BANCO CENTRAL DE RESERVA DEL PERÚ

SFX Interventions, Financial Intermediation, and External Shocks in Emerging Economies

Alex Carrasco*, David Florián Hoyle* y Rafael Nivín*

* Banco Central de Reserva del Perú

DT. N°. 2019-022
Serie de Documentos de Trabajo
Working Paper series
Diciembre 2019

SFX Interventions, Financial Intermediation, and External Shocks in Emerging Economies*

By Alex Carrasco[†], David Florian Hoyle[‡], and Rafael Nivin[§]

This Draft: December 2019

In this document, we study the role of sterilized foreign exchange (SFX) interventions as an additional monetary policy instrument for emerging market economies in response to external shocks. We develop a model in order to analyze SFX interventions as a balance sheet policy induced by a financial friction in the form of an agency problem between banks and depositors. The severity of the bank's agency problem depends directly on a measure of currency mismatch at the bank level. Moreover, credit and deposit dollarization co-exists in equilibrium as endogenous variables. In this context, SFX interventions can lean against the response of the bank's lending capacity and ultimately the response of real variables by moderating the response of the exchange rate.

Furthermore, we take the model to data by calibrating it to replicate some financial steady-state targets for the Peruvian banking system as well as matching the impulse responses of the macroeconomic model to the impulse responses implied by an SVAR model. Our results indicate that SFX interventions successfully reduce GDP and investment volatility by about 6% and 14%, respectively, when compared to a flexible exchange rate regime. Moreover, SFX interventions reduce the response of GDP to foreign interest rate and commodity price shocks by around 11 and 22 percent, respectively. Hence, this policy produces significant welfare gains when responding to external shocks: if the Central Bank does not intervene in the Forex market in the face of external shocks, there would be a welfare loss of 1.1%.

Keywords: Sterilized Forex Interventions, External Shocks, Financial Cycle, Dollarization, Monetary Policy.

Small open economies face volatile foreign shocks that has shaped capital flows and exchange rate dynamics since the end of the Bretton Woods agreement and more recently due to global financial integration. Foreign shocks have different sources or fundamentals which can be summarized in terms of three main interconnected components: global demand, world interest rates, and commodity prices. Capital flows heavily respond to foreign shocks affecting domestic financial conditions and credit growth through the availability of external funds (mostly in foreign currency) and exchange rate fluctuations. For instance, three relatively recent

*Preliminary draft. Comments and suggestions will be greatly appreciated. The views expressed in this paper are those of the authors and do not necessarily represent the views of the Central Reserve Bank of Peru (BCRP). We thank the Financial Stability and Development Network, the Inter-American Development Bank, the State Secretariat for Economic Affairs/Swiss Government (SECO), and the Central Reserve Bank of Peru for support during the different stages of this research. Specially, we are indebted to Lawrence Christiano, Roberto Chang, Paul Castillo, Carlos Montoro and Marco Ortiz for insightful discussions. We also thank seminar participants at the BCRP 2019 annual conference. All remaining errors are ours.

[†]Central Reserve Bank of Peru. Email: alex.carrasco@bcrp.gob.pe

[‡]Central Reserve Bank of Peru. Email: david.florian@bcrp.gob.pe

[§]Central Reserve Bank of Peru. Email: rafael.nivin@bcrp.gob.pe

events might be identified as external shocks with the above mentioned effects to emerging market economies: 1) the global commodity boom originated by China's strong external demand during 2000s, 2) expansionary monetary policies of major advanced economies as a response to the Global Financial Crises, and 3) the normalization of the Federal Reserve highly accommodative monetary policy, also known as the "Taper Tantrum". In emerging market economies (EMEs), these events resulted in significant capital flows and exchange rate pressures that in some cases placed the financial system in a more fragile situation.

China's unprecedented growth rates implied a persistent surge in mineral commodity prices that started around 2003, showed a temporary slowdown during the global financial crisis in 2009 and reached its peak around 2011. The commodity prices boom that most emerging market commodity exporters experienced during the time before the financial crisis generated significant capital inflows, persistent appreciatory exchange rate pressures and strong domestic credit growth. The exceptional expansionary monetary policies of major advanced economies in the aftermath of the Global Financial Crisis was followed by a similar set of events which ultimately led to significant capital inflows to emerging markets. Later on, the FED's taper tantrum generated the opposite effects during 2013 and 2014: strong capital outflows and depreciation pressures for the exchange rate.

Many central banks, especially those in emerging market economies, responded to these events accumulating foreign reserves during capital inflow episodes. In this sense, those central banks were considered to be in a good position to deal with capital reversals and effectively they sold those accumulated reserves during capital outflow episodes. Specifically, EMEs have relied on SFX interventions (i.e official purchases or sales of foreign currency that leave domestic liquidity unaffected) to smooth the effects of rapidly shifting capital flows and to reduce exchange rate volatility while providing businesses and households with insurance against exchange rate risks. Moreover, foreign currency debt in emerging markets economies have been increasing, making those countries more exposed to global financial flows, and therefore financial stability has become an important motive for SFX interventions. The mix of policy tools taken by policy makers in EMEs also includes macro-prudential measures and capital controls.[1] Unfortunately, the effectiveness of these tools are still under debate and more research is needed to have a better assessment on the use of these policy tools as a complement to conventional interest rate policy.

The purpose of this paper is to develop a macroeconomic model in order to analyze SFX interventions as a complementary monetary policy tool that takes on attributes of a financial stability instrument in the presence of foreign shocks. For this purpose, we adopt a perspective that views SFX intervention as a non conventional monetary policy tool motivated by the existence of financial frictions in the domestic banking sector. In particular, when the relevant financial friction binds, leverage constraints restrict the balance sheet capacity of banks and limits to arbitrage emerge together with interest rate spreads. Only in the financially constrained equilibrium, SFX interventions affect the equilibrium real allocation since it relaxes or tighten the financial constraint that banks face. In our framework, the latter is accomplished by two reinforcing effects: The stabilization of the exchange rate and the crowding out of lending capacity induced by the sterilization process of the SFX intervention (similar to the empirical findings of Hofmann et al. (2019)). Thus, we study SFX interventions as an interaction between the real exchange rate and financial constraints. In this context, SFX interventions lean against the response of banking credit by moderating the response of real exchange rate to external shocks.[2]

[1] See Céspedes et al. (2014) for a discussion of recent LATAM central banks experiences.

[2] See Céspedes et al. (2017), Chang (2019), and Céspedes and Chang (2019) for similar frameworks that introduce SFX interventions as an unconventional policy tool.

We build a general equilibrium model for a commodity exporting small open economy where SFX interventions are relevant for the equilibrium allocation. In our framework, the central bank follows a Taylor rule to set its monetary policy rate (conventional monetary policy) but also "leans against the wind" with respect to exchange rate fluctuations. The model is an extension of Aoki et al. (2018) (hence ABK) where banks face an agency problem that constrains their ability to obtain funds from domestic households and international financial markets as well. Funds obtained domestically are denominated in domestic currency while funds borrowed from abroad are denominated in foreign currency. Similar to Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Gertler et al. (2012), and Gertler and Karadi (2013), the agency problem introduces an endogenous leverage constraint which have the effect to relate credit flows to banks' net worth and ultimately makes the balance sheet of the banking sector a critical determinant of the cost of credit that borrowers face. In this context, unconventional monetary policies or balance sheet policies have real effects.

Similar to Chang (2019), we argue in this paper that SFX interventions might be understood as an unconventional monetary policy tool in the sense that the size and composition of the central bank balance sheet as well as financial intermediaries' balance sheets matter for determining asset prices, including the exchange rate. At the same time, we also argue that the financial friction view of SFX interventions differs from the unconventional monetary policy for closed economies in several aspects. First, the unconventional monetary policy literature emphasizes that the conventional instrument is active until it reaches the effective zero lower bound. Only in those cases, central banks might deploy balance sheet policies such as QE, LSAP or credit policies. On the contrary, we argue that for inflation targeters in emerging market economies, SFX interventions might be considered a balance sheet policy that is active in normal times as well as sudden stops or credit crunch episodes. Second, we consider that for emerging market economies, financial constraints are always binding, even in "normal" times. In contrast to Chang (2019) what really matters for emerging market economies is how tight financial constraints are and not necessarily if those constraints bind or not.

Our model departs from ABK in three key fundamental aspects. First, banks are allowed to extend loans to domestic agents not only in domestic currency (pesos) but also in foreign currency (dollars). The asset composition of banks is given by loans in domestic and foreign currency in addition to holdings of bonds issued by the central bank used for sterilization purposes. As a result, the banking system is partially dollarized in both sides of its balance sheet and exposed to potential currency mismatches. Thereby, credit and deposit dollarization co-exist in equilibrium as endogenous variables that depend on expected interest rate spreads among other variables. Second, the severity of the bank's agency problem depends directly on a measure of currency mismatch given by the difference between dollar denominated liabilities and assets as a fraction of total assets. But not all assets enter symmetrically into the bank's incentive compatibility constraint that characterize the agency problem. In particular, central bank assets are harder to divert than private loans. Third, the central bank "leans against the wind" with regard to exchange rate pressures that occur due to foreign shocks but in a sterilized manner. In our setting, a sterilized exchange rate intervention is a balance sheet operation that takes place when the central bank sells or buys dollars from the banking system in exchange of domestic denominated assets. But in a way that completely offsets any change in the supply of domestic liquidity by using domestic bonds issued by the central bank.

Accordingly, the model economy predicts the existence of different interest rate spreads (excess returns) that limit the ability of banks to borrow. When the incentive constraint binds the return on the bank's assets exceeds the return on deposits but also the return on domestic deposits exceeds the return on foreign borrowing. The latter implies an endogenous deviation from the standard uncovered interest parity equation. In any case, banks would be willing to borrow more from households and from international financial markets. Therefore, limits to arbitrage emerge in equilibrium.

In this setting, we study the transmission of foreign shocks to domestic financial conditions assessing the role of SFX interventions to "lean against the wind" with respect to exchange rate fluctuations and stabilize the response of interest rate spreads and bank lending. Foreign shocks are transmitted to the domestic economy through changes in the exchange rate, interest rate spreads and bank's net worth. For example, a persistent commodity boom generates a domestic economic expansion that among other things, rises commodity exports significantly. A large fraction of the revenues from commodity exports is kept in the economy producing a persistent appreciation of the exchange rate that less than partially offsets the effects over net exports due to a fall of non-commodity exports. The appreciation of the exchange rate relaxes the agency problem that banks face by increasing net worth and the intermediation capacity of banks that after the shock are less exposed to foreign currency liabilities. The latter effect is reinforced by a persistent decline in the banking system currency mismatch that feedbacks to relax the financial constraint even more. By the same token, the interest rate spreads of bank's assets over deposits move towards inducing banks to lend more in both currencies. It is noticeable that the persistent appreciation of the exchange rate increases credit dollarization but reduces deposit dollarization.

When SFX interventions are active, the central bank accumulates foreign exchange reserves and allocates central bank riskless bonds to the banking system as a response to a commodity boom. Given the binding agency problem, accumulating foreign exchange reserves after a persistent increase in commodity prices reduces significantly the appreciation of the exchange rate, limiting the expansion of credit by banks and the consequent expansion in macroeconomic aggregates such as consumption and investment. Besides exchange rate stabilization and its direct effects over intermediation, our framework implies an additional channel for FX interventions associated to the sterilization process. The associated sterilization operation increases the supply of central bank bonds to be absorbed by banks. The latter generates a crowding – out effect in bank's balance sheet that reduces bank intermediation as well. Notice that both effects are consistent with the empirical findings in Hofmann et al. (2019) and with the stylized model in Chang (2019).

We take the model to the data in order to quantify the transmission mechanism of external shocks and the role of SFX interventions in mitigating its effects over the domestic economy. We consider commodity price shocks as described above but also shocks to the foreign interest rate and external demand. Our experiment is intended to quantify the differences in the response of the economy to external shocks when SFX interventions are activated compared to exchange rate flexibility. We also conduct a standard welfare analysis exercise to analyze weather SFX interventions yield welfare gains in the presence of external shocks. We calibrate most of the parameters associated to the banking block of the model to replicate some financial steady-state targets for the Peruvian banking system. The rest of the parameterization is done by matching the impulse responses of the economic model to the impulse responses implied by an SVAR model with two blocks estimated for the Peruvian economy. The external block (exogenous block) consists of Global GDP, the Fed funds rate and a metal export index relevant for Peru while the domestic block consists of domestic demand components, aggregate bank lending, the real exchange rate and the trade balance. The identification scheme for the SVAR model relies on the small open economy assumption for the Peruvian economy: the foreign block affects the domestic block but there is no feedback of the domestic block to the foreign block at any point in time.

According to the calibrated model, SFX interventions successfully reduce macroeconomic volatility. For instance, credit, investment, and output unconditional volatilities by around 6 percent, 14 percent, and 6 percent, respectively. Moreover, conditional on an increase of 20 basis points in the foreign interest rate, a sterilized sell of foreign exchange reserves reduces the response of aggregate bank lending and investment by around 44 percent and 34 percent, respectively. Likewise, when the economy faces a commodity boom (an increase of 6.31% in the commodity export index), a sterilized buy of foreign exchange reserves limits the increase in bank lending in about 63 percent after one year of the initial shock. Consequently, the response of investment and GDP is also muted by around 45 percent and 22 percent, respectively. Hence, our quantitative results suggest that a SFX intervention policy generates significant welfare gains when responding to external shocks. Using standard welfare analysis, we find that if the Central Bank does not intervene in the foreign exchange market in the face of external shocks, there would be a welfare loss of 1.1% in consumption given the standard parameterization of the Taylor rule for the conventional interest rate instrument.

Furthermore, we explore additional numerical experiments. First, we relax three assumptions of our basic formulation of the model that may be viewed as strong and restrictive with the objective to study our setting under more general assumptions. In the first generalization of the model, the three assets that banks can hold enter with equal weights into the incentive compatibility constraint. Then, central bank bonds have a higher impact on the total amount of divertable funds and ultimately on bank's lending capacity. As a result, SFX interventions are more effective in this case than in our baseline model. In our second generalization, banks are allowed to lend only in domestic currency so that the banking system does not exhibit credit dollarization. Thus, in equilibrium banks are more exposed to real exchange rate movements since the size of the steady state currency mismatch on their balance sheet is higher than in the baseline case. Therefore, SFX interventions are more effective in smoothing the response of financial as well as macroeconomics variables to external shocks. Finally, in the third modification, banks do not internalize the effects of higher borrowing in foreign currency with respect to the industry measure of currency mismatch. In this case, banks act as if they are not constrained in terms of obtaining funding from abroad, implying that the standard UIP condition holds without any risk premium. As a result, SFX interventions are less effective in stabilizing the economy in the presence of external shocks.

Second, we examine the transmission mechanism of an unexpected disruption in the financial intermediation of banks. In this scenario, a credit crunch occurs since the economy faces and exogenous increase in the fraction of assets that banks are able to divert which ultimately generates tighter financial conditions for banks. We recalibrate some of the parameters of the model in order to replicate the contraction in bank lending, GDP and investment that the Peruvian economy experienced during the third and fourth quarters of 1998 in the course of the Russian crisis episode. Our findings suggest that an SFX intervention policy mitigates the consequences of a credit crunch shock when compared to exchange rate flexibility.

The remainder of the paper is organized as follows. Section 1 briefly reviews the literature related to SFX interventions in macroeconomic models. Section 2 describes the general equilibrium model with a special emphasis in the financial system and the implementation SFX interventions. In Section 3 the calibration strategy is presented including the specification and identification assumptions for the SVAR model. The main results are presented in Section 4. Section 5 studies the effects of external shocks on some generalizations of our basic formulation of the model. Finally, Section 6 concludes with some final remarks.

1 Brief Literature Review

This paper is related to the literature on foreign exchange interventions. We divide this literature into three broad stages. Pioneered by Kouri (1976), Branson et al. (1977), and Henderson and Rogoff (1982), the first strand of this literature emphasizes the portfolio balance channel which indicates that when domestic and foreign assets are imperfect substitutes, foreign exchange intervention is an additional and effective tool to the central bank because it can change the relative stock of assets and with it the exchange rate risk premium that affects arbitrage possibilities between the rate of return of domestic currency denominated assets and foreign currency denominated assets. However, the models built during this stage were characterized by the lack of solid micro-foundations, preventing a rigorous normative analysis. Additional research studies within the portfolio balance approach without micro-foundations are Krugman (1981), Obstfeld (1983), Dornbusch (1980), Branson and Henderson (1985), and Frenkel and Mussa (1985).

Relying on micro-founded general equilibrium models, the second strand of this literature states that sterilized foreign exchange interventions have no effect on equilibrium prices and quantities. The seminal work in this strand is due to Backus and Kehoe (1989) that not only study the effectiveness of these type of interventions under complete markets but also some types of market incompleteness. That paper points out that when portfolio decisions are frictionless, the imperfect substitutability between domestic and foreign assets postulated by the portfolio balance channel is not enough for sterilized interventions to affect prices and quantities in general equilibrium. After the publication of this work, the academy adopted a pessimistic view with respect to the effectiveness of the foreign exchange interventions, generating a long-lasting dissonance with the policy practice since policy makers have ignored the prescriptions from research and have intervened, frequently and intensely, in the foreign exchange market.

Recently, there has been a resurgence in academic interest in relation to the relevance of SFX interventions based on micro-founded macroeconomic models which parallel the literature on unconventional monetary policy. In this sense, the portfolio balance approach have experienced a recent comeback in studies such as Kumhof (2010), Gabaix and Maggiori (2015), Liu and Spiegel (2015), Benes et al. (2015), Montoro and Ortiz (2016), Cavallino (2019), and Castillo et al. (2019). This literature argues that foreign exchange intervention can affect the exchange rate when domestic and external assets are imperfect substitutes. In this case, sterilized intervention increases the relative supply of domestic assets, driving risk premium up and exercising depreciation pressures on the exchange rate.

The third strand of the literature is the so called financial intermediation view of SFX interventions. The general equilibrium relevance of SFX interventions rely on a financial friction of the type associated to the unconventional monetary policy literature in closed economies. Céspedes et al. (2017) and Chang (2019) build models for an open economy with domestic banks subject to occasionally binding collateral constraints and find that SFX interventions have an impact on macroeconomic aggregates only when the relevant financial constraint is binding. When financial markets are frictionless, domestic banks are able to accommodate SFX interventions borrowing less or more from domestic depositors as well as from foreign financial markets. In the latter case, the general equilibrium is left undisrupted. Also, Fanelli and Straub (2019) finds that by including a pecuniary externality in a partial segmented domestic and foreign bond markets results in an excessively volatile exchange rates in response of capital inflows, making foreign exchange interventions desirable.

Empirical evidence on the effectiveness of SFX interventions has been particularly difficult to find because of endogeneity problems that made difficult the identification of its effects, especially on the exchange rate. While individual country studies report mixed results on the effectiveness of SFX intervention, cross-country studies generally find some effectiveness in curbing financial conditions and exchange rate dynamics (see Ghosh et al. (2018), Villamizar-Villegas and Perez-Reyna (2017), and Fratzscher et al. (2018)). Recent empirical findings have shed some light on how SFX intervention reduces the impact of capital flows on domestic financial conditions. For instance, Blanchard et al. (2015) show that capital flow shocks have significantly smaller effects on exchange rates and capital accounts in countries that intervene in Forex markets on a regular basis. According to Hofmann et al. (2019), SFX intervention has two mutually reinforcing effects. On one hand, during a period of easing of global financial condition SFX can be used to lean against the increase in bank lending after a dollar appreciation (the risk-taking channel of the exchange rate). On the other hand, there is a "crowding out" effect of bank lending due to the sterilization of FX interventions which increases the supply of domestic bonds absorbed by banks. The aggregate impact of SFX interventions results from the mixture of these two effects. By curbing domestic credit, SFX intervention will have an impact on the real economy.

2 A General Equilibrium Model

We build a medium-scale small open economy New Keynesian model extended with banks, foreign exchange market interventions, and a commodity sector. Following ABK, banks are allowed to finance their assets using two type of liabilities: domestic deposits and external borrowing from international financial markets. Nevertheless, banks lend not only in domestic currency (pesos) but also in foreign currency (dollars). Sterilized Foreign Exchange (SFX) intervention is introduced in order to study the role of this tool over financial intermediation, macroeconomic stabilization, and exchange rate volatility.

The rest of the model follows very closely the standard small open economy New Keynesian framework with the exception of two main features. First, we introduce an endogenous commodity sector in order to analyze the effect of commodity booms and busts in domestic financial conditions. We assume that the representative commodity producer accumulate its own capital facing standard capital adjustment costs and does not need external funding or any form of borrowing in order to produce. Second, we assume that intermediate good producers must borrow from banks before producing. In addition, we suppose the small open economy is financially dollarized, hence intermediate good producers demand a bundle of loans that consists in a combination of domestic and foreign currency denominated loans. Below, further details about the model are presented. For the rest of the document, small letters characterize individual variables while capital letters denote aggregates.

2.1 The Financial System

We follow Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) to introduce a banking sector in an otherwise standard infinite horizon macroeconomic model of a small open economy. In this setting, the representative household consists of a continuum of bankers and workers of measure unity. Workers supply labor and return labor income to the household. Workers, save in terms of bank deposits denominated in domestic currency while foreign agents lend to banks in foreign currency. Workers and foreign agents are precluded from lending directly to non-financial firms. All financial contracts between agents are short-term, non-contingent and thus, risk-less. An agency problem constraints the ability of banks to obtain funds from households and foreigners. How tight is the financial constraint that banks face depends on a measure of currency mismatch at the bank level. In this section, we focus the attention on bankers while workers are described in detail in section 2.3.

Banks. In a given household, each banker member manages a bank until she retires with probability $1-\sigma$. The retired bankers transfers any earnings back to the household in the form of dividends. Retired bankers are replaced by an equal number of workers randomly becoming bankers and keeping the relative proportion of each type of household members constant. New bankers receive a fraction ξ of total asset from the household as start-up funds.

Additionally, banks provide funding to producing firms without any financial friction. Hence, the only financially constrained agents in the model are banks due to a moral hazard problem between the bank and its depositors. Bank lending to firms is denominated in domestic and foreign currency denoted by l_t and l_t^* respectively. Banks' assets are also composed by central bank bonds, b_t, considered to be the only financial instruments used in the associated sterilization process of any SFX intervention. Bank investments are financed by home deposits, denominated in domestic currency d_t, by borrowing from foreigners in foreign currency d_t^*, or by using its own net worth n_t. A bank's balance sheet expressed in real terms is,

$$l_t + e_t l_t^* + b_t = n_t + d_t + e_t d_t^* \tag{1}$$

where e_t is the real exchange rate. Table 1 illustrates the typical balance sheet of a bank in the model.

Table 1. Bank's Balance Sheet

Assets	**Liabilities**
l_t	d_t
$e_t l_t^*$	$e_t d_t^*$
b_t	n_t

Net worth is accumulated through retained earnings and it is defined as the difference between the gross return on assets and the cost of liabilities:

$$n_{t+1} = R_{t+1}^l l_t + R_{t+1}^{l*} e_{t+1} l_t^* + R_{t+1}^b b_t - R_{t+1} d_t - e_{t+1} R_{t+1}^* d_t^* \tag{2}$$

where $\{R_t^b, R_t^l, R_t^{l*}\}$ denote the real gross returns to the bank on central bank bonds, domestic currency denominated loans, and foreign currency denominated loans respectively. Similarly, R_t and R_t^* are the real gross interest rate paid by the bank on domestic deposits and on foreign borrowing respectively.[3]

Agency Problem. With the purpose of limiting the bank's ability to raise domestic and foreign funds, we assume that at the beginning of the period, the banker may choose to divert funds from the assets it holds and transfer the proceeds to its own household. If bank managers operate honestly then assets will be held until payoffs are realized in the next period and repay their liabilities to creditors (domestic and foreign). On the contrary, if bank managers decide to divert funds then assets will be secretly channel away from investment and consumed by

[3] These rates are expressed as ex-post real interest rates. In this sense, R_t equals $\frac{1+i_{t-1}}{1+\pi_t}$ where i_t is the nominal policy rate.

the household of which they are members. In this framework, it is optimal for bank managers to retain earnings until exiting the industry. The objective of the banker is to maximize the expected discounted stream of profits that are transferred back to the household, i.e, its expected terminal wealth, given by

$$V_t = \mathbb{E}_t \left[\sum_{j=1}^{\infty} \Lambda_{t,t+j} \sigma^{j-1} (1-\sigma) n_{t+j} \right]$$

where $\Lambda_{t,t+j}$ is the stochastic discount factor of the representative household from $t+j$ to t and $\mathbb{E}_t[.]$ is the expectation operator conditional on information set at t. Notice that using $\Lambda_{t,t+j}$ to properly discount the stream of bank profits means that the household effectively owns the banks that its banker members manage. Bank managers will abscond funds if the amount they are capable to divert exceeds the continuation value of the bank V_t. Accordingly, for creditors to be willing to supply funds to the banker, any financial arrangement between them must satisfy the following incentive constraint:

$$V_t \geq \Theta(x_t) \left[\varpi l_t + \varpi^* e_t l_t^* + \varpi^b b_t \right] \tag{3}$$

where $\Theta_t(x)$ is assumed to be strictly increasing[4] and x_t is a measure of currency mismatch at he bank level defined and discussed below. We assume that it is harder to divert some assets than others. Specifically, the banker can divert a fraction $\Theta(x_t)\varpi$ of domestic currency loans, a fraction $\Theta(x_t)\varpi^*$ of foreign currency loans, and a fraction $\Theta(x_t)\varpi^b$ of the total amount of central banks bonds, where $\varpi, \varpi^*, \varpi^b \in [0, \infty)$. For instance, whenever $\varpi^b = 0$, bankers cannot divert sterilized bonds and buying them does not tighten the incentive constraint. As a consequence, a fraction of the interest rate spread on b_t may be arbitrage away, leaving R_t^b lower than R_t^l. In our setting, the three type of assets banks hold do not enter with equal weights into the incentive constraint reflecting that for some assets the constraint on arbitrage is weaker. We calibrate the ϖ, ϖ^*, and ϖ^b to match the average gross returns for each asset type in the Peruvian economy. In Section 3, we show that those targets are consistent with the fact that central bank bonds are much harder to divert than loans, that is, the calibrated ϖ^b is very close to zero. In Section 5 we relax this assumption and assume that all assets enter the incentive constraint with equal weights.

We assume that the banker's ability to divert funds depends upon the size of currency mismatch at the bank level expressed as a fraction of total assets. In this sense, we define x_t to be

$$x_t = \frac{e_t d_t^* - e_t l_t^*}{l_t + e_t l_t^* + b_t} \tag{4}$$

A higher currency mismatch at the bank level implies that the banker is able to divert a higher fraction of its assets, ultimately increasing the severity of the incentive constraint. In this regard, x_t measures the exposure of the bank's balance sheet to abrupt exchange rate movements and foreign capital reversals. A significant degree of currency mismatch in the balance sheet for a bank, places it in a more vulnerable position with respect to foreign shocks, particularly to shocks generating unexpected depreciations. From this perspective and as long as the incentive constraint is binding, an increase in x_t will require an increase in V_t in order to have domestic depositors and foreign lenders to be willing to continue lending funds

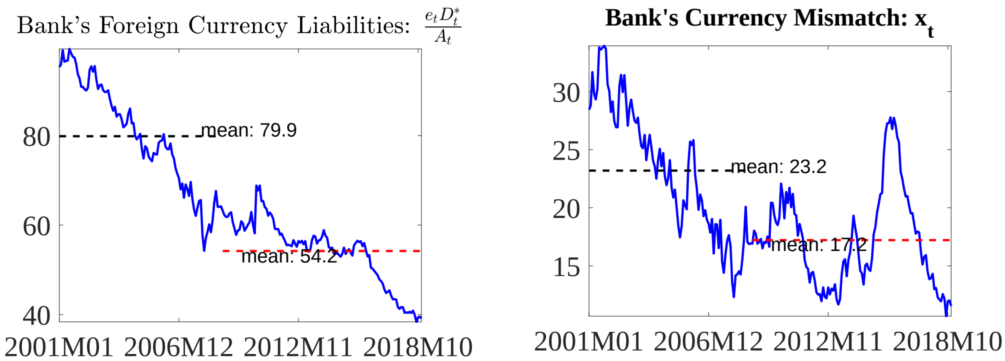
[4] Specifically, we use the following convex function:

$$\Theta(x) = \theta \left(1 + \frac{\varkappa}{2} x^2\right)$$

to the the bank. In the basic formulation of the model, we assume that x_t is internalized by each bank. In Section 5, we assume that x_t is external to the individual bank representing an aggregate measure of currency mismatch of the the banking system as a whole.

Figure 1 plots both the evolution of foreign currency liabilities and currency mismatch for the Peruvian banking system.[5] Foreign currency deposits as a fraction of total assets have been steadily decreasing since 2001 from an average of 79.9% during 2001-2008 to an average of 54.2% ever since. However, we can realize that bank's currency mismatch is much lower, but still positive, than the previous variable and also seems to be stable around 17.2% since 2006. In Section 3, we explain how these series are used to discipline the model.

FIGURE 1. CURRENCY MISMATCH IN PERUVIAN DATA, %



Bank's Recursive problem. Given a function $\Theta(x)$, a vector of interest rates, government policies, and n_t (state variable), each bank chooses its balance sheet components $(l_t, l_t^*, b_t, d_t, d_t^*)$ to maximize the franchise value:

$$V_t = \max_{l_t, l_t^*, b_t, d_t, d_t^*} \mathbb{E}_t \left[\Lambda_{t,t+1} \left\{ (1-\sigma) n_{t+1} + \sigma V_{t+1} \right\} \right]$$

subject to (1), (2), (3), and (4).

The bank's objective function as well as its balance sheet and the incentive constraint it faces can be expressed as a fraction of net worth. Moreover, using the definition of x_t, the problem of the bank can be written in terms of choosing each of the assets it holds as a fraction of net worth together with the optimal size of its currency mismatch x_t. Consequently, the problem of the bank is to choose (ϕ_t, ϕ_t^*, ϕ_t^b, x_t) to maximize its value as a fraction of net worth:

$$\psi_t = \max_{\phi_t^l, \phi_t^{l*} \phi_t^b, x_t} \quad \mu_t^l \phi_t^l + (\mu_t^{l*} + \mu_t^{d*}) \phi_t^{l*} + \mu_t^b \phi_t^b + \mu_t^{d*} \left(\phi_t^l + \phi_t^{l*} + \phi_t^b \right) x_t + \upsilon_t \tag{5}$$

subject to:

$$\psi_t - \Theta(x_t) \left[\varpi \phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b \right] \geq 0 \tag{6}$$

where $\psi_t = \frac{V_t}{n_t}$, $\phi_t = \frac{l_t}{n_t}$, $\phi_t^* = \frac{e_t l_t^*}{n_t}$, $\phi_t^b = \frac{b_t}{n_t}$, $\upsilon_t = \mathbb{E}_t \left[\Omega_{t+1} R_{t+1} \right]$, and

$$\mu_t^l = \mathbb{E}_t \left[\Omega_{t+1} \left(R_{t+1}^l - R_{t+1} \right) \right]$$

[5] We take the consolidated balance sheet of the banking system to data using Peruvian data. On the one hand, we take banking credit in domestic currency as L_t, banking credit in foreign currency as L_t^*, and baking investments as B_t. On the other hand, total net worth of banks is taken as N_t and the sum of foreign currency deposits with foreign currency external liabilities is considered as D_t^*. Finally, the counterpart for D_t is obtained as a residual.

$$\mu_t^{l*} = \mathbb{E}_t\left[\Omega_{t+1}\left(\frac{e_{t+1}}{e_t}R_{t+1}^{l*} - R_{t+1}\right)\right]$$
$$\mu_t^{b} = \mathbb{E}_t\left[\Omega_{t+1}\left(R_{t+1}^{b} - R_{t+1}\right)\right]$$
$$\mu_t^{d*} = \mathbb{E}_t\left[\Omega_{t+1}\left(R_{t+1} - \frac{e_{t+1}}{e_t}R_{t+1}^{*}\right)\right]$$

Ω_{t+1} is the shadow value of a unit of net worth to the bank at $t+1$, given by

$$\Omega_{t+1} = \Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1})$$

Let λ_t^b be the Lagrangian multiplier for the incentive constraint faced by the bank, eq. (6). Then, the first order conditions are characterized by the slackness condition associated to eq. (6) and:[6]

$$\mu_t^{l} + \mu_t^{d*}x_t = \frac{\lambda_t^b}{1+\lambda_t^b}\varpi\Theta(x_t) \tag{7}$$
$$\mu_t^{l*} + \mu_t^{d*}(1+x_t) = \frac{\lambda_t^b}{1+\lambda_t^b}\varpi^*\Theta(x_t) \tag{8}$$
$$\mu_t^{b} + \mu_t^{d*}x_t = \frac{\lambda_t^b}{1+\lambda_t^b}\varpi^b\Theta(x_t) \tag{9}$$
$$\mu_t^{d*}\left(\phi_t^l + \phi_t^{l*} + \phi_t^b\right) = \frac{\lambda_t^b}{1+\lambda_t^b}\left(\varpi\phi_t^l + \varpi^*\phi^{l*} + \varpi^b\phi_t^b\right)\frac{\partial\Theta(x_t)}{\partial x} \tag{10}$$

When the incentive constraint is not binding then $\lambda_t^b = 0$, the discounted excess returns or interest rate spreads are zero. Consequently, under this equilibrium, financial markets are frictionless implying that the standard arbitrage condition holds: banks will acquire assets to the point where the discounted return on each asset equals the discounted cost of deposits (i.e., $\mu_t^l = \mu_t^{l*} = \mu_t^b = 0$). In addition, there is no cost advantage of foreign borrowing over domestic deposits (i.e., $\mu_t^{d*} = 0$, the UIP conditions holds).

When the incentive constraint is binding, $\lambda_t^b > 0$, banks are restricted to obtain funds from creditors. In this context, limits to arbitrage emerge in equilibrium, leading to interest rate spreads. It is important to highlight that excess returns increase with how tightly the incentive constraint binds. The latter is measured by λ_t^b. The intuition behind the above first order conditions is that banks invest in each asset to the point where the marginal benefit of acquiring an additional unit of each asset is equal to its marginal cost. The marginal benefit of each asset is composed by its own discounted excess value plus the excess value associated to the advantage cost of funding it by foreign borrowing which is ultimately influenced by the size of the currency mismatch[7]. For instance, a fraction x_t of an extra unit of l_t or b_t is funded by d_t^*. Similarly, a portion $1+x_t$ of an additional investment in l_t^* is financed by d_t^* a portion $1+x_t$, i.e., banks use more foreign currency funds and less home deposits for each

[6] A complete derivation of the bank's optimality conditions are presented in Appendix D.1

[7] Note that the marginal benefit for each asset can be rewritten in terms of interest rate spreads as

$$\mu_t^{l} + \mu_t^{d*}x_t = \mathbb{E}_t\left[\Omega_{t+1}\left(R_{t+1}^{l} - \left\{\frac{e_{t+1}}{e_t}R_{t+1}^{*}x_t + R_{t+1}(1-x_t)\right\}\right)\right]$$
$$\mu_t^{b} + \mu_t^{d*}x_t = \mathbb{E}_t\left[\Omega_{t+1}\left(R_{t+1}^{b} - \left\{\frac{e_{t+1}}{e_t}R_{t+1}^{*}x_t + R_{t+1}(1-x_t)\right\}\right)\right]$$
$$\mu_t^{l*} + \mu_t^{d*}(1+x_t) = \mathbb{E}_t\left[\Omega_{t+1}\left(R_{t+1}^{l*} - \left\{\frac{e_{t+1}}{e_t}R_{t+1}^{*}(1+x_t) + R_{t+1}(-x_t)\right\}\right)\right]$$

Then, it is clear that x_t strongly influences the fraction financed by foreign currency deposits for each asset.

unit of foreign currency loans. On the other hand, the marginal cost associated to each asset is given by the marginal cost of tightening the incentive constraint times the total fraction of the asset that the bank may actually divert.

Limits to arbitrage emerge from the restriction that the incentive constraint places on the size of a bank's portfolio relative to its net worth. A form of leverage ratio for a bank can be obtained by combining eq. (5), eq. (6), and and the above first order conditions,

$$\Phi_t n_t \geq \varpi l_t + \varpi^* e_t l_t^* + \varpi^b b_t \tag{11}$$

$$\Phi_t = \frac{\varpi v_t}{\varpi \Theta(x_t) - \left(\mu_t^l + \mu_t^{d*} x_t\right)} \tag{12}$$

Gertler and Karadi (2013) argued that Φ_t can be interpreted as the maximum ratio of weighted assets to net worth that the bank may hold without violating the incentive constraint. The weight applied to each asset is the proportion of the asset that the bank is able to divert.

When the incentive constraint binds the weighted leverage ratio Φ_t is increasing in two factors: 1) the savings of deposits costs from another unit of net worth given by v_t and 2) the discounted marginal benefit of lending in domestic currency. As discussed in Gertler et al. (2012), both factors raise the value of a Bank inducing its creditors to be willing to lend more. The leverage ratio also varies inversely with exchange risk perceptions: whenever the currency mismatch rises, bankers are more exposed to real exchange movements and its creditors restrict external funding. Notice that in a closed economy setting, μ_t^{d*} is zero and Φ_t constant. In this case, eq. (12) reduces to Gertler and Karadi (2013) set up for the bank's leverage ratio.

The leverage ratio can be expressed as a collateral constraint consistent with Kiyotaki and Moore (1997) as follows:

$$l_t \leq \theta_t n_t \qquad \text{and} \qquad \theta_t = \frac{\Phi_t}{\varpi} - \frac{\varpi^*}{\varpi}\phi_t^* - \frac{\varpi^b}{\varpi}\phi_t^b$$

where $\phi_t^* = \frac{e_t l_t}{n_t}$ and $\phi_t^b = \frac{b_t}{n_t}$. Recently, Céspedes et al. (2017) and Chang (2019) use a similar collateral constraints to capture foreign debt limits faced by domestic banks of emerging economies. However, in our more general framework, θ_t is not a parameter but an endogenous variable that depends on a measure of currency mismatch at the bank level. In our setting, similar collateral constraints for l_t^* and b_t can be obtained straightforwardly[8].

2.2 The Central Bank and SFX Interventions

The related literature on SFX interventions (for example, Chang (2019)) agrees in defining it as the following situation: whenever a central bank sells or buys foreign exchange and at the same time it also buys or sells an equivalent amount of securities denominated in domestic currency. Under this policy, the net credit position of the central bank changes. Without sterilization, buying or selling foreign exchange, would directly affect the supply of domestic liquidity. The latter implies difficulties for the central bank in keeping its interbank

[8] These collateral constraints are:

$$e_t l_t^* \leq \theta_t^* n_t \qquad \text{and} \qquad \theta_t^* = \frac{\Phi_t}{\varpi^*} - \frac{\varpi}{\varpi^*}\phi_t^l - \frac{\varpi^b}{\varpi^*}\phi_t^b$$

$$b_t \leq \theta_t^b n_t \qquad \text{and} \qquad \theta_t^b = \frac{\Phi_t}{\varpi^b} - \frac{\varpi^*}{\varpi^b}\phi_t^* - \frac{\varpi}{\varpi^b}\phi_t^l$$

interest rate target which ultimately is determined by a Taylor rule. Nevertheless, there is less agreement in the literature about the implementation of the sterilization leg of an SFX intervention. This is a reflection of different practices at central banks that intervene in forex markets.

In our framework the sterilization operations associated to any SFX intervention is implemented by changing the supply of central bank bonds in the banking system. Recall that central bank bonds are risk less one period bonds issued by the monetary authority. Accordingly, SFX intervention denotes the following: If the central bank buys (sells) foreign exchange, for example dollars, from (to) the domestic banking system, a simultaneous raise (fall) in official foreign reserves would occur. At the same time, the central bank will completely offset the effect over domestic liquidity by issuing (retiring) central bank bonds to (from) the banking system. The central bank's balance sheet is given by

$$B_t = e_t F_t \tag{13}$$

where B_t denotes central bank bonds and F_t official foreign exchange reserves. Notice that eq. (13) serves both as a sterilization rule and as accounting identity for the central bank's balance sheet. In this setting, SFX interventions induce the central bank to produce operational losses or quasi-fiscal deficit since it is assumed that official foreign exchange reserves are invested abroad at the foreign interest rate R_t^* while central bank bonds pay R_t^b. Then, the Central Bank's quasifiscal deficit is:

$$CB_t = \left(R_t^b - \frac{e_t}{e_{t-1}} R_t^* \right) B_{t-1} \tag{14}$$

As long as $R_t^b > R_t^*$ the central bank produce operational losses associated to the sterilization process which ultimately represents the fiscal costs of SFX interventions. We assume that any operational losses are transferred to the central government and financed through lump sum taxes on households.

Furthermore, in addition to the standard policy rate rule, the central bank implements the following SFX intervention rule written in terms of the supply of central bank bonds responding to deviations of the exchange rate with respect to its steady-state value:

$$\ln B_t = (1 - \rho_B) B + \rho_B \ln B_{t-1} - \upsilon_e (\ln e_t - \ln e) \tag{15}$$

with $\upsilon_e > 0$ and $0 < \rho_B < 1$ being parameters that measures the intensity in which FX interventions respond to exchange rate movements and its persistence respectively. Under this rule, the central bank sells official foreign exchange reserves in response to a real depreciation (i.e, whenever the real exchange rate is above its steady state value). As mentioned before, the counterpart of selling reserves is to withdraw central bank bonds from the balance sheet of banks, eq. (13). Consequently, SFX interventions present two potential transmission mechanisms in our framework: 1) when selling official foreign exchange reserves to the banking system, the exchange rate is stabilized and 2) when sterilizing the effect over domestic liquidity, the central bank frees resources from domestic banks to extend additional loans to firms. Moreover, the exchange rate stabilization effect potentially affects the size of the currency mismatch at the bank level. For instance, ceteris paribus, stabilizing a depreciatory pressure on the exchange rate may lead to reduce the size of the currency mismatch at the bank level compared to the situation without SFX intervention. If this is the case, the incentive constraint (more specifically, its the degree of tightening) may be relaxed even further, stimulating even more domestic financial conditions.

One key aspect of our model is that SFX interventions are relevant for determining the general equilibrium allocation only when the incentive constraint binds, as in Céspedes et al.

(2017) and Chang (2019). Whenever the incentive constraint is not binding financial markets are frictionless meaning there is no leverage constraint for banks nor interest rate spreads. Therefore, balance sheet policies such as SFX interventions are irrelevant since the size and composition of balance sheets, for both the banking system and the central bank, do not matter for equilibrium. In particular, under frictionless financial markets the sterilization process associated to SFX interventions does not have real effects: the exchange rate as well as domestic financial conditions are determined without any consideration to balance sheets. More important, in our framework and differently to Chang (2019), domestic banks can accommodate the accumulation of foreign exchange reserves by the central bank during "normal" times (non-binding incentive constraint), by increasing domestic deposits, foreign borrowing or both since banks are indifferent between domestic or foreign currency funding. Therefore, when the incentive constraint is not binding and the central bank accumulates foreign exchange reserves it does not necessary mean that banks will end up more exposed to foreign borrowing.

We consider that for emerging markets, such as the Peruvian economy, financial constraints are always binding even in "normal" times. The difference between normal times and a financial crisis, is how tight financial constraints bite. In our framework, the degree of financial constraint tightening depends on the size of the currency mismatch presented in the balance sheet of banks which ultimately respond to foreign shocks. In this context, SFX Interventions are meant to be an additional policy instrument for the central bank aimed to smooth the response of domestic financial conditions to foreign shocks via the stabilization of the exchange rate.

2.3 Households

Workers supply labor and return the labor income to the household they belong to. The household use labor income and profits from firm ownership to consume non-commodity goods and save in terms of bank deposits denominated in domestic currency[9]. Finally, as it is standard in the literature on financial and labor market frictions, it is assumed that within the family there is perfect consumption insurance in order to keep the representative agent assumption. Following Miao and Wang (2010) and Gertler et al. (2012), the household's preference structure is

$$\mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \frac{1}{1-\gamma} \left(C_{t+j} - \mathcal{H} C_{t+j-1} - \frac{\zeta_0}{1+\zeta} H_{t+j}^{1+\zeta} \right)^{1-\gamma} \right] \tag{16}$$

where C_t is consumption and H_t is the labor effort given by hours worked. The subjective discount factor is given by $\beta \in (0,1)$, $\gamma > 0$ measures the elasticity of intertemporal substitution, while ζ_0 controls the dis-utility of labor. Also, the Frisch elasticity is mainly determined by the interaction $\zeta > 0$ an the degree of internal habit formation, $\mathcal{H} \in [0,1)$. For instance, if there is no habit formation, i.e. $\mathcal{H} = 0$, this specification abstracts from wealth effects on labor supply as in Greenwood et al. (1988), and the Frisch elasticity is $1/\zeta$.[10]

Bank deposits are assumed to be one-period riskless real assets that pay a gross real return of R_t from period $t-1$ to t. Let D_t be the total quantity of real bank deposits of households,

[9]We consider that home residents do not hold foreign bonds because the real interest rate tends to be lower in foreign country than our emerging market economy (unless the incentive to insure against exchange risk is large).

[10]For a complete examination of the labor supply function in the general case $\mathcal{H} \in [0,1)$, see Appendix E.1.

w_t the real wage, Π_t net payouts to the household from ownership of both financial and non-financial firms, and T_t, lump-sum taxes. Hence, the household budget constraint is written as

$$C_t + D_t + T_t = w_t H_t + \Pi_t + R_t D_{t-1} \tag{17}$$

Notice that Π_t includes the net transfer to its members that become bankers at the beginning of the period as it is written as

$$\Pi_t = \underbrace{\Pi_t^g}_{\text{Good Producer}} + \underbrace{\Pi_t^k}_{\text{Capital Producer}} + \underbrace{(1-\sigma)N_t}_{\text{Retiring bankers}} \underbrace{-\xi\left(R_t^l L_{t-1} + R_t^{l*} e_t L_{t-1}^* + R_t^b B_{t-1}\right)}_{\text{Bankers' start-up funds}}$$

Hence, the representative worker chooses consumption, labor supply, and bank deposits, to maximize eq. (16) subject to eq. (17). Let u_{ct} denote the marginal utility of consumption and $\Lambda_{t,t+1}$ the household's stochastic discount factor, then the household's first order conditions for labor supply and consumption/saving decision are

$$\mathbb{E}_t u_{ct} w_t = \zeta_0 H_t^{\zeta} \left(C_t - \mathcal{H} C_{t-1} - \frac{\zeta_0}{1+\zeta} H_t^{1+\zeta} \right)^{-\gamma} \tag{18}$$

$$1 = \mathbb{E}_t \left[R_{t+1} \Lambda_{t,t+1} \right] \tag{19}$$

with

$$u_{ct} = \left(C_t - \mathcal{H} C_{t-1} - \frac{\zeta_0}{1+\zeta} H_t^{1+\zeta} \right)^{-\gamma} - \mathcal{H}\beta \mathbb{E}_t \left(C_{t+1} - \mathcal{H} C_t - \frac{\zeta_0}{1+\zeta} H_{t+1}^{1+\zeta} \right)^{-\gamma}$$

$$\Lambda_{t,t+1} = \beta \frac{u_{c,t+1}}{u_{ct}}$$

2.4 The production sector

There are four types of non-financial firms that constitute the production side of the model economy: 1) Non-commodity final good producers, 2) Intermediate good producers, 3) Capital good producers, and 4) The commodity production sector that takes as given world commodity prices and external demand.

Non-Commodity Final Good Producers. Final goods in the non-commodity sector are produced under perfect competition and using a variety of differentiated intermediate goods y_{jt}^{nc}, with $j \in [0,1]$, according to the following constant returns to scale technology

$$Y_t^{nc} = \left(\int_0^1 y_{jt}^{nc\,\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} \tag{20}$$

where $\eta > 1$ is the elasticity of substitution across goods. The representative firm chooses y_{jt}^{nc} to maximize profits subject to the production function eq. (20) with profits given by:

$$P_t^{nc} Y_t^{nc} - \int_0^1 p_{jt}^{nc} y_{jt}^{nc} dj,$$

The first order conditions for the jth input are

$$y_{jt}^{nc} = \left(\frac{p_{jt}^{nc}}{P_t^{nc}} \right)^{-\eta} Y_t^{nc}$$

$$P_t^{nc} = \left(\int_0^1 p_{jt}^{nc\,1-\eta} dj \right)^{\frac{1}{1-\eta}}$$

The final homogeneous good can be used either for consumption or to produce capital goods. In addition, part of the final good production is exported for foreign consumption.

Intermediate Good Producers. There is a continuum of monopolistically competitive firms, indexed by $j \in (0,1)$, producing differentiated intermediate goods that are sold to final good producers. Each firm manufactures a single variety, face nominal rigidities in the form of price adjustment costs as in Rotemberg (1982) and pay for their capital expenditures in advance of production with funds borrowed from banks. Each intermediate good producer operates the following constant return to scale technology with three inputs: capital k_{t-1}^{nc}, imported goods m_t, and labor l_t

$$y_{jt}^{nc} = A_t^{nc} \left(\frac{k_{j,t-1}^{nc}}{\alpha_k} \right)^{\alpha_k} \left(\frac{m_{jt}}{\alpha_m} \right)^{\alpha_m} \left(\frac{h_{jt}}{1-\alpha_k-\alpha_m} \right)^{1-\alpha_k-\alpha_m} \tag{21}$$

where α_k, α_m, and $\alpha_k + \alpha_m \in (0,1)$. Also, A_t^{nc} denotes a neutral technology process that follows

$$\ln A_t^{nc} = (1-\rho_{A^{nc}}) \ln A^{nc} + \rho_{A^{nc}} \ln A_{t-1}^{nc} + u_t^{A^{nc}} \tag{22}$$

We assume that intermediate good producers must borrow from banks in order to acquire capital for production. After obtaining bank loans, each intermediate good producer buys capital from capital good producers at the unitary price q_t^{nc}. Furthermore, in order to reflect the presence of credit dollarization in some emerging economies and the fact that partially dollarized economies might be more vulnerable to external shocks, we assume that an intermediate good producer need a combination of domestic, and foreign currency loans in order to buy capital. The combination of both types of loans is achieved assuming a CES technology that yields a unit measure of disposable fund, $\mathcal{F}$. Thus, the loan bundle that an intermediate good producer need to buy capital is the following:

$$\mathcal{F}_t = A_t^e \left[(1-\delta^f)(l_{jt})^{\frac{\vartheta_l - 1}{\vartheta_l}} + \delta^f (e_t l_{jt}^*)^{\frac{\vartheta_l - 1}{\vartheta_l}} \right]^{\frac{\vartheta_l}{\vartheta_l - 1}} \tag{23}$$

Where A_t^e is the productivity level for aggregating loans, ϑ_l measures the elasticity of substitution of foreign and domestic currency loans, and the parameter δ^f controls the degree of credit dollarization in the economy. Finally, at the end of the period, intermediate good producers sell the undepreciated capital, $\lambda_{nc} k_{jt-1}^{nc}$, to capital good producers.

First order conditions for intermediate good producers are presented in three groups[11], each associated to the following production stages: cost minimization, borrowing from banks, and price setting. The cost minimization stage yields the standard conditional demands for each input:

$$z_t = \alpha_k mc_t \frac{y_{jt}^{nc}}{k_{jt-1}^{nc}} \tag{24}$$

$$e_t = \alpha_m mc_t \frac{y_{jt}^{nc}}{m_{jt}} \tag{25}$$

$$mc_t = \frac{1}{A_t^{nc}} z_t^{\alpha_k} e_t^{\alpha_m} w_t^{1-\alpha_k-\alpha_m} \tag{26}$$

[11] See appendix E.2 for a detail derivation of the following equations.

The borrowing stage is characterized by a non-arbitrage condition (see eq. (27) below) that defines the return on capital and loan demands in domestic and foreign currency (eq. (28) and eq. (29)):

$$R_t^k = \frac{z_t + \lambda_{nc} q_t^{nc}}{q_{t-1}^{nc}} \tag{27}$$

$$l_{jt} = (1-\delta^f)\left(\frac{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^k}{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^l}\right)^{\vartheta_l} (A^e)^{\vartheta_l - 1} q_t^{nc} k_{jt}^{nc} \tag{28}$$

$$e_t l_{jt}^* = \delta^f \left(\frac{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^k}{\mathbb{E}_t \Lambda_{t,t+1} \frac{e_{t+1}}{e_t} R_{t+1}^{l*}}\right)^{\vartheta_l} (A^e)^{\vartheta_l - 1} q_t^{nc} k_{jt}^{nc} \tag{29}$$

The demand schedules for domestic and foreign currency loans depend directly on the expected return on capital as well as on the current value of acquired capital by each firm and inversely on the expected interest rate cost of each type of credit. Therefore, in equilibrium the degree of credit dollarization is an endogenous variable that depends on domestic financial conditions.

Finally, the price setting stage is characterized by the following New Keynesian Phillips curve:

$$(1+\pi_t)\pi_t = \frac{1}{\kappa}(1-\eta+\eta mc_t) + \mathbb{E}_t\left[\Lambda_{t,t+1}(1+\pi_{t+1})\pi_{t+1}\frac{Y_{t+1}^{nc}}{Y_t^{nc}}\right] \tag{30}$$

Capital Good Producers. There is a continuum of capital producers operating in a competitive market. Each capital good producer use as inputs the final good in the form of non-commodity investment as well as the undepreciated capital bought from intermediate good producers. New capital is produced using the following technology:

$$K_t^{nc} = I_t^{nc} + \lambda_{nc} K_{t-1}^{nc} \tag{31}$$

where K_t^{nc} is sold to intermediate good producers at the price q_t^{nc}. Producing capital implies an additional cost of $\Phi^{nc}\left(\frac{I_t^{nc}}{I_{t-1}^{nc}}\right) I_t^{nc}$ which represent adjustment cost of investment. The latter assumption is introduces to replicate some empirical moments [12]. Given that households own the capital good firm, the objective of a capital producer is to choose $\{I_{t+j}^{nc}\}_{j\geq 0}$ to solve:

$$\mathbb{E}_t\left[\sum_{j=0}^{\infty} \Lambda_{t,t+j}\left(q_{t+j}^{nc} I_{t+j}^{nc} - \left[1+\Phi^{nc}\left(\frac{I_{t+j}^{nc}}{I_{t+j-1}^{nc}}\right)\right] I_{t+j}^{nc}\right)\right]$$

Profit maximization implies that the price of capital goods is equal to the marginal cost of investment goods production as follows:

$$q_t^{nc} = 1 + \Phi^{nc}\left(\frac{I_t^{nc}}{I_{t-1}^{nc}}\right) + \left(\frac{I_t^{nc}}{I_{t-1}^{nc}}\right)\partial\Phi_t^{nc} - \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{I_{t+1}^{nc}}{I_t^{nc}}\right)^2 \partial\Phi_{t+1}^{nc}\right] \tag{32}$$

where $\partial\Phi_t^{nc}$ denotes the derivative of $\Phi^{nc}(.)$ evaluated at $\frac{I_t^{nc}}{I_{t-1}^{nc}}$.

Commodity Sector. Commodity price movements play a major role in commodity-exporting emerging market economies. The conventional wisdom is that fluctuations in terms of trade constitute an important driver of business cycle fluctuations in emerging market economies. In particular, commodity booms generate real as well as credit booms.[13]

[12] The function $\Phi^{nc}()$ must satisfy some restrictions: $\Phi^{nc}(1) = (\Phi^{nc})'(1) = 0$ and $(\Phi^{nc})''(.) > 0$.

[13] For empirical evidence on this fact, see Fornero et al. (2015), Shousha (2016), Fernández et al. (2017), Garcia-Cicco et al. (2017), and Drechsel and Tenreyro (2018).

We introduce a commodity sector with a representative firm that produces a homogeneous commodity good taking as given world commodity prices and external demand. We assume this firm is owned in part by foreign agents and in part by domestic agents. Commodity production is entirely exported abroad and is conducted using capital specific to this sector as the only input. Capital is acquired directly from final good producers and it is used to produce commodity-sector capital without any lending from the banking system. Technology in this sector is

$$Y_t^c = A^c (K_{t-1}^c)^{\alpha_c} \tag{33}$$

Where Y_t^c is the commodity production, K_t^c is the specific capital for commodity sector, and A^c is the productivity level for this sector.

The representative commodity producer faces investment adjustment costs of $\Phi^c\left(\frac{I_t^c}{I_{t-1}^c}\right)$. Thus, capital accumulation is done through the following equation:

$$K_t^c = I_t^c + \lambda_c K_{t-1}^c \tag{34}$$

The problem of the representative producer in the commodity sector is to choose $\{K_{t+s}^c\}_{s\geq 0}$ and $\{I_{t+s}^c\}_{s\geq 0}$ to maximize:

$$\sum_{s=0}^{\infty} \Lambda_{t,t+s}^c \left(p_{t+s}^c A^c (K_{t+s-1}^c)^{\alpha_c} - \left[1 + \Phi^c\left(\frac{I_{t+s}^c}{I_{t+s-1}^c}\right)\right] I_{t+s}^c \right)$$

subject to eq. (33). The first order conditions for the above problem are

$$q_t^c = \left[1 + \Phi^c\left(\frac{I_t^c}{I_{t-1}^c}\right)\right] + \left(\frac{I_t^c}{I_{t-1}^c}\right) \partial\Phi_t^c - \mathbb{E}_t\left[\Lambda_{t,t+1}^c \left(\frac{I_{t+1}^c}{I_t^c}\right)^2 \partial\Phi_{t+1}^c\right] \tag{35}$$

$$1 = \mathbb{E}_t\left[\Lambda_{t,t+1}^c \frac{\alpha_c p_{t+1}^c \frac{Y_{t+1}^c}{K_{t+1}^c} + q_{t+1}^c \lambda^c}{q_t^c}\right] \tag{36}$$

where $\partial\Phi_t^c$ denotes the derivative of $\Phi^c(.)$ evaluated at $\frac{I_t^c}{I_{t-1}^c}$ and q_t^c is the shadow price for the commodity specific stock of capital.

Finally, we assume that a fraction $(1 - \chi^c)$ of the profits is transfered abroad to foreign owners. The aggregate profit in the commodity sector is given by

$$\Pi_t^c = p_t^c A^c (K_{t-1}^c)^{\alpha_c} - \left[1 + \Phi^c\left(\frac{I_t^c}{I_{t-1}^c}\right)\right] I_t^c \tag{37}$$

It is worth mentioning that in our framework a commodity boom directly raises the demand for domestic final goods since non commodity investment is used as an input for producing specific capital to the commodity sector. The latter occurs independently of the standard wealth effect that surges in commodity prices generates when this sector is modeled as an exogenous endowment. Furthermore, the demand for credit also increases as a response to both, the wealth effect and the increase in the production of intermediate goods needed to support the higher demand for final goods.

2.5 External Sector

We assume that foreign demand for non-commodity final goods is a decreasing function of relative price $\frac{1}{e_t}$ but increasing with the foreign income Y_t^* as

$$Y_t^{nc,x} = e_t^{\varphi_1} (Y_t^*)^{\varphi_2} \tag{38}$$

where $\varphi_1 > 0$ is the price elasticity while $\varphi_2 > 0$ is the foreign output elasticity for non-commodity exports.

The foreign sector block hast its own dynamic outside the domestic macroeconomic equilibrium and does not have feedback from domestic variables. We consider as external variables the foreign output Y_t^*, foreign interest rate R_t^*, and the commodity price index p_t^{wc}, and collect these variables in the vector $\hat{\mathbf{X}}_t$ which captures the cyclical movements of these variables, i.e.,

$$\hat{\mathbf{X}}_t = \begin{bmatrix} \hat{Y}_t^* \\ \hat{R}_t^* \\ \hat{p}_t^* \end{bmatrix}$$

where $\hat{Y}_t^* = \ln \frac{Y_t^*}{Y^*}$, $\hat{R}_t^* = R_t^* - R^*$, and $\hat{p}_t^* = \ln \frac{p_t^{wc}}{p^{wc}}$. Then, we assume that $\hat{\mathbf{X}}_t$ follows a vector autoregressive equation written as

$$\hat{\mathbf{X}}_t = \mathbf{C}\hat{\mathbf{X}}_{t-1} + \mathbf{B}\mathbf{u}_t^X \tag{39}$$

where $\mathbf{C}$ and $\mathbf{B}$ are 3×3 matrices that rule the dynamics of the vector $\hat{\mathbf{X}}_t$, and $\mathbf{u}_t^X$ is the vector of external structural shocks from which we analyze its consequences. Section 3 presents further details in the way we calibrate eq. (39) and identify its structural shocks.

2.6 Central Government

The consolidated government collects taxes from household and receives a fraction χ^c of commodity firms' profits. These resources are then used to finance public consumption G_t and central bank operational losses CB_t:

$$\chi^c \Pi_t^c + T_t = CB_t + G_t \tag{40}$$

where G_t is modeled as a first order autoregressive process written as

$$G_t = (1 - \rho_G)G + \rho_G G_{t-1} + u_t^G \tag{41}$$

where ρ_G controls the persistence of public expenditure dynamics. It is worthy to notice that eq. (40) indicates that either commodity price cycles or central bank's operational losses will strongly affect household's decisions through variations in lump-sum taxes.

We also assume that the monetary authority sets the short-term nominal interest i_t rate according to a simple feedback rule belonging to an standard Taylor-type rule:

$$i_t - i = \rho_i(i_{t-1} - i) + (1 - \rho_i)\omega_\pi \pi_t + u_t^i \tag{42}$$

where ρ_i measures nominal policy rate smoothing, ω_π controls the degree in the response of the policy rate to inflation rate variations, and u_t^i represents monetary policy shocks. In order to have a stable equilibrium, this rule should satisfy the Taylor principle, i.e., $\omega_\pi > 1$.

2.7 Market Equilibrium

The non-commodity output is either consumed, invested, exported, or used to pay the cost of adjusting prices, the cost of changing investment decisions, and the resources wasted after aggregating funds at the intermediate good producer level,

$$Y_t^{nc} = C_t + G_t + \left[1 + \Phi^c\left(\frac{I_t^c}{I_{t-1}^c}\right)\right] I_t^{nc} + \left[1 + \Phi^c\left(\frac{I_t^c}{I_{t-1}^c}\right)\right] I_t^c + Y_t^{x,nc} + REST_t \tag{43}$$

where

$$REST_t = \frac{\kappa}{2}\pi_t^2 Y_t^{nc} + L_t + e_t L_t^* - q_t^{nc} K_t^{nc}$$

The Gross Domestic Product (GDP) equals to the sum of the added value of non-commodity sector and the added value of commodity sector, all priced at constant prices:

$$GDP_t = Y_t^{nc} - eM_t + p^c Y_t^c \tag{44}$$

where p^c and e are the steady state level for commodity price index and real exchange rate, respectively. Then GDP_t uniquely captures real output movements and is not affected by valuation effects.

The aggregate net foreign asset position B_t^*, which equals to the foreign exchange official reserves less the aggregate level of foreign currency liabilities in the baking system, i.e. $F_t - D_t^*$, evolves through the trade balance net the fraction of commodity firm's profits transfered abroad and the financial income of net foreign assets from the previous period,

$$e_t \left[B_t^* - R_t^* B_{t-1}^*\right] = Y_t^{x,nc} + p_t^c Y_t^c - e_t M_t - (1-\chi^c)\Pi_t^c \tag{45}$$

Finally, since optimal bank's decisions does not depend on bank-specific factors, the aggregation for the baking system variables is straightforward. In appendix D.2, we show that the total net worth evolves according to:

$$\begin{aligned} N_t = &\left[(\sigma+\xi)R_t^l - \sigma R_t\right] L_{t-1} + \left[(\sigma+\xi)\frac{e_t}{e_{t-1}} R_t^{l*} - \sigma R_t\right] e_{t-1} L_{t-1}^* \\ &+ \left[(\sigma+\xi)R_t^b - \sigma R_t\right] B_{t-1} + \sigma\left[R_t - \frac{e_t}{e_{t-1}} R_t^*\right] e_{t-1} D_{t-1}^* + \sigma R_t N_{t-1} \end{aligned} \tag{46}$$

3 Calibration Strategy

We discipline the model in order to replicate some relevant unconditional as well as conditional moments for the Peruvian economy. We calibrate a subset of the parameters to be consistent with some steady-state targets associated to historical means. Additionally, we follow Schmitt-Grohe and Uribe (2018) to calibrate another subset of parameters by using a limited information method based on an impulse response matching function estimator. For this, we estimate an SVAR with two recursive blocks and estimate some parameters of the economic model by minimizing the distance between the structural impulse responses implied by the macroeconomic model and the corresponding empirical impulse responses implied by the SVAR model. Let Ξ be the subset of parameters to be estimated by matching the impulse responses to foreign shocks, $\mathcal{M}^{\text{data}}$ the corresponding empirical impulse responses from the SVAR model, and $\mathcal{M}^{\text{model}}$ the theoretical counterpart of $\mathcal{M}^{\text{data}}$. Then we set Ξ to be the solution to the following problem

$$\Xi^* = arg \min_{\Xi} \quad \sum_{i=1}^{k} \varrho_i \times abs(\mathcal{M}_i^{\text{model}}(\Xi) - \mathcal{M}_i^{\text{data}}) \tag{47}$$

where $abs(.)$ is the absolute value function and ϱ_i denoted the width of the 66% confidence interval associated with the ith variable in $\mathcal{M}^{\text{data}}$.

Empirical VAR Specification. We consider an SVAR model with two blocks similar to Canova (2005), Cushman and Zha (1997), and Zha (1999). Let $\mathbf{X}_t$ denote the vector of foreign variables and $\mathbf{D}_t$ the vector of domestic variables. In the baseline specification, each block is composed by the following variables:

$$\mathbf{X}_t = \begin{bmatrix} Y_t^* \\ R_t^* \\ p_t^{wc} \end{bmatrix}, \qquad \mathbf{D}_t = \begin{bmatrix} tb_t \\ GDP_t \\ C_t \\ I_t \\ L_t \\ eL_t^* \\ e_t \end{bmatrix}$$

The external variables Y_t^*, R_t^* and p_t^{wc} denote the real GDP index for the G20 group of countries, the Baa U.S corporate spread and a metal export price index relevant for Peru. The domestic variables GDP_t, C_t, I_t, L_t, and $e_tL_t^*$ denote real indexes for Peruvian GDP, consumption, investment, real bank lending in domestic currency, and real bank lending in foreign currency respectively, while e_t denote the bilateral real exchange rate and tb_t the ratio of the trade balance to GDP. Following Canova (2005), the baseline specification considers $\mathbf{X}_t$ as an exogenous block, with no feedback dynamics from the domestic block, $\mathbf{D}_t$, at any point in time. Therefore, like much of the related literature, the main identification assumption is that an emerging small open economy as Peru, takes as given world prices and quantities. The baseline specification assumes that all variables are expressed in log-levels. The only variables expressed in percentage terms are R_t^* and tb_t. Therefore, we consider an SVAR in levels with zero restrictions between blocks and a linear or quadratic time trend in order to capture the SOE assumption of the Peruvian economy as well as to control for time trends. It is important to mention that shocks within each block are identified recursively with zero contemporaneous restrictions.

Formally, consider the following restricted block VAR model with deterministic trend:

$$\begin{bmatrix} \mathbf{X}_t \\ \mathbf{D}_t \end{bmatrix} = \begin{bmatrix} \Phi_X \\ \Phi_D \end{bmatrix} G(t) + \begin{bmatrix} \Phi_{XX}^1(L) & \mathbf{0} \\ \Phi_{DX}^1(L) & \Phi_{DD}^1(L) \end{bmatrix} \begin{bmatrix} \mathbf{X}_{t-1} \\ \mathbf{D}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_t^X \\ \mathbf{v}_t^D \end{bmatrix} \tag{48}$$

where $G(t)$ measures a deterministic time trend[14]. Φ_X, Φ_D are vectors of ones, $\mathbf{v}_t^X \sim N(0, \Sigma_{\mathbf{v}^F})$ and $\mathbf{v}_t^D \sim N(0, \Sigma_{\mathbf{v}^D})$. Hence, the underlying SVAR model is

$$\begin{aligned} \begin{bmatrix} \Theta_{XX}^0 & \mathbf{0} \\ \Theta_{DX}^0 & \Theta_{DD}^0 \end{bmatrix} \begin{bmatrix} \mathbf{X}_t \\ \mathbf{D}_t \end{bmatrix} &= \begin{bmatrix} \Theta_X \\ \Theta_D \end{bmatrix} G(t) \\ &\quad + \begin{bmatrix} \Theta_{XX}^1(L) & \mathbf{0} \\ \Theta_{DX}^1(L) & \Theta_{DD}^1(L) \end{bmatrix} \begin{bmatrix} \mathbf{X}_{t-1} \\ \mathbf{D}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{u}_t^X \\ \mathbf{u}_t^D \end{bmatrix} \end{aligned} \tag{49}$$

The data has a quarterly frequency and goes from 1994Q1 to 2017Q2 for the domestic block and from 1980Q1 to 2017Q2 for the foreign block. Following Fernández et al. (2017) we first estimate the foreign block separately and impose the corresponding estimated parameters in the estimation of the domestic block.

Calibration based on previous literature. The subjective discount factor β is calibrated in order to set the annual interest rate at 4% as in the steady state. Non-Commodity capital share α_k equals 0.275 which is in line to some traditional calibration for the Peruvian Economy. The demand elasticity η is taken from Castillo et al. (2009) and the price adjustment cost parameter κ is set in order to implicitly simulate a 75% non-adjuster firms in a Calvo Price Setting. Moreover, the Phillips Curve's slope $\frac{\eta}{\kappa}$ is around 0.12 which is consistent

[14] Similar to the SVAR model, the DSGE model considers deterministic time trends that are removed before the matching procedure.

to estimations for the Peruvian economy (see Winkelried (2013)). The domestic share for commodity sector profits χ^c is set to 0.60 as in Garcia-Cicco et al. (2017) . The fraction of divertable domestic currency loans ϖ is normalized to 1 while Taylor Rule coefficients are set in line with the New Keynesian literature.

Steady-State Targets. For workers, (ζ_0, ζ, χ) are calibrated in order to attain $K^{nc} = 20$, a given IFE value, and $\frac{K^{nc,b}}{K^{nc}} = 0.80$ which is the fraction of non-commodity capital financed by households. Regarding the production sector, α_m is calibrated in order to set aggregate hours worked equal to 1/3 while (α_c, A^c) achieve a commodity/non-commodity investment ratio equal to 0.16 and 60% of commodity exports relative to total exports. Concerning the financial sector, the vector $(\delta^f, \varpi^*, \varpi^b, \xi, \theta, \varkappa)$ is calibrated to be consistent with the following steady-state targets: credit dollarization rate of 40%, domestic currency loan return of 5%, foreign currency loan return of 3.5%, central bank bond return of 4.0%, domestic currency leverage of 3.30, and dollar deposits to total assets ratio of 55%. Based on these targets, the currency mismatch at the steady state is around 22% which lays in the range $17-23\%$ suggested in Figure 1.

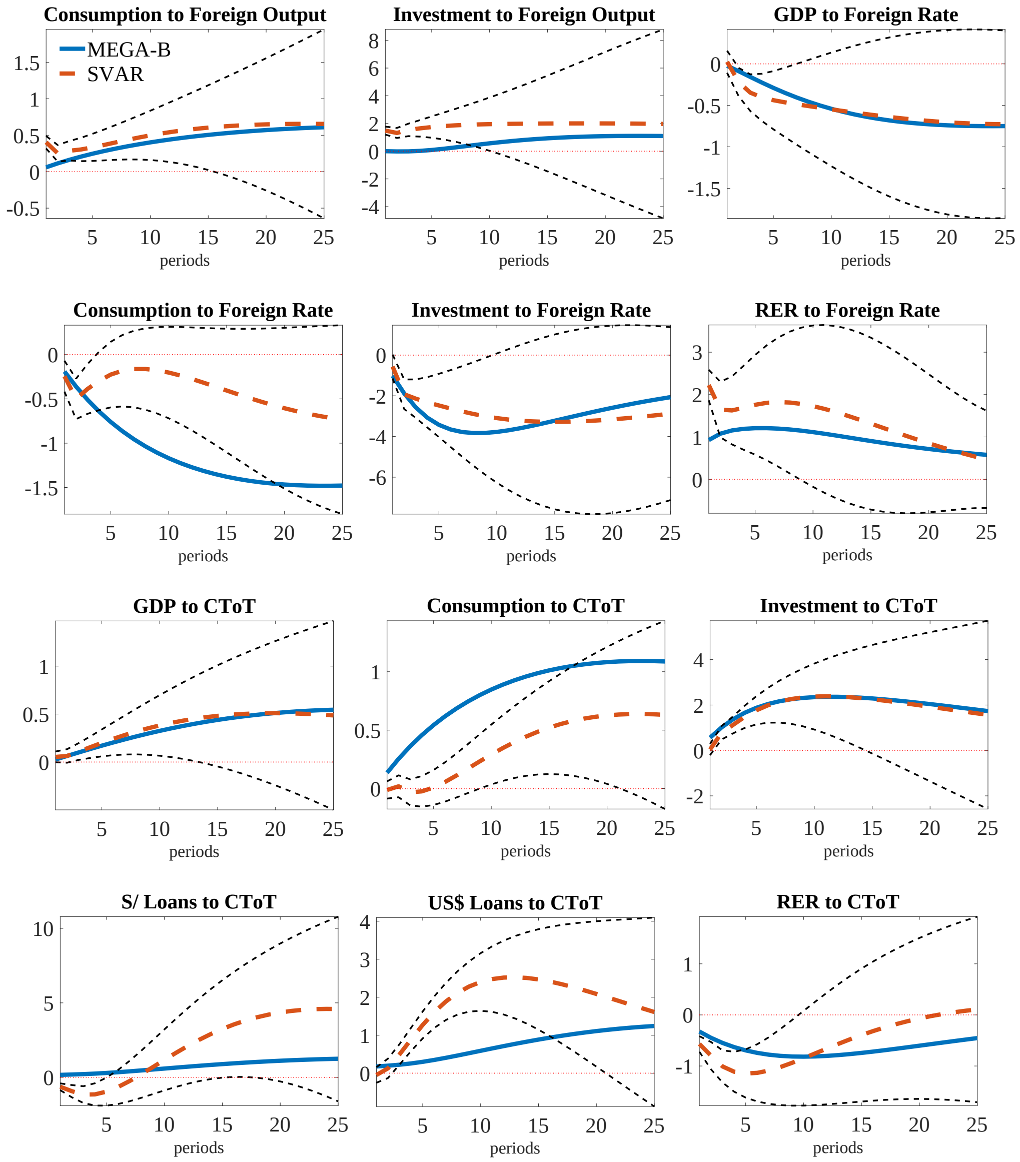
Impulse Response Matching. The rest of the parameters, including exogenous variables persistences and variances, are calibrated to match impulses responses to foreign shocks between the SVAR and the DSGE model.

Our baseline calibration is summarized in Table 2. As a summary, Fig. 2 compares the corresponding impulse-responses associated to the empirical and structural models. The theoretical responses follow their empirical counterpart very closely. Finally, the second column of Tab. 7 resumes the steady-state values for some key variables.

Table 2. Calibration

	Description	Value	Target/Source
	Financial Sector		
δ^f	Foreign Currency Loans Bias	0.35	Credit Dollarization: 40%
ϖ	Moral Hazard Parameter 1	1	Normalization
ϖ^*	Moral Hazard Parameter 2	1.36	Foreign Currency Loan's Return: 3.5% (p.a.)
ϖ^b	Moral Hazard Parameter 3	0.31	Central Bank Bond's Return: 4.0% (p.a.)
ξ	Households to Banks Transfers	0.07	Domestic Currency Loan's Return: 5.0% (p.a.)
θ	Θ's Parameter 1	0.13	Domestic Currency Loan's Leverage, $\phi_l = 3.5$
$\varkappa$	Θ's Parameter 2	7.49	Foreign Liability - Assets Ratio, $\frac{eD^*}{A} = 55\%$
	Government		
ρ_i	Policy Rate Smoothing	0.70	Standard in literature
ω_π	Policy Rate Response to Inflation	1.50	Standard in literature
B	Central Bank Bond's at SS	4.4	Official exchange reserves/GDP ratio, $\frac{B}{4GDP} = 20\%$
G	Gov. Expenditure at SS	0.83	Public Expenditure/GDP ratio, $\frac{G}{4GDP} = 15\%$

Figure 2. Impulse Response Matching



4 Numerical Experiments

In this section we perform several simulations designed to analyze how SFX Interventions affect the response of the model economy to external shocks. Specifically, we focus on the transmission of a sudden increase in the foreign interest rate and a global commodity boom.[15] The foreign block estimated in the SVAR is used in the DSGE model as the joint process for external variables.

We begin our analysis by analyzing the responses of aggregate variables to external shocks under different exchange rate regimes: Flexible exchange rate regime vs SFX intervention regime. Under the SFX intervention regime, the central bank "leans against the wind" with respect to real exchange rate fluctuations by implementing eq. (15) but also its monetary policy rule is activated. Next, we simulate an exogenous, sufficiently large unanticipated and

[15] In Appendix C we also show an increase in global GDP.

permanent accumulation (purchase) of foreign exchange reserves and study its transmission mechanism. Finally, we measure the effectiveness of SFX interventions through a standard welfare analysis.

Quantitatively our results suggest that under the SFX intervention regime, macroeconomic volatility is reduced relative to the flexible exchange rate regime (see Tab. 3). As expected, the unconditional volatility of the real exchange rate is reduced by 26.4% while the corresponding unconditional volatilities of total credit and currency mismatch are reduced by 5.9% and 6.2% respectively. Similarly, the unconditional volatility of different interest rate spreads, such as $\frac{e}{e_{-1}}R^{l*} - R^{l}$ and $\frac{e}{e_{-1}}R^{*} - R$, are reduced by 64.2% and 57.4% accordingly. Simultaneously, the unconditional volatility of macroeconomic variables also fall significantly reflecting that SFX interventions may play the role of an external shock absorber. For instance, investment volatility is reduced by 14.4% while consumption volatility falls by 4.6%. Hence, the unconditional volatility of GDP is reduced by 6.4 percent under the SFX intervention regime.

Table 3. Unconditional Volatilities

	$\upsilon_e > 0$	$\upsilon_e = 0$	Stabilization (%)
Total Credit	16.07	17.07	5.9
RER	8.02	10.90	26.4
Δe	2.47	7.88	68.7
Credit Dollarization	1.93	2.56	24.7
Deposit Dollarization	15.79	18.97	16.8
Mismatch	16.51	17.59	6.2
$\frac{e}{e_{-1}}R^{l*} - R^{l}$	2.58	7.22	64.2
$\frac{e}{e_{-1}}R^{*} - R$	3.80	8.91	57.4
Inflation	2.16	2.37	8.9
GDP	7.57	8.09	6.4
Investment	23.00	26.85	14.4
Consumption	13.54	14.19	4.6
NC Exports	4.91	6.61	25.8
C Exports	3.95	4.70	15.9

4.1 Foreign Interest Rate Shock

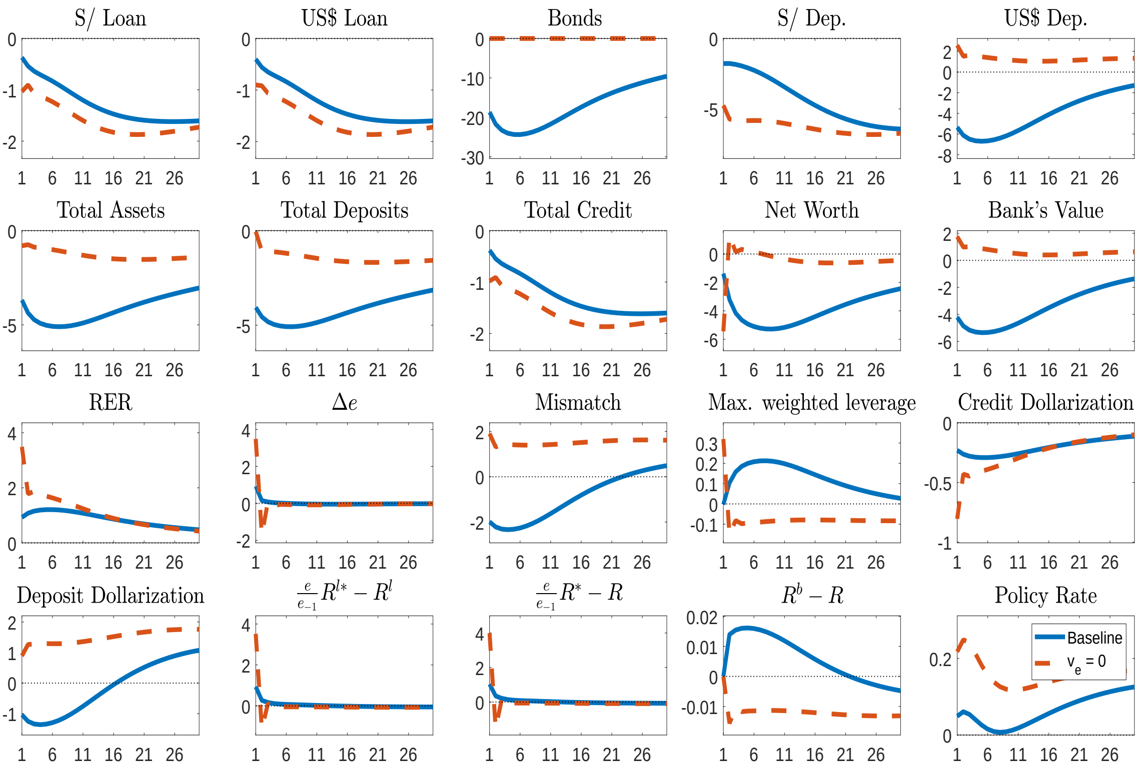
Figure 3 and Fig. 4 display responses of the model economy to an unexpected increase of 20 basis points in the foreign interest rate.[16] The dotted line reports responses under flexible exchange rates (i.e., $\upsilon_e = 0$) while the solid line represents the economy under the SFX intervention policy. We first describe the transmission mechanism under exchange rate flexibility.

Initially, the real exchange rate depreciates by 3.7% and non-commodity exports increase by 2.3% reflecting the standard exchange rate trade channel. However, the economy experiences a contractionary financial effect that overpowers the net export response. Since banks are exposed to currency mismatches on their balance sheets, the real exchange depreciation negatively affects bank's net worth, total credit and ultimately generates a recession in the non-commodity sector. Net worth declines at impact but shows a fast recovery and

[16] All variables, including lending and borrowing in foreign currency, are expressed in terms of non-commodity goods. Moreover, the response of each quantity or index variable (e.g., GDP, credit, and real exchange rate) is presented in percent deviation from its steady-state, while the response of any rate variable (e.g., interest rates, spreads, and depreciation rates) is displayed in percentage point deviations from its steady-state.

then stabilizes below zero. Although the real exchange rate depreciates immediately after the shock, agents expect an exchange rate appreciation. The expected appreciation of the exchange rate modifies the relative costs and returns of borrowing and lending in foreign currency with respect to domestic currency, changing the composition of bank's balance sheet. Thus, banks realize that borrowing in foreign currency is cheaper than in domestic deposits and also that lending in foreign currency has become less profitable than lending in domestic currency. Consequently, banks increase foreign borrowing and reduce lending in foreign currency implying higher deposit dollarization (around 1 percentage points on impact) but lower credit dollarization (-0.8 percentage points on impact). Hence, under a flexible exchange rate regime, the exchange rate depreciation induced by the increase in the foreign interest rate raises the size of the currency mismatch reducing the intermediation capacity of banks: lending in both currencies declines by around 1 percent.

FIGURE 3. RESPONSES OF FINANCIAL VARIABLES TO A FOREIGN INTEREST RATE SHOCK

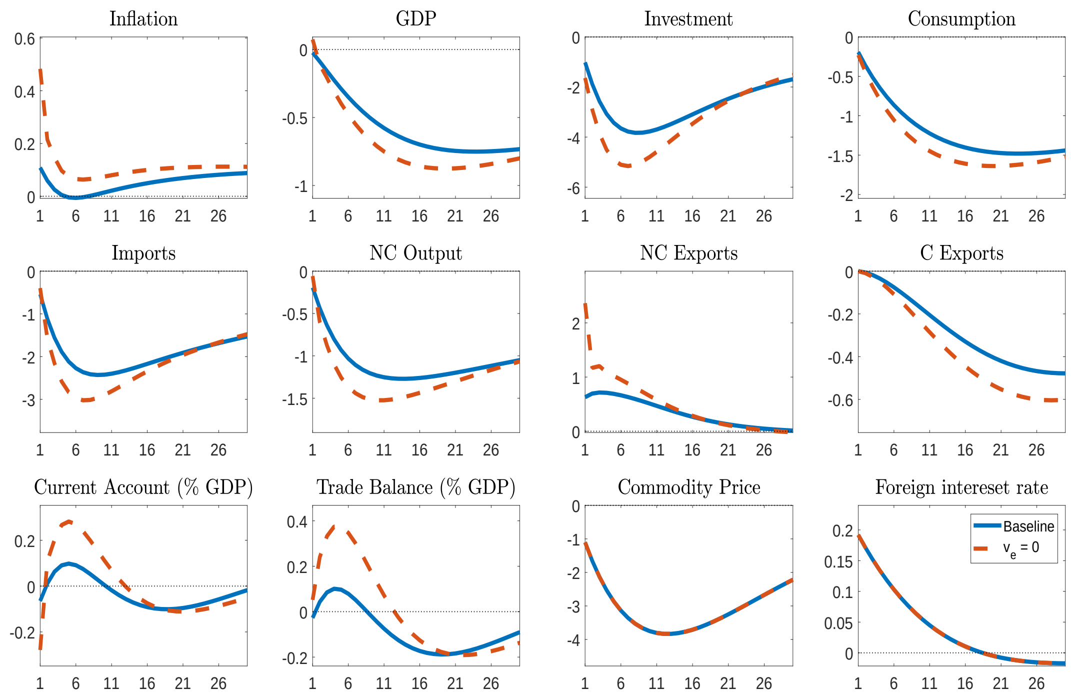


Financial conditions are reflected on interest rate spreads and macroeconomic variables. In particular, right after the foreign interest rate increases, the excess return of foreign currency lending relative to domestic currency lending increases in 3.6 p.p. while the interest rate spread of foreign borrowing over domestic deposits increase in 4 percentage points. Then, investment, consumption, and imports fall by 1.8%, 0.2%, and 0.4% respectively, generating a persistent recession with GDP falling by 0.8% at the through of it. Finally, the depreciation of the exchange rate raises inflation by 0.5% on impact since the marginal cost of intermediate good producers depends on an imported input. The increase in inflation leads to a higher interest rate.

When the central bank responds to a foreign interest rate shock implementing SFX interventions together with its standard monetary policy rule, both financial and macroeconomic variables are stabilized. The effect of SFX interventions over the transmission mechanism of any external shock operates through two main channels: the exchange rate smoothing channel and the balance sheet substitution channel.

The Exchange Rate Smoothing Channel. When the incentive constraint binds, SFX interventions modify the net asset foreign position of the economy as well as the interest rate spread between foreign borrowing and domestic deposits. In particular, the central bank responds to an increase in the foreign interest by selling foreign currency, i.e., a decline in official foreign exchange reserves. Therefore, exchange rate dynamics change relative to the flexible exchange rate regime. First, at impact the real exchange rate depreciates by 1% under the SFX intervention regime instead of 3.7% under the flexible exchange rate regime. Second, a hump-shape response emerges for the real exchange rate when SFX interventions are active. Under free floating, the exchange rate depreciates only at impact, overshooting its long run equilibrium level. After the impact, the exchange rate is above its steady-state level but it appreciates every period since. Hence, exchange rate expectations differ between the two exchange rate regimes since, under SFX intervention regime, agents expect a real exchange depreciation for approximately six quarters.

FIGURE 4. RESPONSES OF MACROECONOMIC VARIABLES TO A FOREIGN INTEREST RATE SHOCK



As a result of smoothing the response of the real exchange rate, at impact, the bank's net worth declines by less under the SFX intervention regime (2% instead of 5.5% under the flexible exchange rate regime). The hump-shape pattern for the real exchange rate modifies the cost of borrowing in foreign currency with respect to domestic deposits. In particular, under the SFX intervention regime, the interest rate spread of lending in foreign currency over lending in domestic currency rises in 0.9 percentage points instead of 3.8 percentage points under free floating exchange rate.

Contrary to the free-floating regime, banks increase domestic deposits rather than foreign currency, implying that deposit dollarization declines by one percentage points at impact. Similarly, the expected depreciation that occurs under SFX intervention implies that foreign currency loans are less profitable than domestic currency loans, implying that the dollarization of credit falls but not as much as under exchange rate flexibility.

The Balance Sheet Substitution Channel. This channel is associated with the sterilization operation that the central bank implements to keep domestic liquidity constant right after selling foreign currency. The central bank buys bonds that are in the bank's balance sheets, ultimately affecting its size and composition. Consequently, this operation frees funds for banks which are used for lending in both currencies. In this sense, SFX interventions are similar to credit policy in the non-conventional monetary policy literature for closed economies.

Quantitatively, our results suggest that the sterilization leg of selling foreign currency implies that central bank bonds in the balance sheet of banks decline by 20% on impact and in 25% after 6 quarters of SFX intervention. As a result, lending in domestic and foreign currency decline less than under exchange rate flexibility. In particular, at impact, domestic currency loans fall by 0.4% when SFX interventions are active instead of declining in 1% under free floating. Similarly, lending in foreign currency falls by 0.4% under SFX interventions and by 1% under exchange rate flexibility.

Table 4. Pass-Through for foreign interest rate shocks

	Year 1			Year 2		
	$\upsilon_e > 0$	$\upsilon_e = 0$	SFX	$\upsilon_e > 0$	$\upsilon_e = 0$	SFX
Total Credit	-3.51	-6.27	44.0	-5.57	-8.83	36.9
RER	6.69	13.67	51.1	8.81	14.72	40.1
Δe	1.83	2.69	31.9	1.13	1.43	20.9
Current Account (% GDP)	0.15	0.46	66.8	0.40	1.24	67.8
Credit Dollarization	-1.63	-3.23	49.5	-2.14	-3.50	38.9
Mismatch	-13.61	9.33	-45.9	-17.04	11.24	-51.5
$\frac{e}{e_{-1}}R^{l*} - R^l$	2.27	3.70	38.6	1.68	2.13	21.2
$\frac{e}{e_{-1}}R^{*} - R$	2.69	3.87	30.5	2.00	2.20	9.0
Inflation	0.30	1.45	79.0	0.18	1.17	84.7
GDP	-0.76	-0.85	10.8	-1.92	-2.50	23.0
Investment	-13.14	-19.83	33.8	-22.41	-31.99	30.0
Consumption	-2.62	-3.22	18.4	-5.13	-6.26	17.9
NC Exports	4.23	8.96	52.8	5.14	9.14	43.8
C Exports	-0.09	-0.13	31.2	-0.40	-0.57	30.1
Trade Balance (% GDP)	0.31	1.53	79.8	0.43	2.19	80.2

In Table 4, we compute the pass-through of different endogenous variables to a foreign interest rate shock after 1 and 2 years for each exchange rate regime. We also compute the percentage difference between the pass-through of different variables of interest for the two exchange rate regimes as a way to quantify differences in the dynamic responses to any foreign shock.[17] Our result suggest that, after the first year of the shock, total credit falls by 3.51%

[17] We measure the effectiveness of SFX interventions conditional on one specific shock comparing the j-year horizon pass-through between an economy under SFX intervention regime and another under exchange rate flexibility:

$$\mathtt{SFX}_j = -100 \times \left[\frac{abs(\mathtt{PT}_j|_{\upsilon_e>0}) - abs(\mathtt{PT}_j|_{\upsilon_e=0})}{abs(\mathtt{PT}_j|_{\upsilon_e=0})} \right]$$

where $\mathtt{PT}_j$ is the j-year pass-through and is defined as

$$\mathtt{PT}_j = \frac{\sum_{k=1}^{4j} \mathtt{IRF}_k}{\sum_{k=1}^{4j} \mathtt{IRF}_k}$$

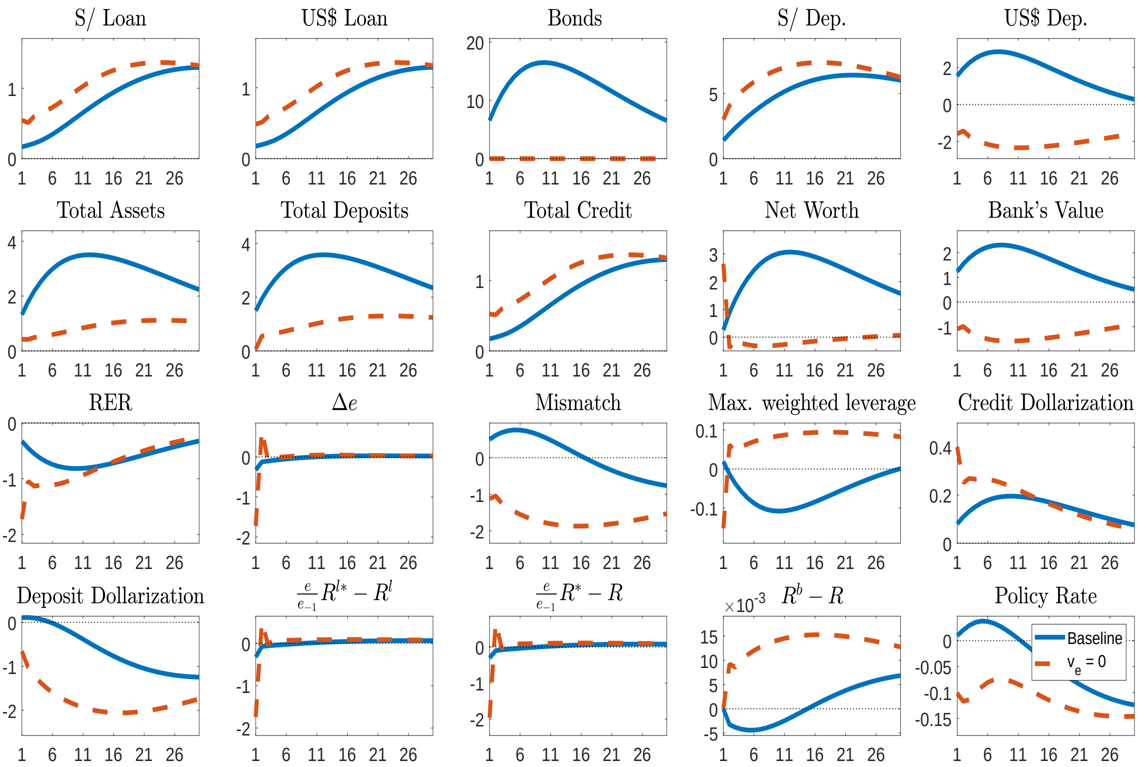
Hence, the conditional stabilization index, **SFX**, reads as the pass-through for the concerning variables, in absolute values, falls by $\mathtt{SFX}_j$ percent when the central bank start implementing SFX interventions.

under the SFX intervention regime, instead of 6.27% under exchange rate flexibility. Therefore, SFX interventions reduce the response of aggregate bank lending in 44 percent. Moreover, the implementation of this policy reduces real exchange rate depreciation from 2.69 to 1.83 (`SFX` around 32 percent) and the response of credit dollarization from -3.23 to -1.63 (`SFX` around 50 percent). Similarly, the SFX intervention regime also reduces macroeconomic responses. In particular, the response of investment and GDP fall by around 34% and 11% after one year, respectively. The stabilization properties of SFX intervention remain active for the second year after the shock.

4.2 Commodity Price Shock

Emerging market economies face volatile commodity prices that shape capital flows and domestic financial conditions. In this section, we simulate a persistent increase in commodity prices and compare the transmission mechanism of this shock under exchange rate flexibility and SFX intervention. Figure 5 shows the responses of financial variables while Figure 6 presents the responses of key macroeconomic variables. The dotted line is consistent with the flexible exchange rate regime.

Figure 5. Responses of financial variables to a commodity price shock

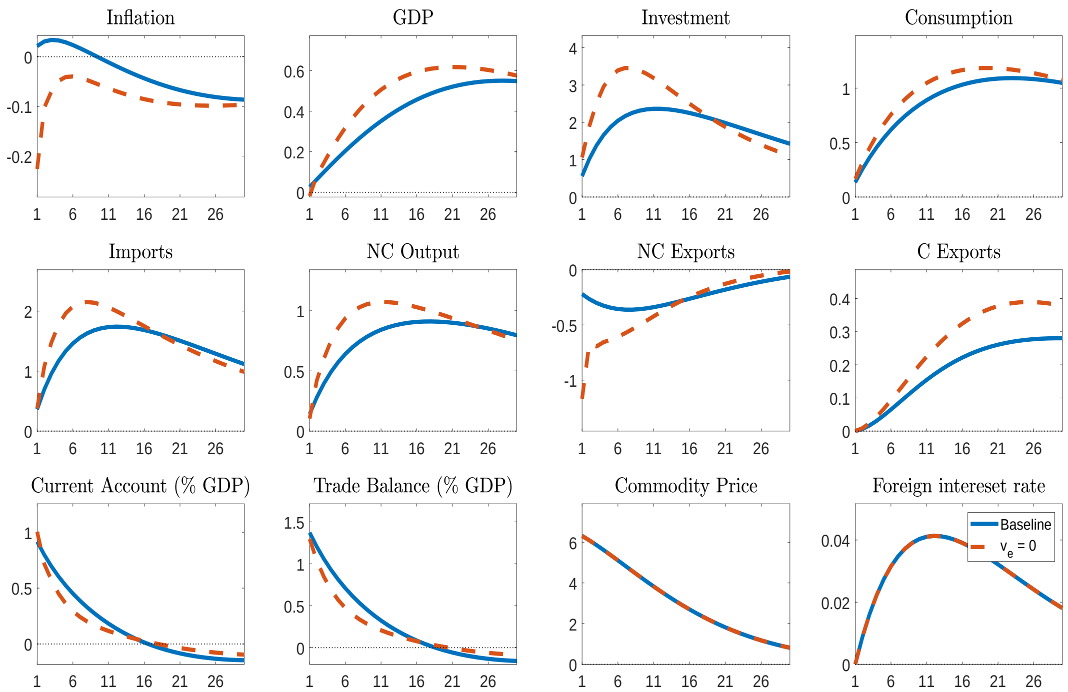


Under exchange rate flexibility, a persistent increase in commodity prices raises commodity exports by around 5% producing a persistent current account and trade balance surpluses of 1.5 and 1 percentage points respectively. A large fraction of the revenues from commodity exports is kept in the economy leading to a persistent exchange rate appreciation of around 1.7% at impact. The commodity sector experiences a prolonged economic boom that is spread to the rest of the economy through a significant wealth effect and a higher demand of investment goods.

The appreciation of the exchange rate relaxes the agency constraint that banks face by increasing net worth in 2.6% together with a significant fall in the currency mismatch of 1 percentage point right after the shock occurs. The latter is an expansionary financial effect that occurs even though non commodity exports decline significantly due to the appreciation of the real exchange rate. Hence, lending in both currencies rises by around 0.5 percent at impact. Under exchange rate flexibility, agents expect a depreciation of the real exchange rate implying that banks realize borrowing in foreign currency is more expensive than in domestic currency while lending in foreign currency is more profitable than in domestic currency. The change in the composition in the balance sheet of banks is consistent with an increase of 0.4 p.p. in credit dollarization and a reduction of one percentage point in deposit dollarization at impact.

The commodity boom together with the consequent expansionary financial conditions it generates, modify the dynamics of interest rate spreads and real macroeconomic variables. Specifically, the excess return of foreign currency lending relative to domestic currency lending falls in 1.7 percentage points while the interest rate spread of foreign borrowing with respect to domestic currency deposits falls in 2 percentage points. Then, investment and consumption increase persistently by around 3.5% and 1.2% at the peak of their responses, respectively. The commodity boom under flexible exchange rates induce a period of persistent economic expansion with GDP reaching an increase of 0.6% after four years.

Figure 6. Responses of macroeconomic variables to a commodity price shock



When SFX interventions are active, the central bank accumulates foreign exchange reserves and allocates central bank riskless bonds to the banking system as a response to higher commodity prices and the appreciatory pressures on the real exchange rate. Given the binding agency problem, accumulating foreign exchange reserves reduces significantly the appreciation of the exchange rate, limiting the expansion of credit by banks and the consequent expansion in macroeconomic aggregates such as consumption, investment and GDP. As mentioned before, SFX interventions operate through the exchange rate smoothing channel and the balance sheet substitution channel.

The Exchange Rate Smoothing Channel. The central bank responds to the commodity price shock by buying foreign exchange reserves and thus modifying the net foreign asset position of the economy. As a result, exchange rate dynamics change relative to the flexible exchange rate regime. At impact, the real exchange rate appreciates by 0.4% instead of 1.7%. Similarly, as in the foreign interest rate shock, SFX intervention change the dynamics of the real exchange rate, generating a hump-shape response with consecutive appreciations for about 10 quarters. Instead, under exchange rate flexibility, the real exchange rate only appreciates at impact, then it depreciates until reaches its new equilibrium level. Therefore, real exchange rate expectations differ between regimes. Under free floating agents expect a depreciation but under the SFX intervention regime agents expect appreciations.

Consequently, at impact bank net worth increases by less than under free floating (0.2% instead of 2.6%). Moreover, the hump-shape pattern of real exchange rate modifies the costs and returns of borrowing and lending in foreign currency. When the central bank implements SFX interventions, banks increase foreign borrowing together with domestic deposits implying higher deposit dollarization relative to the flexible exchange rate regime. Likewise, the expected real exchange rate appreciation that arises under SFX interventions signal banks that lending in foreign currency is more profitable than lending in domestic currency. Credit dollarization increases but less than under exchange rate flexibility.

Table 5. Pass-Through for commodity price shocks

	Year 1			Year 2		
	$\upsilon_e > 0$	$\upsilon_e = 0$	SFX	$\upsilon_e > 0$	$\upsilon_e = 0$	SFX
Total Credit	0.04	0.09	63.0	0.05	0.12	55.6
RER	-0.08	-0.21	61.1	-0.11	-0.21	47.2
Δe	-0.03	-0.05	43.6	-0.02	-0.02	24.6
Current Account (% GDP)	0.13	0.11	-9.9	0.11	0.09	-22.5
Credit Dollarization	0.02	0.05	60.0	0.03	0.05	46.2
Mismatch	0.11	-0.20	47.0	0.12	-0.25	50.8
$\frac{e}{e_{-1}} R^{l*} - R^l$	-0.02	-0.05	56.4	-0.01	-0.02	32.6
$\frac{e}{e_{-1}} R^* - R$	-0.02	-0.05	54.9	-0.02	-0.02	23.4
Inflation	0.00	-0.02	73.5	0.00	-0.01	68.0
GDP	0.01	0.02	22.3	0.03	0.04	32.2
Investment	0.19	0.35	45.2	0.30	0.50	41.0
Consumption	0.05	0.06	19.4	0.09	0.11	18.4
NC Exports	-0.05	-0.13	65.2	-0.06	-0.13	54.3
C Exports	0.00	0.00	28.6	0.01	0.01	29.9
Trade Balance (% GDP)	0.19	0.16	-17.9	0.17	0.13	-28.1

The Balance Sheet Substitution Channel. When the central bank responds to the commodity price shock by accumulating foreign exchange reserves a sterilization operation is implemented simultaneously. The latter consists in selling central bank bonds in order to maintain domestic liquidity constant. As a result, the composition and size of bank's balance sheets change, ultimately generating a crowding-out effect that limit resources for lending. In particular, banks allocate their increasing available funds into central bank bonds instead of lending. Banks increase their holding of central bank bonds in 8 percent at the moment of the commodity shock and by around 16 percent after 10 quarters of the shock. Accordingly, lending in domestic and foreign currency increase by less than under exchange rate flexibility, showing a response that is always below the one associated to the flexible exchange rate regime. The muted response of aggregate credit under the SFX intervention regime is reflected in the response of interest rate spreads. Figure 5 shows that the interest rate spread of lending in foreign currency over lending in domestic currency falls in 0.3 p.p. when the central bank responds by accumulating foreign exchange reserves instead of 1.7 p.p. under exchange rate flexibility.

In Table 5, we quantify the effect of SFX interventions in stabilizing the response of the economy to a commodity price shock by computing the pass-through of different macroeconomic and financial variables. The SFX intervention regime stabilize the dynamics of the real exchange rate, total credit, investment and GDP by around 61%, 63%, 45% and 22% respectively after the first year of the initial shock.

Therefore, our results suggest that the accumulation of foreign exchange reserves serves as an additional monetary policy tool that soften the expansionary response of domestic financial conditions when the economy faces a commodity boom.

4.3 The transmission of a permanent accumulation of foreign exchange reserves

In this section, we analyze the impact of an exogenous SFX intervention shock in order to obtain insight about its transmission mechanism. We assume the SFX intervention rule is given by the following exogenous autoregressive process:

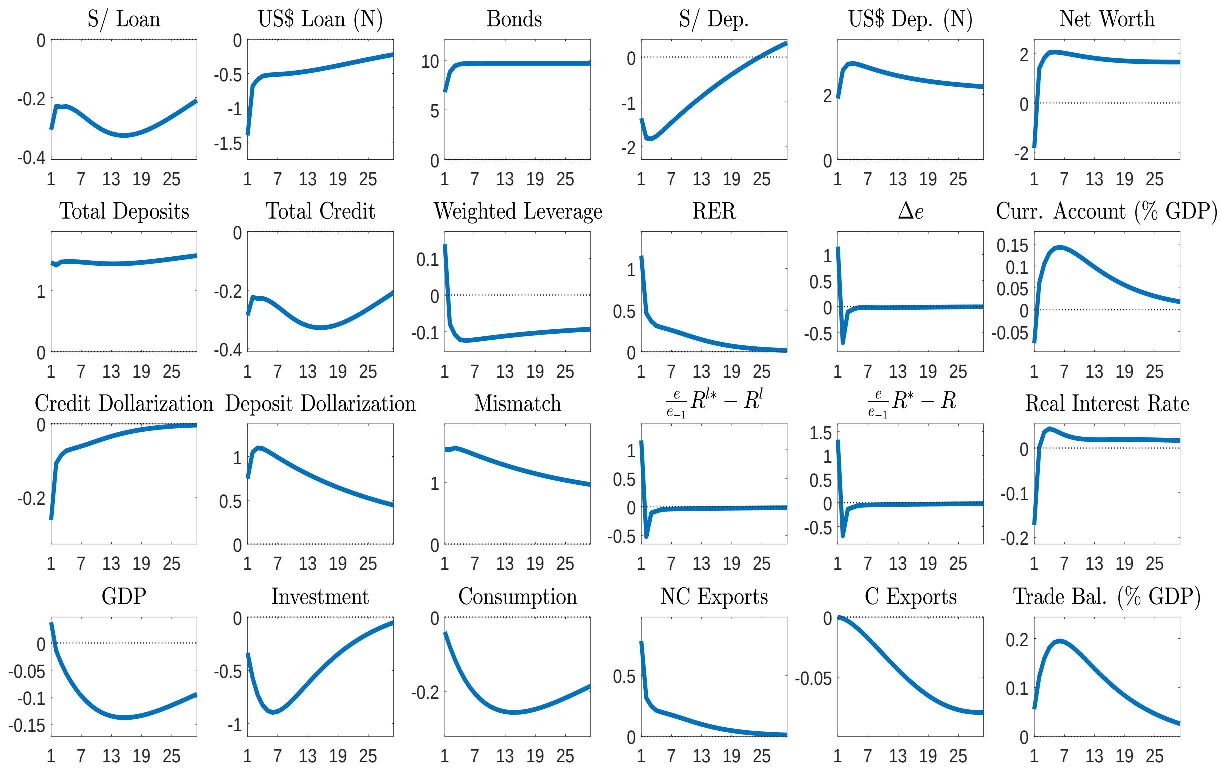
$$\Delta \ln B_t = \rho_{\Delta B} \Delta \ln B_{t-1} + u_t^B \tag{50}$$

where u_t^B is interpreted as an unanticipated central bank purchase of foreign exchange reserves. Under the above process, an accumulation shock of foreign exchange reserves has permanent effects over central bank bonds in hands of the banking system. Figure 7 present responses to a sufficiently persistent unanticipated purchase of foreign exchange reserves together with its corresponding sterilization operation: the selling of central bank bonds to the banking system. The central bank accumulation of official reserves induces an initial depreciation in the real exchange rate of around 1.2% that raises inflation and the monetary policy rate as well. The trade channel triggers a persistent rise in non-commodity exports with a corresponding trade balance surplus. The balance sheet substitution channel is such that the sterilization operation modifies the asset composition of bank's balance sheet to less lending and more central bank bonds. Finally, the purchase of foreign reserves by the central bank induce a financial channel. The real exchange rate depreciation reduces bank's net worth and raises the size of the currency mismatch at the bank level.

Consequently, domestic financial conditions worsen which are reflected in higher interest rate spreads and lower credit. The dynamics of the real exchange rate are such that agents expect an appreciation right after the shock occurs. Therefore, deposit dollarization increases while credit dollarization falls. The financial and the balance sheet substitution channels outweighs the trade channel. As a result, the persistent and exogenous purchase of foreign exchange reserves push the economy to a credit crunch generating a prolonged recession.

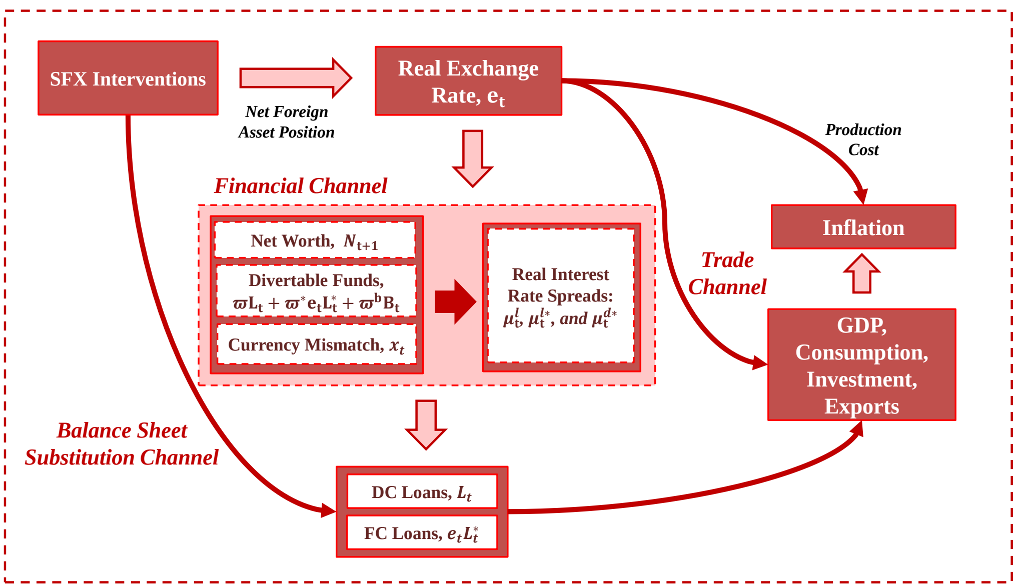
It is worth mentioning that the financial channel as well as the balance sheet substitution channel amplify the initial exogenous shock. On the contrary, both channels work as a stabilization mechanism when SFX interventions are implemented as a response to external shocks. Figure 8 summarizes the main transmission mechanisms through which SFX interventions stabilize financial and macroeconomic volatility.

Figure 7. Response to a Permanent Purchase of Foreign Exchange Reserves



Note: Except for lending and borrowing in foreign currency, which are denominated in foreign goods, all variables are expressed in terms of non-commodity goods.

Figure 8. Stabilization Channels of SFX interventions



4.4 Welfare Analysis

We conduct a policy evaluation exercise by computing the welfare gains/costs of one policy regime relative to a different regime. Each policy regime is characterized by its own time-invariant stochastic equilibrium allocation. In particular, we follow Schmitt-Grohe and Uribe (2007) and define two policy regimes denoted by r_1 and r_2. Our benchmark regime r1 is such that the central bank has two policy instruments: the monetary policy interest rate and SFX interventions. On the other hand, the alternative regime r2, assumes the central bank responds to shocks with only its interest rate policy.

We define the welfare associated with the equilibrium allocation implied by our benchmark policy regime r_1 conditional on a particular state of the economy in period 0 as

$$W^{\text{benchmark}} \equiv \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left(C_t^{r_1} - \mathcal{H} C_{t-1}^{r_1} - \frac{\zeta_0}{1+\zeta} (H_t^{r_1})^{1+\zeta} \right)^{1-\gamma} \right] \tag{51}$$

where $\{C_t^{r_1}, H_t^{r_1}\}_{t\geq 0}$ is a contingent plan for consumption and hours under the policy regime r_1. The distinct policy regimes that we consider only change the dynamics of the model economy but not its nonstochastic steady state. Therefore, we compute the welfare associated to each policy regime conditional on the initial state being the nonstochastic steady state of the model economy. The latter ensures that the economy begins from the same initial point under all possible policies. In particular, we compute the welfare gain of regime r_2 relative to the benchmark policy regime r_1. Let ς_{cond} denote the welfare gain/cost of adopting policy regime r_2 instead of the benchmark policy regime r_1 conditional that the economy is at nonstochastic steady state at time zero. The parameter ς_{cond} measures the fraction of the benchmark regime consumption process that a household would be willing to accept (or give up) to be as well off under the alternative policy regime r_2 as under regime r_1. Thus, ς_{cond} is implicitly defined by

$$W^{\text{benchmark}} = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left((1-\varsigma_{cond}) \left[C_t^{r_2} - h C_{t-1}^{r_2} \right] - \frac{\zeta_0}{1+\zeta} (H_t^{r_2})^{1+\zeta} \right)^{1-\gamma} \right] \tag{52}$$

where $\{C_t^{r_2}, H_t^{r_2}\}_{t\geq 0}$ is the corresponding contingent plan for consumption and hours under the policy regime r_2. Hence, if $\varsigma_{cond} > 0$ there is a welfare gain while if $\varsigma_{cond} < 0$ then there is a welfare loose under the alternative regime r_2. We approximate ς_{cond} up to a second order of accuracy.

Table 6 shows the welfare gains of different combinations of monetary and SFX intervention policy regimes. We change parameters ω_π and v_e in order to study the consequences of implementing different policy rules.

Table 6. Welfare Analysis: $\varsigma_{cond} \times 100\%$ - External Shocks

$\boldsymbol{\omega_\pi \backslash v_e}$	0	5	10	**20**	25	30	50	75
1.25	−3.1	−2.0	−1.4	−0.9	−0.8	−0.8	−1.3	−2.3
1.50	−1.1	−0.4	−0.1	**0.0**	−0.0	−0.1	−0.8	−1.8
2.00	−0.5	0.0	0.2	0.2	0.2	0.0	−0.6	−1.6
3.00	−0.2	0.2	0.3	0.3	0.2	0.1	−0.6	−1.6
5.00	−0.1	0.3	0.4	0.3	0.2	0.1	−0.6	−1.6
10.00	0.0	0.3	0.4	0.4	0.3	0.1	−0.6	−1.6

There are three relevant remarks from Tab. 6. First, SFX interventions and interest rate policies are effective in reducing macroeconomic volatility and increasing social welfare in a big region of the parameter space associated to the policy regimes (i.e., v_e and ω_π). Second, given our baseline calibration for ω_π, the calibrated degree of SFX intervention response to the real exchange rate, $v_e = 20$, seems to be optimal in the sense that higher or lower values of v_e reduce social welfare. The latter result is virtually independent of the Taylor coefficient, ω_π. Finally, our calibrated model suggests that, given the Taylor coefficient at its baseline level, not responding to the real exchange rate by implementing SFX interventions ($v_e = 0$) would cause a welfare loose of 1.1 percent in consumption. These remarks justify the actively use of SFX interventions as a additional monetary policy tool aimed to smooth real exchange rate dynamics.

5 Further Quantitative Results

5.1 Generalizations of the Baseline Framework

We relax some assumptions of our baseline framework in order to study if the effectiveness of SFX interventions as a response to external shocks depend on those assumptions. We compare the baseline model with the following three extensions:

Case 1: Bank assets enter the incentive constraint with equal weights. In other words, domestic currency loans and central bank bonds become perfect substitutes as in Chang (2019).

Case 2: The banking system does not exhibit credit dollarization. Banks are allowed to lend only in domestic currency but they still can borrow in foreign currency.

Case 3: The size of the currency mismatch affecting the banker's ability to divert funds is assumed to be an aggregate measure of the banking system and therefore it is taken as given at the individual level.

Each of these generalizations presents key features that affect the steady state of the model as well as the effectiveness of SFX interventions to mitigate the response of the economy to external shocks. The steady state equilibrium for each case, including the baseline model, is presented in Table 7. The differences between steady states mainly arise from the financial block of the model – the steady state for the real sector hardly changes. We study the implications of each case in relation to the properties of SFX interventions to smooth the response of the economy to external shocks. We focus on the pass-through of key endogenous variables to external shocks. More specifically, we focus on the percentage difference between the pass-through of different variables under the flexible exchange regime and the SFX intervention regime.[18]

The calibration of the baseline model implies that central bank bonds are harder to deviate relative to loans (i.e., $\varpi^* > \varpi > \varpi^b$). Since central bank bonds are the only sterilization instrument that the central bank is able to use, the role played by SFX interventions in mitigating the effects of external shocks is limited by the value of ϖ^b. However, when the all assets that banks can hold enter with equal weights into the incentive compatibility constraint (i.e., $\varpi^* = \varpi = \varpi^b = 1$), central bank bonds have a higher impact on the total amount of

[18] Impulse responses and tables associated to these results are shown in appendix B.

Table 7. Steady State Equilibrium

Variable	Baseline	Case 1 $\varpi^* = \varpi^b = 1$	Case 2 $\delta^f = 0$	Case 3 Aggregate x_t	Notation
Financial System Rates					
Capital return	7.50	7.30	7.92	7.50	$400(R^k - 1)$
Loan's return (S/)	5.00	5.01	4.81	5.00	$400(R^l - 1)$
Loan's return (US $)	3.50	3.00		3.50	$400(R^{l*} - 1)$
Bonds return	4.00	5.01	3.85	4.00	$400(R^b - 1)$
Foreign interest rate	2.00	2.00	2.00	4.00	$400(R^\star - 1)$
Deposit interest rate	4.00	4.00	4.00	4.00	$400(R - 1)$
Bank Leverage (CB Bonds)	1.28	1.27	1.31	1.29	ϕ^b
Bank leverage (Peso Loans)	3.50	3.50	5.87	3.50	ϕ^l
Bank leverage (Dollar Loans)	2.33	2.34	0.00	2.33	ϕ^{l*}
Currency Mismatch	22.21	21.84	29.09	22.25	$100 \times x$
Credit Dollarization Rate	40.00	40.00	0.00	40.00	$100 \times \frac{eL^*}{L+eL^*}$
Deposit Dollarization Rate	63.99	63.63	33.80	63.98	$100 \times \frac{eD^*}{D+eD^*}$
Sectorial Rates					
Commodity/Total Exports	60.00	60.03	59.84	60.00	$100 \times \frac{Y^{x,c}}{Y^x}$
Commodity/Total Investment	13.79	13.72	13.73	13.79	$100 \times \frac{I^c}{I}$
Non Commodity Capita/GDP	1.23	1.24	1.22	1.23	$\frac{K^{nc}}{4\times GDP^{nc}}$
Commodity Capita/GDP	1.08	1.08	1.08	1.05	$\frac{K^c}{4\times Y^{x,nc}}$
Stock Rates					
Foreign Reserves/GDP	20.00	19.94	20.03	20.00	$100 \times \frac{B}{4\times GDP}$
Foreign Asset Position/GDP	-41.00	-41.01	-12.00	-40.69	$100 \times \frac{B^*}{4\times GDP}$
Stock of Capital/GDP	1.19	1.20	1.18	1.18	$\frac{K}{4\times GDP}$
Real Demand Rates					
Investment/GDP	20.94	21.06	20.73	20.81	$100 \times \frac{I}{GDP}$
Public Consumption/GDP	15.00	14.96	15.02	15.00	$100 \times \frac{G}{GDP}$

divertable funds and ultimately on bank's lending capacity. As a result, SFX interventions are more effective as a external shock absorber in this case than in our baseline model. In order to measure the quantitative consequences between the model under case 1 and the baseline case, we focus on the percentage difference between the responses of key macroeconomic variables to an external shock under the SFX intervention regime and exchange rate flexibility. We compute percentage differences between the responses under different exchange rate regimes for each model extension as well as for the baseline case (recall that we label this percentage difference as SFX).

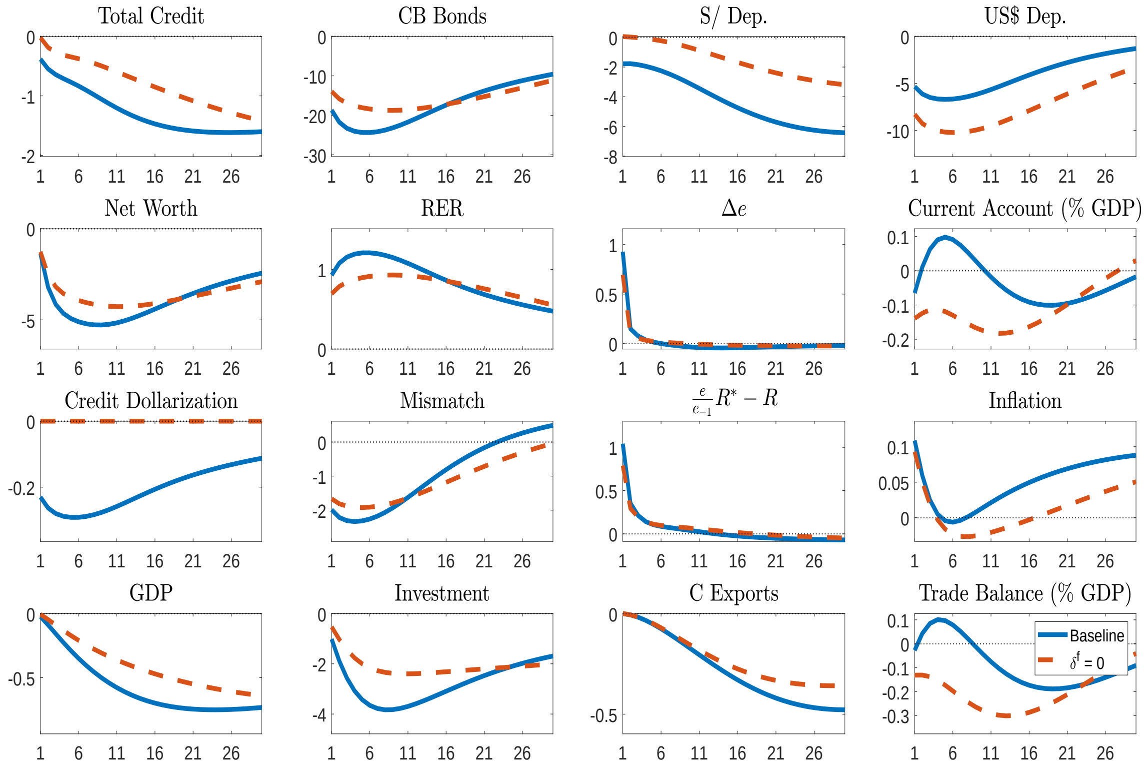
Quantitatively, our results suggest that under case 1, SFX interventions relative to exchange rate flexibility stabilize the responses of total credit, investment and GDP to a foreign interest rate shock, by around 66.4%, 59.9% and 39.4% after one year respectively. In contrast, when we simulate the baseline economy the responses of those variables are stabilized by 44.0%, 33.8% and 10.8% accordingly (see Table 8 in Appendix A). Similarly, Table 9 in Appendix A presents comparative results for the commodity price shock. We find that under case 1, SFX interventions are more effective to reduce the response of financial and macroeconomic variables to a persistent increase in commodity prices. In particular, in case 1, SFX interventions stabilize total credit and GDP in 91% and 52.8% respectively when compared to exchange rate flexibility while in the baseline case both variables are stabilized in 63% and in 22.3%, respectively.

When banks are not allowed to lend in foreign currency (case 2), the size of the steady state currency mismatch for the banking system is higher than in the baseline case (29% under case 2 rather than 22.2% under baseline, see Table 7). Thus, in equilibrium banks are more exposed to real exchange rate movements. In particular, bank's net worth is more sensitive to exchange rate movements in case 2. Consequently, SFX interventions are more effective in smoothing the response of financial as well as macroeconomic variables to external shocks. For example, after one year of a foreign interest rate shock, SFX interventions stabilize GDP in 42% relative to exchange rate flexibility in the economy without foreign currency loans. Under the baseline case, SFX interventions stabilize GDP relative to exchange rate flexibility only in 10.8%. Similar stabilization implications are found when the economy faces a commodity price shock (see Table 9 in Appendix A).

Moreover, in the model economy consistent with case 2, non financial firms are only affected by real exchange rate movements through the price of imported inputs but not through foreign currency denominated loans. Thus, although the banking system is more exposed to exchange rate fluctuations due to higher currency mismatch, firms are not financially vulnerable since their bank debt is denominated in domestic currency. In figure 9, we compare the impulse responses to a foreign interest rate shock between the baseline model and the model economy with no credit dollarization ($\delta^f = 0$, represented by the dotted line). First, notice that in the economy with deposit dollarization but no credit dollarization, the depreciation of the real exchange rate is not as large as in the case with credit and deposit dollarization (at impact the real exchange rate increases by 0.7% in case 2 while it increases by 1% in our baseline case). Thus, in case 2, SFX interventions are more effective in reducing the depreciation of the real exchange rate when the economy faces a foreign interest rate shock. Similarly, bank's net worth, total credit and bank's currency mismatch decline more in the baseline case when compared to the economy with no credit dollarization. As a result, the recession that occurs, in terms of GDP, consumption and investment, is much deeper under the baseline model which ultimately is a result that is related to having a real sector with no financial exposure to exchange rate fluctuations even though the banking sector may be more exposed, at least at the steady-state. Table 8 and Table 9 in Appendix A present pass-through calculations for this case.

In case 3, banks do not internalize the effects of higher borrowing in foreign currency with respect to the industry measure of currency mismatch. The main insight in this case is that banks, when choosing d_t^*, act as if they are not constrained in terms of obtaining funding from abroad. In other words, the first order conditions in case 3, imply that the UIP condition for banks holds, i.e., $\mu_t^{d*} = 0$. In this case, since banks do not internalize the incentive constraint SFX interventions are irrelevant in determining the dynamics of exchange rate. However, the sterilization operations associated to SFX interventions have effects on the composition of banks liabilities. This result is similar to the non-binding collateral constraint presented in Chang (2019). As a result, under case 3, SFX interventions are less effective in stabilizing the economy in the presence of external shocks (see Table 8 and Table 9 in Appendix A for further quantitative results).

Figure 9. Foreign Interest Rate Shock: Baseline vs Case 2



5.2 A Credit Crunch Simulation

We examine the transmission mechanism of an unexpected disruption in the financial intermediation of banks. To that end, we simulate a financial crisis scenario where the disturbance is an exogenous increase in the fraction of assets that banks are able to divert. We simulate a shock in the function $\Theta(x)$ which now is given by

$$\Theta(x_t) = \theta \exp(\texttt{crunch}_t)(1 + 0.5\varkappa x^2) \tag{53}$$

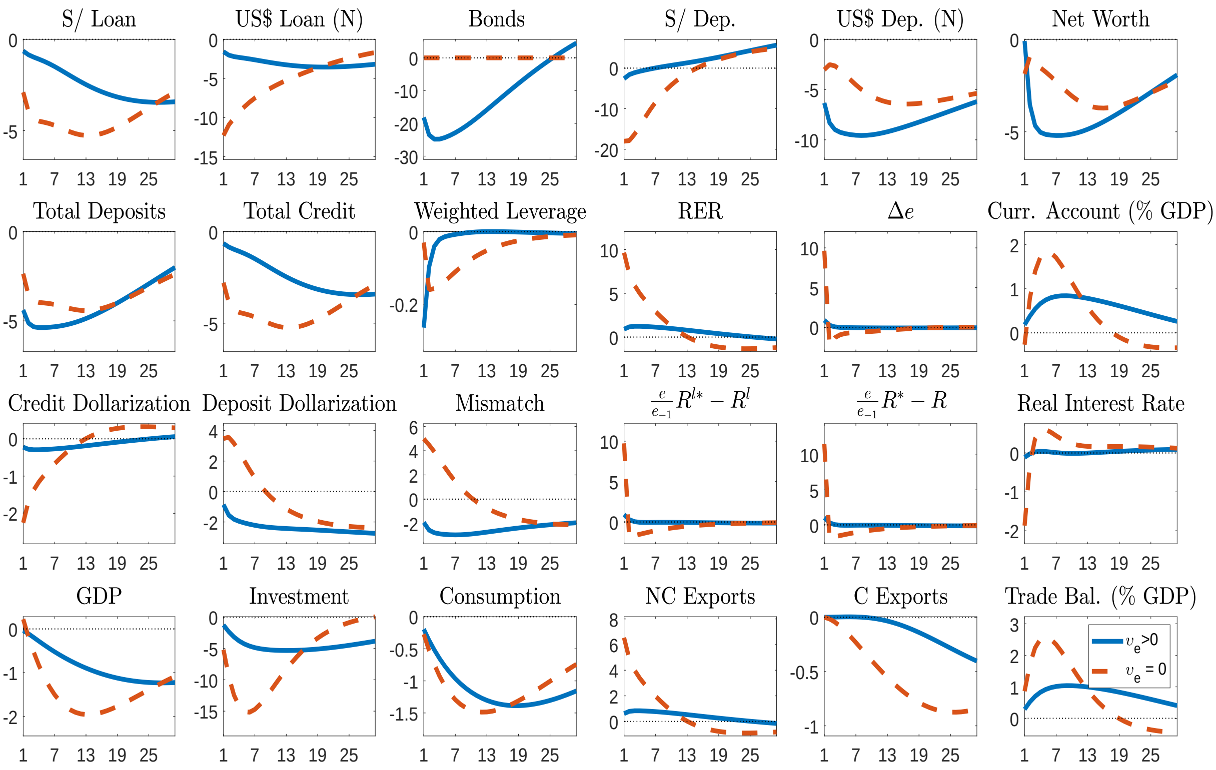
where $\texttt{crunch}_t$ is an exogenous variable that follows an autoregressive process. Whenever $\texttt{crunch}_t$ has a positive innovation, it generates tighter financial conditions for banks. In particular, an exogenous increase in $\Theta(x_t)$ reflects an exogenous tightening of financial conditions for domestic banks. From the point of view of a banker, an increase in $\Theta(x_t)$ means that now is harder to obtain funds domestically as well as internationally. From the point of view of bank lenders, now they prefer to lend less to banks.

We recalibrate some parameters in order to take the model to an scenario where bank's bankruptcies are more frequent events and it is easier for bankers to divert central bank bonds. In particular, the survival probability for bankers, σ, is reduced in almost 60 percentage points, and bank's assets enter the incentive constraint with equal weights, i.e., $\varpi^* = \varpi = \varpi^b = 1$. The size of the credit crunch shock is calibrated to generate, in an economy under a flexible exchange rate regime, a contraction in domestic bank's lending, GDP, and investment close to the contraction during the period 1998-2000 in the Peruvian economy.[19] We choose this

[19] In 1998, the Peruvian annual GDP growth decreased from 1.1 in 1998Q3 to -1.5 in 1998Q4, implying a contraction in the economic activity growth in 2.5 percentage points. Similarly, for the same period, the annual growth for investment and credit in dollars felt in around 16 and 9 percentage points. Moreover, the real exchange rate depreciated by 9.2% during the same window.

period because it is a period characterized by a significant financial disruption in Peru that originated due to external financial shocks such as the Asian and Russian financial crisis.

Figure 10. Responses to a Credit Crunch Experiment



Note: Except for lending and borrowing in foreign currency, which are denominated in foreign goods, all variables are expressed in terms of non-commodity goods.

Figure 10 compares the responses to a credit crunch shock in an economy under free floating (red dotted lines) and an economy under the SFX intervention regime (blue continuous line). In the two regimes, the tighter financial constraint restricts the bank's lending capacity and reduces the demand for deposits which ultimately pushes the domestic real interest rate, R_t, down. Under exchange rate flexibility, the real exchange rate overshoots on impact and further real appreciations are expected to be consistent with a given borrowing interest rate spread, μ_t^d. Then, the currency mismatch increases and consequently the incentive constraint gets even tighter which ultimately reinforces the initial financial intermediation disruption. The calibrated model indicates that, under the free floating regime, the real exchange rate depreciates by around 10% at impact while total credit falls by 5% after thirteen quarters. Moreover, expectations on real exchange rate appreciation limits the reduction of foreign currency deposits. Even though the initial real exchange rate depreciation induces a surplus in the trade balance, the contraction in financial conditions generate a persistent recession with GDP, investment and consumption declining in 2, 15 and 1.5 percent respectively at the trough of their responses.

On the other hand, SFX intervention policy mitigates the consequences of the credit crunch shock. The central bank responds to depreciation pressures over the exchange rate by selling foreign exchange reserves, and, consequently, shaping the response of the real exchange rate. Hence, under the SFX intervention regime, the domestic currency depreciates but not as much as under a free floating exchange rate, and a hump-shape pattern for the real exchange rate response emerges. The contraction in foreign currency deposits is larger under the SFX regime than under the flexible exchange rate regime since agents continue to expect consecutive real exchange rate depreciations rather than appreciations when SFX interventions are not active. Therefore, the currency mismatch in bank's balance sheet falls instead of increasing which means that the central bank is leaning against the tighter financial constraint by responding with SFX interventions. Furthermore, the associated sterilization operation activate the balance sheet substitution channel, enhancing the mitigation of the financial disruption in the domestic economy.

Quantitatively, under the SFX intervention regime, the real exchange rate depreciates by 1% instead of 10% under the flexible exchange rate regime. Similarly, when the central bank intervenes in the foreign exchange market, the interest rate spreads for borrowing and lending in foreign currency relative to do it in domestic currency increases in 1 percentage point rather than in 10 percentage points under free floating. On impact, the less contractionary financial conditions are finally reflected in the reduction of total credit and investment by 0.5 and 1 percent instead of 3.5 and 5 percent under the free exchange rate flexibility.

6 Concluding Remarks

We study the role of SFX interventions as an additional monetary policy instrument for emerging market economies as a response to external shocks. To that end, we propose a macroeconomic model in order to analyze sterilized foreign exchange interventions as a balance sheet policy induced by a financial friction in the form of an agency problem between banks and depositors. In this setting, we examine the effectiveness of SFX invention policies in mitigating the effects of external shocks over the domestic macroeconomic allocation. We find that the effect of SFX interventions over the transmission mechanism of the external shocks operates through two main mechanisms: the exchange rate smoothing channel and the balance sheet substitution channel. On the one hand. the exchange rate smoothing channel is related to lean against the trade channel, balance sheet channel, and the tightness of financial constraints faced by banks. On the other hand, because they are implemented through the variations of the supply of central bank bonds, then SFX interventions have a direct effect over the composition of assets held by banks.

We take the model to the data in order to quantify the transmission mechanism of external shocks and the role of SFX interventions in mitigating its effects over the domestic economy. Our experiment is intended to quantify the differences in the response of the economy to external shocks when SFX interventions are activated compared to the case where they are not. In the calibrated model, a sterilized sell of foreign exchange reserves reduces the response of aggregate bank lending and investment by around 44 percent and 34 percent respectively whenever the economy faces an increase of around 20 basis points in the foreign interest rate. When the economy faces a commodity boom (an increase of 6.31% in the metal export index), a sterilized buy of foreign exchange reserves limits the increase in bank lending in about 63 percent after one year of the initial shock. Consequently, the response of investment and GDP is also muted by around 45 percent and respectively. Moreover, SFX interventions successfully reduce credit, investment, and output unconditional volatility by around 6 percent, 14 percent, and 6 percent, respectively. Hence, these results indicate that SFX interventions produce significant welfare gains when responding to external shocks. If the Central Bank does not intervene in the foreign exchange market in the face of external shocks, there would be a welfare loss of 1.1% in consumption.

Furthermore, we explore additional numerical experiments. First, we relax three assumptions of our basic formulation of the model that may be viewed as strong and restrictive with the objective to study our setting under more general assumptions. In the first generalization of the model, the three assets that banks can hold enter with equal weights into the incentive compatibility constraint. In our second generalization, banks are allowed to lend only in domestic currency so that the banking system does not exhibit credit dollarization. For these two first extensions, we find that SFX interventions are significantly more effective due to stronger impacts over the incentive compatibility constraint and a higher currency mismatch in bank's balance sheet, respectively. Finally, in the third modification, banks do not internalize the effects of higher borrowing in foreign currency with respect to the industry measure of currency mismatch. In this case, banks act as if they are not constrained in terms of obtaining funding from abroad, implying that the standard UIP condition holds without any risk premium. As a result, SFX interventions are less effective in stabilizing the economy in the presence of external shocks.

Second, we examine the transmission mechanism of an unexpected disruption in the financial intermediation of banks. In this scenario, a credit crunch occurs since the economy faces and exogenous increase in the fraction of assets that banks are able to divert which ultimately generates tighter financial conditions for banks. We recalibrate some of the parameters of the model in order to replicate the contraction in bank lending, GDP and investment that the Peruvian economy experienced during the third and fourth quarters of 1998 in the course of the Russian crisis episode. Our findings suggest that an SFX intervention policy mitigates the consequences of a credit crunch shock when compared to exchange rate flexibility.

We consider that the financial friction view of SFX interventions needs further research. For instance, it differs from the unconventional monetary policy framework for closed economies in several ways. First, it seems that SFX interventions have been active and effective even in normal times for emerging market economies, contrary to the unconventional monetary policy tools studied in the context of closed economies. In the latter case, once the zero lower bound is reached, unconventional tools may be deployed. Second, what really matters for emerging market economies is how tight financial constraints are and not necessarily if they bind or not. Third, in practice, the communication of SFX interventions is at odds with the communication of unconventional policies in closed economies. For example, it seems that there is much less forward guidance associated to SFX interventions than with QE or LSAP. Finally, the effective lower bound for EMEs may not only be related to the nominal interest rate but also to a non negative amount of official foreign reserves needed to combine SFX interventions within an inflation targeting regime.

References

Aoki, K., Benigno, G., and Kiyotaki, N. (2018). Monetary and Financial Policies in Emerging Markets. Technical report, Mimeo.

Backus, D. K. and Kehoe, P. J. (1989). On the denomination of government debt : A critique of the portfolio balance approach. *Journal of Monetary Economics*, 23(3):359–376.

Benes, J., Berg, A., Portillo, R., and Vavra, D. (2015). Modeling Sterilized Interventions and Balance Sheet Effects of Monetary Policy in a New-Keynesian Framework. *Open Economies Review*, 26(1):81–108.

Blanchard, O., Adler, G., and de Carvalho Filho, I. (2015). Can Foreign Exchange Intervention Stem Exchange Rate Pressures from Global Capital Flow Shocks? Working Paper Series WP15-18, Peterson Institute for International Economics.

Branson, W., Halttunen, H., and Masson, P. (1977). Exchange rates in the short run: The dollar-dentschemark rate. *European Economic Review*, 10(3):303–324.

Branson, W. H. and Henderson, D. W. (1985). Chapter 15 the specification and influence of asset markets. volume 2 of *Handbook of International Economics*, pages 749 – 805. Elsevier.

Canova, F. (2005). The transmission of US shocks to Latin America. *Journal of Applied Econometrics*, 20(2):229–251.

Castillo, P., Lama, R., and Medina, J. P. (2019). Liability dollarization and foreign exchange intervention in emerging economies. Mimeo.

Castillo, P., Montoro, C., and Tuesta, V. (2009). A Dynamic Stochastic General Equilibrium Model with Dollarization for the Peruvian Economy. Working Papers 2009-003, Banco Central de Reserva del Perú.

Cavallino, P. (2019). Capital Flows and Foreign Exchange Intervention. *American Economic Journal: Macroeconomics*, 11(2):127–170.

Chang, R. (2019). Foreign Exchange Intervention Redux. In Álvaro Aguirre, Brunnermeier, M., and Saravia, D., editors, *Monetary Policy and Financial Stability: Transmission Mechanisms and Policy Implications*, volume 26 of *Central Banking, Analysis, and Economic Policies Book Series*, chapter 7, pages 205–247. Central Bank of Chile.

Céspedes, L. F. and Chang, R. (2019). Optimal foreign reserves and central bank policy under financial stress. Working in progress.

Céspedes, L. F., Chang, R., and Velasco, A. (2014). Is Inflation Targeting Still on Target? The Recent Experience of Latin America. *International Finance*, 17(2):185–208.

Céspedes, L. F., Chang, R., and Velasco, A. (2017). Financial intermediation, real exchange rates, and unconventional policies in an open economy. *Journal of International Economics*, 108:S76 – S86. 39th Annual NBER International Seminar on Macroeconomics.

Cushman, D. O. and Zha, T. (1997). Identifying monetary policy in a small open economy under flexible exchange rates. *Journal of Monetary Economics*, 39(3):433–448.

Dornbusch, R. (1980). Exchange rate risk and the macroeconomics of exchange rate determination. Working Paper 493, National Bureau of Economic Research.

Drechsel, T. and Tenreyro, S. (2018). Commodity booms and busts in emerging economies. *Journal of International Economics*, 112(C):200–218.

Fanelli, S. and Straub, L. (2019). A theory of foreign exchange interventions. Revise and resubmit at Review of Economic Studies.

Fernández, A., Schmitt-Grohé, S., and Uribe, M. (2017). World shocks, world prices, and business cycles: An empirical investigation. *Journal of International Economics*, 108:S2 – S14. 39th Annual NBER International Seminar on Macroeconomics.

Fornero, J., Kirchner, M., and Yany, A. (2015). Terms of Trade Shocks and Investment in Commodity-Exporting Economies. In Caputo, R. and Chang, R., editors, *Commodity Prices and Macroeconomic Policy*, volume 22 of *Central Banking, Analysis, and Economic Policies Book Series*, chapter 5, pages 135–193. Central Bank of Chile.

Fratzscher, M., Lo Duca, M., and Straub, R. (2018). On the international spillovers of us quantitative easing. *The Economic Journal*, 128(608):330–377.

Frenkel, J. A. and Mussa, M. L. (1985). Asset markets, exchange rates and the balance of payments. In Jones, R. W. and Kenen, P. B., editors, *Handbook of International Economics*, volume 2 of *Handbook of International Economics*, chapter 14, pages 679–747. Elsevier.

Gabaix, X. and Maggiori, M. (2015). International Liquidity and Exchange Rate Dynamics. *The Quarterly Journal of Economics*, 130(3):1369–1420.

Garcia-Cicco, J., Kirchner, M., Carrillo, J., Rodríguez, D., Perez, F., Gondo, R., Montoro, C., and Chang, R. (2017). Financial and real shocks and the effectiveness of monetary and macroprudential policies in Latin American countries. BIS Working Papers 668, Bank for International Settlements.

Gertler, M. and Karadi, P. (2011). A model of unconventional monetary policy. *Journal of Monetary Economics*, 58(1):17–34.

Gertler, M. and Karadi, P. (2013). QE 1 vs. 2 vs. 3. . . : A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool. *International Journal of Central Banking*, 9(1):5–53.

Gertler, M. and Kiyotaki, N. (2010). Financial intermediation and credit policy in business cycle analysis. In Friedman, B. M. and Woodford, M., editors, *Handbook of Monetary Economics*, volume 3, chapter 11, pages 547–599. Elsevier, 1 edition.

Gertler, M., Kiyotaki, N., and Queralto, A. (2012). Financial crises, bank risk exposure and government financial policy. *Journal of Monetary Economics*, 59(S):17–34.

Ghosh, A., Ostry, J., and Qureshi, M. (2018). *Taming the Tide of Capital Flows: A Policy Guide*, volume 1. The MIT Press, 1 edition.

Greenwood, J., Hercowitz, Z., and Huffman, G. (1988). Investment, capacity utilization, and the real business cycle. *The American Economic Review*, (3):402–417.

Henderson, D. W. and Rogoff, K. (1982). Negative net foreign asset positions and stability in a world portfolio balance model. *Journal of International Economics*, 13(1-2):85–104.

Hofmann, B., Shin, H. S., and Villamizar-Villegas, M. (2019). FX intervention and domestic credit: evidence from high-frequency micro data. Borradores de Economia 1069, Banco de la Republica de Colombia.

Kiyotaki, N. and Moore, J. (1997). Credit Cycles. *Journal of Political Economy*, 105(2):211–248.

Kouri, P. J. K. (1976). The exchange rate and the balance of payments in the short run and in the long run: A monetary approach. *The Scandinavian Journal of Economics*, 78:280 – 304. Proceeding of a Conference on Flexible Exchange Rates and Stabilization Policy.

Krugman, P. R. (1981). Consumption Preferences, Asset Demands, and Distribution Effects in International Financial Markets. NBER Working Papers 0651, National Bureau of Economic Research, Inc.

Kumhof, M. (2010). On the theory of sterilized foreign exchange intervention. *Journal of Economic Dynamics and Control*, 34(8):1403–1420.

Liu, Z. and Spiegel, M. M. (2015). Optimal Monetary Policy and Capital Account Restrictions in a Small Open Economy. *IMF Economic Review*, 63(2):298–324.

Miao, J. and Wang, P. (2010). Credit Risk and Business Cycles. Boston University - Department of Economics - Working Papers Series WP2010-033, Boston University - Department of Economics.

Montoro, C. and Ortiz, M. (2016). Foreign exchange intervention and monetary policy design: a market microstructure analysis. Working Papers 2016-008, Banco Central de Reserva del Perú.

Obstfeld, M. (1983). Exchange rates, inflation, and the sterilization problem: Germany, 1975-1981. *European Economic Review*, 21(1-2):161–189.

Rotemberg, J. (1982). Monopolistic price adjustment and aggregate output. *Review of Economic Studies*, 49(4):517–531.

Schmitt-Grohe, S. and Uribe, M. (2007). Optimal simple and implementable monetary and fiscal rules. *Journal of Monetary Economics*, 54(6):1702–1725.

Schmitt-Grohe, S. and Uribe, M. (2018). How important are term of trade shocks. *International Economic Review*, 59(1):86–111.

Shousha, S. (2016). Macroeconomic effects of commodity booms and busts: The role of financial frictions. Unpublished manuscript.

Villamizar-Villegas, M. and Perez-Reyna, D. (2017). A Theoretical Approach To Sterilized Foreign Exchange Intervention. *Journal of Economic Surveys*, 31(1):343–365.

Winkelried, D. (2013). Modelo de Proyección Trimestral del BCRP: Actualización y novedades. *Revista Estudios Económicos*, (26):9–60.

Zha, T. (1999). Block recursion and structural vector autoregressions. *Journal of Econometrics*, 90(2):291–316.

A Additional Tables

Table 8. One-Year Pass-Through for Foreign Interest Rate Shock

	Baseline			*Case 1*: $\varpi^* = \varpi = \varpi^b = 1$			*Case 2*: $\delta^f = 0$			*Case 3*: Aggregate x_t		
	$v_e > 0$	$v_e = 0$	SFX	$v_e > 0$	$v_e = 0$	SFX	$v_e > 0$	$v_e = 0$	SFX	$v_e > 0$	$v_e = 0$	SFX
Total Credit	-3.51	-6.27	44.0	-2.05	-6.09	66.4	-1.22	-5.56	78.1	-3.67	-4.33	15.1
RER	6.69	13.67	51.1	4.20	13.88	69.7	4.91	11.07	55.7	6.65	6.41	-3.8
Δe	1.83	2.69	31.9	1.20	2.74	56.1	1.34	2.10	35.9	1.45	1.37	-5.7
Current Account (% GDP)	0.15	0.46	66.8	-0.55	0.46	-18.8	-0.75	-0.03	-2319.3	-0.65	-0.69	5.5
Credit Dollarization	-1.63	-3.23	49.5	-1.04	-3.27	68.0	-0.00	-0.00	54.0	-1.66	-1.47	-12.6
Mismatch	-13.61	9.33	-45.9	-7.92	9.26	14.5	-11.19	3.08	-263.7	-13.02	6.01	-116.7
$\frac{e}{e_{-1}}R^{l*} - R^l$	2.27	3.70	38.6	1.89	3.19	40.6	1.81	3.02	40.3	2.18	-2.33	6.7
$\frac{e}{e_{-1}}R^* - R$	2.69	3.87	30.5	1.93	3.95	51.2	2.18	3.08	29.4	0.00	0.00	-
Inflation	0.30	1.45	79.0	-0.06	1.52	95.8	0.24	0.92	73.6	0.09	0.20	55.4
GDP	-0.76	-0.85	10.8	-0.51	-0.85	39.4	-0.42	-0.72	41.7	-0.55	-0.54	-1.6
Investment	-13.14	-19.83	33.8	-7.93	-19.77	59.9	-7.32	-16.41	55.4	-14.56	-13.95	-4.4
Consumption	-2.62	-3.22	18.4	-2.01	-3.29	38.8	-2.05	-2.51	18.4	0.01	-0.07	85.9
NC Exports	4.23	8.96	52.8	2.54	9.10	72.1	3.02	7.19	58.0	4.20	4.04	-4.1
C Exports	-0.09	-0.13	31.2	-0.08	-0.13	41.7	-0.09	-0.11	23.1	-0.09	-0.09	6.5
Trade Balance (% GDP)	0.31	1.53	79.8	-0.73	1.57	53.3	-0.85	0.67	-26.1	-0.53	-0.58	8.6

Note. We measure the effectiveness of SFX interventions conditional on one specific shock comparing the j-year horizon pass-through between an economy under SFX intervention regime and another under exchange rate flexibility:

$$\texttt{SFX}_j = -100 \times \left[\frac{abs(\texttt{PT}_j|_{v_e>0}) - abs(\texttt{PT}_j|_{v_e=0})}{abs(\texttt{PT}_j|_{v_e=0})} \right]$$

where $\texttt{PT}_j$ is the j-year pass-through and is defined as

$$\texttt{PT}_j = \frac{\sum_{k=1}^{4j} \texttt{IRF}_k}{\sum_{k=1}^{4j} \texttt{IRF}_k}$$

Hence, the conditional stabilization index, SFX, reads as the pass-through for the concerning variables, in absolute values, falls by $\texttt{SFX}_j$ percent when the central bank start implementing SFX interventions.

Table 9. One-Year Pass-Through for Commodity Price Index Shock

	Baseline			*Case 1*: $\varpi^* = \varpi = \varpi^b = 1$			*Case 2*: $\delta^f = 0$			*Case 3*: Aggregate x_t		
	$\upsilon_e > 0$	$\upsilon_e = 0$	SFX	$\upsilon_e > 0$	$\upsilon_e = 0$	SFX	$\upsilon_e > 0$	$\upsilon_e = 0$	SFX	$\upsilon_e > 0$	$\upsilon_e = 0$	SFX
Total Credit	0.04	0.09	63.0	0.01	0.09	91.2	0.02	0.11	85.5	0.02	0.04	42.3
RER	-0.08	-0.21	61.1	-0.04	-0.21	79.7	-0.07	-0.20	64.2	-0.04	-0.04	-2.1
Δe	-0.03	-0.05	43.6	-0.02	-0.05	67.8	-0.02	-0.04	47.0	-0.01	-0.01	-3.9
Current Account (% GDP)	0.13	0.11	-9.9	0.14	0.12	-22.3	0.13	0.11	-18.3	0.15	0.14	-2.2
Credit Dollarization	0.02	0.05	60.0	0.01	0.05	78.8	0.00	0.00	63.2	0.01	0.01	-8.8
Mismatch	0.11	-0.20	47.0	0.02	-0.20	91.0	0.09	-0.12	22.8	-0.01	-0.12	90.9
$\frac{e}{e_{-1}} R^{l*} - R^l$	-0.02	-0.05	56.4	-0.01	-0.04	70.8	-0.02	-0.05	59.4	-0.02	-0.00	-682.0
$\frac{e}{e_{-1}} R^* - R$	-0.02	-0.05	54.9	-0.01	-0.05	81.1	-0.02	-0.04	57.2	0.00	0.00	-
Inflation	0.00	-0.02	73.5	0.01	-0.02	53.0	0.01	-0.01	24.0	0.02	0.02	0.8
GDP	0.01	0.02	22.3	0.01	0.02	52.8	0.01	0.02	42.2	0.01	0.01	14.1
Investment	0.19	0.35	45.2	0.09	0.34	74.0	0.15	0.38	59.8	0.17	0.19	8.4
Consumption	0.05	0.06	19.4	0.04	0.06	40.5	0.05	0.06	16.0	-0.01	-0.01	-15.0
NC Exports	-0.05	-0.13	65.2	-0.02	-0.13	85.0	-0.04	-0.13	68.7	-0.02	-0.02	-3.4
C Exports	0.00	0.00	28.6	0.00	0.00	35.4	0.00	0.00	24.7	0.00	0.00	3.9
Trade Balance (% GDP)	0.19	0.16	-17.9	0.21	0.16	-30.2	0.20	0.16	-23.4	0.22	0.22	-1.5

Note. We measure the effectiveness of SFX interventions conditional on one specific shock comparing the j-year horizon pass-through between an economy under SFX intervention regime and another under exchange rate flexibility:

$$\mathtt{SFX}_j = -100 \times \left[\frac{abs(\mathtt{PT}_j|_{\upsilon_e>0}) - abs(\mathtt{PT}_j|_{\upsilon_e=0})}{abs(\mathtt{PT}_j|_{\upsilon_e=0})} \right]$$

where $\mathtt{PT}_j$ is the j-year pass-through and is defined as

$$\mathtt{PT}_j = \frac{\sum_{k=1}^{4j} \mathtt{IRF}_k}{\sum_{k=1}^{4j} \mathtt{IRF}_k}$$

Hence, the conditional stabilization index, SFX, reads as the pass-through for the concerning variables, in absolute values, falls by $\mathtt{SFX}_j$ percent when the central bank start implementing SFX interventions.

B Additional Figures

B.1 Foreign Interest Rate Shock

Figure 11. Case 1: Perfect Subsitution in Bank's assets

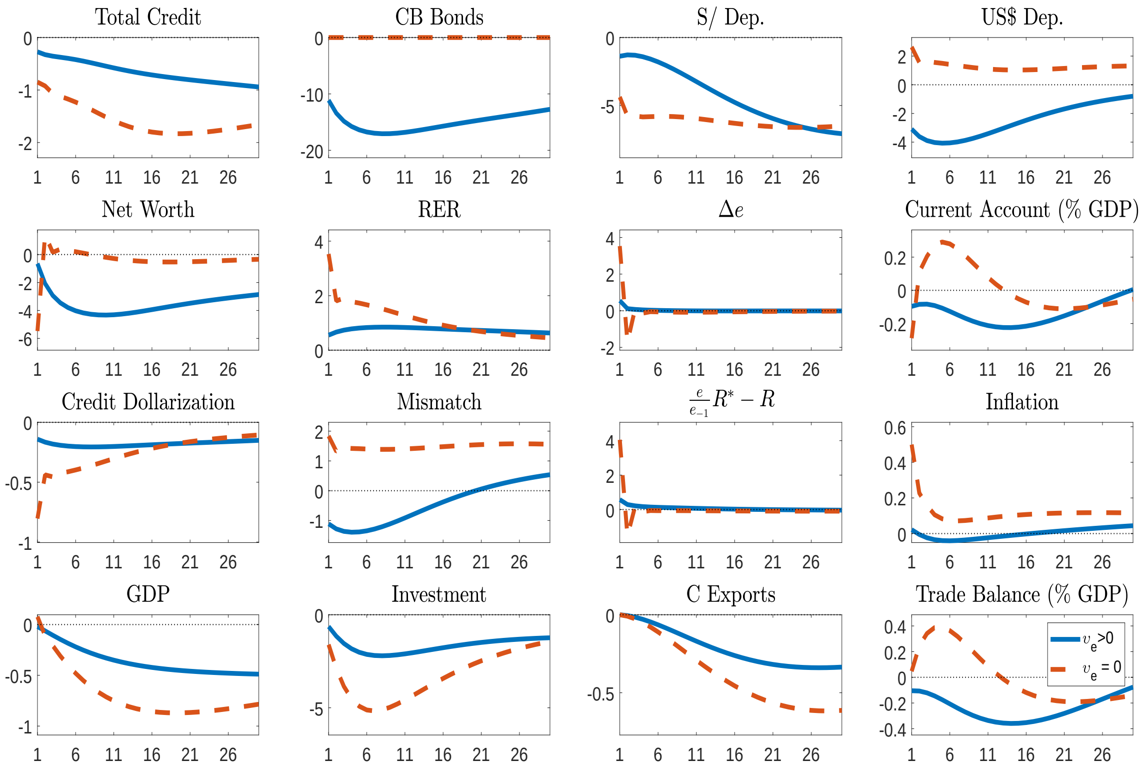


Figure 12. Case 2: No credit Dollarization

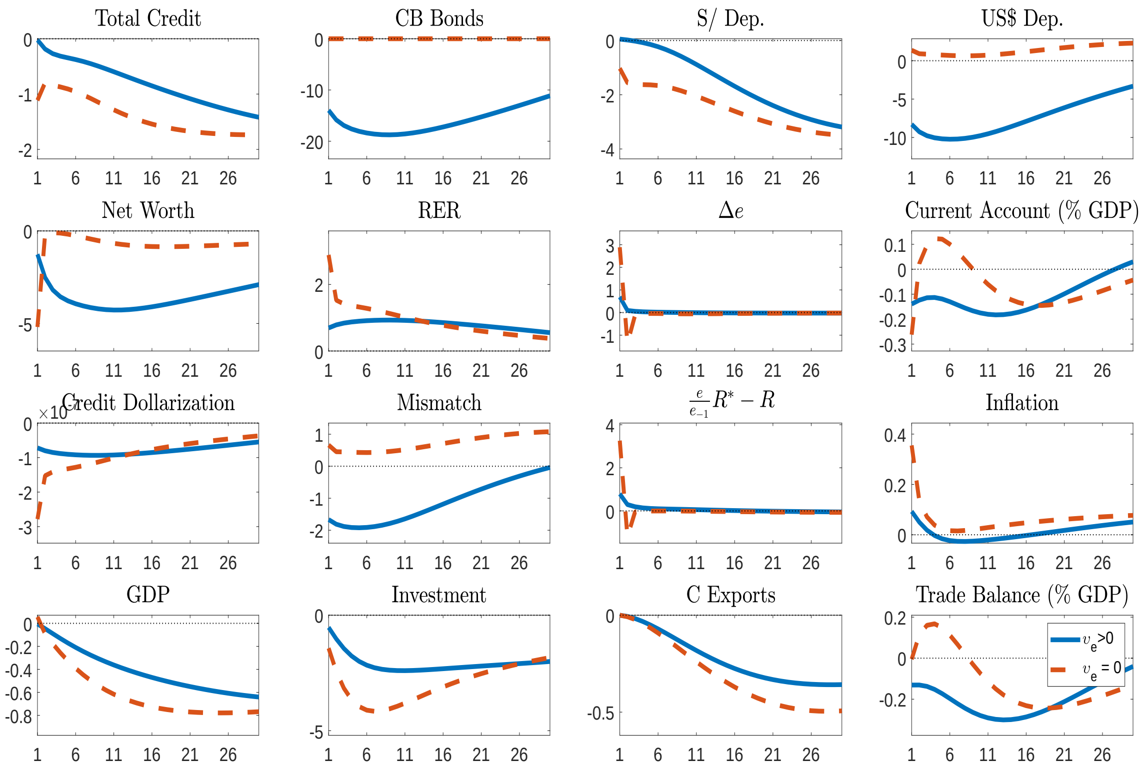
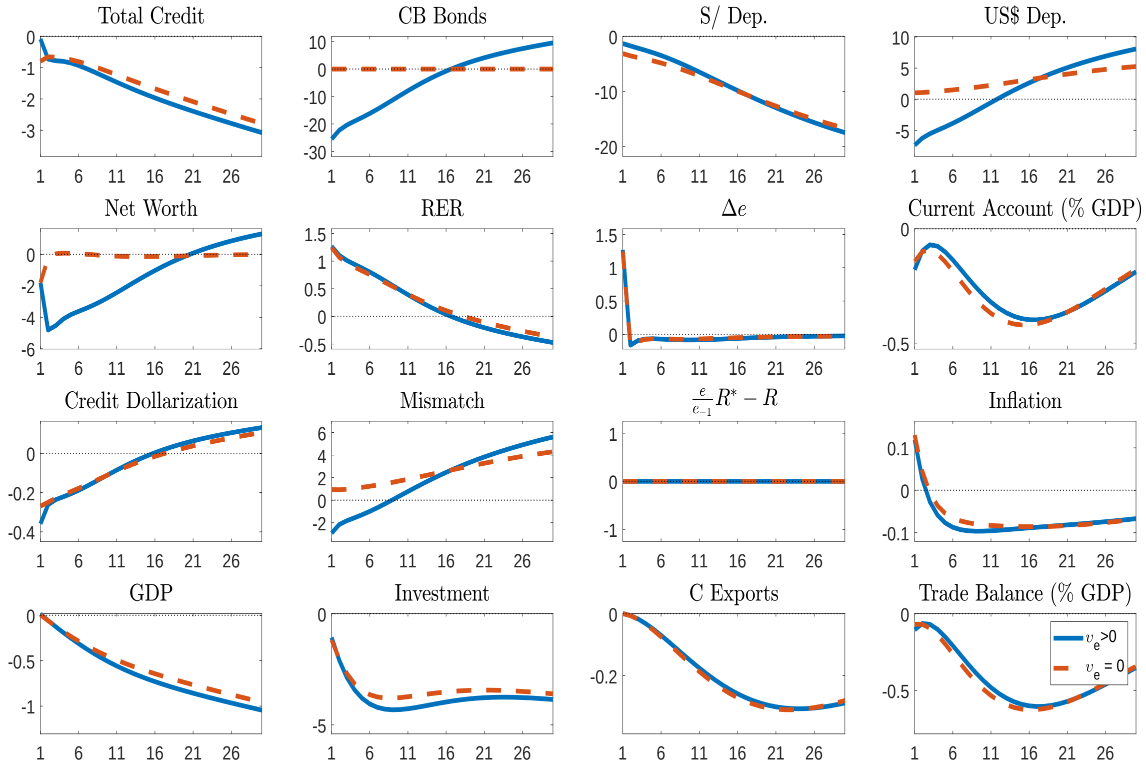


Figure 13. Case 3: Aggregate Currency Mismatch



B.2 Commodity Price Shocks

Figure 14. Case 1: Perfect Subsitution in Bank's assets

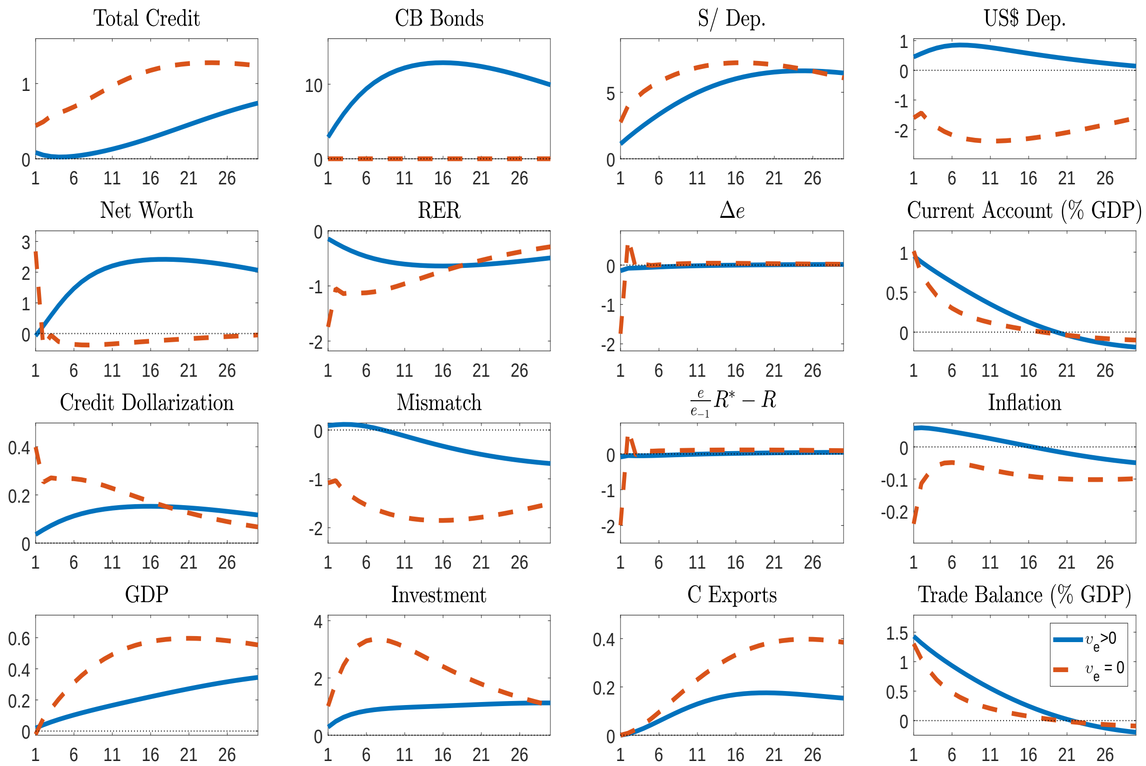


FIGURE 15. CASE 2: NO CREDIT DOLLARIZATION

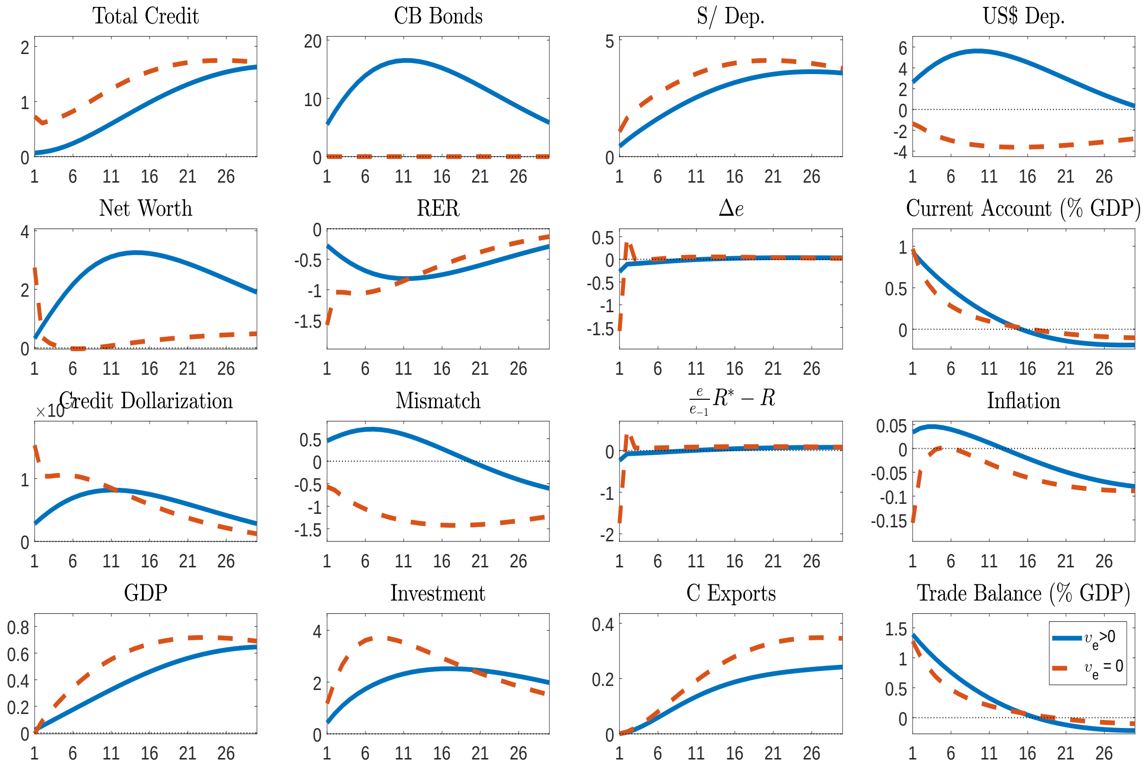


FIGURE 16. CASE 3: AGGREGATE CURRENCY MISMATCH

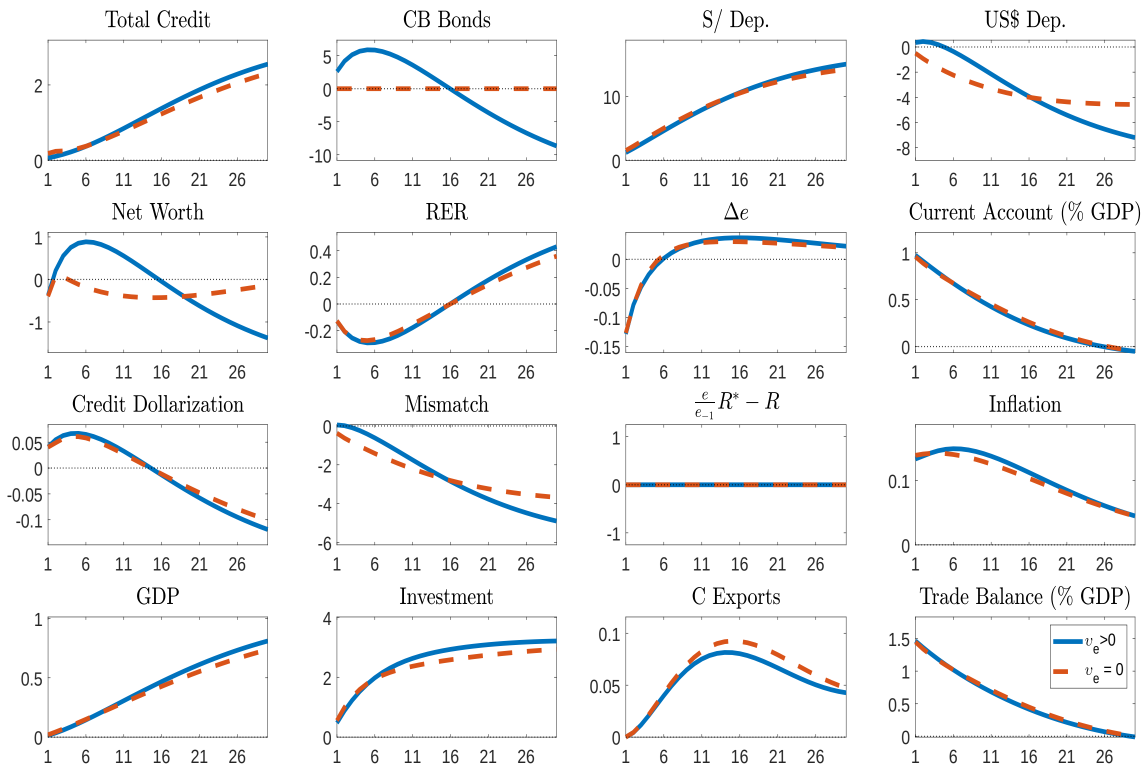
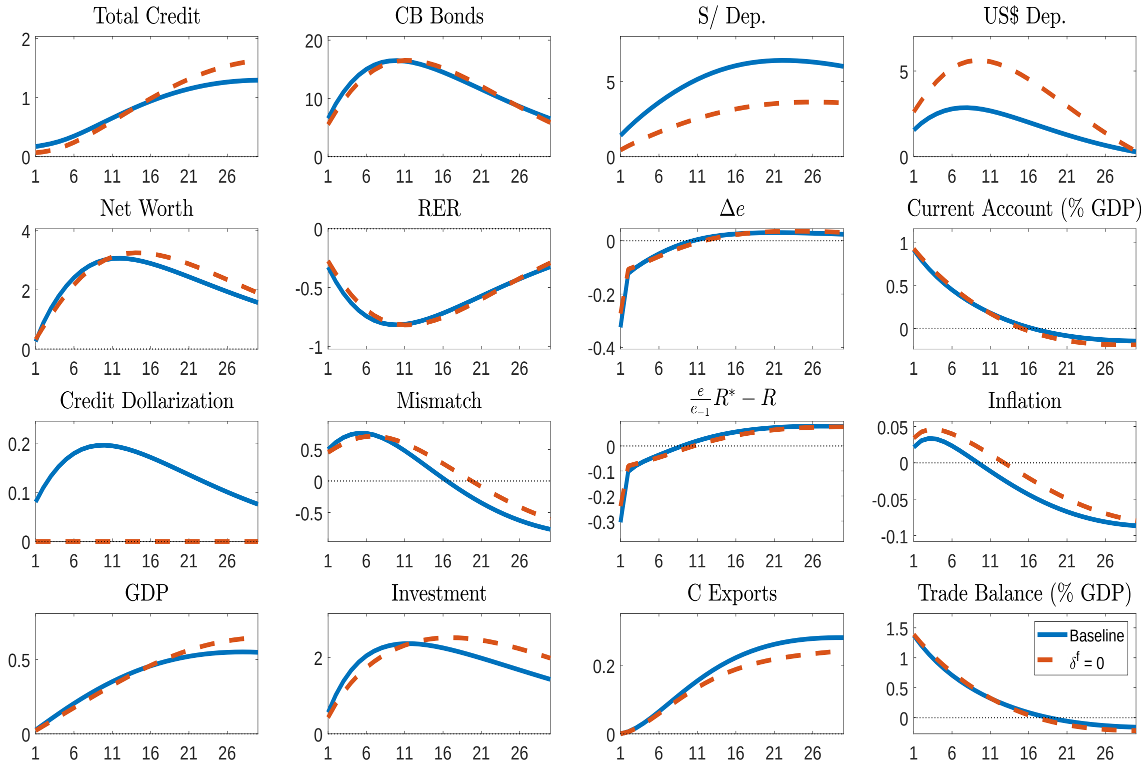


Figure 17. Baseline vs Case 2



C Foriegn Demand Shock

Figure 18. Baseline

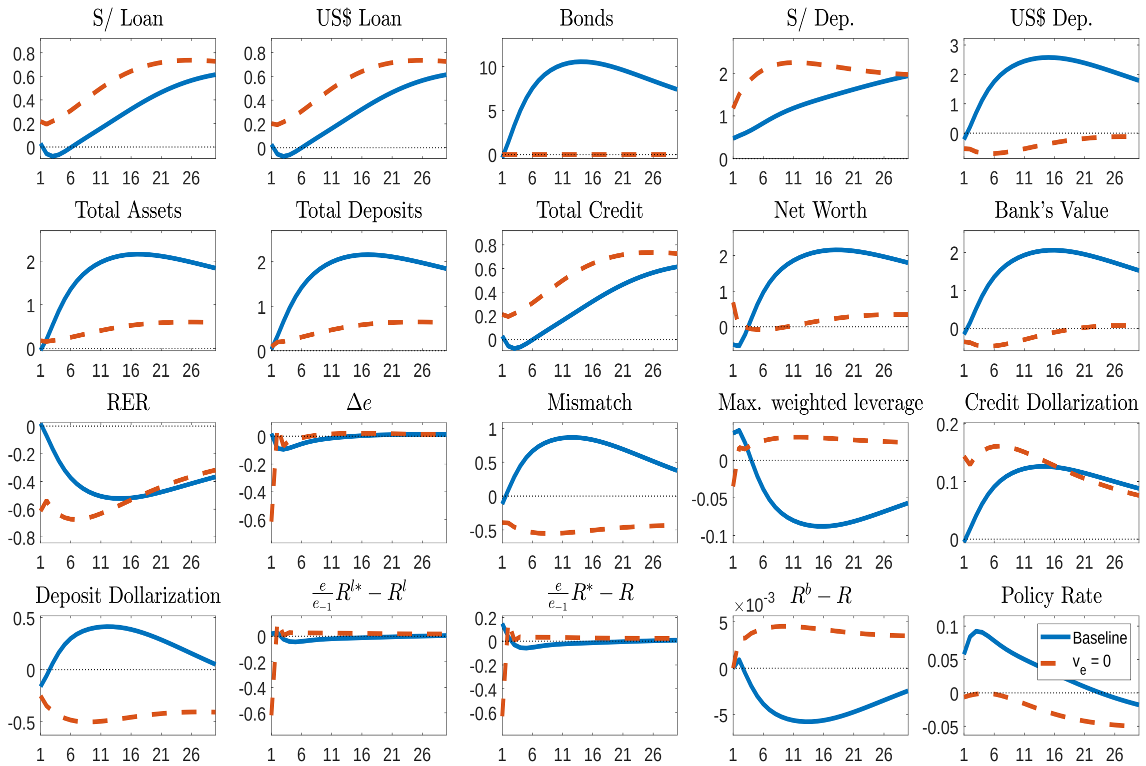


Figure 19. Baseline

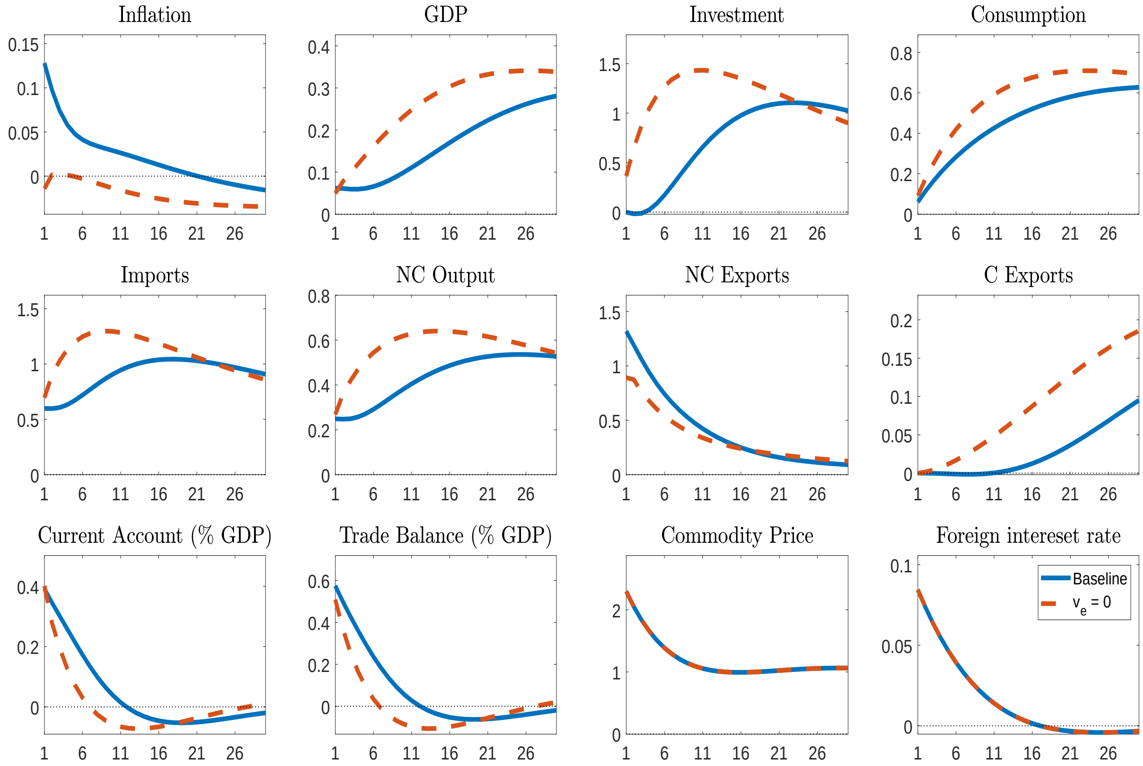


Table 10. Pass-Through in the Baseline Model

	Year 1			Year 2		
	$v_e > 0$	$v_e = 0$	SFX	$v_e > 0$	$v_e = 0$	SFX
Total Credit	-0.10	0.52	80.7	-0.04	0.72	94.8
RER	-0.28	-1.44	80.7	-0.67	-1.69	60.2
Δe	-0.15	-0.38	61.0	-0.15	-0.22	33.0
Current Account (% GDP)	0.78	0.60	-29.5	0.64	0.36	-77.9
Credit Dollarization	0.07	0.34	80.1	0.16	0.40	59.4
Mismatch	0.42	-1.05	59.7	1.15	-1.30	11.6
$\frac{e}{e_{-1}}R^{l*} - R^l$	-0.01	-0.27	96.1	-0.06	-0.12	52.8
$\frac{e}{e_{-1}}R^{*} - R$	0.06	-0.25	76.4	-0.03	-0.10	67.5
Inflation	0.21	-0.01	-3620.6	0.17	-0.01	-1823.8
GDP	0.15	0.21	29.8	0.17	0.34	49.2
Investment	-0.00	1.76	99.7	0.30	2.70	89.0
Consumption	0.33	0.50	34.3	0.58	0.86	32.7
NC Exports	2.71	1.92	-40.9	2.44	1.75	-39.2
C Exports	-0.00	0.01	94.4	-0.00	0.03	93.6
Trade Balance (% GDP)	1.12	0.78	-43.6	0.90	0.46	-98.0

Figure 20. Case 1: Perfect Subsitution in Bank's assets

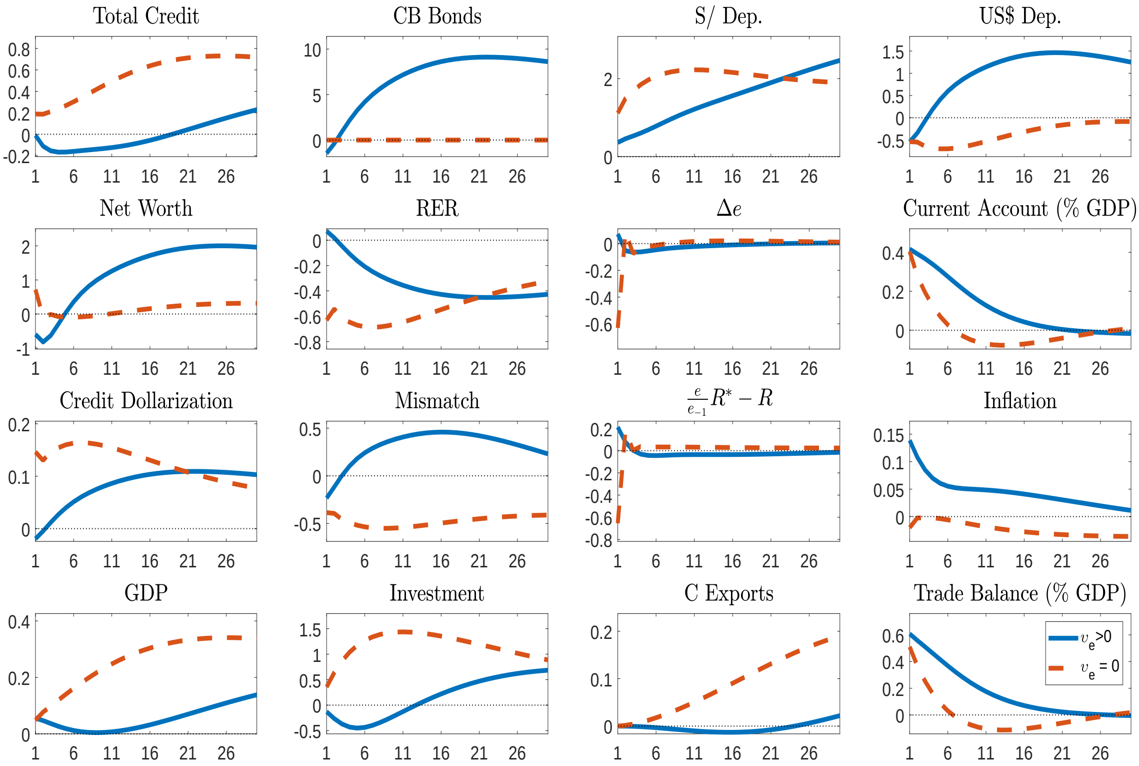


Table 11. Pass-Through for Case 1

	Year 1			Year 2		
	$v_e > 0$	$v_e = 0$	SFX)	$v_e > 0$	$v_e = 0$	SFX)
Total Credit	-0.27	0.49	46.4	-0.35	0.70	49.7
RER	-0.04	-1.46	97.5	-0.32	-1.72	81.4
Δe	-0.06	-0.39	83.4	-0.09	-0.23	59.0
Current Account (% GDP)	0.91	0.60	-51.0	0.85	0.36	-139.1
Credit Dollarization	0.01	0.35	97.5	0.08	0.41	80.9
Mismatch	-0.15	-1.04	85.7	0.26	-1.29	79.6
$\frac{e}{e_{-1}}R^* - R$	0.16	-0.26	39.5	0.03	-0.10	68.5
Inflation	0.24	-0.02	-1380.6	0.21	-0.02	-1004.4
GDP	0.10	0.21	51.6	0.07	0.34	79.6
Investment	-0.72	1.74	58.8	-0.94	2.68	65.0
Consumption	0.19	0.51	62.5	0.35	0.88	60.5
NC Exports	2.87	1.91	-50.7	2.68	1.74	-54.6
C Exports	-0.00	0.01	87.7	-0.01	0.03	81.6
Trade Balance (% GDP)	1.29	0.78	-65.3	1.17	0.45	-161.2

Figure 21. Case 2: No credit Dollarization

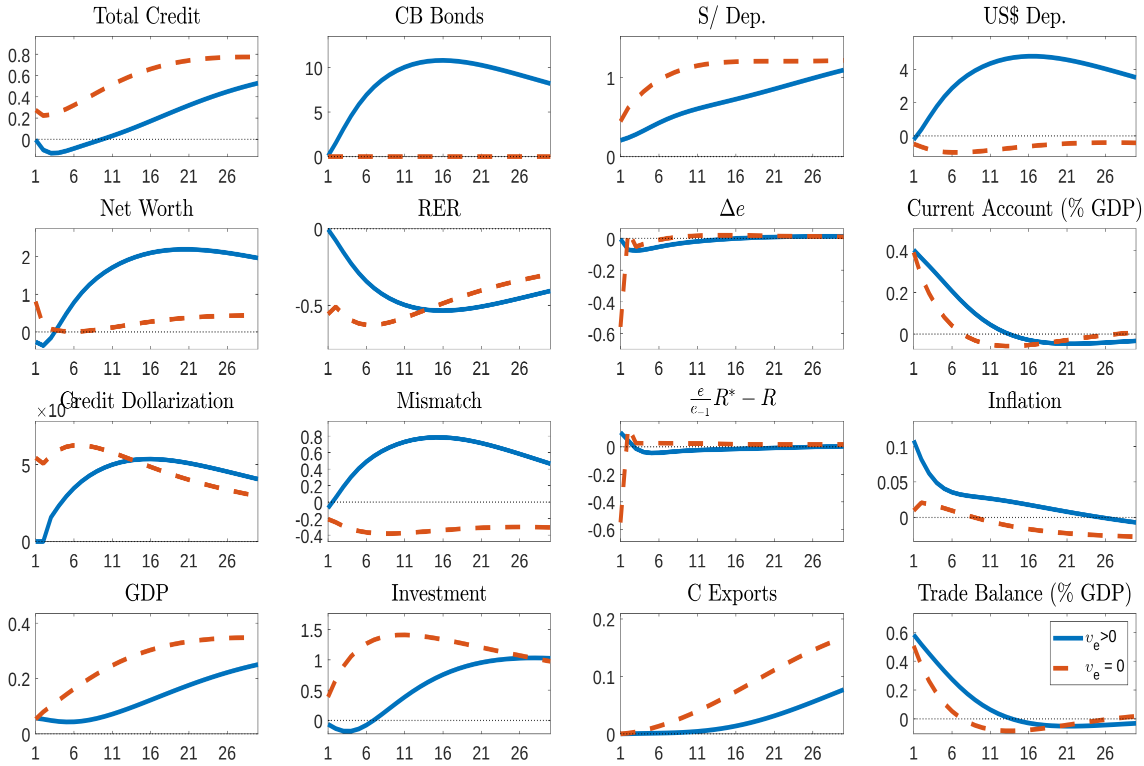


Table 12. Pass-Through for Case 2

	Year 1			Year 2		
	$v_e > 0$	$v_e = 0$	SFX	$v_e > 0$	$v_e = 0$	SFX
Total Credit	-0.22	0.59	63.6	-0.22	0.78	72.2
RER	-0.28	-1.34	79.4	-0.63	-1.57	59.7
Δe	-0.14	-0.36	62.2	-0.14	-0.21	31.8
Current Account (% GDP)	0.83	0.60	-39.1	0.71	0.37	-92.0
Credit Dollarization	0.00	0.00	79.0	0.00	0.00	59.1
Mismatch	0.28	-0.64	55.9	0.85	-0.85	0.1
$\frac{e}{e_{-1}}R^* - R$	0.06	-0.21	70.8	-0.02	-0.08	75.1
Inflation	0.18	0.04	-363.1	0.15	0.03	-357.3
GDP	0.12	0.21	42.7	0.13	0.35	63.0
Investment	-0.33	1.81	81.7	-0.22	2.72	91.9
Consumption	0.31	0.45	31.1	0.55	0.78	30.1
NC Exports	2.71	1.99	-36.0	2.47	1.84	-34.6
C Exports	0.00	0.01	86.4	0.00	0.02	90.5
Trade Balance (% GDP)	1.17	0.79	-47.9	0.98	0.49	-101.0

FIGURE 22. CASE 3: AGGREGATE CURRENCY MISMATCH

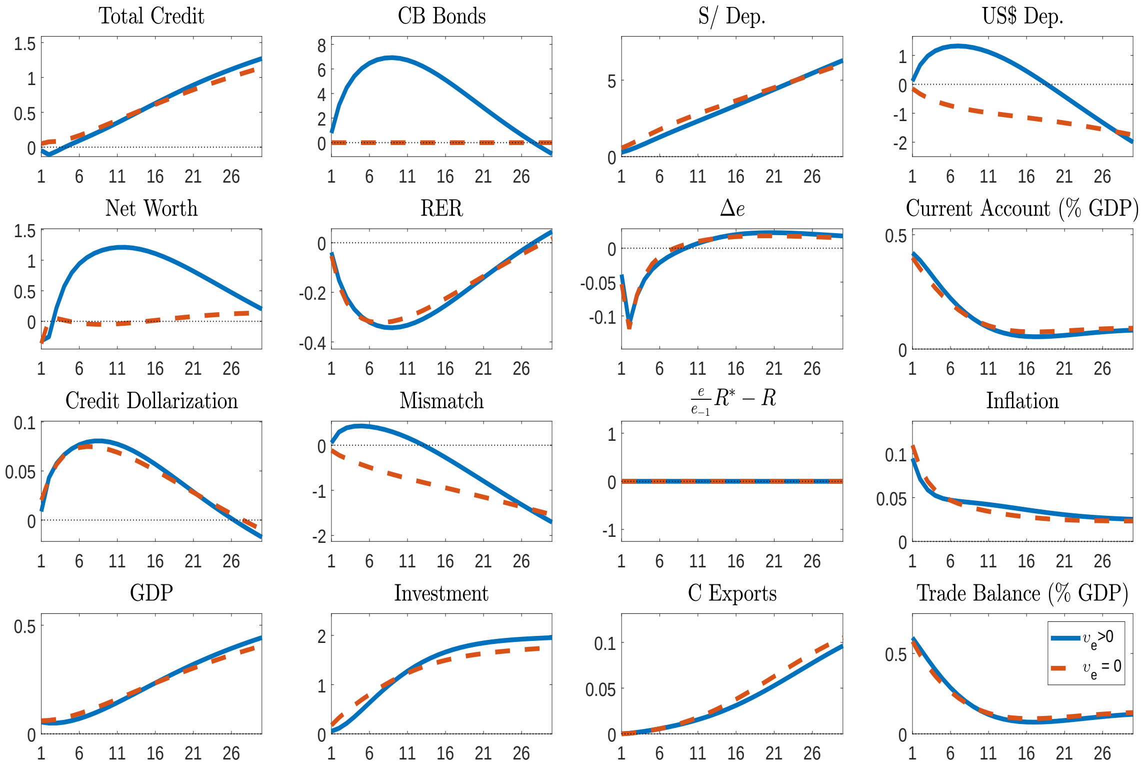
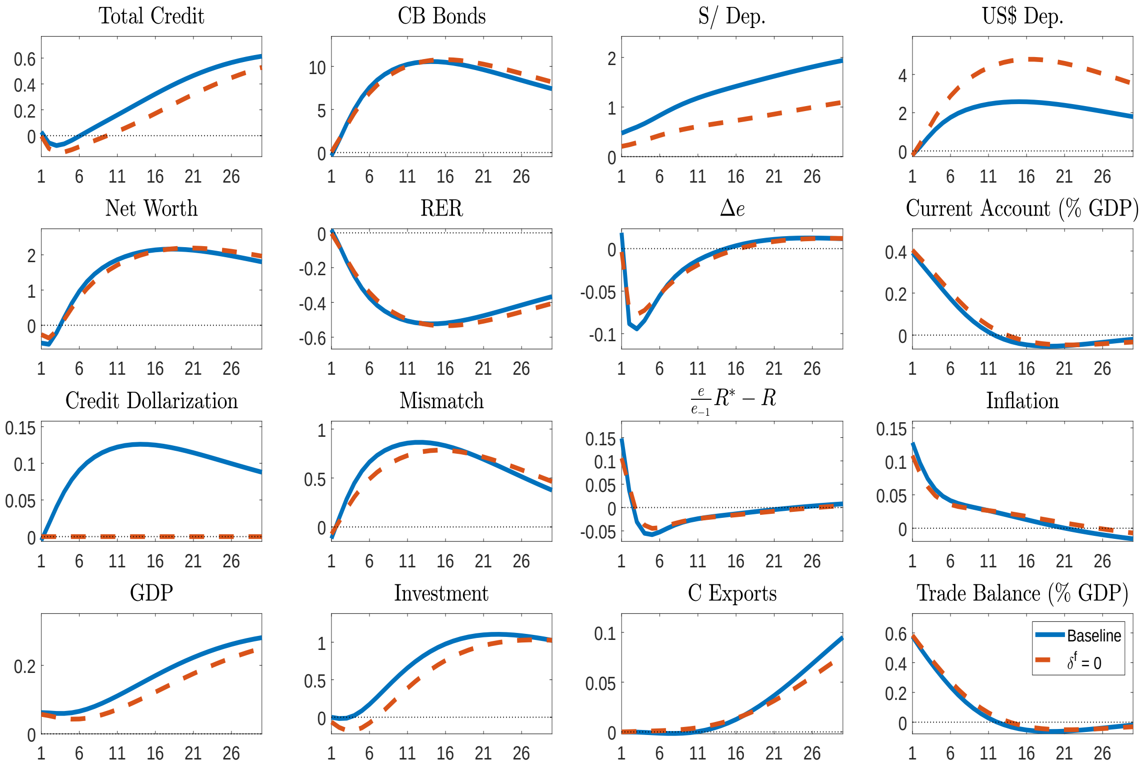


TABLE 13. PASS-THROUGH FOR CASE 3

	Year 1			Year 2		
	$v_e > 0$	$v_e = 0$	SFX	$v_e > 0$	$v_e = 0$	SFX
Total Credit	-0.13	0.19	29.5	0.09	0.36	76.0
RER	-0.41	-0.45	9.1	-0.66	-0.67	1.9
Δe	-0.16	-0.17	4.9	-0.11	-0.11	-6.1
Current Account (% GDP)	0.88	0.80	-10.0	0.76	0.69	-10.1
Credit Dollarization	0.11	0.11	4.4	0.16	0.16	-1.4
Mismatch	0.69	-0.60	-15.2	0.91	-1.02	11.1
$\frac{e}{e_{-1}}R^* - R$	0.00	0.00	91.6	0.00	0.00	92.1
Inflation	0.17	0.19	13.7	0.15	0.17	7.8
GDP	0.12	0.16	21.4	0.17	0.22	22.2
Investment	0.43	0.93	53.2	1.17	1.64	28.6
Consumption	-0.02	-0.00	-536.3	-0.02	0.01	-75.5
NC Exports	2.62	2.59	-1.1	2.45	2.45	-0.4
C Exports	0.00	0.00	-6.8	0.01	0.01	1.2
Trade Balance (% GDP)	1.20	1.11	-7.9	1.02	0.94	-8.2

Figure 23. Baseline vs Case 2



D The Financial System

D.1 Solving Bank's Problem

Recursive version for banker's problem:

$$V_t = \max_{l_t, l_t^*, b_t, d_t, d_t^*} \mathbb{E}_t \left[\Lambda_{t,t+1} \left\{ (1-\sigma) n_{t+1} + \sigma V_{t+1} \right\} \right]$$

subject to:

$$l_t + e_t l_t^* + b_t = n_t + d_t + e_t d_t^*$$

$$n_{t+1} = R_{t+1}^l l_t + R_{t+1}^{l*} e_{t+1} l_t^* + R_{t+1}^b b_t - R_{t+1} d_t - e_{t+1} R_{t+1}^* d_t^*$$

$$x_t = \frac{e_t d_t^* - e_t l_t^*}{l_t + e_t l_t^* + b_t}$$

$$V_t \geq \Theta(x_t) \left[\varpi l_t + \varpi^* e_t l_t^* + \varpi^b b_t \right]$$

Let $\psi_t = \frac{V_t}{n_t}$, $\phi_t^l = \frac{l_t}{n_t}$, $\phi_t^l = \frac{e_t l_t^*}{n_t}$, and $\phi_t^b = \frac{b_t}{n_t}$, then the objective function can be rewritten as

$$\psi_t = \mathbb{E}_t \left[\Lambda_{t,t+1} (1 - \sigma + \sigma \psi_{t+1}) \frac{n_{t+1}}{n_t} \right]$$

Using the law of motion for bank's net worth, we can rearrange:

$$\frac{n_{t+1}}{n_t} = R_{t+1}^l \frac{l_t}{n_t} + R_{t+1}^{l*} \frac{e_{t+1}}{e_t} \frac{e_t l_t^*}{n_t} + R_{t+1}^b \frac{b_t}{n_t} - R_{t+1} \frac{d_t}{n_t}$$
$$- R_{t+1}^* \frac{e_{t+1}}{e_t} \frac{(l_t + e_t l_t^* + b_t)}{n_t} \frac{(e_t d_t^* - e_t l_t^* + e_t l_t^*)}{l_t + e_t l_t^* + b_t}$$

$$
\begin{aligned}
\frac{n_{t+1}}{n_t} &= R^l_{t+1}\phi^l_t + \frac{e_{t+1}}{e_t}(R^{l*}_{t+1} - R^*_{t+1})\phi^{l*}_t + R^b_{t+1}\phi^b_t - R_{t+1}\left[\frac{l_t}{n_t} + \frac{e_t l^*_t}{n_t}\right. \\
&\quad \left. + \frac{b_t}{n_t} - 1 - \frac{e_t d^*_t}{n_t}\right] - \frac{e_{t+1}}{e_t}R^*_{t+1}\left[\phi^l_t + \phi^{l*}_t + \phi^b_t\right]x_t \\
\frac{n_{t+1}}{n_t} &= R^l_{t+1}\phi^l_t + \frac{e_{t+1}}{e_t}(R^{l*}_{t+1} - R^*_{t+1})\phi^{l*}_t + R^b_{t+1}\phi^b_t \\
&\quad - R_{t+1}\left[\phi^l_t + \phi^b_t - 1 - \left[\phi^l_t + \phi^{l*}_t + \phi^b_t\right]x_t\right] - \frac{e_{t+1}}{e_t}R^*_{t+1}\left[\phi^l_t + \phi^{l*}_t + \phi^b_t\right]x_t \\
\frac{n_{t+1}}{n_t} &= \left[R^l_{t+1} - R_{t+1}\right]\phi^l_t + \left[\frac{e_{t+1}}{e_t}(R^{l*}_{t+1} - R^*_{t+1})\right]\phi^{l*}_t \\
&\quad + \left[R^b_{t+1} - R_{t+1}\right]\phi^b_t + \left[R_{t+1} - \frac{e_{t+1}}{e_t}R^*_{t+1}\right]\left(\phi^l_t + \phi^{l*}_t + \phi^b_t\right)x_t + R_{t+1}
\end{aligned}
$$

Thus, bank's problem can be rewritten as the following form:

$$
\begin{aligned}
\psi_t &= \max_{\phi^l_t,\phi^{l*}_t\phi^b_t,x_t} \quad \mu^l_t\phi^l_t + (\mu^{l*}_t + \mu^{d*}_t)\phi^{l*}_t + \mu^b_t\phi^b_t + \mu^{d*}_t\left(\phi^l_t + \phi^{l*}_t + \phi^b_t\right)x_t + v_t \\
&\qquad \text{subject to:} \\
\psi_t &- \Theta(x_t)\left[\varpi\phi^l_t + \varpi^*\phi^{l*}_t + \varpi^b\phi^b_t\right] \geq 0
\end{aligned}
$$

where

$$
\begin{aligned}
\mu^l_t &= \mathbb{E}_t\left[\Omega_{t+1}\left(R^l_{t+1} - R_{t+1}\right)\right] && (1) \\
\mu^{l*}_t &= \mathbb{E}_t\left[\Omega_{t+1}\left(\frac{e_{t+1}}{e_t}R^{l*}_{t+1} - R_{t+1}\right)\right] && (2) \\
\mu^b_t &= \mathbb{E}_t\left[\Omega_{t+1}\left(R^b_{t+1} - R_{t+1}\right)\right] && (3) \\
\mu^{d*}_t &= \mathbb{E}_t\left[\Omega_{t+1}\left(R_{t+1} - \frac{e_{t+1}}{e_t}R^*_{t+1}\right)\right] && (4) \\
v_t &= \mathbb{E}_t\left[\Omega_{t+1}R_{t+1}\right] && (5) \\
\Omega_{t+1} &= \Lambda_{t,t+1}(1 - \sigma + \sigma\psi_{t+1}) && (6)
\end{aligned}
$$

We can interpret Ω_{t+1} as the stochastic discount factor of the banker, μ^l_t as the excess return of domestic currency loans over home deposit, μ^l_t is the excess return of foreign currency loans over home deposit, μ^b_t the excess return of sterilized bonds over home deposit, and μ^{d*}_t as the cost advantage of foreign currency debt over home deposit. Note that at the optimal ratios, the following equation will be satisfied:

$$
\psi_t = \mu^l_t\phi^l_t + (\mu^{l*}_t + \mu^{d*}_t)\phi^{l*}_t + \mu^b_t\phi^b_t + \mu^{d*}_t\left(\phi^l_t + \phi^{l*}_t + \phi^b_t\right)x_t + v_t \qquad (7)
$$

Let λ^b_t be the Lagrange multiplier of the associated incentive restriction, then the problem becomes:

$$
\begin{aligned}
\mathcal{L}_t &= \max_{\phi^l_t,\phi^{l*}_t,\phi^b_t,x_t} \mu^l_t\phi^l_t + (\mu^{l*}_t + \mu^{d*}_t)\phi^{l*}_t + \mu^b_t\phi^b_t + \mu^{d*}_t\left(\phi^l_t + \phi^{l*}_t + \phi^b_t\right)x_t + v_t \\
&\quad + \lambda^b_t\Big[\mu^l_t\phi^l_t + (\mu^{l*}_t + \mu^{d*}_t)\phi^{l*}_t + \mu^b_t\phi^b_t + \mu^{d*}_t\left(\phi^l_t + \phi^{l*}_t + \phi^b_t\right)x_t + v_t \\
&\quad - \Theta(x_t)\left(\varpi\phi^l_t + \varpi^*\phi^{l*}_t + \varpi^b\phi^b_t\right)\Big] \\
\mathcal{L}_t &= \max_{\phi^l_t,\phi^{l*}_t,\phi^b_t,x_t}(1 + \lambda^b_t)\left[\mu^l_t\phi^l_t + (\mu^{l*}_t + \mu^{d*}_t)\phi^{l*}_t + \mu^b_t\phi^b_t + \mu^{d*}_t\left(\phi^l_t + \phi^{l*}_t + \phi^b_t\right)x_t + v_t\right]
\end{aligned}
$$

$$- \lambda_t^b \Theta(x_t) \left(\varpi \phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b \right)$$

Then, the first order conditions (FOCs) for this problem are:

$$\begin{aligned}
\phi_t^l :& \quad (1+\lambda_t^b)[\mu_t^l + \mu_t^{d*} x_t] - \varpi \lambda_t^b \Theta(x_t) = 0 \\
\phi_t^{l*} :& \quad (1+\lambda_t^b)[\mu_t^{l*} + \mu_t^{d*} + \mu_t^{d*} x_t] - \varpi^* \lambda_t^b \Theta(x_t) = 0 \\
\phi_t^b :& \quad (1+\lambda_t^b)[\mu_t^b + \mu_t^{d*} x_t] - \varpi^b \lambda_t^b \Theta(x_t) = 0 \\
x_t :& \quad (1+\lambda_t^b)\mu_t^{d*} \left(\phi_t^l + \phi_t^{l*} + \phi_t^b \right) - \lambda_t^b \left(\varpi \phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b \right) \partial_x \Theta(x_t) = 0 \\
\text{slackness} :& \quad \lambda_t^b \left[\psi_t - \Theta(x_t) \left(\varpi \phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b \right) \right] = 0
\end{aligned}$$

We assume that $\lambda_t^b > 0$ and the incentive constraint is binding. Thus

$$\begin{aligned}
\mu_t^l + \mu_t^{d*} x_t &= \frac{\lambda_t^b}{1+\lambda_t^b} \varpi \Theta(x_t) \\
\mu_t^{l*} + \mu_t^{d*}(1+x_t) &= \frac{\lambda_t^b}{1+\lambda_t^b} \varpi^* \Theta(x_t) \\
\mu_t^b + \mu_t^{d*} x_t &= \frac{\lambda_t^b}{1+\lambda_t^b} \varpi^b \Theta(x_t) \\
\mu_t^{d*} \left(\phi_t^l + \phi_t^{l*} + \phi_t^b \right) &= \frac{\lambda_t^b}{1+\lambda_t^b} \left(\varpi \phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b \right) \partial_x \Theta(x_t) \\
\Theta(x_t) \left(\varpi \phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b \right) &= \left(\mu_t^l + \mu_t^{d*} x_t \right) \phi_t^l + \left(\mu_t^{l*} + \mu_t^{d*} x_t \right) \phi_t^{l*} + \left(\mu_t^b + \mu_t^{d*} x_t \right) \phi_t^b + \upsilon_t
\end{aligned}$$

Dividing the first condition by the second and third:

$$\mu_t^{l*} = \frac{\varpi^*}{\varpi} \mu_t^l - \left[\left(1 - \frac{\varpi^*}{\varpi} \right) x_t + 1 \right] \mu_t^{d*} \tag{8}$$

$$\mu_t^b = \frac{\varpi^b}{\varpi} \mu_t^l - \left(1 - \frac{\varpi^b}{\varpi} \right) \mu_t^{d*} x_t \tag{9}$$

Considering the incentive constraint we can rearrange to obtain:

$$\varpi \phi_t^l = \Phi_t - \varpi^* \phi_t^{l*} - \varpi^b \phi_t^b \tag{10}$$

$$\Phi_t = \frac{\varpi \upsilon_t}{\varpi \Theta(x_t) - (\mu_t^l + \mu_t^{d*} x_t)} \tag{11}$$

Note Φ_t defines the maximum weighted leverage ratio induced by the moral hazard problem[20]. We can see that, whenever $\varpi, \varpi^*, \varpi^b > 0$, private loans and sterilized bonds are substitutes in the portfolio of banks.

Using the fourth optimality condition:

$$\mu_t^{d*} \left(\phi_t^l + \phi_t^{l*} + \phi_t^b \right) = \left(\mu_t^l + \mu_t^{d*} x_t \right) \frac{\partial_x \Theta(x_t)}{\varpi \Theta(x_t)} \Phi_t$$

[20]Note that this restriction can be rewritten as:

$$l_t \leq \theta_t n_t$$

where $\theta_t = \Phi_t - \varpi^* \phi_t^{l*} - \varpi^b \phi_t^b$. This type of collateral constraint were popularized by Kiyotaki and Moore (1997) and is used in Chang (2019) to capture foreign debt limits that are faced by the financial system in emerging economies.

$$
\mu_t^{d*}\left(\frac{1}{\varpi}\Phi_t+\left(1-\frac{\varpi^*}{\varpi}\right)\phi_t^{l*}+\left(1-\frac{\varpi^b}{\varpi}\right)\phi_t^b\right)=\left(\mu_t^l+\mu_t^{d*}x_t\right)\frac{\partial_x\Theta(x_t)}{\varpi\Theta(x_t)}\Phi_t
$$

$$
\mu_t^{d*}\left(\Phi_t+(\varpi-\varpi^*)\,\phi_t^{l*}+\left(\varpi-\varpi^b\right)\phi_t^b\right)=\left(\mu_t^l+\mu_t^{d*}x_t\right)\frac{\partial_x\Theta(x_t)}{\Theta(x_t)}\Phi_t
$$

$$
\mu_t^{d*}(\varpi-\varpi^*)\phi_t^{l*}+\mu_t^{d*}(\varpi-\varpi^b)\phi_t^b=\left[\left(\mu_t^l+\mu_t^{d*}x_t\right)\frac{\partial_x\Theta(x_t)}{\Theta(x_t)}-\mu_t^{d*}\right]\Phi_t
$$

Hence, the fifth equation for solving bank's problem is[21]:

$$
(\varpi-\varpi^*)\phi_t^{l*}+(\varpi-\varpi^b)\phi_t^b=\left[\left(\frac{\mu_t^l}{\mu_t^{d*}}+x_t\right)\frac{\partial_x\Theta(x_t)}{\Theta(x_t)}-1\right]\Phi_t \tag{12}
$$

D.2 Aggregation

We have solved the problem for an individual bank but not for the aggregate banking sector. From eq. (8), we see that the determination of the foreign debt - weighted asset ratio does not depend on bank-specific factors, then this equation is also satisfied at entire banking sector. The same logic applies for eq. (9), eq. (10), eq. (12). Then,

$$
\phi_t^l=\frac{L_t}{N_t} \tag{13}
$$

$$
\phi_t^{l*}=\frac{e_tL_t^*}{N_t} \tag{14}
$$

$$
\phi_t^b=\frac{B_t}{N_t} \tag{15}
$$

$$
x_t=\frac{e_tD_t^*-e_tL_t^*}{L_t+e_tL_t^*+B_t} \tag{16}
$$

Since the aggregate level of sterilized bonds B_t are determined by the monetary authority and N_t is a state variable, then, in the whole financial system, ϕ_t^b is given. However, now the vector (R_t^l, R_t^{l*}, R_t^b) is not given anymore. The equations which help in the determination of this vector is the law of motion of the aggregated bank's net worth and credit demand functions. The aggregate net worth of banks evolves according to

$$
N_{t+1}=\sigma\left(R_{t+1}^lL_t+R_{t+1}^{l*}e_{t+1}L_t^*+R_{t+1}^bB_t-R_{t+1}D_t-e_{t+1}R_{t+1}^*D_t^*\right)
$$

$$
+\,\xi\left(R_{t+1}^lL_t+R_{t+1}^{l*}e_{t+1}L_t^*+R_{t+1}^bB_t\right)
$$

$$
N_{t+1}=(\sigma+\xi)\left(R_{t+1}^lL_t+R_{t+1}^{l*}e_{t+1}L_t^*+R_{t+1}^bB_t\right)-\sigma R_{t+1}D_t-\sigma e_{t+1}R_{t+1}^*D_t^* \tag{17}
$$

D.3 Aggregate Currency Mismatch

Given x_t and n_t,

$$
V_t=\max_{l_t,l_t^*,b_t,d_t,d_t^*}\quad\mathbb{E}_t\left[\Lambda_{t,t+1}\left\{(1-\sigma)n_{t+1}+\sigma V_{t+1}\right\}\right]
$$

[21] Note that if $1=\varpi^*$ and $1=\varpi^b$, we arrive to the a similar solution of Aoki et al. (2018) :

$$
1=\left(\frac{\mu_t^l}{\mu_t^{d*}}+x_t\right)\frac{\partial_x\Theta(x_t)}{\Theta(x_t)}
$$

If $\varpi^*=\varpi^b=1$, we get the same solution of Aoki et al. (2018) for the whole financial system since returns are the same across different types of assets.

subject to:

$$l_t + e_t l_t^* + b_t = n_t + d_t + e_t d_t^*$$

$$n_{t+1} = R_{t+1}^l l_t + R_{t+1}^{l*} e_{t+1} l_t^* + R_{t+1}^b b_t - R_{t+1} d_t - e_{t+1} R_{t+1}^* d_t^*$$

$$V_t \geq \Theta(x_t) \left[\varpi l_t + \varpi^* e_t l_t^* + \varpi^b b_t \right]$$

Let $\psi_t = \frac{V_t}{n_t}$, $\phi_t^l = \frac{l_t}{n_t}$, $\phi_t^l = \frac{e_t l_t^*}{n_t}$, and $\phi_t^b = \frac{b_t}{n_t}$, then the objective function can be rewritten as

$$\psi_t = \mathbb{E}_t \left[\Lambda_{t,t+1} (1 - \sigma + \sigma \psi_{t+1}) \frac{n_{t+1}}{n_t} \right]$$

Moreover, let $\phi_t^{d*} = \frac{e_t d_t^*}{n_t}$

$$\frac{n_{t+1}}{n_t} = R_{t+1}^l \phi_t^l + R_{t+1}^{l*} \frac{e_{t+1}}{e_t} \phi_t^* + R_{t+1}^b \phi_t^b - R_{t+1} \frac{d_t}{n_t} - R_{t+1}^* \frac{e_{t+1}}{e_t} \phi_t^{d*}$$

$$\frac{n_{t+1}}{n_t} = R_{t+1}^l \phi_t^l + R_{t+1}^{l*} \frac{e_{t+1}}{e_t} \phi_t^* + R_{t+1}^b \phi_t^b - R_{t+1} \left[\phi_t^l + \phi_t^{l*} + \phi_t^b - 1 - \phi_t^{d*} \right] - R_{t+1}^* \frac{e_{t+1}}{e_t} \phi_t^{d*}$$

$$\frac{n_{t+1}}{n_t} = [R_{t+1}^l - R_{t+1}] \phi_t^l + \left[R_{t+1}^{l*} \frac{e_{t+1}}{e_t} - R_{t+1} \right] \phi_t^* + [R_{t+1}^b - R_{t+1}] \phi_t^b$$
$$+ \left[R_{t+1} - R_{t+1}^* \frac{e_{t+1}}{e_t} \right] \phi_t^{d*} + R_{t+1}$$

Then, the bank's problem can be rewritten as

$$\psi_t = \max_{\phi_t^l, \phi_t^{l*} \phi_t^b, \phi_t^{d*}} \quad \mu_t^l \phi_t^l + \mu_t^{l*} \phi_t^{l*} + \mu_t^b \phi_t^b + \mu_t^{d*} \phi_t^{d*} + v_t$$

subject to:

$$\psi_t - \Theta(x_t) \left[\varpi \phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b \right] \geq 0$$

FOCs

$$\begin{aligned}
\phi_t^l : &\quad (1 + \lambda_t^b) \mu_t^l - \varpi \lambda_t^b \Theta(x_t) = 0 \\
\phi_t^{l*} : &\quad (1 + \lambda_t^b) \mu_t^{l*} - \varpi^* \lambda_t^b \Theta(x_t) = 0 \\
\phi_t^b : &\quad (1 + \lambda_t^b) \mu_t^b - \varpi^b \lambda_t^b \Theta(x_t) = 0 \\
\phi_t^{d*} : &\quad (1 + \lambda_t^b) \mu_t^{d*} = 0 \\
\text{slackness} : &\quad \lambda_t^b \left[\psi_t - \Theta(x_t) \left(\varpi \phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b \right) \right] = 0
\end{aligned}$$

Rearranging

$$\mu_t^{d*} = 0$$

$$\mu_t^{l*} = \frac{\varpi^*}{\varpi} \mu^l$$

$$\mu_t^b = \frac{\varpi^b}{\varpi} \mu^l$$

thus

$$\psi = \frac{\mu^l}{\varpi} \Phi_t + v_t \tag{18}$$

Hence,

$$\Phi_t = \frac{\varpi v_t}{\varpi \Theta(x_t) - \mu^l} \tag{19}$$

D.4 Simplified version

No Externality. Assume that there exists only one asset L_t, $\varpi = 1$, and $x_t = \frac{e_t d_t^*}{l_t}$. Then, given μ_t^l and μ^{d*t}, the solution for the individual problem $(x_t, \phi_t^l, \lambda_t^b)$ is characterizes by

$$\begin{aligned}
\mu_t^l + \mu_t^{d*} x_t &= \frac{\lambda_t^b}{1+\lambda_t^b}\Theta(x_t) \\
\phi_t^l &= \frac{v_t}{\Theta(x_t) - \mu_t^l - \mu_t^{d*} x_t} \\
\Theta(x_t) &= \left(\frac{\mu_t^l}{\mu_t^{d*}} + x_t\right)\partial_x \Theta(x_t)
\end{aligned}$$

Note that we assume that $0 < \mu_t^l + \mu_t^{d*} x_t < \Theta(x_t)$. Given e_t, R_t^l, R_t, and R_t^*, the entire financial system variables are determined by

$$\lambda_t^b = \frac{\mu_t^l + \mu_t^{d*} x_t}{\Theta(x_t) - \mu_t^l - \mu_t^{d*} x_t} \tag{20}$$

$$\phi_t^l = \frac{v_t}{\Theta(x_t) - \mu_t^l - \mu_t^{d*} x_t} \tag{21}$$

$$\Theta(x_t) = \left(\frac{\mu_t^l}{\mu_t^{d*}} + x_t\right)\partial_x \Theta(x_t) \tag{22}$$

$$\psi_t = \Theta(x_t)\phi_t^l \tag{23}$$

$$\Omega_{t+1} = \Lambda_{t,t+1}(1 - \sigma + \sigma\psi_{t+1}) \tag{24}$$

$$v_t = \mathbb{E}_t[\Omega_{t+1} R_{t+1}] \tag{25}$$

$$\mu_t^l = \mathbb{E}_t[\Omega_{t+1}(R_{t+1}^l - R_{t+1})] \tag{26}$$

$$\mu_t^{d*} = \mathbb{E}_t\left[\Omega_{t+1}\left(R_{t+1} - \frac{e_{t+1}}{e_t} R_{t+1}^*\right)\right] \tag{27}$$

Externality. Given x_t and μ_t^l, then the solution for $(\phi_t^{d*}, \phi_t^l, \lambda_t^b)$,

$$\begin{aligned}
\mu_t^l &= \frac{\lambda_t^b}{1+\lambda_t^b}\Theta(x_t) \\
\mu_t^{d*} &= 0 \\
\phi_t^l &= \frac{v_t}{\Theta(x_t) - \mu_t^l}
\end{aligned}$$

Note that we assume that $0 < \mu_t^l < \Theta(x_t)$. Given x_t, R_t^l, R_t, and R_t^*, the entire financial system variables are determined by

$$\mu_t^l + \mu_t^{d*} x_t = \frac{\lambda_t^b}{1+\lambda_t^b}\Theta(x_t) \tag{28}$$

$$\phi_t^l = \frac{v_t}{\Theta(x_t) - \mu_t^l - \mu_t^{d*} x_t} \tag{29}$$

$$\mu_t^{d*} = 0 \tag{30}$$

$$\psi_t = \Theta(x_t)\phi_t^l \tag{31}$$

$$\Omega_{t+1} = \Lambda_{t,t+1}(1 - \sigma + \sigma\psi_{t+1}) \tag{32}$$

$$v_t = \mathbb{E}_t[\Omega_{t+1} R_{t+1}] \tag{33}$$

$$\mu_t^l = \mathbb{E}_t[\Omega_{t+1}(R_{t+1}^l - R_{t+1})] \tag{34}$$

$$\mu_t^{d*} = \mathbb{E}_t\left[\Omega_{t+1}\left(R_{t+1} - \frac{e_{t+1}}{e_t} R_{t+1}^*\right)\right] \tag{35}$$

E General Equilibrium Model

E.1 Solving Worker's Problem

Let λ_t^h be the Lagrange multiplier associated to eq. (17), then the optimality conditions are:

$$
\begin{aligned}
0 &= \left(C_t - \mathcal{H}C_{t-1} - \frac{\zeta_0}{1+\zeta}H_t^{1+\zeta}\right)^{-\gamma} - \mathcal{H}\beta\mathbb{E}_t\left(C_{t+1} - \mathcal{H}C_t - \frac{\zeta_0}{1+\zeta}H_{t+1}^{1+\zeta}\right)^{-\gamma} - \lambda_t^h \\
0 &= -\zeta_0 H_t^{\zeta}\left(C_t - \mathcal{H}C_{t-1} - \frac{\zeta_0}{1+\zeta}H_t^{1+\zeta}\right)^{-\gamma} + w_t\lambda_t^h \\
0 &= -\lambda_t^h + \beta\mathbb{E}_t\left[R_{t+1}\lambda_{t+1}^h\right]
\end{aligned}
$$

Moreover, note that the stochastic discount factor of the representative household is

$$
\Lambda_{t,t+j} = \beta^j\frac{\lambda_{t+j}^h}{\lambda_t^h} \tag{1}
$$

Note on GHH preferences with internal habits. Define $X_t = C_t - \mathcal{H}C_{t-1} - \frac{\zeta_0}{1+\zeta}H_t^{1+\zeta}$, then log-linearazing supply labor equation:

$$
\begin{aligned}
w\lambda^h(\hat{w}_t + \hat{\lambda}_t^h) &= \zeta_0 H^{\zeta}X^{-\gamma}\zeta\hat{H}_t - \zeta_0 H^{\zeta}\gamma X^{-\gamma-1}(-\zeta_0 H^{1+\zeta})\hat{H}_t \\
&\quad - \gamma\zeta_0 H^{\zeta}X^{-\gamma-1}\left(C\hat{C}_t - \mathcal{H}C\hat{C}_{t-1}\right) \\
\hat{w}_t + \hat{\lambda}_t^h &= \left(\zeta + \frac{\gamma\zeta_0 H^{1+\zeta}}{X}\right)\hat{H}_t - \frac{\gamma C}{X}\left(\hat{C}_t - \mathcal{H}\hat{C}_{t-1}\right)
\end{aligned}
$$

Log-linearazing the stochastic discount factor equation:

$$
\begin{aligned}
\lambda^h\hat{\lambda}_t^h &= -\gamma X^{-\gamma-1}\left(C\hat{C}_t - \mathcal{H}C\hat{C}_{t-1} - \zeta_0 H^{1+\zeta}\hat{H}_t\right) \\
&\quad + \gamma\mathcal{H}\beta X^{-\gamma-1}\left(C\mathbb{E}_t\hat{C}_{t+1} - \mathcal{H}C\hat{C}_t - \zeta_0 H^{1+\zeta}\mathbb{E}_t\hat{H}_{t+1}\right) \\
\hat{\lambda}_t^h &= -\frac{\gamma}{(1-\mathcal{H}\beta)X}\left(C\hat{C}_t - \mathcal{H}C\hat{C}_{t-1} - \zeta_0 H^{1+\zeta}\hat{H}_t\right) \\
&\quad + \frac{\gamma\mathcal{H}\beta}{(1-\mathcal{H})X}\left(C\mathbb{E}_t\hat{C}_{t+1} - \mathcal{H}C\hat{C}_t - \zeta_0 H^{1+\zeta}\mathbb{E}_t\hat{H}_{t+1}\right)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\hat{w}_t &= \left(\zeta + \frac{\gamma\zeta_0 H^{1+\zeta}}{X}\right)\hat{H}_t - \frac{\gamma C}{X}\left(\hat{C}_t - \mathcal{H}\hat{C}_{t-1}\right) \\
&\quad - \Bigg[-\frac{\gamma}{(1-\mathcal{H}\beta)X}\left(C\hat{C}_t - \mathcal{H}C\hat{C}_{t-1} - \zeta_0 H^{1+\zeta}\hat{H}_t\right) \\
&\quad + \frac{\gamma\mathcal{H}\beta}{(1-\mathcal{H})X}\left(C\mathbb{E}_t\hat{C}_{t+1} - \mathcal{H}C\hat{C}_t - \zeta_0 H^{1+\zeta}\mathbb{E}_t\hat{H}_{t+1}\right)\Bigg] \\
\hat{w}_t &= \left(\zeta + \frac{\gamma\zeta_0 H^{1+\zeta}}{X} - \frac{\gamma\zeta_0 H^{1+\zeta}}{(1-\mathcal{H}\beta)X}\right)\hat{H}_t - \left(\frac{\gamma C}{X} - \frac{\gamma C}{(1-\mathcal{H}\beta)X}\right)\left(\hat{C}_t - \mathcal{H}\hat{C}_{t-1}\right) \\
&\quad - \frac{\gamma\mathcal{H}\beta}{(1-\mathcal{H})X}\left(C\mathbb{E}_t\hat{C}_{t+1} - \mathcal{H}C\hat{C}_t - \zeta_0 H^{1+\zeta}\mathbb{E}_t\hat{H}_{t+1}\right)
\end{aligned}
$$

$$
\begin{aligned}
\hat{w}_t = &\left(\zeta - \frac{\mathcal{H}\gamma\zeta_0 H^{1+\zeta}}{(1-\mathcal{H}\beta)X}\right)\hat{H}_t + \frac{\mathcal{H}\gamma C}{(1-\mathcal{H}\beta)X}\left(\hat{C}_t - \mathcal{H}\hat{C}_{t-1}\right)\\
&- \frac{\gamma\mathcal{H}\beta}{(1-\mathcal{H})X}\left(C\mathbb{E}_t\hat{C}_{t+1} - \mathcal{H}C\hat{C}_t - \zeta_0 H^{1+\zeta}\mathbb{E}_t\hat{H}_{t+1}\right)
\end{aligned}
$$

Hence, the inverse of Frisch Elasticity (IFE) is given by:

$$
\text{IFE} = \zeta - \frac{\mathcal{H}\gamma\zeta_0 H^{1+\zeta}}{(1-\mathcal{H}\beta)X} \tag{2}
$$

E.2 Price Setting

Given k_{t-1}^{nc}, l_{t-1}, l_{t-1}^*, and p_{t-1}^{nc}, a representative intermediate good producer chooses $\{k_{jt+s}^{nc}$, l_{jt+s}, l_{jt+s}^* h_{jt+s}, m_{jt+s}, p_{t+s}^{nc}, $y_{t+s}^{nc}\}_{s\geq 0}$ to maximize

$$
\begin{aligned}
\max\ \mathbb{E}_t\Big[\sum_{s=0}^{\infty}\Lambda_{t,t+s}\Big\{&\frac{p_{t+s}^{nc}}{P_{t+s}^{nc}}y_{jt+s}^{nc} - w_{t+s}h_{jt+s} - e_{t+s}m_{jt+s} - \Theta_{t+s}\left(\frac{p_{t+s}^{nc}}{p_{t+s-1}^{nc}}\right)\\
&+ q_{t+s}^{nc}\lambda_{nc}k_{jt+s-1}^{nc} - R_{t+s}^{l}l_{jt+s-1} - R_{t+s}^{l*}e_{t+s}l_{jt+s-1}^{*}\Big\}\Big]
\end{aligned}
$$

subjec to:

$$
\begin{aligned}
0 &= y_{jt}^{nc} - A_t^{nc}\left(\frac{k_{j,t-1}^{nc}}{\alpha_k}\right)^{\alpha_k}\left(\frac{m_{jt}}{\alpha_m}\right)^{\alpha_m}\left(\frac{h_{jt}}{1-\alpha_k-\alpha_m}\right)^{1-\alpha_k-\alpha_m}\\
0 &= q_t^{nc}k_{jt}^{nc} - A_t^e\Big[(1-\delta^f)^{1/\vartheta_l}(l_{jt})^{\frac{\vartheta_l-1}{\vartheta_l}} + (\delta^f)^{1/\vartheta_l}(l_{jt}^*)^{\frac{\vartheta_l-1}{\vartheta_l}}\Big]^{\frac{\vartheta_l}{\vartheta_l-1}}\\
0 &= y_{jt}^{nc} - \left(\frac{p_{jt}^{nc}}{P_t^{nc}}\right)^{-\eta}Y_t^{nc}
\end{aligned}
$$

Denote that Lagrangian multiplier: mc_t, $\mathcal{L}_{1t}$, and $\mathcal{L}_{2t}$ respectively, then

$$
\begin{aligned}
0 = &-\mathcal{L}_{1t}q_t^{nc} + \mathbb{E}_t\Lambda_{t,t+1}\Bigg[\lambda_{nc}q_{t+1}^{nc}\\
&+ mc_{t+1}A_{t+1}^{nc}(k_{jt}^{nc})^{\alpha_k-1}\left(\frac{m_{jt+1}}{\alpha_m}\right)^{\alpha_m}\left(\frac{h_{jt+1}}{1-\alpha_k-\alpha_m}\right)^{1-\alpha_k-\alpha_m}\Bigg]\\
0 = &\ \mathcal{L}_{1t}A_t^e\Big[(1-\delta^f)^{1/\vartheta_l}(l_{jt})^{\frac{\vartheta_l-1}{\vartheta_l}} + (\delta^f)^{1/\vartheta_l}(l_{jt}^*)^{\frac{\vartheta_l-1}{\vartheta_l}}\Big]^{\frac{1}{\vartheta_l-1}}(1-\delta^f)^{1/\vartheta_l}l_{jt}^{-\frac{1}{\vartheta_l}} - \mathbb{E}_t\Lambda_{t,t+1}R_{t+1}^l\\
0 = &\ \mathcal{L}_{1t}A_t^e\Big[(1-\delta^f)^{1/\vartheta_l}(l_{jt})^{\frac{\vartheta_l-1}{\vartheta_l}} + (\delta^f)^{1/\vartheta_l}(l_{jt}^*)^{\frac{\vartheta_l-1}{\vartheta_l}}\Big]^{\frac{1}{\vartheta_l-1}}(\delta^f)^{1/\vartheta_l}(l_{jt}^*)^{-\frac{1}{\vartheta_l}} - \mathbb{E}_t\Lambda_{t,t+1}R_{t+1}^{l*}e_{t+1}\\
0 = &-w_t + mc_tA_t^{nc}\left(\frac{k_{j,t-1}^{nc}}{\alpha_k}\right)^{\alpha_k}\left(\frac{m_{jt}}{\alpha_m}\right)^{\alpha_m}h_{jt}^{-\alpha_k-\alpha_m}\\
0 = &-e_t + mc_tA_t^{nc}m_{jt}^{\alpha_m-1}\left(\frac{k_{j,t-1}^{nc}}{\alpha_k}\right)^{\alpha_k}\left(\frac{h_{jt}}{1-\alpha_k-\alpha_m}\right)^{1-\alpha_k-\alpha_m}\\
0 = &\ \frac{1}{P_t^{nc}}y_{jt}^{nc} - \frac{1}{p_{t-1}^{nc}}\Theta_t' - \mathcal{L}_{2t}\eta(p_t^{nc})^{-\eta-1}\left(\frac{1}{P_t^{nc}}\right)^{-\eta}Y_t^{nc} + \mathbb{E}_t\left[\Lambda_{t,t+1}\frac{p_{t+1}^{nc}}{(p_t^{nc})^2}\Theta_{t+1}'\right]\\
0 = &\ \frac{p_t^{nc}}{P_t^{nc}} - mc_t - \mathcal{L}_{2t}
\end{aligned}
$$

Let

$$
z_t = mc_tA_t(k_{jt-1}^{nc})^{\alpha_k-1}\left(\frac{m_{jt}}{\alpha_m}\right)^{\alpha_m}\left(\frac{h_{jt}}{1-\alpha_k-\alpha_m}\right)^{1-\alpha_k-\alpha_m} \tag{3}
$$

then

$$\mathcal{L}_{1t} = \mathbb{E}_t \Lambda_{t,t+1} \frac{[z_{t+1} + \lambda_{nc} q^{nc}_{t+1}]}{q^{nc}_t} \tag{4}$$

$$z_t = \alpha_k mc_t \frac{y^{nc}_{jt}}{k^{nc}_{jt-1}} \tag{5}$$

$$e_t = \alpha_m mc_t \frac{y^{nc}_{jt}}{m_{jt}} \tag{6}$$

$$l_{jt} = (1-\delta^f) \left(\frac{\mathcal{L}_{1t}}{\mathbb{E}_t \Lambda_{t,t+1} R^l_{t+1}} \right)^{\vartheta_l} (A^e)^{\vartheta_l - 1} q^{nc}_t k^{nc}_{jt} \tag{7}$$

$$l^*_{jt} = \delta^f \left(\frac{\mathcal{L}_{1t}}{\mathbb{E}_t \Lambda_{t,t+1} e_{t+1} R^{l*}_{t+1}} \right)^{\vartheta_l} (A^e)^{\vartheta_l - 1} q^{nc}_t k^{nc}_{jt} \tag{8}$$

$$mc_t = \frac{1}{A^{nc}_t} z_t^{\alpha_k} e_t^{\alpha_m} w_t^{1-\alpha_k-\alpha_m} \tag{9}$$

$$\mathcal{L}_{2t} = \frac{p^{nc}_t}{P^{nc}_t} - mc_t \tag{10}$$

Moreover,

$$\begin{aligned}
&\frac{1}{P^{nc}_t} \left(\frac{p^{nc}_t}{P^{nc}_t} \right)^{-\eta} Y^{nc}_t - \frac{\eta}{p^{nc}_t} \left(\frac{p^{nc}_t}{P^{nc}_t} - mc_t \right) \left(\frac{p^{nc}_t}{P^{nc}_t} \right)^{-\eta} Y^{nc}_t \\
&\qquad - \frac{\kappa}{p^{nc}_{t-1}} \left(\frac{p^{nc}_t}{p^{nc}_{t-1}} - 1 \right) Y^{nc}_t + \kappa \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{p^{nc}_{t+1}}{(p^{nc}_t)^2} \left(\frac{p^{nc}_{t+1}}{p^{nc}_t} - 1 \right) Y^{nc}_{t+1} \right] = 0
\end{aligned}$$

Considering the symmetric equilibrium $p^{nc}_{jt} = P^{nc}_t$ for all $j \in [0,1]$ and denoting $\pi_t = \frac{P^{nc}_t}{P^{nc}_{t-1}} - 1$, then

$$\begin{aligned}
0 &= \frac{1}{P^{nc}_t} Y^{nc}_t - \frac{\eta}{P^{nc}_t} (1 - mc_t) Y^{nc}_t - \frac{\kappa}{P^{nc}_{t-1}} \left(\frac{P^{nc}_t}{P^{nc}_{t-1}} - 1 \right) Y^{nc}_t \\
&\qquad\qquad + \kappa \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{P^{nc}_{t+1}}{(P^{nc}_t)^2} \left(\frac{P^{nc}_{t+1}}{P^{nc}_t} - 1 \right) Y^{nc}_{t+1} \right] \\
0 &= Y^{nc}_t - \eta (1 - mc_t) Y^{nc}_t - \kappa (1+\pi_t) \pi_t Y^{nc}_t + \kappa \mathbb{E}_t \left[\Lambda_{t,t+1} (1+\pi_{t+1}) \pi_{t+1} Y^{nc}_{t+1} \right] \\
0 &= 1 - \eta (1 - mc_t) - \kappa (1+\pi_t) \pi_t + \kappa \mathbb{E}_t \left[\Lambda_{t,t+1} (1+\pi_{t+1}) \pi_{t+1} \frac{Y^{nc}_{t+1}}{Y^{nc}_t} \right]
\end{aligned}$$

Rearranging we obtain the Phillips Curve equation:

$$(1+\pi_t)\pi_t = \frac{1}{\kappa}(1 - \eta + \eta mc_t) + \mathbb{E}_t \left[\Lambda_{t,t+1} (1+\pi_{t+1}) \pi_{t+1} \frac{Y^{nc}_{t+1}}{Y^{nc}_t} \right] \tag{11}$$