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Bank Risk-Taking in a Small Open Economy

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Abstract

I develop an open economy model with banks facing foreign borrowing limits. The interaction of banks' limited liability and deposit insurance leads banks into socially excessive risk-taking, which involves credit volume and not the type of credit. The novel result is that, under a realistic calibration, a lower foreign interest rate reduces the excessive bank risk-taking. Since the foreign borrowing limit is binding, this lower rate does not boost banks' credit, but rather decreases it, since for a given capital the lower rate reduces the default probability of banks, which diminishes their risk-taking incentives. Through the same mechanism, a greater access to the international credit markets reduces the excessive risk-taking by banks. Hence, less banking regulation to achieve socially efficient risk-taking is required after a foreign rate reduction and a higher foreign borrowing limit.

Keywords: Macroprudential policies, financial stability, monetary policy and bank risk-taking.

JEL Classification: E44, E52, F41, G01, G21, G28.

1 Introduction

As emerging economies become more integrated into the international credit markets, their banking systems' dependence on foreign funds becomes more important (see figure 1.a-b), which make them more vulnerable to foreign shocks. For instance, Advjiev et al. (2017), who splits debt inflows into four borrowing sectors, government, central bank, banks, and corporates, show that the average banks' external debt as a share of total external debt for 34 emerging economies has been around 30% in the last two decades.[1]

*This is the first chapter of my doctoral thesis defended at Universitat Pompeu Fabra. The views expressed are those of the author and do not necessarily reflect those of the Central Reserve Bank of Peru.

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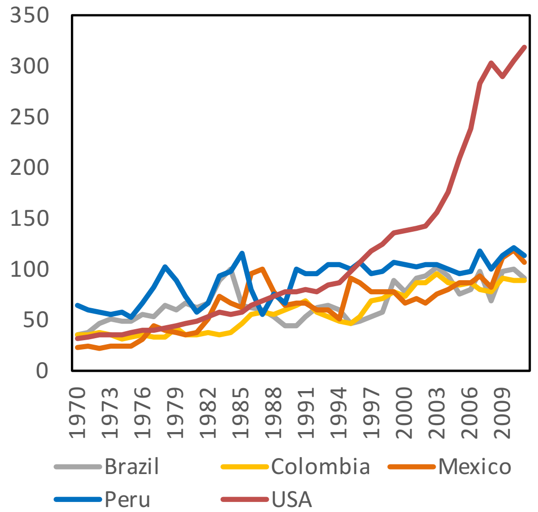
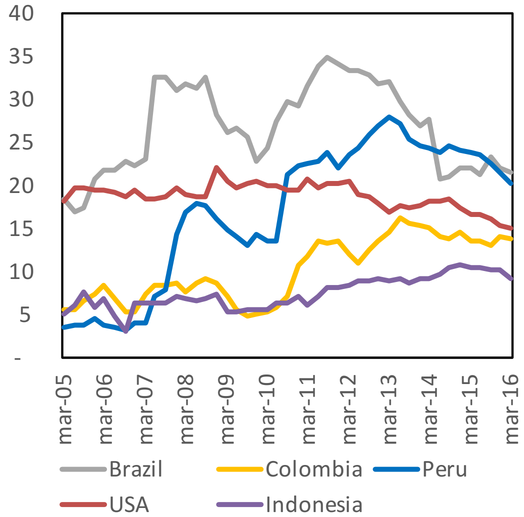
[1]This number for the case of 25 advanced economies is 42%.

In addition, it is expected that the still undeveloped and small banking system in the emerging economies will become more important and sophisticated (see figure 1.c)

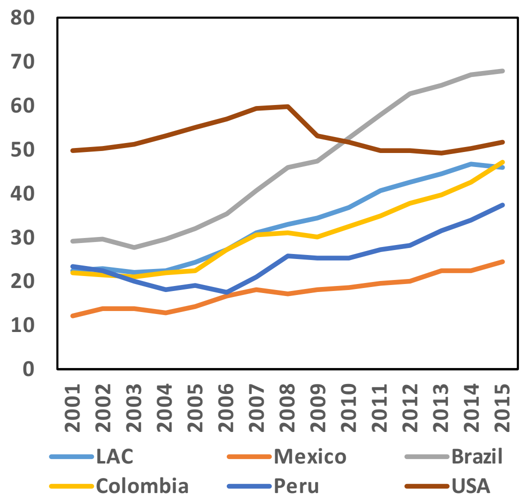
Figure 1: Macroeconomic Indicators



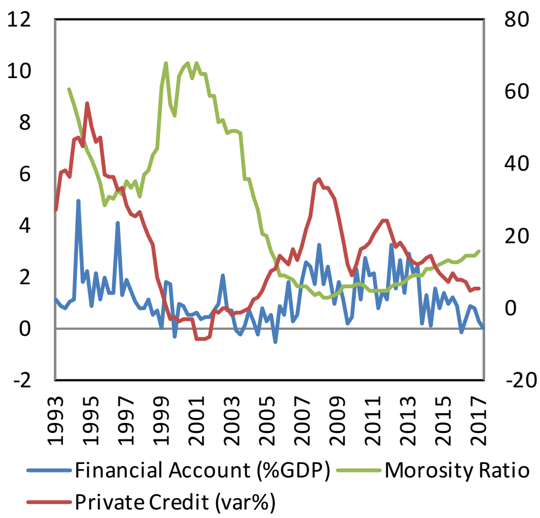
(a) Assets and Liabilities (%GDP)



(b) Bank External Debt (% External Debt)



(c) Bank Private Credit (%GDP)



(d) Peru (Credit - Right axis)

Source: IMF, World Bank (Quarterly External Debt Statistics and World Development Indicators), BIS. (A+L)/GDP: Cross border assets + cross border liabilities to GDP ratio, Lane and Milesi-Ferreti (2007). The external debt is the debt owed to nonresidents where the debtors can be the government, corporations or private households.

In this context, it is crucial to monitor the banking system's exposure to the international credit market. Considering the large and volatile capital flows to emerging markets economies, it becomes imperative to study the effects of foreign shocks, such as foreign interest rates and access to the international credit market, on the risk-taking behavior of banks. For instance, large credit booms and capital inflows seem to be followed by a deep crisis (see, e.g., Benigno et al., 2015; Caballero, 2014). In particular, figure 1.d shows that in the emerging economy of Peru, capital flows are strongly positively correlated with private credit growth and negatively correlated with an indicator of the quality of the banking system's loans. Hence, to formulate appropriate macroprudential policies it is necessary to explore the macroeconomic effects of capital inflows on the probability of a banking crisis. Although there has been a large amount of research into the impact of domestic policy rates on the degree of bank risk-taking, known as the "risk-taking channel" (term coined by Borio and Zhu, 2012), less attention has been devoted to studying the effects of foreign monetary policy and the access to foreign credit on the excessive bank risk-taking.

In this sense, this paper aims to study and compare the effects of interest rates (domestic and foreign) and access to the international credit market on domestic banks' excessive risk-taking. This allows the paper to shed light on the prudential policy suitable for a small open economy. To do this, this paper develops a two-period small partially open economy model with domestic banks, and domestic and foreign depositors. As commonly assumed in the relevant literature, the foreign risk-free interest rate will be lower than the domestic risk-free interest rate.

Another important assumption in this document is that financial intermediaries face a limit on borrowing. This borrowing limit tries to capture any informational friction that might exist between banks and depositors. Specifically, this paper assumes an exogenous borrowing limit only on foreign debt. This is justified by the fact that domestic depositors might be better informed than foreign depositors about domestic banks' business. Hence, it is easier to enforce repayment if the creditor is domestic. This creates imperfect substitutability between domestic and foreign borrowing. The limit on the aggregate external borrowing is emphasized in the emerging literature (see, e.g., Atkenson and Rios-Rull, 1996; Bulow and Rugoff, 1987; Caballero and Krishnamurthy, 2011). As in those studies, here this type of constraint aims to capture any friction between the emerging economy and international creditors.[2]

Two additional frictions in the model are limited liability and deposit insurance. Their interaction results in banks overestimating the expected net present value of future dividends, since they cannot internalize the effects of higher credit on the interest rates because interest rates are risk-insensitive. Due to this, banks underestimate the effective marginal cost of capital. Since this paper assumes diminishing marginal returns to capital, the aggregate credit and the bank risk-taking are going to be inefficiently high.[3] Here, bank risk-taking involves the volume of bank credit rather than the type of credit. In

[2]For example, Atkenson and Rios-Rull (1996) imposes a foreign borrowing (a) by citizens of the for $a<\bar{a}$, where $\bar{a}$ represents the portion of the endowment of a citizen that can be sized if the citizen does not pay the debt.

[3]Since the foreign borrowing limit is binding, the excessive level of credit is funded purely with domestic debt and hence there is an inefficiently low foreign debt participation in equilibrium.

particular, bank loans are 3.53% inefficiently high. The model is calibrated observing the 2000-2013 average data of small open economies. Specifically, the parameters are set such that the model with limited liability and deposit insurance yields a credit to GDP ratio of 30%, a bank leverage ratio of 9, a foreign debt participation 30% and a default probability of 3.0%.[4] The benchmark model corresponds to the model with unlimited liability, which delivers a socially efficient allocation, as in the domestic social planner's problem.[5]

The literature on risk-taking commonly suggests that a lower domestic interest rate increases bank risk-taking (see, e.g., Jiménez et al., 2014). Under a realistic calibration, we find the same result for the domestic interest rate, but the opposite for the foreign interest rate. Interestingly, the model suggests that a lower foreign interest rate and greater access to the international credit market reduce excessive levels of capital and hence excessive domestic bank risk-taking. In particular, a 100 basis points (bps) reduction in the foreign rate reduces banks' default probability from 3% to 2.95%, and bank loans become 3.48% inefficiently high. In addition, a 100% increase in foreign borrowing limits reduces this probability by 15 bps, and bank loans become 3.38% inefficiently high. Since the foreign borrowing limit is binding, banks' marginal debt is domestic and thus the foreign rate does not boost banks' credit. Thus, for a given capital, the lower foreign rate reduces banks' default probability. This lower default probability reduces banks' incentives to take excessive risk. Hence, a lower foreign rate only indirectly increases the effective marginal cost of capital by reducing banks' incentives to take excessive risk.[6] Similarly, greater access to foreign funds only indirectly affects the effective marginal cost of capital through the banks' incentives to take excessive risk, which are reduced since the greater access to cheap funds reduces banks' default probability.

Regulatory capital requirements that limit the amount of bank loans per unit of bank equity restore efficiency. The model suggests that after a foreign rate decline, the optimal policy intervention is diminished. This is because when the foreign borrowing limit binds, the foreign rate reduction does not create any credit boom since the marginal cost of credit is the domestic interest rate, but reduces bank default probability and hence excessive bank risk-taking. Similarly, a greater access to international credit market, which reduces excessive bank risk-taking, leads to a less policy intervention. In addition, the policy intervention is stronger when the foreign debt limit does not bind, since capital is higher when the foreign borrowing does not bind.

[4]To obtain this I set the subjective discount factor of domestic households to 0.93, the capital's share in output to 0.32, the foreign borrowing limit to 0.048, the initial level of bank's equity 0.02, the mean of the log of the productivity level to -0.15 and the standard deviation of it to 0.58.

[5]The domestic social planner here aims to maximize domestic households' welfare. As will be shown, the allocation under limited liability and in the absence of deposit insurance is also efficient.

[6]Indeed, this lower capital produces a higher marginal productivity of capital which creates banks' incentives to reduce again the excessive bank risk-taking and thus the excessive capital, etc.

The indirect effect of the foreign rate on the marginal cost of capital also exists for the domestic rate changes. In other words, ceteris paribus the lower domestic interest rate reduces bank default probability. This pushes bank incentives to take excessive risk down, which in turn increases the effective marginal cost of capital as in the case of the foreign interest rate cut. Hence, as the foreign interest rate, the domestic interest rate also affects indirectly the marginal cost of loans. A lower domestic interest rate can, in addition, directly affect the marginal cost of capital since the marginal bank loan is funded by domestic deposits and thus boost capital. This latter pushes the probability of default upward and increases banks' incentives to take excessive risk. In other words, more capital reduces the marginal returns of capital and thus it reduces banks' profits at good states, which creates banks' incentives to take excessive risk. In the calibration presented here, the direct effect dominates, and hence an expansionary monetary policy results in higher excessive bank risk-taking, as commonly suggested by the literature.

The model predicts that some capital inflows produce consumption booms while others produce investment booms. When the foreign borrowing limit binds, capital inflows are modeled as a more relaxed foreign borrowing limit. Hence, after capital inflows, banks substitute expensive domestic deposits for cheap foreign deposits. Thus, when the foreign borrowing limit binds, capital inflows produce a consumption boom that is associated with a reduction in excessive bank risk-taking. When the foreign borrowing limit does not bind, capital inflows are better modeled as a foreign debt decline. Hence, after a foreign interest rate decline, there is a reduction on the marginal cost of bank funding since now the marginal deposit is foreign. This leads to higher bank loans (funded by more foreign debt) and excess bank risk-taking, as in the case of the domestic interest rate cut when the foreign borrowing limit binds. Thus, when the foreign borrowing limit does not bind, capital inflows produce investment booms that are associated with higher excessive bank risk-taking.

Another interesting result of the model is that this indirect effect of the domestic and foreign interest rates is stronger when the banks' leverage on domestic and foreign debt, respectively, is higher.[7] This is because a higher foreign debt emphasizes the negative effect on the banks' probability of default after a foreign rate reduction. Similarly, a higher domestic debt accentuates the negative effect on banks' default probability of default of a lower domestic rate.[8] This is in line with figure 6 from Dell'Ariccia et al. (2014), which shows a positive relationship between the domestic policy rate and bank risk-taking over a period of highly leveraged banks: 2007Q4 to 2009Q3.

This paper proceeds as follows: Section 2 presents the literature review. Section 3

[7]Banks' leverage on domestic (foreign) debt is defined as the amount of banks' domestic (foreign) debt to total assets ratio.

[8]For the closed economy version, for instance, I conclude that when bank is highly leveraged, a lower domestic interest rate decreases the excessive bank risk-taking.

describes the model and key assumptions. Section 4 shows the closed economy version of the model. Section 5 moves to the open version of the model. Section 6 presents the main model when the foreign borrowing limit is binding. Section 7 calibrates the parameters using data from emerging economies and presents the numerical results and some comparative statics exercises. Section 8 describes the implementation of the efficient allocation and the optimal policy behavior. Section 9 discusses some assumptions of the model. Section 10 concludes.

2 Literature review

This paper follows a branch of the literature where the need for prudential policies arises from the interaction of limited liability and another friction. Similar to some papers (see, e.g., Collard et al., 2017; Agur and Demertzis, 2012, 2015; De Nicolò et al., 2012), in this paper, the interaction of the limited liability and the deposit insurance is used to explain the socially excessive bank risk-taking. In contrast to Collard et al. (2017), however, the default probability of banks is endogenous, which allows me to properly measure the effects of the interest rates on the excessive bank risk-taking.[9] This paper attempts to contribute to this branch of the literature by studying the effects of interest rates changes on the excessive bank risk-taking and by presenting an open framework to study the role of capital requirements in the presence of foreign markets.

In other papers, limited liability and a moral hazard problem cause excessive bank risk-taking. According to Sinn (2003), since depositors are not able to perfectly observe banks' risk choices, they are unable to monitor banks' actions ex-ante. Therefore, depositors are unable to anticipate these actions with an appropriate interest demand and hence the deposit rate is risk-insensitive. In this way, banks may get stuck in an inefficient equilibrium, where they all choose an excessively high-risk level. Christiano and Ikeda (2013) also show that binding capital requirements increase welfare by reducing bank leverage, which in turn reduces the risk to the creditors who cannot obverse banks' efforts. The role of capital requirements in mitigating the inefficiencies created by government bailouts is studied in Nguyen (2014).[10]

This paper is also related to the large number of studies on role of macroprudential policies as stabilizers of the real and financial sectors in an open economy.[11] Caballero

[9] Collard et al. (2017) develops an extension that incorporates the risk-taking channel of the monetary policy. By construction, a lower domestic rate increases the excessive bank risk-taking.

[10] Also, Begenau (2019) shows, in a DSGE model, that capital requirements can reduce banks' funding costs and increase lending when households have preferences for safe and liquid assets.

[11] These models typically assume borrowing constraints which are key to explain the inefficiencies or an exogenous structure for the spread of the domestic and foreign interest rate called country risk premium (see, e.g., Schmitt-Grohe and Uribe, 2003). They justify the presence of these constraints due to some micro-funded moral hazard problems which are not modeled in theses.

and Krishnamurthy (2001) develop a model with domestic and international collateral constraints. A policy, oriented to reducing ex-ante foreign debt, reduces the distortions that might create a binding international foreign constraint and avoids any contagion on the domestic collateral constraint. Bianchi (2011), Bianchi and Mendoza (2011) and Korinek (2011) state also that a policy, oriented to reducing the level of foreign debt, increases domestic welfare. In this literature, the inefficient high level of debt is because agents do not internalize the negative effects on the endogenous foreign debt collateral of choosing high levels of foreign debt.[12] In contrast to this literature, here the source of the inefficiency is not the foreign borrowing limit since this is exogenous (and thus there is not foreign over-borrowing), but rather the interaction of the limited liability and deposit insurance that creates domestic over-borrowing. Even though in Bianchi (2011) a higher foreign rate also increases the size of the inefficiencies, the mechanism is not the same as the one described here.[13]

This work is related to the group of papers devoted to studying the different channels through which monetary policy can affect bank risk-taking (see, e.g., Agur and Demertzis, 2012, 2015; Dell'Ariccia et al., 2014, 2016). They mainly suggest two channels: the *profit* channel and the *leverage* channel. According to the *profit* channel, a lower rate increases banks' profits at good states and reduces banks' incentives to take risk. The *leverage* channel suggests that the lower rate makes leverage less expensive. This means that the bank internalizes less of its risk-taking and increases its risk-taking incentives. Dell'Ariccia et al. (2014) conclude that when leverage is endogenous, low interest rates lead to higher bank risk-taking. However, if the leverage ratio is exogenous, the effect depends on the leverage level as follows: the higher the leverage, the higher the probability that a lower rate reduces bank risk-taking.[14] The latter is also observed in this paper.[15] In contrast to Dell'Ariccia et al. (2014), this work focuses on excessive bank risk-taking. To this end, it builds a simple model to quantitatively measure this excessive risk-taking. It proposes an optimal macroprudential policy and looks at the effects of domestic and foreign interest rates on excessive risk-taking and on the optimal policy.

This paper is also related to the literature that studies capital inflow surges and the probability of a financial crisis (see, e.g., Benigno et al., 2015; Caballero, 2014). Caballero (2014) suggests that surges in inflows increase crisis probability even in the absence of

[12] A higher foreign debt chosen at period t reduces the consumption of tradable goods at period $t+1$ putting downward pressure on the price of nontradable goods.

[13] In other words, a higher foreign rate amplifies the negative effect of the foreign debt on future tradable consumption. It results in a higher reduction in future non-tradable prices and a tighter future financial constraint.

[14] Dell'Ariccia et al. (2014) assumes banks' limited liability and asymmetric information, depositors cannot observe ex-ante the bank's risk-taking level. It also studies the effects of different degrees of deposit insurance.

[15] Reinforcing this idea, an empirical work using Federal Reserve's survey of terms of business lending over the period 1997 to 2011 by Dell'Ariccia et al. (2016) concludes that the negative relationship between bank risk-taking and short-term interest rates is less pronounced for periods of low bank capital.

lending booms. Here, this paper complements this literature by suggesting a mechanism by which capital inflows bonanzas might reduce the probability of a banking crisis. In addition, as in the capital control literature, which suggests that capital account openness has a positive effect on firms' credit rating (see, e.g., Prati et al., 2012), this paper suggests that greater access to foreign markets reduces excessive bank risk-taking.

Finally, this paper is related to the large empirical literature that studies the risk-taking channel of monetary policy, which typically suggests that excessive bank risk-taking increases after a reduction in the policy rate. In Jiménez, Ongena, Peydró and Saurina (2014), using data from Spain, conclude that a lower short-term interest rate increases the level of risk of the loans. In the same way, Maddaloni and Peydró (2011) show that lending standards deteriorate after a reduction in the short-term interest rate. Here, under the calibration presented, I find similar results when considering the effects of the domestic interest rate, but the opposite for the foreign domestic rate. Recently, Chen et al. (2017), using a panel-data from more than 1000 banks in 29 emerging economies during 2000-2012, find that bank's riskiness increases when the monetary policy is eased.

3 Description of the model

I develop a two-period model with a continuum of measure one of identical domestic financial intermediaries (banks), domestic investors (domestic households), and foreign investors. Domestic households own banks. Domestic and foreign investors make domestic and foreign deposits, respectively, into banks. Banks use identical exogenous initial equity and deposits to fund their risky investments.[16]

There are two key assumptions: limited liability faced by banks and deposit insurance. In order to capture the fact that the risk-free interest rate in emerging economies is higher than in developed economies, I assume that the opportunity cost of domestic investors is higher than the opportunity cost of the foreign investors. In addition, I assume banks have borrowing limits only on foreign debt. The source of the borrowing can be motivated by some informational problem between banks and depositors and also by asymmetric information among domestic and foreign depositors (see, e.g., Coval and Moskwitz, 1999; Choe et al., 2001).

For simplicity, I also make the following assumptions: households do not have access to the international credit market, which makes the economy partially open; banks can, without cost, identify if a depositor is domestic or foreign; depositors invest in risky assets only through banks; all the agents are risk-neutral; banks are not able to issue equity

[16] I am assuming the initial equity is exogenous without abstracting too much from reality since it is well known that to raise new equity is a long-term process and if the bank is going to face binding capital requirements, it will mainly reduce loans rather than increases equity.

and there is only one type of risky investment. These simplifying assumptions will not affect the main results of the model. Further, the deposit insurance is funded by the government through lump-sum taxes on domestic households.

The timing of the model is as follows: At t=0 investors make bank's deposits and banks fund their risky lending activities with deposits and an exogenous initial equity. At t=1 the outcome of banks' investment is realized. Since banks have limited liability, it transfers non-negative dividends to domestic households, since these own banks.[17] Also, as the value of the banks' final equity (or future banks' dividends) cannot be negative, then each time that at t=1 banks' obligations are higher than banks' revenues from the risky investments, banks are not able to fully repay depositors and thus banks default. Due to the deposit insurance, If banks default, the government collects enough lump-sum taxes from households and complements banks' payments so that depositors are fully repaid.

As in Gertler and Kiyotaki (2015), each bank intermediates capital, K_0, in period t=0. In period t=1, there is a payoff of $Z_1 K_0^\alpha$, plus the leftover capital $(1-\delta)K_0$, where $0<\alpha<1$ since I assume diminishing marginal returns to capital, and Z_1 is the multiplicative aggregate shock to productivity. I assume Z_1 has a lognormal distribution, $ln(Z_1) \sim N(\mu_z, \sigma_z^2)$, F is the cumulative density function and f is the probability density function of Z_1. For simplicity, I assume capital is fully depreciated.

The problem of the representative domestic household is straightforward. Since this is risk-neutral, it maximizes the domestic utility that has the following form: $U_0 = C_0 + \beta E_0\{C_1\}$, where C_0 and C_1, respectively, denote the household's consumptions at t=0 and t=1, subject to the budget constraints at t=0, $C_0 = Y_0 - D_0$, and at t=1, $C_1 = \bar{R}_0^D D_0 + \Pi_1 - T_1$, where β, Y_0, D_0, $\bar{R}_0^D$, Π_1 and T_1, respectively, denote the household's exogenous discount factor, an exogenous initial endowment, the domestic deposits, the gross rate of return on domestic deposits, the banks' dividends and the government's lump-sum taxes.

In the benchmark model (unlimited liability), domestic deposits are risk-free and hence their gross rate of return is going to be the same as the gross rate of return on the government bonds, R_0^B, which I assume are risk-free, i.e., $\bar{R}_0^D = R_0^B$. More importantly, under limited liability, the equilibrium condition $\bar{R}_0^D = R_0^B$ still holds since the domestic deposits are fully protected by deposit insurance. Since domestic utility is linear on D_0 and to avoid any corner solution, I assume $R_0^B = \frac{1}{\beta}$. Hence, households are indifferent to the amount they deposit in banks. It follows that the deposit supply facing banks is perfectly elastic at the interest rate of R_0^B.

In the next section, this paper studies the closed economy equilibrium and then the open economy equilibrium. This is going to help us to see how inefficiency looks after

[17] In this two-period model, the bank's dividends are identical to the equity at t=1.

opening the economy and to explore how access to the foreign credit market and foreign interest rate might affect the size of the inefficiency. In particular, moving from a closed to an open economy shows more clearly the mechanism for how a lower interest rate might decrease the excessive bank risk-taking.

4 Closed economy

In a closed economy, the individual bank can only fund its loans or business investments, K_0, with domestic deposits, D_0, and the exogenous initial equity, N_0. The balance sheet of the bank is,

$$K_0 = D_0 + N_0. \tag{1}$$

I start by presenting the equilibrium where banks have unlimited liability (ULL), which leads to the socially efficient allocation, and then adding the limited liability (LL) and the deposit insurance assumptions. This is in order to explain and measure the inefficiencies and welfare losses, caused by the interaction of limited liability and deposit insurance.

4.1 Unlimited liability

The final net worth of a bank is the difference between the bank's revenues, $Z_1 K_0^{\alpha}$, and the payments to domestic depositors, $R_0^B D_0$,

$$N_1 = Z_1 K_0^{\alpha} - R_0^B D_0.$$

Recall the bank faces a perfectly elastic supply of domestic deposits at the interest rate R_0^B. Since the bank has unlimited liability, it might transfer negative dividends to the bank's owners (households). While this assumption is unrealistic, it will serve as the benchmark model. Hence, when $N_1 \geq 0$, the bank transfers positive dividends to the bank's owners; otherwise, $N_1 < 0$ and thus the bank's owners receive negative dividends.

Since banks are owned by households, the objective of a bank is to maximize the expected present value of future dividends. In this two-period model, the only future dividend is the one at period t=1 and it is going to be the same as the final equity, N_1. The expected present value of future bank dividends is given by,

$$V_0 = \mathbb{E}_0\{\beta(Z_1 K_0^{\alpha} - R_0^B D_0)\}, \tag{2}$$

where β is the discount factor of domestic households. The representative bank optimally chooses the level of domestic deposits, D_0, to maximize V_0, subject to (1). The first order condition for D_0 yields,

$$\beta(\bar{Z}\alpha K_0^{\alpha-1} - R_0^B) = 0, \tag{3}$$

where $\bar{Z}$=$E_0\{Z_1\}$. The condition (3) requires that the marginal product of capital equals the marginal cost of capital represented by the domestic gross interest rate. Hence, the optimal level of capital and domestic deposits in equilibrium can be found directly from (3),

$$K_0 = \left(\frac{\bar{Z}\alpha}{R_0^B}\right)^{\frac{1}{1-\alpha}}, \tag{4}$$

and then the deposit level from (1), D_0=$K_0 - N_0$. While this is not a novel result, it will work as a benchmark. It can be seen from (3) or (4) that the domestic risk-free interest rate directly affects the marginal cost of capital in equilibrium. Due to this direct effect, a lower domestic rate motivates the bank to increase capital due to the diminishing marginal returns assumption.

4.2 Limited liability

The unlimited liability assumption is very far from being realistic. Recall the bank faces a perfectly elastic supply of deposits at the interest rate R_0^B. Since now the bank faces limited liability, the final equity cannot take negative values. Hence, the final equity becomes,[18]

$$N_1 = max\{0, Z_1 K_0^\alpha - R_0^B D_0\}.$$

This means that the bank cannot transfer negative dividends to households. For a given K_0 there is going to be a Z^* such that,

$$0 = Z^* K_0^\alpha - R_0^B D_0. \tag{5}$$

This means that if $Z_1 \geq Z^*$, the bank does not default; otherwise, the bank is not able to fully honor its obligations and consequently it defaults and $N_1 = 0$. It follows that the endogenous probability of a bank defaulting is given by,

$$p_0 = F(Z^*).$$

The expected present value of the future terminal dividend or final net worth under limited liability is,

$$V_0 = \mathbb{E}_0\{\beta \left(max\{0, Z_1 K_0^\alpha - R_0^B D_0\}\right)\}. \tag{6}$$

Hence, when a bank has limited liability, it cares only about the states of nature where its revenues are higher than all its obligations. Since bank deposit returns are risk-insensitive due to the deposit insurance, the bank cannot internalize the effects of its risk-taking' decision through the return of the deposits. In other words, a higher capital level, which

[18]Note that since Z_1 has a log-normal distribution and capital fully depreciate, the limited liability must be binding by construction.

increases the bank's default probability, is not going to increase the domestic deposit returns and hence it does not reduce the bank's profits when the bank does not default. In the absence of deposit insurance, deposit returns are risk-sensitive and hence V_0 looks like as equation (2). As a result, the optimality condition under limited liability and in the absence of deposit insurance is going to be the same as under unlimited liability, i.e., the limited liability by itself does not create any inefficiency in this model.[19]

The individual bank seeks to maximize (6), subject to the bank balance sheet, (1). To understand the bank's incentives when there is limited liability and deposit insurance, I rewrite (6) as,

$$V_0 = \mathbb{E}_0\{\beta(Z_1 K_0^{\alpha} - R_0^B D_0)\} + \int_0^{Z^*} \beta(R_0^B D_0 - Z_1 K_0^{\alpha}) dF(Z_1), \tag{7}$$

where recall $Z^* = R_0^B D_0 / K_0^{\alpha}$. The first term of (7) is the discounted expected final equity, given that the bank services its deposits under all circumstances. The second term appears due to the presence of limited liability and deposit insurance. From an individual bank's perspective, this represents an advantage resulting from the fact that the individual bank does not fully service its bonds under all circumstances, but only in cases of non-default. Each time the bank defaults, it can avoid paying back that part of the promised deposit repayment that exceeds its revenues, $R_0^B D_0 - Z_1 K_0^{\alpha}$, and this advantage (from the bank's perspective) contributes to the final equity to the extent of the probability that it happens, $f(Z_1)$, for each $Z_1 < Z^*$. Hence, the first term of (7) delivers the same trade-off discussed in the unlimited liability case and the second term motivates the bank for a higher D_0, since it produces a positive marginal benefit, as is shown later. The first order condition for D_0 yields,

$$\beta(\alpha \bar{Z} K_0^{\alpha-1} - R_0^B) + \int_0^{Z^*} \beta(R_0^B - \alpha Z_1 K_0^{\alpha-1}) dF(Z_1) + \beta(R_0^B D_0 - Z^* K_0^{\alpha}) f(Z^*) \frac{\partial Z^*}{\partial D_0} = 0.$$

By (5), $Z^* K_0^{\alpha} - R_0^B D_0 = 0$, the optimality condition becomes,

$$\beta(\alpha \bar{Z} K_0^{\alpha-1} - R_0^B) + \int_0^{Z^*} \beta(R_0^B - \alpha Z_1 K_0^{\alpha-1}) dF(Z_1) = 0, \tag{8}$$

where in equilibrium the domestic deposits' gross return is higher that the capital marginal productivity when the bank defaults, i.e., $R_0^B - \alpha Z_1 K_0^{\alpha-1} > 0$, $\forall\ Z_1 < Z^*$. This is because in equilibrium the domestic deposits' gross return equals the expected capital marginal productivity conditional to the non-default events, i.e., by (8), $R_0^B = \alpha Z^+ K_0^{\alpha-1}$, where $Z^+ = E_0\{Z_1 | Z_1 > Z^*\}$.

A comparison between the optimality conditions (8) and (3) shows that the bank's

[19] The formal proof is in Appendix A.

choices are indeed distorted. The first term of (8) in the unlimited liability case gives zero, which yields to the optimal decision of domestic debt. When there is limited liability and deposit insurance, this decision is no longer optimal. This is because increasing the domestic deposits has an additional advantage (additional positive marginal benefit) represented by the second term of (8).

It is not feasible to provide a form closed solution for capital or domestic debt. For illustrative purposes, I can rewrite (8) as,

$$\beta(\bar{Z}\alpha K_0^{\alpha-1} - R_0^B + \theta_0) = 0. \tag{9}$$

Finally,

$$K_0 = \left(\frac{\bar{Z}\alpha}{R_0^B - \theta_0}\right)^{\frac{1}{1-\alpha}}, \tag{10}$$

where,

$$\theta_0 = \int_0^{Z^*} (R_0^B - \alpha Z_1 K_0^{\alpha-1}) dF(Z_1), \tag{11}$$

is positive and the effective marginal cost of capital, $R_0^B - \theta_0$, is also positive (proof in Appendix B) and $D_0 = \left(\frac{\bar{Z}\alpha}{R_0^B - \theta_0}\right)^{\frac{1}{1-\alpha}} - N_0$. By comparing (4) and (10), the interaction of the limited liability and the deposit insurance generates an inefficient additional marginal benefit of capital, $\theta_0 > 0$. This leads to an inefficiently high capital under LL. This inefficient additional marginal benefit of capital, θ_0, is the source of the bank risk-taking and thus it can be considered as a measure of excessive risk-taking.[20] Another way to understand it is that the LL equilibrium is equivalent to the ULL equilibrium where the banker makes a mistake and considers a wrong domestic risk-free interest rate, $R_0^B - \theta_0$.

4.3 Domestic welfare losses

Since domestic households own banks, the welfare of the domestic economy can be measured by the utility of domestic households, U_0. The domestic welfare losses (WL_0) under limited liability, if there exists, can be represented thus as,

$$WL_0 = U_0^{ULL} - U_0^{LL}, \tag{12}$$

where the superscripts ULL and LL represent, respectively, the equilibrium values for the unlimited liability and limited liability scenarios. The domestic welfare losses can be

[20] Indeed, a more appropriate measure for the bank risk-taking is the difference capital under LL and capital under ULL, i.e., $K_0^{LL} - K_0^{ULL}$. However, I mostly focus on θ_0 since I am mainly interested in the source of excessive bank risk-taking.

rewritten as,[21]

$$WL_0 = \beta\bar{Z}\left[\left(K_0^{ULL}\right)^{\alpha} - \left(K_0^{LL}\right)^{\alpha}\right] - (K_0^{ULL} - K_0^{LL}). \tag{13}$$

These are composed of a first negative part, $\left(K_0^{ULL}\right)^{\alpha} - \left(K_0^{LL}\right)^{\alpha}$, which represents the fact that under limited liability there is an inefficiently high level of capital and thus output. The second positive part of the welfare losses, $-(K_0^{ULL} - K_0^{LL})$, represents the fact that under limited liability the bank is borrowing more resources and thus incurs in higher costs. As is suggested next, under limited liability the additional borrowing costs (second positive part) dominate the additional output (the first negative part).

As shown in Appendix C, it is possible to rewrite WL_0 as,

$$WL_0 = \left(\frac{\alpha\bar{Z}}{R_0^B}\right)^{\frac{1}{1-\alpha}} \left\{\left(\frac{1}{\alpha} - 1\right) - \left(\frac{1}{\alpha}y^{\alpha} - y\right)\right\},$$

where $y = \left(\frac{R_0^B}{R_0^B - \theta_0}\right)^{\frac{1}{1-\alpha}}$. It is proved also in Appendix C that the welfare losses, WL_0, are positive, which means that the allocation under limited liability is inefficient. In other words, the allocation under limited liability leads to lower domestic welfare compared with the unlimited liability case.

4.4 Effects of changes in the domestic interest rate:

Under unlimited liability, a higher domestic risk-free interest rate clearly reduces the level of capital. This is because a higher domestic rate directly increases the marginal cost of capital, which leads to lower capital due to the diminishing returns assumption, as is observed in (3).

However, under limited liability the assessment of the effects of the domestic rate changes is not trivial. In particular, it is not trivial to appreciate the final effect on excessive bank risk-taking. By observing (9), as in the unlimited liability case, there is a direct effect of a change of the domestic interest rate on the effective marginal cost of capital, $R_0^B - \theta_0$. The novelty, under LL, is the indirect effect of a change of R_0^B on the effective marginal cost. This indirect effect takes place through the inefficient additional marginal benefit of capital, θ_0, which is the source of the discrepancy between capital allocations under LL and ULL,

$$\beta(\bar{Z}\alpha K_0^{\alpha-1} - \underbrace{R_0^B}_{\text{Direct effect source}} + \underbrace{\theta_0}_{\text{Indirect effect source}}) = 0.$$

As can be seen by (11), the inefficient additional marginal benefits of capital, θ_0, depend

[21]The proof for the expression of WL_0 is in Appendix C.

positively on the bank's default probability through Z^* and positively on the bank's marginal net benefits in the event of default, $R_0^B - \alpha Z_1 K_0^{\alpha-1}$.[22] On the one hand, through the indirect effect, it is expected that, for given capital, a higher R_0^B increases the bank's obligations and hence it increases Z^* and the probability of default. A higher R_0^B also increases the bank's marginal net benefits when the bank defaults, $R_0^B - \alpha Z_1 K_0^{\alpha-1}$. Hence, through the indirect effect, a higher R_0^B increases excessive bank risk-taking, θ_0, and capital.

On the other hand, through the direct effect, a higher domestic interest rate reduces the capital level which in turn decreases Z^* and the probability of default. In addition, the lower capital reduces $R_0^B - \alpha Z_1 K_0^{\alpha-1}$. Hence, through the direct effect, a higher R_0^B reduces excessive bank risk-taking, θ_0. Finally, the net effect of a higher R_0^B on excessive bank risk-taking and on aggregate capital is ambiguous.

In the particular case of fully leveraged banks, i.e., N_0=0, it can be proven analytically that,[23]

$$(i)\; 0 < \frac{\partial \theta_0}{\partial R_0^B} = \frac{\theta_0}{R_0^B} < 1, \qquad (ii)\; \frac{\partial Z^*}{\partial R_0^B} = 0, \qquad (iii)\; \frac{\partial K_0^{LL}}{\partial R_0^B} = -\frac{1}{1-\alpha}\frac{K_0^{LL}}{R_0^B} < 0,$$

Expression (i) concludes that a higher domestic interest rate increases the size of the distortions and hence increases the excessive bank risk-taking. Expression (ii) states that the positive effect of domestic rate on Z^* is perfectly canceled out by the negative effect of a lower K_0 on Z^*, due to the direct effect of domestic rate on the marginal cost of capital. Hence, the bank's default probability is independent of changes in domestic rate. In addition, according to expression (iii), after an increase in domestic rate, capital decreases. This is because the net effect on the effective marginal cost of capital, $R_0^B - \theta_0$, is positive, i.e., the positive increment of R_0^B dominates the positive increase in θ_0.

Since in reality banks are not fully leveraged, in the calibration presented here the initial equity is positive and a higher domestic interest rate produces a reduction in capital. This result is very robust to different calibrations, but it is difficult to prove the partial derivative of capital with respect to domestic interest rate is negative. However, the effect on the excessive bank risk-taking measure, θ_0, is not very robust. Appendix E proposes a calibration for this closed economy and makes some robustness checks for the sign of the partial derivative $\frac{\partial \theta_0}{\partial R_0^B}$, varying some parameter values. It can be concluded that for low levels of bank leverage (for example 6.8 or below, defined as bank assets to equity ratio), the domestic interest rate has a negative effect on excessive bank risk-taking, θ_0; otherwise, the effect is positive. This is because the higher the bank leverage, the higher the magnitude of the indirect effect. In particular, higher leverage makes Z^*

[22]Recall these are marginal net benefits form the bank's perspective in the sense that the bank avoids assuming these marginal net losses when it defaults.

[23]Proof in Appendix D.

and banks' default probability more sensitive to interest rate changes and thus banks face a higher increment in the probability of default after the interest rate increase.[24]

The positive effect found for low levels of leverage is in line with the literature that suggests that an expansionary monetary policy increases the bank risk-taking in an excessive way (see, e.g., Jiménez et al., 2014). However, it will be interesting to study the effect of changes in funding costs when controlling for the direct effect of the domestic interest rate on the marginal cost of credit or by the loan size. Later, this paper shows that in an open economy an expansionary foreign monetary policy that does not have a direct effect on the marginal cost of domestic credit reduces the excessive risk-taking of domestic banks.

Next, this document examines the open economy version where the direct effect on the effective marginal cost of capital of the foreign risk-free interest rate is naturally turned off and only the indirect effect, through θ_0, takes places. Hence, I can focus on the effects of the funding costs on capital and on the excessive bank risk-taking when controlling for the direct effect on the effective marginal cost and taking into account only the indirect effect through θ_0.

5 Open economy

Now financial intermediaries can fund their loans also with foreign deposits, D_0^F. The balance sheet of the bank becomes,

$$K_0 = D_0 + D_0^F + N_0. \tag{14}$$

I assume foreign investors have an exogenous opportunity cost of R_0^F. In particular, I assume,

$$R_0^F < R_0^B,$$

where R_0^F can be interpreted as the gross return of safe foreign government bonds. Since foreign deposits are also fully protected by deposit insurance and foreign investor are risk-neutral, the bank also faces a perfectly elastic supply of foreign debt at the interest rate R_0^F. Recall that, for simplicity, I have assumed that households cannot borrow directly from foreign investors and banks can identify if the investor is domestic or foreign.

I further assume the following exogenous borrowing constraint on foreign debt,

$$D_0^F \leq \phi, \tag{15}$$

where $\phi>0$ is a parameter. Regarding this assumption I state the following: First, this

[24]The leverage cutoff depends on economy's characteristics. This is investigated in Appendix G

foreign borrowing limit can arise due to some informational frictions that might exist between the domestic bank and foreign creditors, but this paper does not model these frictions explicitly.[25] Further, I assume that this friction is independent of the credit risk and hence of the default probability of banks (driven by fundamentals).

Second, this borrowing limit on only foreign debt captures the plausible assumption that the borrowing limit is tighter on foreign borrowing than on domestic borrowing, i.e., that foreign debt requires relatively more collateral or that this collateral is relatively less available compared with the domestic one (see, e.g., Caballero and Krishnamurthy, 2001).[26] Here, for simplicity, I assume an ad hoc borrowing limit on only foreign borrowing.

I implicitly assume that the agency problem (if any) between domestic banks and domestic investors is less severe compared with the agency problem between the domestic banks and foreign investors. In the context of the moral hazard problem developed by Gertler and Kiyotaki (2015), I implicitly assume that in the margin it is more difficult for bankers to divert assets funded by foreign deposits than by domestic deposits. This might be because foreign depositors have less ability to persuade bankers not to divert assets, since they might have less information (compared to the domestic depositors) about the asset value or it might be more expensive for foreigners to monitor domestic banks. The "home bias" puzzle might support this assumption, since it argues that home equity preferences can be explained by information asymmetries between domestic and foreign investors. Coval and Moskowitz (1999) state that investors may have easier access to information about the companies located near them. Local investors can talk to employees, managers, and suppliers of the firm, all of whom may provide them with an information advantage. Choe et al. (2001) find evidence that domestic individual investors have a short-lived private information advantage.

Finally, the borrowing limit also captures a friction between the domestic government and foreign investors. This friction arises because, even though foreign deposits are fully insured, the government might decide not to pay foreign depositors if banks default. This is explained because domestic depositors might have more ability than foreign investors to enforce domestic government to honor their obligations.

In contrast to the small open economy literature, the exogenous collateral value avoids that the constraint generate additional inefficiencies, as in Bianchi (2011), to the one generated by the constraint itself. In other words, the exogenous collateral constraint, proposed in this paper, does not yield any pecuniary externality extensively studied in

[25]Kiyotaki and Moore (1997), Bianchi (2011) and Mendoza (2010) also introduce a borrowing constraint that aims to capture some financial friction between the domestic economy and foreign creditors. For instance, Bianchi (2011) states that these informational frictions can be associated with monitoring costs, limited enforcement, asymmetric information, and imperfections in the judicial system.

[26]In Gertler et al. (2012) and Akinci and Queralto (2014) the asymmetry of the moral hazard problem is not with respect to the residence of the bank's creditors but to the type of bank's liability.

the literature. For the purpose of this paper, this simple form for the foreign borrowing constraint is convenient since it allows me to focus on the inefficiency caused by the interaction of limited liability and deposit insurance.

I also assume that the bank cannot be a net lender to domestic depositors, i.e.,

$$0 \leq D_0.$$

This could be because it is difficult to monitor domestic depositors directly and they prefer to invest in firms or projects where financial information is more public. The previous condition ensures that for high values of ϕ, the bank cannot exhaust all their foreign debt capacity and hence cannot make profits by borrowing from abroad and lending to domestic depositors.

Note that there exist $\bar{\phi}_u^{ULL}$ and $\bar{\phi}_u^{LL}$, defined in Appendix H, where $0<\bar{\phi}_u^{ULL}<\bar{\phi}_u^{LL}$ such that if $\bar{\phi}_u^{ULL}<\phi$, the foreign collateral constraint does not bind under unlimited liability; otherwise, it does. If $\bar{\phi}_u^{LL}<\phi$ the foreign collateral constraint does not bind under limited liability; otherwise, it does.

For illustrative purposes, I start assuming that the foreign collateral constraint is not binding, for instance,

$$\phi = +\infty.$$

Since by assumptions $R_0^B>R_0^F$ and domestic agents cannot borrow from banks, $D_0=0$, then capital is funded only by the initial equity and foreign funds, and banks will borrow from abroad as much as they want. Hence, when the foreign constraint is not binding, the equilibriums under the unlimited and limited liability are basically the same as their corresponding equilibriums in the closed economy framework with only two variations: first, domestic debt becomes foreign debt; and second, domestic interest rate becomes foreign interest rate. Even the structure of the inefficiency θ_0 is going to look similar.

Under the unlimited liability equilibrium, capital and foreign debt are, respectively,

$$K_0 = \left(\frac{\bar{Z}\alpha}{R_0^F}\right)^{\frac{1}{1-\alpha}}, \quad \textit{and} \quad D_0^F = \left(\frac{\bar{Z}\alpha}{R_0^F}\right)^{\frac{1}{1-\alpha}} - N_0. \tag{16}$$

As with risk-free domestic rate in a closed economy, the risk-free foreign rate has a direct effect on the marginal cost of capital since in equilibrium the marginal deposit is foreign. As expected under ULL, domestic welfare increases when opening the economy.[27]

Under the limited liability equilibrium, capital, foreign debt and the excessive bank

[27]Proof in Appendix I.

risk-taking measure are, respectively,

$$K_0 = \left(\frac{\bar{Z}\alpha}{R_0^F - \theta_0}\right)^{\frac{1}{1-\alpha}}, \quad \textit{and} \quad D_0^F = \left(\frac{\bar{Z}\alpha}{R_0^F - \theta_0}\right)^{\frac{1}{1-\alpha}} - N_0, \tag{17}$$

where θ_0 is now,

$$\theta_0 = \int_0^{Z^*} \left(R_0^F - \alpha Z_1 K_0^{\alpha-1}\right) dF,$$

and $0{=}Z^* K_0^\alpha {-} R_0^F D_0^F$ and $\{\theta_0,\, R_0^F {-}\theta_0\}{>}0$.[28]

The level of capital is going to be higher when banks face limited liability. The foreign rate changes affect directly and indirectly the marginal cost of funding and the level of capital in a very similar way as does the domestic risk-free interest rate in a closed economy, section (4.4). The direct effect is due to the presence of R_0^F in the denominator of (17) since the marginal deposit is foreign. The indirect effect on the marginal cost of capital is through the inefficient additional marginal benefit of capital, θ_0.[29]

As in the closed economy, when the foreign constraint does not bind, it is not possible to control for the direct effect of the interest rate change on the marginal cost of bank credit since the marginal deposit is foreign. The next section presents the case when the foreign collateral is binding, and in that case, I naturally control for the direct effect of the foreign interest rate change on the marginal cost of credit.

6 Binding foreign collateral constraint

In this case, I assume that the foreign collateral constraint always binds. Specifically, that the constraint is tight enough for banks to find it optimal at the margin to demand some domestic deposits. In other words, the marginal productivity of capital, being capital $\phi{+}N_0$, is higher than the cost of domestic deposits. Hence, in equilibrium, there is going to be a positive value of domestic deposits.

There exist $\bar{\phi}_d^{ULL}$ and $\bar{\phi}_d^{LL}$, defined in Appendix H, where $0{<}\bar{\phi}_d^{ULL}{<}\bar{\phi}_d^{LL}$, such that if $\phi{\leq}\bar{\phi}_d^{ULL}$, the constraint binds and domestic deposits are positive under unlimited liability. If $\phi{\leq}\bar{\phi}_d^{LL}$, the constraint binds and domestic deposits are positive under limited liability.

In what follows, I assume that $\phi{\leq}\bar{\phi}_d^{ULL}$. Hence, the foreign constraint binds and then in equilibrium $D_0^F{=}\phi$. Since the constraint is tight enough, $D_0{>}0$.[30] The balance of the

[28] $\{\theta_0,\, R_0^F{-}\theta_0\}{>}0$ can be proven in a similar way as I proved that in a closed economy $\{\theta_0,\, R_0^B{-}\theta_0\}{>}0$ (Appendix B).

[29] As in the closed economy, if I assume the deposits are not insured, the limited liability has not distortionary effects on the economy. The formal proof is in Appendix J. Also, welfare losses, $U_0^{ULL}{-}U_0^{LL}$, are proved to be positive in Appendix C and hence the allocation under LL is inefficient.

[30] For completeness: Under unlimited liability: if $\bar{\phi}_d^{ULL}{\leq}\phi{\leq}\bar{\phi}_u^{ULL}$, $\{K_0{=}\phi + N_0,\, D_0^F{=}\phi,\, D_0{=}0\}$, i.e., banks do not want to issue any additional unit of loans since its marginal cost, R_0^B, is larger than its marginal benefit (marginal productivity of capital); if $\bar{\phi}_u^{ULL}{\leq}\phi{\leq}\bar{\phi}_u^{LL}$, the solution is given by (16).

bank is given by,

$$K_0 = D_0 + D_0^F + N_0. \tag{18}$$

6.1 Unlimited liability

The final equity under unlimited liability is,

$$N_1 = Z_1 K_0^\alpha - R_0^B D_0 - R_0^F D_0^F,$$

As usual, the bank seeks to maximize $V_0{=}E_0\{\beta N_1\}$, subject to $D_0^F{\leq}\phi$ and the bank's balance sheet, (18). The first order conditions for D_0 and D_0^F are, respectively,

$$\beta\left(\alpha\bar{Z}K_0^{\alpha-1} - R_0^B\right) = 0, \quad \beta\left(\alpha\bar{Z}K_0^{\alpha-1} - R_0^F\right) - \lambda = 0. \tag{19}$$

λ is the LM associated with the binding foreign collateral constraint. The level of capital is,

$$K_0 = \left(\frac{\bar{Z}\alpha}{R_0^B}\right)^{\frac{1}{1-\alpha}}.$$

In addition, by construction now the foreign collateral is binding and hence $D_0^F{=}\phi$ and then the domestic deposits are $D_0{=}K_0 - D_0^F - N_0$. In the case of an open economy with limited liability and with a binding collateral constraint, only the domestic interest rate affects the marginal cost of capital, since the marginal deposit is domestic.

6.2 Limited liability

The final equity becomes,

$$N_1 = max\left\{0, Z_1 K_0^\alpha - R_0^B D_0 - R_0^F D_0^F\right\},$$

and there is a Z^* such that,

$$0 = Z^* K_0^\alpha - R_0^B D_0 - R_0^F D_0^F. \tag{20}$$

Hence, if $Z_1{<}Z^*$ the bank defaults; otherwise, the bank does not default. Hence, $p_0{=}F(Z^*)$ is going to be the default probability of the representative bank. Recall that the bank faces perfectly elastic supplies of domestic and foreign debt. The expected net

Under limited liability: if $\bar{\phi}_d^{LL}{\leq}\phi{\leq}\bar{\phi}_u^{LL}$, $\{K_0{=}\phi + N_0,\ D_0^F{=}\phi,\ D_0{=}0\}$, i.e., banks do not want to issue any additional unit of loans since its marginal cost, R_0^B, is larger than its marginal benefit (marginal productivity of capital); if $\bar{\phi}_u^{ULL}{\leq}\phi{\leq}\bar{\phi}_u^{LL}$, the solution is given by (23).

present value of future positive dividends, N_1, is,

$$V_0 = \mathbb{E}_0\{\beta(max\left\{0, Z_1 K_0^\alpha - R_0^B D_0 - R_0^F D_0^F\right\})\}.$$

This can be rewritten as,

$$V_0 = \mathbb{E}_0\{\beta(Z_1 K_0^\alpha - R_0^B D_0 - R_0^F D_0^F)\} + \beta \int_0^{Z^*} \left(R_0^B D_0 + R_0^F D_0^F - Z_1 K_0^\alpha\right) dF(Z_1). \quad (21)$$

The bank seeks to maximize (21) subject to $D_0^F \leq \phi$, the expression for the Z^*, (20), and the balance sheet equation, (18). The first order condition for D_0 yields,

$$\begin{aligned} - \quad & \beta(\alpha \bar{Z} K_0^{\alpha-1} - R_0^B) + \beta \int_0^{Z^*} (R_0^B - \alpha Z_1 K_0^{\alpha-1}) dF(Z_1) \\ + \quad & \beta \left(R_0^B D_0 + R_0^F D_0^F - Z^* K_0^\alpha\right) f(Z^*) \frac{\partial Z^*}{\partial D_0} = 0, \end{aligned}$$

Since $Z^* K_0^\alpha - R_0^B D_0 - R_0^F D_0^F = 0$,

$$\beta(\alpha \bar{Z} K_0^{\alpha-1} - R_0^B) + \beta \int_0^{Z^*} (R_0^B - \alpha Z_1 K_0^{\alpha-1}) dF(Z_1) = 0. \quad (22)$$

As in the closed economy, $R_0^B - \alpha Z_1 K_0^{\alpha-1} > 0$ since from equation (22) $R_0^B = \alpha Z^+ K_0^{\alpha-1}$, where $Z^+ = E_0\{Z_1 | Z_1 > Z^*\}$. Similarly, the first order condition for D_0^F is,

$$\beta \int_{Z^*}^{+\infty} (\alpha Z_1 K_0^{\alpha-1} - R_0^F) dF(Z_1) - \lambda = 0,$$

where λ is the Lagrange multiplier associated with the binding foreign borrowing constraint. By comparing (19) and (22), it can be seen that the bank's choice of capital is distorted. The second term of (22) shows the additional advantage from the bank's perspective. I rewrite (22) as,

$$\beta(\bar{Z} \alpha K_0^{\alpha-1} - R_0^B + \theta_0) = 0, \quad (23)$$

Hence,

$$K_0^{LL} = \left(\frac{\alpha \bar{Z}}{R_0^B - \theta_0}\right)^{\frac{1}{1-\alpha}},$$

where,

$$\theta_0 = \int_0^{Z^*} (R_0^B - Z_1 \alpha K_0^{\alpha-1}) dF(Z_1).$$

As before, θ_0 represents the inefficient additional marginal benefit of increasing one unit of capital and $R_0^B - \theta_0$ represents the effective marginal cost of capital. Since $\theta_0 > 0$, the

LL yields to an inefficiently high level of capital, and level of domestic debt, i.e.,

$$K_0^{LL} > K_0^{ULL}, \quad D_0^{LL} > D_0^{ULL}, \quad D_0^{F,LL} = D_0^{F,ULL} = \phi.$$

Since the returns of domestic and foreign debt are risk-insensitive due to deposit insurance, the bank does not internalize the effects of higher capital (or risk) on households' utility. This results in an inefficiently higher capital and bank risk-taking. It is proved in Appendix L that under limited liability and in the absence of deposit insurance, the allocation under LL is the same as the ULL solution and thus is efficient.[31]

It can be seen by (23) that the foreign interest rate has no direct effect on the marginal cost of capital, as in the closed economy (or in the case of a non-binding foreign constraint), since the marginal debt is domestic rather than foreign. In this case, the foreign rate affects capital only through its effects on the inefficient additional marginal benefits, θ_0. This economic structure will help me consider the effects of the interest rates when there is no direct effect on the marginal cost of credit and when the indirect effect is just because of the presence of the inefficiency. Hence, when the foreign constraint binds, it is possible to control for the direct effect of the foreign rate change on the marginal cost of the bank's credit.

A higher foreign interest rate, as will be shown in the numerical results, leads to higher excessive bank risk-taking, or to higher excessive capital. The intuition is that a higher foreign rate is not going to have a direct effect on the marginal cost of credit and hence will not affect the level of credit directly as the domestic rate is going to, and hence a higher foreign rate will only affect the marginal cost indirectly through θ_0. Hence, the higher foreign rate will increase the default probability of banks and, with this, banks are going to be more inefficiently biased to issue more credit, since they will feel they are avoiding paying the full obligations a higher number of times.

Before presenting the numerical results, next section describes how this open economy model is calibrated.

7 Quantitative analysis

7.1 Parameters values

Here, I calibrate the open economy version with limited liability and deposit insurance with a binding foreign borrowing limit, which is the one that interests us. The foreign risk-free gross interest rate, R_0^F, is calibrated following the suggestion of Kydland and Prescott (1982) and Prescott (1986) for the annual real interest rate in the US, i.e R_0^F=1.04. The

[31] Appendix M shows that the welfare losses (WL_0) are positive and hence the allocation under limited liability and deposit insurance is inefficient.

domestic households' discount factor, β, is set to 0.93, which is a relatively standard value in the literature, in order to obtain a real interest rate differential, $R_0^B - R_0^F$, of 3.5%, which is relatively close to the observed average in (virtually open) upper middle income countries and higher than in other (virtually open countries), see table 3.

Table 1: Parameters

Description	Parameter	Value	
Risk-free foreign interest rate	R_0^F	1.04	Kydland and Prescott (1982), Prescott (1986)
Parameters set to match the data			
Discount of domestic HHs	β	0.93	Targeted to match $R_0^B - R_0^F$
Capital's share in output	α	0.32	Targeted to match credit to GDP ratio
Foreign borrowing collateral	ϕ	0.048	Targeted to match foreign debt ratio
Initial level of bank's equity	N_0	0.02	Targeted to match leverage ratio
Mean of log Z_1	μ_z	-0.15	Targeted to match default probability
Std. Dev. of log Z_1	σ_z	0.58	

Table 2: Model

Description		ULL	LL
Credit to GDP ratio (%)	$K_0/(\bar{Z}K_0^{\alpha}).100$	29.7	30
Banks' leverage	K_0/N_0	8.6	9
Foreign debt participation (%)	$D_0^F/(D_0 + D_0^F).100$	31.5	30
Default probability of banks	p_0	2.7%	3.0%
Relative excess loans	$(K_0^{LL}/K_0^{ULL}$ -1)	-	3.53%
Inefficient additional marginal benefits	θ_0	-	2.50%

The other five parameters, $\{\alpha, \phi, N_0, \mu_z, \sigma_z\}$, are set to make the following variables of the model consistent with average data for small open economies, 2000-2013: the bank leverage ratio (K_0/N_0), the credit to GDP ratio ($K_0/(\bar{Z}K_0^{\alpha})$), a proxy for a measure of the bank's foreign debt participation ratio ($D_0^F/(D_0+D_0^F)$), and the bank's default probability (p_0). The average data for (virtually open) countries grouped by income level and geographic location is presented in table 3. I use those values as references to calibrate this small open economy model.

In particular, α is mainly set to obtain a credit to GDP ratio of 0.30, which is similar to countries in LAC and SSA. It results in α=0.32, which is standard in the literature. N_0 is essentially calibrated to obtain a bank leverage equal to 9.0. This is a conservative number and indeed only LI countries report a smaller bank leverage ratio. The foreign borrowing limit, ϕ, is set to obtain a foreign debt participation of 30%. This value is

Table 3: Average ratios for annual country data, 2000-2013, where CC≤0.60

	HI	UMI	LMI	LI	LAC	MENA	SSA	ECA	EAP
Leverage	15.6	9.6	11.1	8.7	10.3	11.3	10.9	15.5	14.8
BankCred-GDP (%)	104	36.4	23.9	11.7	33.4	53.0	28.9	102	109
ForDebt Part (%)	46.1	32.7	51.7	17.7	42.1	64.3	17.7	38.7	60.8
Rate differential (%)	2.0	3.3	1.4	-	2.6	1.9	2.3	2.7	2.1
Prob. BC (%) *	2.2	2.5	2.9	3.0	3.8	2.4	2.8	2.9	-
Prob. Being BC (%) *	9.6	10.1	7.6	2.4	11.6	5.2	7.5	11.4	-

Leverage: The bank's asset to capital ratio. ForDebt Part (%): Domestic Private Debt Securities to Total Domestic and International Debt Securities. Prob. BC (%): Probability of starting a Banking Crisis. Prob. Being BC: Probability of being in a Banking Crisis. CC: Capital control measure Fernandez et al. (2015). 0: virtually open and 1: virtually closed economy. HI: High income. UMI: Upper middle income. LMI: Lower middle income. LI: Low income. LAC: Latin America & Caribbean. MENA: Middle East & North Africa. SSA: Sub-Saharan Africa. ECA: Europe & Central Asia. EAP: East Asia & Pacific. Source: IMF, World Bank, BIS. *I build these using the data of Laeven-Valencia (2013). It covers the period of 1970 to 2011 for 162 countries subject to average CC (1995-2013)≤0.60; however, observations for 70 countries are dropped since there are not CC information for them. If I assume these 70 countries have an average CC≤0.60, the probabilities are slightly lower. Rate differential: Real policy rate of the country - real shadow policy rate for the U.S. economy.

below the one observed in countries in LAC and between the values reported in LMI and UMI countries. μ_z and σ_z are mainly set to make p_0=3%, as in Collard et al. (2017). In general, this small value is consistent probability of a banking crisis built from the database of Leaven-Valencia (2013). In particular, this is closer to the banking crisis probability in LMI and LI countries. The calibration is summarized in table 2.

7.2 Numerical results

Figure 2 shows the solution for the main variables of the open economy under unlimited and limited liability. For illustrative purposes, I allow foreign borrowing limit, ϕ, to take values from 0 to 0.35, being 0.048 the calibrated value, and keep the other parameters constant in order to observe the solutions of the model when the foreign constraint binds (low values of ϕ) and when it does not bind (high values of ϕ).

Since the calibrated banks' default probability is small, p_0=3.0%, the inefficient additional marginal benefits are also small, θ_0=2.5%. When comparing the latter with the marginal cost of net capital of the undepreciated capital rate, $R_0^B-(1-\delta)$=107.5%, which determines the capital level in equilibrium, it is relatively smaller.[32] Hence, the inefficient level of capital under LL is going to be 3.53% higher than the efficient one under ULL, and it results in very low domestic welfare losses: bank's welfare under LL is just 0.01%

[32]Note that for simplicity I assume δ=1.

lower than under ULL.[33]

The capital level under limited liability is inefficiently higher whatever the binding status is. Also, capital is higher when the constraint does not bind since $R_0^B > R_0^F$. This higher capital leads to higher banks' obligations and higher default probability, p_0. Hence, the size of the inefficiency, θ_0, and the welfare losses, WL, are higher when the constraint does not bind (i.e., for higher values of ϕ) as shown in figure 2.

Most importantly, figure 2 shows the effects of the foreign borrowing limit on key variables. When the constraint binds, as in the calibration presented here, greater access to the foreign credit market reduces the excessive bank risk-taking, θ_0, and with this there is going to be a lower K_0. The mechanism is the following: For given capital, a higher ϕ reduces the bank's obligations since it substitutes expensive domestic debt for cheap foreign debt. It follows that the bank's default probability decreases and this in turn lowers excessive bank risk-taking. Since a higher ϕ affects only indirectly the marginal cost of credit through the inefficient additional marginal benefits, it reduces excessive bank risk-taking and the excessive capital level. As the economy is more open, at some point, the foreign constraint is no longer binding and thus ϕ does not affect the equilibrium.

Quantitatively, a 100% increase in the foreign borrowing limit (being the foreign constraint still binding) decreases capital by only 0.2%, reduces the default probability of banks, p_0, by only 15 bps, reduces the inefficient additional marginal benefits, θ_0, by only 14 bps and bank loans become 3.38% inefficiently high. In addition, it reduces the domestic welfare losses by only 0.1%.[34] The magnitude of the effects of a ϕ change depends essentially on the difference between the domestic and the foreign risk-free interest rates. It is expected that a higher spread will increase the significance of the effects.

The effects of movements in the foreign rate are shown in figure 3. Interestingly, when the constraint binds, a lower foreign rate decreases the excessive capital level and thus excessive bank risk-taking. A lower R_0^F affects only indirectly the marginal cost of capital through the inefficient additional marginal benefits, θ_0, since the marginal debt is domestic. Thus, a lower R_0^F mainly reduces the bank's obligations and then the bank's default probability and the inefficient additional marginal benefit of capital, θ_0.[35]

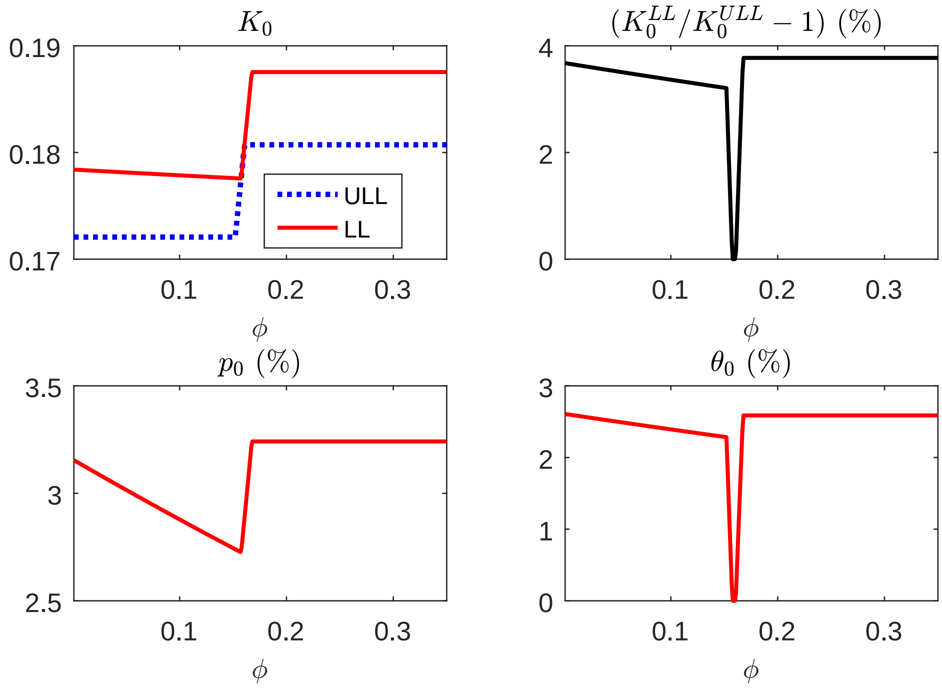
Quantitatively, a 100 bps reduction in R_0^F reduces capital by only 0.05%, decreases the default probability by only 5 bps, reduces the inefficient additional marginal benefits

[33] To properly compute the percentage reduction of domestic welfare under LL I need to calibrate Y_0. For simplicity, I calibrate this as $Y_0 = \bar{Z} K_0^{LL}$.

[34] I detail description of the effects on the welfare losses of the foreign collateral, the foreign interest rate and the domestic interest rate are presented in Appendix O.

[35] When the foreign constraint does not bind, the effects of the foreign rate are similar to the effects of domestic rate in a closed economy, see Appendix E. Consequently, higher leverage (or a higher leverage on foreign debt) increases the probability that the foreign rate affects positively the excessive risk-taking, θ_0.

Figure 2: Non-Binding and Binding Foreign Constraint Equilibriums

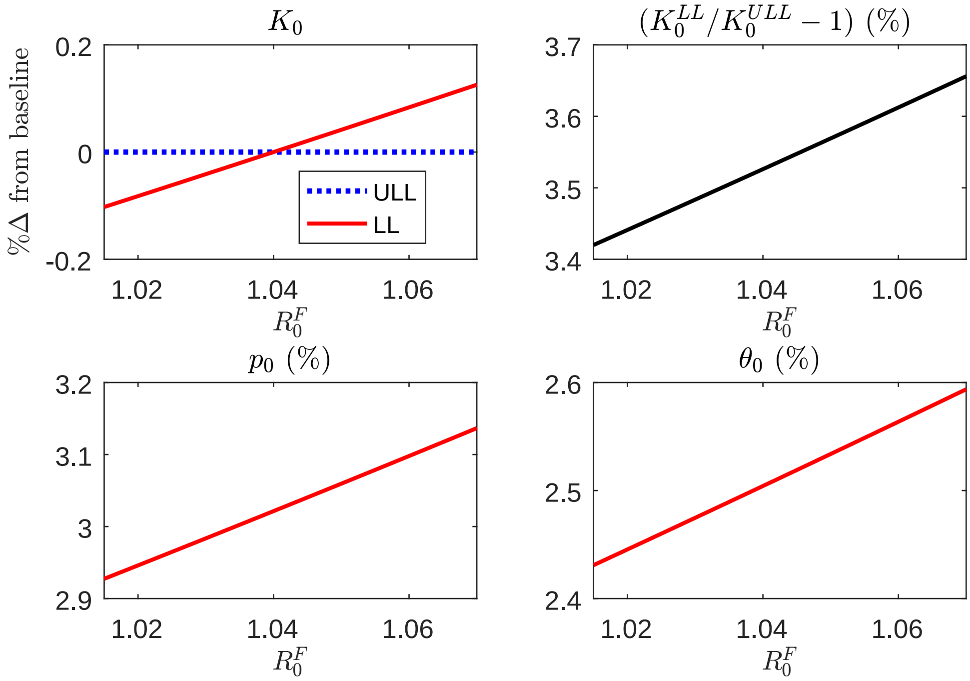


by only 3 bps and bank loans becomes 3.48% inefficiently high. In addition, it decreases welfare losses by 2.5% only. The size of the effects mainly depends on the size of the foreign debt, ϕ. Indeed, as is shown in figure 10.d, in Appendix N, the higher the ϕ the more significant the effects.

Figure 4 shows the results for the domestic rate changes. As in the case of a lower foreign rate, for given capital a lower domestic rate pushes the probability of default down (the indirect effect). In addition, a lower domestic rate also directly affects the marginal cost of capital since the marginal debt is domestic. Due to this direct effect, a lower domestic rate reduces the marginal cost and thus increases capital, which in turn pushes the probability up. Under the realistic calibration presented here, the net effect on the bank's default probability and on excessive bank risk-taking, θ_0, is positive rather than negative.

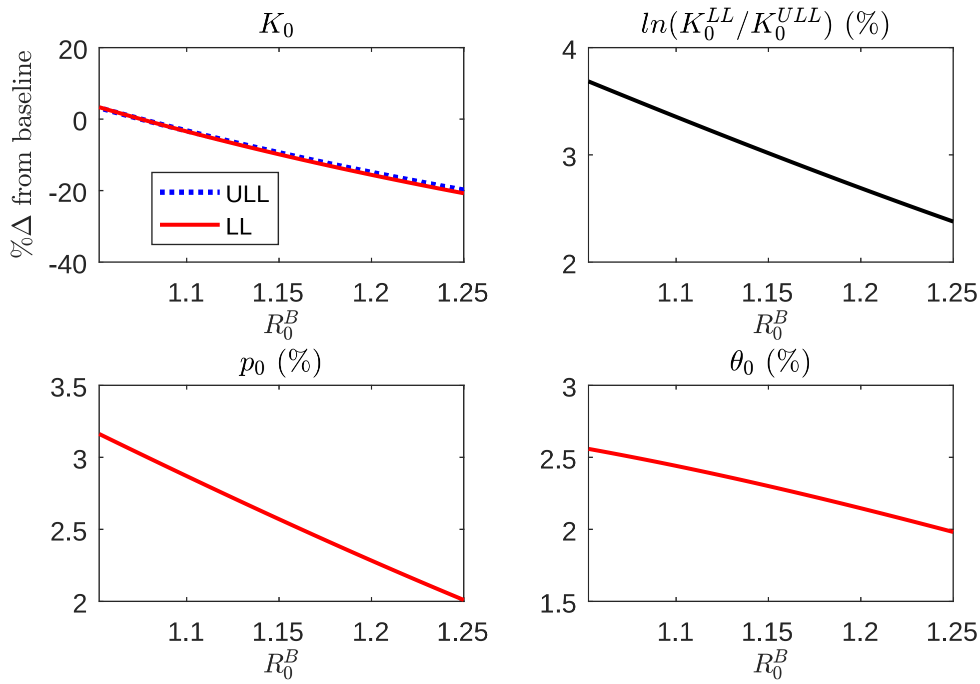
To sum up, the negative effect of a lower foreign interest rate and greater access to foreign funds on excessive bank risk-taking is not very intuitive since it might be expected that after better funding conditions, the excessive bank risk-taking would increase. The intuition is that since the foreign borrowing limit is binding and thus banks' marginal debt is domestic, the foreign rate does not directly affect the effective marginal cost of capital and thus does not boost credit. Hence, for given capital, the lower foreign rate reduces banks' default probability. This lower default probability reduces banks' incentive to take excessive risk. When funding cost also directly affects the marginal cost of capital (as domestic deposits' costs do), the marginal return of capital decreases (due to the diminishing returns assumption), which reduces banks' profits when it does not default, and hence produces the opposite effect, i.e., it creates incentives to take more

Figure 3: Changes of R_0^F and Binding Foreign Constraint



%Δ from baseline: Percentage deviation from the baseline calibration, i.e. R_0^F=1.04.

Figure 4: Changes of R_0^B and Binding Foreign Constraint



%Δ from baseline: Percentage deviation from the baseline calibration, i.e. R_0^F=1.04.

risk. The direct effect, under the calibration presented here, offsets the indirect effect in an open economy.

The importance of the positive relationship between the foreign rate and excessive bank risk-taking and the negative relationship between the latter and the foreign borrowing limit depends on the foreign borrowing limit and on the spread between the domestic and the foreign interest rates, respectively. In fact, banks' higher foreign debt participations and higher spreads are expected to be observed mainly on capital inflows, which are very common in emerging economies (see, e.g., Calvo et al., 1996; Ahmed et al., 2014; Advjiev et al., 2017).[36]

The model predicts that some capital inflows produce consumption booms while other produces investments booms. When the foreign borrowing limit binds, capital inflows are modeled as a more relaxed foreign borrowing limit, i.e., a higher ϕ. Hence, after capital inflows, banks substitute expensive domestic deposits for cheap foreign deposits. Thus, when the foreign borrowing limit binds, capital inflows produce a consumption boom that is associated with a reduction in excessive bank risk-taking. When the foreign borrowing limit does not bind, capital inflows are better modeled as a foreign debt decline. Hence, after a foreign interest rate decline, there is a reduction on the marginal cost of bank funding since now the marginal deposit is foreign. This leads to higher bank loans (funded by more foreign debt) and excess bank risk-taking, as in the case of the domestic interest rate cut when the foreign borrowing limit binds. Thus, when the foreign borrowing limit does not bind, capital inflows produce investment booms that are associated with higher excessive bank risk-taking.

7.3 Robustness

In this subsection I explore the extent to which the main results found are robust to changes in the model specification. I assume banks issue loans to competitive firms and non full capital depreciation rate.[37] With undepreciated capital, one part of loan payments is risk-free, $(1-\delta)K_0$, and the other is risky, $\alpha Z_1(K_0)^\alpha$. The undepreciated capital reduces the relative importance of risky banks revenues. And the presence of firms makes that only a fraction α of output goes to banks as loan payments (interest and principal), while a fraction $1-\alpha$ goes to households as wages. This makes bank profits and hence bank default probability more sensitive to changes on bank obligations. Hence, qualitatively, the effects on the excess bank risk-taking of changes in foreign interest rate, domestic interest rate and foreign borrowing limit hold; however, quantitatively,

[36]Calvo et al. (1996) highlights that one cause for capital inflows is the sustained decline in the world interest rate. Ahmed et al. (2014) state that interest rate differentials are a determinant of private capital flows to emerging economies. Advjiev et al. (2017) find that the bank's external borrowing is procyclical.

[37]Appendix P describes the new equilibrium conditions, calibration and numerical results.

the impacts are amplified. For instance, a 20% increase of the foreign borrowing limit reduces the relative excess loans by 84 bps, while in the original specification it goes down by 3 bps. Also, a 50 bps reduction of the foreign interest rate decreases the relative excess loans by 62 bps, while in the original specification it decreases by 3 bps. And, a 50 bps reduction of the domestic interest rate increases the relative excess loans by 19 bps, while in the benchmark economy it increases by 2 bps.

8 Implementation of the efficient allocation

This section aims to restore the socially efficient allocation by introducing some macro-prudential policies in the open economy with banks facing limited liability. Also, it assesses how the severity of the regulation might change with higher or lower access to international markets and an expansionary or contractionary domestic and foreign monetary policy.

Here, I assess whether or not a simple bank capital requirement constraint (CRC) of the form,

$$\kappa K_0 \leq N_0,$$

where κ is the capital requirement ratio (CRR) set by the regulatory authority. It says that banks for each unit of loans should hold at least κ units of equity. A binding CRC leads banks to two alternatives: (1) To reduce the level of loans, K_0, and/or (2) to increase the level of equity, N_0. Since the equity is fixed and endogenous, a binding CRC forces banks to reduce the level of loans. Hence, the effect of the CRC on loans is full.

By imposing this CRC on the financial intermediaries, I aim to find (if it exists) the optimal capital requirement ratio (CRR), κ^*, which is the one that restores the socially efficient allocation in the decentralized competitive economy, i.e., the one that restores the efficient level of loans.

Banks now seek to maximize (21) subject to foreign borrowing constraints, $D_0^F \leq \phi$, the expression for the Z^*, (20) and to this CRC, $\kappa K_0 \leq N_0$. The first order conditions for D_0 and D_0^F yield, respectively,

$$\beta(\alpha \bar{Z} K_0^{\alpha-1} - R_0^B) + \beta \int_0^{Z^*} (R_0^B - \alpha Z_1 K_0^{\alpha-1}) dF(Z_1) - \mu\kappa = 0, \tag{24}$$

$$\beta \int_{Z^*}^{+\infty} (\alpha Z_1 K_0^{\alpha-1} - R_0^F) dF(Z_1) - \lambda = 0, \tag{25}$$

where μ and λ are the Lagrange multipliers associated with the CRC and binding foreign borrowing constraint. Clearly, a binding CRC makes credit more expensive compared with an unregulated banking sector. In particular, if the regulator sets $\kappa^* = N_0 / K_0^{ULL}$, the CRC is binding, since bank credit under limited liability is going to be higher than under unlimited liability. As a result, the obvious result is that bank credit is constrained up to the level of K_0^{ULL} and thus this is going to be the optimal level chosen by banks. Since the foreign debt level is already efficient, this simple capital requirement constraint leads to the socially efficient allocation in the domestic economy.

The CRC can be interpreted as a limit on the bank's leverage level. Hence, this optimal policy requires that for each unit of equity the bank must reduce the amount of issued credit by,

$$\tau_0 = \frac{K_0^{LL}}{N_0} - \frac{K_0^{ULL}}{N_0},$$

and under the calibration presented here it can be easily proved that $\frac{\partial \tau_0}{\partial R_0^B} < 0$.[38] It means that an expansionary monetary policy increases the severity of the policy intervention. The intuition is that the expansionary monetary policy increases the excessive bank risk-taking, θ_0, and with this the policy intervention must be stronger.[39]

Also, a lower foreign rate reduces the strictness of the regulator intervention, i.e., $\frac{\partial \tau_0}{\partial R_0^F}$>0. This is because a lower foreign interest rate reduces excessive bank risk-taking, as explained, without directly affecting the marginal cost of credit of the bank, and hence policy intervention must be less strong. Finally, greater access to foreign international credit markets reduces the severity of the capital regulation, i.e., $\frac{\partial \tau_0}{\partial \phi}$<0. Again, this is because the greater access reduces excessive bank risk-taking.[40]

The consequences of a wrong macroprudential policy can lead to inefficient allocation and thus to lower domestic social welfare. For instance, let's say that τ_0^1 is the current optimal intervention size on banks' leverage. In other words, the regulator is asking banks to reduce their privately optimal leverage levels by τ_0^1. After a reduction of the foreign interest rate since $\frac{\partial \tau_0}{\partial R_0^F}$>0, the new optimal intervention size, τ_0^2, must be smaller, i.e., τ_0^2<τ_0^1, and this is because the privately optimal banks' leverage level is closer to the socially efficient one. However, if the regulator's response to the foreign rate reduction is the opposite, τ_0^3, i.e., if the regulator wrongly believes that the reduction of the foreign interest rate creates higher excessive bank risk-taking, it will intervene greater severity by asking banks for a higher reduction on their privately optimal levels of leverage, i.e., τ_0^1<τ_0^3. Clearly, this results in inefficiently low levels of bank credit, K_0.

[38]Proof in Appendix Q.

[39]This results hold for a closed economy.

[40]The proof of $\frac{\partial \tau_0}{\partial R_0^F}$<0 and $\frac{\partial \tau_0}{\partial R_0^F}$<0 is presented in Appendix Q. For completeness, in the case of a open economy with $\bar{\phi}_u^{ULL}$<ϕ (a non-binding foreign constraint), $\frac{\partial \tau_0}{\partial R_0^B}$=0. This is because domestic deposits are zero. In the particular case of having an economy with $\bar{\phi}_d^{LL}$≤ϕ≤$\bar{\phi}_u^{ULL}$, the allocations for domestic and foreign debts is the same under both the limited and unlimited liability assumptions. Hence, there is no need for policy intervention.

9 Discussion of some issues

Unanticipated ex-post capital injection: In this simple model, from the perspective of the owners of banks there is not an incentive to inject capital at t=1 to avoid bank defaults. Also, the domestic social planner will not improve domestic welfare by injecting capital to banks at t=1 to avoid defaulting since what determines the level of domestic welfare are the domestic and foreign debt levels determined in equilibrium at t=0.[41] Hence, in this two-period model the inefficiencies created by the limited liability and the fact that the individual bank cannot manipulate the interest rates cannot be eliminated by unanticipated ex-post policies. When the ex-post capital injection is anticipated, banks know that they will not default and hence will internalize the losses suffered by their owners. Hence, this is similar to the unlimited liability case. However, in reality, the interests of banks' managers and banks' owners are not aligned and hence banks' managers will mostly to keep their excessive risk-taking since their compensations are going to be higher as they do not assume any losses when the bank defaults. Unfortunately, this paper does not model this problem, but it takes into account that anticipated ex-post capital injections might not lead to the efficient allocation.

Liquidity coverage ratio: Does a policy that forces banks to hold some amount of domestic safe assets (S_0) restore the social allocation? In this model it does not.

In the benchmark model (banks under unlimited liability) if you allow banks to hold risk safe assets (i.e., domestic government bonds that give a risk-free gross return of R_0^B), S_0, the balance sheet of the bank becomes S_0+K_0=D_0+D_0^F+N_0. However, the equilibrium condition does not change and hence in equilibrium it must hold that D_0-S_0=K^{ULL}-ϕ-N_0. Hence, D_0 and S_0 are indeterminate. In the model with banks facing limited liability and deposit insurance, the conclusion is similar, i.e., in equilibrium it must hold that D_0-S_0=K^{LL}-ϕ-N_0, and then D_0 and S_0 are indeterminate. Most importantly, policies of the form (i) $\bar{S}_0 \leq S_0$ and (ii) $\bar{s}_k \leq \frac{S_0}{K_0}$, where $\bar{S}_0$ and $\bar{s}_k$ are given exogenously, do not lead to the efficient allocation.[42]

The intuition regarding the policy (i), $\bar{S}_0 \leq S_0$, is that it does not affect the marginal cost or marginal benefit of capital. Hence, it will not affect private optimal allocation. The intuition regarding the policy (ii), $\bar{s}_k \leq \frac{S_0}{K_0}$, is that this is not necessarily binding since recall the value of S_0 is indeterminate. Even if this is it binding, it affects in the same proportion the marginal cost of capital and the marginal benefit of it such that the capital level is not affected.

The exogenous initial equity: Since I assume the initial equity is exogenous, the binding capital requirements can be only satisfied by reducing the level of loans.

[41] This statement, for instance, cannot be longer true if it is assumed that if the bank defaults some resources are destroyed.

[42] Similarly, a policy as $\bar{s}_D \leq \frac{S_0}{D_0}$, where $\bar{s}_D$ is exogenous, cannot lead to the efficient allocation.

However, in an infinite-period bank's equity might become endogenous. Hence, to satisfy the capital requirement constraint the individual bank might reduce the level of capital or to ex-ante ensure higher equity.

The excessive bank risk-taking: Here the excessive bank risk-taking involves the volume of bank's credit and not the type of the credit as in Collard et al. (2017). This is line with current literature, particularly, the monetary policy literature, that commonly views the excessive bank risk-taking in terms of the aggregate volume of credit (see, e.g., Borio and Zhu, 2008). In Appendix R I model excessive bank risk-taking by considering the type rather than the volume of the credit in a simple open economy model in the spirit of Sinn (2003). When the probability of default of bank is linear on the risk-taking measure, clearly the results are the same to the model presented here. However, if the relationship is not linear, another condition is needed.

Outside equity: For simplicity, I assume banks cannot issue outside equity to domestic investors. If I drop this assumption, the main results in the model do not change. There is going to be also an inefficiently high level of capital and the partial derivative are going to have the same signs as long as the in equilibrium $D_0>0$. Since outside equity's return is state-contingent, it works as a hedging instrument and might reduce default probability of banks and then the inefficient allocation.

10 Conclusions

This paper develops a two-period partially open economy model with domestic banks, and domestic and foreign investors. Investors make bank deposits and banks that are subject to a foreign borrowing limit intermediate capital. The presence of excessive risk-taking by banks, which involves the volume of credit, is due to the interaction of limited liability and deposit insurance. These two features mean that banks underestimate the marginal cost of funding, which leads to an inefficiently high level of capital and excessive bank risk-taking. The main novel result, in contrast to what is commonly suggested in the literature, is that under a realistic calibration a lower foreign interest rate reduces excessive risk-taking by banks. And this reduction is more important the higher the level of foreign debt. In addition, the model suggests that a higher foreign borrowing limit reduces the excessive level of capital and excessive bank risk-taking. Consequently, the lower the foreign interest rate and the higher the foreign borrowing limit, the less rigorous the intervention of optimal policy needs to be.

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A Closed economy: limited liability and risky bank debt

Under limited liability and in the absence of deposit insurance, the return of domestic debt, $\bar{R}_0^D$, is risk-sensitive. Since the bank defaults with a positive probability, households require a gross return for theirs bank deposits higher than the return of safe assets, i.e., $\bar{R}_0^D > R_0^B$. Hence, $\bar{R}_0^D$ has to be high enough to compensate the reduced payment each time the bank defaults. Since households are risk neutral, the expected repayment of bank's deposits has to be equal to the repayment of domestic safe assets, which corresponds to the alternative investment for domestic investors, i.e.,

$$R_0^B = \mathbb{E}_0\{x_1^D \bar{R}_0^D\}, \tag{26}$$

where x_1^D is known as the endogenous recovery ratio (see Gertler and Kiyotaki, 2015). This is defined as the fraction of the promised return that depositors receive in the event of default or, equivalently, as the fraction of bank agreed payment that is recovered by the domestic depositors. Equation (26) represents the deposit supply curve faced by banks. If at t=1 the bank does not default, x_1^D=1 since depositors do receive the full agreed payment; however, if the bank defaults, depositors only receive an endogenous fraction, x_1^D<1, of the agreed payment. In equilibrium, if the bank defaults, the recovery ratio must satisfy,

$$0 = Z_1 K_0^\alpha - x_1^D \bar{R}_0^D D_0,$$

which means that the payment recovered by the domestic depositors, $x_1^D \bar{R}_0^D D_0$, is equal to the bank's realized income, $Z_1 K_0^\alpha$. In general, I can rewrite x_1^D as,

$$x_1^D = min\left\{1, \frac{Z_1 K_0^\alpha}{\bar{R}_0^D D_0}\right\}.$$

Therefore, $x_1^D \bar{R}_0^D$ represents the effective gross return of domestic deposits. The objective function of the bank facing limited liability and no deposit insurance is,

$$V_0 = \mathbb{E}_0\{\beta(max\{0, Z_1 K_0^\alpha - \bar{R}_0^D D_0\})\}.$$

Note that now the return of deposits is risk-sensitive. The bank seeks to maximize V_0 subject to supply curve of deposits, equation (26), which is not longer perfectly elastic. Hence, the bank is going to internalizes the effects of its decisions on the promised interest rate of deposits, $\bar{R}_0^D$. V_0 can be rewritten as,

$$V_0 = \int_{Z^*}^{+\infty} \beta(Z_1 K_0^\alpha - \bar{R}_0^D D_0) dF(Z_1),$$

where Z^* is solved in $0{=}Z^* K_0^\alpha - \bar{R}_0^D D_0$. This means that if $Z_1{<}Z^*$ the bank default, otherwise, do not. The supply curve of domestic deposits, faced by banks, equation (26), can be rewritten as,

$$R_0^B = \int_0^{Z^*} \frac{Z_1 K_0^\alpha}{\bar{R}_0^D D_0} \bar{R}_0^D dF(Z_1) + \int_{Z^*}^{+\infty} \bar{R}_0^D dF(Z_1).$$

Inserting the supply curve of deposits into V_0, the latter results in,

$$V_0 = \int_{Z^*}^{+\infty} \beta Z_1 K_0^\alpha dF(Z_1) + \beta \left(\int_0^{Z^*} \frac{Z_1 K_0^\alpha}{\bar{R}_0^D D_0} \bar{R}_0^D dF(Z_1) - R_0^B \right) D_0.$$

Solving, V_0 yields,

$$V_0 = E_0\{\beta(Z_1 K_0^\alpha - R_0^B D_0)\}.$$

Indeed, this is the same objective function of the bank when there is unlimited liability, equation (2). Therefore, the bank's problem under limited liability and no deposit insurance is equivalent to the one faced under unlimited liability. Consequently, allocation under limited liability and deposit insurance in this two-period model is socially efficient as under unlimited liability.

B Closed and open economy: $R_0^B - \theta_0 > 0$

Here, I aim to prove that, $R_0^B - \theta_0 > 0$. Since,

$$R_0^B - \theta_0 = R_0^B - \int_0^{Z^*} (R_0^B - \alpha Z_1 K_0^{\alpha-1}) dF(Z_1),$$

solving,

$$R_0^B - \theta_0 = R_0^B - F(Z^*) R_0^B + \int_0^{Z^*} \alpha Z_1 K_0^{\alpha-1} dF(Z_1).$$

Finally,

$$R_0^B - \theta_0 = (1 - F(Z^*)) R_0^B + \int_0^{Z^*} \alpha Z_1 K_0^{\alpha-1} dF(Z_1) > 0.$$

C Closed economy: domestic welfare losses

The households' utility is given by $U_0{=}C_0{+}\beta\mathbb{E}_0\{C_1\}$. It can be rewrite as,

$$U_0 = Y_0 - D_0 + \beta\mathbb{E}_0\{R_0^B D_0^{ULL} + \Pi_1 - T_1\}.$$

Under unlimited liability T_1^{ULL}=0 $\forall$ Z_1 and Π_1^{ULL}=N_1^{ULL}=$Z_1 K_0^{ULL}$−$R_0^B D_0^{ULL}$. Hence, U_0 can be rewritten as,

$$U_0^{ULL} = Y_0 + \beta \bar{Z} (K_0^{ULL})^{\alpha} - D_0^{ULL}.$$

Under limited liability, T_1^{LL}=$max\{0, R_0^B D_0^{LL} - Z_1 (K_0^{LL})^{\alpha}\}$
and Π_1^{LL}=N_1^{LL}=$max\{0, Z_1 (K_0^{LL})^{\alpha} - R_0^B D_0^{LL}\}$. Hence, U_0 can be rewritten as,

$$U_0^{ULL} = Y_0 + \beta \bar{Z} (K_0^{LL})^{\alpha} - D_0^{LL}.$$

Since households are the owners of banks, society welfare can be represented by households' utility. Hence, welfare losses under unlimited liability and deposit insurance are given by,

$$WL_0 = U_0^{ULL} - U_0^{LL} = \beta \bar{Z} \left[\left(K_0^{ULL}\right)^{\alpha} - \left(K_0^{LL}\right)^{\alpha} \right] - (D_0^{ULL} - D_0^{LL}).$$

Since $D_0 = K_0 - N_0$,

$$WL_0 = U_0^{ULL} - U_0^{LL} = \beta \bar{Z} \left[\left(K_0^{ULL}\right)^{\alpha} - \left(K_0^{LL}\right)^{\alpha} \right] - (K_0^{ULL} - K_0^{LL}).$$

Recalling the expressions for K_0^{ULL} and K_0^{LL}, equations (4) and (10), respectively, WL_0 are rewritten as,

$$WL_0 = \beta \bar{Z} \left[\left(\frac{\alpha \bar{Z}}{R_0^B} \right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{\alpha \bar{Z}}{R_0^B - \theta_0} \right)^{\frac{\alpha}{1-\alpha}} \right] - \left[\left(\frac{\alpha \bar{Z}}{R_0^B} \right)^{\frac{1}{1-\alpha}} - \left(\frac{\alpha \bar{Z}}{R_0^B - \theta_0} \right)^{\frac{1}{1-\alpha}} \right].$$

Then,

$$WL_0 = \left(\frac{\alpha \bar{Z}}{R_0^B} \right)^{\frac{1}{1-\alpha}} \left\{ \beta \bar{Z} \left[\frac{R_0^B}{\alpha \bar{Z}} - \frac{R_0^B - \theta_0}{\alpha \bar{Z}} \left(\frac{R_0^B}{R_0^B - \theta_0} \right)^{\frac{1}{1-\alpha}} \right] - \left[1 - \left(\frac{R_0^B}{R_0^B - \theta_0} \right)^{\frac{1}{1-\alpha}} \right] \right\},$$

solving,

$$WL_0 = \left(\frac{\alpha \bar{Z}}{R_0^B} \right)^{\frac{1}{1-\alpha}} \left\{ \frac{1}{\alpha} - \frac{R_0^B - \theta_0}{\alpha R_0^B} \left(\frac{R_0^B}{R_0^B - \theta_0} \right)^{\frac{1}{1-\alpha}} - 1 + \left(\frac{R_0^B}{R_0^B - \theta_0} \right)^{\frac{1}{1-\alpha}} \right\}.$$

Then,

$$WL_0 = \left(\frac{\alpha \bar{Z}}{R_0^B} \right)^{\frac{1}{1-\alpha}} \left\{ \frac{1}{\alpha} - \frac{1}{\alpha} \left(\frac{R_0^B}{R_0^B - \theta_0} \right)^{\frac{\alpha}{1-\alpha}} - 1 + \left(\frac{R_0^B}{R_0^B - \theta_0} \right)^{\frac{1}{1-\alpha}} \right\}.$$

Finally, I rewrite WL_0 as,

$$WL_0 = \left(\frac{\alpha\bar{Z}}{R_0^B}\right)^{\frac{1}{1-\alpha}} \left\{\left(\frac{1}{\alpha}-1\right) - \left(\frac{1}{\alpha}y^\alpha - y\right)\right\},$$

where $y = \left(\frac{R_0^B}{R_0^B-\theta_0}\right)^{\frac{1}{1-\alpha}}$. Hence, it is easy to see that the y that maximizes $\left(\frac{1}{\alpha}y^\alpha - y\right)$ and hence minimizes WL_0 is y=1. In particular, WL_0=0 at y=1. Therefore, since in equilibrium y>1, then $\left(\frac{1}{\alpha}-1\right) - \left(\frac{1}{\alpha}y^\alpha - y\right) > 0$ and then WL_0>0.

D Closed economy: comparative statics, N_0=0 or fully leveraged banks

When N_0=0, in equilibrium for a closed economy, it must hold that,

$$0 = \int_{Z^*}^{+\infty} \beta\left(\alpha Z_1 K_0^{\alpha-1} - R_0^B\right) dF(Z_1), \tag{27}$$

$$Z^* = R_0^B K_0^{1-\alpha}. \tag{28}$$

Taking the partial derivative of (28) with respect to R_0^B,

$$\frac{\partial Z^*}{\partial R_0^B} = K_0^{1-\alpha} + R_0^B(1-\alpha)K_0^{-\alpha}\frac{\partial K_0}{\partial R_0^B}. \tag{29}$$

Taking the partial derivative of (27) with respect to R_0^B,

$$0 = \int_{Z^*}^{+\infty} Z_1 dF(Z_1)\alpha(\alpha-1)K_0^{\alpha-2}\frac{\partial K_0}{\partial R_0^B} - (1-F(Z^*)) - (\alpha Z^* K_0^{\alpha-1} - R_0^B)f(Z^*)\frac{\partial Z^*}{\partial R_0^B}.$$

Using (27) and (28), I can rewrite the above expression as,

$$0 = \frac{\alpha-1}{K_0}(1-F(Z^*))R_0^B\frac{\partial K_0}{\partial R_0^B} - (1-F(Z^*)) + (1-\alpha)Z^*K_0^{\alpha-1}f(Z^*)\frac{\partial Z^*}{\partial R_0^B}.$$

Inserting (29) into the above expression,

$$\begin{aligned} 0 &= \frac{\alpha-1}{K_0}(1-F(Z^*))R_0^B\frac{\partial K_0}{\partial R_0^B} - (1-F(Z^*)) \\ &+ (1-\alpha)K_0^{\alpha-1}Z^*f(Z^*)\left(K_0^{1-\alpha} + R_0^B(1-\alpha)K_0^{-\alpha}\frac{\partial K_0}{\partial R_0^B}\right). \end{aligned}$$

Solving,

$$0 = \frac{(\alpha-1)R_0^B}{K_0}\frac{\partial K_0}{\partial R_0^B} - 1 + \frac{(1-\alpha)Z^*f(Z^*)}{1-F(Z^*)} + \frac{(1-\alpha)Z^*f(Z^*)}{1-F(Z^*)}\frac{(1-\alpha)R_0^B}{K_0}\frac{\partial K_0}{\partial R_0^B}.$$

Then,

$$0 = \left[-1 + \frac{(1-\alpha)Z^* f(Z^*)}{1-F(Z^*)}\right] + \frac{(1-\alpha)R_0^B}{K_0}\left[\frac{(1-\alpha)Z^* f(Z^*)}{1-F(Z^*)} - 1\right]\frac{\partial K_0}{\partial R_0^B}.$$

Finally,

$$\frac{\partial K_0}{\partial R_0^B} = -\frac{K_0}{(1-\alpha)R_0^B} < 0. \tag{30}$$

In addition, replacing (30) into (29),

$$\frac{\partial Z^*}{\partial R_0^B} = K_0^{1-\alpha} + (1-\alpha)R_0^B K_0^{-\alpha}\left(-\frac{K_0}{(1-\alpha)R_0^B}\right),$$

and solving, I obtain,

$$\frac{\partial Z^*}{\partial R_0^B} = 0. \tag{31}$$

Recalling that,

$$\theta_0 = \int_0^{Z^*}\left(R_0^B - \alpha Z_1 K_0^{\alpha-1}\right)dF(Z_1),$$

and taking the partial derivative of θ_0 with respect to R_0^B,

$$\frac{\partial \theta_0}{\partial R_0^B} = F(Z^*) - \int_0^{Z^*}\alpha(\alpha-1)Z_1 K_0^{\alpha-2}dF(Z_1)\frac{\partial K_0}{\partial R_0^B} + (R_0^B - \alpha Z^* K_0^{\alpha-1})f(Z^*)\frac{\partial Z^*}{\partial R_0^B}.$$

Inserting (30) and (31) into the above expression,

$$0 < \frac{\partial \theta_0}{\partial R_0^B} = \frac{\theta_0}{R_0^B} < 1,$$

since $R_0^B - \theta_0 > 0$ (Proof in Appendix B).

E Closed economy: calibration and robustness

In the closed economy model there are five parameters, two are very well-know, $\{\beta, \alpha\}$, other two are related with the distribution of the productivity shock $\{\mu_z, \sigma_z\}$, and the later is the exogenous initial equity, N_0. I calibrate β=0.93 that is a relatively standard value in literature and I set the other four parameters to achieve a bank's leverage ratio, a credit to GDP ratio and a default probability of banks in the model similar to those observed in the data in virtually closed economies.

The average data for different groups of countries (grouped by the income level and by geographic location) is presented in table 4. Only data of virtually closed economy is considered to calibrate the model. I use two different ways to define a virtually closed economy: The foreign bank debt participation and a capital control measure. With this

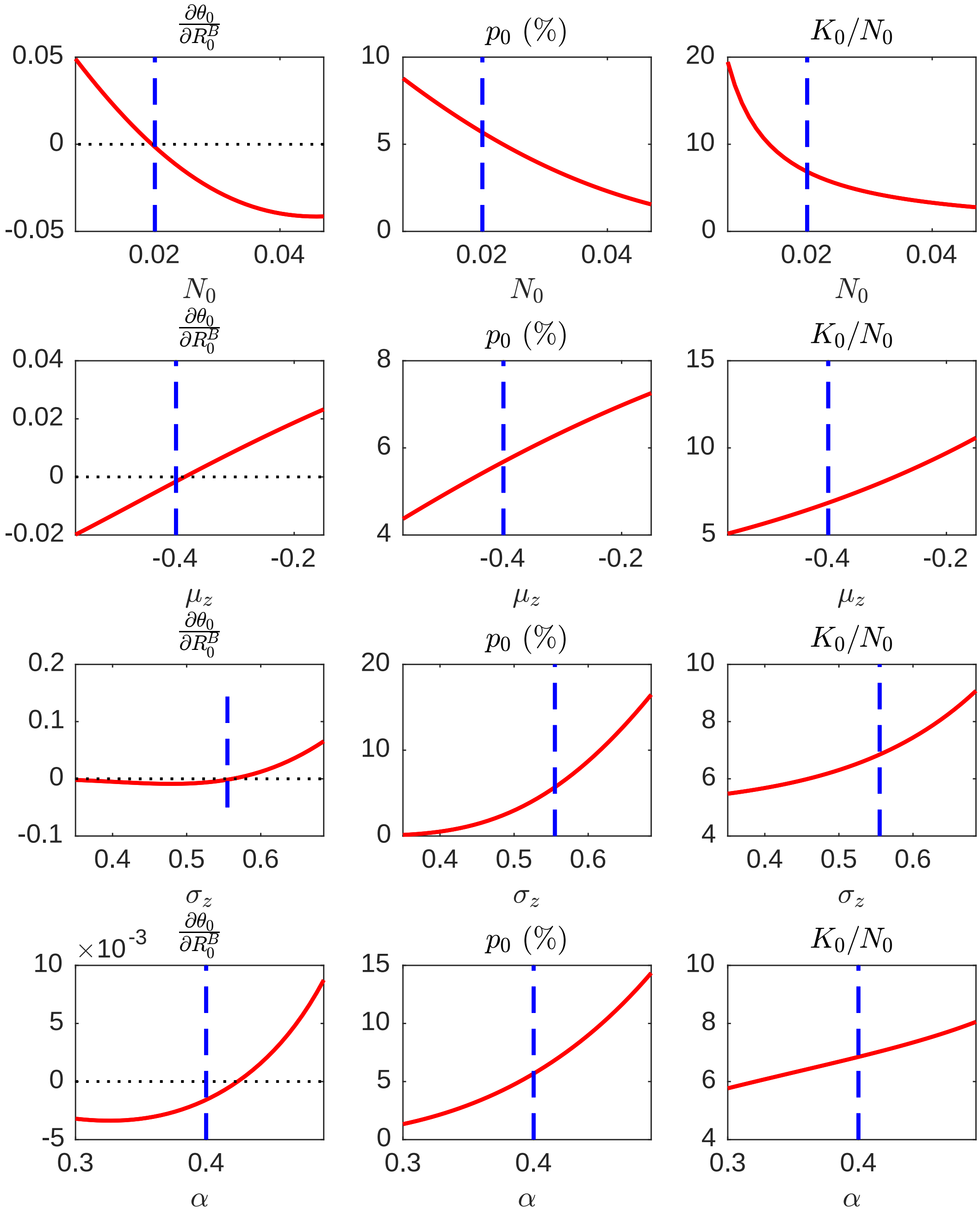
Table 4: Average of annual country data, 2000-2013

	HI	UMI	LMI	LI	LAC	MENA	SSA	ECA	EAP
(a) Conditional to $0.60 \leq$ CC									
Leverage	12.3	10.4	10.6	10.4	9.6	11.2	10.9	9.9	11.7
BankCred-GDP (%)	72.3	45.3	28.7	14.1	48.2	53.0	17.0	52.3	52.6
ForDebt Part (%)	47.8	42.5	51.8	42.7	40.0	57.7	38.3	36.1	56.9
(b) Conditional to $0.60 \leq$ average CC									
Leverage	12.0	10.6	10.7	10.5	8.9	11.2	11.3	9.5	11.6
BankCred-GDP (%)	56.4	46.2	28.8	14.1	48.3	54.7	17.1	40.4	49.1
ForDebt Part (%)	48.2	43.3	52.2	42.2	41.7	57.3	37.9	32.8	57.9
Prob. BC (%) *	2.4	2.4	2.1	1.8	4.8	1.8	1.5	2.4	-
Prob. Being BC (%) *	7.1	6.1	6.7	5.4	9.5	6.5	5.7	6.7	-
(c) Conditional to: average ForDebt Part (%) ≤ 0.35									
Leverage	13.2	9.7	-	10.9	10.3	-	10.9	12.7	-
BankCred-GDP (%)	92.3	38.2	35.8	19.1	37.9	43.7	28.0	81.9	28.5
ForDebt Part (%)	17.8	25.9	22.6	23.0	22.4	32.1	23.8	18.8	19.1

Leverage: The bank's asset to capital ratio. ForDebt Part (%): International Private Debt Securities to Total Domestic and International Debt Securities. Prob. BC (%): Probability of starting a Banking Crisis. Prob. Being BC: Probability of being in a Banking Crisis. CC: Capital control measure Fernandez et al. (2015). 0: virtually open economy and 1: virtually closed economy. HI: High income. UMI: Upper middle income. LMI: Lower middle income. LI: Low income. LAC: Latin America & Caribbean. MENA: Middle East & North Africa. SSA: Sub-Saharan Africa. ECA: Europe & Central Asia. EAP: East Asia & Pacific. Source: IMF, World Bank, BIS. *I build these variables using the data from Laeven-Valencia (2013), it covers the period of 1970 to 2011 for 162 countries subject to 0.60≤ average CC 1995-2013; however, observations for 70 countries are dropped since there are not CC information for them. When subject to ForDebt Part (%) 1990-2013 ≤0.35, there are very few observations to compute the Prob of BC and the Prob. Being BC.

in mind, in the baseline calibration, I set the parameters so that the model delivers a bank leverage ratio, K_0/N_0, of 6.8, which is a very conservative value, a credit to GDP ratio, $K_0/(\bar{Z}K_0^\alpha)$, of 39%, which is slightly smaller than what is found in the data, and a default probability of banks, p_0, of 5.7%, which is between the range of 1.5 and 9.5 found in the data. This results in α=0.40, μ_z=−0.400, σ_z=0.555 and N_0=0.02.

Figure 5: Robustness of $\frac{\partial \theta_0}{\partial R_0^B}$



Under this calibration $\frac{\partial \theta_0}{\partial R_0^B}$ is negative. In other words, a domestic interest rate cut leads to lower excess marginal benefits of capital and hence lower excessive bank risk-taking. Next, I show how $\frac{\partial \theta_0}{\partial R_0^B}$ might change if I allow the parameters $\{N_0, \mu_z, \sigma_z, \alpha\}$ to take different values around their calibrated ones. Results suggest that the negative sign of $\frac{\partial \theta_0}{\partial R_0^B}$, found in the baseline calibration, is not robust for small changes of some parameters.

Figure 5 shows in red lines $\frac{\partial \theta_0}{\partial R_0^B}$, the default probability of banks and the leverage level for an interval of a certain parameter, keeping the other parameters unchanged. The vertical blue dashed lines correspond to the baseline calibration. It can be seen that a slightly lower N_0, higher μ_z, higher σ_z can switch the sign of the partial derivative from negative to positive.

The intuition to explain these results is that the lower N_0 or higher μ_z or higher σ_z push the leverage level up, which in turn leads to larger relative participation of domestic deposits in banks' liabilities. This latter, clearly, amplifies the positive impact of higher domestic deposit costs on banks' default probability. And this increases the likelihood that in equilibrium the inefficient additional marginal benefits and excessive bank risk-taking move up after a higher domestic interest rate.

F Closed economy: comparative statics, $N_0 > 0$

When $N_0 = 0$ in equilibrium for a closed economy it must hold that,

$$0 = \int_{Z^*}^{+\infty} \beta \left(\alpha Z_1 K_0^{\alpha-1} - R_0^B \right) dF, \tag{32}$$

$$Z^* = R_0^B K_0^{1-\alpha} \frac{1}{\omega}, \tag{33}$$

where $\omega = \frac{K_0}{D_0}$. Taking the partial derivative of (33) with respect to R_0^B,

$$\frac{\partial Z^*}{\partial R_0^B} = K_0^{1-\alpha} \frac{1}{\omega} + R_0^B \left(1 - \frac{\alpha}{\omega}\right) K_0^{-\alpha} \frac{\partial K_0}{\partial R_0^B}. \tag{34}$$

Taking the partial derivative of (32) with respect to R_0^B,

$$0 = \int_{Z^*}^{+\infty} Z_1 dF(Z_1) \alpha(\alpha-1) K_0^{\alpha-2} \frac{\partial K_0}{\partial R_0^B} - (1 - F(Z^*)) - (\alpha Z^* K_0^{\alpha-1} - R_0^B) f(Z^*) \frac{\partial Z^*}{\partial R_0^B}.$$

Using (32) and (33), I can rewrite the above expression as,

$$0 = \frac{\alpha-1}{K_0}(1 - F(Z^*)) R_0^B \frac{\partial K_0}{\partial R_0^B} - (1 - F(Z^*)) + (\omega - \alpha) K_0^{\alpha-1} Z^* f(Z^*) \frac{\partial Z^*}{\partial R_0^B}.$$

Inserting (34) into the above expression,

$$\begin{aligned} 0 &= \frac{\alpha-1}{K_0}(1 - F(Z^*)) R_0^B \frac{\partial K_0}{\partial R_0^B} - (1 - F(Z^*)) \\ &+ (\omega - \alpha) K_0^{\alpha-1} Z^* f(Z^*) \left(\frac{1}{\omega} K_0^{1-\alpha} + \left(1 - \frac{\alpha}{\omega}\right) R_0^B K_0^{-\alpha} \frac{\partial K_0}{\partial R_0^B} \right). \end{aligned}$$

Solving,

$$0 = -\frac{1-\alpha}{K_0} R_0^B \frac{\partial K_0}{\partial R_0^B} - 1 + \left(1 - \frac{\alpha}{\omega}\right) \frac{Z^* f(Z^*)}{1 - F(Z^*)} + \frac{(\omega - \alpha) R_0^B}{K_0} \left(1 - \frac{\alpha}{\omega}\right) \frac{Z^* f(Z^*)}{1 - F(Z^*)} \frac{\partial K_0}{\partial R_0^B}.$$

Then,

$$0 = \left[\left(1-\frac{\alpha}{\omega}\right)\frac{Z^* f(Z^*)}{1-F(Z^*)} - 1\right] + \frac{(\omega-\alpha)R_0^B}{K_0}\left[\left(1-\frac{\alpha}{\omega}\right)\frac{Z^* f(Z^*)}{1-F(Z^*)} - \frac{1-\alpha}{\omega-\alpha}\right]\frac{\partial K_0}{\partial R_0^B}$$

Rewriting (32) as $\alpha Z^+ K_0^\alpha = R_0^B K_0$ where $Z^+ = \mathbb{E}_0\{Z_1 | Z_1 > Z^*\}$, and combining it with (33), it can be obtained, $\frac{\alpha}{\omega} = \frac{Z^*}{Z^+}$, and replacing it into the above expression,

$$\frac{\partial K_0}{\partial R_0^B} = -\frac{K_0}{(\omega-\alpha)R_0^B}\frac{\Sigma - 1}{\Sigma - \frac{1-\alpha}{\omega-\alpha}}, \tag{35}$$

where,

$$\Sigma = \left(1-\frac{Z^*}{Z^+}\right)\frac{Z^* f(Z^*)}{1-F(Z^*)}.$$

Regarding the partial derivative of the excessive bank risk-taking measure, since

$$\theta_0 = \int_0^{Z^*} \left(R_0^B - \alpha Z_1 K_0^{\alpha-1}\right) dF(Z_1), \tag{36}$$

the partial derivative of θ_0 with respect to R_0^B yields,

$$\frac{\partial \theta_0}{\partial R_0^B} = (R_0^B - \alpha Z^* K_0^{\alpha-1})\frac{\partial Z^*}{\partial R_0^B} + F(Z^*) + \int_0^{Z^*} Z_1 dF(Z_1)(1-\alpha)\alpha K_0^{\alpha-2}\frac{\partial K_0}{\partial R_0^B}.$$

Using (33), the previous expression becomes,

$$\frac{\partial \theta_0}{\partial R_0^B} = \left(1-\frac{\alpha}{\omega}\right) R_0^B \frac{\partial Z^*}{\partial R_0^B} + F(Z^*) + \int_0^{Z^*} Z_1 dF(Z_1)(1-\alpha)\alpha K_0^{\alpha-2}\frac{\partial K_0}{\partial R_0^B}.$$

Using (34), it yields,

$$\begin{aligned}\frac{\partial \theta_0}{\partial R_0^B} = \left(1-\frac{\alpha}{\omega}\right) R_0^B \left(K_0^{1-\alpha}\frac{1}{\omega} + R_0^B\left(1-\frac{\alpha}{\omega}\right)K_0^{-\alpha}\frac{\partial K_0}{\partial R_0^B}\right) + F(Z^*) \\ + \int_0^{Z^*} Z_1 dF(Z_1)(1-\alpha)\alpha K_0^{\alpha-2}\frac{\partial K_0}{\partial R_0^B}.\end{aligned}$$

Solving,

$$\begin{aligned}\frac{\partial \theta_0}{\partial R_0^B} = \left(1-\frac{\alpha}{\omega}\right)\frac{R_0^B}{\omega}K_0^{1-\alpha} + F(Z^*) \\ + \left((R_0^B)^2\left(1-\frac{\alpha}{\omega}\right)^2 K_0^{-\alpha} + \int_0^{Z^*} Z_1 dF(Z_1)(1-\alpha)\alpha K_0^{\alpha-2}\right)\frac{\partial K_0}{\partial R_0^B}.\end{aligned}$$

Then, using (35),

$$\frac{\partial \theta_0}{\partial R_0^B} = F(Z^*) + \frac{(\omega-1)\frac{(R_0^B)^2}{\omega^2}K_0^{1-\alpha} - \int_0^{Z^*} Z_1 dF(Z_1)\frac{(1-\alpha)\alpha}{(\omega-\alpha)}K_0^{\alpha-1}(\Sigma-1)}{R_0^B\left(\Sigma - \frac{1-\alpha}{\omega-\alpha}\right)}.$$

Finally, using (33),

$$\frac{\partial \theta_0}{\partial R_0^B} = F(Z^*) + \frac{\left(1-\frac{1}{\omega}\right)Z^* - \int_0^{Z^*} Z_1 dF(Z_1)\frac{(1-\alpha)\alpha}{(\omega-\alpha)\omega Z^*}(\Sigma-1)}{\Sigma - \frac{1-\alpha}{\omega-\alpha}}. \tag{37}$$

G Closed economy: leverage cutoff

From equation (37) in Appendix F,

$$\frac{\partial \theta_0}{\partial R_0^B} = F(Z^*) + \frac{\left(1-\frac{1}{\omega}\right)Z^* - \int_0^{Z^*} Z_1 dF(Z_1)\frac{(1-\alpha)\alpha}{(\omega-\alpha)\omega Z^*}(\Sigma-1)}{\Sigma - \frac{1-\alpha}{\omega-\alpha}}, \tag{38}$$

where,

$$\Sigma = \left(1 - \frac{Z^*}{Z^+}\right)\frac{Z^* f(Z^*)}{1-F(Z^*)}.$$

In order to find the cutoff value of the leverage ratio, I make $\frac{\partial \theta_0}{\partial R_0^B}$=0. This latter can be rewritten as $g(Z^*, \alpha, \mu_z, \sigma_z)$=0, where Z^*=$Z^*(N_0, \beta, \alpha, \mu_z, \sigma_z)$. So, equation (38) helps me to find the combination of parameters that yields $\frac{\partial \theta_0}{\partial R_0^B}$=0. I perform the following three numerical exercises:

- I keep β, μ_z and σ_z fixed, calibrated as in Appendix E, and solve for the combination (α, N_0) that makes $g(.)$=0, (see figure 6).
- I keep β, α and σ_z fixed, calibrated as in Appendix E, and solve for the combination (μ_z, N_0) that makes $g(.)$=0 (see figure 7).
- I keep β, μ_z and α fixed, calibrated as in Appendix E, and solve for the combination (σ_z, N_0) that makes $g(.)$=0, (see figure 8).

Numerical results suggest that for economies with higher capital's shares, the leverage cutoff is lower and hence these economies are more likely that the domestic rate and the excessive bank risk-taking are positively correlated. This is because a high α makes total income more sensible to the domestic debt, which makes the effect of domestic rate on excessive bank risk-taking for a given capital (indirect effect) more sensible. Hence, a lower leverage is required so that indirect effect becomes less strong in order to be canceled out by the direct effect.

Figure 6: g(α, N_0)=0 keeping fixed {β, μ_z, σ_z}

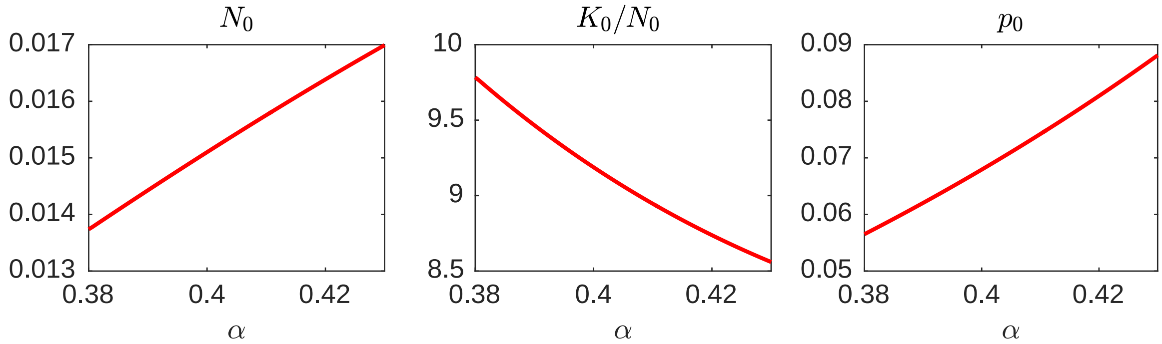
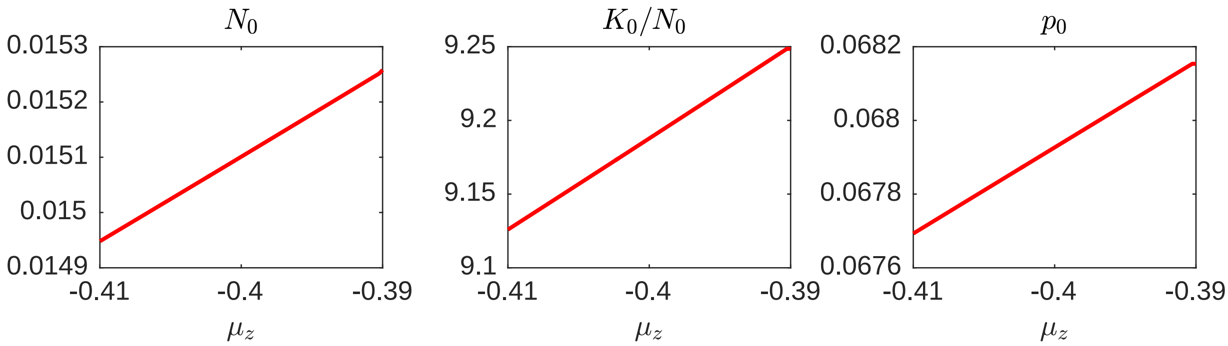


Figure 7: g(μ_z, N_0)=0 keeping fixed {β, α, σ_z}



In addition, a higher mean of the productivity shock increases the marginal productivity of capital, which pushes capital and leverage up. This means that the higher μ_z amplifies the direct effect of the domestic interest rate on capital, which has a positive impact on the default probability and on the inefficient additional marginal benefits of capital. Therefore, a larger leverage is required (i.e., a higher leverage cutoff) so that the indirect effect of the foreign rate on the default probability becomes stronger and offsets the direct effect. This suggests that economies with higher μ_z are more likely to exhibit a negative correlation between the domestic interest rate and excessive bank risk-taking.

Finally, economies with higher uncertainty face a lower leverage cutoff, which results in a higher likelihood of facing a positive correlation between the domestic interest rate and excessive bank risk-taking. This is because the higher the uncertainty of the productivity, the larger the reaction of the default probability to bank obligations and hence the stronger the indirect effect of the domestic interest rate on the effective marginal cost of capital.

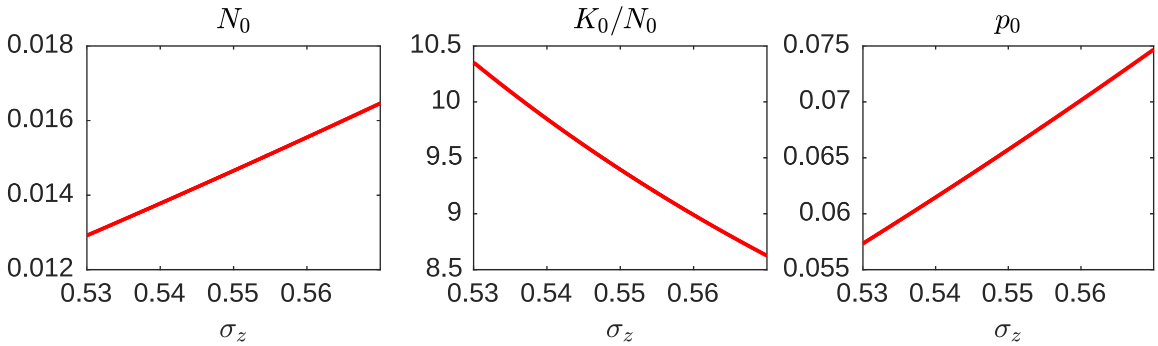
H Boundaries of ϕ

The boundaries $\bar{\phi}_u^{ULL}$, $\bar{\phi}_u^{LL}$, $\bar{\phi}_d^{ULL}$ and $\bar{\phi}_d^{LL}$ are, respectively, defined as,

$$\bar{\phi}_u^{ULL} = k_u^{ULL} - N_0,$$

$$\bar{\phi}_u^{LL} = k_u^{LL} - N_0,$$

Figure 8: g(σ_z, N_0)=0 keeping fixed $\{\beta,\ \mu_z,\ \alpha\}$



$$\bar{\phi}_d^{ULL} = k_d^{ULL} - N_0,$$

$$\bar{\phi}_d^{ULL} = k_d^{LL} - N_0,$$

where,

$$k_u^{ULL} = \left(\frac{\alpha\bar{Z}}{R_0^F}\right)^{\frac{1}{1-\alpha}}, k_u^{LL} = \left(\frac{\alpha\bar{Z}}{R_0^F - \theta_u}\right)^{\frac{1}{1-\alpha}}, k_d^{ULL} = \left(\frac{\bar{Z}\alpha}{R_0^B}\right)^{\frac{1}{1-\alpha}}, k_d^{LL} = \left(\frac{\bar{Z}\alpha}{R_0^B - \theta_d}\right)^{\frac{1}{1-\alpha}},$$

$$\theta_u = \int_0^{Z_u^*} \left(R_0^F - \alpha Z_1 (k_u^{LL})^{\alpha-1}\right) dF(Z_1) > 0, \qquad Z_u^*(k_u^{LL})^\alpha = R_0^F(k_u^{LL} - N_0),$$

$$\theta_d = \int_0^{Z_d^*} \left(R_0^B - \alpha Z_1 (k_d^{LL})^{\alpha-1}\right) dF(Z_1), \qquad Z_d^*(k_d^{LL})^\alpha = R_0^B(k_d^{LL} - N_0) - (R_0^B - R_0^F)\phi,$$

I Domestic welfare gains from opening the economy under unlimited liability with non-binding constraint

The society welfare is measured by the household's utility. As it was shown in Appendix C, in a closed economy model society welfare, $U_0^{ULL,C}$, is,

$$Y_0 + \beta\left(\bar{Z}\left(K_0^{ULL,C}\right)^\alpha - R_0^B D_0^{ULL,C}\right) = Y_0 + \beta\left[\bar{Z}\left(\frac{\alpha\bar{Z}}{R_0^B}\right)^{\frac{\alpha}{1-\alpha}} - R_0^B\left(\left(\frac{\alpha\bar{Z}}{R_0^B}\right)^{\frac{1}{1-\alpha}} - N_0\right)\right]$$

Similarly, in an open economy model, the domestic welfare, $U_0^{ULL,O}$, is,

$$Y_0 + \beta\left(\bar{Z}\left(K_0^{ULL,O}\right)^\alpha - R_0^F D_0^{F,ULL,O}\right) = Y_0 + \beta\left[\bar{Z}\left(\frac{\alpha\bar{Z}}{R_0^F}\right)^{\frac{\alpha}{1-\alpha}} - R_0^F\left(\left(\frac{\alpha\bar{Z}}{R_0^F}\right)^{\frac{1}{1-\alpha}} - N_0\right)\right],$$

where the superscript C and O refer to a closed and open economy, respectively. Then,

$$U_0^{ULL,O} - U_0^{ULL,C} = \beta\left[\bar{Z}\left(\left(\frac{\alpha\bar{Z}}{R_0^F}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{\alpha\bar{Z}}{R_0^B}\right)^{\frac{\alpha}{1-\alpha}}\right) - R_0^F\left(\frac{\alpha\bar{Z}}{R_0^F}\right)^{\frac{1}{1-\alpha}} + R_0^B\left(\frac{\alpha\bar{Z}}{R_0^B}\right)^{\frac{1}{1-\alpha}}\right]$$

Solving,

$$U_0^{ULL,O} - U_0^{ULL,C} = \left(\frac{\alpha\bar{Z}}{R_0^B}\right)^{\frac{1}{1-\alpha}} \left[\left(R_0^F\beta\right)^{\frac{-\alpha}{1-\alpha}} - 1\right]\left[\frac{1}{\alpha} - 1\right].$$

Since by assumption $R_0^F\beta<1$, $U_0^{ULL,O}-U_0^{ULL,C}>0$.

J Open economy and non-binding constraint: limited liability and risky bank debt

First, since foreign debt is cheap and banks do not have a borrowing limit on it, they will use only foreign debt (in addition to the initial exogenous equity) to fund their lending activities.

As in a closed economy, if there is not deposit insurance, the foreign interest rate is risk-sensitive. Then, the supply curve of foreign deposits faced by the bank is,

$$R_0^F = \mathbb{E}_0\{x_1^F\}\bar{R}_0^F,$$

where,

$$x_1^F = min\left\{1, \frac{Z_1K_0^\alpha}{\bar{R}_0^F D_0^F}\right\}.$$

The objective function for the bank facing limited liability and not deposit insurance is

$$V_0 = \mathbb{E}_0\{\beta(max\{0, Z_1K_0^\alpha - \bar{R}_0^F D_0\})\},$$

which can be rewritten as,

$$V_0 = \int_{Z^*}^{+\infty} \beta(Z_1K_0^\alpha - \bar{R}_0^F D_0)dF(Z_1),$$

I can rewrite $R_0^F = E_0\{x_1^F\}\bar{R}_0^F$ as,

$$R_0^F = \int_0^{Z^*} \frac{Z_1K_0^\alpha}{\bar{R}_0^F D_0^F}\bar{R}_0^F dF(Z_1) + \int_{Z^*}^{+\infty} \bar{R}_0^F dF(Z_1). \tag{39}$$

Hence, bank maximizes V_0 subject to the supply curve of foreign deposits, (39). Inserting (39) into V_0,

$$V_0 = \int_{Z^*}^{+\infty} \beta Z_1K_0^\alpha dF(Z_1) + \beta\left(\int_0^{Z^*} \frac{Z_1K_0^\alpha}{\bar{R}_0^F D_0^F}\bar{R}_0^F dF(Z_1) - R_0^F\right)D_0^F,$$

and solving,

$$V_0 = \mathbb{E}_0\{\beta(Z_1K_0^\alpha - R_0^F D_0^F)\},$$

which looks as the objective function of the bank under unlimited liability. Hence, the allocation under unlimited liability is going to be the same to the allocation under limited liability in the absence of deposit insurance.

K Open economy and non-binding constraint: domestic welfare losses

Domestic welfare is given by the households' utility,

$$U_0 = Y_0 - D_0 + \beta \mathbb{E}_0\{R_0^B D_0 + \Pi_1 - T_1\}.$$

Under unlimited liability $T_1^{ULL}{=}0 \ \forall \ Z_1$ and $\Pi_1^{ULL}{=}N_1^{ULL}{=}Z_1 K_0^{ULL}{-}R_0^F D_0^{F,ULL}$. Hence, U_0 can be rewritten as,

$$U_0^{ULL} = Y_0 + \beta \bar{Z}(K_0^{ULL})^\alpha - \beta R_0^F D_0^{F,ULL}.$$

Under limited liability $T_1^{LL}{=}max\{0, R_0^F D_0^{F,LL}{-}Z_1(K_0^{LL})^\alpha\}$
and $\Pi_1^{LL}{=}N_1^{LL}{=}max\{0, Z_1(K_0^{LL})^\alpha{-}R_0^F D_0^{F,LL}\}$. Hence, U_0 can be rewritten as,

$$U_0^{ULL} = Y_0 + \beta \bar{Z}(K_0^{LL})^\alpha - \beta R_0^F D_0^{F,LL}.$$

Hence, the welfare losses are,

$$WL_0 = U_0^{ULL} - U_0^{LL} = \beta \bar{Z}\left[\left(K_0^{ULL}\right)^\alpha - \left(K_0^{LL}\right)^\alpha\right] - \beta R_0^F (D_0^{F,ULL} - D_0^{F,LL}).$$

Since $D_0^F = K_0 - N_0$,

$$WL_0 = U_0^{ULL} - U_0^{LL} = \beta \bar{Z}\left[\left(K_0^{ULL}\right)^\alpha - \left(K_0^{LL}\right)^\alpha\right] - \beta R_0^F (K_0^{ULL} - K_0^{LL}).$$

Recalling the expressions for K_0^{ULL} and K_0^{LL}, WL_0 is rewritten as,

$$WL_0 = \beta \bar{Z}\left[\left(\frac{\alpha \bar{Z}}{R_0^F}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{\alpha \bar{Z}}{R_0^F - \theta_0}\right)^{\frac{\alpha}{1-\alpha}}\right] - \beta R_0^F \left[\left(\frac{\alpha \bar{Z}}{R_0^F}\right)^{\frac{1}{1-\alpha}} - \left(\frac{\alpha \bar{Z}}{R_0^F - \theta_0}\right)^{\frac{1}{1-\alpha}}\right].$$

Then,

$$WL_0 = \left(\frac{\alpha \bar{Z}}{R_0^F}\right)^{\frac{1}{1-\alpha}} \left\{\beta \bar{Z}\left[\frac{R_0^F}{\alpha \bar{Z}} - \frac{R_0^F - \theta_0}{\alpha \bar{Z}}\left(\frac{R_0^F}{R_0^F - \theta_0}\right)^{\frac{1}{1-\alpha}}\right] - \beta R_0^F \left[1 - \left(\frac{R_0^B}{R_0^B - \theta_0}\right)^{\frac{1}{1-\alpha}}\right]\right\},$$

solving,

$$WL_0 = \left(\frac{\alpha\bar{Z}}{R_0^F}\right)^{\frac{1}{1-\alpha}} \left\{\frac{\beta R_0^F}{\alpha} - \frac{R_0^F - \theta_0}{\alpha R_0^B}\left(\frac{R_0^F}{R_0^F - \theta_0}\right)^{\frac{1}{1-\alpha}} - \beta R_0^F + \beta R_0^F \left(\frac{R_0^F}{R_0^F - \theta_0}\right)^{\frac{1}{1-\alpha}}\right\}.$$

Finally,

$$WL_0 = \left(\frac{\alpha\bar{Z}}{R_0^F}\right)^{\frac{1}{1-\alpha}} \beta R_0^F \left\{\left(\frac{1}{\alpha} - 1\right) - \left(\frac{1}{\alpha}y^\alpha - y\right)\right\},$$

where $y{=}\left(\frac{R_0^F}{R_0^F - \theta_0}\right)^{\frac{1}{1-\alpha}}$. Hence, it is easy to see that the y that maximizes $\left(\frac{1}{\alpha}y^\alpha - y\right)$ and hence minimizes WL_0 is $y{=}1$. In particular, $WL_0{=}0$ at $y{=}1$. Therefore, since in equilibrium $y{>}1$, then $\left(\frac{1}{\alpha} - 1\right){-}\left(\frac{1}{\alpha}y^\alpha - y\right){>}0$ and then $WL_0{>}0$.

L Open economy and binding constraint: limited liability and risky bank debt

Since domestic and foreign depositors are risk-neutral and in the absence of the deposit insurance, they require gross returns for the domestic and foreign deposits respectively which satisfy the following conditions,

$$R_0^B D_0 = \mathbb{E}_0\{x_1^D \bar{R}_0^D\} D_0, \tag{40}$$

$$R_0^F D_0^F = \mathbb{E}_0\{x_1^F \bar{R}_0^F\} D_0^F. \tag{41}$$

They say that $\bar{R}_0^D$ and $\bar{R}_0^F$ has to be high enough to compensate the reduced payment when the bank defaults and that the expected payment for domestic and foreign investors need to be equal to their corresponding opportunity costs, R_0^B and R_0^F, respectively, where x_1^D and x_1^F are the domestic and foreign endogenous recovery ratios, respectively. If the bank defaults, x_1^D and x_1^F have to satisfy,

$$0 = Z_1 K_0^\alpha - x_1^D \bar{R}_0^D D_0 - x_1^F \bar{R}_0^F D_0^F, \tag{42}$$

Clearly, if the bank does not default, $x_1^D{=}x_1^F{=}1$. Hence, $x_1^D \bar{R}_0^D$ and $x_1^F \bar{R}_0^F$ represent the effective gross return of domestic and foreign deposits, respectively. The bank's objective function under limited liability in the absence of deposit insurance is,

$$V_0 = \mathbb{E}_0\{\beta(max\{0, Z_1 K_0^\alpha - \bar{R}_0^D D_0 - \bar{R}_0^F D_0^F\})\},$$

which can be rewritten as,

$$V_0 = \int_{Z^*}^{+\infty} \beta(Z_1 K_0^\alpha - \bar{R}_0^D D_0 - \bar{R}_0^F D_0^F) dF(Z_1).$$

Recall bank seeks to maximize V_0 subject to the supply curve of domestic and foreign deposits, equations (40) and (41), respectively. From (40) and (41), I can obtain,

$$\int_{Z^*}^{+\infty}(\bar{R}_0^D D_0 + \bar{R}_0^F D_0^F)dF(Z_1) = \int_0^{Z^*}(x_1^D \bar{R}_0^D D_0 + x_1^F \bar{R}_0^F D_0^F)dF(Z_1) - R_0^B D_0 - R_0^F D_0^F. \tag{43}$$

For convenience, I rewrite V_0 as,

$$V_0 = \int_{Z^*}^{+\infty}\beta Z_1 K_0^\alpha dF(Z_1) - \beta\int_{Z^*}^{+\infty}(\bar{R}_0^D D_0 + \bar{R}_0^F D_0^F)dF(Z_1). \tag{44}$$

Inserting (43) into (44) yields,

$$V_0 = \int_{Z^*}^{+\infty}\beta Z_1 K_0^\alpha dF(Z_1) - \beta\left[\int_0^{Z^*}(x_1^D \bar{R}_0^D D_0 + x_1^F \bar{R}_0^F D_0^F)dF(Z_1) - R_0^B D_0 - R_0^F D_0^F\right]. \tag{45}$$

Inserting (42) into (45) results in,

$$V_0 = \mathbb{E}_0\{\beta(Z_1 K_0^\alpha - R_0^B D_0 - R_0^F D_0^F)\},$$

which looks as the objective function of the bank under unlimited liability. Hence, the optimality condition is going lead to the efficient allocation.

Note that this argument holds for any assumption regarding the structure of the recovery ratios, x_1^D and x_1^F. In other words, the argument holds for any assumption about the seniority of the domestic and foreign deposits.

M Open economy and binding constraint: domestic welfare losses

Domestic welfare is given by the households' utility,

$$U_0 = Y_0 - D_0 + \beta\mathbb{E}_0\{R_0^B D_0 + \Pi_1 + T_1\}.$$

Under unlimited liability $T_1^{ULL}=0\ \forall\ Z_1$ and $\Pi_1^{ULL}=N_1^{ULL}=Z_1 K_0^{ULL}-R_0^B D_0^{ULL}-R_0^F D_0^{F,ULL}$. Hence, U_0 can be rewritten as,

$$U_0^{ULL} = Y_0 + \beta\bar{Z}(K_0^{ULL})^\alpha - D_0^{ULL} - \beta R_0^F D_0^{F,ULL}.$$

Under limited liability $T_1^{LL}=max\{0, R_0^B D_0^{LL}+R_0^F D_0^{F,LL}-Z_1(K_0^{LL})^\alpha\}$
and $\Pi_1^{LL}=N_1^{LL}=max\{0, Z_1(K_0^{LL})^\alpha-R_0^B D_0^{LL}-R_0^F D_0^{F,LL}\}$. Hence, U_0 can be rewritten as,

$$U_0^{ULL} = Y_0 + \beta\bar{Z}(K_0^{LL})^\alpha - D_0^{LL} - \beta R_0^F D_0^{F,LL}.$$

Since $D_0^{F,LL}$=$D_0^{F,ULL}$ =ϕ and D_0=K_0−ϕ−N_0, the welfare losses are,

$$WL_0 = U_0^{ULL} - U_0^{LL} = \beta \bar{Z} \left[\left(K_0^{ULL} \right)^{\alpha} - \left(K_0^{LL} \right)^{\alpha} \right] - \left(K_0^{ULL} - K_0^{LL} \right).$$

Recalling the expressions of K_0^{ULL} and K_0^{LL}, WL_0 is rewritten as,

$$WL_0 = \beta \bar{Z} \left[\left(\frac{\alpha \bar{Z}}{R_0^B} \right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{\alpha \bar{Z}}{R_0^B - \theta_0} \right)^{\frac{\alpha}{1-\alpha}} \right] - \left[\left(\frac{\alpha \bar{Z}}{R_0^B} \right)^{\frac{1}{1-\alpha}} - \left(\frac{\alpha \bar{Z}}{R_0^B - \theta_0} \right)^{\frac{1}{1-\alpha}} \right].$$

Then,

$$WL_0 = \left(\frac{\alpha \bar{Z}}{R_0^B} \right)^{\frac{1}{1-\alpha}} \left\{ \beta \bar{Z} \left[\frac{R_0^B}{\alpha \bar{Z}} - \frac{R_0^B - \theta_0}{\alpha \bar{Z}} \left(\frac{R_0^B}{R_0^B - \theta_0} \right)^{\frac{1}{1-\alpha}} \right] - \left[1 - \left(\frac{R_0^B}{R_0^B - \theta_0} \right)^{\frac{1}{1-\alpha}} \right] \right\},$$

solving,

$$WL_0 = \left(\frac{\alpha \bar{Z}}{R_0^B} \right)^{\frac{1}{1-\alpha}} \left\{ \frac{1}{\alpha} - \frac{R_0^B - \theta_0}{\alpha R_0^B} \left(\frac{R_0^B}{R_0^B - \theta_0} \right)^{\frac{1}{1-\alpha}} - 1 + \left(\frac{R_0^B}{R_0^B - \theta_0} \right)^{\frac{1}{1-\alpha}} \right\}.$$

Finally,

$$WL_0 = \left(\frac{\alpha \bar{Z}}{R_0^B} \right)^{\frac{1}{1-\alpha}} \left\{ \left(\frac{1}{\alpha} - 1 \right) - \left(\frac{1}{\alpha} y^{\alpha} - y \right) \right\},$$

where $y = \left(\frac{R_0^B}{R_0^B - \theta_0} \right)^{\frac{1}{1-\alpha}}$. Hence, it is easy to see that the y that maximizes $\left(\frac{1}{\alpha} y^{\alpha} - y \right)$ and hence minimizes WL_0 is y=1. In particular, WL_0=0 at y=1. Therefore, since in equilibrium y>1, then $\left(\frac{1}{\alpha} - 1 \right)$−$\left(\frac{1}{\alpha} y^{\alpha} - y \right) > 0$ and then WL_0>0.

N Elasticities

Figure 9: Changes of ϕ

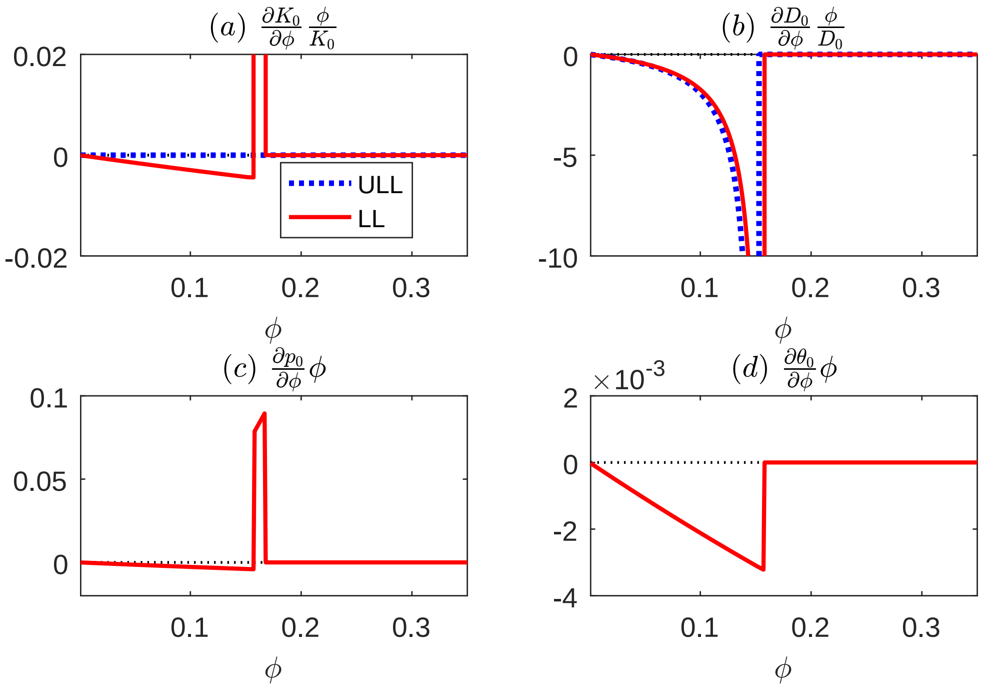


Figure 10: Changes of the risk-free foreign rate

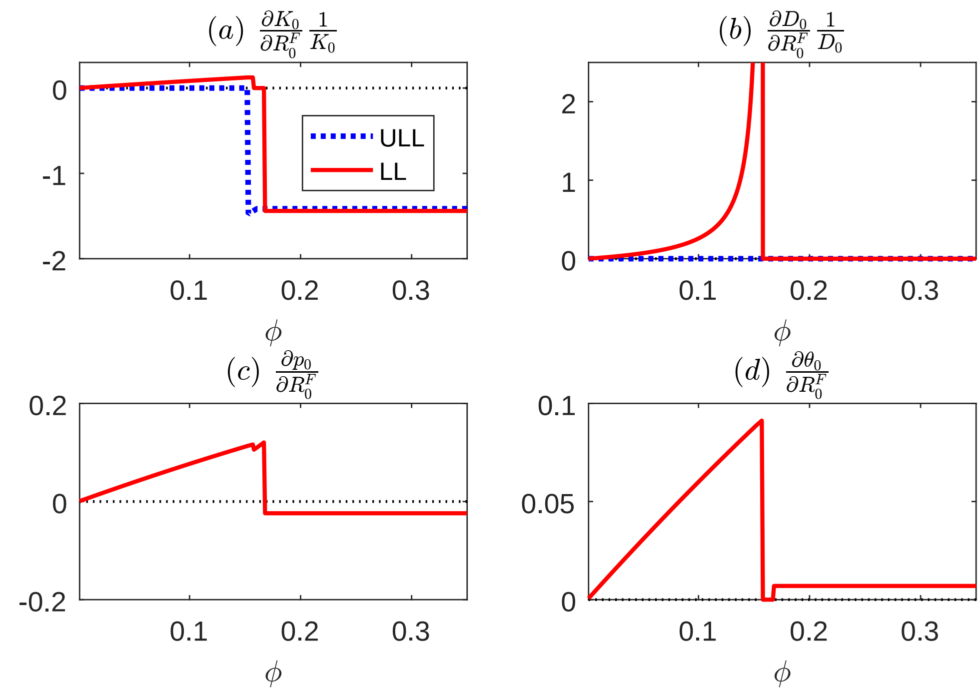
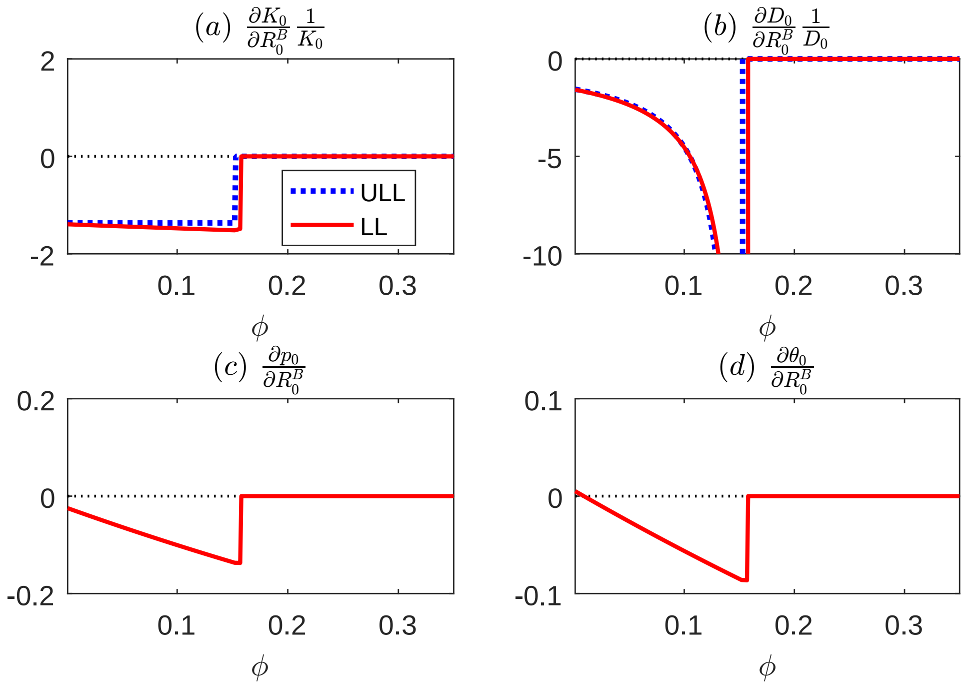


Figure 11: Changes of the risk-free domestic rate

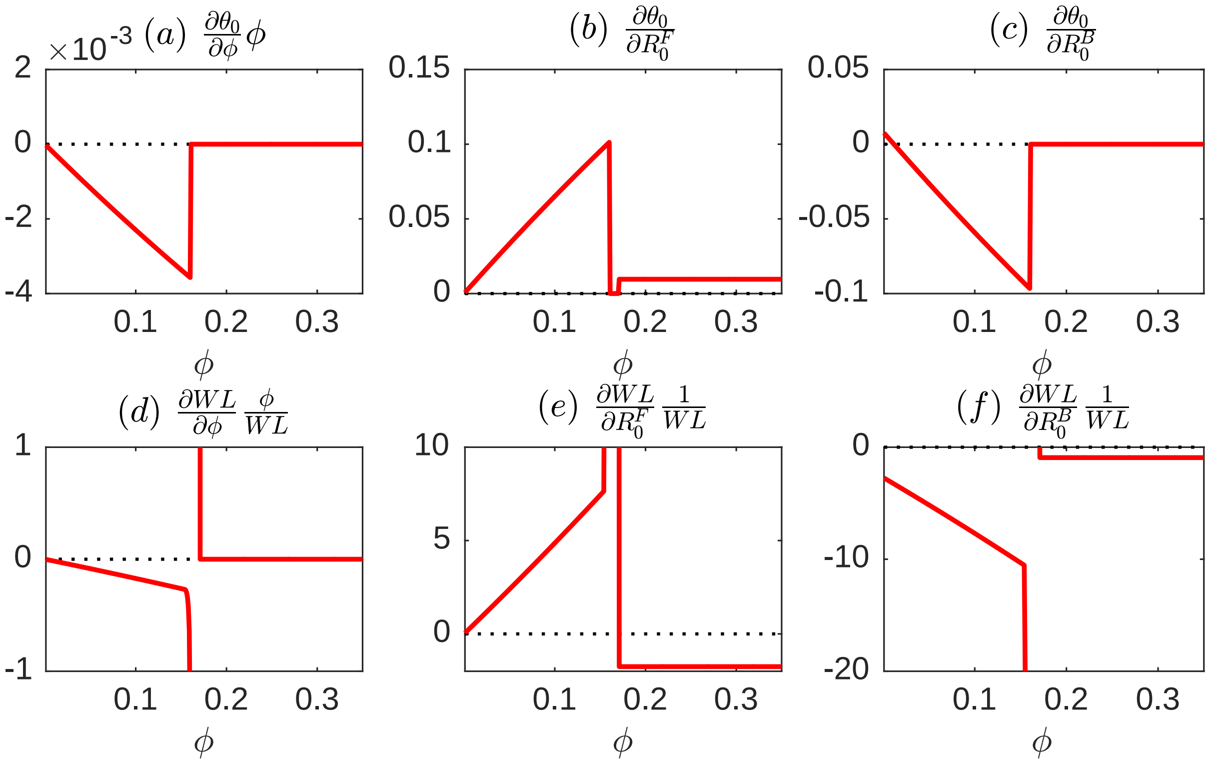


O Welfare effects

Intuitively, it is expected that the higher the inefficient additional marginal benefits, θ_0, or the excessive bank risk-taking, the higher the welfare losses, WL_0. As figure 12 shows this argument is true for the calibration presented here. Hence, a lower foreign rate, a higher domestic rate and a higher ϕ reduces θ_0 and domestic welfare losses, WL_0.

However, from figure 12 it can be seen that for some calibrations this argument does not necessarily hold. For instance, when ϕ is high enough (non-binding foreign constraint) a lower foreign debt decreases the excessive bank risk-taking measure, θ_0, but increases the domestic welfare losses, and a lower domestic rate will have no effect on θ_0 since domestic debt is zero, but will increase welfare losses since the discounted factor is higher. When ϕ=0 (closed economy), a lower domestic rate decreases θ_0, but welfare losses increases.

Figure 12: Effects on Welfare Losses



P Robustness

Here, I incorporate firms to the model by assuming that banks lend to them who then use bank loans to buy capital. At $t{=}0$ firms buy capital, K_0, funded by bank loans, L_0, that are demanded to banks, $L_0{=}K_0$. Firms use capital and labor, H_1, demanded at $t{=}1$ for the production of goods using a Cobb-Douglas technology,

$$Y_1 = Z_1(K_0)^{\alpha}(H_1)^{1-\alpha},$$

where $0{<}\alpha{<}1$ and Z_1 is the productivity level, which is known at $t{=}1$ and has a log normal distribution as in the original specification. Firm profits at $t{=}1$ are,

$$\Pi_1 = (1-\delta)K_0 + Y_1 - R_1^L L_0 - W_1 H_1,$$

where $\delta{<}1$ is the capital depreciation rate, R_1^L is the state-contingent lending interest rate. Since there is an infinite number of firms, they take prices as given. The demand of loans of firms is found by maximizing the discounted value of future profits at $t{=}0$,

$V_0 = \mathbb{E}_0\{\beta\Pi_1\}$, where $L_0 = K_0$.[43] The first order condition for K_0 is,

$$0 = \mathbb{E}_0\left\{\beta(R_1^K - R_1^L)\right\},$$

where $R_1^K = 1 - \delta + \alpha Z_1(K_0)^{\alpha-1}(H_1)^{1-\alpha}$ is the marginal productivity of capital. At $t=1$ the productivity level is realized and firms demand labor until the marginal product of labor equals the wage, i.e., $W_1 = (1-\alpha)Z_1(K_0)^{\alpha}(H_1)^{-\alpha}$. I assume a non-negative condition for the realized profits, i.e., $0 \leq \Pi_1$. This implies that the foc for K_0 yields $R_1^L = R_1^K$. I assume households supply inelastically one unit of labor. Then, in equilibrium the lending rate yields,

$$R_1^L = 1 - \delta + \alpha Z_1(K_0)^{\alpha-1}.$$

Final equity of banks is given now by,

$$N_1 = max\{0, R_1^L K_0 - R_0^B D_0\}.$$

It is easy to verify that the first order condition for D_0, equation (22), now yields,

$$\beta(R_1^L - R_0^B) + \beta\int_0^{Z^*}(R_0^B - R_1^L)dF(Z_1) = 0.$$

I rewrite it as,

$$\beta(1 - \delta + \bar{Z}\alpha K_0^{\alpha-1} - R_0^B + \theta_0) = 0, \tag{46}$$

where,

$$\theta_0 = \int_0^{Z^*}(R_0^B - (1-\delta) - Z_1\alpha K_0^{\alpha-1})dF(Z_1),$$

and Z^* is solved in,

$$0 = (1-\delta)K_0 + \alpha Z^* K_0^{\alpha} - R_0^B D_0 - R_0^F D_0^F. \tag{47}$$

Compared to the equilibrium condition in the original specification, this is different in three aspects:

1. According to equation (46), ceteris paribus the non full depreciation rate positively affects the marginal return of capital.

2. Ceteris paribus the undepreciated capital, $(1-\delta)K_0$, increases bank profits and hence according to equation (47) reduces bank default probability and thus the size of the inefficiencies. This implies that in order to avoid a reduction in the default probability it is required a higher uncertainty on Z_1, i.e., a larger σ_z.

[43] Since firms are owned by risk-neutral domestic households, I multiply Π_1 by the impatient parameter.

3. Risky revenues of bank are reduced to a fraction α. In other words, due to the presence of firms, only a fraction α of total production, $\alpha Z_1(K_0)^{\alpha}$, goes to banks as loan payments (interest and principal), while a fraction $1-\alpha$, $(1-\alpha)Z_1(K_0)^{\alpha}$, goes to households as wages. Then, according to the second term of equation (47), Z^* becomes more sensitive to movements in bank obligations (caused by changes of R_0^F, D_o^F or R_0^B). In other words, since bank revenues are less sensitive to the risky revenues, it is required a stronger change of Z^* so that total bank revenues, $(1-\delta)K_0+\alpha Z^* K_0^{\alpha}$, vary enough to compensate bank obligations movement. Thus, ceteris paribus a reduction of bank obligations is now accompanied by a stronger reduction of the bank default probability than in the original specification.

Table 5 reports the some parameter values for the original specification (CE). Only these parameters are re-calibrated such that the economy with firms and non full capital depreciation (denoted by CE*) also matches the same targets as in the original specification. The re-calibrated σ_z is relatively high, as suggested before, and the re-calibrated α is very small. In order to assess the implications of having a small α, the recalibration procedure in CE** (that also represents an economy with firms and non full capital depreciation) is as in CE* but without matching the credit to GDP ratio and so α is not re-calibrated. Capital depreciation rate is set to 8%.[44]

The table also reports the relative excess loans, which is relatively similar in all specifications. The inefficient additional marginal benefits of capital are significantly smaller. This is because the presence of the δ=0.08, makes capital more sensitive to changes in its marginal productivity and hence on the marginal cost of capital. As a result, in equilibrium it is needed a relatively small inefficient marginal benefit in order to produce a 3% default probability.

[44]From Penn World Table version 9.1 I find an average capital depreciation rate of 4.3% with a standard deviation of 1.3% for 182 countries between 1950 and 2017. In order to ensure a positive default probability, in the worst state of nature bank revenues must be smaller than bank obligations, i.e., $\delta>1-R_0^B+(R^B-R_0^F)(1/(1/For\_share-1))(1/(Lev)+R_0^B(1/Lev)$, where For_share= foreign debt to total bank debt ratio and Lev=bank loans to bank net worth ratio. This yields a lower bound for δ. Since the lower bound results higher that the 4.3% observed in the data, the δ is calibrated a bit higher.

Table 5: Robustness exercise

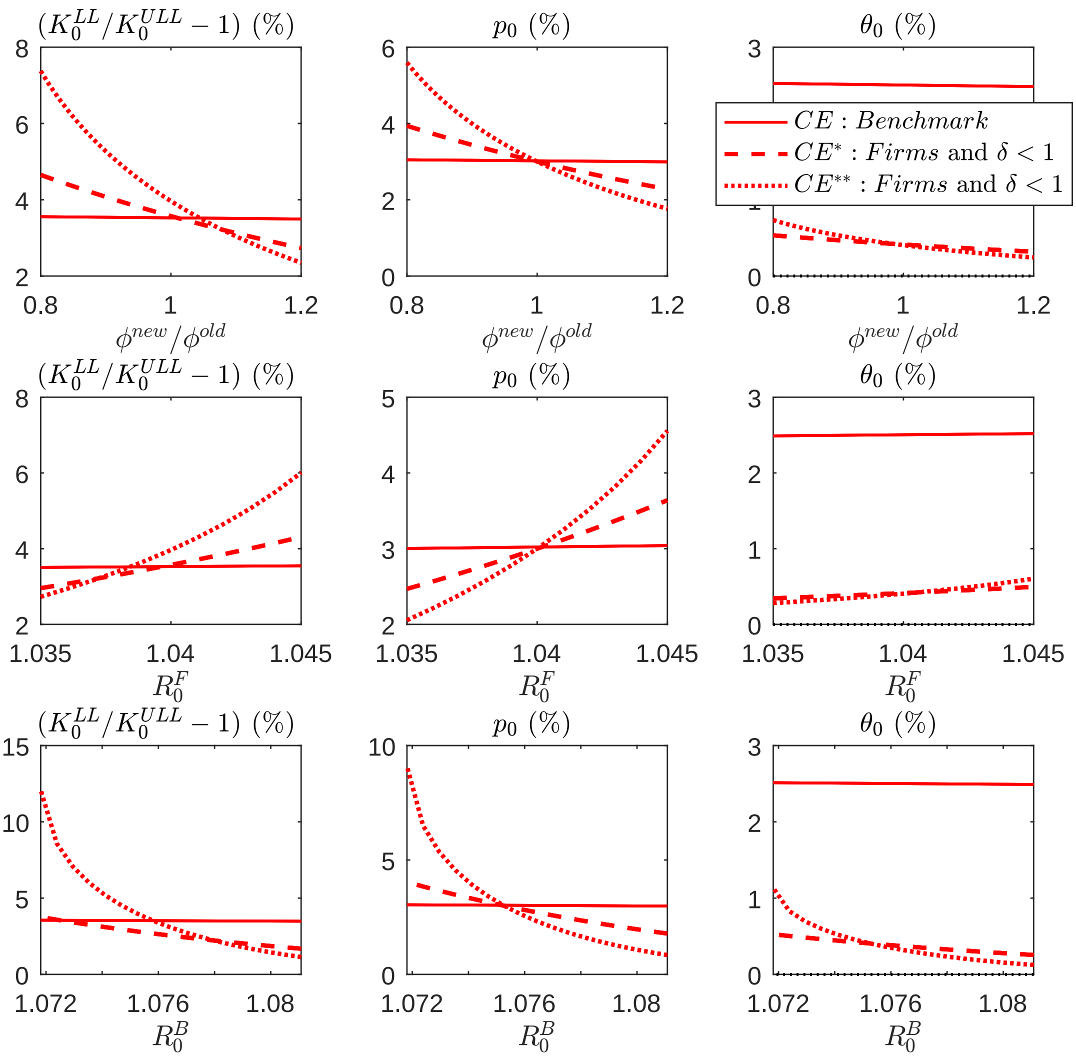
Description		CE (1)	CE* (2)	CE** (3)
Parameters set to match the data				
Capital depreciation rate	δ	1.00	0.08	0.08
Capital's share in output	α	0.32	0.05	0.32
Foreign borrowing limit	ϕ	0.048	0.10	1.00
Initial bank equity	N_0	0.02	0.04	0.42
Std. Dev. of log Z_1	σ_z	0.58	0.77	0.77
Relative excess loans	$\frac{K_0^{LL}}{K_0^{ULL}}$-1	3.53%	3.57%	3.96%
Inefficient additional MB	θ_0	2.5%	0.4%	0.4%
Credit to GDP ratio	$K_0/(\bar{Z}K_0)^{\alpha}$	30.4%	30.4%	211.6%

MB: marginal benefit. CE: the benchmark economy. In both cases (CE* and CE**) I assume there are firms and non full capital depreciation rate. The only difference is that in CE** I do not match the targeted credit to GDP ratio and hence α=0.32 as in CE.

According to figure 13, as expected, in any case (CE* and CE**) the responses of the relative excess loans (or capital) are significantly amplified. In the case of CE*, a 20% increase of the foreign borrowing limit reduces the relative excess loans from 3.57% to 2.73%, while in the benchmark economy it moves from 3.53% to 3.50%. Also, a 50 bps reduction of the foreign interest rate decreases the relative excess loans from 3.57% to 2.95%, while in the benchmark economy it moves from 3.53% to 3.50%. And, a 50 bps reduction of the domestic interest rate increases the relative excess loans from 3.57% to 3.76%, while in the benchmark economy it moves from 3.53% to 3.55%.

Note that in CE**, α is higher than in CE*, but responses of relative excess loans are larger. This is because in CE** the credit-to-GDP is substantially larger leading to a smaller risky bank revenues, $\alpha Z_1(K_0)^{\alpha}$, to total revenues, $(1-\delta)K_0 - \alpha Z_1(K_0)^{\alpha}$, ratio than in CE*, which in turn drives larger responses.

Figure 13: Robustness exercise



ϕ^{new}/ϕ^{old} : ϕ^{old} does not change, while ϕ^{new} changes, for instance a value of 1.2 means a 20% increase of ϕ.

Q Excessive leverage level

Recall that,

$$\tau_0 = \frac{K_0^{LL}}{N_0} - \frac{K_0^{ULL}}{N_0} = \frac{1}{N_0}\left(\left(\frac{\alpha\bar{Z}}{R_0^B - \theta_0}\right)^{\frac{1}{1-\alpha}} - \left(\frac{\alpha\bar{Z}}{R_0^B}\right)^{\frac{1}{1-\alpha}}\right).$$

Taking the partial derivative of τ_0 with respect to R_0^B,

$$\frac{\partial\tau_0}{\partial R_0^B} = \frac{1}{N_0}\left\{\frac{1}{1-\alpha}\left(\frac{\alpha\bar{Z}}{R_0^B - \theta_0}\right)^{\frac{\alpha}{1-\alpha}}\frac{\alpha\bar{Z}}{(R_0^B - \theta_0)^2}\left(1 - \frac{\partial\theta_0}{\partial R_0^B}\right) - \frac{1}{1-\alpha}\left(\frac{\alpha\bar{Z}}{R_0^B}\right)^{\frac{\alpha}{1-\alpha}}\frac{\alpha\bar{Z}}{(R_0^B)^2}\right\}.$$

Solving,

$$\begin{aligned}\frac{\partial\tau_0}{\partial R_0^B} &= \frac{1}{N_0}\left\{\frac{1}{1-\alpha}\left(\frac{\alpha\bar{Z}}{R_0^B-\theta_0}\right)^{\frac{\alpha}{1-\alpha}}\frac{\alpha\bar{Z}}{(R_0^B-\theta_0)^2}\frac{\partial\theta_0}{\partial R_0^B}\right.\\ &- \left.\frac{(\alpha\bar{Z})^{\frac{1}{1-\alpha}}}{1-\alpha}\left(\frac{1}{(R_0^B-\theta_0)^{\frac{2-\alpha}{1-\alpha}}}-\frac{1}{(R_0^B)^{\frac{2-\alpha}{1-\alpha}}}\right)\right\}.\end{aligned}$$

Since under the calibration presented here $\frac{\partial\theta_0}{\partial R_0^B}<0$, then $\frac{\partial\tau_0}{\partial R_0^B}<0$. Taking the partial derivative of τ_0 with respect to R_0^F,

$$\frac{\partial\tau_0}{\partial R_0^F}=\frac{1}{N_0(1-\alpha)}\left(\frac{\alpha\bar{Z}}{R_0^B-\theta_0}\right)^{\frac{1}{1-\alpha}}\left(\frac{1}{R_0^B-\theta_0}\right)\frac{\partial\theta_0}{\partial R_0^F}.$$

Since under the calibration presented here $\frac{\partial\theta_0}{\partial R_0^F}>0$, then $\frac{\partial\tau_0}{\partial R_0^F}>0$. Taking the partial derivative of τ_0 with respect to ϕ,

$$\frac{\partial\tau_0}{\partial\phi}=\frac{1}{N_0(1-\alpha)}\left(\frac{\alpha\bar{Z}}{R_0^B-\theta_0}\right)^{\frac{1}{1-\alpha}}\left(\frac{1}{R_0^B-\theta_0}\right)\frac{\partial\theta_0}{\partial\phi}.$$

Since under the calibration presented here $\frac{\partial\theta_0}{\partial\phi}<0$, then $\frac{\partial\tau_0}{\partial\phi}<0$. Note: In a closed economy with fully leveraged banks, i.e N_0=0, since $\frac{\partial\theta_0}{\partial R_0^B}=\frac{\theta_0}{R_0^B}$, it is easy to see that $\frac{\partial\tau_0}{\partial R_0^B}>0$.

R A simple model of the excessive bank risk-taking

Here, I present a simple model where bank risk-taking involves the type rather than the volume of credit.

Description and assumptions: The excessive bank risk-taking is due to the presence of limited liability and deposit insurance. Since interest rates are risk-insensitive, the banks cannot manipulate the return of domestic deposits. Safe domestic assets have a fixed rate of returns of R_0^B-1, such as domestic government savings bonds; and safe foreign assets have a fixed rate of returns of R_0^F-1, such as foreign government savings bonds. Business loans pay a target rate, q-1, if the business succeeds with probability $p(q)$, where $p'(q)<0$, but pay no return and incur the total loss of capital if the business fails. R_0^B and R_0^F are exogenous and I assume $R_0^F<R_0^B$, but q is explained endogenously. Banks face an inelastic demand for funds, K_0, from private firms. N_0 is the exogenous initial bank equity. K_0 is funded by domestic deposits, D_0, foreign debt, D_0^F, and the exogenous equity, i.e., K_0=D_0+D_0^F+N_0. Banks a foreign borrowing limit, $D_0^F\leq\phi$, where $\phi>0$ is low enough to make this binding. Since $R_0^F<R_0^B$, in equilibrium it must be that $D_0^F=\phi$ and $D_0=K_0-\phi-N_0$. The expected profit of the bank choosing a project with a

target return of size q is,

$$V_0 = \beta(p(q)qK_0 - R_0^B D_0 - R_0^F D^F).$$

Under unlimited liability, the first order condition for q is,

$$(p'(q)q + p(q))K_0 = 0.$$

Under limited liability, the expected bank profits become,

$$V_0 = \mathbb{E}_0\{\beta max(qK_0 - R_0^B D_0 - R_0^F D_0^F, 0)\},$$

which can be rewritten as,

$$V_0 = \beta \left(p(q)qK_0 - R_0^B D_0 - R_0^F D_0^F + (1 - p(q))(R_0^B D_0 + R_0^F D_0^F)\right),$$

and thus the first order condition for q is,

$$(p'(q)q + p(q))K_0 - p'(q)(R_0^B D_0 + R_0^F D_0^F) = 0.$$

Finally, considering $D_0^F{=}\phi$ and $D_0{=}K_0 - \phi - N_0$,

$$(p'(q)q + p(q))K_0 - p'(q)(R_0^B K_0 - (R_0^B - R_0^F)\phi - R_0^B N_0) = 0. \tag{48}$$

Clearly, under limited liability the risk choices are distorted. In particular, q is inefficiently high under limited liability since $-p'(q){>}0$.

The excessive bank risk-taking: The measure of risk-taking is given by q. Let be q^{ULL} and q^{LL} the measures of bank risk-taking under unlimited and limited liability respectively. Thus, the excessive bank risk-taking is given by $\theta_0{=}q^{LL} - q^{ULL}$.

Comparative statics: Taking the partial derivate to (48) with respect to R_0^F:

$$(p''q + p' + p')K_0\frac{\partial q}{\partial R_0^F} - p''(R_0^B K_0 - (R_0^B - R_0^F)\phi - R_0^B N_0)\frac{\partial q}{\partial R_0^F} - p'\phi = 0,$$

$$\left[p''q + 2p' - \frac{p''}{p'}(p'q + p)\right] K_0\frac{\partial q}{\partial R_0^F} - p'\phi = 0,$$

$$\frac{\partial q}{\partial R_0^F} = \frac{\phi}{K_0(2 - \frac{p''p}{(p')^2})},$$

The intuition is the following: From (48), on the one hand, a higher cost of funding, R_0^F, motivates banks to take more risk since they avoid to paid a greater amount of debt if they default; on the other hand, a riskier position might reduce, $-p'(q)$, i.e., it might reduce the marginal increment of the probability of default and thus reduces the benefits of inefficiently increasing the bank risk-taking. Clearly, if p is linear on q, this second effect is null and a higher R_0^F increases the bank risk taking, i.e., $\frac{\partial q}{\partial R_0^F} = \frac{\phi}{2K_0}$.

If $p(q)$ takes the form $p(q) = bq^{-\alpha}$, $b>0$ and $\alpha>0$, the this second effects is not null since $-p''(q)<0$. In this case, $\frac{p''p}{(p')^2} = 1 + \frac{1}{\alpha}$ and thus,

$$\frac{\partial q}{\partial R_0^F} = \frac{\phi}{2K_0\left(1 - \frac{1}{\alpha}\right)}. \tag{49}$$

If $\alpha>1$, a higher elasticity of p with respect to q (i.e., a higher α) increases the severity of the second effect. According to (49), if $\alpha>1$, $\frac{\partial q}{\partial R_0^F}>0$; otherwise, $\frac{\partial q}{\partial R_0^F}<0$.

In addition, it can be obtained that,

$$\frac{\partial \theta}{\partial R_0^F} = \frac{\partial q^{LL}}{\partial R_0^F} - \frac{\partial q^{ULL}}{\partial R_0^F} = \frac{\phi}{K_0}\frac{1}{2\left(1 - \frac{1}{\alpha}\right)},$$

and,

$$\frac{\partial \theta}{\partial R_0^B} = \frac{D_0}{K_0}\frac{1}{2\left(1 - \frac{1}{\alpha}\right)}, \qquad \frac{\partial \theta}{\partial \phi} = -\frac{\left(R_0^B - R_0^F\right)}{2K_0\left(1 - \frac{1}{\alpha}\right)},$$

since $\frac{\partial q^{ULL}}{\partial R_0^F}=\frac{\partial q^{ULL}}{\partial R_0^B}=\frac{\partial q^{ULL}}{\partial \phi}=0$. When $\alpha>1$, domestic and foreign interest rates have a positive effect on the excessive bank risk-taking, and this effect is stronger the higher the domestic deposits to capital ratio and foreign deposits to capital ratio, respectively. These results are in line with the results of the main model.