children of young agent in pyramid 4	0.63
θ	habit persistence	0.95
ρ	habit stock	0.50

Table 6. Parameter configuration

3.6 Results

I use the first order conditions from the contrained optimization problem, together with the market-clearing conditions specified above, to feed a non-linear system with 36 equations in 36 unknowns. Then I use the Levenberg-Marquardt algorithm to solve for $(q^*, \mathbf{c}^*) \in \mathbb{R}^{36}_{+}$ (since the constraints that ensure that the slopes of the indifference curves are negative at all times happen to be non-binding at the stationary equilibrium, the non-linear algorithm suffices).

The equilibrium prices and interest rates, together with the MY ratio, are presented in Table 7:

	Pyramids			
	Δ_1	Δ_2	Δ_3	Δ_4
q_i	0.42	1.25	1.51	0.39
q_i^e	166.15	336.53	213.59	85.59
PE_i	15.03	30.44	19.29	7.74
$\tilde{r}_i$	8.95	-2.23	-4.06	9.91
MY_i	0.93	1.50	1.08	0.67

Table 7. Prices in a stationary equilibrium

Again, these results can be better understood with the help of Figure 2, which uses the property that the equilibrium sequences are arithmetic modulo 4. Again, stock prices and the price-earnings ratio are in phase with the MY ratio (see Figure 2, quadrants 2.1 and 2.2), but bond prices and the MY ratio do not comove and consequently real interest rates are no longer in reverse phase with the MY ratio (see Figure 2, quadrant 2.3).

In a stationary equilibrium, the predicted peak-to-trough ratio of stock prices is 4.0 and the range of real interest rates goes from -4.1 to 9.9 percent. Finally, the model predicts that the price-earnings ratio should vary between 7.7 and 30.4.

As in the previous section, a high proportion of households in their prime saving years leads to high stock prices in response to excess demand, as stocks are in fixed net supply, and viceversa. There are other ingredients in this version of the model though. On the one hand, the introduction of altruism (i.e. the fact that parents provide for the consumption of their children) may tend to weaken the demand for stocks, as parents now need more resources to bring up their children. Inherited tastes work in the same direction, as parents struggle to cope up with the standards of living determined in childhood by their own parents. However, the presence of consumption persistence works in the opposite way, because parents may save more in the early stages of life in order to keep up with previous consumption levels and the habit stock.

The calibrated parameters $(\rho, \theta) = (0.5, 0.95)$ from Table 6 suggest not only that young parents' instantaneous utility is highly attached to current and past consumption, but also that the effect of inherited tastes (that operate through the habit stock) is small. Because the complementarity of dated goods supposedly increases, now large swings in stock prices are required to clear the markets. Not surprisingly, the intertemporal elasticity of substitution (IES) between adjacent goods does not remain constant, even though the functional form is of the power-utility type[4].

[4] For simplicity, consider the case in which $W(c_i^t, c_i^{t+1}) = (1-\alpha)^{-1}\left[(c_i^t)^{1-\alpha} + \beta(c_i^{t+1} - \rho c_i^t)^{1-\alpha}\right]$. The standard formula for the intertemporal elasticity of substitution is:

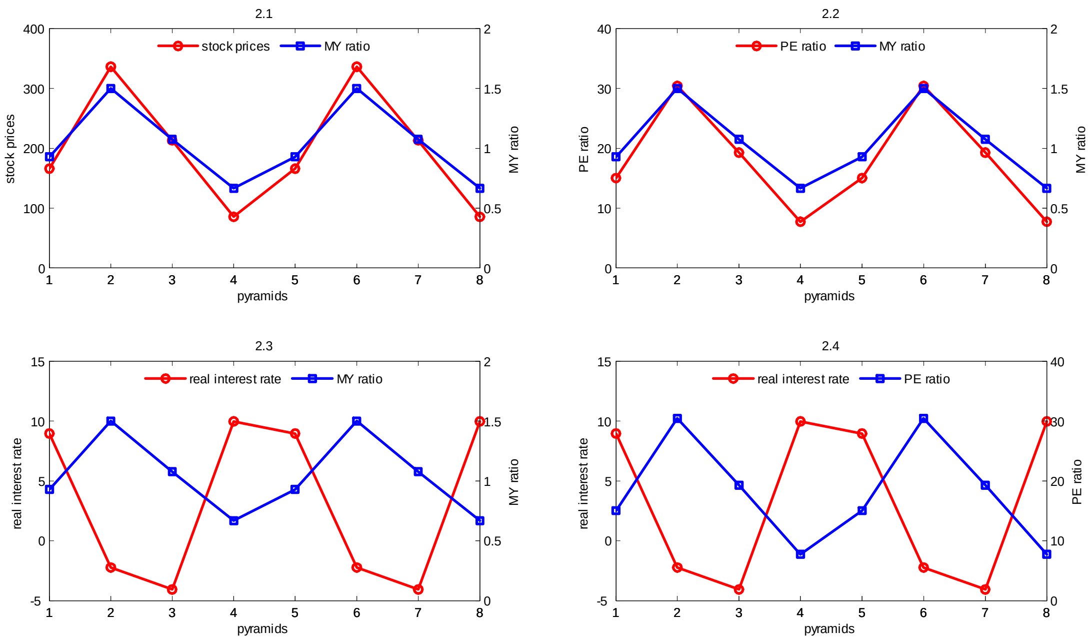

Figure 2. Stock prices, price-earning ratio, long-term real interest rates and MY ratio in a stationary equilibrium under exogenous changes in fertility, aspirations and consumption habits.

In order to finish the story, I compute in Table 8 the well-being of the young parent that begins the economic life in pyramid i. It is still the case that baby boomers (i.e. young adults in large cohorts n_3 and n_4) are worse off than the Depression babies (i.e. young adults in small cohorts n_1 and n_2). Put it differently, the model with second-order effects still relates the economic fortune of a cohort to its relative size. This result is consistent with the fact that the number of offspring depends on the relative well-being of the young parent.

$$IES_{\left(c_i^t, c_i^{t+1}\right)} = \begin{cases} \frac{1}{\alpha} & \text{, if } \rho = 0 \\ \dfrac{\frac{1}{c_i^t W_1} + \frac{1}{c_i^{t+1} W_2}}{-\frac{W_{11}}{(W_1)^2} + \frac{2W_{12}}{W_1 W_2} - \frac{W_{22}}{(W_2)^2}} & \text{, if } \rho > 0 \end{cases}$$

where W_i is the first derivative of W with respect to the i-th variable, W_{ii} is the second derivative and W_{ij} is the cross derivative.

	Cohort i			
	n_1	n_2	n_3	n_4
Utility level $U\left[\mathbf{c}_i, c_{i+2}^0\right]$	-1.39	-1.28	-1.68	-1.96

Table 8. Economic fortune of parents and cohort size

Now it is possible to further characterize the behavior of stock prices observed in Figure 2. To begin with, parents with an adverse exposure to financial terms of trade must put aside a lot more resources into financing consumption when old in the presence of habits, and hence generate a higher excess demand for stocks. This magnifies the peak-to-trough ratio of stock prices, because in pyramid 2 (and pyramid 3 to a lesser extent) parents between 40 and 60 years of age are relatively poor and have fewer children, while parents between 40 and 60 years of age in pyramid 4 (and pyramid 1 to a lesser extent) are relatively wealthy and have more children. Interest rates move accordingly to prevent any arbitrage opportunities.

4 Empirical evidence

In this section I contrast the theoretical findings of the previous 2 sections with U.S. postwar data. The MY ratio is calculated yearly from 1950 to 2008 as the size of the cohort aged 40-59 to the size of the cohort aged 20-39, based on estimates and information provided by the U.S. Census Bureau. Nominal yields on U.S. Treasury bonds for various maturities from 1954 to 2008 are obtained from *economagic*. Finally, I download monthly data from January 1950 to December 2008 on (i) consumer price index, (ii) nominal composite Standard and Poor's index, and (iii) (cyclically adjusted) price-earnings ratio, from Robert Shiller's webpage at Yale University[5].

Price-earnings ratio. Figure 3 depicts the relation between the price-earnings ratio and the MY ratio. There is a clear comovement between the 2 series in terms of long-run trends, with a phase shift of 4 years. Quantitatively speaking, the price-earnings ratio increases from a low of 7.5 in 1950 to around 20.0 in the mid 1960s, and then decreases in the following two decades to 7.8, after which it increases to 33.9 in 2002. The model predicts that the lowest values of the price-earnings ratio vary between 6.6 and 7.7, while the highest values vary between 16.3 and 30.4.

Geanakoplos et al. (2004) go beyond the eyeball inspection and run an Engle-Granger cointegration test. They demonstrate that the price-earnings ratio and the MY ratio are indeed cointegrated, when the sample period is either 1945-2002 or 1965-2002.

[5] In order to construct the real counterparts, I fix March 2010 as the base month. Then I take annual averages to construct yearly data.

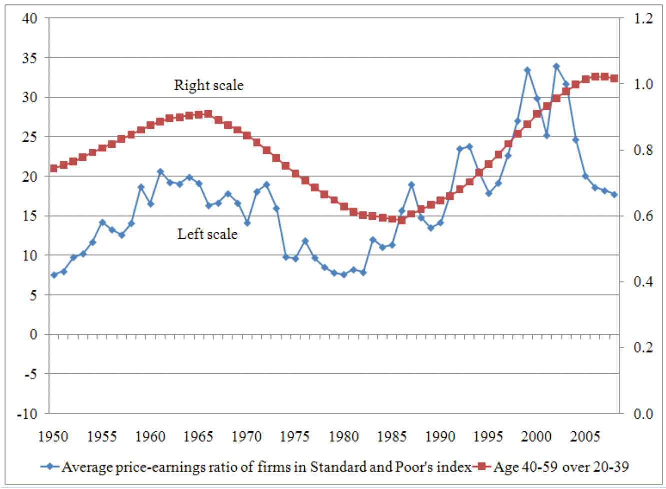

Figure 3. Price-earnings ratio and the MY ratio.

Stock prices. Figure 4 depicts the historical relationship between real stock prices and the MY ratio. Clearly, the MY is in phase with the long-run trend of stock prices. Moreover, the lowest peak-to-trough ratio is of the order of 2.3 in the data, and the highest ratios are of the order of 5 or 6. The model predicts peak-to-trough ratios between 2.5 and 4.0.

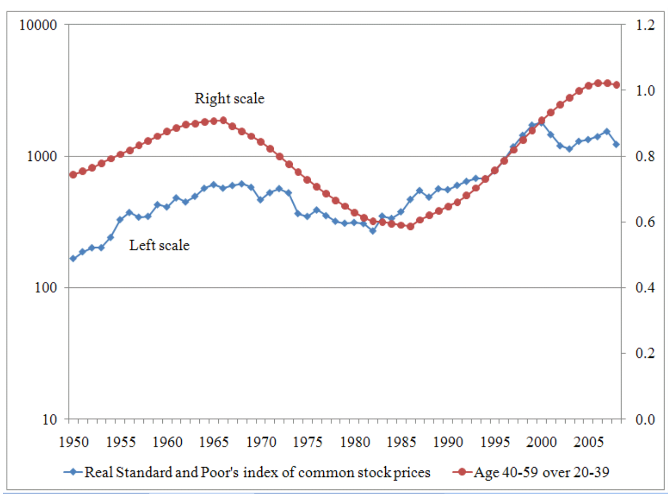

Figure 4. Standard and Poor's real index of common stock prices, and the MY ratio.

Long-term real interest rates. Figure 4 depicts the MY ratio and real yields on Treasury bonds for various maturities. First of all, the yield curve is upward sloping and hence it is not the case that long-term rates lie below the short-term rates on the ascending phase of the MY ratio and above them on the descending phase, as predicted by Geanakoplos et al. (2004). Nevertheless, the data is consistent with the model's prediction, in the sense that changes in the trend of real interest rates are inversely related to changes in the MY ratio, except after the peak in the MY ratio (and before the MY ratio reaches its bottom). The first peak in the MY ratio occurs in 1966, and real interest rates decline during most of the 1970s. The second peak occurs in 2006, and recent financial information suggests that long-term real interest rates are negative and going down steadily.

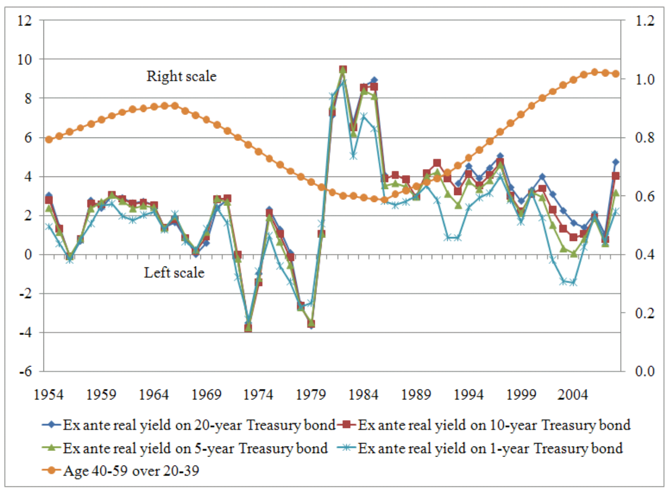

Figure 5. Real yields on Treasury bonds (various maturities) and the MY ratio.

Also note that according to U.S. postwar data, long-term interest rates in the 1970s are of the order of -3.5 and -3.8 percent, which then increase to something between 8.6 and 9.5 percent in the 1980s. Nowadays, long-term interest rates are as low 0 percent or even negative in real terms. On the other hand, the model predicts that the lowest real interest rates should vary between -0.4 and -4.1, while the highest interest rates should vary between 8.5 and 9.9.

5 Final remarks

This paper has presented a deterministic equilibrium model that embraces the Easterlin hypothesis and matches qualitatively the long-run trends in real interest rates and stock prices in the U.S. postwar era. Unlike Geanakoplos et al. (2004), this model does not predict that stock prices should move inversely with real interest rates, at least from a long-run perspective. On the contrary, this model shows that in a stationary cyclic equilibrium there are independent movements in stock and bond prices, which are necessary to prevent arbitrage opportunities.

The full-blown version of the model does a good job at assessing quantitatively the impact of demographic change on stock prices and interest rates. The introduction of aspirations and habits increases the aversion to consumption variability, and consequently the model requires in equilibrium greater movements in prices to clear markets. The fact that some generations are luckier than others completes the story, as prices move in the desired direction and magnitude. Currently I am implementing the techniques developed by Kubler and Schmmeders (2010a, 2010b) to show whether the stationary cyclic equilibrium is unique.

Now, could it be possible to get the same predictions by lowering the elasticity of substitution in the model laid out in section 2? Unfortunately, the answer is no. For instance, when the elasticity of substitution is as low as 0.13 (or, equivalently, 1/8), the model with pure demographic effects predicts that (i) the peak-to-trough ratio of stock prices should be 4.7, (ii) real interest rates should vary between -2.8 and 12.7, and (iii) the lowest price-earnings ratio should be of the order of 4.5, and the highest should be 21.4. Clearly the ingredients introduced in section 3 are desirable to alter the IES across generations and thus improve the outcomes.

Notice that I ignore the role of immigration in my model, because Geanakoplos et al. (2004) argue that immigration in the U.S. in the last century has not changed dramatically the composition of the MY ratio. Furthermore, note that because the environment is deterministic, it is not possible to track the household's portfolio composition at different periods in life. Actually, this task is beyond the scope of this paper.

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