Monetary Policy Under Currency Substitution*

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Abstract

We use a tractable fully micro-founded general equilibrium New Keynesian model with endogenous currency substitution to study how monetary policy should be conducted in dual-currency economies. Our results are as follows: first, as the degree of currency substitution increases, domestic interest rate smoothing becomes less desirable as a target for the central bank. Second, contrary to the common view that exchange rate smoothing might be justified in economies with currency substitution, we find that this is not the case. On the contrary, as the degree of currency substitution increases, exchange rate smoothing generates higher welfare losses. Third,currency substitution increases the set of parameters that allow determinacy of the rational expectations equilibrium under contemporaneous domestic inflation Taylor rules, in particular allows the central bank to react more strongly to output gap fluctuations.

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1 Introduction

How monetary policy should be conducted in an economy with currency substitution (CS from now on)?, should the central bank put more weight on exchange rate stabilization than inflation and output gap stabilization?, is it the interest rate chanel weaker in this type of economies?, does CS precludes the central bank from controlling inflation?. These are question at the center of the monetary policy agenda of many emerging market economies where, even after several years of low and stable inflation, CS [1]. These questions are of particular interest for central banks that face CS and that have adopted the inflation-targeting framework since this framework heavily relies on forecasting and information of the structure of the economy [2].

Besides its relevance, very few papers have addressed the implications of CS for monetary policy in a context of the new generation of fully micro-founded New Keynesian models[3]. This paper intents to contribute in this direction by providing a tractable and fully micro-founded general equilibrium model of a small open economy with endogenous CS, that can be used to study monetary policy design.

We depart from much of the recent literature on monetary policy rules that uses general equilibrium models, as in Benigno and Benigno (2003), Gali and Monacelli (2005), Sutherland (2003) and Woodford (2003) among others, by explicitly assuming that transaction frictions are important in the economy, therefore money plays a relevant role in our model. The aforementioned papers neglect the role of money, by either assuming that money has only effects on the money market, or does not exist, i.e, cashless economies. These assumptions are justified for develop economies on empirical grounds since the indirect effects of money seem to be relatively small in those economies[4]. However, the case for a cashless economy is much harder to make for developing economies where the advantages of using money are larger since a much narrow set of alternative medium of payments is available for transactions than in develop ones[5]. This observation motivates the introduction of transaction frictions in our model.

In order to capture CS we assume that the domestic (pesos) and foreign currency (dollars)

[1] We define currency substitution as the partial replacement of the domestic currency by a foreign one in its function of medium of payment.

[2] Peru is the first country that has adopted an inflation target regime and faces currency substitution. However, many other countries with similar features, as Uruguay, Bolivia, and Costa Rica are planning to follow this path. see Armas and Grippa (2005) for a detailed account of the inflation target framework adopted in Peru

[3] Felices and Tuesta (2005) and Batini, et al (2006) are the exception. Both papers use a New Keynesian general equilibrium model with non separable preferences on money holdings to study the effects of currency substitution on the dynamics of the economy and determinacy of interest rate rules

[4] See Ireland (2001) and Woodford (1999) for a detail discussion on this issue, and Nelson (2002) for a critical view

[5] In Peru for instance less than 50 percent of the population participates on the financial system, similarly for Bolivia.

are imperfect substitutes as medium of payment. More precisely, both can be used to purchased consumptions goods, but at cost, which is proportional to the value of the transaction and vary with the good being purchased. For purchasing some goods it is cheaper to pay with pesos, and for others with dollars. Also, we assume that households can not purchase consumption goods without the use of a particular type of medium of payment. This modeling strategy allows as determining endogenously the degree of CS. Alternative setups to model CS include shopping time and money in utility models [6].

Under our modeling strategy, households choose optimally the composition of their currency holdings by equalizing at the margin the sum of the transaction and the opportunity costs, among alternative currencies. This condition completely pin downs the degree of CS, as an increasing function of the spread between the domestic and the foreign nominal interest rates. This equilibrium implies that the degree of CS will be endogenously higher in economies with relatively high levels of inflation, since in those economies the domestic nominal interest rates are persistently higher than foreign ones. Our set up is flexible enough to generate positive levels of CS even in economies with zero domestic inflation.

We evaluate the implications of CS for the design of monetary policy using a very tractable, fully micro-founded model of a small open economy with sticky prices, similar to the ones used in De Paoli (2004), Felices and Tuesta (2005) and Gali and Monacelli (2005) but extended to consider transaction frictions. We show that in an economy with CS the foreign interest rate distorts consumption, saving and labor supply decisions by generating a stochastic gap between the marginal utility of consumption and that of income. The relative impact of the foreign interest rate in this gap is increasing on the degree of CS.

The log linear version of the model economy with CS admits a canonical representation analogous to their counterparts without CS, but differ from the latter ones in several important dimensions: first: the foreign nominal interest rates appears as an endogenous cost push in the Phillips curve and on the dynamic IS curve, where the magnitude of its effect on inflation and output gap depends on the degree of CS. Thus, an increase in the foreign nominal interest rate reduces output and increase inflation in an economy with CS[7]. This additional determinant of the inflation dynamics makes impossible for the central bank to stabilize simultaneously domestic inflation and the output gap. Also, the impact of the domestic foreign interest rate in the aggregate demand falls as the degree of CS increases.

Second, the micro-founded loss function for the central bank for an economy with CS has

[6] For models with non separable money in utility functions, see, Woodford, chapter 3 for a close economy, and Felices and Tuesta (2005) for an open economy.

[7] This implication provides some rationale for the empirical findings of Agenor, et al (2000), Neumeyer and Perri (2005) and Uribe and Yue (2004) who report a negative correlation between the foreign interest rate and output for emerging economies.

some new features. In particular, as the degree of CS increases it become less costly for the central bank to allow more volatility on the domestic interest rate. Similarly, the welfare costs of exchange rate smoothing increases, as the degree of CS rises. Thus, CS does not justify "fear of floating", in the terminology of Calvo and Reinhart (2001).

These new features of an economy with currency substitution have implications for the conduction of monetary. First, we show that in economies with CS, interest rate rules that allow for a flexible exchange rate outperform, in terms of welfare, those that generate some degree of exchange rate smoothing. Second, interest rate rules with some degree of persistence are desirable, although the gains of interest rate smoothing decrease with the degree of CS.

Also, CS increases the area of determinacy for the rational expectations equilibrium under contemporaneous domestic inflation Taylor rules. In the limit, when there is full substitution of the domestic currency, the area of determinacy coincides with the one of a cashless economy, therefore the Taylor Principle holds. Under all other cases, the set of parameters that allow its determinacy is smaller. In particular, for a given response of the central bank respect to inflation deviations, the central bank should not react too much to output gap in order to guarantee the determinacy of the equilibrium.

However, it is important to highlight that even though CS increases the area of determinacy, the equilibrium achieved under those rules is more volatile than the one without CS. In particular, we show that both domestic inflation and output gap volatility monotonically increase with the degree of CS increases.

This paper is related to some previous work on CS and monetary policy: Felices and Tuesta, (2005), use a small open economy model with non-separable money in utility function that depend on both domestic and foreign currency to analysis the effects of dollarisation on monetary policy. The non-separable money in utility function assumption of Felices and Tuesta (2006), allows them to obtain a similar condition to determine the degree of CS to the one in this paper. We differ from Felices and Tuesta (2006) in considering a flexible cash-in-advance model instead of money in utility to generate endogenous CS. Also, Uribe (1997) uses a model with trading frictions but in an economy with flexible prices to analyze the persistency of CS. Gillman (1992) and Den Hann (1990) use models with transaction frictions but in closed economies and to measure the welfare implications of inflation, and Woodford (2003) studies the implications of transaction frictions for optimal monetary policy in the context of a one-currency close economy.

The rest of the paper is organized as follows: In section 2, we detail the model economy, although the derivations are presented in the appendixes. Section 3 discuses the implications of CS for the steady-state and flexible prices equilibrium and it presents the canonical representation of the small open economy under CS. Section 4, analyzes the implications of CS for monetary policy. Section 5 presents some concluding remarks.

2 The Model

The economy is composed by households, firms that produce consumption goods, firms that produce intermediate goods, foreigners and the central bank. Following De Paoli (2004) and Sutherland (2002) we model an small open economy, SMO from now on, as the limiting case of a two country general equilibrium model, but with the particular feature that domestic households can freely choose between two imperfectly competitive medium of payments, a domestic and a foreign currency, the "dollar" and the "peso", respectively[8]. Besides choosing the composition of their money holdings, households consume a bundle of final consumption goods, supply labor to intermediate goods producers through a competitive labor market, and save using a complete set of stage contingent bonds.

Final goods producers combine domestic and foreign produced intermediate goods as inputs to produce consumption goods. They operate in a perfectly competitive market. On the other hand, intermediate good producers use labor as production input and operated in a monopolistic competitive market. They fix prices in advance and face an exogenous probability of changing prices, as in Calvo (1983).. Trade of goods is done using only intermediate goods. The central bank implements monetary policy through interest rate rules.

2.1 Households

2.1.1 Preferences

Households receive utility from the consumption of a bundle of final goods and disutility from working. Their preferences are described by the following utility function:

$$U_t = E_t \left[\sum_{k=0}^{\infty} \beta^k \left(\frac{C_{t+k}^{1-\sigma}}{1-\sigma} \right) - \frac{1}{1+\varphi} L_{t+k}^{1+\varphi} \right] \tag{2.1}$$

Where E_t represents the expectations operator, conditional on information in period t, $\beta \in \langle 0, 1 \rangle$, the household subjective discount factor, $\sigma > 0$, the coefficient of risk aversion and $\varphi > 0$, the inverse of the Frish labor supply elasticity, L_t the number of hours that household work and C_t the composite of a continuum of final consumption goods denoted by $c_t(s)$ where

[8] De Paoli (2004) studies derives a micro-founded loss function for a central bank in a SOE to study optimal monetary policy, whereas Sutherland (2000), studies the implications of adopting the inflation targeting regime in SOE.

$s \in [0,1]$ indexes final consumption goods.

$$\ln C_t = \left(\int_0^1 \ln C_t\,(s)\, d\,(s) \right) \qquad (2.2)$$

Agents in the foreign economy have a similar set of preferences [9]. In what follows we adopt the convention of denoting foreign variables using an asterisks, i. e, C_t^* and L_{t+k}^*, represent consumption and working hours in the foreign economy, respectively.

2.1.2 Transaction Technology

Households in the domestic economy are required to use cash for transactions, but in contrast to the early cash-in-advance models of Lucas and Stockey (1983) and Svenson (1985), here,as in Uribe (1997) and Gillman (1993), households can choose freely among two imperfectly substitute medium of payments: pesos and dollars[10].

Transactions with each currency are subject to a particular real cost that depends on the type of good being purchased and the amount of the transaction. We assume that this transaction costs are charged by final goods producers and then rebated to consumers as dividends[11]. We index goods by s. and denote by $\tau(s)$ and $g(s)$ the proportional costs per good that consumers pay when buying good s with dollars and pesos, respectively. These transaction cost functions intend to capture some particular features of the trading environment in economies with CS, for instance, the short supply of foreign notes and coins, which makes more costly the use the foreign currency for small transactions[12], and the exchange rate differentials, when paying in domestic currency for goods with prices in foreign currency.

We assume that , $\tau(s) \succeq 0$, $\frac{\partial \tau(s)}{\partial s} > 0$, $g(s) \succeq 0$, $\frac{\partial g(s)}{\partial s} > 0$, $g(\dot{0}) >$,$\tau(0)$ and $\frac{\partial \tau(s)}{\partial s} > \frac{\partial g(s)}{\partial s}$. This is a set of minimum assumptions on the trading environment that fully characterizes the composition of the household money holdings[13]. Under these assumptions, there exist a threshold good, $\overline{s}_t$ such that goods with index lower than $\overline{s}_t$ are purchased with dollar, whereas,

[9] See Appendix A for a description of the foreign economy.

[10] Uribe (1997) uses a flexible CIA model for domestic and foreign currency to study the determinants of the persistence of currency substitution. On the other hand, Gillman (1993) uses an endogenous cash-credit model to evaluate the welfare effects of inflation.

[11] This assumption is harmless to our results and it is made only on the sake of simplicity. It does not affect the substitution effects that transaction costs generate in the economy.

[12] The sub optimal distribution of foreign notes and coins is natural, since the unitary cost of transporting from abroad notes is decreasing in its denomination.

[13] These assumptions are not restrictive, we could alternatively assume that, $\tau(s) \succeq 0$, $\frac{\partial \tau(s)}{\partial s} < 0$, and, $g(s) \succeq 0$ and $\frac{\partial g(s)}{\partial s} > 0$. and our results would not change. We only need to guarantee that $\tau(s)$ and $g(s)$ intersect only once.

goods with index higher than $\overline{s}_t$ are purchase with pesos. Households choose this threshold level, as part of their optimization problem, and it represents the measure of the degree of CS used in this paper.

The timing of transactions is as follows: at the beginning of every period t, households enter to the asset market with the stock of wealth carried over from the previous period, ϖ_t, plus a transfer TR_t of domestic currency from the government, and their corresponding wage payments, W_tL_t. At this time, household observe all the shocks in the economy and choose their holdings of state contingents bonds, B_t, and pesos and dollars M_t and D_t . Furthermore, we denote by e_t the nominal exchange rate, pesos per dollar and by $\xi_{t,t+1}$, the price of the state contingent bonds that pays one unit of domestic currency in the next period . The household budget constraint in the financial market, expressed in terms of pesos, is given by:

$$M_t + D_t e_t + E_t\left(\xi_{t+1} B_{t+1}\right) = \varpi_t + TR_t + W_t L_t \tag{2.3}$$

After the financial market is close, the goods market opens, there, households and firms meet to trade consumption goods by pesos or dollars. In this market, households can pay for each good with any currency, but transactions are subject to their corresponding transaction cost. Rational households choose to hold the composition of currency that minimizes transaction costs and the corresponding opportunity cost of holding both currencies. Also in this market they receive dividends from the firms, Ξ_t. At the end of period t households receive the income from the state contingent bonds they purchased in the morning[14]. Therefore, household's stock of wealth at the end of period t, is given by:

$$\begin{aligned} \varpi_{t+1} &= B_t + \Xi_t + \; M_t - \int_{\overline{s}_t}^{1} P_t(s) C_t(s)\left(1 + g(s) d(s)\right. \\ &\quad + e_t D_t - \int_{0}^{\overline{s}_t} P_t(s) C_t(s)\left(1 + \tau(s)\right) d(s) \end{aligned} \tag{2.4}$$

Also, the household decision is restricted by the following cash-in-advance constraints that determines the demand for pesos and dollars, respectively:

$$M_t = \int_{\overline{s}_t}^{1} P_t(s) C_t(s)\left(1 + g(s) d(s)\right. \quad e_t D_t = \int_{0}^{\overline{s}_t} P_t(s) C_t(s)\left(1 + \tau(s)\right) d(s) \tag{2.5}$$

[14]Note that the timing of transactions in this model is similar to Lucas and Stockey, (1983) who assume that the asset market open first. Svensson (1985) instead considers that the goods market open first.

Notice however, that since $\overline{s_t}$ is a choice variable for households, they can flexibility choose the composition of their money holdings between pesos and dollars. Furthermore, we restrict household decisions to satisfy the following transversality condition.

$$\lim_{n \rightharpoonup \infty} E_t\left(\varpi_{t+n} Q_{t+n}\right) \geqslant 0$$

2.1.3 Household Optimality Conditions

Each household maximizes her utility function given by equation (2.1) subject to the cash-in-advance constraints, equations in (2.5) and the flow budget constraint, equation (2.4). The first order conditions of the household problem are given by the following set of equations:

Degree of Currency Substitution The first order conditions for the optimal level of consumption of good s is given by the following set of equations :

$$U_{c,t} \frac{\partial c_t}{\partial c_t(s)} = P_t(s)\lambda_t\left(1+\frac{q_t}{\lambda_t}\right)(1+g(s)) \text{ for } s \geqslant \overline{s}_t \tag{2.6}$$

$$U_{c,t} \frac{\partial c_t}{\partial c_t(s)} = P_t(s)\lambda_t\left(1+\frac{n_t}{\lambda_t}\right)(1+\tau(s)) \text{ for } s < \overline{s}_t \tag{2.7}$$

where, λ_t, q_t and n_t, represent the lagrange multipliers of the budget constraint and the two CIA constraints, respectively. The optimal condition for consumption of low index goods, $s < \overline{s}_t$, implies that at the optimum, the marginal utility of consumption of this particular good has to be equal to the marginal cost of income adjusted by the opportunity cost for holding money plus its corresponding transaction cost, $\left(1+\frac{n_t}{\lambda_t}\right)(1+\tau(s))$. Similarly, for goods with high index, $s > \overline{s}_t$, optimality implies that the marginal utility of consumption has to equal the marginal utility of income, adjusted by the corresponding transaction cost, $(1+g(s))$, and the opportunity cost of holding domestic currency, $\left(1+\frac{q_t}{\lambda_t}\right)$. From the corresponding first order conditions for holdings of pesos and dollars we obtain:

$$\frac{q_t}{\lambda_t} = 1 - E_t\left(Q_{t,t+1}\right) = 1 - \frac{1}{(1+i_t)} \tag{2.8}$$

$$\frac{n_t}{\lambda_t} = 1 - E_t\left(Q_{t+1}\frac{e_{t+1}}{e_t}\right) = 1 - \frac{1}{(1+i_t^*)} \tag{2.9}$$

Since at the optimum, the household has to be indifferent between using domestic or foreign currency to purchase the marginal good $\overline{s}_t$, it has to be true that for this particular good it holds:

$$\frac{1+\tau\left(\overline{s}_t\right)}{1+g(\overline{s}_t)}=\frac{1-\frac{1}{(1+i_t)}}{1-\frac{1}{(1+i_t^*)}} \tag{2.10}$$

This latter condition determines the degree of CS, $\overline{s}_t$. Condition (2.10) is very similar to the one derived by Baumol (1958), in which the optimal demand for money is obtained when the transaction cost of exchanging bonds by money equalizes the nominal interest rate, its opportunity cost. It is also in line with the condition derived by Eichenbaum and Wallace (1985) where the optimal demand for different types of money is given at the point where marginal transaction costs are equalized across currencies.

The equilibrium is determined by the intersection of the curves, $(1+\tau\left(\overline{s}_t\right))\left(1-\frac{1}{(1+i_t^*)}\right)$ and $(1+g(\overline{s}_t))\left(1-\frac{1}{(1+i_t)}\right)$ that represent the total marginal cost of using dollars and pesos, respectively. As figure 1 shows, when the nominal interest rate increases, the g curve shifts up increasing the degree of CS, $\overline{s}_t$. On the contrary, when i_t^* increases, the τ curve shifts down making marginally more expensive the trading using dollars, and consequently the degree of CS falls, in equilibrium. We parameterize these two transaction cost functions as follows,

$$\tau\left(s_t\right)=\exp(\Psi_o+\Psi_1 s_t)-1 \quad g\left(s_t\right)=\exp\left(n_o+n_1 s_t\right))-1 \tag{2.11}$$

Where, we restrict that, $\Psi_1>n_1$ and $n_o>\Psi_o$ in order to guarantee a well defined equilibrium of CS. Using condition (2.10), the degree of CS will be determined by,

$$\overline{s}_t=\frac{\left(n_0-\Psi_0+\log\left(\frac{2-\frac{1}{(1+i_t)}}{2-\frac{i^*}{(1+i^*)}}\right)\right)}{(\Psi_1-n_1)} \tag{2.12}$$

Notice that since we assume that $\tau'\left(\overline{s}_t\right)-g'(s)>0$, the fraction of goods purchased with foreign currency, $\overline{s}_t$will be increasing on the level of domestic interest rates, i_t, and decreasing on the foreign interest rate, i_t. Moreover, even when the domestic and the foreign interest rate are the same, there exist a minimum degree of CS given by,

$$\overline{s}_o=\frac{(n_0-\Psi_0)}{(\Psi_1-n_1)} \tag{2.13}$$

This minimum degree of CS is related to the particular assumptions we made on the trading environment. In particular, we assumed that even with zero nominal interest rates, there exist a set of goods for which is cheaper to use dollar for transactions, $n_0-\Psi_0>0$. Only, in the case where, $n_0=\Psi_0=0$, we have that equal nominal interest rate in the domestic and foreign economy guarantees that the degree of CS is zero.

The equilibrium with positive levels of currency substitution and low inflation levels is consistent with the experience of CS in countries like Bolivia and Peru in recent years.

In order to analyse the implications of CS in the economy, we aggregate the optimal conditions for the demand of final goods, given by equations (2.6) and (2.7). As it is shown in detail in appendix C, it is possible to write the marginal utility of consumption as follows:

$$U_{c,t} = \lambda_t \left(1 + \Upsilon_t\right) \tag{2.14}$$

where,λ_t represents the shadow value of income, and Υ_t a distortion associated with transaction costs that depend on both the domestic and foreign nominal interest rates, and the degree of CS, through function, $\Gamma\left(\overline{s}_t\right)$ in the following way:

$$\left(1 + \Upsilon_t\right) = \left(1 + \frac{i_t}{\left(1 + i_t\right)}\right)\left(1 + \Gamma\left(\overline{s}_t\right)\right) \tag{2.15}$$

using the functional forms for the transaction costs in dollars and pesos, defined in equation (2.11), $\left(1 + \Gamma\left(\overline{s}_t\right)\right)$ can be written as the following exponential function on $\overline{s}_t$[15] :

$$1 + \Gamma\left(\overline{s}_t\right) = \exp\left(\frac{n_1}{2} + n_o - \left(\Psi_1 - n_1\right)\frac{\overline{s}_t^2}{2}\right) \tag{2.16}$$

Observing equation (2.14) it is easy to understand how CS affects the equilibrium of the economy. As this condition shows, transaction costs, Υ_t create a wedge between the marginal utility of consumption and that of income that distorts the efficient allocation of consumption and labor. This distortion is increasing in both the domestic and the foreign nominal interest rate.

Interestingly, when keeping fixed the foreign interest rates, the marginal effect of i_t on Υ_t is decreasing, since, when CS is allowed, agents can freely substitute domestic currency for foreign currency. Thus, by allowing CS, agents can reduce the welfare cost that higher nominal and inflation rates generate. Figure 2 illustrates this latter point, by showing that function Υ_t is concave on the nominal interest rate i_t

[15] See appendix, C, for details of this derivation

Figure 1: Υ_t and the nominal interest rate

Notice that Υ_t is minimized when $i = 0$, although, this condition does not guarantee that transaction frictions are fully eliminated. As we mentioned previously, only when, $n_0 = \Psi_0 = 0$,a zero nominal interest rate guarantee zero transaction costs.

On the other hand, using the CIA constraints, equation (2.5), and equation (2.14) we can write the corresponding money demands for domestic and foreign currency as follows,

$$\frac{M_t}{P_t} = C_t \frac{(1-\overline{s}_t)(1+\Upsilon_t)}{2-\frac{1}{1+i_t}} \quad \frac{e_t D_t}{P_t} = C_t \frac{\overline{s}_t(1+\Upsilon_t)}{2-\frac{1}{1+i_t^*}} \tag{2.17}$$

These two money demand functions exhibit standard properties, both are increasing in the level of domestic consumption, and M_t is decreasing (increasing) on $i_t\,(i_t^*)$, whereas, D_t is decreasing (increasing) on $i_t^*\,(i_t)$. Furthermore, taking a log quadratic approximation of the two previous equations around their corresponding steady-states, it is easy to show that M_t is decreasing and a convex function of i_t thus, the model implies that an increase in the volatility of the opportunity cost of holding money would lead to higher money demand.

Figure 3 plots $\frac{M_t}{P_t}$ and $\frac{e_t D_t}{P_t}$ for different values of the domestic interest rate, holding fixed C_t and ${}_t^*$,

Figure 2: Money Demand Functions

Saving and Portfolio Decisions Savings and the portfolio decision of households are determined by the usual Euler conditions. At the optimum households are indifferent among allocating wealth in any period, since the expected present discounted value of the marginal utility of wealth is the same across periods:

$$\frac{1}{(1+R_t)} = E_t\left(\frac{\beta\lambda_{t+1}}{\lambda_t\,(1+\pi_{t+1})}\right) \tag{2.18}$$

Notice that since, $\lambda_t = \frac{U_{c,t}}{(1+\Upsilon_t)}$, the saving decisions of agents will depend, besides the level of the real interest rate, on the degree of CS. Furthermore, since markets are complete, it also holds that the price of an state contingent bond that delivers one unit of consumption in foreign currency is given by:

$$\frac{1}{(1+R_t^*)} = E_t\left(\frac{\beta\lambda_{t+1}e_{t+1}}{e_t\lambda_t\,(1+\pi_{t+1})}\right) \tag{2.19}$$

Combining equations (2.18) and (2.19) , we obtain the uncovered interest parity condition (UIP):

$$\frac{(1+R_t)}{(1+R_t^*)} = \frac{E_t\left(\frac{\lambda_{t+1}e_{t+1}}{e_t\lambda_t(1+\pi_{t+1})}\right)}{E_t\left(\frac{\lambda_{t+1}}{\lambda_t(1+\pi_{t+1})}\right)} \tag{2.20}$$

Labor Supply Households supply labor in equilibrium up to the point where the marginal cost of working equalizes its marginal benefit:

$$U_{h,t} = \lambda_t \frac{W_t}{P_t} \tag{2.21}$$

The marginal benefit $\lambda_t W_t$ depends, among other things, on the level of nominal interest rates and on the degree of currency substitution through λ_t. This is a second channel through which currency substitution affects the economy. Since, real wages affect marginal cost of firms and through the Phillips curve, inflation, the degree of CS, as we discuss in detail in the next sections, will affect inflation dynamics.

Risk Sharing Condition The complete markets assumption implies that the price of the state contingent bond domestically and abroad have to be same, therefore, we have that the following condition it must hold:

$$\xi_{t+1} = \frac{\beta\lambda_{t+1}^* P_t^*}{\lambda_t^* P_{t+1}^*} = \frac{\beta\lambda_{t+1}P_t e_{t+1}}{\lambda_t e_t P_{t+1}} \tag{2.22}$$

Denoting by Q_t the real exchange rate, the relative price of foreign goods in terms of domestic goods, $Q_t = \frac{P_t^* e_t}{P_t}$ we can transform the previous expression into the following condition:

$$Q_{t+1} = \frac{\lambda_{t+1}^*}{\lambda_{t+1}}\frac{\lambda_t}{\lambda_t^*}Q_t \tag{2.23}$$

Following Chari, Kehoe and McGratan (2001) we iterate backwards the previous equation to get the following risk sharing condition[16]:

$$Q_t = \varsigma_0 \frac{\lambda_t^*}{\lambda_t} \tag{2.24}$$

[16] Chari, Kehoe and McGratan (2001) use a model of an open economy with complete markets to analize the role of price stickiness in explaining the volatility of the real exchange rate.

where the constant ς_0 is defined as follows:

$$\varsigma_0 = \frac{\lambda_0}{\lambda_0^*} Q_0 \tag{2.25}$$

2.2 Firms

2.2.1 Final Good Producers

There is a continuum of final good producers of mass n indexed by q in the domestic economy, which operate under perfect competition, whereas a mass $1-n$, of final goods producers is allocated in the foreign economy. Domestic final goods producers use home, $Y_{H,t}$, and foreign, $Y_{F,t}$, intermediate goods as inputs into the following production function:

$$Y_t^q = \left[(1-\alpha)^{\frac{1}{\eta}} \left(Y_{H,t} \right)^{\frac{\eta-1}{\eta}} + (\alpha)^{\frac{1}{\eta}} \left(Y_{F,t} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{1-\eta}} \tag{2.26}$$

$$Y_{H,t}^q = \left(\left(\tfrac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n Y_{H,t}\left(z\right)^{\frac{\varepsilon-1}{\varepsilon}} d\left(z\right) \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad Y_{F,t}^q = \left(\left(\tfrac{1}{1-n}\right)^{\frac{1}{\varepsilon}} \int_n^1 Y_{F,t}\left(z\right)^{\frac{\varepsilon-1}{\varepsilon}} d\left(z\right) \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.27}$$

where $\eta > 0$ is the elasticity of substitution between domestic and foreign intermediate goods, whereas, $\epsilon > 1$, is the elasticity of substitution across varieties of intermediate goods. Then the cost minimizing demand functions by firm q of each type of differentiated good is given by the following two conditions:

$$Y_{H,t}^q(z) = (1-\alpha) \left(\frac{P_{H,t}(z)}{P_{H,t}} \right)^{-\epsilon} \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} Y_t^q \tag{2.28}$$

$$Y_{F,t}^q(z) = \alpha \left(\frac{P_{F,t}(z)}{P_{F,t}} \right)^{-\epsilon} \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} Y_t^q \tag{2.29}$$

The price level charged by final good producers is equal to its marginal cost and it is given by:

$$P_t = \left((1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{2.30}$$

where:

$$P_{H,t} = \left(\frac{1}{n} \int_0^n P_{H,t}^{1-\epsilon}\left(z\right) dz \right)^{\frac{1}{1-\epsilon}} \quad P_{F,t} = \left(\frac{1}{n} \int_0^n P_{F,t}^{1-\epsilon}\left(z\right) dz \right)^{\frac{1}{1-\epsilon}} \tag{2.31}$$

Final goods producers in the foreign economy have a similar technology to those used by domestic intermediate producers, see appendix for A details on the foreign economy.

2.2.2 Intermediate Good Producers

There is a continuum of intermediate good producers of mass n allocated in the domestic economy and of mass $1-n$, in the foreign economy that operate under monopolistic competition. Each firm uses a constant returns to scale technology to produce a particular variety of intermediate goods. This technology takes labor as production input as follows :

$$Y_{H,t}(z) = A_t L_t(z) \tag{2.32}$$

where A_t represents an aggregate productivity shock that follows the following $AR(1)$ process:

$$\ln(A_t) = \chi \ln(A_{t-1}) + \zeta_t \tag{2.33}$$

with $\zeta_t \sim N\left(0, \sigma_\zeta^2\right)$. Similarly, the foreign intermediate goods producers uses a constant returns to scale production function given by:

$$Y_{F,t}(z) = A_t^* L_t^*(z) \tag{2.34}$$

where A_t^* representing the foreign productivity shock that follows an autoregressive process:

$$\ln(A_t^*) = \chi^* \ln(A_{t-1}^*) + \zeta_t^* \tag{2.35}$$

with $\zeta_t^* \sim N\left(0, \sigma_{\zeta^*}^2\right)$.The aggregated demand for the intermediate good z is obtained by aggregating the demand of both the home and foreign final goods producers for this good, as follows:

$$Y_{H,t}(z) = \int_0^n Y_{H,t}^q(z)d(q) + \int_n^1 \left(Y_{H,t}^q\right)(z)d(q) \tag{2.36}$$

In this economy the law of one price holds for a particular good z, therefore we have that: $P_{H,t}(z) = e_t P_{H,t}^*(z)$, and $P_{F,t}(z) = e_t P_{F,t}^*(z)$, consequently, the aggregate demand for home intermediated good z is written as follows:

$$Y_{H,t}(z) = \left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\epsilon} \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left((1-\alpha)\overline{Y}_t + \frac{(1-\alpha^*)(1-n)}{n} Q_t^\eta \overline{Y}_t^*\right) \tag{2.37}$$

Where, $\overline{Y}_t = \int_0^n Y_t^q d(q)$, and $\overline{Y}_t^* = \int_n^1 Y_t^{q*} d(q)$, represent the aggregated production level of final goods at the domestic and foreign economy, respectively. Using a similar derivation for the foreign economy we obtain $Y_{F,t}(z)$ as follows:

$$Y_{F,t}(z) = \left(\frac{P_{F,t}(z)}{P_{F,t}}\right)^{-\epsilon}\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}\left(\frac{n}{1-n}\alpha\overline{Y}_t + \alpha^* Q^\eta \overline{Y}_t^*\right) \tag{2.38}$$

2.2.3 The Small Open Economy

Following Sutherland (2001), we parameterize the participation of foreign inputs in the production of home and foreign final goods, α , α^*, respectively as follows:

$$\alpha = (1-n)\gamma \quad 1-\alpha^* = n\gamma$$

where n represents the size of the home economy, and γ its degree of openness[17]. This particular parametrization implies that as the economy becomes more open the fraction of imported goods used in domestic production increases, whereas as the economy becomes larger, this fraction falls. The parametrization defined previously allow us to obtain the SOE as the home economy when its size approach to zero, $n \to 0$. In this case we have that $\alpha \to \gamma$ and $\alpha^* \to 1$. Also, in this limiting case the foreign economy does not use any home produced intermediated good for production of foreign final goods. Thus, changes in home aggregate demand have a nil impact on the foreign economy, this is $\frac{\partial Y_{F,t}}{\partial \overline{Y}_t} = 0$. Furthermore, in this limiting case, $P^* = P_F^*$. and:

$$Y_{H,t}(z) = \left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\epsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left((1-\gamma)\overline{Y}_t + \gamma Q_t^\eta \overline{Y}_t^*\right) \tag{2.39}$$

$$Y_{F,t}(z) = \left(\frac{P_{F,t}(z)}{P_{F,t}}\right)^{-\epsilon}\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}\left(Q_t^\eta \overline{Y}_t^*\right) \tag{2.40}$$

In order to save notation, we denote by

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left((1-\gamma)\overline{Y}_t + \gamma Q_t^\eta \overline{Y}_t^*\right) \tag{2.41}$$

$$Y_{F,t} = \left(\frac{P_{F,t}}{P_t}\right)^{-\eta}\left(Q_t^\eta \overline{Y}_t^*\right) \tag{2.42}$$

thus the demand facing individual intermediate goods producing firms can be simply expressed as:

$$Y_{H,t}(z) = \left(\tfrac{P_{H,t}(z)}{P_{H,t}}\right)^{-\epsilon} Y_{H,t} \quad Y_{F,t}(z) = \left(\tfrac{P_{F,t}(z)}{P_{F,t}}\right)^{-\epsilon} Y_{F,t} \tag{2.43}$$

[17] Sutherland (2001) derives the SOE in a model where final consumption goods are used for trade, here in contrast, we derive the SOE in a model where domestic and foreign firms trade intermediate goods.

2.2.4 Price Setting

Each period t intermediate goods producers face an exogenous probability of changing prices given by $(1-\theta)$. Following Calvo (1983) and Yun (1996), we assume that this probability is independent of the price level chosen by the firm in previous periods and on the last period the firm changed its price. Thus a typical firm choose an optimal price $P^{o}_{H,t}(z)$ to maximize the present discounted value of its expect flow of profits, given by:

$$E_t\left[\sum_{k=0}^{\infty}(\theta\beta)^k\left(\lambda_{t+k}\left(\frac{P^{o}_{H,t}(z)}{P_{H,t+k}}-mc_{t+k}\right)\widetilde{Y}_{H,t+k}(z)\right)\right] \tag{2.44}$$

let´s denote by Ψ_{t+k} the inverse of the cumulative domestic inflation level as follows:

$$\Psi_{t+k}=\frac{P_{H,t}}{P_{H,t+k}} \tag{2.45}$$

and by $\widetilde{Y}_{H,t+k}(z)$ the demand of intermediate good z conditioned on that its price has kept fixed at $P^{o}_{H,t}(z)$:

$$\widetilde{Y}_{H,t+k}(z)=\left(\frac{P^{o}_{H,t}(z)}{P_{H,t}}\right)^{-\epsilon}\Psi^{-\epsilon}_{t+k}Y_{H,t+k} \tag{2.46}$$

The first order condition that maximizes equation (2.44) is given by:

$$E_t\left[\sum_{k=0}^{\infty}(\theta\beta)^k\left(C^{-\sigma}_{t+k}\left(\frac{P^{o}_{H,t}(z)}{P_{H,t}}\Psi_{t+k}-\frac{\epsilon}{(\epsilon-1)}mc_{t+k}\right)\widetilde{Y}_{H,t+k}(z)\right)\right]=0 \tag{2.47}$$

As it is shown in appendix B, from this first order condition we can derive a non linear recursive representation of the Phillips curve given by the following three equations:

$$N_t=\mu\lambda_t mc_t Y_{H,t}+\theta\beta\pi^{\epsilon}_{H,t+1}N_{t+1} \tag{2.48}$$

$$D_t=\lambda_t Y_{H,t}+\theta\beta\pi^{\epsilon-1}_{H,t+1}D_{t+1} \tag{2.49}$$

$$\theta\left(\pi_{H,t}\right)^{\epsilon-1}=1-(1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\epsilon} \tag{2.50}$$

Where, N_t and D_t are auxiliary variables defined in this appendix. A similar set of equations characterizes the Phillips curve in the foreign economy.

2.2.5 Real Exchange Rate and Terms of Trade

Next we define some identities that are helpful in describing the dynamic equilibrium of an open economy. First we define the terms of trades, T_t, as the relative price of foreign goods in terms of domestic goods as follows:

$$T_t = \frac{P_{F,t}}{P_{H,t}} \tag{2.51}$$

since the domestic economy is small and the law of one price holds, the price of foreign goods is $P_{F,t} = e_t P_t^*$, therefore the equation for terms of trade can be written as follows:

$$T_t = \frac{Q_t}{\widetilde{P}_{H,t}} \tag{2.52}$$

where $\widetilde{P}_{H,t} = \frac{P_{H,t}}{P_t}$. Furthermore, using the definition of the consumer price indices for the home and foreign economy and the small open economy assumption, we have that:

$$\left(\frac{Q_t}{T_t}\right)^{\eta-1} = (1-\gamma) + \gamma T_t^{1-\eta} \tag{2.53}$$

Using this last identity, we can define a relationship between CPI inflation and home inflation as follows:

$$\left(\frac{\pi_t}{\pi_{H,t}}\right)^{1-\eta} = \frac{(1-\gamma) + \gamma T_t^{1-\eta}}{(1-\gamma) + \gamma T_{t-1}^{1-\eta}} \tag{2.54}$$

2.3 Monetary Policy

The central bank sets monetary policy by choosing the nominal interest rate according to a Taylor rule. We consider the following generic type of Taylor rule,

$$(1+i_t) = \overline{(1+i)}\,(1+i_{t-1})^{\rho_i} \left(\frac{\pi_{i,t}}{\overline{\pi}_t}\right)^{\phi_\pi(1-\rho_i)} \left(\frac{y_t}{\overline{y}_t}\right)^{\phi_x(1-\rho_i)} \left(\frac{e_t}{e_{t-1}}\right)^{\phi_e(1-\rho_i)}$$

where $i = \{H, CPI\}$, $\phi_\pi > 1$, $\phi_x > 0$ and $\phi_e > 0$.and $\overline{\pi}_t$ represent the inflation target of the domestic central bank and $\overline{y}$, the natural level of output in the domestic economy.

2.4 Parametrization

The model is calibrated with standard parameter values for small open economies. In particular, we choose, $\sigma = \eta = 1$, to mitigate the effects of terms of trade on the dynamic equilibrium of the economy, as in Gali and Monacelli (2005). The parameter β is set to 0.99, which implies a annual real interest rate of 4 percent. The inverse of the elasticity of labor supply, φ, is set to 3, consistent with micro studies that report low elasticities of labor supply. The parameter θ is set to 0.75, which implies that firms keep prices unchanged on average four quarters. The degree of openness of the domestic economy $1-\gamma$ is set to 0.7, whereas, ϵ is set to 6, which implies a mark up over marginal cost of 20 percent. The persistence of all shocks is set to 0.95 and the variance of their innovations to 0.0071^2. The parameters that characterize the transaction frictions are calibrated to generate a relatively low steady-state level of CS under zero inflation, thus we set $\Psi_0 = 0.01$, $\Psi_1 = 1.1$, $n_0 = \log(2-\beta) + 0.151$ and $\psi_1 = 0.01.$, which implies a 15 percent degree of CS.

3 Dynamic Equilibrium and Currency Substitution

In order to highlight the effects of CS on the economy we choose a parametrization where the intertemporal elasticity of substitution and the elasticity of substitution between domestic and foreign intermediate goods are equal to 1, i.e. $\sigma = \eta = 1$. In this case, the welfare effects of terms of trade are completely eliminated, since the income and substitution effects that terms of trade generate perfectly cancel out each other. Consequently, domestic and foreign shocks do not affect the current account of the economy. This simplification makes easier to characterize analytically the implications for welfare and optimal monetary policy of CS. Moreover, in this particular case, the SOE with CS is determined by the following set of non linear equations,

$$N_t = \mu \left(\frac{Y_t}{Y_t^*}\right)^{\gamma} \left(\frac{1+\gamma\Upsilon_t}{1+\Upsilon_t}\right)^{1-\gamma} MC_t + \theta\beta E_t \left(\Pi_{H,t+1}^{\varepsilon} N_{t+1}\right) \tag{3.1}$$

$$D_t = \left(\frac{Y_t}{Y_t^*}\right)^{\gamma} \left(\frac{1+\gamma\Upsilon_t}{1+\Upsilon_t}\right)^{1-\gamma} + \theta\beta E_t \left(\Pi_{H,t+1}^{\varepsilon-1} D_{t+1}\right) \tag{3.2}$$

$$\theta\Pi_{H,t+1}^{\varepsilon-1} = 1 - (1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\varepsilon} \tag{3.3}$$

$$MC_t = \frac{Y_t^{1+\varphi}}{A_t^{1+\varphi}} \left(\frac{1+\Upsilon_t}{1+\gamma\Upsilon_t}\right) \Delta_t^{\varphi} \tag{3.4}$$

$$\Delta_t = \theta\Delta_{t-1} + (1-\theta)\left(\frac{1-\theta\Pi_{H,t}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.5}$$

$$\frac{1}{(1+i_t)} = \beta E_t \left(\left(\frac{Y_{t+1}}{Y_t}\right)^{-1} \left(\frac{1+\gamma\Upsilon_{t+1}}{1+\Upsilon_{t+1}}\right) \left(\frac{1+\Upsilon_t}{1+\gamma\Upsilon_t}\right) \frac{1}{\Pi_{H,t+1}} \right) \tag{3.6}$$

$$C_t = Y_t^{1-\gamma} (Y_t^*)^{\gamma} \left(\frac{1}{1+\gamma\Upsilon_t}\right)^{1-\gamma} \frac{1}{(1+\Upsilon_t)^{\gamma}} \tag{3.7}$$

Equations from (3.1) to (3.5) determine the Phillips Curve, therefore the dynamics of inflation, whereas equation (3.6) the aggregate demand. Notice that besides the usual determinants of inflation and output, a variable that appears on both the aggregate demand and the Phillips curve is Υ_t. This variable, as we discussed previously, measures the distortion that transaction frictions generate on the marginal utility of income, and it depends on both the domestic and the foreign nominal interest rates, as it is established by equations (2.15), (2.15) and (2.12).

It is through this variable that CS affects the economy, when transaction frictions are not present, $\Upsilon_t = 0$, the economy collapses to a standard cashless SOE, as the one analysed by Gali and Gertler (2001). However, when $\Upsilon_t > 0$ there are additional channels through which both the domestic and the foreign nominal interest rate affect the economy and CS plays a role.

In particular, transaction frictions, Υ_t, act as a stochastic tax for holding cash that breaks the equality between the marginal utility of income and consumption. This stochastic tax affects, by making more costly to transform income into consumption, the dynamics of both the aggregate demand and of inflation. The role that CS plays in this mechanism is to determine the weights of both the domestic and the foreign nominal interest rates on Υ_t. In the coming subsections we explain in detail the effects of CS for the steady-state, the flexible and the sticky price equilibrium. In section 4 we address the implications of CS for optimal monetary policy and for the determinacy of the equilibrium of Taylor rules. From now on, we adopt the convention of denoting by capital letters without time subscript, the corresponding steady-state value variables, and by lower case letters their log deviations from their steady-states, i.e. X is the steady-state of X_t and $x_t = \log(\frac{X_t}{X})$.

3.1 Currency Substitution and the Steady State

We analyze a deterministic steady-state where all shocks take their unconditional means, and where both domestic and foreign inflation rates are equal to zero. Since at the steady state, $MC = \frac{1}{\mu}$, from the corresponding analog steady-state equation (3.4) we obtain the following expression for the level of domestic output,

$$Y_H = (1 - \Phi)^{\frac{1}{1+\varphi}} \tag{3.8}$$

where $1 - \Phi = \frac{(1-\tau)(1+\gamma\Upsilon)}{\mu(1+\Upsilon)}$ accounts for the overall distortions that affect the steady-state level of output. As equation (3.8) shows, the level of output is below its optimal level of 1. Two factors distort output at the steady-state, the degree of monopolistic competition, measured by the degree of mark-up, μ, that induce firms to produce below its efficient level, and transaction frictions, measured by Υ, that rise the marginal cost of firms. The size of this second distortion is positively related to the degree of CS, thus, when the degree of CS increases, Υ also increases, inducing firms to produce a lower level of output in steady-state. However, notice that since the degree of CS is endogenously determined by the spread of domestic and foreign nominal interest rates, and this spread is determined mainly by the spread of domestic and foreign inflation, Υ can be also interpreted as the welfare cost of domestic inflation. Thus, in economies where domestic inflation is relatively high, the degree of CS and consequently, the distortion that transaction frictions generate on output will also be high.

Importantly, when CS is allowed the cost that transaction frictions generate are increasing on the nominal interest rate but at decreasing rates. As the next figure shows, when CS is permitted, the degree of concavity of Υ increases.

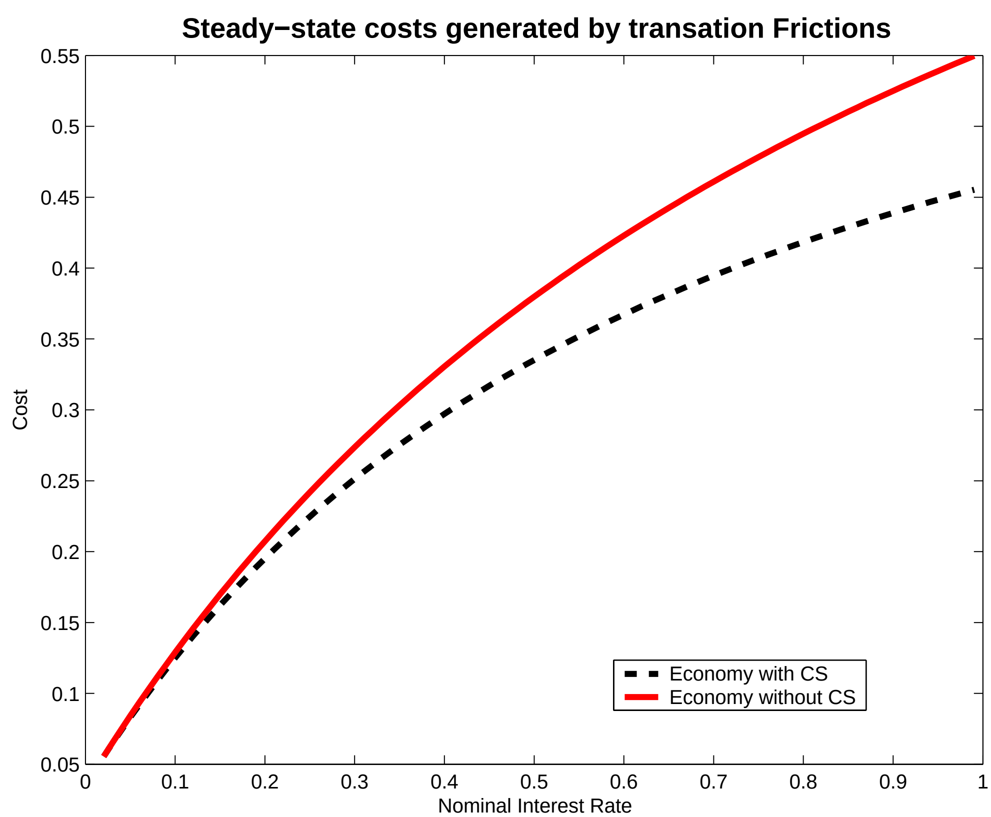


Figure 3: The Benefits of Currency Substitution

Thus the previous results shows that by allowing CS, it is possible to reduce the welfare effects of high inflation, since when CS is allowed, households can optimally avoid transaction frictions, by shifting their money demand towards a foreign currency.

3.2 Currency Substitution and the Flexible Price Equilibrium

In contrast with the monopolistic distortion that only affects the steady-state, transaction frictions also distort the dynamic equilibrium of the economy, in particular, they induce an inefficiently low output level. As equation (3.9) shows, transaction frictions generate a gap between the output under flexible prices and domestic productivity.

$$y_t^n = a_t - \frac{1-\gamma}{1+\varphi}\vartheta \upsilon_t \tag{3.9}$$

where, $\upsilon_t = \log(\frac{1+\Upsilon_t}{1+\Upsilon})$ and $\vartheta = \frac{1}{(1+\gamma\Upsilon)}$ [18]. This gap is increasing on both the domestic and the foreign nominal interest rates. The degree of CS determines the weight that each nominal interest rate has on υ_t,

$$\upsilon_t = \omega\left((1-s)\, i_t + s i_t^*\right) \tag{3.10}$$

where, $\omega = \frac{1}{2(1+i)-1}$. Thus in economies where CS is high, it is the foreign interest rate the variable that has a larger impact on distorting the dynamic behavior of output and not the domestic one.

The efficient output level in this economy is achieved when $\upsilon_t = 0$. However, this allocation is not feasible under neither a policy of zero inflation nor a policy of zero domestic nominal interest rates. When inflation is zero, both the nominal interest rate and the degree of CS are positive, therefore, $\upsilon_t \neq 0$. Similarly, when the domestic interest rate is fixed to zero, as equation (2.12) shows, the degree of CS is not necessarily equal to zero, thus, $\upsilon_t \neq 0$.

To achieve the efficient allocation we assume, similarly to Woodford (2003), that the Central bank has additional instruments, in particular we assume that the central bank can pay interest on money holdings, i_t^m and that it can tax the holdings of foreign currency,τ_t^m [19].These two additional instruments can be used to make $\upsilon_t = 0$. Under these assumptions, υ_t and s are determined by the following two equations,

$$\upsilon_t = \omega\left((1-s)\left(i_t - i_t^m\right) + s\left(i_t^* + \tau_t^m\right)\right) \tag{3.11}$$

$$\overline{s} = \frac{\left(n_0 - \Psi_0 + \log\left(\frac{1+\frac{i-i^m}{(1+i)}}{1+\frac{i^*+\tau^m}{(1+i^*)}}\right)\right)}{(\Psi_1 - n_1)} \tag{3.12}$$

It follows from equations (3.11) and (3.12) that by setting, $i_t = i_t^m$, $i = i^m$ the central bank can eliminate the distortion generated by the domestic nominal interest rate, and by making

[18] Equation (3.9) is obtained by takin a log linear approximation of equation (3.4), details of this derivation are provided in appendix F

[19] Woodford (2003) studies models with transaction frictions but with only one currency for close economies and Walsh (2004) studies models with working capital.

$\tau^m = \frac{1}{\beta}(2 - \exp(n_0 - \Psi_0)) - 1$, the corresponding one to the foreign nominal interest rate, thus $\upsilon_t = 0$. Therefore, the efficient level of domestic output can be achieved, $y_t^e = a_t$.and equation (3.9) can be written as follows,

$$y_t^n = y_t^e - \frac{1-\gamma}{1+\varphi}\vartheta\upsilon_t \tag{3.13}$$

In what follows, we assume that the economy exhibits some degree of transaction frictions in steady-state, thus, we set $\tau^m = 0$ and $i - i^m$ to be small. In this case, $\upsilon_t \neq 0$, thus we assume that the flexible price equilibrium and the efficient one do not coincide. This discrepancy affects how the central bank implements monetary policy in a fundamental way. In the next section, we show in detail this issue.

3.3 Currency Substitution and the Equilibrium under Sticky Prices

In order to analyze the effects of CS on the dynamic equilibrium under price stickiness, we take a log linear approximation of equations from (3.1) to (3.5) around the deterministic steady-state. It turns out that the economy exhibits a canonical representation of three equations: a dynamic aggregate demand, a Phillips curve and an interest rate policy rule. These three equations are presented next:

$$x_t = E_t x_{t+1} - (i_t - E_t \pi_{H,t+1} - r_t^n) + \sigma_i E_t \Delta i_{t+1} + \sigma_{i^*} E_t \Delta i_{t+1}^* \tag{3.14}$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa x_t + \kappa_i i_t + \kappa_f i_t^* \tag{3.15}$$

$$i_t = \phi_\pi \pi_{H,t} + \phi_x x_t \tag{3.16}$$

where, x_t represents the gap between output under sticky prices and its efficient level. $x_t = y_t - y_t^e$, and r_t^n the natural interest rate, which is function only of structural shocks. The new set of parameters are defined as follows:

Table 1: Definition of Parameters

$\sigma_i = \omega\vartheta(1-\gamma)(1-s)$	$\kappa = \lambda(1+\varphi)$
$\sigma_{i^*} = \vartheta\omega(1-\gamma)s$	$\kappa_i = \omega\vartheta\lambda(1-\gamma)(1-s)$
$\kappa_f = \vartheta\lambda\omega(1-\gamma)s$	

It is apparent from its canonical representation that the economy with CS exhibits two new features. First, the foreign interest rate shows up in the Phillips curve, equation (3.15), as a cost-push shock, where the magnitude of its impact on inflation depends on the degree of CS. Only when the degree of CS is zero, $s = 0$, the foreign interest rate does not affect the dynamics of inflation. In this case, the economy behaves similarly to the one analyzed by Woodford(2003)[20].

The mechanism that generates this additional channel by which i_t^* appears in the Phillips curve works as follows: transaction costs create a gap between the marginal utility of consumption and that of income, given by υ_t. This gap, for a given degree of CS, is increasing in both the domestic and the foreign nominal interest rates,

$$\lambda_t = -c_t - \upsilon_t \tag{3.17}$$

Consequently, as interest rates go up, the real value of a given real wage in terms of consumption falls, since more real resources have to be allocated for transforming wage income into consumption, thus inducing workers to cut their labor supply. This in turn pushes real wages up and accordingly marginal cost rises. The next equation makes explicit this link between transaction and marginal costs,

$$mc_t = (1+\varphi)(y_t - a_t) + \vartheta(1-\gamma)\upsilon_t \tag{3.18}$$

The degree of CS determines the relative weight that the domestic and the foreign interest rate have on marginal costs. Equation (3.19) shows how the presence of transaction frictions, υ_t distorts the proportionality between the real marginal cost and the output gap that models without CS exhibit.

$$mc_t = (1+\varphi)x_t + (1-\gamma)\upsilon_t \tag{3.19}$$

Therefore, a central bank that targets $x_t = 0$, can not stabilize the marginal cost of firms, since zero output gap does not imply zero transaction costs, $\upsilon_t = 0$[21]. If the central bank does not stabilize marginal costs, can neither stabilize inflation. Consequently, in an economy with CS, it would be impossible for the central bank to simultaneously achieve zero inflation and zero output gap.

The second new feature of this type of economies is a negative effect of i_t^* on aggregate demand. This effect is different to the one based on the intertemporal substitution mechanism. As equation (3.20) shows, its impact on aggregate demand is given by σ_{i^*}, which is increasing

[20] Woodford (2003) analyzes a model of a close economy where transaction frictions affect the equilibrium of the economy. He finds that in this type of economies, the domestic interest rate affects directly the dynamics of inflation, similarly to our model.

[21] Note that transaction costs reach their minimun value only when the domestic nominal interest rate is set close to zero.

on the degree of CS. Also, notice that under CS, the partial response of the output gap to an increase on the domestic nominal interest rate becomes, $-(1+\sigma_i)$, therefore, as the degree of CS increases, σ_i falls, and consequently the output gap become less responsive to changes in the domestic interest rate:

$$x_t = E_t x_{t+1} - (i_t - E_t \pi_{H,t+1} - r_t^n) - \sigma_i i_t - \sigma_{i^*} i_t^* + \sigma_i E_t i_{t+1} + \sigma_{i^*} E_t i_{t+1}^* \tag{3.20}$$

Under CS, when an agent decides to postpone one unit of income for future consumption, the cost of her decision in period t, is given not only by the marginal utility of consumption but also by the transaction cost, υ_t. Similarly, the next period benefits of that decision includes, besides the present discounted value of the marginal utility of consumption, the corresponding expected value of the transaction cost, $E_t \upsilon_{t+1}$. Since all shocks in the model are transitory, it holds that $-\upsilon_t + E_t \upsilon_{t+1} < 0$. Thus, when nominal interest rate increases, the associated transaction cost rises, making more expensive to consume in period t relative to future periods. The interaction of these two effects induces agents to reduce their consumption levels. The effect of transaction costs on savings decisions can be seen more easily by observing the following representation of the Euler equation that results after λ_t is replaced by equation (3.17),

$$c_t = E_t c_{t+1} - \upsilon_t + E_t \upsilon_{t+1} - (i_t - E_t \pi_{t+1}) \tag{3.21}$$

In order to illustrate the effects of these new mechanisms on the rational expectations equilibrium, we solve for it, by considering that there exist only one shock in the economy, the foreign nominal interest rate. This assumption help us to obtain simple analytical solutions. Furthermore, we assume that i_t^* follows the following autorregressive process:

$$i_t^* = \rho i_{t-1}^* + \varepsilon_t$$

Under this assumption, the rational expectation equilibrium of equations (3.14), (3.15) and (3.16) is given by the following two equations:

$$x_t = -b i_t^* \tag{3.22}$$

$$\pi_{H,t} = \frac{\kappa_f - b\kappa}{1 - \beta\rho - \kappa_i \phi_\pi} i_t^* \tag{3.23}$$

where,

$$b = \frac{\left(\frac{\phi_\pi - \rho}{(1-\rho)} + \sigma_i \phi_\pi\right) \kappa_f + \sigma_{i^*} \left((1 - \beta\rho - \kappa_i \phi_\pi)\right)}{\left(1 + \left(\frac{1}{(1-\rho)} + \sigma_i\right) \phi_x\right) (1 - \beta\rho - \kappa_i \phi_\pi) + \left(\frac{\phi_\pi - \rho}{(1-\rho)} + \sigma_i \phi_\pi\right) (\kappa + \kappa_i \phi_x)}$$

For most parameterizations, $b > 0$ and $\kappa_f - b\kappa > 0$. Therefore, an increase in foreign interest rate leads to a fall in output gap and to an increase on the domestic inflation rate. However, it is important to highlight that the inflation responses to the foreign nominal interest rate is smaller than that of output gap, since the fall in output gap through the standard aggregate demand channel partially offset the direct impact of i_t^* on inflation in the Phillips curve.

These implications are confirmed in figures 4 and 5 that shows the impulse response functions of domestic inflation, output gap and the nominal interest rate to a positive foreign interest rate shock. These responses were obtained under the benchmark parametrization, for two different levels of the steady-state degree of CS.

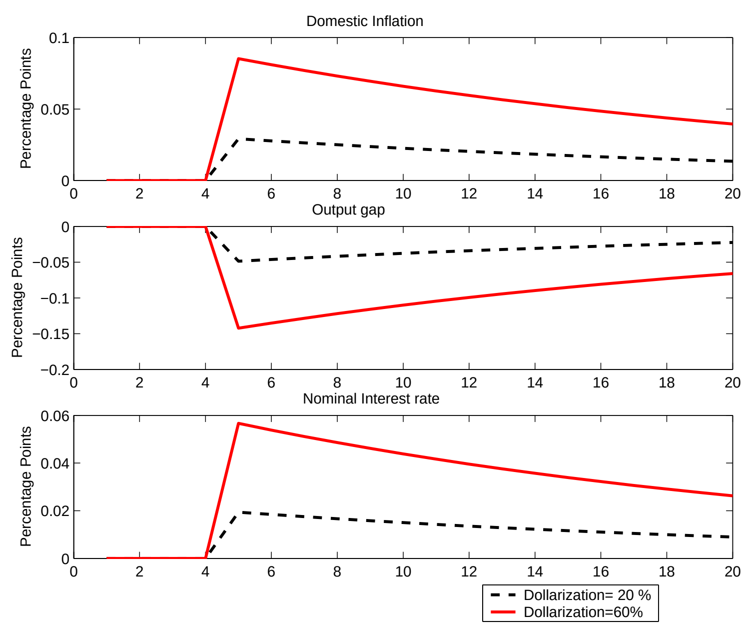


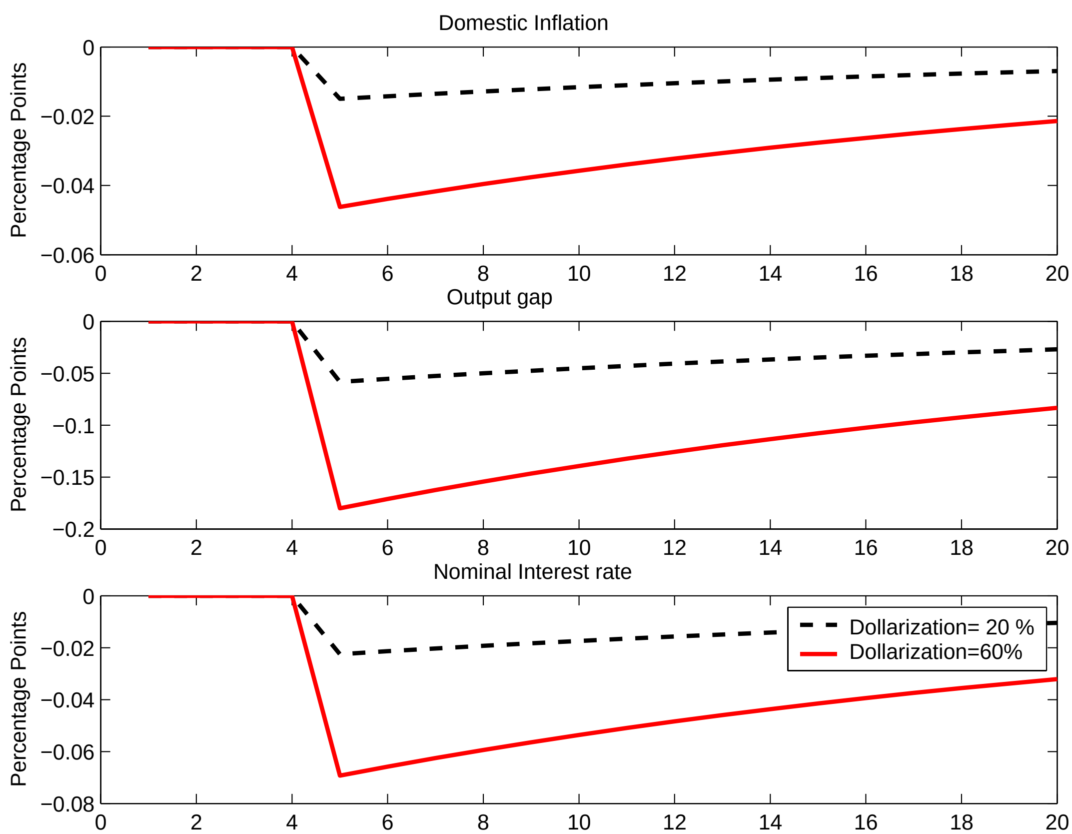
Figure 4: Impulse response functions to a foreign nominal interest rate shock, $\phi_x > 0$

As figure 2 shows, the response of the three variables is stronger to the foreign nominal interest rate shock when the degree of CS is higher. It is also interesting to highlight that, as the analytical solution of the equilibrium indicated, the response of inflation is much lower than the one of output gap, since the endogenous response of the domestic nominal interest rates and the output gap partially offsets the initial impact of i_t^* on inflation.

Notice that when the weight that central bank puts on the output gap stabilization is large enough, $\phi_x \to \infty$, the response of this variable to i_t^*, measured by b shrinks towards zero. In this case, the effect of i_t^* on domestic inflation reaches its maximum value,

$$\pi_{H,t} = \frac{\kappa_f}{1 - \beta\rho - \kappa_i \phi_\pi} i_t^*$$

On the contrary, when the central bank does not react to the output gap, $\phi_x = 0$, the fall in output more than compensate the direct effect of the foreign nominal interest rate on inflation, $\kappa_f - b\kappa < 0$, thus, both the domestic inflation and the nominal interest rate falls in equilibrium. As in the previous case, here as well, the response of the three variables, output gap, inflation and the domestic nominal interest rate are increasing on the degree of CS. Figure 5 shows the impulse responses of these three variables when $\phi_x = 0$ for two different levels of steady-state CS.



A direct implication of our model is that economies with CS should be more sensitive to foreign nominal interest shocks than economies without CS. This result is in line with the empirical evidence reported by Agenor and Prasat (2000), Neumerry and Perri (2005) and Uribe and Yue (2001), who report a negative correlation between domestic output and the foreign nominal interest rate for emerging markets, where CS is more frequent. The next section explores the implications CS for the design of monetary policy; in particular we derive the micro-founded loss function of the central bank and used it to evaluate the performance of different interest rate rules. Also, we analyze the implications of CS for the determinacy of the rational expectations equilibrium.

4 Monetary Policy Under Currency Substitution

In this section we analyze how CS affects monetary policy. In particular, we discuss the implications of CS for the convenience of exchange rate smoothing and for inflation determination. Although, there exist empirical evidence that shows that many central banks in emerging economies, in particular in economies with dollarization, tend to actively intervene in the exchange rate market to reduce the volatility of their nominal exchange rates, it is not clear cut why they behave in this way. Authors like Calvo and Reinhart (2002), emphasize the role of financial dollarization. However, Cespedes, Chang and Velasco (2004), and Gertler, Natalucci and Massino (2004), find that even with financial dollarization, a flexible exchange rate outperforms a fixed one. More recently, Castillo (2006) shows that in economies with sector specific productivity shocks it is possible to sustain an equilibrium with price dollarization, where it is optimal for the central bank to allow some degree of exchange rate smoothing.

In order to evaluate the benefits of exchange rate smoothing in economies with CS we derive a micro-founded loss function of the central bank. As in Woodford (2003) and Benigno and Woodford (2004), this loss function comes from a second order approximation of the utility function of the representative household, around a particular steady-state. We choose a steady-state where the effects of terms of trade is eliminated, but where we allow for transaction frictions. In particular, we choose an steady-state of zero inflation, but where, $i-i^m$ is relatively small, thus both domestic and the foreign nominal interest rate distort the dynamics of the economy.

4.1 The Loss Function of the Central Bank

As we show in appendix F, the loss function for a central bank in a SOE with CS has the following form,

$$L = \frac{\Omega}{2} \sum_{t=0}^{t=\infty} \beta^t \left[(1-s) \left(\Lambda_i i_t + \Lambda_{ii} i_t^2 + s \Lambda_{ii^*} i_t i_t^* \right) + \Lambda x_t + \pi_{H,t}^2 \right] \tag{4.1}$$

where, Λ_i, Ω, Λ_{ii}, Λ and Λ_{ii^*} are positive parameters. Notice that this lost function differs, in at least two dimensions, from those obtained for economies where transaction frictions are not allowed[22].First, in an economy with CS, both the domestic and the foreign nominal interest rates generate welfare losses. In particular, the central bank has an incentive to keep domestic interest rates low and stable, but also to induce a negative correlation between domestic and foreign interest rates.

To understand why the central bank has this incentive, notice that when CS is positive, $s > 0$, the foreign nominal interest rate also generates transaction costs for households, therefore a central bank, which aims at maximizing households welfare, would have the incentive to move i_t in the opposite direction of i_t^* to compensate the costs generated by fluctuations in the foreign interest rate. Remarkably, this incentive is larger, as the degree of CS increases.

This cross term between domestic and foreign interest on the central bank loss function, also has implications for the convenience of exchange rate smoothing. Since, smoothing exchange rate implies that the central bank has to move the domestic interest rate to mimic the path of foreign domestic, the welfare loss that exchange rate smoothing turns out to be increasing on the degree of CS. thus, we can argue that CS does not provide a rational for fear of floating.

Second, the incentives of the central bank to smooth fluctuations in the domestic nominal interest rate are decreasing on the degree of CS. In the limit , when $s = 1$, neither the domestic nor the foreign interest rate generate welfare losses, thus interest rate smoothing, thus the central banks has no incentives to smooth domestic nominal interest rate fluctuations On the contrary, when $s = 0$, the loss function collapse to the one derived by Woodford(2003a)[23].

Next, we use the microfounded loss function, equation (4.1), to rank different interest rate rules. In particular, we compare the performance of domestic inflation and consumer price inflation interest rate rules under different degrees of interest rate and exchange rate smoothing, the policy rules are parameterized as follows,

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\left[\phi_\pi \pi_{H,t} + \phi_x x_t + \phi_e E_t \Delta e_t\right] \tag{4.2}$$

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\left[\phi_\pi \pi_t + \phi_x x_t + \phi_e E_t \Delta e_t\right] \tag{4.3}$$

A policy rule would outperform another, if it generates a rational expectations equilibrium that implies a lower expected welfare loss. In order to calculate the expected welfare loss for

[22]For instance Woodford (2003), obtains, for an economy with transaction frictions, a loss function that depends on quadratic terms of inflation, output gag, and the nominal interest rate. He shows that this latter term justifies some degree of interest rate smoothing.

[23]Woodford (2003a), shows that in a close economy where there exist transaction frictions, the expected loss function of the central bank depends, besides the variance of output gap and inflation, on the variance of the nominal interest rates.

each interest rate rule, we solve up to second order the rational expectations equilibrium of the economy, using equations from (3.1) to (3.6), plus the interest rate rule defined previously. The rational expectations equilibrium is calculated for a set of economies, each of one defined for a particular value of ϕ_e and ρ_i.

Figure 6 shows the main results. Welfare is decreasing on the degree of exchange rate smoothing, indexed by the value of ϕ_e, for both rules, however, welfare losses are higher when the rule that targets the consumer price inflation, is used. Thus, as in Galí and Monacelli (2005), targeting domestic price inflation allows the central bank to deliver a superior outcome in terms of welfare, since, the terms of trade channel is not operating in our model economy.

Next, we perform the same exercise but this time we vary continuously the degree of persistence of nominal interest rate, ρ_i.As it is expected, we find that welfare losses decreases as ρ_i increases, thus, interest rate smoothing is a desirable objective for the central bank. The intuition of this result is simple, a lower variability on domestic nominal interest rates makes more predictable the costs generated by transaction frictions, thus welfare losses fall.

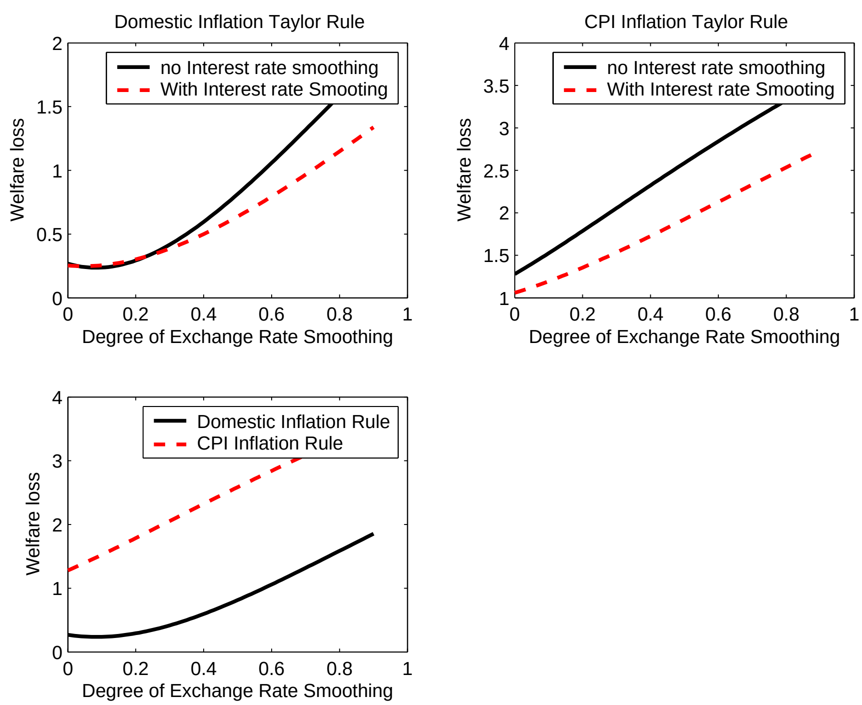


Figure 6: Welfare Ranking of Interest Rate Rules

Notice that in this paper instead of using the set of log linear structural equations to evaluate the loss function, (4.1), we use the structural equations approximated up to second order.

This strategy allow us to consider the case of economies where the degree of CS is not too small, as it is indeed the case of economies with CS and still keep the magnitude of approximation errors in our analysis up to a second order. Also, by using a second order approximation of the structural equations we can evaluate monetary policy that uses implementable rules in a consistent way. To sum up, in small open economies with CS the central bank should allow the exchange rate to float and put less weight on interest rate smoothing, in comparison to economies without CS. Also, the steady-state degree of CS does not affect the relative weights the central bank put on output gap and domestic inflation stabilization, it only affects the weight on interest rate smoothing.

4.2 Determinacy of Equilibrium

In this subsection we analyze the implications of CS for the determinacy of rational expectations equilibrium. We limit our analysis to the case of the benchmark parametrization and domestic inflation Taylor Rules. Following Woodford, (2003), we write the canonical representation of the model economy as follows:

$$E_t z_{t+1} = A z_t + a e_t$$

where: $z_t = \begin{bmatrix} \pi_{H,t} \\ x_t \end{bmatrix}$, and

$$A = \begin{bmatrix} \frac{(1-\kappa_i \phi_\pi)}{\beta} & -\left(\frac{\kappa+\kappa_i\phi_x}{\beta}\right) \\ \frac{(1+\sigma_i)\phi_\pi - (1+\sigma_i\phi_\pi)\frac{(1-\kappa_i\phi_\pi)}{\beta}}{1+\sigma_i\phi_x} & \frac{1+(1+\sigma_i)\phi_x+(1+\sigma_i\phi_\pi)\frac{(\kappa+\kappa_i\phi_x)}{\beta}}{1+\sigma_i\phi_x} \end{bmatrix} \quad (4.4)$$

Since there are two forward looking variables in the model, the rational expectations equilibrium is uniquely determined when both eigenvalues of matrix A are outside the unit circle. As it is detailed in Woodford (2003), the necessary and sufficient conditions for this to hold are:

$$\det A > 1 \quad (4.5)$$

$$\det A + trace(A) \ > -1 \quad (4.6)$$

$$\det A - trace(A) \ > -1 \quad (4.7)$$

Writing Conditions (4.5),(4.6) and (4.7) in terms of the parameters of the model we obtain:

$$\left(\frac{1}{\beta} - 1\right) + \left(\frac{1+\sigma_i(1-\beta)}{\beta}\right)\phi_x + \left(\frac{\kappa + (\beta\sigma_i\kappa - \kappa_i)}{\beta}\right)\phi_\pi > 0 \quad (4.8)$$

$$(1-\beta-\kappa_i)\,\phi_x + \kappa\,(\phi_\pi - 1) > 0 \quad (4.9)$$

$$\frac{2(1+\beta)}{\beta}+\left(\frac{(1+\beta)}{\beta}(1+2\sigma_i)+\frac{\kappa_i}{\beta}\right)\phi_x+\left(\frac{\kappa}{\beta}+\frac{2}{\beta}(\sigma_i\kappa-\kappa_i)\right)>0 \qquad (4.10)$$

It turns out that conditions (4.8) and (4.10) hold for any pair of positive values of ϕ_x and ϕ_π when the inverse of the elasticity of substitution is large enough. In particular when it satisfies the following inequality[24].

$$\varphi>\frac{1-\beta}{\beta} \qquad (4.11)$$

Therefore, under this parametrization the only condition that the parameters of the Taylor rule have to satisfy in order to guarantee determinacy is (4.9). Notice that this condition coincides with the Taylor principle when $\kappa_i = 0$. This occurs when the degree of CS is 1, $(s = 1)$, since, $\kappa_i = \lambda(1-\gamma)(1-s)$. Interesting, the model implies that the conditions for determinacy under full CS coincides with those of a cashless economy, panel (a) of figure 5.

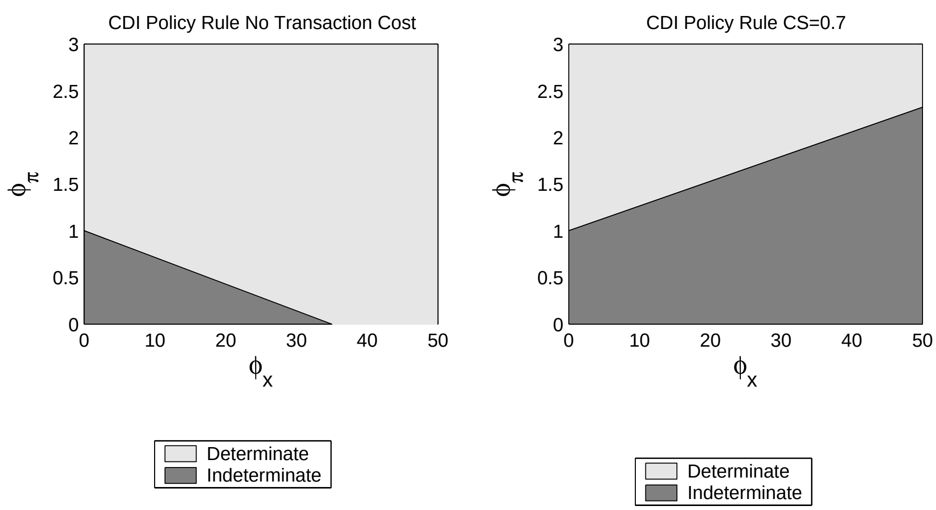


Figure 7: Determinancy Without and With CS

In a cashless economy, the domestic nominal interest rate affects the economy only through its effect on the dynamic IS curve, the same happens when $s = 1$ in an economy with CS. In contrast, when $s \neq 1$, the area of determinacy is much smaller, panel b, figure 5. In this case, the domestic interest rate have a direct effect on inflation through the wedge that transaction cost generates between marginal cost and the output gap. The effect of the domestic interest

[24]To understand why this is so, notice that condition (4.8) holds when $(\beta\sigma_i\kappa-\kappa_i)>0$. Since, under our benchmark parametrization, $\sigma_i=(1-\gamma)(1-s)$, $\kappa_i=\lambda(1-\gamma)(1-s)$ and $\kappa=\lambda(1+\varphi)$. we have that

$$(\beta\sigma_i\kappa-\kappa_i)=(1-\gamma)(1-s)\lambda(\beta(1+\varphi)-1)$$

which is positive for values of φ that satisfy condition (4.11). Similarly, condition (4.10), holds when $(\sigma_i\kappa-\kappa_i)=\lambda(1-\gamma)(1-s)\varphi>0$, which is always true when $\varphi>0$. Therefore, the only condition that determines the set of parameter values for ϕ_x and ϕ_π that render the equilibrium determine is condition (4.9) rate on inflation will be larger, as the degree of CS decreases, κ_i will be smaller. This additional effect of the domestic interest rate on inflation generates the possibility for indeterminacy of the equilibrium. To see how this mechanism for indeterminacy works, let´s suppose that the central bank observes a negative output gap, through the Taylor rule the central bank would reduce the interest rate, however, this reduction on nominal interest rates, leads to a lower inflation, through, κ_i. This second round effect generates a further reduction on the nominal interest rate, when ϕ_x is large enough, the direct effect of nominal interest rates on inflation is larger than the indirect effect on output gap, therefore, the central bank will not be able to stabilize the economy, this cycle leads to the indeterminacy of the equilibrium. Therefore, in economies with CS as the degree of CS falls the area of determinacy for the rational expectations equilibrium shrinks, this is shown in figure 6, panel (a) .

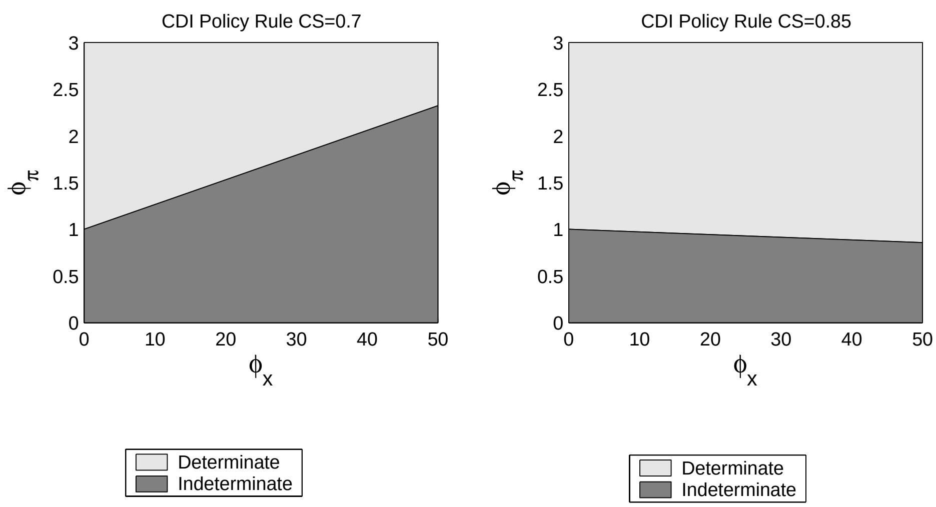


Figure 8: Determinacy under different degrees of CS

On the contrary, when the degree of CS increases, κ_i falls and the area for determinacy of the rational expectations equilibrium increases, figure 6, panel (b). This however does not imply that the RE delivers a more stable economy. As we discussed in the previous section, the volatility of both inflation and output gap increases when the degree of CS increases. Therefore, even though CS allows the central bank to react more aggressively to stabilize output gap and guarantee the determinacy of the rational expectations equilibrium, it cannot reduce the higher volatility that higher degrees of CS generates.

5 Concluding Remarks

In this paper we have developed a very tractable and fully micro-founded model of a small open economy with CS that can be used for monetary policy analysis. The model economy have a canonical representation, analogous to their counterparts without CS, but differ from the latter ones in two important dimensions: first, the foreign nominal interest rates appears as an endogenous cost push in the Phillips curve, where the magnitude of its effect on inflation depends on the degree of CS. Second, the domestic nominal interest rates has a direct effect on inflation, making less effective the use of the nominal interest rate for the control of inflation. Currency substitution emerges as an endogenous response of agents to environments of high relatively inflation, however, the model economy is general enough to nest the case of an economy without CS when inflation is sufficiently low

These new features that CS adds to a standard small open economy model have interesting implications for monetary policy. First, the level of inflation target in economies with a history of currency substitution may need to be lower than that of the foreign economy to induce a reduction on the steady-state level of CS. Second, the central bank faces a trade off between stabilizing inflation and the efficient level of output gap, where the magnitude of this trade off depends on the degree of CS. In particular, as the degree of CS increases, the central bank have to accept a higher volatility of output gap to maintain the volatility of inflation.

Third, CS increases the volatility of inflation, both domestic and CPI, and output gap under a variety typical interest rate target rules. Moreover, rules that smooth the nominal exchange rate perform worse than those that allow more flexibility on the exchange rate. In particular, the volatility of inflation and output gap increases with the degree of smoothness of the exchange rate, similarly to the case of economies without CS. Therefore, CS does not justify "fear of floating", smoothness of the exchange rate.

Finally, CS increases the area of determinacy for the rational expectations equilibrium under contemporaneous domestic inflation Taylor rules. In the limit, when there is full substitution of the domestic currency, the area of determinacy coincides with the one of a cashless economy, therefore the Taylor Principle holds. In contrast, when there is no CS, but money matters in the dynamic equilibrium, the set of parameters that allow its determinacy shrinks. In particular, for a given reaction of the central bank to inflation deviations, the central bank cannot react too much to output gap to guarantee equilibrium determinacy.

The model can be extended to several directions. For instance, the assumption that transaction costs are rebated to households can be relaxed to address issues related to the welfare effects of inflation. Also, the terms of trade distortion can be included to analysis the interaction between this channel and CS We leave these extensions for future research.

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A The foreign Economy

Agents in the foreign economy, have similar preferences to those of agents in the domestic economy.

$$U_t = E_t \left[\sum_{k=0}^{\infty} \beta^k \left(\frac{C_{t+k}^{*1-\sigma}}{1-\sigma} \right) - \frac{1}{1+\varphi} L_{t+k}^{*1+\varphi} \right] \tag{A.1}$$

Where E_t represents the expectations operator, conditional on information in period t, $\beta \in \langle 0, 1 \rangle$, the household subjective discount factor, $\sigma > 0$, the coefficient of risk aversion and $\varphi > 0$, the inverse of the Frish labor supply elasticity, L_t^* the number of hours that household work and C_t^* the composite of a continuum of final consumption goods, $c_t(s)$ indexed by $s \in [0, 1]$

$$\ln C_t^* = \left(\int_0^1 \ln C_t^* \left(s \right) d \left(s \right) \right) \tag{A.2}$$

However, for simplicity we assume that the foreign economy is a cashless one. Also, the technology of final good producers in the foreign economy is given by:

$$Y_t^{q*} = \left(\left(\alpha^* \right)^{\frac{1}{\eta}} \left(Y_{H,t}^* \right)^{\frac{\eta-1}{\eta}} + \left(1 - \alpha^* \right)^{\frac{1}{\eta}} \left(Y_{F,t}^* \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{A.3}$$

$$Y_{H,t}^{q*} = \left(\left(\tfrac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n Y_{H,t}^* \left(z \right)^{\frac{\epsilon-1}{\epsilon}} d \left(z \right) \right)^{\frac{\epsilon}{\epsilon-1}} \quad Y_{F,t}^{q*} = \left(\left(\tfrac{1}{1-n} \right)^{\frac{1}{\epsilon}} \int_n^1 Y_{F,t}^* \left(z \right)^{\frac{\epsilon-1}{\epsilon}} d \left(z \right) \right)^{\frac{\epsilon}{\epsilon-1}} \tag{A.4}$$

Which in turn implies that the corresponding demands for domestic and foreign intermediate goods are given by:

$$Y_{H,t}^{q*}(z) = \alpha^* \left(\frac{P_{H,t}^*(z)}{P_{H,t}^*} \right)^{-\varepsilon} \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} Y_t^{q*} \tag{A.5}$$

$$Y_{F,t}^{q*}(z) = (1 - \alpha^*) \left(\frac{P_{H,t}^*(z)}{P_{H,t}^*} \right)^{-\varepsilon} \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\eta} Y_t^{q*} \tag{A.6}$$

where the corresponding price indices are defined as follows:

$$P_t^* = \left(\alpha^* \left(P_{H,t}^* \right)^{1-\eta} + (1 - \alpha^*) \left(P_{F,t}^* \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{A.7}$$

and

$$P_{H,t}^{*}=\left(\frac{1}{n}\int_{0}^{n}\left(P_{H,t}^{*}\left(z\right)\right)^{1-\epsilon}dz\right)^{\frac{1}{1-\varepsilon}}\quad P_{F,t}^{*}=\left(\frac{1}{n}\int_{0}^{n}\left(P_{F,t}^{*}\left(z\right)\right)^{1-\varepsilon}dz\right)^{\frac{1}{1-\epsilon}} \tag{A.8}$$

The set of non-linear equations that describe the behavior of the foreign economy is given by:

Table A.1: Non Linear equations

Phillips Curve	Marginal Cost
$\theta\left(\pi_t^*\right)^{\varepsilon-1}=1-\left(1-\theta\right)\left(\frac{N_t^*}{D_t^*}\right)^{1-\epsilon}$	$mc_t^*=\frac{Y_t^{*\varphi}}{\lambda_t^* A_t^{*1+\varphi}}$
$N_t^*=\mu\left(Y_t^*\right)^{-\sigma}mc_t^* Y_t^*+\theta\beta\left(\pi_{t+1}^*\right)^{\epsilon}N_{t+1}^*$	Taylor Rule
$D_t^*=\left(Y_t^*\right)^{-\sigma}Y_t^*+\theta\beta\left(\pi_{t+1}^*\right)^{\epsilon-1}D_{t+1}^*$	$\left(1+R_t^*\right)=\overline{R}\left(\frac{\pi_t^*}{\overline{\pi}^*}\right)^{\phi_\pi}\left(\frac{y_t^*}{\overline{y}_t^*}\right)^{\phi_y}\exp\left(\upsilon_t^*\right)$
Euler Equation	Marginal Utility
$\frac{1}{1+R_t^*}=\beta E_t\left(\frac{\lambda_{t+1}^*}{\lambda_t^*}\frac{1}{\pi_{t+1}^*}\right)$	$Y_t^{*-\sigma}=\lambda_t^*$

B The Phillips Curve

The typical intermediate good producer choose price, $P_{H,t}^{o}(z)$ to maximize the following profit function:

$$E_t\left[\sum_{k=0}^{\infty}(\theta\beta)^k\left(\lambda_{t+k}\left(\frac{P_{H,t}^{o}(z)}{P_{H,t+k}}-mc_{t+k}\right)\widetilde{Y}_{H,t+k}(z)\right)\right] \tag{B.1}$$

let's define the cumulative domestic inflation by

$$\Psi_{t+k}=\frac{P_{H,t}}{P_{H,t+k}} \tag{B.2}$$

and

$$\widetilde{Y}_{H,t+k}(z)=\left(\frac{P_{H,t}\left(z\right)}{P_{H,t}}\right)^{-\epsilon}\Psi_{t+k}^{-\epsilon}Y_{H,t+k} \tag{B.3}$$

Using the previous notation the first order condition of firms problem can be written as follows:

$$E_t\left[\sum_{k=0}^{\infty}(\theta\beta)^k\left(C_{t+k}^{-\sigma}\left(\frac{P_{H,t}^{o}(z)}{P_{H,t}}\Psi_{t+k}-\frac{\epsilon}{(\epsilon-1)}mc_{t+k}\right)\widetilde{Y}_{H,t+k}(z)\right)\right]=0 \tag{B.4}$$

Rearranging this expression properly, we obtain the optimal price as function of future expected marginal costs follows:

$$\frac{P^o_{H,t}(z)}{P_{H,t}} = \frac{\epsilon}{(\epsilon-1)} \frac{E_t\left[\sum_{k=0}^{\infty} (\theta\beta)^k \left(\lambda_{t+k} mc_{t+k} \Psi_{t+k}^{-\epsilon} Y_{H,t+k}\right)\right]}{E_t\left[\sum_{k=0}^{\infty} (\theta\beta)^k \left(\lambda_{t+k} \Psi_{t+k}^{1-\epsilon} Y_{H,t+k}\right)\right]} \tag{B.5}$$

Since all firms face the same cost structure, those firms that can adjust prices, choose the same price $P^o_{H,t}(z)$., whereas those that can not adjust prices maintain the previous price level, $P_{H,t-1}$.. Using the definition of the final good price index, we can derive the following no linear condition for the determination of domestic inflation:

$$\theta\left(\pi_{H,t}\right)^{\epsilon-1} = 1-(1-\theta)\left(\frac{P^o_{H,t}(z)}{P_{H,t}}\right)^{1-\epsilon} \tag{B.6}$$

Following Benigno and Woodford (2005), we define the following auxiliary variables N_t and D_t

$$N_t = \mu E_t\left[\sum_{k=0}^{\infty} (\theta\beta)^k \left(\lambda_{t+k} mc_{t+k} \Psi_{t+k}^{-\epsilon} Y_{H,t+k}\right)\right] \tag{B.7}$$

and

$$D_t = E_t\left[\sum_{k=0}^{\infty} (\theta\beta)^k \left(\lambda_{t+k} \Psi_{t+k}^{1-\epsilon} Y_{H,t+k}\right)\right] \tag{B.8}$$

that allow to write the optimal price of intermediate good producers in a more convenient way as follows:

$$\frac{P^o_{H,t}(z)}{P_{H,t}} = \frac{N_t}{D_t} \tag{B.9}$$

where:

$$N_t = \mu\lambda_t mc_t Y_{H,t} + \theta\beta\pi^{\epsilon}_{H,t+1} N_{t+1} \tag{B.10}$$

$$D_t = \lambda_t Y_{H,t} + \theta\beta\pi^{\epsilon-1}_{H,t+1} D_{t+1} \tag{B.11}$$

Therefore, the dynamic equation that determines domestic inflation can be written as follows:

$$\theta\left(\pi_{H,t}\right)^{\epsilon-1} = 1-(1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\epsilon} \tag{B.12}$$

C Aggregating consumption decisions

We start from the conditions that define the optimal allocation of consumption across type of goods

$$U_{c,t}\frac{\partial c_t}{\partial c_t(s)} = P_t(s)\lambda_t\left(1+\frac{q_t}{\lambda_t}\right)(1+g(s)) \text{ for } s \geqslant \overline{s}_t \tag{C.1}$$

$$U_{c,t}\frac{\partial c_t}{\partial c_t(s)} = P_t(s)\lambda_t\left(1+\frac{n_t}{\lambda_t}\right)(1+\tau(s)) \text{ for } s < \overline{s}_t \tag{C.2}$$

$$\frac{1+\tau(\overline{s}_t)}{1+g(\overline{s}_t)} = \frac{1+\frac{q_t}{\lambda_t}}{1+\frac{n_t}{\lambda_t}} = \frac{1+\frac{R_t}{(1+R_t)}}{1+\frac{R_t^*}{(1+R_t^*)}} \tag{C.3}$$

Notice that using the consumption aggregator defined as:

$$\ln C_t = \left(\int_0^1 \ln C_t(s)\, d(s)\right) \tag{C.4}$$

and the fact that prices of final goods are the same in equilibrium, equations (C.1) and (C.2) can be written as:

$$U_{c,t} = P_t\frac{c_t(s)}{c_t}\lambda_t\left(1+\frac{q_t}{\lambda_t}\right)(1+g(s)) \text{ for } s \geqslant \overline{s}_t \tag{C.5}$$

$$U_{c,t} = P_t\frac{c_t(s)}{c_t}\lambda_t\left(1+\frac{n_t}{\lambda_t}\right)(1+\tau(s)) \text{ for } s < \overline{s}_t \tag{C.6}$$

Taking logs to equations (C.5) and (C.6), we and integration over type of consumption goods we obtain the following condition:

$$\log U_{c,t} = \log(P_t\lambda_t) + (1-\overline{s}_t)\log\left(1+\tfrac{q_t}{\lambda_t}\right) + \int_{\overline{s}_t}^{1}\log(1+g(s))\,ds$$
$$+\overline{s}_t\log\left(1+\tfrac{\eta_t}{\lambda_t}\right) + \int_0^{\overline{s}_t}\log(1+\tau(s))\,ds$$

Taking logs to equation (C.3) we can eliminate, $\frac{\eta_t}{\lambda_t}$, from the previous equation as follows:

$$\log\left(1+\frac{\eta_t}{\lambda_t}\right) = \log\left(1+\frac{q_t}{\lambda_t}\right) - \log(1+\tau(\overline{s}_t)) + \log(1+g(\overline{s}_t)) \tag{C.7}$$

therefore, we can have:

$$\log U_{c,t} = \log\left(P_t \lambda_t\right) + \log\left(1 + \tfrac{q_t}{\lambda_t}\right) +$$
$$\int_{\overline{s}_t}^{1} \log \tfrac{(1+g(s))}{(1+g(\overline{s}_t))} ds + \int_{0}^{\overline{s}_t} \log \tfrac{(1+\tau(s))}{(1+\tau(\overline{s}_t))} ds + \log\left(1 + g\left(\overline{s}_t\right)\right)$$

Taking antilog to the previous equation we obtain the marginal utility of consumption upon aggregation:

$$U_{c,t} = P_t \lambda_t \left(1 + \Upsilon_t\right) \tag{C.8}$$

where,

$$\left(1 + \Upsilon_t\right) = \left(2 - \frac{1}{1 + i_t}\right)\left(1 + \Gamma\left(\overline{s}_t\right)\right)$$

$$\begin{aligned}(1 + \Gamma(\overline{s}_t)) \quad &= \quad \exp(\int_{\overline{s}_t}^{1} \log \frac{(1 + g(s))}{(1 + g(\overline{s}_t))} ds + \\ &\quad \int_{\overline{s}_t}^{1} \log \frac{(1 + \tau(s))}{(1 + \tau(\overline{s}_t))} ds + \log(1 + g(\overline{s}_t)))\end{aligned}$$

$$\tau\left(s_t\right) = \exp(\Psi_o + \Psi_1 s_t) - 1 \tag{C.9}$$

$$g\left(s_t\right) = \exp\left(n_o + n_1 s_t\right)) - 1 \tag{C.10}$$

where, $\Psi_1 > n_1 \; n_o > \Psi_o$, thus we obtain

$$\begin{aligned}\int_{\overline{s}_t}^{1} \log \frac{(1 + g(s))}{(1 + g(\overline{s}_t))} ds \quad &= \quad \int_{\overline{s}_t}^{1} n_1 (s_t - \overline{s}_t) ds = \\ \left[\Psi_1 \left(\frac{s_t^2}{2} - s_t \overline{s}_t\right)\right]_{\overline{s}_t}^{1} \quad &= \quad n_1 \left(\frac{1}{2} - \overline{s}_t + \frac{\overline{s}_t^2}{2}\right)\end{aligned}$$

Similarly, we have,

$$\int_{0}^{\overline{s}_t} \log \frac{(1 + \tau(s))}{(1 + \tau(\overline{s}_t))} ds = \int_{\overline{s}_t}^{1} \Psi_1 (s_t - \overline{s}_t) ds = -\Psi_1 \frac{\overline{s}_t^2}{2} \tag{C.11}$$

$$\log\left(1 + g\left(\overline{s}_t\right)\right) = n_o + n_1 \overline{s}_t \tag{C.1}$$

Therefore, we have that

$$1+\Gamma\left(\overline{s}_t\right)=\exp\left(\frac{n_1}{2}+n_o-\left(\Psi_1-n_1\right)\frac{\overline{s}_t^2}{2}\right) \tag{C.13}$$

D The Non Linear Economy

The home economy is fully characterized by the following set of non-linear difference equations:

Table D1: Non linear equations

Phillips Curve	Terms of Trade
$\theta\left(\Pi_{H,t}\right)^{\epsilon-1}=1-(1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\epsilon}$ $N_t=\mu\lambda_t mc_t Y_t+\theta\beta\left(\Pi_{H,t+1}\right)^{\epsilon}N_{t+1}$ $D_t=\lambda_t Y_t+\theta\beta\left(\Pi_{H,t+1}\right)^{\epsilon-1}D_{t+1}$	$\left(\frac{Q_t}{T_t}\right)^{\eta-1}=(1-\gamma)+\gamma T_t^{1-\eta}$ CPI inflation $\left(\frac{\Pi_t}{\Pi_{H,t}}\right)^{1-\eta}=\frac{(1-\gamma)+\gamma T_t^{1-\eta}}{(1-\gamma)+\gamma T_{t-1}^{1-\eta}}$
Euler Equation	Taylor Rule
$\frac{1}{1+i_t}=\beta E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\Pi_{t+1}}\right)$	$(1+i_t)=\overline{i}\left(\frac{\Pi_{H,t}}{\overline{\pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\overline{Y}_t}\right)^{\phi_x}$
Aggregate Demand	Marginal Utility of Consumption
$Y_t=\left(\frac{Q_t}{T_t}\right)^{-\eta}\left((1-\gamma)C_t+\gamma Q_t^{\eta}Y_t^*\right)$	$C_t^{-\sigma}=\lambda_t\left(1+\Upsilon_t\right)$
Risk Sharing	Transaction cost distortion
$Q_t=\varsigma_0\frac{\lambda_t^*}{\lambda_t}$	$1+\Upsilon_t=\left(1+\frac{R_t}{(1+R_t)}\right)\left(1+\Gamma\left(\overline{s}_t\right)\right)$
Marginal Cost	CS distortion
$mc_t=\frac{T_t Y_t^{\varphi}}{\lambda_t Q_t A_t^{1+\varphi}}$	$1+\Gamma\left(\overline{s}_t\right)=\exp\left(\frac{n_1}{2}+n_o-\left(\Psi_1-n_1\right)\frac{\overline{s}_t^2}{2}\right)$
Demand for money	Cost of dollar
$\frac{M_t}{P_t}=\int_0^{\overline{s_t}}C_t(s)\,d(s)$	$\tau\left(s_t\right)=\exp(\Psi_o+\Psi_1 s_t)-1$
Cost of Peso	CS condition
$g\left(s_t\right)=\exp\left(n_o+n_1 s_t\right))-1$	$\frac{1+\tau(\overline{s}_t)}{1+g(\overline{s}_t)}=\frac{1+\frac{i_t}{(1+i_t)}}{1+\frac{i_t^*}{(1+i_t^*)}}$

We use the marginal utility of consumption to eliminate, λ_t from the Euler equation, the marginal cost and the risk sharing condition, thus we obtain,

$$\frac{1}{1+i_t}=\beta E_t\left(\left(\frac{C_{t+1}}{C_t}\right)^{-1}\frac{1+\Upsilon_t}{1+\Upsilon_{t+1}}\frac{1}{\Pi_{t+1}}\right) \tag{D.1}$$

$$MC_t=\frac{T_t Y_t^{\varphi}C_t\left(1+\Upsilon_t\right)}{Q_t A_t^{1+\varphi}} \tag{D.2}$$

$$Q_t = \frac{C_t \left(1 + \Upsilon_t\right)}{Y_t^*} \tag{D.3}$$

Also, the equations of terms of trade and CPI inflation converge to the following conditions when $\eta = 1$,

$$Q_t = T_t^{1-\gamma} \tag{D.4}$$

$$\Pi_t = \Pi_{H,t} \left(\frac{T_t}{T_{t-1}}\right)^{\gamma} \tag{D.5}$$

We use equation (D.3) to simplify the aggregate demand equation, thus we obtain,

$$Y_t = \left(\frac{Q_t}{T_t}\right)^{-1} \left(1 + \gamma \Upsilon_t\right) C_t \tag{D.6}$$

and equation (D.4), to write the previous equation only in terms of terms of trade, consumption and transaction frictions,

$$Y_t = T_t^{\gamma} \left(1 + \gamma \Upsilon_t\right) C_t \tag{D.7}$$

additionally by plugging in equation (D.7) and (D.5) into equation (D.1), and after simplifying we obtain equation (3.6) of the main text,

$$\frac{1}{1 + i_t} = \beta E_t \left(\left(\frac{Y_{t+1}}{Y_t}\right)^{-1} \frac{1 + \Upsilon_t}{1 + \Upsilon_{t+1}} \frac{1 + \gamma \Upsilon_{t+1}}{1 + \gamma \Upsilon_t} \frac{1}{\Pi_{H,t+1}} \right) \tag{D.8}$$

Next, we use equation (D.7) and (D.4) to eliminate, C_t and Q_t and T_t from equation (D.2), we obtain the following condition for the marginal costs,

$$MC_t = \frac{Y_t^{1+\varphi}}{A_t^{1+\varphi}} \left(\frac{1 + \gamma \Upsilon_t}{1 + \Upsilon_t}\right) \Delta_t^{\varphi} \tag{D.9}$$

which corresponds to equation (3.4) of the main text. Notice that, Δ_t measures price dispersion generated by price stickiness,

$$\Delta_t = \int_0^n \left(\frac{P_t(z)}{P_t}\right)^{-\theta} dz$$

from equation (D.2), we can obtain consumption in terms of the real exchange rate,

$$C_t = \frac{Q_t Y_t^*}{\left(1 + \Upsilon_t\right)}$$

plugging in the consumption level obtained in the previous equation into equation (D.7), we obtain

$$Y_t = T_t^{\gamma}\left(1+\gamma\Upsilon_t\right)\frac{Q_t Y_t^*}{\left(1+\Upsilon_t\right)}$$

since $Q_t = T_t^{1-\gamma}$, it follows immediately from the previous equation that terms of trade can be determined from,

$$T_t = \frac{Y_t\left(1+\Upsilon_t\right)}{Y_t^*\left(1+\gamma\Upsilon_t\right)} \tag{D.10}$$

Notice that when, $\Upsilon_t = 0$, terms of trade are determined only by relative levels of output, as in standard SOE models. We use equations (D.10) and (D.7) to derive an expression that determines the level of consumption in terms of domestic and foreign output and the transaction distortion.

$$C_t = Y_t^{1-\gamma}\left(Y_t^*\right)^{\gamma}\left(\frac{1}{1+\gamma\Upsilon_t}\right)^{1-\gamma}\frac{1}{\left(1+\Upsilon_t\right)^{\gamma}} \tag{D.11}$$

Finally, using the definition of λ_t we obtain,

$$\lambda_t = \frac{1}{Y_t^{1-\gamma}\left(Y_t^*\right)^{\gamma}}\left(\frac{1+\gamma\Upsilon_t}{\left(1+\Upsilon_t\right)}\right)^{1-\gamma} \tag{D.12}$$

We plug in this expression in equations that define the Phillips curve to obtain equations (3.1) and (3.2) of the main text.

E The log linear equations

By log linearizing equations (3.1), (3.2) and (3.3) of section 3, we obtain,

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \lambda mc_t \tag{E.1}$$

where, $\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}$. Similarly, the log linear approximation of equation 3.4 is given by,

$$mc_t = (1+\phi)\left(y_t - a_t\right) + (1-\gamma)\,\vartheta\upsilon_t \tag{E.2}$$

by combining equations (E.1) and (E.2) we obtain the Phillips curve,

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa\left(y_t - a_t\right) + \lambda\left(1-\gamma\right)\vartheta\upsilon_t \tag{E.3}$$

On the other hand, the dynamic aggregate demand condition in its log linear form is obtained from equation (3.6),

$$i_t = E_t \Delta y_{t+1} + \pi_{H,t+1} + (1-\gamma)\vartheta \Delta \upsilon_{t+1} \tag{E.4}$$

E.1 The flexible price allocation

Under flexible prices it holds that $mc_t^n = 0$, thus from equation (E.2), we obtain the natural level of output, which is defined as follows,

$$y_t^n = a_t - \frac{(1-\gamma)\,\vartheta}{1+\phi}\upsilon_t \tag{E.5}$$

This equation corresponds to equation (3.9) in section 3.1. Similarly, using equation (E.4) we obtain a law of motion for the natural interest rate,

$$r_t^n = E_t \Delta y_{t+1}^n + (1-\gamma)\vartheta \Delta \upsilon_{t+1} \tag{E.6}$$

Clearly, the allocation obtained in the equilibrium under flexible prices, is not efficient. To achieve the efficient allocation we assume, similarly to Woodford (2003), that the Central bank has additional instruments, in particular we assume that, there exist a nominal interest rate on domestic money holdings, i_t^m and a tax to holdings of foreign currency,τ_t^m that can be used to eliminate trading frictions on the steady-state and flexible prices allocation. Under these assumptions, υ_t and s will be given by,

$$\upsilon_t = \omega\left((1-s)\left(i_t - i_t^m\right) + s\left(i_t^* + \tau_t^m\right)\right) \tag{E.7}$$

$$\overline{s} = \frac{\left(n_0 - \Psi_0 + \log\left(\frac{1+\frac{i-i^m}{(1+i)}}{1+\frac{i^*+\tau^m}{(1+i^*)}}\right)\right)}{(\Psi_1 - n_1)} \tag{E.8}$$

By setting, $i_t = i_t^m$, $i = i^m$ and $\tau^m = \frac{1}{\beta}\left(2 - \exp\left(n_0 - \Psi_0\right)\right) - 1$, it holds that, $\overline{s} = 0$, and consequently that, υ_t. When these conditions hold, transaction frictions do not affect the dynamic equilibrium under flexible prices, thus, the level of output become efficient,

$$y_t^e = a_t$$

This latter equation corresponds to equation (3.13) in section 3.2. By defining the efficient output gap, as $x_t = y_t - a_t$, and by using equations (E.3) and (E.7), we obtain equation (3.15) of section 3.3 . Similarly, by subtracting equations (E.4) and (E.6) we obtain equation (3.14) of section 3.3.

E.2 The approximated Loss function

We approximate, using a second order Taylor expansion around the following generic utility function of the representative agent,

$$U_t = E_t \left[\sum_{k=0}^{\infty} \beta^k \left[U \left(C_{t+k} \right) - V \left(L_{t+k} \right) \right] \right] \tag{E.9}$$

First we approximate, up to second order of accuracy, the $U(C_t)$. We take the log-quadratic approximation of this function around a steady-state where the degree of CS is positive but small. We choose this particular steady-state since our goal is to analyze how monetary policy should be conducted in an economy with CS. In order to obtain this approximation we use equation (E.10), derived in section, xx, that relates domestic consumption to domestic and foreign output, and to an stochastic tax, generated by transaction frictions,

$$C_t = Y_t^{1-\gamma} \left(Y_t^* \right)^{\gamma} G \left(\Upsilon_t \right) \tag{E.10}$$

where,

$$G \left(\Upsilon_t \right) = \left(\frac{1}{1 + \gamma \Upsilon_t} \right)^{1-\gamma} \left(\frac{1}{(1 + \Upsilon_t)} \right)^{\gamma} \tag{E.11}$$

Υ_t is defined by equations (2.15), (2.16) and (2.12), and L_t is defined as follows,

$$L_t = \frac{Y_t \Delta_t}{A_t} \tag{E.12}$$

where,

$$\Delta_t = \int_0^n \left(\frac{P_{H,t}(z)}{P_t} \right)^{-\theta} dz \tag{E.13}$$

We first approximate $U(C_t)$. Since $U(C_t) = \ln(C_t)$, after plugging equations (E.10) and (E.11) into equation (E.9), we obtain,

$$U(C_t) = (1-\gamma) \ln(Y_t) + \gamma \ln Y_t^* - (1-\gamma) \ln(1 + \gamma \Upsilon_t) - \gamma \ln \left(1 + \Upsilon_t \right) \tag{E.14}$$

a log quadratic approximation of this equation is given by,

$$U(C_t) = (1-\gamma) y_t + \gamma y_t^* - (1-\gamma) \frac{\gamma \left(1 + \Upsilon \right)}{\left(1 + \gamma \Upsilon \right)} \left(\upsilon_t + \frac{1 - \sigma_{\Upsilon}}{2} \upsilon_t^2 \right) - \gamma \upsilon_t + o \left(\left\| \Upsilon, \epsilon \right\|^3 \right) \tag{E.15}$$

where $\upsilon_t = \ln\left(\frac{1+\Upsilon_t}{1+\Upsilon}\right)$. We simplify equation (E.10), thus we obtain,

$$U(C_t) = (1-\gamma)y_t + \gamma y_t^* - (1-\gamma)\lambda_\Upsilon\gamma\left(\lambda_\upsilon \upsilon_t + \frac{1-\sigma_\Upsilon}{2}\upsilon_t^2\right) + o\left(\|\Upsilon, \epsilon\|^3\right) \tag{E.16}$$

where we have defined, ,$\sigma_\Upsilon = \frac{\gamma(1+\Upsilon)}{(1+\gamma\Upsilon)}$and $\lambda_\upsilon = \left(\frac{1+\Upsilon+((1-\gamma))}{(1-\gamma)(1+\Upsilon)}\right)$. Thus, the previous equation implies that both the level and the volatility of transaction frictions, υ_t, affect negatively household welfare by making more costly for them to transform income into consumption. Furthermore, by noticing that $U_c C = 1$, we can write equation (E.16) as follows,

$$U(C_t) = (1-\gamma)U_c C\left(y_t - \sigma_\Upsilon\left(\lambda_\upsilon \upsilon_t + \frac{1-\sigma_\Upsilon}{2}\upsilon_t^2\right)\right) + tip + o\left(\|\Upsilon, \epsilon\|^3\right) \tag{E.17}$$

Next, we determine, υ_t in terms of the domestic and the foreign interest rate,

$$\upsilon_t = \ln\left(\frac{2-\frac{1}{1+i_t}}{2-\frac{1}{1+i}}\right) + \ln\left(\frac{1+\Gamma(\overline{s}_t)}{1+\Gamma(\overline{s})}\right) \tag{E.18}$$

the first component of the previous condition can be approximated up to second order as follows,

$$\ln\left(\frac{2-\frac{1}{1+i_t}}{2-\frac{1}{1+i}}\right) = \frac{1}{(2(1+i)-1)}i_t + o\left(\|\Upsilon, \epsilon\|^2\right)$$

we define $\omega = \frac{1}{(2(1+i)-1)}$, thus the previous equation can be written as follows,

$$\ln\left(\frac{2-\frac{1}{1+i_t}}{2-\frac{1}{1+i}}\right) = \omega i_t + o\left(\|\Upsilon, \epsilon\|^2\right) \tag{E.19}$$

Next, we take a log linear approximation of $\ln\left(\frac{1+\Gamma(\overline{s}_t)}{1+\Gamma(\overline{s})}\right)$, since,

$$\ln(1+\Gamma(\overline{s}_t)) = \exp\left(\frac{n_1}{2} + n_o - (\Psi_1 - n_1)\frac{\overline{s}_t^2}{2}\right) \tag{E.20}$$

approximating around $\overline{s}$, we have that,

$$\widehat{\Gamma}(\overline{s}_t) = -(\Psi_1 - n_1)s^2\widehat{\overline{s}}_t$$

where,

$$\widehat{\overline{s}}_t = \frac{1}{s(\Psi_1 - n_1)}\omega(i_t - i_t^*) + o\left(\|\Upsilon, \epsilon\|^2\right)$$

thus we obtain the following expression $\widehat{\Gamma}\left(\overline{s}_t\right)$for now we obtain the approximation of s_t,

$$\widehat{\Gamma}\left(\overline{s}_t\right) = -s\omega\left(i_t - i_t^*\right) + o\left(\left\|\Upsilon, \epsilon\right\|^2\right) \tag{E.21}$$

therefore, we have that,

$$\upsilon_t = \omega i_t - s\omega\left(i_t - i_t^*\right) + o\left(\left\|\Upsilon, \epsilon\right\|^2\right) \tag{E.22}$$

simplifying this expression we obtain,

$$\upsilon_t = \omega(1-s)i_t + s\omega i_t^* + o\left(\left\|\Upsilon, \epsilon\right\|^2\right) \tag{E.23}$$

Therefore, the utility that consumption generates, up to second order, is determined by,

$$\begin{aligned} U(C_t) \quad = \quad & (1-\gamma)U_c C\left(y_t - \sigma_\Upsilon \omega(1-s)\lambda_\upsilon i_t\right) \\ & -(1-\gamma)U_c C(1-s)\sigma_\Upsilon\omega\left[\left(\frac{1-\sigma_\Upsilon}{2}\right)\omega(1-s)i_t^2 + \left(1-\sigma_\Upsilon\right)\omega s i_t i_t^*\right] \\ & +tip + o\left(\left\|\Upsilon, \epsilon\right\|^3\right) \end{aligned} \tag{E.24}$$

Next we take a second order expansion of $v(h_t)$, we use equation (E.12) to define the aggregate level of labor in terms of output, productivity shocks and price dispersion. Therefore, the second order approximation of the desutility of labor is given by,

$$\begin{aligned} v(L_t) \quad = \quad & v(\Delta_t \frac{Y_t}{A_t}) = \overline{v} + \overline{v}_\Delta\left(\Delta_t - 1\right) + \overline{v}_y\left(Y_t - \overline{Y}\right) + \overline{v}_A\left(A_t - 1\right) \\ & +\frac{1}{2}\left[\overline{v}_{yy}\left(Y_t - \overline{Y}\right)^2 + \overline{v}_{\Delta\Delta}\left(\Delta_t - 1\right)^2 + \overline{v}_{AA}\left(A_t - 1\right)^2\right] + \\ & +\overline{v}_{y\Delta}\left(Y_t - \overline{Y}\right)\left(\Delta_t - 1\right) + \overline{v}_{yA}\left(Y_t - \overline{Y}\right)\left(A_t - 1\right) \\ & +\overline{v}_{\Delta A}\left(\Delta_t - 1\right)\left(A_t - 1\right) + o\left(\left\|\varepsilon\right\|^3\right) \end{aligned} \tag{E.25}$$

Notice that $\widehat{\Delta}_t$ depends only on second order term, where, $\widehat{\Delta}_t$ is determined by the following law of motion,

$$\widehat{\Delta}_t = \theta\widehat{\Delta}_{t-1}\Pi_{H,t}^{\varepsilon} + (1-\theta)\left(\frac{1-\theta\Pi_{H,t}^{\varepsilon-1}}{1-\theta}\right)^{\frac{-\varepsilon}{1-\varepsilon}} \tag{E.26}$$

and its second order approximation by:

$$\widehat{\Delta}_t = \theta\widehat{\Delta}_{t-1} + \frac{\theta\varepsilon}{1-\theta}\frac{\Pi_{H,t}^2}{2} + o\left(\left\|\epsilon\right\|^3\right) \tag{E.27}$$

Notice that under the assumption that $\widehat{\Delta}_{-1}$ is of order $o\left(\|\epsilon\|^3\right)$, $\widehat{\Delta}_t$ is of order $o\left(\|\epsilon\|^2\right)$, thus we can eliminate all cross terms of $\widehat{\Delta}_t$, from equation ( ), therefore,

$$\begin{aligned} v(L_t) &= \overline{v} + \overline{v}_\Delta \widehat{\Delta}_t + \overline{v}_y \overline{Y}\left(\widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2\right) + \overline{v}_A \widehat{A}_t + \\ & \frac{1}{2}\left[\overline{v}_{yy}\overline{Y}^2 \widehat{Y}_t^2\right] + \overline{v}_{yA}\overline{Y}\widehat{Y}_t \widehat{A}_t + t.i.p + o\left(\|\varepsilon\|^3\right) \end{aligned} \tag{E.28}$$

Furthermore, since we eliminate the terms of trade distortion, under this allocation, it holds that,

$$(1-\gamma)\,\overline{u}_c C = \overline{v}_y Y \tag{E.29}$$

we have that:

$$\begin{aligned} u\left(C_t\right) - v(h_t) &= (1-\gamma)U_c C\left(y_t - \sigma_\Upsilon \omega(1-s)\lambda_\upsilon i_t\right) - \\ & -(1-\gamma)U_c C(1-s)\sigma_\Upsilon \omega\left[\left(\frac{1-\sigma_\Upsilon}{2}\right)\omega(1-s)i_t^2 + (1-\sigma_\Upsilon)\,\omega s i_t i_t^*\right] \quad \text{(E.30)} \\ & \overline{v}_y \overline{Y}\left(\frac{\overline{v}_\Delta}{\overline{v}_y \overline{Y}}\widehat{\Delta}_t + \widehat{Y}_t + \frac{1}{2}(1+\varphi)\,\widehat{Y}_t^2 + \frac{\overline{v}_{y\varepsilon}}{\overline{v}_y}\widehat{Y}_t \widehat{A}_t\right) \\ & + t.i.p + o\left(\|\varepsilon\|^3\right) \quad \text{(E.31)} \end{aligned}$$

Thus, simplifying the previous expression we obtain, :

$$\begin{aligned} &= -\overline{v}_y \overline{Y}\omega(1-s)\sigma_\Upsilon\left(\lambda_\upsilon i_t + \left(\frac{1-\sigma_\Upsilon}{2}\right)\omega(1-s)i_t^2 + (1-\sigma_\Upsilon)\,\omega s i_t i_t^*\right) \\ & \overline{v}_y \overline{Y}\left(-\frac{1}{2}\left((1+\varphi)\right)\widehat{Y}_t^2 - \frac{\overline{v}_\Delta}{\overline{v}_y \overline{Y}}\widehat{\Delta}_t - \frac{\overline{v}_\varepsilon}{\overline{v}_y}\widehat{Y}_t \widehat{A}_t\right) \quad \text{(E.32)} \\ & + t.i.p + o\left(\|\epsilon\|^3\right) \quad \text{(E.33)} \end{aligned}$$

Let´s define the following new set of parameters:

$$u_{yy} = -\left(1+\varphi\right) \tag{E.34}$$

$$u_{y\varepsilon} = \frac{\overline{v}_\varepsilon}{\overline{v}_y} \tag{E.35}$$

$$u_\Delta = \frac{\overline{v}_\Delta}{\overline{v}_y \overline{Y}} \tag{E.36}$$

We can now write the utility function of the representative agent as follows:

$$
\begin{aligned}
u\left(C_t\right)-v(h_t) \;=\;& -\overline{v}_y\overline{Y}\omega\left(1-s\right)\sigma_{\Upsilon}\left(\lambda_{\upsilon}i_t+\left(\frac{1-\sigma_{\Upsilon}}{2}\right)\omega(1-s)i_t^2+\left(1-\sigma_{\Upsilon}\right)\omega s i_t i_t^*\right)\\
& -\overline{v}_y\overline{Y}\left(\frac{1}{2}u_{yy}\widehat{Y}_t^2+u_{\Delta}\widehat{\Delta}_t+u_{yA}\widehat{Y}_t\widehat{A}_t\right)+t.i.p+o\left(\left\|\varepsilon\right\|^3\right) \qquad \text{(E.37)}
\end{aligned}
$$

By iterating forward equation ( ) we obtain the following equation that is useful for eliminating $\widehat{\Delta}_t$ from the previous condition,

$$
\sum_{t=0}^{t=\infty}\beta^t\widehat{\Delta}_t \;=\; \theta\widehat{\Delta}_{t-1}+\frac{\theta\varepsilon}{1-\theta}\frac{\pi_{H,t}^2}{2}+\beta\left(\theta\widehat{\Delta}_t+\frac{\theta\varepsilon}{1-\theta}\frac{\pi_{H,t+1}^2}{2}\right)+ \qquad \text{(E.38)}
$$

$$
\beta^2\left(\theta\widehat{\Delta}_{t+1}+\frac{\theta\varepsilon}{1-\theta}\frac{\pi_{H,t+2}^2}{2}\right).... \qquad \text{(E.39)}
$$

Simplifying this expression we obtain:

$$
\sum_{t=0}^{t=\infty}\beta^t\widehat{\Delta}_t=\frac{\theta\varepsilon}{\left(1-\theta\right)\left(1-\beta\theta\right)}\sum_{t=0}^{t=\infty}\beta^t\frac{\pi_{H,t}^2}{2} \qquad \text{(E.40)}
$$

Therefore, second order approximated welfare function can be written as follows:

$$
-\overline{v}_Y\overline{Y}\sum_{t=0}^{t=\infty}\beta^t\left(+\frac{1}{2}u_{yy}\widehat{Y}_t^2+u_{\pi}\frac{\pi_t^2}{2}+u_{\Delta}\widehat{Y}_t\widehat{A}_t\right) \qquad \text{(E.41)}
$$

$$
-\overline{v}_y\overline{Y}\omega\sigma_{\Upsilon}\left(1-s\right)\sum_{t=0}^{t=\infty}\beta^t\left(\lambda_{\upsilon}i_t+\left(\frac{1-\sigma_{\Upsilon}}{2}\right)\omega(1-s)i_t^2+\left(1-\sigma_{\Upsilon}\right)\omega s i_t i_t^*\right) \qquad \text{(E.42)}
$$

Using the preferences functions defined in the text obtain the following set of parameters:

$$
u_{yy}=\left(1+\varphi\right) \qquad \text{(E.43)}
$$

$$
u_{\Delta}=1 \qquad \text{(E.44)}
$$

$$
u_{yA}=-\left(1+\varphi\right) \qquad \text{(E.45)}
$$

we have that:

$$
u_{\pi}=\frac{\theta\varepsilon u_{yA}}{\left(1-\theta\right)\left(1-\beta\theta\right)}=\frac{\theta\varepsilon}{\left(1-\theta\right)\left(1-\beta\theta\right)}=\frac{\varepsilon}{\lambda} \qquad \text{(E.46)}
$$

Rewriting appropriately the quadratic terms we have:

$$\frac{1}{2}u_{yy}\widehat{Y}_t^2 + u_\Delta \widehat{Y}_t\widehat{A}_t = \frac{1}{2}\left((1+\varphi)\left(\widehat{Y}_t^2 - 2\widehat{Y}_t\widehat{A}_t + \widehat{A}_t^2\right)\right) \tag{E.47}$$

since we have eliminated all the distortions of the steady-state equilibrium, the quadratic terms of the approximated lost function of the central bank can be written as follows:

$$\frac{1}{2}u_{yy}\widehat{Y}_t^2 + u_\Delta \widehat{Y}_t\widehat{A}_t = \frac{1}{2}(1+\varphi)\widehat{x}_t^2$$

where: $\widehat{x}_t = \widehat{Y}_t - \widehat{Y}_t^e$, and $\widehat{Y}_t^e$ represent the efficient level of output. Therefore, the lost function for a central bank in an economy with currency substitution is given by:

$$-\frac{\varepsilon}{2\lambda}\overline{V}_h\overline{Y}\sum_{t=0}^{t=\infty}\beta^t\left(\frac{(1+\varphi)\lambda}{\varepsilon}\widehat{x}_t^2 + \pi_{H,t}^2\right) \tag{E.48}$$

$$-\frac{\lambda}{2\varepsilon}\overline{V}_h\overline{Y}\sigma_\Upsilon\omega(1-s)\sum_{t=0}^{t=\infty}\beta^t\left[2\lambda_\upsilon i_t + (1-\sigma_\Upsilon)\omega(1-s)i_t^2 + 2(1-\sigma_\Upsilon)\omega s i_t i_t^*\right] \tag{E.49}$$

denoting by $\Omega = \frac{\varepsilon}{\lambda}\overline{V}_h\overline{Y}$, and by $\Lambda_i = \frac{2\lambda\sigma_\Upsilon\omega\lambda_\upsilon}{\varepsilon}$, $\Lambda_{ii} = \frac{2\lambda\sigma_\Upsilon(1-\sigma_\Upsilon)\omega^2(1-s)}{\varepsilon}$, $\Lambda_{ii^*} = \frac{\lambda(1-\sigma_\Upsilon)\omega^2}{\varepsilon}$ and $\Lambda = \frac{(1+\varphi)\lambda}{\varepsilon}$, we define the following lost function for the central bank

$$-\frac{\Omega}{2}\sum_{t=0}^{t=\infty}\beta^t\left[(1-s)\left(\Lambda_i i_t + \Lambda_{ii}i_t^2 + s\Lambda_{ii^*}i_t i_t^*\right) + \Lambda x_t + \pi_{H,t}^2\right] \tag{E.50}$$

This last equation corresponds to equation (4.1) in section 4.