TANK meets ELB: Gains From Wage Flexibility

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Abstract

We evaluate the welfare implications of wage flexibility in a two-agent new Keynesian model in which monetary policy is occasionally limited by the effective lower bound on nominal interest rates. The model incorporates sticky wages and prices, yet it is tractable enough that finding a global solution to the non-linear equilibrium conditions is feasible. Thus, we are able to provide accurate calculations of the model dynamics and welfare, as well as different measures associated with the effective lower bound's frequency. We find that wage flexibility amplifies the welfare cost when monetary policy responses are restricted by the effective lower bound. In fact, gains from higher wage flexibility, such as output gap stability, are far outweighed by the welfare cost induced by the rise in the volatility of prices and wages. Moreover, welfare loss does not disappear when the effective lower bound regime ends. Instead, it systematically stays creating long-run inefficiencies. The latter finding has been ignored in the literature, yielding systematic understatements of the welfare cost if the effective lower bound is a policy restriction.

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1 Introduction

There is a widespread belief that wage flexibility is a desirable feature for an economy. As Hall (2005) and Shimer (2005, 2012) discuss, wage flexibility acts as a macroeconomic stabilizer because it potentially offsets to some degree, the undesirable effects of adverse shocks. Conversely, wage rigidity tends to amplify the negative effects of these shocks and, as Blanchard and Galí (2007, 2010) explain, it also magnifies policy trade-offs leading to higher welfare costs.

However, in models where demand plays a more relevant role, macroeconomic stability depends more on the management of aggregated demand than wage flexibility. This idea was discussed in Keynes (1936) General Theory. More recently, Galí (2013) and Galí and Monacelli (2016), in a New Keynesian (NK) framework, argued that gains from wage flexibility are strongly tied to the behavior of monetary policy. Precisely, if the Central Bank (CB) follows a rule that calls for aggressive responses to inflation (or if optimal policy with commitment can be implemented), increased wage flexibility tends to improve welfare. In contrast, if the response to inflation is weak or limited, the benefits from increased wage flexibility (such as a more stable output gap) will be small compared to the welfare losses from increased volatility of price and wage inflation.

Another related branch in the NK literature introduces 'rule-of-thumb' households. Such households were introduced with the spirit of matching empirical estimates of fiscal multipliers. For instance, Amato and Laubach (2003) and Gali, Gertler and Lopez-Salido (2007) show that one method of improving the size of fiscal multipliers embedded into the NK model is to introduce agents whose consumption is directly determined by their income (hand-to-mouth households). In a later work, Kaplan et al. (2015) explain that the benefits of adding this type of restricted household to a model goes beyond gaining accuracy regarding fiscal policy–it also improves the model's ability to explain monetary phenomena. Specifically, Kaplan et al. (2015) show evidence suggesting that traditional NK models understate the relevancy of the direct demand channel for the monetary transmission mechanism; this problem is overcome if hand-to-mouth households are added the model.

In this paper, we investigate the extent to which high wage flexibility affects economic stability and welfare if monetary policy is occasionally restricted by the effective lower bound (ELB). To do this, we extend the representative agent new Keynesian (RANK) model in Erceg et al. (2000) to a two agent new Keynesian (TANK) model and approximate the nonlinear solution when the policy rule is truncated at the ELB. In our model, unrestricted families (the representative agent in Erceg et al. (2000)) coexist with restricted households (hand-to-mouth as in Gali, Gertler and Lopez-Salido (2007)). Both families supply labor, but only unrestricted households can access financial markets and benefit from firms' profits. As a result, restricted households consume only their labor income, thus, providing a more central role to the dynamics of aggregated demand. Besides, the ELB acts as a natural limitation in the CB's ability to manage the cycle, yielding a model that is particularly susceptible to the volatility of price and wage inflation.

The paper makes three contributions to the literature. First, we build a tractable, yet relevant, model. Its relevance derives from the introduction of some degree of heterogeneity among agents, yielding a more precise description of the effects and limitations of monetary policy. In this line, Ravn and Sterk (2016) shows that a TANK model, like the one in this paper, can capture some of the main highlights of a full HANK model, as in Kaplan et al. (2014, 2015), but in a more parsimonious set-up. The model was built to maintain tractability while remaining consistent with traditional linearized TANK models.[1]

Second, we approximate the model's full non-linear solution, which is uncommon in the ELB literature. Specifically, we follow Fernández-Villaverde et al. (2012) and implement the Smolyak algorithm to approximate the global solution. One advantage of having the non-linear solution is its superior accuracy relative to any local lineal solution method.[2] But, perhaps the most important benefit of the non-linear solution is that it allows us to keep the entry to, duration of, and exit from the ELB as endogenous events. Hence, we can numerically approximate the likelihood of such events and the effects of having restricted households or highly flexible wages.

Third, we use an explicit approximation of the solution for welfare loss, which is useful for evaluating the effects of having either more flexible wages or restricted households in the economy. There are two common approaches to measuring welfare loss in a DSGE model. The first is a second-order approximation to welfare as in Colciago (2011), Walsh (2014), Debortoli et al. (2017), and in the chapter one of this dissertation within the TANK framework.[3] The second is, when the second-order approximation is not feasible, by calculating the additional units of consumption required to equate the unconditional expected welfare to its value at the steady-state, as in Galí and Monacelli (2016). The approach taken here represents a novel, easy-to-implement alternative that has the advantage of inheriting the accurate properties of the global solution.

When linearized, our model is similar to Colciago (2011), which is a TANK sticky-price, sticky-wage model with perfect risk-sharing of consumption and where all agents

[1]For instance, price and wage rigidity is modeled with adjustment cost à la Rotemberg (1982), yielding a nonlinear system with only two endogenous state variables. If, instead, we followed the more conventional sticky-price, sticky-wage setting à la Calvo (1983), the equivalent non-linear system would have at least three more endogenous state variables.

[2]Levintal (2018) evaluates accuracy in the NK framework, although not for the ELB. He shows that in models with long-run risk and recursive preferences, the Smolyak algorithm provides more accurate solutions when compared against perturbation solution of orders one to five.

[3]For more traditional presentations of second-order approximation in RANK models, see Woodford (2003), Galí (2015), and Walsh (2017*a*)

supply labor.[4] The two types of households in our model behave as two big families in Ortigueira and Siassi (2013) with intra-family transfers to hedge fully against idiosyncratic labor income risk.[5] Consequently, our model lacks the cross sectional dispersion of labor income that is the focus in the first chapter of this dissertation. Our model embeds standard TANK features, such as having a response to income distribution in the aggregated Euler equation, as in Amato and Laubach (2003), Galí, López-Salido and Vallés (2007), Colciago (2011) and **?**. Unlike these traditional models, the aggregated wage Phillip curve responds to the distribution of labor income as in the first chapter of this dissertation. This effect is absent in Colciago (2011) who assumed all agents provide all types of labor, receiving the same labor income regardless of the group to which they belong. However, in our paper, and in the first chapter of this dissertation, individuals set wages based in their marginal rate of substitution; hence, the aggregate will display systematic differences between wages of restricted and unrestricted households.

Given our interest in understanding the dynamics under ELB, our model is closely related to Billi and Galí (2018).[6] Employing a RANK framework, Billi and Galí (2018) assume a linear approximation in either regime (ELB or normal times) provides an adequate approximation of the model's dynamics.[7] This assumption yields tractability to the extent that they can easily evaluate optimal policy under commitment. However, this approach has limitations when inferring the likelihood and duration of ELB instances. As shown in Fernández-Villaverde et al. (2012), accounting for a full non-linear solution allow us to calculate relatively accurate measures related to the frequency of the ELB. Unlike Billi and Galí (2018), we focus on a TANK model where monetary policy is conducted through a Taylor rule, thus we provide a more central role to direct demand channels and do not investigate optimal policy. However, we calculate the non-linear solution of the model and get relatively accurate calculations for the effects of wage flexibility in all likelihoods related to the ELB's frequency. From this exercise, we find that higher wage flexibility increases the likelihood and duration of the ELB, yielding larger welfare costs. This suggests that welfare loss could be understated in Billi and Galí (2018).

The rest of the paper is organized as follows. Details of the model are displayed

[4]Examples with a sharper differentiation between restricted and unrestricted households are Walsh (2017*b*) and Broer et al. (2016). In those papers, only restricted households (or workers) supply labor while unrestricted households (or capitalist) consume only firms' profits.

[5]Similar assumptions are implicitly taken in Christiano et al. (1999) and **?**.

[6]Our paper is also related to recent contributions to ELB literature along different dimensions, including: optimal monetary policy (e.g., Jung et al. (2005), Adam and Billi (2007), Nakata (2016)), forward guidance (e.g., Eggertsson and Woodford (2003)), multiple steady states (e.g. Benhabib et al. (2001, 2002), Mertens and Ravn (2014), Nakata (2017)), fiscal policy effectiveness (Christiano et al. (2011),Eggertsson (2011)), among others.

[7]Specifically, their solution method is equivalent to a two-regime first-order perturbation, where one of the regimes corresponds to the ELB.

in section (2). The solution method is presented and discussed in Section (3). In Section (4), we present the calibration of the model. In Section (5), we perform the quantitative analysis related to: (i) dynamic properties, (ii) steady-state, and (iii) likelihoods related to the ELB's frequency. Finally, we summarize and present our conclusions in Section (6).

2 A non-linear TANK

The economy is composed of two household types (restricted and unrestricted), final goods-producing firms, a labor packer, a central bank (CB), and a fiscal authority. Restricted households are equivalent to non-Ricardian households in Galí, López-Salido and Vallés (2007) or hand-to-mouth households in Colciago (2011)—that is, they do not benefit from firms profits nor have access to financial markets; hence, they fund their consumption through labor income.

Unrestricted households own all firms and smooth consumption by purchasing bonds in financial markets. Both household types face costs of adjusting wages à la Rotemberg (1982), which induces inefficient variations of wage inflation. A natural by-product of the Rotemberg wage setting is that, within type, households completely share idiosyncratic consumption risk. This is equivalent to assuming there are two families (restricted and unrestricted) that pool labor income and, hence, achieve the same level of consumption within family.

Final goods-producing firms are standard in the NK literature. They face a price adjustment cost à la Rotemberg (1982) and a common productivity shock. Firms hire homogeneous units of labor from a centralized labor market where labor is supplied to them by labor packers. Labor packers build composite units of labor by aggregating individual units of labor provided by restricted and unrestricted households. Among the economy's three markets, only the financial market is competitive while the labor and final goods market are characterized by monopolistic competition.

As in Fernández-Villaverde et al. (2012), the CB implements policy through a Taylor rule truncated by the ELB. Finally, the government implements fiscal policy aimed at maximizing aggregated welfare at the deterministic steady-state and to eliminating the inefficiencies caused by imperfect competition (as in **?**).

We are interested in studying the model's dynamics when the ELB is occasionally binding. Therefore, we implement only two exogenous shocks: productivity and households' impatience shocks. We abstract from markup shocks (wages or prices), as it is generally believed that they cannot trigger an ELB regime.

2.1 Households

There is a one-unit measure of households indexed by $s \in [0,1]$, where household s is restricted (i.e., of type $\ell(s) = r$) if $s \in [0,n]$, or unrestricted (i.e., of type $\ell(s) = u$) if $s \in (n,1]$. We also assume that households have identical preferences and face the same aggregated shocks. Household s welfare is given by

$$\mathbb{E}_t \sum_{i=0}^{\infty} \left[\prod_{j=t+1}^{t+i} \beta_j \right] \left(\frac{C_{t+i}^{\ell(s)}(s)^{1-\sigma}}{1-\sigma} + \gamma \frac{H_{t+i}^{\ell(s)}(s)^{1+\eta}}{1+\eta} \right), \tag{2.1}$$

where $C^{\ell(s)}(s)$ and $H^{\ell(s)}(s)$ are consumption and hours of household s of type $\ell(s)$. Following Fernández-Villaverde et al. (2012), we assume the discount factor is exogenous and time-varying. The stochastic process followed by the impatience factor is

$$\beta_t = \beta^{1-\rho_b} \beta_{t-1}^{\rho_b} \exp\left(\sigma_b \epsilon_{b,t}\right). \tag{2.2}$$

As in Rotemberg (1982), it is assumed that households will expend resources in updating wages if necessary. It is standard to assume the adjustment cost (in units of aggregated labor) is quadratic; hence, the total and marginal wage adjustment costs are

$$\mathcal{A}_{w,t}(s) = \frac{\phi_w^{rot}}{2} \left(\frac{\Pi_t}{\bar{\Pi}} \frac{W_t^{\ell(s)}(s)}{W_{t-1}^{\ell(s)}(s)} - 1 \right)^2 \text{ and } \mathcal{A}'_{w,t}(s) = \phi_w^{rot} \left(\frac{\Pi_t}{\bar{\Pi}} \frac{W_t^{\ell(s)}(s)}{W_{t-1}^{\ell(s)}(s)} - 1 \right),$$

respectively. Variables, Π and $W^{\ell(s)}(s)$ are the inflation rate and the wage set by household s of type $\ell(s)$. Notice it is assumed there is complete indexation to the inflation target $\bar{\Pi}$. To have an equivalence to the relatively more standard price setting à la Calvo (1983), the Rotemberg coefficient ϕ_w^{rot} should be defined as follows:

$$\phi_w^{rot} \equiv \frac{\phi_w^{cal}(1+\eta\theta_w)\theta_w W_{ss}}{(1-\phi_w^{cal})(1-\phi_w^{cal}\beta)}, \tag{2.3}$$

where ϕ_w^{cal} is the Calvo coefficient that represents the likelihood of not being able to update prices at any period, θ_w is the labor demand elasticity as in Dixit and Stiglitz (1977), and W_{ss} is the non-stochastic steady-state value of aggregated wages.

The budget constraint of restricted household s is

$$C_t^r(s) + T_t^r = (1+\tau_w) W_t^r(s) H_t^r(s) - \mathcal{A}_{w,t}(s) H_t + M_t^r(s).$$

That is, restricted household s finances consumption and lump-sum taxes with labor income $W^r(s)H^r(s)$ after the payroll subsidy τ_w and after discounting the cost of updating wages $\mathcal{A}_w(s)H$. Restricted households trade contingent assets between members of the restricted group $M^r(s)$, allowing them to share risk regarding consumption. This asset-holding nets out when aggregating: $\int_0^n M_t^r(s) = 0$.

As such, restricted household optimal wage schedule is given by

$$\begin{aligned} MRS_t^r =& \frac{(\theta_w - 1)(1+\tau_w)}{\theta_w} W_t^r \\ &+ \frac{1}{\theta_w}\left(\frac{W_t^r}{W_t}\right)^{\theta_w}\left[\mathcal{A}'_{wr,t}\frac{\Pi_t^{w,r}}{\bar{\Pi}} - \mathbb{E}_t SDF^r_{t,t+1}\mathcal{A}'_{wr,t+1}\frac{\Pi_{t+1}^{w,r}}{\bar{\Pi}}\frac{H_{t+1}}{H_t}\right] \end{aligned} \tag{2.4}$$

where MRS^r, SDF^r, $\Pi^{w,r}$ and $\mathcal{A}'_{wr}$ are the restricted household values of the marginal rate of substitution between leisure and consumption, the stochastic discount factor, wage inflation, and marginal wage adjustment cost, respectively. In addition, W^r and W are wages set by restricted households and aggregated wages, respectively. These variables are defined as follows

$$\Pi_t^{w,r} \equiv \Pi_t \frac{W_t^r}{W_{t-1}^r},\ MRS_t^r \equiv \gamma (H_t^r)^\eta (C_t^r)^{-\sigma},\ SDF^r_{t,t+1} \equiv \beta_{t+1}\left[\frac{C_{t+1}^r}{C_t^r}\right]^{-\sigma}$$
$$\mathcal{A}_{wr,t} \equiv \frac{\phi_w^{rot}}{2}\left(\Pi_t^{w,r} - 1\right)^2 \text{ and } \mathcal{A}'_{wr,t} \equiv \phi_w^{rot}\left(\Pi_t^{w,r} - 1\right).$$

Equation (2.4) is the Phillips curve for restricted households' wages. It yields the same implicit dynamics as in the Calvo setting: if restricted households expect future increments of wage inflation, they prefer raising current wages. However, the mechanism is different. In the Rotemberg setting, the incentive to raise current wages is driven by the potential resource saving brought by avoiding a sharp increase in wages in the future. In the Calvo setting, however, the incentive to increase current wages is motivated by the chance that the agent will not be able to adjust wages when required in the future.

On the other hand, the unrestricted households' budget constraint is given by

$$\begin{aligned} C_t^u(s) + B_{t+1}(s) + T_t^u =& (1+\tau_w) W_t^u(s) H_t^u(s) + \frac{R_{t-1}^{nom}}{\Pi_t} B_t(s) \\ &- \mathcal{A}_{w,t}(s) H_t + \mathbb{P}_t - M_t^u(s) \end{aligned}$$

that is, unrestricted household s finance consumption, savings, and lump-sum taxes with labor income after the payroll subsidy $(1+\tau_w)W^u(s)H^u(s)$, savings returns and firms profits $\mathbb{P}$, discounted by the cost of updating wages $\mathcal{A}_w(s)H$. Similar to restricted households, unrestricted households trade contingent assets between members of the unrestricted group $M_t^u(s)$, which allows them to share risk regarding consumption. This asset-holding nets out when aggregating: $\int_n^1 M_t^u(s) = 0$.

Optimality conditions regarding bond holdings and wage settings are

$$
\begin{aligned}
1 =& R_t^{nom}\mathbb{E}_t \frac{SDF_{t,t+1}^u}{\Pi_{t+1}} && (2.5)\\
MRS_t^u =& \frac{(\theta_w - 1)(1+\tau_w)}{\theta_w} W_t^u \\
&+ \frac{1}{\theta_w}\left(\frac{W_t^u}{W_t}\right)^{\theta_w}\left[\mathcal{A}'_{wu,t}\frac{\Pi_t^{w,u}}{\bar{\Pi}} - \mathbb{E}_t SDF_{t,t+1}^u \mathcal{A}'_{wu,t+1}\frac{\Pi_{t+1}^{w,u}}{\bar{\Pi}}\frac{H_{t+1}}{H_t}\right] && (2.6)
\end{aligned}
$$

where MRS^u, SDF^u, $\Pi^{w,u}$ and $\mathcal{A}'_{wu}$ are the unrestricted household values of marginal rate of substitution between leisure and consumption, stochastic discount factor, wage inflation and wage adjustment cost, respectively. These variables are calculated as set out below

$$
\begin{aligned}
&\Pi_t^{w,u} \equiv \Pi_t \frac{W_t^u}{W_{t-1}^u},\ MRS_t^u \equiv \gamma (H_t^u)^\eta (C_t^u)^{-\sigma},\ SDF_{t,t+1}^u \equiv \beta_{t+1}\left[\frac{C_{t+1}^u}{C_t^u}\right]^{-\sigma}\\
&\mathcal{A}_{wu,t} \equiv \frac{\phi_w^{rot}}{2}\left(\frac{\Pi_t^{w,u}}{\bar{\Pi}} - 1\right)^2 \text{ and } \mathcal{A}'_{wu,t} \equiv \phi_w^{rot}\left(\frac{\Pi_t^{w,u}}{\bar{\Pi}} - 1\right)
\end{aligned}
$$

Equation (2.5) is a standard Euler equation, which, given the model's TANK structure, only models the consumption behavior of unrestricted households. Equation (2.6) is the Phillips curve for the wage inflation of unrestricted households, which parallels Equation (2.4) for restricted households.

2.2 Aggregation in the labor market

Labor packers build a composite unit of labor from each household labor supply. The aggregation is made through a CES technology as in Dixit and Stiglitz (1977)

$$
H_t = \left[\int_0^1 H_t^{\ell(s)}(s)^{\frac{\theta_w - 1}{\theta_w}} \mathrm{d}s\right]^{\frac{\theta_w}{\theta_w - 1}}, \tag{2.7}
$$

where $H^{\ell(s)}(s)$ is hours supplied by household $s \in [0,1]$ of type $\ell(s) \in \{r, u\}$. Let's define average hours supplied by restricted (H^r) and unrestricted (H^u) as

$$
H_t^r = \left[\frac{1}{n}\int_0^n H_t^r(s)^{\frac{\theta_{w,t}-1}{\theta_{w,t}}}\mathrm{d}s\right]^{\frac{\theta_{w,t}}{\theta_{w,t}-1}} \text{ and } H_t^u = \left[\frac{1}{1-n}\int_1^n H_t^u(s)^{\frac{\theta_{w,t}-1}{\theta_{w,t}}}\mathrm{d}s\right]^{\frac{\theta_{w,t}}{\theta_{w,t}-1}};
$$

hence, (2.7) can be written as follows:

$$
H_t = \left[n(H_t^r)^{\frac{\theta_{w,t}-1}{\theta_{w,t}}} + (1-n)(H_t^u)^{\frac{\theta_{w,t}-1}{\theta_{w,t}}}\right]^{\frac{\theta_{w,t}}{\theta_{w,t}-1}}. \tag{2.8}
$$

Consequently, the bundle (H_t^r, H_t^u) that minimizes expenditure in $nW_t^r H_t^r + (1-n)W_t^u H_t^u$ subject to (2.8) (i.e., optimal allocation) is given by

$$H_t^r = \left(\frac{W_t^r}{W_t}\right)^{-\theta_{w,t}} H_t \text{ and } H_t^u = \left(\frac{W_t^u}{W_t}\right)^{-\theta_{w,t}} H_t, \tag{2.9}$$

with aggregated wage determined by

$$W_t = [n(W_t^r)^{-(\theta_{w,t}-1)} + (1-n)(W_t^u)^{-(\theta_{w,t}-1)}]^{-\frac{1}{\theta_{w,t}-1}}. \tag{2.10}$$

2.3 Firms

There is a one-unit mass of retail firms that maximize the discounted present value of profits. Firms hire labor at a given wage and face a Rotemberg type of price adjustment cost. All firms face two restrictions: technology and oriented demand. Technology is given by the following production function

$$Y_t(j) = Z_t H_t(j)^{1-a}, \tag{2.11}$$

where $Y(j)$ and $H(j)$ are firm j supply of output and demand for labor. The variable Z is composed by productivity and the fixed amount of capital. The stochastic process of productivity is

$$Z_t = Z_{ss}^{1-\rho_z} Z_{t-1}^{\rho_z} \exp\left(\sigma_z \epsilon_{z,t}\right); \tag{2.12}$$

The second restriction is given by the demand oriented to variety j

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta_p} Y_t \tag{2.13}$$

All firms face a price-adjusting cost à la Rotemberg (1982), which is specified by the following functions

$$\mathcal{A}_{p,t}(j) = \frac{\phi_p^{rot}}{2}\left(\frac{P_t(j)}{\bar{\Pi}P_{t-1}(j)} - 1\right)^2 \text{ and } \mathcal{A}'_{p,t}(j) = \phi_p^{rot}\left(\frac{P_t(j)}{\bar{\Pi}P_{t-1}(j)} - 1\right)$$

where, $P(j)$ is the price set by firm j. To keep consistency with the Calvo setting, the Rotemberg coefficient ϕ_p^{rot} should be defined as follows:

$$\phi_p^{rot} = \frac{\phi_p^{cal}(1+\xi_p)(\theta_p - 1)}{(1-\phi_p^{cal})(1-\beta\phi_p^{cal})} \text{ with } \xi_p = \frac{a\theta_p}{1-a}. \tag{2.14}$$

Therefore, firms' optimal price setting is given by

$$RMC_t = \frac{\theta_p - 1}{\theta_p} - \frac{1}{\theta_p}\left[\mathbb{E}_t SDF_{t,t+1}^u \mathcal{A}'_{p,t+1}\frac{\Pi_{t+1}}{\bar{\Pi}}\frac{Y_{t+1}}{Y_t} - \mathcal{A}'_{p,t}\frac{\Pi_t}{\bar{\Pi}}\right], \tag{2.15}$$

where RMC is the real marginal cost of production and

$$RMC_t \equiv \frac{1-\tau_p}{1-a}\frac{W_t}{Z_t^{\frac{1}{1-a}}Y_t^{-\frac{a}{1-a}}},\ H_t \equiv Z_t^{-\frac{1}{1-a}}Y_t^{\frac{1}{1-a}},$$

$$\mathcal{A}_{p,t} \equiv \frac{\phi_p^{rot}}{2}\left(\frac{\Pi_t}{\bar{\Pi}}-1\right)^2 \text{ and } \mathcal{A}'_{p,t} \equiv \phi_p^{rot}\left(\frac{\Pi_t}{\bar{\Pi}}-1\right)$$

Equation (2.15) is the Phillips curve for the inflation rate. Notice that, similar to the case of both Phillips curves for wages, the Phillips curve for prices yields a similar dynamic as that of a Calvo set-up. That is, if future marginal costs are expected to increase (indirectly determined by future production), then current inflation should rise. In our set-up, the incentive to raise current inflation responds to the potential saving in adjustment cost of avoiding a future large rise in inflation. However, in the Calvo set-up, the incentive to increment current inflation responds to the potential firms' inability to adjust prices when necessary.

2.4 Fiscal and monetary policy

The aim of fiscal policy is to eliminate the inefficiencies induced by imperfect competition and households' heterogeneity. Two different fiscal instruments are assumed: aggregate transfers and subsidies.

Motivated by the outcome of a hypothetical central planner, the fiscal authority will attempt to achieve the maximum aggregated welfare by equating the steady-state level of the marginal utility of consumption of both household types (as in **?**). With additively separable utility, this is achieved by choosing transfers $\mathcal{T}^r$ and $\mathcal{T}^u$, such that consumption of either household type are equal at the steady-state.

From the steady-state consumption of restricted households, we know that $C_{ss}^r = \psi Y_{ss} + \mathcal{T}^r$, where $\psi \equiv \frac{W_{ss}^r H_{ss}^r}{C_{ss}^r}$. Likewise, from the steady-state of the market-clearing condition (to be explained in Section (2.5), see Equation (2.22)), steady-state of unrestricted households is given by $C_{ss}^u = \frac{1-n\psi}{1-n}Y_{ss} - \frac{n}{1-n}\mathcal{T}^r$. Hence, after setting $C_{ss}^r = C_{ss}^u = Y_{ss}$ and $n\mathcal{T}^r + (1-n)\mathcal{T}^u = 0$, we obtain

$$\mathcal{T}^r = (1-\psi)Y_{ss} \text{ and } \mathcal{T}^u = -\frac{n(1-\psi)}{1-n}Y_{ss}. \tag{2.16}$$

To tackle the inefficiencies of imperfect competition, the government subsidizes production to eliminate all markups (as in Woodford (1999), Erceg et al. (2000), **?** and Walsh (2017*a*)). To minimize the government's influence on aggregated demand, these subsidies are funded by charging lump-sum taxes to specific groups of agents that benefit from the subsidy (as in **?**).

These subsidy rates are τ_w and τ_p. We define lump-sum taxes as t_t^r and t_t^u, which are expressed as a proportion of steady-state output. Then, the lump-sum taxes charged to restricted households ($Y_{ss}t_t^r$) and to unrestricted households ($Y_{ss}t_t^u$) are

$$Y_{ss}t_t^r = \tau_w W_t^r H_t^r \text{ and } Y_{ss}t_t^u = \tau_w W_t^u H_t^u + \frac{\tau_p}{1-n} W_t H_t.$$

Restricted households are taxed only by the average amount of the subsidy received when signing a labor contract. Unrestricted households, however, experience an extra charge deriving from the average subsidies to goods-producing firms.

Define T^r and T^u as the taxes charged every period to restricted and unrestricted households, respectively. These are determined by summing all taxes and subsidies described above

$$\begin{aligned} T_t^r =& \tau_w W_t^r H_t^r - (1-\psi) Y_{ss} \\ T_t^u =& \tau_w W_t^u H_t^u + \frac{\tau_p}{1-n} W_t H_t + \frac{n(1-\psi)}{1-n} Y_{ss} \end{aligned} \tag{2.17}$$

We assume the CB implements policy through a truncated Taylor rule as follows

$$R_t^{mp} = \frac{\bar{\Pi}}{\beta} \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\varphi_\pi} \left(\frac{Y_t}{Y_{f,t}} \right)^{\varphi_x}, \tag{2.18}$$

$$R_t^{nom} = \max\{R_t^{mp}, \underline{R}\} \tag{2.19}$$

where R^{mp} is the interest rate suggested by the Taylor rule while R^{nom} is the nominal interest rate in the market. The Taylor rule commands deviations from the steady-state interest rate (given by $\bar{\Pi}/\beta$) in response to deviations of the inflation rate and output relative to the inflation target and flexible output (Y_f), respectively. Flexible output is the output level that would prevail with flexible prices and wages (i.e., if $\phi_p^{rot} = \phi_w^{rot} = 0$). The interest rate that affects the bonds market is R^{nom}, which is restricted by the ELB denoted by $\underline{R}$.

2.5 Aggregation and market clearing conditions

The aggregation of the budget constraints of unrestricted households (with $\int_n^1 B_t(s)ds = \int_n^1 B_{t-1}(s)ds = \int_n^1 M_t^u(s)ds = 0$) yields

$$C_t^u + T_t^u = (1+\tau_w) W_t^u H_t^u - \mathcal{A}_{wu,t} H_t + \frac{1}{1-n} \mathbb{P}_t.$$

Given that firms' aggregated profits are $\mathbb{P}_t = (1-\mathcal{A}_{p,t})Y_t - (1-\tau_p)W_t H_t$, the above expression becomes

$$C_t^u = (1+\tau_w) W_t^u H_t^u - \mathcal{A}_t^u H_t + \frac{1}{1-n}((1-\mathcal{A}_{p,t})Y_t - (1-\tau_t^p)W_t H_t) - T_t^u. \tag{2.20}$$

Similarly, aggregated restricted households' budget constraints, with $\int_0^n M_t^r(s)ds = 0$, is

$$C_t^r = (1+\tau_w)W_t^r H_t^r - \mathcal{A}_{wr,t}H_t - T_t^r \tag{2.21}$$

As a result, replacing (2.17) in (2.20) and (2.21), and calculating aggregated consumption as $C_t \equiv nC_t^r + (1-n)C_t^u$ yields the market-clearing condition

$$(1-\mathcal{A}_{p,t})Y_t = C_t + (n\mathcal{A}_{wr,t} + (1-n)\mathcal{A}_{wu,t})H_t. \tag{2.22}$$

That is, aggregated supply Y_t matches aggregated demand for consumption C_t after spending resources in price and wage adjustment costs. Finally, by replacing (2.17) in (2.21), we obtain an explicit expression for restricted households' consumption after the fiscal intervention:

$$C_t^r = (1-\psi)Y + W_t^r H_t^r - \mathcal{A}_{wr,t}H_t, \tag{2.23}$$

i.e., the average restricted household finances expenditure in consumption and wage adjustment with labor income and with the fiscal maximizing transfer.

2.6 Welfare loss

Society's welfare ($\mathbb{W}$) can be calculated by aggregating Equation (2.1) over both type of households

$$\mathbb{W}_t = \mathbb{E}_t \sum_{i=0}^{\infty} \left[\prod_{j=t+1}^{t+i} \beta_j \right] \int_0^1 \left(\frac{C_{t+i}^{\ell(s)}(s)^{1-\sigma}}{1-\sigma} + \gamma \frac{H_{t+i}^{\ell(s)}(s)^{1+\eta}}{1+\eta} \right) \mathrm{d}i.$$

Given perfect income risk sharing, consumption and labor income are common across households within group; hence, $(C^{\ell(s)}(s), H^{\ell(s)}(s)) = (C^r, H^r)$ if $s \in [0, n]$ and $(C^{\ell(s)}(s), H^{\ell(s)}(s)) = (C^u, H^u)$ if $s \in (n, 1]$. Consequently, distributing the integral in the equation above and writing it recursively yields

$$\begin{aligned} \mathbb{W}_t =& \frac{n(C_t^r)^{1-\sigma} + (1-n)(C_t^u)^{1-\sigma}}{1-\sigma} - \gamma \frac{n(H_t^r)^{1+\eta} + (1-n)(H_t^u)^{1+\eta}}{1+\eta} \\ &+ \mathbb{E}_t \beta_{t+1} \mathbb{W}_{t+1}. \end{aligned}$$

Fiscal policy is set up to eliminate inefficiencies derived from household heterogeneity and imperfect competition. Therefore, nominal stickiness is the only reason for the economy to not be Pareto optimal. Define efficient welfare ($\mathbb{W}_e$) as the level of welfare that would prevail under jointly flexible prices and wages. Given that $\mathbb{W}_e$

corresponds to a Pareto optimal version of our economy, the second fundamental theorem of welfare applies to it. Consequently, $\mathbb{W}_e \geq \mathbb{W}$ and society's welfare loss can be calculated with

$$\mathbb{L}_t = \mathbb{W}_{e,t} - \mathbb{W}_t \geq 0. \tag{2.24}$$

In the first chapter of this dissertation an explicit second-order approximation of an expression equivalent to (2.24) is calculated. In this approximation, it is shown that, in addition to the regular determinants of welfare loss (i.e., deviations of output gap and inflation rates), variations of consumption and wage inequality also contribute to welfare loss. They also show that this result remains even if perfect income risk sharing is assumed. Consequently, it will be instrumental to define consumption and wage inequality (of unrestricted relative to restricted households) as

$$c_t^u - c_t^r = \log \frac{C_t^u}{C_t^r} \text{ and } w_t^u - w_t^r = \log \frac{W_t^u}{W_t^r}, \tag{2.25}$$

respectively.

3 Solution method

For this section, we adopt the notation introduced by Judd (1996), where variables are categorized as follows: innovations ($\mathbf{u}$), jumping endogenous ($\mathbf{y}$), and state ($\mathbf{x}$) variables, and the latter can be further decomposed in endogenous ($\mathbf{x}_1$) and exogenous ($\mathbf{x}_2$) states. For the model to be as parsimonious as possible, we group all other variables not categorized in Judd (1996) as definitions ($\mathbf{z}$) that can be calculated with $\mathbf{z} = k(\mathbf{y}, \mathbf{x})$ where $k(\cdot)$ is known. In Appendix (A), we display the system of equations that determines the equilibrium for the jumping variables $\mathbf{y} := \{\log Y_{f,t},\ \log W_{f,t}^r,\ \log Y_t,\ \log \Pi_t,\ \log W_t^r,\ \log W_t^u\}$, endogenous states $\mathbf{x}_1 := \{\log W_{t-1}^r,\ \log W_{t-1}^u\}$, and exogenous states $\mathbf{x}_2 := \{\log \beta_t,\ \log Z_t\}$.

Using this notation the model can be written as

$$\mathbb{E}_{\mathbf{u}} f(\mathbf{y}', \mathbf{x}', \mathbf{y}, \mathbf{x}, \mathbf{u}') = 0 \text{ where } \mathbf{x}_2' = h_2(\mathbf{x}_2, \mathbf{u}') \text{ with } f(\cdot) \text{ and } h_2(\cdot) \text{ known.} \tag{3.1}$$

where $\mathbf{u}'$ is a vector of stochastic i.i.d. innovations. In Equation (3.1), we use apostrophes as shortcut notation for the lead operator (i.e., if $x = z_t$ then $x' = z_{t+1}$). The function $f(\cdot)$ is given by equations (A.4-A.11) displayed in Appendix (A) while $h_2(\cdot)$ is given by equations (A.12) and (A.13) in the same appendix.

The solution to (3.1) is given by functions $g(\cdot)$ and $h_1(\cdot)$ such that

$$\mathbf{y} = g(\mathbf{x}) \text{ and } \mathbf{x}_1' = h_1(\mathbf{x}) \tag{3.2}$$

We follow Fernández-Villaverde et al. (2012) and implement the Smolyak algorithm to find an approximation to the functions $g(\cdot)$ and $h_1(\cdot)$. To accomplish this, we stack variables $\mathbf{y}$ and $\mathbf{x}_1'$ in a single vector $\mathbf{Y} = [\mathbf{y}^T,\ (\mathbf{x}_1')^T]^T$. As a result, the function to approximate is $G(\cdot) = [g(\cdot)^T,\ h_1(\cdot)^T]^T$.

As in Judd et al. (2014), we approximate $G(\cdot)$ as a basis of orthogonal Chebychev polynomial $\hat{G}(\cdot|\hat{\Phi}) = [\hat{g}(\cdot|\hat{\Phi}_g)^T\ \hat{h}_1(\cdot|\hat{\Phi}_h)^T]^T$, where the coefficients to weight the basis $\hat{\Phi} = [\hat{\Phi}_g^T\ \hat{\Phi}_h^T]^T$ are such that

$$\mathbb{E}_{\mathbf{u}} f\left(\hat{g}\left(\begin{bmatrix}\hat{h}_1(\mathbf{x}|\hat{\Phi}_h)\\ h_2(\mathbf{x}_2,\mathbf{u}')\end{bmatrix}\middle|\hat{\Phi}_g\right), \begin{bmatrix}\hat{h}_1(\mathbf{x}|\hat{\Phi}_h)\\ h_2(\mathbf{x}_2,\mathbf{u}')\end{bmatrix}, \hat{g}(\mathbf{x}|\hat{\Phi}_g), \mathbf{x}, \mathbf{u}'\right) \to 0 \tag{3.3}$$

at all nodes of an isotropic Smolyak grid of $\mathbf{x}$ as described in Judd et al. (2014). Where expectations are calculated with a nine-weighted-nodes quadrature approximation, as suggested by Judd et al. (2011).

We have four state variables ($\#\mathbf{x} = 4$) and we implement a third-order Smolyak grid; therefore, there are 137 nodes in the hypercube of state variables. We approximate eight functions contained in $\hat{G}(\cdot|\hat{\Phi})$, i.e., one function per variable within $\mathbf{y}$ and $\mathbf{x}_1$. As a result, $\hat{\Phi} \in \mathbb{R}^{8\times 137}$ and Equation (3.3) require the calculation of 1096 coefficients.

4 Calibration

We calibrate the model with standard values in the literature. We set $\beta = 0.995$ to match a real interest rate at the steady-state of 2% on an annual basis, $\eta = 4$ to set the Frisch elasticity at 25%, and $\sigma = 1$ (i.e., log-utility), which is standard in the literature. As in Gali, Gertler and Lopez-Salido (2007), the proportion of restricted households is set at $n = 0.5$. The Cobb-Douglas coefficient is calibrated at $a = 0.25$, consistent with Colciago (2011).

We set $\bar{\Pi} = 1.005$, which is equivalent to a 2% inflation rate target on an annual basis. Rotemberg coefficients are set at $\phi_p^{rot} = 372.82$ and $\phi_w^{rot} = 3990.29$ for the baseline calibration, which is consistent with Calvo coefficients of $\phi_p^{cal} = \phi_w^{cal} = 0.75$ as in Erceg et al. (2000). This equivalence is given by Equations (2.14) and (2.3). The elasticity of substitution among good and labor varieties is set at $\theta_p = \theta_w = 6$, which implies steady-state markups of 12.5% in the goods and labor market.

Coefficients in the Taylor rule are conventional, $\varphi_\pi = 1.5$ and $\varphi_x = 0.25$. The ELB is set at $\underline{R} = 1$, which is a zero lower bound of the interest rate. We set $Z_{ss} = 2.28$ and $\gamma = 546.75$, such that the steady-state values of output and hours are $Y_{ss} = 1$ and $H_{ss} = 1/3$. In this set-up, fiscal policy is set to get a steady-state of $C_{ss}^r = C_{ss}^u = 1$, $H_{ss}^r = H_{ss}^u = 1/3$ and $W_{ss}^r = W_{ss}^u = W_{ss} = 2.25$; hence, $\psi = \frac{W_{ss}^r H_{ss}^r}{C_{ss}^r} = 1 - a = 0.75$.

The stochastic processes' persistence are set at $\rho_b = \rho_z = 0.90$ and the innovations standard deviations at $\sigma_b = \sigma_z = 0.0025$. The TFP shock calibrated variance is smaller than typical values in the literature; however, this reflects a lower volatility that is consistent with the Great Moderation. A similar calibration of the TFP shock is used in Fernández-Villaverde et al. (2012).

As part of the quantitative analysis, we evaluate the change in the dynamics and ergodic steady-state under different levels of nominal wage rigidity. The different alternative calibrations of the Rotemberg coefficient considered correlates with the easier-to-interpret Calvo coefficient as depicted in Table (1).

	More Flex.	Baseline	Stickier
ϕ_w^{cal}	0.65	0.75	0.85
ϕ_w^{rot}	1774.38	3990.29	12399.56

Table 1: Different calibrations of ϕ_w^{rot}

5 Quantitative evaluation

In this section, we discuss the model's dynamic properties and the instances in which the ELB is binding. We also investigate numerically how wage flexibility and the inclusion of restricted households affect the likelihood of reaching and leaving the ELB, as well as the properties of the long-run equilibrium.

5.1 Ergodic steady-state

The non-linearity introduced by the ELB creates a significant difference between the deterministic steady-state (DSS) and the ergodic steady-state (ESS).[8] We calculate the ESS as a fixed-point problem once the solution in Equation (3.2) is approximated. Algorithm (1) describes the process for computing the ESS.

Algorithm 1 (Pseudo code to calculate the ESS) Given that all exogenous state variables $\mathbf{x}_2$ follow linear stochastic processes, their DSS and ESS coincide. Denote the fixed point for exogenous state variables as $\mathbf{x}_{2,ss}$, then the recursion to obtain the ESS of the model follows:

1. Initialize the vector of endogenous state variables $\mathbf{x}_{1,ess}^{(0)}$ and build the vector of state variables $\mathbf{x}_{ess}^{(0)} = [(\mathbf{x}_{1,ess}^{(0)})^T \ \mathbf{x}_{2,ss}^T]^T$.

[8] The DSS, also known as the non-stochastic steady-state, is the resting point of the economy when all shocks in the model lack their stochastic components. On the other hand, the ESS, also referred to as the risky steady-state (see Coeurdacier et al. (2011)), is the resting point of the economy when the distribution of the shocks are considered.

2. Update the vector of state variables $\mathbf{x}_{ess}^{(i)}$ with the approximated solution $\hat{h}_1(\cdot)$: calculate $\mathbf{x}_{1,ess}^{(i)} = \hat{h}_1(\mathbf{x}_{ess}^{(i-1)}|\hat{\Phi}_h)$ and build $\mathbf{x}_{ess}^{(i)} = [(\mathbf{x}_{1,ess}^{(i)})^T \ \mathbf{x}_{2,ss}^T]^T$.

3. If $||\mathbf{x}_{ess}^{(i)} - \mathbf{x}_{ess}^{(i-1)}|| < \epsilon$, go to the next step. Otherwise repeat step 2 until this condition is met.

4. Calculate the ESS of all jumping endogenous variables $\mathbf{y}_{ess}$ and definitions $\mathbf{z}_{ess}$ according to $\mathbf{y}_{ess} = \hat{g}(\mathbf{x}_{ess}|\hat{\Phi}_g)$ and $\mathbf{z}_{ess} = k(\mathbf{y}_{ess}, \mathbf{x}_{ess})$.

Table (2) displays a summary of the ESS under different degrees of wage stickiness in the TANK ($n = 0.5$) and RANK ($n = 0$) model. The likelihood of reaching the ELB is also reported in the last two rows. The calculation of this is explained below in Algorithm (2). From Table (2) we claim our first result:

Result 1 (ESS and the likelihood of reaching the ELB) Greater wage flexibility or a larger proportion of restricted households increases the likelihood of reaching the ELB, the distance between the ESS and the DSS, and welfare loss at the long-run equilibrium.

	n	DSS	ESS		
			0.65	0.75	0.85
$400r^{nom}$	TANK	4	3.1835	3.4408	3.8809
	RANK		3.5365	3.7586	3.8899
$100(y - y_e)$	TANK	0	-0.0220	-0.0095	-0.0432
	RANK		-0.0501	-0.0530	-0.0345
400π & $400\pi^w$	TANK	2	1.4703	1.6335	1.9494
	RANK		1.7244	1.8744	1.9496
$100(c^u - c^r)$	TANK	0	-0.0089	-0.0105	-0.0051
$100(w^u - w^r)$			0.0062	0.0010	-0.0031
$\mathbb{L}$	TANK	0	1.3825	1.0519	0.3216
	RANK		0.8580	0.5564	0.3118
$100p$(ELB)	TANK	–	11.49	5.07	0.56
	RANK		5.93	1.75	0.40

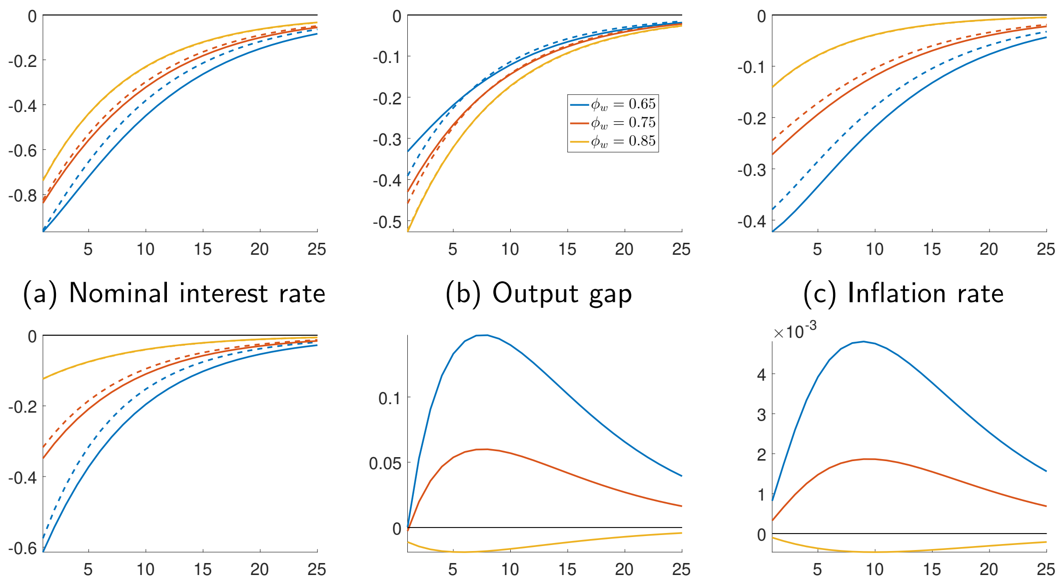
Table 2: Ergodic steady-state under different degrees of wage stickiness in the RANK and TANK model

Table (2) shows that when the ELB is occasionally binding the CB will experience difficulties in achieving its inflation and output gap targets, even in the long-run. It is worth noting that as the likelihood of reaching the ELB falls with greater wage rigidity, both the long-run equilibrium of the output gap and inflation approach their targets. Similarly, long-run values of consumption and wage inequality also approach zero as wage flexibility reduces. Long-run welfare loss is significantly larger than zero at the ESS. It is larger in the TANK model and increases with wage flexibility.

5.2 Economic dynamics

To understand the model's specific features, we first show its dynamic properties under different calibrations. Figures (1) and (2) display the economy's dynamic responses to demand and productivity shocks, respectively. These shocks have a magnitude of one standard deviation. In both figures, continuous lines represent responses in the TANK model (i.e., $n = 0.5$) while dashed lines correspond to responses in the model's RANK version (i.e., $n = 0$). Responses under the baseline calibration of wage rigidity are printed in red ($\phi_w = 0.75$). We also display responses under more flexible wages, which are in blue ($\phi_w = 0.65$), and under more rigid wages, which are shown in yellow lines ($\phi_w = 0.85$).

A sudden increment in the households' impatience factor (or a decrease of the impatience rate) is equivalent to a negative demand shock. That is, if households are suddenly more patient, they will postpone consumption, which will reduce demand. As expected, this shock is contractive with the same mechanism applying in both the RANK and the TANK model.



(a) Nominal interest rate (b) Output gap (c) Inflation rate

(d) Wage inflation (e) Consumption inequality (f) Wage inequality

Figure 1: Responses to a negative demand shock

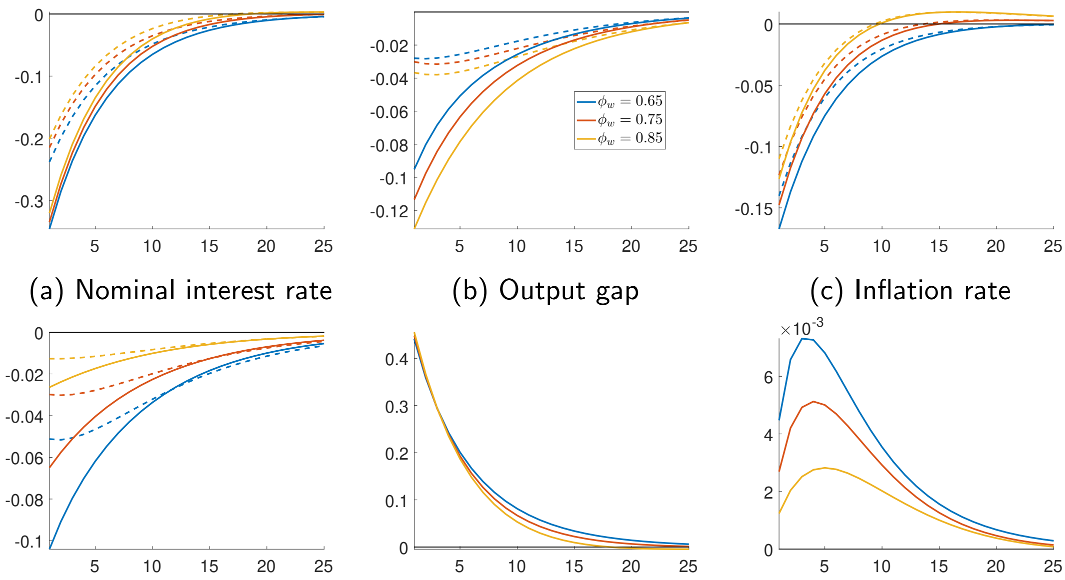
In this economy, firms' production is driven by aggregated demand; hence, reduced demand results in a fall in output (Subfigure (1b)) and labor demand. The latter reduces wages in equilibrium, which causes wage inflation to fall (Subfigure (1d)). As firms' production falls below its potential and wages are low, marginal costs of production are diminished, yielding a fall in the inflation rate (Subfigure (1c)). The CB reacts by reducing the interest rate (Subfigure (1a)) to boost the economy. Monetary policy is not yet binded by the ELB, as the fall in interest rate is only of eight basic points. However, the policy response (given by Equation (2.18)) is not enough to offset the effects of the shock, despite not being binded by the ELB.

Generally, the TANK model is slightly more sensitive to the demand shock, which is explained by the additional demand channel introduced by restricted households. Concerning wage rigidity, it can be seen that more wage flexibility brings some stability to the output gap (Subfigure (1b)); however, inflation rates become more responsive to the demand shock if wages are more flexible.

Consumption and wage inequality are features specific to the TANK model and they are calculated according to (2.25). It is noticeable in Subfigures (1e) and (1f) that under the benchmark calibration (red) and in the case of more flexible wages (blue), the negative demand shock plays in favor of unrestricted households as both measures of inequality are positive; however, the sign is reverted in the case of more rigid wages (yellow).

Responses to positive productivity shock are displayed in Figure (2). In this case, flexible output is immediately increased by the shock.[9] However, as price and wage stickiness introduce inertia into the model, output cannot match the rise of its flexible counterpart, yielding a negative output gap (Subfigure (2b)). As firms are more productive, but production is limited by demand, marginal costs diminish leading to a drop in the inflation rate (Subfigure (2c)). This fall in price inflation dominates the dynamics of wage inflation, which is composed by the inflation rate and the variation in wages (Subfigure (2d)). Given a negative output gap and inflation rate, the CB intervenes by cutting the interest rate (Subfigure (2a)). The fall in the interest rate is of 12 basic points; hence, the CB is not yet restricted by the ELB.

[9] Flexible output is defined as the level of production that would prevail under flexible prices and wages. Given the fiscal policy set-up, flexible output coincides with efficient output or the level of output that would prevail under flexible prices and wages and perfect competition.



(a) Nominal interest rate (b) Output gap (c) Inflation rate

(d) Wage inflation (e) Consumption inequality (f) Wage inequality

Figure 2: Responses to a productivity shock

Responses to a productivity shock show substantial differences between the TANK and RANK model. As wages do not adjust quickly enough, neither will labor income, limiting the consumption of restricted households. As a result, aggregated demand falls noticeably more in the TANK model. Because of the reduced demand, firms have to lessen production by reducing labor demand even more, which leads to a further fall in wages.

A similar pattern as the one displayed in the responses to a demand shock can be observed regarding the influence of wage flexibility. If wages are more flexible, the output gap becomes more stable; however both inflation rates become more unstable (see the blue line in Subfigures (2b), (2c) and (2d)). A sudden increase in productivity raise firms' dividends, which are distributed among unrestricted households. Consequently, consumption inequality increases (Subfigure (2e)) for all calibrations of wage flexibility. Given the increment of unrestricted households' consumption, their associated marginal rate of substitution also rises more than the increment in the rate of substitution of restricted households. As a result, an increase in wage inequality is also a consequence of the productivity shock (Subfigure (2f)). It is noticeable that wage flexibility mostly affects wage inequality and affects consumption inequality slightly. Given that wages adjust faster when more flexible, wage inequality is more pronounced in that case.

The responses discussed above are standard in the literature. Similar responses can be seen in the linear model in the first chapter of this dissertation. As we have calculated the non-linear solution of the model, responses are not proportional when the magnitudes of the shocks are larger than one standard deviation. In what follows, we will investigate the responses when the shocks are large enough to push the economy to the ELB for a year. It is worth emphasizing that such shocks are unlikely to happen. In fact, this constitutes our second result.

Result 2 (ELB reached under certain states of nature) It is unlikely that the ELB will be reached by the realization of a single large shock. Instead, the ELB will be reached when regular shocks happen under specific states of nature.

Taking the ergodic steady-state as a starting point, it is unlikely to observe shocks that are large enough to move the economy to the ELB. Figures (3) and (4) display the responses when the shocks are large enough to trap the economy at the ELB for one year. However, the size of those shocks are 5.45 and 19.63 standard deviations of the impatience factor and productivity shock, respectively. As both shocks are normally distributed, the likelihood of such events occurring is effectively zero. However, despite such extreme events, the impulse responses are relatively accurate. In fact, the worst Euler error associated with these responses is in the order of $10^{-2.5}$. All subfigures shown in Figures (3) and (4) display deviations relative to the ESS; however, for presentation convenience, each line in Subfigures (3b), (3f), (4b) and (4f) converge to their own ESS.[10]

Figure (3) displays the economy's response to a demand shock large enough to trap it at the ELB for one year at the benchmark calibration (red line). The first remarkable observation is that wage flexibility influences the ELB's duration. Compared to the duration under the benchmark, the ELB under more flexible wages (blue line) lasts three more quarters while, under stickier wages (yellow line), it lasts one quarter less (Subfigure (3b)).

Such an extreme event, depicted by Subfigure (3a), is heavily contractive. Noticeably, wage flexibility stabilizes the output gap (Subfigure (3c)); however, it strongly destabilizes both inflation rates (blue lines in Subfigure (3d) and (3e) are further away from zero). Despite the output gap stabilization, welfare loss is larger under more flexible wages, as shown in Subfigure (3f). It can be seen in Subfigures (3g) and (3h) that both measures of inequality deviate significantly from zero and the deviations are more pronounced if wages are more flexible, adding to a higher welfare cost.

[10] This exercise contrasts with Billi and Galí (2018) in two aspects. First, the responses displayed in their paper are in deviations from the DSS. Second, their economy is at the ELB as long as the shock that triggers it is active. There are no internal drivers in their model solution that determines the entry to and duration of the ELB.

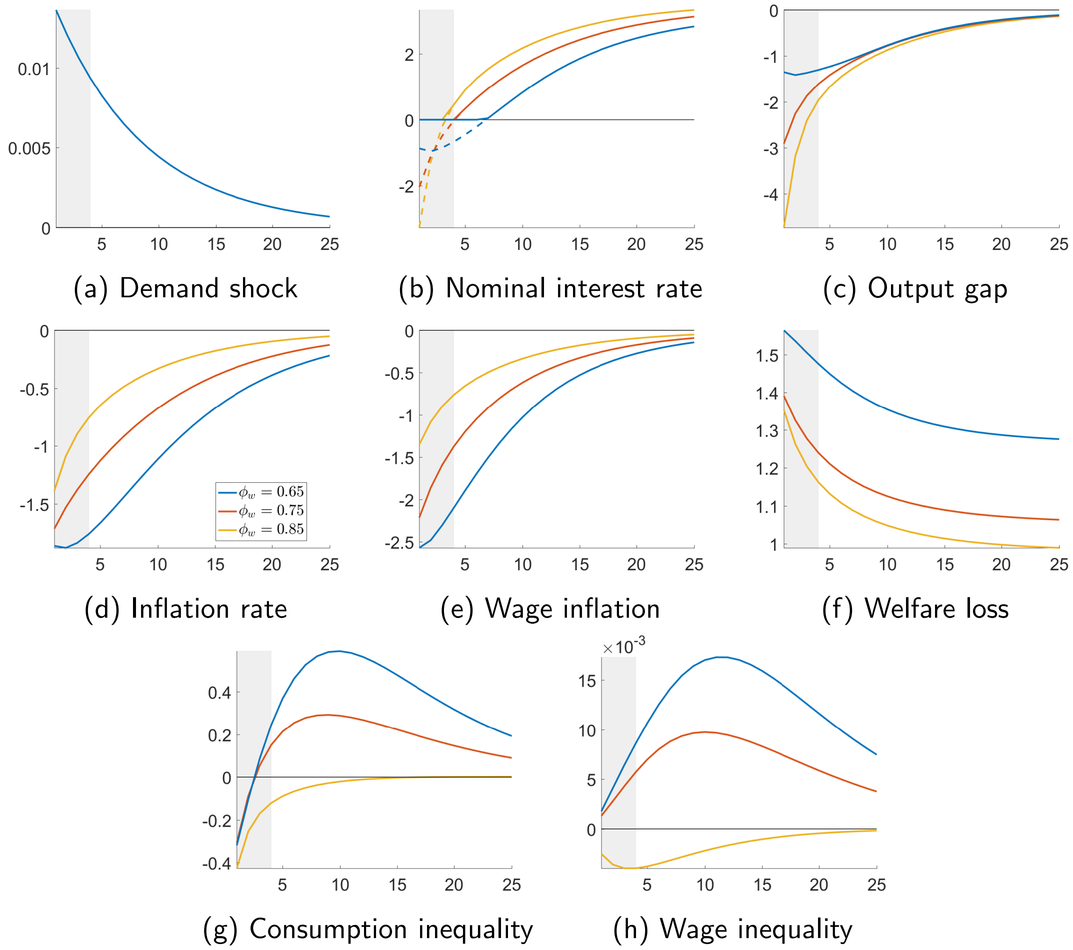
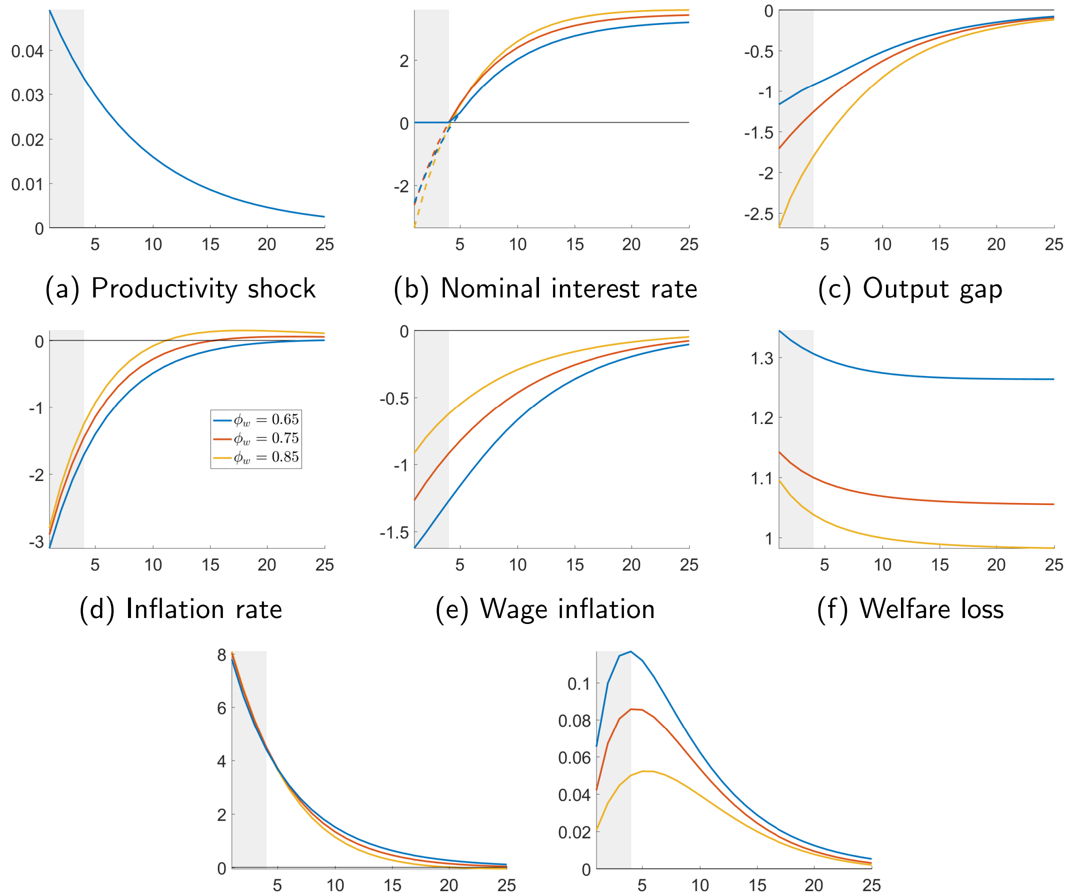


Figure 3: Responses to a 'large enough' negative demand shock

Responses to a productivity shock large enough to trap the economy at the ELB for one year at the benchmark calibration is depicted in Figure (4). Unlike the responses to a demand shock, duration is not affected significantly by wage flexibility Subfigure (4b); however, it is noteworthy that the nominal rate under more flexible wages is still close to zero in the fifth quarter. The shock causes a large contraction of the economy, and a similar pattern regarding wage flexibility can be observed. That is, while the output gap stabilizes with wage flexibility, inflation rates (prices and wages) and wage inequality destabilize significantly (Subfigures (3d), (3e) and (3g)). All this adds up to a large welfare loss, which is larger when wages are more flexible.



(a) Productivity shock (b) Nominal interest rate (c) Output gap

(d) Inflation rate (e) Wage inflation (f) Welfare loss

(g) Consumption inequality (h) Wage inequality

Figure 4: Responses to a 'large enough' positive productivity shock

5.3 Duration of the ELB

We calculate duration and all likelihoods related to the ELB frequency through simulations of the model. Algorithm (2) summarizes these calculations:

Algorithm 2 (ELB events) We simulate $T = 300000$ draws of the model under all alternative calibrations and calculate the following:

- The likelihood of reaching the ELB

$$p(\{r_t^{nom} = 0\}) = \int \mathcal{I}_{\{0\}}[r_t^{nom}]\mu(\mathbf{x})\mathrm{d}\mathbf{x} \approx \frac{1}{T}\sum_{i=1}^{T}\mathcal{I}_{\{0\}}[r^{nom}(\mathbf{x}^{(i)})] \tag{5.1}$$

where $\mathbf{x}^{(i)}$ is the i-th draw of the state vector and $\mathcal{I}_{\{C\}}[m(x)]$ is a function that equals 1 if $m(x) \in C$ and 0 otherwise.

- The likelihood of staying at the ELB for exactly s consecutive periods once the ELB is reached.
  Define the event of s consecutive periods at the ELB as $\mathcal{R}_{t-1,s} = \{r^{nom}_{t-1} = 0, ..., r^{nom}_{t-s} = 0, r^{nom}_{t-s-1} > 0\}$. Hence, staying exactly s consecutive periods has a likelihood of

$$p(\{r^{nom}_t > 0, \mathcal{R}_{t-1,s}\}) \approx \frac{1}{T_{elb}} \sum_{i=1}^{T_{elb}} \#(\{r^{nom}_t > 0, \mathcal{R}_{t-1,s}\}) \tag{5.2}$$

  where T_{elb} is the number of periods at the ELB.

- Expected duration

$$Dur = \sum_{i=1}^{k} i \times p(\{r^{nom}_t > 0, \mathcal{R}_{t-1,i}\})$$

- The likelihood of staying at the ELB for at least one additional period after s consecutive periods.

$$\begin{aligned} p(\{r^{nom}_t = 0\}|\mathcal{R}_{t-1,s}) &= \int_{\mathbf{x}} \mathcal{I}_{\{0\}}[r^{nom}_t]\mu(\mathbf{x}|\mathcal{R}_{t-1,s})\mathrm{d}\mathbf{x} \\ &\approx \frac{1}{T_s} \sum_{i=1}^{T_s} \mathcal{I}_{\{0\}}[r^{nom}(\mathbf{x}^{(i)}|\mathcal{R}_{t-1,s})] \end{aligned} \tag{5.3}$$

  where $\mathbf{x}^{(i)}|\mathcal{R}_{t-1,s}$ collect all states realized after the economy was trapped at the ELB for s periods and $T_s = \#(\mathbf{x}^{(i)}|\mathcal{R}_{t-1,s})$.

- The likelihood of leaving the ELB at the next period.

$$p(\{r^{nom}_t > 0\}|\mathcal{R}_{t-1,s}) = 1 - p(\{r^{nom}_t = 0\}|\mathcal{R}_{t-1,s})$$

From Algorithm (2) we build Tables (3) and (4), based on which we claim our third result.[11]

[11]We have few simulations with spells at the ELB of duration 10 or longer. Hence, the last rows of tables (3) and (4) are subject to a non-negligible numerical noise.

Result 3 (Duration of the ELB) The expected ELB duration increases with the proportion of restricted households and with wage flexibility.

In Table (3) we display the likelihood of staying at the ELB for a number of consecutive periods according to Equation (5.2). The number of periods are displayed in the first column of the table. Relative to the TANK model, the RANK model accumulates most of the likelihood in the first four quarters and with a larger likelihood. Hence, it is more likely to observe shorter durations in RANK models. Concerning wage rigidity, under more flexible wages (either the RANK or TANK model) the likelihood of a duration of a small number of quarters is always reduced. As a result, *wage flexibility tends to increase the duration of the shock*. This claim is confirmed by our calculated expected duration of the ELB in the last row in the table.

	TANK			RANK		
ϕ_w	0.650	0.750	0.850	0.650	0.750	0.850
1	37.982	46.791	64.185	44.339	51.965	65.106
2	17.316	18.555	19.517	18.966	21.279	18.014
3	11.243	12.101	7.948	11.803	10.870	7.801
4	7.754	7.081	4.024	7.419	6.229	4.681
5	5.180	4.034	1.911	4.561	3.679	1.986
6	4.103	3.066	1.207	3.340	2.174	1.702
7	3.123	2.277	0.503	2.634	0.836	0.142
8	2.423	1.560	0.402	1.847	0.920	0.426
> 8	10.877	4.536	0.302	5.091	2.048	0.142
Exp. Dur.	3.714	2.725	1.696	2.858	2.195	1.692

Table 3: Likelihood of staying at the ELB a number of consecutive periods and expected duration of the ELB

Table (4) reports the likelihood of exiting the ELB at the next period according to Equation (5.3). This likelihood decreases in the RANK model from 37.4% to 32.7% in the first four quarters from the benchmark calibration and begins increasing afterwards. In the TANK model, however, it decreases monotonically from 26% to 19.3% in the 12 quarters displayed. This observation is even more pronounced when wages are more flexible. As a result, it is more difficult to leave the ELB if there are restricted households in the economy and it is even harder if wages are more flexible.

	TANK			RANK		
ϕ_w	0.650	0.750	0.850	0.650	0.750	0.850
1	10.228	17.169	37.841	15.515	23.676	38.474
2	10.389	16.439	37.023	15.710	25.406	34.605
3	11.290	19.245	35.909	17.399	26.096	34.375
4	11.703	18.595	37.825	17.654	26.981	41.905
5	11.068	16.264	36.122	16.473	27.278	38.251
6	11.830	17.714	42.857	17.333	26.598	63.717
7	11.915	18.653	36.458	19.288	16.260	17.073
8	11.994	17.952	52.459	19.151	24.411	70.588

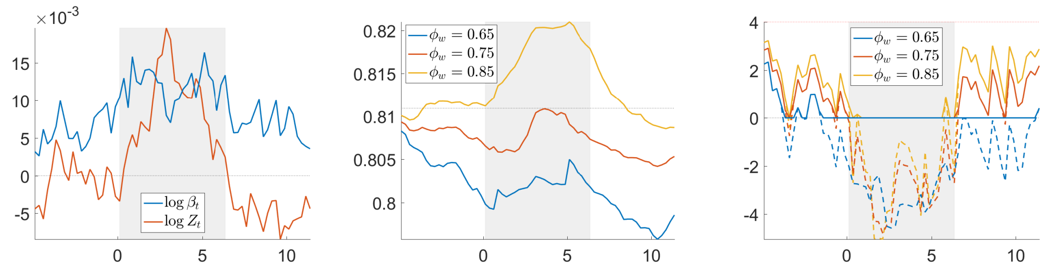
Table 4: Likelihood of leaving the ELB at the next period.

5.4 A six-year spell at the ELB

The last occurrence of an ELB spell in the USA lasted eight years.[12] To compare a similar instance in our model, we simulate an ELB occurrence of long duration (6 years) which we display in figures (5) and (6).

The ELB is triggered by a sequence of positive innovations to the two shocks. As shown by equations (2.2) and (2.12), these shocks follow persistent stochastic processes; therefore, they tend to accumulate consecutive innovation of the same sign. In the example in Subfigure (5a), despite experiencing a sequence of regular innovations, the demand shock reaches a deviation of 7.2 standard errors, and of 6 standard errors in the case of the productivity shock. For the endogenous state variables, these shocks cause a fall in wages, which is more pronounced when wages are more flexible (Subfigure (5b)). In Subfigure (5c), we can observe the dynamic of the interest rate during such an event. The dashed line is the interest rate suggested by the policy rule (2.18) while the continuous line is the nominal interest rate given by the truncated policy rule (2.19). It is remarkable that the same sequence of shocks leads to different lengths of the ELB regime depending on the degree of wage flexibility. Relative to the benchmark (red line), the ELB lasts almost five more years when wages are more flexible (blue line), while it lasts four fewer quarters under stickier wages (yellow line).

[12]From Dec. 07 of 2008 to Dec. 16 2015.



(a) Exogenous states (b) Wages (c) Nominal interest rate

Figure 5: A six-year spell at the effective lower bound

Greater wage flexibility yields a more stable output gap as can be seen in Subfigure (6a). The output gap exhibits a large drop during the ELB episode, which is more than 6% under the the stickier wages case (yellow line). Despite the output gap being more stable under more flexible wages, both inflation rates (subfigures (6b) and (6c)) show a large drop for this case, reaching levels of negative 2% for the inflation rate (blue line). Wage stickiness, on the other hand, prevents such large drops in either measure of inflation (yellow lines).

(a) Output gap (b) Price inflation (c) Wage inflation

(d) Welfare loss (e) Consumption inequality (f) Wage inequality

Figure 6: Responses to a demand shock

The degree of wage flexibility also affects our measures of inequality (Subfigures (6e) and (6f)). The pattern is similar to the inflation rates, as more flexibility tends to be associated with larger deviations from zero. Noticeably, during the ELB episode both measures of inequality are large and mostly positive which means that the recession affects more severely to restricted households. Worse yet, the outcome for restricted households worsens with wage flexibility. The gain in welfare associated with a more stable output gap under more flexible wages is more than offset by the instability induced to both inflation rates and both measures of inequality. It can be seen in Subfigure (6f) that the welfare cost is significantly higher under the model calibrated with more wage flexibility (blue line).

6 Conclusions

We constructed a tractable two-agent, sticky-price, sticky-wage model and obtained its full non-linear solution. In the model, Ricardian families coexist with restricted (or hand-to-mouth) families. The latter group activates a demand channel that ties labor income to aggregated demand, thus, providing more relevancy to the model. The system discussed in this paper is consistent with standard NK models in the literature. In fact, when log-linearized, it is equivalent to Colciago (2011) and under the extreme calibration of no restricted households, it converges to Erceg et al. (2000). Price and wage rigidity is modeled with adjustment cost à la Rotemberg (1988), yielding a parsimonious non-linear specification of the model dynamics and welfare. This model allows for studying the effects of wage flexibility in a context where monetary policy is occasionally restricted by the ELB.

We evaluate the effects of different degrees of wage flexibility in two versions of the model: the RANK and TANK model. In the RANK model, the economy is populated with only unrestricted (or Ricardian) households. On the other hand, in the TANK model, we assume that half the population is restricted (or hand-to-mouth) households. Regarding the model's dynamics, there are no major differences between the RANK and TANK model when responding to a one standard deviation of the demand shock. However, responses to a productivity shock yields some important differences as the output gap and wage inflation are more responsive in the TANK version. In all cases, a similar pattern is recognized regarding the degree of wage flexibility—if wages are more flexible, the output gap becomes more stable while price and wage inflation become more responsive to either shock.

We computed a nonlinear approximation of the solution to our model; hence, the estimated responses to the shocks are not proportional to the size of the shock. As a result, without losing model accuracy, we can infer the effect of shocks large enough to push the economy to the ELB. In general, wage flexibility helps stabilize the output gap but the responsiveness of both inflation rates increases with wage flexibility when the ELB is binding. However, shocks large enough to push the economy to the ELB are unlikely if the starting point is the ESS. Our simulations suggest the ELB is reached when a sequence of shocks happen under very specific states of nature.

The chief benefit of having the approximated nonlinear solution is that we can calculate the likelihood of entry and exit to the ELB, as well as its duration, which are endogenous events in our model. Noticeably, wage flexibility tends to increase the likelihood of reaching the ELB and its duration. It also reduces the likelihood of exiting the ELB in a small number of periods. This has important long-run implications, as it causes non negligible deviations between the ergodic and deterministic steady-state. Specifically, the resting point for inflation and output gap fall below their long-run targets. As a result, a systematic welfare loss is created, which is exacerbated if wages are more flexible or if there are restricted families in the economy.

Our findings suggest that wage flexibility is undesirable in an economy with strong, yet plausible, demand channel and with an occasionally limited monetary policy. Not only for the well known negative consequences of an ELB episode but also because wage flexibility increases the likelihood of such episodes occurring. If welfare cost is taken as a metric, we can infer the benefits from wage flexibility (as output-gap stabilization) are far outweighed by the stability loss in both inflation rates and in both measures of inequality. Worse yet, the welfare cost does not dissipate as ELB regime ends. In fact, it prevails in the long-run equilibrium.

There are many directions in which this study could be extended. For instance, a more relevant role of precautionary saving could be added if we introduce capital into the model. A financial friction could be introduced to add more insight into ELB entry, or optimal policy could be considered. However, we consider this paper as one step forward to a better understanding of the economic dynamics and welfare when the ELB is a relevant policy constraint.

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A The complete non-linear system

What follows replicates the system in Ledesma&Walsh after linearizing
Core variables

$$\mathbf{y} := \{y_{f,t}, w^r_{f,t}, y_t, \pi_t, w^r_t, w^u_t\} \quad \#\mathbf{y} = 6 \tag{A.1}$$

$$\mathbf{x}_1 := \{\breve{w}^r_{t-1}, \breve{w}^u_{t-1}\} \quad \#\mathbf{x}_1 = 2 \tag{A.2}$$

$$\mathbf{x}_2 := \{\varepsilon_{b,t}, \varepsilon_{z,t}\} \quad \#\mathbf{x}_2 = 2 \tag{A.3}$$

Core system:
Optimality conditions from the unrestricted households provide the Euler equation and one of the Phillip curve for wages, equations (2.5) and (2.6) in the main text, where $\tau_w = \frac{1}{\theta_w - 1}$ has been replaced in the Phillips curve:

$$1 = R^{nom}_t \mathbb{E}_t \frac{SDF^u_{t,t+1}}{\Pi_{t+1}} \text{ and} \tag{A.4}$$

$$MRS^u_t = W^u_t + \frac{1}{\theta_w}\left(\frac{W^u_t}{W_t}\right)^{\theta_w}\left[\mathcal{A}'_{wu,t}\frac{\Pi^{w,u}_t}{\bar{\Pi}} - \mathbb{E}_t SDF^u_{t,t+1}\mathcal{A}'_{wu,t+1}\frac{\Pi^{w,u}_{t+1}}{\bar{\Pi}}\frac{H_{t+1}}{H_t}\right]. \tag{A.5}$$

Restricted households' optimal wage setting gives the second Phillips curve regarding wage inflation, equation (2.4) in the text:

$$MRS^r_t = W^r_t + \frac{1}{\theta_w}\left(\frac{W^r_t}{W_t}\right)^{\theta_w}\left[\mathcal{A}'_{wr,t}\frac{\Pi^{w,r}_t}{\bar{\Pi}} - \mathbb{E}_t SDF^r_{t,t+1}\mathcal{A}'_{wr,t+1}\frac{\Pi^{w,r}_{t+1}}{\bar{\Pi}}\frac{H_{t+1}}{H_t}\right]. \tag{A.6}$$

From the production side, firms reset prices by following the Phillip curve given by equation (2.15) in the text:

$$RMC_t = \frac{\theta_p - 1}{\theta_p} + \frac{1}{\theta_p}\left[\mathcal{A}'_{p,t}\frac{\Pi_t}{\bar{\Pi}} - \mathbb{E}_t SDF^u_{t,t+1}\mathcal{A}'_{p,t+1}\frac{\Pi_{t+1}}{\bar{\Pi}}\frac{Y_{t+1}}{Y_t}\right]. \tag{A.7}$$

The flexible price economy comes from setting both marginal rate of substitutions (for restricted and unrestricted households) equal to the marginal product of labor (determined by the condition $RMC_{f,t} = 1$):

$$W^u_{f,t} = \gamma(H^u_{f,t})^\eta (C^u_{f,t})^\sigma \tag{A.8}$$

$$W^r_{f,t} = \gamma(H^r_{f,t})^\eta (C^r_{f,t})^\sigma \tag{A.9}$$

A technicality in Judd (1996) notation, require us to close the economy by setting the lead of the endogenous states equal to their mirroring jumping variables

$$\breve{w}^r_t = w^r_t \tag{A.10}$$

$$\breve{w}^u_t = w^u_t \tag{A.11}$$

Exogenous processes

All exogenous variables follow AR(1) processes as described in equations (2.2) and (2.12), which are displayed here in logs:

$$\varepsilon_{z,t+1} = (1-\rho_z)\log Z_{ss} + \rho_z \varepsilon_{z,t} + \sigma_z \epsilon_{z,t+1} \tag{A.12}$$

$$\varepsilon_{b,t+1} = (1-\rho_b)\log \beta + \rho_b \varepsilon_{b,t} + \sigma_b \epsilon_{b,t+1} \tag{A.13}$$

Transformations

In order to have a closed model, the system (A.4-A.11) should depend exclusively on jumping and state variables as depicted in equations (A.1-A.3). As a result, all variables involved in the core system, different from those declared as jumping or states, should be defined as function of them.

Consider $\mathbf{z}$ as the vector of all required definitions, then it is composed by

$$\mathbf{z} := \left\{ \begin{array}{cccccccc} Y_{f,t} & W^r_{f,t} & Y_t & \Pi_t & W^r_t & W^u_t & \breve{W}^r_t & \breve{W}^u_t \\ \beta_t & Z_t & H_{f,t} & W_{f,t} & W^u_{f,t} & H^r_t & H^u_t & C^r_t \\ C^u_t & W_t & \Pi^{w,r}_t & \Pi^{w,u}_t & \Pi^w_t & \mathcal{A}_{wr,t} & \mathcal{A}'_{wr,t} & \mathcal{A}_{wu,t} \\ \mathcal{A}'_{wu,t} & \mathcal{A}_{p,t} & \mathcal{A}'_{p,t} & H_t & H^r_t & H^u_t & C_t & C^r_t \\ C^u_t & MRS^r_t & MRS^u_t & SDF^r_{t,t+1} & SDF^u_{t,t+1} & RMC_t & R^{mp}_t & R^{nom}_t \\ \mathbb{U}_t & \mathbb{U}_{f,t} & \mathbb{L}_t & & & & & \end{array} \right\}, \tag{A.14}$$

where, variables in $\mathbf{z}$ are built as function of $\{\mathbf{y}, \mathbf{x}, \mathbf{u}'\}$ as follows:

1. From log to levels:

$$Y_{f,t} = \exp(y_{f,t}),\ W^r_{f,t} = \exp(w^r_{f,t}),\ Y_t = \exp(y_t),\ \Pi_t = \exp(\pi_t),\ W^r_t = \exp(w^r_t),$$
$$W^u_t = \exp(w^u_t),\ \breve{W}^r_t = \exp(\breve{w}^r_t),\ \breve{W}^u_t = \exp(\breve{w}^u_t),\ \beta_t = \exp(\varepsilon_{b,t}),\ Z_t = \exp(\varepsilon_{z,t})$$

2. Required definitions for the flexible economy

$$
\begin{aligned}
H_{f,t} =& \left[\frac{Y_{f,t}}{Z_t}\right]^{\frac{1}{1-a}}, \\
W_{f,t} =&(1-a)Z_t^{\frac{1}{1-a}}Y_{f,t}^{-\frac{a}{1-a}} \\
W_{f,t}^u =& \left[\frac{1}{1-n}W_{f,t}^{-(\theta_w-1)} - \frac{n}{1-n}(W_{f,t}^r)^{-(\theta_w-1)}\right]^{-\frac{1}{\theta_w-1}} \\
H_{f,t}^r =& \left(\frac{W_{f,t}^r}{W_{f,t}}\right)^{-\theta_w} H_{f,t}, \\
H_{f,t}^u =& \left(\frac{W_{f,t}^u}{W_{f,t}}\right)^{-\theta_w} H_{f,t} \\
C_{f,t}^r =&(1-\psi)Y + W_{f,t}^r H_{f,t}^r \\
C_{f,t}^u =&\frac{1}{1-n}Y_{f,t} - \frac{n}{1-n}C_{f,t}^r
\end{aligned}
$$

2. Required definitions for the sticky price economy

$$
\begin{aligned}
W_t =& [n(W_t^r)^{-(\theta_w-1)} + (1-n)(W_t^u)^{-(\theta_w-1)}]^{-\frac{1}{\theta_w-1}} \\
\Pi_t^{w,r} =& \Pi_t \frac{W_t^r}{\breve{W}_{t-1}^r},\ \Pi_t^{w,u} = \Pi_t \frac{W_t^u}{\breve{W}_{t-1}^u},\ \Pi_t^w = \Pi_t \frac{W_t}{\breve{W}_{t-1}}, \\
\mathcal{A}_{wr,t} =& \frac{\phi_w^{rot}}{2}\left(\frac{\Pi_t^{w,r}}{\bar{\Pi}} - 1\right)^2,\ \mathcal{A}'_{wr,t} = \phi_w^{rot}\left(\frac{\Pi_t^{w,r}}{\bar{\Pi}} - 1\right) \\
\mathcal{A}_{wu,t} =& \frac{\phi_w^{rot}}{2}\left(\frac{\Pi_t^{w,u}}{\bar{\Pi}} - 1\right)^2,\ \mathcal{A}'_{wu,t} = \phi_w^{rot}\left(\frac{\Pi_t^{w,u}}{\bar{\Pi}} - 1\right) \\
\mathcal{A}_{p,t} =& \frac{\phi_p^{rot}}{2}\left(\frac{\Pi_t}{\bar{\Pi}} - 1\right)^2,\ \mathcal{A}'_{p,t} = \phi_p^{rot}\left(\frac{\Pi_t}{\bar{\Pi}} - 1\right) \\
H_t =& \left[\frac{Y_t}{Z_t}\right]^{\frac{1}{1-a}},\ H_t^r = \left(\frac{W_t^r}{W_t}\right)^{-\theta_{w,t}} H_t,\ H_t^u = \left(\frac{W_t^u}{W_t}\right)^{-\theta_{w,t}} H_t \\
C_t =& (1-\mathcal{A}_{p,t})Y_t - (n\mathcal{A}_{wr,t} + (1-n)\mathcal{A}_{wu,t})H_t \\
C_t^r =& (1-\psi)Y + W_t^r H_t^r - \mathcal{A}_{wr,t} H_t \\
C_t^u =& \frac{1}{1-n} C_t - \frac{n}{1-n} C_t^r \\
MRS_t^r =& \gamma(H_t^r)^\eta (C_t^r)^\sigma,\ MRS_t^u = \gamma(H_t^u)^\eta (C_t^u)^\sigma, \\
SDF_{t,t+1}^r =& \beta_{t+1}\left(\frac{C_{t+1}^r}{C_t^r}\right)^{-\sigma},\ SDF_{t,t+1}^u = \beta_{t+1}\left(\frac{C_{t+1}^u}{C_t^u}\right)^{-\sigma}, \\
RMC_t =& \frac{\theta_p - 1}{\theta_p} \frac{W_t}{(1-a) Z_t^{\frac{1}{1-a}} Y_t^{-\frac{a}{1-a}}}
\end{aligned}
$$

2. Monetary policy is introduced in the system as a definition

$$
\begin{aligned}
R_t^{mp} =& \frac{\bar{\Pi}}{\beta}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\varphi_\pi}\left(\frac{Y_t}{Y_{f,t}}\right)^{\varphi_x}, \\
R_t^{nom} =& \max\{R_t^{mp}, \underline{R}\}
\end{aligned}
$$

3. Required to calculate welfare loss

$$
\begin{aligned}
\mathbb{U}_t =& \frac{n(C_t^r)^{1-\sigma} + (1-n)(C_t^r)^{1-\sigma}}{1-\sigma} - \gamma \frac{n(H_t^r)^{1+\eta} + (1-n)(H_t^u)^{1+\eta}}{1+\eta} \\
\mathbb{U}_{f,t} =& \frac{n(C_{f,t}^r)^{1-\sigma} + (1-n)(C_{f,t}^r)^{1-\sigma}}{1-\sigma} - \gamma \frac{n(H_{f,t}^r)^{1+\eta} + (1-n)(H_{f,t}^u)^{1+\eta}}{1+\eta} \\
\mathbb{L}_t =& \frac{1}{Y_{ss}^{1-\sigma}}[\mathbb{U}_{f,t} - \mathbb{U}_t].
\end{aligned}
$$