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Monetary policy operating procedures, lending frictions, and employment

David Florian[1], Chris Limnios[2] and Carl Walsh[3]

[1] Central Reserve Bank of Peru
[2] Providence College
[3] University of California, Santa Cruz

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Monetary policy operating procedures, lending frictions, and employment*

David Florian Hoyle
Central Reserve Bank of Peru

Chris Limnios
Providence College

Carl E. Walsh
University of California, Santa Cruz

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Abstract

This paper studies a channel system for implementing monetary policy when bank lending is subject to frictions. These frictions affect the spread between the interbank rate and the loan rate. We show how the width of the channel, the nature of random payment flows in the interbank market and the presence of frictions in the loan market affect the propagation of financial shocks that originate either in the interbank market or in the loan market. We study the transmission mechanism of two different financial shocks: 1) An increase in the volatility of the payment shock that banks face once the interbank market has closed and 2) An exogenous termination of loan contracts that directly affects the probability of continuation of credit relationships. Both financial shocks are propagated through the interaction of the marginal value of having excess reserves as collateral relative to other bank assets, the real marginal cost of labor for all active firms and the reservation productivity that selects the mass of producing firms. Our results suggest that financial shocks produce a reallocation of bank assets towards excess reserves as well as intensive and extensive margin effects over employment. The aggregation of those effects produce deep and prolonged recessions that are associated to fluctuations in the endogenous component of total factor productivity that appears as an additional input in the aggregate production function of the economy. We show that this wedge depends on aggregate credit conditions and on the mass of producing firms.

Keywords: *Monetary policy implementation, channel system, central bank, credit frictions.*

JEL Classification: *E4, G21*

*This paper is still considered preliminary work by the authors.

1 Introduction

How does the central bank's operating procedure affect the transmission process of monetary policy? In the 20 years prior to the financial crisis beginning in 2007, this question was little examined. With major central banks directly targeting the interbank interest rate, this single interest rate was viewed as the sole link between actions of the central bank and the real economy. And how the central bank managed discount borrowing and whether it paid interest on reserves were implicitly deemed irrelevant to understanding how changes in the target for the interbank rate affected real economic activity. This view was most explicit in standard new Keynesian models in which the policy interest rate was the sole interest rate appearing in the model and monetary aggregates, including bank reserves, could be ignored.

The financial crisis, the renewed recognition that financial markets are subject to frictions, the constraint imposed by the zero lower bound on the policy interest rate, and the adoption of new procedures for affecting reserve supply call for a reexamination of the links between the central bank's operating procedures in the interbank market, the availability of credit, and the impact of monetary policy on the real economy.

In this paper, we examine these links in a model in which banks hold reserves to meet random fluctuations in settlements, and the central bank pays interest on reserves, lends reserves at a penalty rate, and can independently affect the quantity of reserves and the level of interest rates. Banks make loans to firms in credit markets characterized by matching frictions, and interest rates on loans are set in bilateral bargaining between banks and firms.

The type of monetary policy operating procedure we analyze is often called a corridor or channel system of interest rate control. Such a system is employed by several central banks (e.g., the Reserve Bank of New Zealand) and is the type of system the U.S. Federal Reserve seems likely to employ when interest rates return to historically more normal levels. In a channel system, a central bank offers a lending facility, whereby commercial banks are permitted to borrow against collateral from the central bank at an interest rate that is above the target rate (the penalty or ceiling rate) and a deposit facility, whereby banks can earn overnight interest on their excess reserves at a rate that is below the target rate (the floor rate). The ceiling and floor rates form an interest rate channel (or corridor). In reality, many central banks use what is known as a symmetric channel system for monetary policy implementation, in which the ceiling and floor rates are the same number of basis points (the width) above and below the target rate. The symmetric channel systems used by various central banks differ in many respects. For example, the Bank of England and the ECB institute a relatively wide channel framework with a spread of 100 basis points on each side of the target. Australia and Canada, in contrast, operate narrow channels with a spread of only 25 basis points above and below their targets.[1]

[1] Australia and Canada have no reserve requirements.

There is a small existing literature on channel systems. Woodford (2000, 2001, 2003) discusses how to conduct monetary policy with a vanishing stock of money using the framework of a channel system. Whitesell (2006) evaluates reserves regimes versus channel systems.[2] Berentsen and Monnet (2006, 2008) develop a general equilibrium framework of a channel system and investigate optimal policy. Berentsen, Marchesiani, and Waller (2010) show that a positive spread between the policy rate and the interest rate paid on reserves is optimal. The uncertainty facing banks in these papers arises from a Diamond-Dybvig environment in which depositors are revealed, ex post, to be either patient or impatient. Thus, banks must hold excess reserves to insure against a net payment drain from the entire banking system. In contrast, we assume uncertainty arises from the random distribution of payment flows among banks that result in some banks facing a net outflow while others experience a net inflow. However, the net flow aggregated across the banking system is always zero. Other work related to elements of channel systems include Gaspar, Quiros and Mendizabal (2004), Guthrie and Wright (2000), and Heller and Lengwiler (2003).

In contrast to these papers, we focus on the links between the implementation of monetary policy under a channel system of interest-rate control and credit spreads in the market for bank loans in the face of lending frictions. These lending frictions are captured by a simple search-and-matching framework, with lending interest rates determined by Nash bargaining between lenders (banks) and borrowers (firms). In this environment, the joint surplus to the bank and the firm depends, in part, on the structure of the interbank market as the structure of the interbank market affects the outside opportunity of the bank.

This paper is also related to the literature which studies frictions in credit markets. Most of the work on credit market frictions has focused on issues related to informational asymmetry and moral hazard; early examples include Carlstrom-Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), and Kiyotaki and Moore (1997), while more recent papers include Gertler and Kiyotaki (2010), Gertler and Karadi (2010), and del Negro, Eggertsson, Ferrero, and Kiyotaki (2011).

An additional financial market friction, referred to as "search and entry friction" (Becsi et al, 2000), represents the cost related to the initial participation of firms in credit markets. Recent empirical evidence using U.S disaggregated bank-level data by Contessi and Francis (2011 and 2013), Craig and Haubrich (2006), Dell'ariccia and Garibaldi (2005) and Herrera, Kolar and Minetti (2007/2011) suggest that sizable gross credit flows coexist at the business cycle frequency ,emphasizing the existence of heterogeneous patterns of credit creation and contraction at various phases of the business cycle. For example, Dell'ariccia and Garibaldi (2005) find that in the United States, gross credit flows are by an order of magnitude more volatile than GDP and investment. The empirical evidence for credit flows found in this literature is consistent with predictions made by search models in which

[2] These models are also discussed in Walsh (2010).

the interaction of shocks generates simultaneous expansions and contractions in credit.

On the theoretical side, Den Haan et al. (2003) and Wasmer and Weil (2004) discussed the role of matching frictions in the amplification of macroeconomic volatility. The model of Den Haan et al. (2003) is agency-cost-based, while Wasmer and Weil (2004) employs a Nash bargaining solution. Beauburn-Diant and Tripier (2009) and Xu (2010) also employ an aggregate matching function to characterize the search-and-matching process between borrowers and lenders in the credit market. Petrosky-Nadeau and Wasmer (2012) study matching frictions in credit and labor markets.

In this paper, we incorporate a search-and-matching process between borrowers (firms) and lenders (banks). To produce, an individual firm must be matched with a bank; to lend, an individual bank must be matched with a firm. Banks obtain funds to finance firms by raising retail deposits and then turn to the credit market to seek out a firm with a project in need of financing. As in den Haan, Ramey, and Watson (2003), Wasmer and Weil (2004), and Beaubrun-Diant and Tripier (2009), matched banks and firms decide whether to maintain or sever their credit relationship, depending on the productivity of the firm's project. If the firm and the bank choose to cooperate, Nash bargaining determines how the joint surplus of the match is shared by the bank and the firm in accordance with the financial contract's loan rate.

We extend the literature on search and matching models of credit friction by incorporating a second stage where banks operate in a centralized bond and interbank market. Banks need to settle their balances in the interbank market and, besides interbank lending, banks can borrow or deposit excess reserves through a standing facility administered by the central bank. The structure of the interbank market, the matching process in the loan market, the nature of Nash bargaining, and monetary policy operating procedures affect the lending decisions of banks and the resulting spread between the average lending rate and the central bank's policy rate.

A further contribution of the present paper pertains to the cost channel of monetary policy (Ravenna and Walsh 2006). A typical cost channel impacts the relevant cost of labor by affecting the interest rate firms pay on loans since firms must finance wage payments in advance of production. As a result of the Nash bargaining in this paper however, the role of the loan rate is to split the surplus between the borrower (the firm) and the lender (the bank). The firm chooses employment to maximize the joint surplus of the match, so while there is still a cost channel in the model, it depends on the opportunity cost of funds to the bank and not the interest rate charged on the loan. This implies that the cost channel is directly influenced by the structure of the interbank market. Changes in the policy interest rate, the penalty for borrowing reserves from the central bank, the interest rate paid on reserve deposits at the central bank, the supply of bank reserves by the central bank, and the volatility of settlement payment flows all influence this outside opportunity and therefore affect the equilibrium spread between the average rate on bank loans and the policy interest rate.

Finally, by assuming individual firms are subject to idiosyncratic productivity shocks, the same factors characterizing the interbank market influence the threshold productivity level – the level of productivity at which the firm is able/unable to obtain financing. In particular, we show that a rise in interbank volatility increases the credit spread and, by raising the threshold productivity level the firm needs to obtain financing, reduces the number of firms able to obtain loans. Similarly, monetary policy has effects on employment and output on both the extensive (the fraction of firms receiving loans) and the intensive (the size of loans conditional on obtaining one) margins. The latter arises as a reduction in the cost of funds for banks that makes it optimal for firms with access to credit to expand employment. The former arises because the lower cost of finance makes it profitable for banks to lend to more firms.

The remainder of the paper is arranged as follows. Section 2 presents the basic model setting, describing first the loan market, the bargaining solution that determines the interest rate on loans and the evolution of the number of bank-firm matches. Then, the reserve market under a corridor system is presented together with a description of the consolidated government budget constraint, monetary policy as well as the characterization of the aggregate equilibrium. The results of a numerical analysis are described in section 3. Numerical simulations demonstrate the main transmission mechanism of aggregate shocks that affect the economy. Specifically, two types of financial shocks are studied under a neutral policy response: An increase in the volatility of the payment shock that banks face after the interbank market closes and an exogenous increase in the termination rate of loan contracts. In future versions of the paper, policy experiments, where the central bank responds by using different instruments will be included. Finally, conclusions are given in section 4.

2 The Model

The model economy is populated by households, banks, firms, and a central bank. Households supply labor to firms, hold bonds, cash and bank deposits, and purchase output in the goods market. Firms seek financing, hire labor financed by bank loans and produce output. Banks accept deposits, hold reserves and bonds, and finance the wage bill of firms. The central bank pays interest on reserve deposits and charges a penalty rate on lending to banks.

Three aspects of the model are of crucial importance. First, due to informational asymmetry, households cannot lend directly to firms. We motivate this type of market segmentation by assuming that while banks are able to monitor firms' credit worthiness, it is too costly for households. Second, lending activity involving firms and banks occurs in a decentralized market characterized by search and matching frictions. Third, as a result of participation in the interbank market, individual banks face idiosyncratic and uninsurable risk from random end-of-period settlement flows which must be settled with the central bank.

At the beginning of each period, aggregate shocks are realized and households deposit funds with a bank. The market for deposits is competitive and therefore all banks offer the same interest rate on deposits. In the lending market, firms seek banks to finance their wage payments and banks search for firms to lend to. Firms are subject to aggregate and idiosyncratic productivity shocks which determine both the profitability and scale of operation. Once the loan market closes, firms and workers produce, households consume, and banks invest deposits net of loans into risk-free bonds, lend to or borrow from other banks in the interbank market, and hold deposits with the central bank. Once these markets close, all net payment flows are settled.

Since firm receipts arise from households with deposits at different banks, at the end of the period when all checks must clear, some banks experience a net payment outflow, others an inflow.[3] Banks with a shortage of funds must borrow from the central bank's standing facility; those with an excess of funds can deposit these with the central bank.

We assume a continuum of firms on the unit interval who are either producing or seeking finance and treat the labor market as competitive with firms taking the wage as given in deciding how much labor to employ. Beaubrun-Diant and Tripier, Wasmer and Weil, and Petrosky-Nadeau and Wasmer all treat the opportunity cost of funds to the banks as an exogenous parameter. In contrast, our model focuses on the roles that the interbank market and central bank policy implementation play in affecting the cost of funds for banks, and we provide a complete general equilibrium model.

2.1 Households

Households consume final output and supply labor to maximize

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i U\left(C_{t+i}, 1 - N_{t+i}\right); \; 0 < \beta < 1.$$

The utility function has standard properties. The household enters the period with nominal assets A_{t-1} consisting of the existing stock of government debt B^h_{t-1} and holdings of high powered money HP_{t-1}. These assets are allocated by the household between bank deposits D_t and bond holdings B^h_t:

$$A_{t-1} \equiv B^h_{t-1} + HP_{t-1} = D_t + B^h_t. \tag{1}$$

The household is subject to a cash-in-advance (CIA) constraint that requires initial

[3] In other words, if household "A" purchased their good from a firm which is owned by household "B", then unless households "A" and "B" have an account at the same bank, there will be an outflow from one bank and an inflow to the other in the form of an equal-valued but opposite-signed shock. Poole (1968) assumes a similar repayment shock to the reserves of commercial banks.

deposit balances and current period wage receipts to purchase consumption, or

$$D_t + w_t P_t N_t = A_{t-1} - B_t^h + w_t P_t N_t \geq P_t C_t,$$

where w is the real wage and P is the price level.[4] In real terms the CIA constraint is

$$d_t + w_t N_t - C_t \geq 0, \tag{2}$$

where $d_t = D_t/P_t$. Let i_t^d be the nominal return on bank deposits and i_t^b the nominal return on bonds. The household's end of the period nominal wealth evolves according to

$$A_t = A_{t-1} + i_t^d D_t + i_t^b B_t^h + w_t P_t N_t + \Pi_t^b + \Pi_t^f - P_t C_t - P_t T_t,$$

where Π^i, $i = b, f$ are bank and firm profits and $P_t T_t$ are nominal lump-sum taxes or transfers. Define $a_t = A_t/P_t$, and $b_t^h = B_t^h/P_t$. In real terms, the budget constraint becomes

$$a_t = \left(\frac{1}{1+\pi_t}\right) a_{t-1} + i_t^d d_t + i_t^b b_t^h + w_t N_t + \left(\frac{\Pi_t^b + \Pi_t^f}{P_t}\right) - C_t - T_t. \tag{3}$$

where $a_t \equiv b_t^h + hp_t$ which implies $a_t = d_{t+1} + b_{t+1}^h$.[5]

The representative household maximizes the utility function subject to the CIA constraint 2, the budget constraint 3 and 1 expressed in real terms. The value function for the representative household is defined by

$$V\left(\frac{a_{t-1}}{1+\pi_t}\right) = \max_{d_t, b_t, a_t, C_t, N_t} \left[U\left(C_t, 1 - N_t\right) + \beta \mathrm{E}_t V\left(\frac{a_t}{1+\pi_{t+1}}\right)\right],$$

where $1 + \pi_t = P_t/P_{t-1}$ and the maximization is subject to (2), (3), and, from (1),

$$\left(\frac{1}{1+\pi_t}\right) a_{t-1} - d_t - b_t^h = 0. \tag{4}$$

Let μ and λ be the Lagrangian multipliers associated with the cash-in-advance and

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[4] This constraint could, if one felt it necessary, be motivated by assuming households are anonymous to firms so firms will not sell goods on credit. This assumption, combined with the assumption that banks cannot track the household's future deposit activity would suffice.

[5] Notice that at the end of the period, banks transfer aggregate profits to the representative household, which include the earnings on the holdings of government bonds and cash. Thus, at the end of the period, households must hold the entire supply of high powered money in addition to the supply of government bonds. The household's nominal wealth at the beginning of period $t+1$ is $A_t = HP_t^h + B_t^p$ where $B_t^p = B_t^h + B_t^b$ are government bonds in the hand of the public and B_t^b are bank's holdings of government bonds. By the same token, at the end of the period, $HP_t^h = HP_t^s$, where HP_t^s is the supply of high powered money.

budget constraints. Let φ be the Lagrangian multiplier on the constraint (4). Then the first order necessary conditions for the household's problem of maximizing utility are

$$C_t\colon\ U_C\left(C_t, 1-N_t\right) = \left(\mu_t + \lambda_t\right)$$

$$N_t\colon\ U_N\left(C_t, 1-N_t\right) = w_t\left(\mu_t + \lambda_t\right)$$

$$a_t\colon\ -\lambda_t + \mathrm{E}_t\beta\left(\frac{1}{1+\pi_{t+1}}\right)V'\left(\frac{a_t}{1+\pi_{t+1}}\right) = 0$$

$$d_t\colon\ \mu_t + i_t^d\lambda_t - \varphi_t = 0$$

$$b_t^h\colon\ i_t^b\lambda_t - \varphi_t = 0 \Rightarrow \varphi_t = i_t^b\lambda_t$$

The first two imply

$$\frac{U_N\left(C_t, 1-N_t\right)}{U_C\left(C_t, 1-N_t\right)} = w_t. \tag{5}$$

while the last two imply

$$\mu_t = \varphi_t - i_t^d\lambda_t = \left(i_t^b - i_t^d\right)\lambda_t \tag{6}$$

so that the excess yield of bonds over deposits measures the liquidity services provided by deposits. This in turn implies that

$$U_C\left(C_t, 1-N_t\right) = \mu_t + \lambda_t = \left(1 + i_t^b - i_t^d\right)\lambda_t. \tag{7}$$

From the envelope theorem,

$$V'\left(\frac{a_{t-1}}{1+\pi_t}\right) = \lambda_t + \varphi_t = \left(1 + i_t^b\right)\lambda_t,$$

and the first order condition for a_t can then be written as

$$\lambda_t = \beta\mathrm{E}_t\left(\frac{1}{1+\pi_{t+1}}\right)V'\left(\frac{a_t}{1+\pi_{t+1}}\right) = \beta\mathrm{E}_t\left(\frac{1+i_{t+1}^b}{1+\pi_{t+1}}\right)\lambda_{t+1}. \tag{8}$$

In terms of the marginal utility of consumption, the Euler equation is

$$\frac{U_C\left(C_t, 1-N_t\right)}{1+i_t^b - i_t^d} = \beta E_t\left(\frac{1+i_{t+1}^b}{1+\pi_{t+1}}\frac{U_C\left(C_{t+1}, 1-N_{t+1}\right)}{1+i_{t+1}^b - i_{t+1}^d}\right)$$

and the household stochastic discount factor is defined to be

$$\frac{\beta\lambda_{t+1}}{\lambda_t} = \frac{\beta U_C\left(C_{t+1}, 1-N_{t+1}\right)}{U_C\left(C_t, 1-N_t\right)}\frac{1+i_t^b - i_t^d}{1+i_{t+1}^b - i_{t+1}^d}$$

2.2 The loan market

We assume that the process of finding a credit partner is costly in terms of time and resources, leading to the existence of sunk costs at the time of trading and a surplus to be shared between lenders (banks) and borrowers (firms in the intermediate goods sector). Search and matching frictions prevent instantaneous trading in the loan market, implying that not all market participants will end up matched at a given point in time. We allow for both exogenous and endogenous destruction of credit matches, and a matching technology that determines the aggregate flow of new credit relationships over time as a function of the relative number of lenders and borrowers searching for credit partners. Upon matching successfully (i.e., a match that survives the exogenous and endogenous separation hazards), bilateral Nash bargaining between the parties determines the firm's employment level and interest rate on the loan. The latter is equivalent to choosing the loan size which maximizes the joint surplus to the lender and borrower, while the interest rate determines how the surplus is split between the two partners.

The loan market is populated by a continuum of banks and firms, with the number of banks seeking borrowers varying endogenously and being determined by a free entry condition to the market. We assume that banks have a constant returns to scale technology for managing loans, so that each loan can be treated as a separate match between a bank and a firm. Each firm is endowed with one project and is either searching for external funds or involved in an ongoing credit contract with a bank. If a firm is matched with a bank, then the bank extends the necessary funds to allow the firm to hire workers. There is no possibility of default, all loans are paid back at the end of the period.

2.2.1 The matching process

Firms searching for external funds, f_t, are matched with banks seeking borrowers, b_t^u, according to the following matching function

$$m_t = \mu f_t^{\varphi} (b_t^u)^{1-\varphi}.$$

The function m_t is strictly concave with constant returns to scale and determines the flow of new credit contracts during date t; $0 < \mu < 1$ is a scale parameter that measures the productivity of the matching function and $0 < \varphi < 1$ is the elasticity of match arrival with respect to the mass of searching firms.

Matching rates The variable $\tau_t = f_t/b_t^u$ is a measure of credit market tightness, and corresponds to the standard measure of market tightness arising in search and matching models of the labor market. The probability that a firm with an unfunded project becomes matched with a bank seeking to lend at date t is denoted by p_t^f and is given by

$$p_t^f = \mu \tau_t^{\varphi-1}. \tag{9}$$

Similarly, the probability that a searching bank becomes matched with an unfunded firm at time t is denoted by p_t^b and is given by

$$p_t^b = \mu \tau_t^{\varphi}. \tag{10}$$

Since $\tau_t = p_t^b / p_t^f$, a rise in τ_t implies that it is easier for a bank to find a borrower relative to a firm finding a lender, corresponding to a tighter credit market. An increase (decrease) in τ_t reduces the expected time a bank (firm) must search for a credit partner, lowering the bank's (firm's) expected pecuniary search costs. Since $\tau_t = f_t / b_t^u = p_t^b / p_t^f$, at any date t the number of newly matched banks must equal the number of newly matched firms: $p_t^b b_t^u = p_t^f f_t$.

Separations and the evolution of loan contracts Loan contracts end for exogenous reasons with (time varying) probability δ_t. Contractual parties engaged in a credit relationship that survive this exogenous separation hazard may also decide to dissolve the contract depending on the realization of the productivity of the firm's project, taken to be $z_t \omega_{i,t}$, where z_t is an aggregate productivity component common to all firms (projects) and $\omega_{i,t}$ is a firm-specific idiosyncratic component with a distribution function $G(\omega_{i,t})$. As shown below, the decision to endogenously dissolve a credit relationship is characterized by an optimal reservation policy with respect to $\omega_{i,t}$ and denoted by $\tilde{\omega}_t$. If the realization of the idiosyncratic productivity shock $\omega_{i,t}$ is above the firm-specific productivity reservation, $\omega_{i,t} > \tilde{\omega}_t$, both parties agree to continue the loan contract and contingent on the match surviving the exogenous separation hazard, the firm is able to produce. On the contrary, If the realization of $\omega_{i,t}$ is below $\tilde{\omega}_t$, both parties choose to end the loan contract. Then, the probability of endogenous termination is defined as $\gamma(\tilde{\omega}_t) \equiv prob\,(\omega_{i,t} \leq \tilde{\omega}_t) = G(\tilde{\omega}_t)$. Let $\varphi(\tilde{\omega}_t)$ denote the overall continuation rate, given by

$$\varphi(\tilde{\omega}_t) = (1 - \delta_t)(1 - \gamma(\tilde{\omega}_t))$$

while the overall separation rate is $1 - \varphi(\tilde{\omega}_t) = \delta_t + (1 - \delta_t)\gamma(\tilde{\omega}_t)$. The existence and uniqueness of the optimal reservation policy $\tilde{\omega}_t$ are shown in the appendix.

Let f_{t-1}^m be the measure of intermediate good producers that enter period t matched with a bank. Of those, a fraction $(1 - \delta_t) f_{t-1}^m$ survive the exogenous hazard and a fraction $\gamma(\tilde{\omega}_t)$ of the survivals receive idiosyncratic productivity shocks that are less than $\tilde{\omega}_t$ and as a result, do not produce. The mass of firms that actually produce in period t is $\varphi(\tilde{\omega}_t) f_{t-1}^m$ and the mass of firms in a credit relationship at the end of period t (which begin period $t+1$ as matched), denoted by f_t^m, is given by the number of firms actually producing during time t plus all the new matches formed during the same period. Then, the evolution of f_t^m is expressed as

$$f_t^m = \varphi(\tilde{\omega}_t) f_{t-1}^m + m_t. \tag{11}$$

We normalize the total number of firms in every time period to one and assume that if a credit relationship is exogenously separated at time t, both parties immediately begin searching during the same time period. If the credit relationship survives the exogenous separation hazard but then endogenously separates, then both parties must wait until the following period to start searching for a credit contract. This assumption implies that the number of firms seeking finance during period t, which we denote f_t, is equal to the mass of searching firms at the beginning of time t, $(1 - f^m_{t-1})$ plus the number of firms that started the period matched with a bank and exogenously separated $(\delta_t f^m_{t-1})$. Therefore,

$$f_t = 1 - (1 - \delta_t)\, f^m_{t-1}. \tag{12}$$

Notice that there are still some firms that have been endogenously separated but cannot search in period t. These firms are unmatched but waiting to start searching again next period. The number of new matches during the loan market trading session at time t can be written as

$$m_t = \mu \tau_t^{\varphi-1} \left[1 - (1 - \delta_t)\, f^m_{t-1}\right].$$

Thus the evolution of f^m_t can be written as

$$f^m_t = \varphi\left(\tilde{\omega}_t\right) f^m_{t-1} + \mu \tau_t^{\varphi-1} \left[1 - (1 - \delta_t)\, f^m_{t-1}\right].$$

We also present a different timing assumption regarding separations and the ability to search within the same period of time that the contract separation has occurred. We can close the model by assuming that both types of separations (exogenous as well as endogenous) are able to search during the same period of time that a separation has occurred. Under this assumption, the mass of firms searching for a borrower evolves according to

$$f_t = 1 - \varphi\left(\tilde{\omega}_t\right) f^m_{t-1} \tag{13}$$

with the corresponding changes in the equations for m_t and f^m_t.

Credit Creation and Credit Destruction Our timing assumption implies that the fraction $p^f_t \delta_t f^m_{t-1}$ of matched firms that were exogenously separated during time t are able to find a new credit relationship within the same period of time. Then, credit creation, CC_t, is defined to be equal to the number of newly created credit relationships at the end of time t net of the number of exogenous credit separations that are successfully rematched during the same period. That is

$$CC_t = m_t - p^f_t \delta_t f^m_{t-1}.$$

The credit creation rate, cc_t is

$$cc_t = \frac{m_t}{f^m_{t-1}} - p^f_t \delta_t. \tag{14}$$

On the other hand, credit destruction CD_t is defined as the total number of credit separations at the end of time t, $(1-\varphi(\tilde{\omega}_t))\, b_t^m$ net of the number of exogenous credit separations that are successfully rematched in the same period. That is,

$$CD_t = (1-\varphi(\tilde{\omega}_t))\, f_{t-1}^m - p_t^f \delta_t f_{t-1}^m,$$

and the credit destruction rate cd_t is given by

$$cd_t = (1-\varphi(\tilde{\omega}_t)) - p_t^f \delta_t. \tag{15}$$

Finally, net credit growth is defined as

$$cg_t = cc_t - cd_t$$

If we instead assume that exogenous as well as endogenous credit separations are able to search within the same period of time that separations have occurred then the credit creation and destruction rates are defined to be

$$cc_t = \frac{m_t}{f_{t-1}^m} - p_t^f \varphi(\tilde{\omega}_t). \tag{16}$$

and

$$cd_t = (1-\varphi(\tilde{\omega}_t)) - p_t^f \varphi(\tilde{\omega}_t). \tag{17}$$

2.2.2 Firms and the loan market

In our setting, a credit relationship is a contract between a bank and a firm that allows the latter to operate an specific production technology, hire workers and pay their wage bill in advance of production. As long as the credit contract prevails, the firm will receive sufficient external funds to pay workers in advance of production in all subsequent periods. After selling its output, the firm repays its debt to the bank and transfers all remaining profits to the household. Therefore, as in De Fiore and Tristani (2012), we abstract from the endogenous evolution of net worth by assuming firms do not accumulate internal funds after repaying their debt.

Value functions Firm i is endowed with a production technology given by

$$y_{i,t} = \xi z_t \omega_{i,t} N_{i,t}^{\alpha},\ 0 < \alpha \leq 1, \tag{18}$$

where ξ is a scale technology parameter that serves for calibration purposes, z_t is the aggregate productivity level with mean $\bar{z}$, $\omega_{i,t}$ is the firm-specific idiosyncratic productivity level drawn from a uniform distribution function $G(\omega)$ with support $[\underline{\omega}\ \bar{\omega}]$, and $N_{i,t}$ is firm

i's employment level. Define

$$g(\omega) \equiv \frac{dG(\omega)}{d\omega} = \frac{1}{\overline{\omega} - \underline{\omega}}, \text{ with } \overline{\omega} > \underline{\omega} > 0 \tag{19}$$

and normalize $\bar{z}$ so that the unconditional expectation of $z_t \omega_{i,t}$ is equal to one. If the firm obtains financing and produces, the firm's instantaneous real profit flow is

$$\pi^f(\omega_{i,t}) = y_{i,t} - w_t R_{i,t}^l N_{i,t} - x^f \tag{20}$$

where w_t is the real wage, $R_{i,t}^l$ is the firm-specific gross nominal loan interest rate bilaterally negotiated with a bank and x^f is a fixed cost of production. The labor market is competitive so all firms face the same real wage. The loan principle is $w_t N_{i,t}$ and the loan contract requires the repayment of the total debt with the bank $w_t R_{i,t}^l N_{i,t}$ at the end of the same period.

Firms i's profit π^f depends on the status of the firm, that is, if the firm is searching for external funds or if it is producing. A firm searching for external funds cannot produce and obtains zero real profits $\pi^f = 0$. Assuming costless search for firms, π^f is

$$\pi^f = \begin{cases} \pi^f(\omega_{i,t}) & \text{with external funds} \\ 0 & \text{without external funds} \end{cases}$$

The state of the firm is characterized by two value functions: The value of being matched with a bank and able to produce at date t, denoted by $V^{FP}(\omega_{i,t})$ and the value of searching for external funds at date t, denoted by V_t^{FN}, both measured in terms of current consumption of the final good. Notice that if the firm is producing then its idiosyncratic productivity is common knowledge. On the contrary, if the firm is searching for a lender, then its idiosyncratic productivity is not known yet. Under these assumptions, the value function $V^{FP}(\omega_{i,t})$ is

$$V^{FP}(\omega_{i,t}) = \pi^f(\omega_{i,t}) + \mathrm{E}_t \Delta_{t,t+1} \left\{ \delta_t V_{t+1}^{FN} + (1-\delta_t) \int_{\underline{\omega}}^{\overline{\omega}} \max(V^{FP}(\omega_{i,t+1}), V_{t+1}^{FN}) dG(\omega) \right\}$$

where $\Delta_{t,t+1} = \beta \lambda_{t+1}/\lambda_t$ is the stochastic discount factor. The value of producing is the flow value of current real profits (the firm's real cash flow) plus the expected continuation value. At the beginning of next period, the credit relationship is exogenously dissolved with probability δ_t, and the firm must seek new financing. With probability $(1-\delta_t)$, the firm survives the exogenous separation hazard and faces the new realization of its idiosyncratic productivity level $\tilde{\omega}_{i,t+1}$. If the firm receives a realization which satisfies $\omega_{i,t+1} \geq \tilde{\omega}_{i,t+1}$, then the loan contract continues with the firm obtaining $V^{FP}(\omega_{i,t+1})$, the value of being matched with new external funds for production. In the case that the new realization satisfies $\omega_{i,t+1} < \tilde{\omega}_{i,t+1}$, the loan contract dissolves and the firm obtains V_{t+1}^{FN}, the value of being unmatched and searching for lenders during the next period. The value of searching for external funds V_t^{FN} for a firm at date t expressed in terms of current consumption is

$$V_t^{FN} = p_t^f \mathrm{E}_t \Delta_{t,t+1} \left[\delta_t V_{t+1}^{FN} + (1-\delta_t) \int_{\underline{\omega}}^{\overline{\omega}} \max(V^{FP}(\omega_{i,t+1}), V_{t+1}^{FN}) dG(\omega) \right] + \left(1 - p_t^f\right) V_{t+1}^{FN},$$

where p_t^f is the probability of matching with a bank. Notice that we assume matches made in period t do not produce until $t+1$. With probability $(1-p_t^f)$, the firm does not match and must continue searching for external funds in the following period.

Under Nash bargaining, the reservation productivity level $\tilde{\omega}_t$ that triggers endogenous separation is determined by the point at which the joint surplus of the match is equal to zero. Thus, if $\omega_{i,t+1} < \tilde{\omega}_{t+1}$, both parties agree to end the credit relationship. Notice that given existence and uniqueness of $\tilde{\omega}_{t+1}$, the integral term on the expected continuation value for both $V^{FP}(\omega_{i,t})$ and V_t^{FN} is

$$\begin{aligned} &\int_{\underline{\omega}}^{\overline{\omega}} \max(V^{FP}(\omega_{i,t+1}), V_{t+1}^{FN}) dG(\omega) \\ &= \gamma(\tilde{\omega}_{t+1}) V_{t+1}^{FN} + (1-\gamma(\tilde{\omega}_{t+1})) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V^{FP}(\omega_{i,t+1}) \frac{dG(\omega)}{1-\gamma_t(\tilde{\omega}_t)}. \end{aligned}$$

Therefore, the firm value functions can be written as

$$\begin{aligned} V^{FP}(\omega_{i,t}) &= \pi^f(\omega_{i,t}) \\ &+ \mathrm{E}_t \Delta_{t,t+1} \left\{ (1-\varphi(\tilde{\omega}_{t+1})) V_{t+1}^{FN} + \varphi(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V^{FP}(\omega_{i,t+1}) \frac{dG(\omega)}{1-\gamma(\tilde{\omega}_{t+1})} \right\} \end{aligned} \tag{21}$$

and

$$V_t^{FN} = \mathrm{E}_t \Delta_{t,t+1} \left\{ \begin{array}{c} p_t^f \left[(1-\varphi(\tilde{\omega}_{t+1})) V_{t+1}^{FN} + \varphi(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V^{FP}(\omega_{i,t+1}) \frac{dG(\omega)}{1-\gamma(\tilde{\omega}_{t+1})} \right] \\ + \left(1-p_t^f\right) V_{t+1}^{FN} \end{array} \right\} \tag{22}$$

Let the surplus to a producing firm be defined as $V^{FS}(\omega_{i,t}) = V^{FP}(\omega_{i,t}) - V_t^{FN}$ so that

$$V^{FS}(\omega_{i,t}) = \pi^I(\omega_{i,t}) + \left(1 - p_t^f\right) \mathrm{E}_t \Delta_{t,t+1} \varphi(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V^{FS}(\omega_{i,t+1}) \frac{dG(\omega)}{1 - \gamma(\tilde{\omega}_{t+1})} \tag{23}$$

Thus, the surplus to a firm matched with a bank and being able to produce depends positively on the current flow of profits and on a fraction of the expected continuation value of the credit relationship.

2.2.3 Banks and the loan market

There is a continuum of banks with infinite mass that are owned by the representative household. Banks operate in various centralized markets such as the interbank, bond and deposit market but also operate in the decentralized loan market. Bank activities in the centralized markets include: raising deposits from households, holding excess reserve balances with the central bank, borrowing and lending reserves to and from other banks as part of the payment settlement system and holding government bonds. The existence of search and matching frictions in the lending market implies that banks have to spend time and resources searching for borrowers prior to extending loans. At any point in time a bank may or may not be involved in a credit contract with a firm, so that some banks may not end up with loans on their portfolio. We assume that banks decide to enter the loan market to search for potential borrowers until the expected cost of extending a loan is equal to its expected benefit. At this point, banks will be indifferent between searching for projects or operating in the centralized markets of the economy.

All uncertainty is revealed before loans are extended: loans are made and paid back during the same period. Therefore, loans are not risky and there is no possibility of default. At the end of the period, the bank transfers all its profits to the representative household.

A bank can only form a credit relationship with one firm and cannot search for a different firm until separation occurs. Bank j's balance sheet expressed in nominal terms is

$$\mathbf{1}_{\omega_{i,t}}(j)\, L_{i,t}(j) + B_t^b(j) + I_t(j) + H_t(j) = (1 - \rho) D_t(j) \tag{24}$$

where $\mathbf{1}_{\omega_{i,t}}(j)$ is an indicator function taking the value of one if bank j extends a loan to firm i with idiosyncratic productivity $\omega_{i,t}$ and zero if the bank is searching for a borrower, $L_{i,t}(j)$ are loans to firm i, $B_t^b(j)$ are holdings of government bonds, $I_t(j)$ is (net) lending in the interbank market, $H_t(j)$ are excess reserve holdings, and ρ is the fractional reserve requirement ratio. In this section we focus on the bank's decision regarding $L_{i,t}(j)$ and the potential profits obtained by operating in the loan market, taking the rest of its decision variables as given. In the next section, we explain the decision process for the rest of the variables on the bank's balance sheet.

Value functions Each period, when the loan market opens, a bank may be in a credit relationship with a firm or may be searching for potential borrowers. If a bank extends a loan to a firm whose idiosyncratic productivity realization is $\omega_{i,t}$, profits from doing so are

$$\pi^b(\omega_{i,t}) = \left(R^l_{i,t} - R_t\right) l_{i,t}$$

where $R^l_{i,t} - R_t$ is the spread between the interest rate on the bank's loan to a firm with idiosyncratic productivity $\omega_{i,t}$ and the bank's opportunity cost of funds R_t. The determination of R_t is explained below; it will be shown to be the same for all banks. Loans expressed in real terms are denoted by $l_{i,t}$. We also assume that a bank searching for a borrower incurs a search cost of κ, measured in current consumption units, and earns zero current profits in the loan market while searching for a borrower. Let $\pi^b_t(j)$ be bank j's real profits from operating in the lending market. We show below that $\pi^b_t(j)$ can be written as:

$$\pi^b_t(j) = \mathbf{1}_{\omega_{i,t}}(j)\,\pi^b(\omega_{i,t}) - \left(1 - \mathbf{1}_{\omega_{i,t}}(j)\right)\kappa \tag{25}$$

Under these assumptions the problem of a bank in the loan market can be characterized by two value functions: The value of lending to a firm with productivity $\omega_{i,t}$ at t, denoted by $V^{BL}(\omega_{i,t})$ and the value of searching for a potential borrower at t, denoted by V^{BN}_t. Both value functions are measured in terms of current consumption of the final good and are given by

$$\begin{aligned} V^{BL}(\omega_{i,t}) &= \pi^b(\omega_{i,t}) \qquad\qquad (26)\\ &+ \mathrm{E}_t\Delta_{t,t+1}\left\{(1-\varphi(\tilde{\omega}_{t+1}))\,V^{BN}_{t+1} + \varphi(\tilde{\omega}_{t+1})\int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V^{BL}(\omega_{i,t+1})\frac{dG(\omega)}{1-\gamma(\tilde{\omega}_{t+1})}\right\} \end{aligned}$$

and

$$\begin{aligned} V^{BN}_t &= -\kappa \qquad\qquad (27)\\ &+ \mathrm{E}_t\Delta_{t,t+1}\left\{\begin{array}{c} p^b_t\left[(1-\varphi(\tilde{\omega}_{t+1}))\,V^{BN}_{t+1} + \varphi(\tilde{\omega}_{t+1})\displaystyle\int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V^{BL}(\omega_{i,t+1})\tfrac{dG(\omega)}{1-\gamma(\tilde{\omega}_{t+1})}\right] \\ +\left(1-p^b_t\right)V^{BN}_{t+1} \end{array}\right\} \qquad (28) \end{aligned}$$

The value of extending a loan, $V^{BL}(\omega_{i,t})$, is the current value of real profits plus the expected continuation value. A bank that extends a loan to a firm with idiosyncratic productivity $\omega_{i,t}$ at date t will continue funding the same firm during $t+1$ with probability $\varphi(\tilde{\omega}_{t+1})$. In this event, the bank obtains the future expected value of lending conditional on having $\omega_{i,t+1} \geq \tilde{\omega}_{t+1}$ given by the following conditional expectation: $\int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V^{BL}(\omega_{i,t+1}) (1-\gamma(\tilde{\omega}_{t+1}))^{-1} dG(\omega)$. The credit relationship will be severed at time $t+1$ with probability $\delta_t + 1 - \varphi(\tilde{\omega}_{t+1})$ and the bank will obtain a future value of V_{t+1}^{BN}. On the other hand, the value of a bank searching for a borrower at date t is given by the flow value of the search costs,$-\kappa$, plus the continuation value. A searching bank faces a probability $1 - p_t^b$ of not being matched during time t, obtaining a future value of V_{t+1}^{BN}, while with probability p_t^b the bank matches with a firm. If a searching bank ends up being matched with a firm at time t, then at the beginning of period $t+1$ the bank will face a probability of separation before actually extending the loan.

Free entry condition In equilibrium, free entry of banks into the loan market ensures that $V_t^{BN} = 0$. Using this in (27), the free entry condition can be written as

$$\frac{\kappa}{p_t^b} = \mathrm{E}_t \Delta_{t,t+1} \left\{ \varphi(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V^{BL}(\omega_{i,t+1}) \frac{dG(\omega)}{1-\gamma(\tilde{\omega}_{t+1})} \right\} \tag{29}$$

Banks will enter the loan market until the expected cost of finding a borrower κ/p_t^b is equal to the expected benefit of extending a loan to a firm with idiosyncratic productivity $\omega_{i,t+1} \geq \tilde{\omega}_{t+1}$. If the expected cost of extending a loan is lower than the expected benefits, banks will enter the loan market to search for borrowers and the probability that a searching bank finds a borrower will fall, up to the point where condition (29) is restored. Note that free entry of banks into the loan market modifies the value function $V^{BL}(\omega_{i,t})$ as follows

$$V^{BL}(\omega_{i,t}) = \pi^b(\omega_{i,t}) + \mathrm{E}_t \Delta_{t,t+1} \left\{ \varphi(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V^{BL}(\omega_{i,t+1}) \frac{dG(\omega)}{1-\gamma(\tilde{\omega}_{t+1})} \right\} \tag{30}$$

The net surplus for bank extending a loan is defined as $V^{BS}(\omega_{i,t}) = V^{BL}(\omega_{i,t}) - V_t^{BN}$, and using (29), can be expressed

$$V^{BS}(\omega_{i,t}) = \pi^b(\omega_{i,t}) + \frac{\kappa}{p_t^b}. \tag{31}$$

2.2.4 Employment and the loan contract: Nash bargaining

At any point in time, a matched firm and bank that survive the exogenous and endogenous separation hazards engage in bilateral bargaining to determine the loan interest rate and loan size in order to split the joint surplus that results from the match. This joint surplus of a credit match is defined as $V^{JS}(\omega_{i,t}) = V^{FS}(\omega_{i,t}) + V^{BS}(\omega_{i,t})$ and using (23), (31), (20) and (25), can be written as

$$\begin{aligned} V^{JS}(\omega_{i,t}) &= y_{i,t} - w_t R_t N_{i,t} - x^f \\ &\quad + \left(1 - p_t^f\right) \mathrm{E}_t \Delta_{t,t+1} \varphi(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V^{FS}(\omega_{i,t+1}) \frac{dG(\omega)}{1-\gamma(\tilde{\omega}_{t+1})}. \end{aligned} \tag{32}$$

We assume Nash bargaining with fixed bargaining shares over the loan rate $R_{i,t}^l$ and the corresponding real loan size $l_{i,t}$. Let $\overline{\eta}$ be the firm's share of the joint surplus and $1-\overline{\eta}$ represent the bank's. The Nash bargaining problem for an active credit relationship is

$$\max_{\left\{R_{i,t}^l, l_{i,t}\right\}} \left(V^{FS}(\omega_{i,t})\right)^{\overline{\eta}} \left(V^{BS}(\omega_{i,t})\right)^{1-\overline{\eta}}$$

where $V^{FS}(\omega_{z,t})$ and $V^{BS}(\omega_{z,t})$ are defined above and the firm's demand for funds is given by its wage bill: $l_{i,t} = w_t N_{i,t}$. The first order conditions imply the following optimal sharing rule:

$$\overline{\eta} V^{BS}(\omega_{i,t}) = (1-\overline{\eta}) V^{FS}(\omega_{i,t})$$

and an employment condition that sets the marginal product of labor equal to the marginal cost of labor inclusive of the bank's opportunity cost of funds R_t when extending a loan:

$$\alpha \xi z_t \omega_{i,t} \left(N_{i,t}^*\right)^{\alpha-1} = w_t R_t \tag{33}$$

for all $\omega_{i,t} \geq \tilde{\omega}_t$. The above optimality condition can be written as the optimal loan size negotiated between credit partners:

$$l_{i,t}^* = \left(\frac{\alpha \xi z_t \omega_{i,t}}{w_t^{\alpha} R_t}\right)^{\frac{1}{1-\alpha}}, \tag{34}$$

with the corresponding optimal negotiated loan rate, $R_{i,t}^l$:

$$R_{i,t}^l = (1-\overline{\eta}) \left(\frac{y_{i,t}^* - x^f}{l_{i,t}^*}\right) + \overline{\eta} \left(\frac{R_t w_t N_{i,t}^* - \frac{\kappa p_t^f}{p_t^b}}{l_{i,t}^*}\right)$$

where $y^*_{i,t}$, $N^*_{i,t}$ and $l^*_{i,t}$ denote the optimal output, labor demand and loan size for a firm with idiosyncratic productivity $\omega_{i,t} \geq \tilde{\omega}_t$. Therefore, a firm with external funds in a loan contract with a bank will produce and demand labor according to the following schedules:

$$y^*_{i,t} = (\xi z_t \omega_{i,t})^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w_t R_t}\right)^{\frac{\alpha}{1-\alpha}}$$

$$N^*_{i,t} = \left(\frac{\alpha \xi z_t \omega_{i,t}}{w_t R_t}\right)^{\frac{1}{1-\alpha}}.$$

The effect of the nominal interest rate on the cost of labor is referred to as the cost channel of monetary policy (Ravenna and Walsh 2006) with the relevant rate being that which the firm pays on loans used finance wage payments. In this model, however, the loan interest rate $R^l_{i,t}$ simply ensures the joint surplus generated by a credit relationship is divided optimally between the firm and the bank; here, the relevant interest rate capturing the cost channel is the bank's opportunity cost of funds R_t. As shown below, R_t depends on the interest rate in the interbank market and the marginal value of loans used as collateral. Even though firms will face different interest rates on bank loans, since the loan rate depends on the firms idiosyncratic productivity realization $\omega_{i,t}$, the interest cost relevant for labor demand is the same for all firms. The loan interest rate divides the joint surplus of a credit match in such a manner that a fraction, $1-\overline{\eta}$, of the firm profits relative to the loan size is obtained by the bank while a fraction $\overline{\eta}$ of the bank's opportunity cost of lending, net of search costs and relative to the loan size, is obtained by the firm.

Finally, notice that the credit contract implies that in equilibrium, there will be a distribution in the size of firms such that more productive firms will be able to obtain a greater amount of lending, hire more workers and become larger firms, conditional on surviving.

Since both parties in a credit match have an incentive to maximize the joint surplus, the Nash bargaining protocol discussed above is equivalent to choosing the firm's employment level to solve

$$\max_{N_{i,t}} \left[y_{i,t} - w_t R_t N_{i,t} - x^f \right],$$

taken together with the loan rate which solved the Nash bargaining problem.

These results can be used to rewrite the free entry condition (equation 29), the net surplus for a firm (equation 23) and the joint surplus for a credit relationship (equation 32) as

$$\frac{\kappa}{p^b_t} = (1-\overline{\eta})\,\mathrm{E}_t \Delta_{t,t+1} \varphi\left(\tilde{\omega}_{t+1}\right) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V^{BL}(\omega_{it+1}) \frac{dG(\omega)}{1-\gamma(\tilde{\omega}_{t+1})} \tag{35}$$

$$V^{FS}(\omega_{i,t}) = \pi_t^f(\omega_{i,t}) + \overline{\eta}\left(\frac{1-p_t^f}{1-\overline{\eta}}\right)\frac{\kappa}{p_t^b} \tag{36}$$

$$V^{JS}(\omega_{i,t}) = \left(\pi^{*f}(\omega_{i,t}) + \pi^{*b}(\omega_{i,t})\right) + \left(\frac{1-\overline{\eta}p_t^f}{1-\overline{\eta}}\right)\frac{\kappa}{p_t^b}, \tag{37}$$

where

$$\pi^{*f}(\omega_{i,t}) + \pi^{*b}(\omega_{i,t}) = y_{i,t}^* - R_t w_t N_{i,t}^* - x^f,$$

and $\pi^{*f}(\omega_{i,t})$ is given by equation 20 and $\pi^{*b}(\omega_{i,t})$ by equation 25 evaluated at the optimal level of employment $N_{i,t}^*$ and output $y_{i,t}^*$. Finally, since equation (33) is equivalent to $\alpha y_{i,t}^*/N_{i,t}^* = w_t R_t$, the joint surplus of a credit relationship (equation 37) can be written explicitly as a function of the idiosyncratic productivity shock $\omega_{i,t}$ in order to facilitate the characterization of the loan market equilibrium:

$$V^{JS}(\omega_{i,t}) = (1-\alpha)\left(\xi z_t \omega_{i,t}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{MC_t}\right)^{\frac{\alpha}{1-\alpha}} - x^f + \left(\frac{1-\overline{\eta}p_t^f}{1-\overline{\eta}}\right)\frac{\kappa}{p_t^b} \tag{38}$$

where $MC_t = w_t R_t$ represents real marginal cost, common to all firms. The joint surplus of an active credit contract between a bank and a firm is a direct function of the firm-specific productivity, $z_t\omega_{i,t}$ and an inverse function of the marginal cost of labor. Due to the banks' free entry condition, the term $\frac{\kappa}{p_t^b}$, which measures the expected search cost of extending a loan, is also the expected benefit of forming a financial contract and extending a loan to a firm. If this expected benefit is higher, while keeping p_t^f constant, the joint surplus of the credit relationship increases. In addition, an increase in p_t^f holding p_t^b constant leads to a reduction in the joint surplus. In general equilibrium, both matching rates will change simultaneously. Notice that both matching rates can be written in terms of tightness τ_t; the joint surplus is a function of the firm-specific productivity, the marginal cost of labor and the credit market tightness and can be written as

$$V^{JS}(\omega_{i,t}) = (1-\alpha)\left(\xi z_t \omega_{i,t}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{MC_t}\right)^{\frac{\alpha}{1-\alpha}} - x^f + \left(\frac{1}{\mu\tau_t^{\varphi}} - \frac{\overline{\eta}}{\tau_t}\right)\frac{\kappa}{1-\overline{\eta}}.$$

Up to a first order approximation, the partial equilibrium effect of τ_t over $V^{JS}(\omega_{i,t})$ is negative, assuming that the Hosios condition holds, so that $\overline{\eta} = \varphi$ with $0 < p^f < 1$. Therefore, if the credit market tightens, the joint surplus of a credit relationship falls [6].

[6]The first order approximation of $V_t^{JS}$is given by

$$\widehat{V}_t^{JS} = \left(\frac{(\xi z\omega)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{MC}\right)^{\frac{\alpha}{1-\alpha}}}{V^{JS}}\right)\left(\widehat{\omega}_{i,t} - \widehat{MC}_t\right) - \left(\frac{\frac{\kappa}{1-\overline{\eta}}\frac{\overline{\eta}}{p^b}}{V^{JS}}\right)\left(\varphi - \overline{\eta}p^f\right)\widehat{\tau}_t$$

Where a variable expressed as $\widehat{x}$ denotes log-linear deviation from its steady state. if $\varphi = \overline{\eta}$ then the first

2.2.5 The optimal reservation policy: Endogenous separations

The optimal reservation policy with respect to the idiosyncratic productivity shock implies the following condition:

$$\begin{aligned} if \quad \omega_{i,t} \leq \tilde{\omega}_{i,t} \quad &\Longrightarrow V^{JS}(\omega_{i,t}) \leq 0 \\ if \quad \omega_{i,t} > \tilde{\omega}_{i,t} \quad &\Longrightarrow V^{JS}(\omega_{i,t}) > 0. \end{aligned}$$

Since the joint surplus is a continuous function and strictly increasing in the firm's idiosyncratic productivity level, there exists a unique threshold level, $\tilde{\omega}_t$, for all firms in a credit match, defined by

$$V^{JS}\left(\tilde{\omega}_t\right) = 0,$$

such that the joint surplus is negative for any firm with idiosyncratic productivity $\omega_{i,t} < \tilde{\omega}_t$. The optimal reservation productivity $\tilde{\omega}_t$, is

$$\tilde{\omega}_t = \frac{(MC_t)^{\alpha}}{\xi z_t} H_t, \tag{39}$$

where

$$H_t = \frac{1}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}} \left[x^f - \left(\frac{1 - \overline{\eta}\mu\tau_t^{\varphi-1}}{1-\overline{\eta}} \right) \frac{\kappa}{\mu\tau_t^{\varphi}} \right]^{1-\alpha}.$$

Since $\tilde{\omega}_t$ is independent of i, the cutoff value is the same for all firms and banks. Moreover, it is decreasing in aggregate productivity z_t so that a positive aggregate productivity shock means the number of credit matches that separate endogenously falls and more matched firms produce. Additionally, the cutoff value is increasing in the marginal cost of labor ($MC_t = w_t R_t$) and the firm's fixed cost (x^f).

The bank's opportunity costs of funds R_t influences the level of economic activity at both the extensive and intensive margins. From (39), a rise in R_t increases the threshold productivity level required of firms in order to generate a positive joint surplus of the match. As a result, fewer firms obtain financing and produce: this is the extensive margin effect. Conditional on producing, firms equate the marginal product of labor to $w_t R_t$ (see equation 33), so an increase in R_t reduces labor demand at each level of the real wage: this is the intensive margin effect. Both channels work to reduce aggregate output as R_t rises. In addition, credit market conditions reflected in τ_t directly affect the extensive margin;

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order approximation of V^{JS} can be expressed as

$$\widehat{V}_t^{JS} = \left(\frac{(\xi z \omega)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{MC}\right)^{\frac{\alpha}{1-\alpha}}}{V^{JS}} \right) \left(\widehat{\omega}_{i,t} - \widehat{MC}_t \right) - \left(\frac{\frac{\kappa}{1-\overline{\eta}} \frac{\overline{\eta}}{p^b}}{V^{JS}} \right) \left(1 - p^f \right) \widehat{\tau}_t.$$

Thus, if $p^f < 1$, then an increase in $\widehat{\tau}_t$ generates a fall in $\widehat{V}_t^{JS}$. Of course any value such that $\varphi - \overline{\eta} p^f > 0$ will also result in a negative effect of $\widehat{\tau}_t$ over $\widehat{V}_t^{JS}$.

a rise in τ_t (a credit tightening) increases $\tilde{\omega}_t$ and fewer firms obtain credit. Both interest costs, measured by R_t, and credit conditions, measured by τ, matter for employment and output[7].

2.3 The interbank market

The interbank market is the centralized market for reserves involving the direct participation of commercial banks and the central bank. Net payments between banks must be settled at the end of each period, after the interbank market has closed. The random nature of settlement payment flows from the perspective of an individual bank will generate a demand for excess reserves (reserves in excess of any required reserves). The cost of holding a level of excess reserves that, ex post, is too high or too low will depend on the opportunity costs of, in the first case, holding reserves as deposits at the central bank and, in the second case, borrowing reserves from the central bank. The central bank sets the interest paid on reserves, the rate charged on borrowed reserves, the quantity of reserves, and the haircuts applied to bank assets posted as collateral when borrowing reserves. Not all of these instruments can be set independently.

2.3.1 Banks

Recall[8] that the balance sheet of bank j in nominal terms is

$$\mathbf{1}_{\omega_{i,t}}(j)\, L_{i,t}(j) + B_t^b(j) + I_t(j) + H_t(j) = (1-\rho)D_t(j). \tag{40}$$

During the period, banks make payments to and receive payments from other banks as part of the payment settlement system. Banks can trade reserve balances in the competitive interbank market at the market rate i_t. After the interbank market has closed, banks may experience a net payment shock $\phi_t(j) = \varepsilon_t D_t(j)$, taken to be homogeneous of degree one in the level of the bank's deposit liabilities. The payment shock itself is assumed to be uniformly distributed over the interval $[-\bar{\varepsilon}D_t(j), \bar{\varepsilon}D_t(j)]$.[9] The density and cumulative distribution functions of this shock are $f(\phi) = 1/\left[2\bar{\varepsilon}D_t(j)\right]$ and $F(\phi) = F(\varepsilon D) = (\varepsilon + \bar{\varepsilon})/2\bar{\varepsilon}$. Since $\mathrm{E}\phi = 0$ and $var(\phi) = \bar{\varepsilon}^2 D_t^2(j)/3$, an increase in $\bar{\varepsilon}$ represents a mean preserving spread in the distribution of payment shocks. If $H_t(j) - \phi_t(j) < 0$, the bank must borrow reserves from the central bank to meet its net payment outflow. If $H_t(j) - \phi_t(j) > 0$, the bank can earn interest on its net balances by depositing them with the central bank.

Assume the central bank sets a desired interest rate (the policy rate) i_t^*, remunerates (required or excess) reserve balances at a rate $i_t^* - s$ and lends reserves at a penalty rate

[7] Up to a first order approximation the effect of τ over $\widetilde{\omega}$ is positive if and only if $\varphi - \overline{\eta}p^f > 0$, where p^f is the steady state value of the firm matching rate.

[8] See (24)

[9] Ashcroft, et al 2011 also models unpredictable payment flows as drawn from a uniform distribution.

$i_t^* + s$ (see Woodford 2001, Whitesell 2003, 2006, or Walsh 2006, 2010)[10] The rate paid on reserves places a floor on the interbank rate as no bank will lend to another at a rate less than $i_t^* - s$. And, in the absence of a collateral constraint on borrowing from the central bank, the penalty rate places a ceiling on the interbank rate as no bank will borrow in the interbank market at a rate greater than $i_t^* + s$.[11] In this case, s is the symmetric width of the channel within which the interbank rate is contained. In practice central bank lending is collateralized while interbank lending is unsecured, though the traditional analysis of a channel system (Woodford 2001, Whitesell 2003, 2006) ignores collateral (but see Berentsen and Monnet 2008). We assume the central bank accepts both government bonds and commercial loans as collateral, applying a haircut to each but imposing a larger haircut on loans.[12] If $H_t(j) - \phi_t(j) < 0$, the maximum a bank can borrow from the central bank is $\xi_b B_t^b(j) + \xi_L L_{i,t}(j)$, where $0 < 1 - \xi_L < 1 - \xi_b < 1$ are the haircuts on commercial loans and bonds posted as collateral. For example, the Federal Reserve currently sets $\xi_b = 0.99$ for U.S. bills and bonds with less than 5 years to maturity and $\xi_L = 0.65$ for zero coupon, normal risk-rated commercial loans of 5 years maturity.[13] For simplicity, we assume banks hold collateralizable assets and reserves sufficient to meet all net settlement flows.[14] This requires

$$H_t(j) + \xi_b B_t^b(j) + \xi_L \mathbf{1}_{\omega_{i,t}}(j)\, L_{i,t}(j) + \xi^{bs} \geq \bar{\varepsilon} D_t(j) \tag{41}$$

where ξ^{bs} is a constant that represents alternative assets that can be used as collateral but are not modeled in this paper. This constant will serve for calibration purposes.

Let $i_{i,t}^l$ be the net nominal interest rate on loans if bank j is in a loan contract with firm i, and let x^l be the cost (per dollar) of servicing loans and x^d the cost of servicing deposits. Then nominal profits of bank with household deposits $D_t(j)$ and a loan status

[10] General equilibirum models with channel systems are developed in Berensten and Monnet (2006, 2008), and Berensten, Marchesiani, and Waller (2010). See also Friedman and Kuttner (2010).

[11] In fact, during 2009-2013, the federal funds rate has been below the rate the Federal Reserve pays on reserves. Beck and Klee (2011) explain that this phenomena can arise because Government Sponsored Enterprises (GSE) hold reserves, but cannot earn interest on them from the Federal Reserve. As Furfine (2011) points out, there must be limits to arbitrage that prevent banks from borrowing these fed funds from GSEs and depositing them in their own interest earning reserve accounts.

[12] See Ashcroft, et al 2010.

[13] See the Fed's Discount Window and Payment System Risk Collateral Margins Table, available at http://www.frbdiscountwindow.org/discountwindowbook.cfm?hdrID=14&dtlID=43

[14] This avoids needing to specify the consequences if a bank is unable to meet an extremely large unexpected outflow.

$\mathbf{1}_{\omega_{i,t}}(j)$ can be written as

$$\begin{aligned}\Pi_t^b(j) = &\left(i_{i,t}^l - i_t - x^l\right)\mathbf{1}_{\omega_{i,t}}(j)\,L_{i,t}(j) - \left(1-\mathbf{1}_{\omega_{i,t}}(j)\right)P_t\kappa \\ &+\left[i_t(1-\rho)+(i_t^*-s)\rho - i_t^d - x^d\right]D_t(j) \\ &+\max_{B_t^b,H_t}\left\{\left(i_t^b - i_t\right)B_t^b(j) - i_t H_t(j)\right. \\ &+\int_{-\bar{\varepsilon}D_t(j)}^{H_t(j)}(i_t^*-s)\left[H_t(j)-\phi_t(j)\right]f(\phi)d\phi \\ &\left.+\int_{H_t(j)}^{\bar{\varepsilon}D_t(j)}(i_t^*+s)\left[H_t(j)-\phi_t(j)\right]f(\phi)d\phi\right\},\end{aligned} \tag{42}$$

where (40) has been used to eliminate $I_t(j)$ and the maximization is subject to (41). The first two terms on the right in (42) represent the net interest income on loans with firm i and deposits where $i_t(1-\rho)+(i_t^*-s)\rho - x^d$ is the return on an additional dollar of deposits. Notice that the net income on loans includes the associated search costs of finding a borrower. The next two terms represent the interest income on bond holdings and the opportunity cost of holding excess reserves or bonds rather then lending in the interbank market. The first integral captures the outcome where the net payment shock is such that the bank ends the period with positive excess reserves. These are held in deposits with the central bank and remunerated at rate i_t^*-s. The second integral captures the opposite situation, where the shock is larger than $H_t(j)$, leaving the bank with a negative net position that requires it to borrow through the central bank's lending facility at the penalty rate i_t^*+s.

Let $h_t(j) \equiv H_t(j)/D_t(j)$ and re-write the nominal profit function as well as the collateral constraint in terms of $h_t(j)$[15]. If $\chi_t(j)$ denotes the Lagrangian multiplier on the collateral constraint, the first order conditions for $h_t(j)$ and $B_t^b(j)$ are

$$h_t(j):\ -i_t + (i_t^*-s)\left[\frac{h_t(j)+\bar{\varepsilon}}{2\bar{\varepsilon}}\right] + (i_t^*+s)\left\{1-\left[\frac{h_t(j)+\bar{\varepsilon}}{2\bar{\varepsilon}}\right]\right\} + \chi_t(j) = 0 \tag{43}$$

and

$$B_t^b(j):\ \left(i_t^b - i_t\right) + \xi_b\chi_t(j) = 0. \tag{44}$$

[15] Given $L_{i,t}(j)$ and $D_t(j)$ the bank chooses $\left\{h_t(j), B_t^b(j)\right\}$ consistent with the following static optimization problem expressed in terms of $h_t(j)$:

$$\max_{\left\{h_t(j),B_t^b(j)\right\}}\Pi_t^b(j)$$

s.t

$$h_t(j)\,D_t(j) + \xi_b B_t^b(j) + \xi_L\mathbf{1}_{\omega_{i,t}}(j)\,L_{i,t}(j) + \widehat{\xi}^{bs} \geq \bar{\varepsilon}D_t(j)$$

The optimal choice of excess reserves equates the opportunity cost of holding one more unit of reserves, i_t, with the weighted sum of the marginal costs in expected interest outlay, $(i_t^* + s)[1 - (h_t(j) + \bar{\varepsilon})/2\bar{\varepsilon}]$, resulting from the deficiency in reserves, and the marginal gains in expected interest income, $(i_t^* - s)(h + \bar{\varepsilon})/(2\bar{\varepsilon})$ from holding excess reserves and the collateral value of an extra dollar of reserve holdings $\chi_t(j)$. Equation (44) implies the interest rate on bonds plus their collateral value equals the interbank market rate, or

$$\chi_t(j) = \left(i_t - i_t^b\right)/\xi_b$$

The above equation also means that in equilibrium, the opportunity cost of holding bonds (rather than lending funds on the interbank market) is exactly balanced by the benefit of the collateral service that bonds provide after the interbank closes.

Hence, χ_t is independent of j. From (43), this also implies that h_t is independent of j, and the demand for excess reserves is given by

$$h_t = \left(\frac{\bar{\varepsilon}}{s}\right)\left(i_t^* - i_t + \chi_t\right). \tag{45}$$

Total excess reserve demand is increasing in the volatility of payment flows (measured by $\bar{\varepsilon}$). It is decreasing in the width of the channel (s) and increasing in the spread between the policy rate and the interbank rate $i_t^* - i_t$ and the marginal value of holding excess reserves as collateral, rather than bonds or loans.[16] When χ_t increases, the bank perceives that it

where $\Pi_t^b(j)$ is given by

$$\begin{aligned}
&\left(i_{i,t}^l - i_t - x^l\right)\mathbf{1}_{\omega_{i,t}}(j)\,L_{i,t}(j) - \left(1 - \mathbf{1}_{\omega_{i,t}}(j)\right)P_t\kappa \\
&+ \left[i_t\,(1-\rho) + (i_t^* - s)\,\rho - \left(i_t^d + x^d\right)\right]D_t(j) \\
&+ \max_{\left\{h_t(j),B_t^b(j)\right\}}\left\{\left(i_t^b - i_t\right)B_t^b(j) - i_t h_t(j) D_t(j)\right. \\
&+ \frac{(i_t^* - s)\,D_t(j)}{2\bar{\varepsilon}}\int_{-\bar{\varepsilon}}^{h_t(j)}\left(h_t(j) - \varepsilon_t\right)d\varepsilon_t \\
&\left.+ \frac{(i_t^* + s)\,D_t(j)}{2\bar{\varepsilon}}\int_{h_t(j)}^{\bar{\varepsilon}}\left(h_t(j) - \varepsilon_t\right)d\varepsilon_t\right\}
\end{aligned}$$

and $\widehat{\xi}^{bs}$ represents assets different from h, B^b and L, expressed as a fraction of total deposits, that can be used as collateral.

[16] When the bank profit function is expressed in terms of excess reserves as a fraction of deposits $h_t(j)$, the payment shock $\phi_t(j) = \varepsilon_t D_t(j)$ is expressed also as a fraction of deposits. In this case, $\varepsilon_t = \frac{\phi_t(j)}{D_t(j)} \sim Unif\left(-\bar{\varepsilon}_t, \bar{\varepsilon}_t\right)$ with density $f(\varepsilon_t) = \frac{1}{2\bar{\varepsilon}_t}$ and variance $var(\bar{\varepsilon}_t) = \frac{\bar{\varepsilon}_t^2}{3}$. Notice that we assume that the support of the payment shock $\bar{\varepsilon}_t$ evolves over time according to an exogenous autoregresive process that we specify below.

is more valuable to allocate excess reserves as collateral instead of using government bonds or loans, since $B_t^b(j)$ and $L_{i,t}(j)$ are subject to haircuts. Equation 45 implies that total excess reserve demand has three components: a component representing a precautionary demand, given by the volatility of the payment shock normalized by the corridor width $\bar{\varepsilon}/s$, a component related to the opportunity cost of having to borrow a reserve shortfall from the Fed rather than from the interbank $i_t^* - i_t$, and a component associated with the marginal benefit of having excess reserves, so as to not necessitate collateralized borrowing, χ_t.

Rewriting (43) as

$$i_t = (i_t^* - s)\left(\frac{h_t + \bar{\varepsilon}}{2\bar{\varepsilon}}\right) + (i_t^* + s)\left[1 - \left(\frac{h_t + \bar{\varepsilon}}{2\bar{\varepsilon}}\right)\right] + \chi_t$$

shows that i_t equals a weighted average of the interest rate on central bank deposits $i_t^* - s$ and the rate of borrowing reserves $i_t^* + s$, adjusted for the marginal value of collateral χ_t. Thus,

$$i_t^* - s + \chi_t \leq i_t \leq i_t^* + s + \chi_t.$$

If the collateral constraint does not bind, so that $\chi_t = 0$, then the standard result that the interbank rate is bounded symmetrically between the rate paid on reserves $(i_t^* - s)$ and the rate charged on borrowing $(i^* + s)$ is obtained as there are symmetric opportunity costs around the central bank's target rate. Since collateral must be posted to borrow from the Fed, as considered in Whitsell (2006), a market rate equivalent to borrowing from the Fed would be $i_t^* + s$ plus the cost of providing collateral. Our model accounts for this asymmetry in opportunity costs because when the commercial bank's collateral constraint binds, $\chi_t > 0$.

These bounds on i_t imply

$$-s \leq i_t - i_t^* - \chi_t \leq s,$$

so from (45) reserve demand is also bounded:

$$-\bar{\varepsilon} \leq h_t \leq \bar{\varepsilon}.$$

If the central bank sets $i = i^*$ (the interbank rate equals the central bank's policy rate), then

$$h_t = \left(\frac{\bar{\varepsilon}}{s}\right)\chi_t \geq 0,$$

and excess reserves are positive. In the absence of a collateral constraint, excess reserves would be zero[17]. But if the collateral constraint binds, the slope of the total reserve demand with respect to the interbank rate is positive since χ_t raises whenever i_t is above

[17]This is the case considered, for example, by Whitesell (2006).

i_t^b. Equivalently, if the central bank provides a level of total reserves equal to required reserves so that excess reserves are zero,

$$i_t = i_t^* + \chi_t \geq i_t^*.$$

In this case, the interbank rate exceeds the policy rate (see Berensten and Monnet 2006).

Evaluating $\Pi_t^b(j)$ at $h(j) = h_t^*$ yields:

$$\Pi_t^b(j) = \left(i_{i,t}^l - i_t - x^l\right) \mathbf{1}_{\omega_{i,t}}(j)\, L_{i,t}(j) - \left(1 - \mathbf{1}_{\omega_{i,t}}(j)\right) P_t \kappa \\ + \left(i_t^b - i_t\right) B_t^b(j) + \left[i_t(1-\rho) - \left(i_t^d + x^d\right) - i_t h_t^* + \overline{x}_t\right] D_t(j) \tag{46}$$

where

$$\overline{x}_t = (i_t^* - s)\left[\rho + \left(\frac{(h_t^*)^2}{4\overline{\varepsilon}} + \frac{h_t^*}{2} + \frac{\overline{\varepsilon}}{4}\right)\right] + (i_t^* + s)\left(-\frac{(h_t^*)^2}{4\overline{\varepsilon}} + \frac{h_t^*}{2} - \frac{\overline{\varepsilon}}{4}\right) \tag{47}$$

since the integrals have been solved out as we already know h_t^*[18].

When choosing deposits $D_t(j)$ the bank takes as given its optimal decision on h_t^* and B_t^b but also takes into account the effect of $D_t(j)$ on the collateral constraint. Taking the derivative of $\Pi_t^b(j)$ with respect to $D_t(j)$ yields

$$\frac{\partial \Pi_t(j)}{\partial D_t(j)} = \zeta_t D_t(j),$$

where

$$\zeta_t \equiv i_t(1-\rho) - \left(i_t^d + x^d\right) - i_t h_t^* + \overline{x}_t + \chi_t\left(h_t^* - \overline{\varepsilon}\right).$$

Competition for deposits among banks will ensure that $\zeta_t = 0$, implying an interest rate on deposits of

$$i_t^d = i_t(1-\rho) - x^d - i_t h_t^* + \overline{x}_t + \chi_t\left(h_t^* - \overline{\varepsilon}\right).$$

[18]The integrals evaluated at the optimal demand for excess reserves, $h_t^*(j) = h_t^*$ become:

$$\int_{h_t^*(j)}^{\overline{\varepsilon}} \left(h_t^*(j) - \varepsilon_t\right) d\varepsilon_t = -\frac{(h_t^*)^2}{2} + \overline{\varepsilon} h_t^* - \frac{\overline{\varepsilon}^2}{2}$$

$$\int_{-\overline{\varepsilon}}^{h_t^*} \left(h_t^* - \varepsilon_t\right) d\varepsilon_t = \frac{(h_t^*)^2}{2} + \overline{\varepsilon} h_t^* + \frac{\overline{\varepsilon}^2}{2}$$

Using the definition of $\overline{x}_t$ (equation 47) as well as equation 45, i_t^d can be written as

$$i_t^d = i_t\left(1-\rho\right) + \left(i_t^* - s\right)\rho - x^d + f\left(i_t^* - i_t, \chi_t, \overline{\varepsilon}\right) \tag{48}$$

where

$$f\left(i_t^* - i_t, \chi_t, \overline{\varepsilon}\right) = \frac{1}{2}\frac{\overline{\varepsilon}}{s}\left(i_t^* - i_t + \chi_t\right)^2 - \left(\chi_t + \frac{s}{2}\right)\overline{\varepsilon}.$$

Hence, the deposit rate is a weighted average of the interbank rate and the rate earned on required reserves adjusted for the bank's cost of providing deposits and the effect of deposits on the need for additional collateral and excess reserves. If excess reserves are zero, (45) implies $i_t^* + \chi_t - i_t = 0$ and

$$i_t^d = i_t(1-\rho) + (i_t^* - s)\rho - x^d - \left(\chi_t + \frac{s}{2}\right)\overline{\varepsilon}.$$

In discussing the loan market earlier, the flow value (in real terms) to a bank when operating in the loan market was equal to $\pi_t^b\left(j\right) = \mathbf{1}_{\omega_{i,t}}\left(j\right)\left(R_{i,t}^l - R_t\right)l_{i,t} - \left(1 - \mathbf{1}_{\omega_{i,t}}\left(j\right)\right)\kappa$, where $\mathbf{1}_{\omega_{i,t}}\left(j\right)$ indicates the status of the bank: extending a loan or searching for a borrower. Notice that the above equation is obtained by taking the following steps:

1) Express 46 in real terms:

$$\begin{aligned}\pi_t^b(j) = &\left(i_{i,t}^l - i_t - x^l\right)\mathbf{1}_{\omega_{i,t}}\left(j\right)l_{i,t}(j) - \left(1 - \mathbf{1}_{\omega_{i,t}}\left(j\right)\right)\kappa \\ &+ \left(i_t^b - i_t\right)b_t^b(j) + \left[i_t\left(1-\rho\right) - \left(i_t^d + x^d\right) - i_t h_t^* + \overline{x}_t\right]d_t(j)\end{aligned}$$

where lowercase letters represent real variables.

2) Substitute the equilibrium expression for i_t^d into $\pi_t^b(j)$:

$$\begin{aligned}\pi_t^b(j) = &\left(i_{i,t}^l - i_t - x^l\right)\mathbf{1}_{\omega_{i,t}}\left(j\right)l_{i,t}(j) - \left(1 - \mathbf{1}_{\omega_{i,t}}\left(j\right)\right)\kappa \\ &+ \left(i_t^b - i_t\right)b_t^b(j) - \chi_t\left(h_t^* - \overline{\varepsilon}\right)d_t(j).\end{aligned}$$

Finally, 3) use the collateral constraint expressed in real terms to eliminate $b_t^b(j)$ as well as making use of $\chi_t = \frac{i_t - i_t^b}{\xi_b^b}$:

$$\pi_t^b(j) = \left(i_{i,t}^l - i_t - x^l + \chi_t\xi_L\right)\mathbf{1}_{\omega_{i,t}}\left(j\right)l_{i,t}(j) - \left(1 - \mathbf{1}_{\omega_{i,t}}\left(j\right)\right)\kappa.$$

The term $\chi_t\xi_L$ reflects the collateral value of extending a loan and is thus a component of the net return that a bank obtains when $l_{i,t}(j) > 0$. We define the opportunity cost of extending a loan as

$$R_t = 1 + i_t - \xi_L\chi_t + x^l.$$

Ceteris peribus, an increase in the haircut applied to loans used as collateral with the central bank (a fall in ξ_L) increases the opportunity cost of lending. As a result, the effective cost of labor increases and the demand for labor falls. This negative effect on employment holds for a given interbank rate i_t. In addition, an increase in the marginal value of collateral (a raise in χ_t) increases the opportunity cost of lending. The gross loan rate negotiated between bank j and firm i in the case in which $l_{i,t}(j) > 0$ is defined as $R^l_{i,t} = 1 + i^l_{i,t}$. Therefore, the flow value to a bank for participating in the loan market, expressed in real terms, is given by equation 25.

2.4 The central bank

The central bank sets the required reserve ratio, the width of the channel and the haircuts on bonds and loans used by banks as collateral against loans from the central bank. The central bank can set its policy interest rate i^*_t, its bond holdings and its liabilities (high-powered money) subject to its budget constraint. In nominal terms, the central bank's budget constraint is given by

$$B^{cb}_t - B^{cb}_{t-1} + RCB_t + (i^*_t - s)(\rho D_t + ER_t) + (i^*_t + s)BR_t = i^b_t B^{cb}_t + HP^s_t - HP^s_{t-1}$$

The central bank's revenue is given by the interest rate payments on government debt holding $\left(i^b_t B^{cb}_t\right)$, the change in high-powered money $\left(HP^s_t - HP^s_{t-1}\right)$ and interest payments on total reserves held with the central bank net of interest payments on total borrowed reserves, $((i^*_t - s)(\rho D_t + ER_t) + (i^*_t + s)BR_t)$. The central bank allocates its revenue into purchases of government debt $\left(B^{cb}_t\right)$ and transfers its residual receipts to the treasury (RCB_t). Notice that ER_t and BR_t denote the banking sector's aggregate excess reserves and aggregate borrowed reserves, respectively. Both measures are obtained by aggregating the optimally chosen expected excess reserves and borrowed reserves of each individual bank. Recall that at the beginning of the period, each bank's expected excess reserves $(ER_t(j))$ and borrowed reserves $(BR_t(j))$ are

$$ER_t(j) = \frac{D_t(j)}{2\overline{\varepsilon}} \int_{-\overline{\varepsilon}}^{h_t(j)} (h_t(j) - \varepsilon_t)\, d\varepsilon_t$$

$$BR_t(j) = \frac{D_t(j)}{2\overline{\varepsilon}} \int_{h_t(j)}^{\overline{\varepsilon}} (h_t(j) - \varepsilon_t)\, d\varepsilon_t$$

where the optimal level of excess reserves expressed as a fraction of deposits is given by

$$h^*_t = \frac{\overline{\varepsilon}}{s}(i^*_t - i_t + \chi_t) \quad \text{for all } j \in [0,1].$$

Therefore,

$$ER_t(j) = \frac{D_t(j)}{2\overline{\varepsilon}} \int_{-\overline{\varepsilon}}^{h_t^*} (h_t^* - \varepsilon_t)\, d\varepsilon_t$$
$$= \frac{D_t(j)}{2\overline{\varepsilon}} \left(\frac{(h_t^*)^2}{2} + \overline{\varepsilon} h_t^* + \frac{\overline{\varepsilon}^2}{2} \right),$$

and

$$BR_t(j) = \frac{D_t(j)}{2\overline{\varepsilon}} \int_{h_t^*}^{\overline{\varepsilon}} (h_t^* - \varepsilon_t)\, d\varepsilon_t$$
$$= \frac{D_t(j)}{2\overline{\varepsilon}} \left(-\frac{(h_t^*)^2}{2} + \overline{\varepsilon} h_t^* - \frac{\overline{\varepsilon}^2}{2} \right).$$

Aggregating across all banks yields expressions for ER_t and BR_t that appear on the central bank's budget constraint:

$$ER_t = \left(\frac{(h_t^*)^2}{4\overline{\varepsilon}} + \frac{h_t^*}{2} + \frac{\overline{\varepsilon}}{4} \right) D_t$$

$$BR_t = \left(-\frac{(h_t^*)^2}{4\overline{\varepsilon}} + \frac{h_t^*}{2} - \frac{\overline{\varepsilon}}{4} \right) D_t$$

On the other hand, the treasury's budget constraint is given by

$$P_t T_t + B_t^T - B_{t-1}^T + RCB_t = P_t G_t + i_t^b B_t^T$$

where the left hand side represents treasury revenue consisting of taxes/transfers to or from households (T_t), new issuance of interest-bearing government debt $\left(B_t^T - B_{t-1}^T\right)$ as well as central bank receipt transfers to the treasury (RCB_t). The right hand side represents expenditures composed of government spending in goods and services (G_t) and interest rate payments on government debt $\left(i_t^b B_t^T\right)$. We assume that the total supply of government debt B_t^T is held by households B_t^h, private banks B_t^b and the central bank B_t^{cb}, so that

$$B_t^T = B_t^h + B_t^b + B_t^{cb}.$$

Therefore, the government's consolidated budget constraint can be written as

$$P_t T_t + B_t^p - B_{t-1}^p + HP_t^s - HP_{t-1}^s = P_t G_t + i_t^b B_t^p + X_t \tag{49}$$

where $B_t^p = B_t^h + B_t^b$ denotes holdings of government debt by the private sector and X_t is the central bank's interest on total reserves net of interest payments on total borrowed reserves, given by

$$X_t = (i_t^* - s)(\rho D_t + ER_t) + (i_t^* + s) BR_t. \tag{50}$$

Notice that BR_t is negative, by definition.

In real terms, 50 is

$$b_t^p - \left(\frac{1}{1+\pi_t}\right) b_{t-1}^p + hp_t^s - \left(\frac{1}{1+\pi_t}\right) hp_{t-1}^s = G_t - T_t + i_t^b b_t^p + x_t \tag{51}$$

where

$$x_t = (i_t^* - s)(\rho d_t + er_t) + (i_t^* + s) br_t, \tag{52}$$

$$er_t = \left(\frac{(h_t^*)^2}{4\overline{\varepsilon}} + \frac{h_t^*}{2} + \frac{\overline{\varepsilon}}{4}\right) d_t, \tag{53}$$

$$br_t = \left(-\frac{(h_t^*)^2}{4\overline{\varepsilon}} + \frac{h_t^*}{2} - \frac{\overline{\varepsilon}}{4}\right) d_t. \tag{54}$$

In order to focus on central bank operations, we re-express the consolidated budget constraint in terms of the central bank holdings of government bonds and total government debt:

$$\left(hp_t^s - \left(\frac{1}{1+\pi_t}\right) hp_{t-1}^s\right) - \left(b_t^{cb} - \left(\frac{1}{1+\pi_t}\right) b_{t-1}^{cb}\right) + i_t^b b_t^c = f_t + x_t \tag{55}$$

where f_t is defined as an exogenous fiscal variable given by

$$f_t = G_t - T_t + i_t^b b_t^T - \left(b_t^T - \left(\frac{1}{1+\pi_t}\right) b_{t-1}^T\right).$$

Notice that f_t is equivalent to the real transfers of the central bank's receipts to the treasury $\left(f_t = \frac{RCB_t}{P_t}\right)$. Then, given the policy rate i_t^* and private sector decisions that determine reserve holdings h_t^*, the term x_t is not controlled directly by the central bank, as emphasized by Berensten and Monet (2008). The consolidated budget constraint links changes in the supply of high powered money $hp_t^s - \left(\frac{1}{1+\pi_t}\right) hp_{t-1}^s$ to changes in the central bank's government bond holdings $\left(b_t^{cb} - \left(\frac{1}{1+\pi_t}\right) b_{t-1}^{cb}\right)$ via the effects of open market operations.

It can also be assumed that lump sum taxes T_t adjust in order to fund any changes in hp_t^s and/or changes in x_t. Under this assumption the consolidated budget constraint, 55, is written as

$$T_t + hp_t^s - \left(\frac{1}{1+\pi_t}\right) hp_{t-1}^s = x_t + \widetilde{f}_t, \tag{56}$$

where $\widetilde{f}_t$ is defined as the following exogenous fiscal shock:

$$\widetilde{f}_t = G_t + \left(b_t^{cb} - \left(\frac{1}{1+\pi_t}\right) b_{t-1}^{cb} - i_t^b b_t^{cb}\right) - \left(b_t^T - \left(\frac{1}{1+\pi_t}\right) b_{t-1}^T - i_t^b b_t^T\right).$$

In this case, the central bank does not perform open market operations when changing the supply of high powered money.

2.5 Market Clearing and the Aggregate equilibrium

Equilibrium in the interbank market requires aggregate interbank net lending to cancel out, that is:

$$\int_j \frac{I_t(j)}{P_t} dj = 0$$

and to balance the reserve demand and reserve supply in real terms, so if hp_t^s is the total (exogenous) real supply of high powered money set by the central bank, total reserve demand is $(\rho + h_t^*)\, d_t$, thus:

$$hp_t^s = (h_t^* + \rho)\, d_t.$$

Using (45) yields an expression for the equilibrium interbank interest rate:

$$i_t = i_t^* + \chi_t - \left(\frac{s}{\bar{\varepsilon}}\right)\left(\frac{hp_t^s}{d_t} - \rho\right), \tag{57}$$

which illustrates that the central bank has multiple instruments for achieving a given interbank rate i_t. For a given reserve supply relative to total deposits $\frac{hp_t^s}{d_t}$ and collateral value χ_t, i_t can be increased directly by raising the target policy rate i_t^* or by reducing the width of the corridor s. Holding i_t^*, χ_t, and s constant, a decrease in the reserve supply (relative to deposit liabilities of the banking sector) increases i_t. A further implication of (57) is that if $s > 0$ the equilibrium interbank rate will equal the policy rate only when $\frac{hp_t^s}{d_t} = \rho + (\bar{\varepsilon}/s)\,\chi_t \geq \rho$, that is, only when the central bank supplies a level of reserves greater than the level of required reserves under the situation where the collateral constraint is binding, $\chi_t > 0$.

To summarize, in this setting the central bank has four potential policy instruments: i^*, s, hp^s, and b^{cb}, of which only three can be varied independently consistent with (56).[19]

[19] If the central bank operated a channel system with an asymmetric corridor, then instead of s, the central bank could vary the upper and lower bounds of the corridor around i^* independently. We restrict attention to a a symmetric system.

If there are no open market operations and T_t adjusts endogenously to any change in hp^s, then the central bank has three potential policy instruments: i^*, s, hp^s.

Aggregate output is the number of producing firms times the expected output of each firm, conditional on its realization of $\omega_{i,t}$ exceeding $\tilde{\omega}_t$. Recall that the number of matched firms at the start of period t is f_{t-1}^m and that only a fraction $\varphi(\tilde{\omega}_t) = (1-\delta_t)(1-\gamma(\tilde{\omega}_t))$ of those firms survive both separation hazards and consequently end up producing. Aggregate output is then

$$Y_t = \varphi(\tilde{\omega}_t) f_{t-1}^m \mathrm{E}\left[y^*(\omega_{i,t}) \mid \omega_{i,t} \geq \tilde{\omega}_t\right],$$

where

$$\mathrm{E}\left[y^*(\omega_{i,t}) \mid \omega_{i,t} \geq \tilde{\omega}_t\right] = \int_{\tilde{\omega}_t}^{\overline{\omega}} y^*(\omega_{i,t}) \frac{dG(\omega)}{(1-\gamma(\tilde{\omega}_t))}$$

where individual output for firm i is written explicitly in terms of its idiosyncratic productivity level, $y(\omega_{i,t})$. Using the assumption that ω follows a uniform distribution with density $g(\omega) = dG(\omega) = 1/(\bar{\omega} - \underline{\omega})$ then Y_t is

$$Y_t = (1-\delta_t)\alpha^{\frac{\alpha}{1-\alpha}} \left(\frac{z_t}{(w_t R_t)^{\alpha}}\right)^{\frac{1}{1-\alpha}} \left(\frac{(\overline{\omega})^k - (\tilde{\omega}_t)^k}{k(\overline{\omega} - \underline{\omega})}\right) f_{t-1}^m, \tag{58}$$

where $k \equiv (2-a)/(1-a) > 1$.

Following the same steps, aggregate employment is

$$N_t = (1-\delta_t)\alpha^{\frac{1}{1-\alpha}} \left(\frac{z_t}{w_t R_t}\right)^{\frac{1}{1-\alpha}} \left(\frac{(\overline{\omega})^k - (\tilde{\omega}_t)^k}{k(\overline{\omega} - \underline{\omega})}\right) f_{t-1}^m. \tag{59}$$

Combining (58 and (59),[20]

$$Y_t = \xi z_t F_t^{1-a} N_t^a. \tag{60}$$

where

$$F_t \equiv (1-\delta_t) \left(\frac{\bar{\omega}^k - \tilde{\omega}_t^k}{k(\overline{\omega} - \underline{\omega})}\right) f_{t-1}^m. \tag{61}$$

Equation (60) is the aggregate production function for this economy and illustrates the way in which aggregate output depends on the aggregate productivity shock and employment but also on the number of producing firms and their average idiosyncratic productivity as reflected in F_t. Credit market disruptions that lead to an exogenous rise in match breakups (a rise in δ_t) acts like a negative productivity shock that is further amplified due to endogenous changes in $\tilde{\omega}_t$ and f_{t-1}^m. In addition, an increase in the cutoff productivity

[20] See appendix.

level $\tilde{\omega}_t$ reduces output (given N) by reducing the mass of firms that actually produce. Notice that under a perfectly competitive credit market, the term F_t would not impact the aggregate production function since, in this case, $F_t = 1$. We interpret F_t as an inefficiency wedge associated with credit market imperfections that amplify the impact of exogenous shocks.

The assumption that ω follows a uniform distribution implies the following overall continuation rate:

$$\varphi(\tilde{\omega}_t) = (1 - \delta_t)\left(\frac{\bar{\omega} - \tilde{\omega}_t}{\overline{\omega} - \underline{\omega}}\right). \tag{62}$$

Market equilibrium in the loan market requires $l^*_{i,t} = w_t N^*_{i,t}$ for all active firms or credit contracts, that is, for all i with $\omega_{i,t} > \tilde{\omega}_t$. Aggregating this equilibrium condition across all of those active firms yields the following condition for aggregate loans and aggregate labor income

$$l_t = w_t N_t. \tag{63}$$

The aggregate equilibrium takes into consideration the aggregation of the balance sheet as well as the collateral constraint for banks, given by

$$l_t + b^b_t + h^*_t d_t = (1 - \rho)\, d_t, \tag{64}$$

and

$$h^*_t d_t + \xi_b b^b_t + \xi_L l_t + \xi^{bs} = \bar{\varepsilon} d_t, \tag{65}$$

respectively. The constant term ξ^{bs} has been added to the aggregate collateral constraint in order to account for the different types of assets that may serve as collateral and are not considered in the model. This new parameter will serve for calibration purposes.

As part of the aggregate equilibrium, firms and banks transfer their profits to the representative household at the end of each period. The aggregate real transfer of profits received by the household consists of the following two functions:

$$\pi^f_t = Y_t - R^l_t l_t - \varphi_t(\tilde{\omega}_t)\, f^m_{t-1} x^f$$

$$\pi^b_t = \left(R^l_t - 1\right) l_t + i^b_t b^b_t - i^d_t d_t + x_t - \left(x^l l_t + x^d d_t + \kappa b^u_t\right)$$

where π^f_t denotes aggregate firm profits and π^b_t denotes aggregate bank profits. In deriving aggregate bank profits, we have used the equilibrium condition in the deposit market $d_t = \int_j d_t(j)\, dj$ where the integration is across all banks, the equilibrium condition in the interbank market $\int_j \frac{I_t(j)}{P_t} dj = 0$, and the aggregate balance sheet for the banking sector (40).

Since each loan contract negotiated carries its own interest rate, the average credit spread is defined as the difference between the average loan rate and the bank's opportunity cost of lending:

$$\frac{R_t^l l_t - R_t l_t}{l_t} = (1-\alpha)(1-\overline{\eta}) \frac{Y_t}{\varphi(\tilde{\omega}_t) f_{t-1}^m l_t} - \left(\frac{(1-\overline{\eta}) x^f + \overline{\eta} \frac{\kappa}{\tau_t}}{l_t} \right), \tag{66}$$

where

$$R_t^l l_t = E\left[R^l(\omega_{it}) l^*(\omega_{it}) \mid \omega_{it} \geq \widetilde{\omega}_t \right],$$

and

$$R_t l_t = E\left[R_t l^*(\omega_{it}) \mid \omega_{it} \geq \widetilde{\omega}_t \right].$$

Equilibrium in the goods market requires aggregate expenditures (consumption plus government expenditures) to equalize aggregate household income net of the aggregate fixed costs of production by producing firms, aggregate search costs by the banking sector, and the aggregate costs of managing both loans and deposits. Then the aggregate resource constraint of the economy is characterized by

$$C_t + G_t = Y_t - \left((\varphi_t(\tilde{\omega}_t) f_{t-1}^m x^f + x^l l_t + x^d d_t + \kappa b_t^u \right) \tag{67}$$

where G_t is treated as exogenous and consumption must satisfy the following aggregate CIA constraint:

$$C_t = d_t + w_t N_t + \xi^{cia}. \tag{68}$$

where ξ^{cia} is an intercept term introduced for calibration purposes that represents all other assets that can be used to purchase consumption goods which are not explicitly modeled.

2.5.1 Characterization of the aggregate loan market equilibrium

In this section we characterize the loan market equilibrium in terms of two main equations. The first equation is given by (39) and relates the cutoff idiosyncratic productivity level, $\tilde{\omega}_t$, with our measure of credit market tightness $\tau_t = f_t / b_t^u$, the marginal cost of labor, $MC_t = w_t R_t$, which is common to all producing firms, and the aggregate component of productivity. The second equation is an Euler equation that describes the dynamics of the credit market tightness as a function of $\tilde{\omega}_t$ and a measure of aggregate output net of fixed costs.

The equation for $\tilde{\omega}_t$ is

$$\tilde{\omega}_t = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha} \xi} \frac{(MC_t)^\alpha}{z_t} \left[x^f - \left(\frac{1-\overline{\eta}\mu\tau_t^{\varphi-1}}{1-\overline{\eta}} \right) \frac{\kappa}{\mu\tau_t^\varphi} \right]^{1-\alpha}.$$

Combining the free entry condition for banks, the joint surplus of a credit relationship as well as the equation for aggregate output and the definitions for the matching rates p_t^b and p_t^f yields the following equation that characterizes the dynamics of τ :

$$\begin{aligned} &\frac{\kappa}{\mu\tau_t^\varphi} - E_t\Delta_{t,t+1}\varphi\left(\widetilde{\omega}_{t+1}\right)\left(1-\overline{\eta}\mu\tau_{t+1}^{\varphi-1}\right)\frac{\kappa}{\mu\tau_{t+1}^\varphi} \\ &= \left(1-\overline{\eta}\right) E_t\Delta_{t,t+1}\left[(1-\alpha)\frac{Y_{t+1}}{f_t^m} - \varphi\left(\widetilde{\omega}_{t+1}\right)x^f\right] \end{aligned} \tag{69}$$

2.5.2 Monetary policy

We assume that lump sum taxes adjust whenever the real supply of high powered money (or reserve balances) changes. Therefore, the consolidated government budget constraint is consistent with equation (56). The model is closed by assuming the central bank sets the growth rate for the nominal reserve balances θ_t, as well as the width of the corridor s_t to be exogenous. In addition, the central bank sets its policy rate to be the same as the interbank interest rate:

$$i_t^* = i_t \tag{70}$$

This implies that the interbank interest rate i_t is endogenous and depends on the collateral value $\chi_t = \left(i_t - i_t^b\right)/\xi_b^b$. Recall that in this case, the aggregate demand for excess reserves is $h_t^* = \frac{\bar{\varepsilon}_t}{s}\chi_t$. We assume that the volatility of the payment shock expressed as a fraction of deposits follows an exogenous process given by

$$\left(\frac{\bar{\varepsilon}_t}{\bar{\varepsilon}}\right) = \left(\frac{\bar{\varepsilon}_{t-1}}{\bar{\varepsilon}}\right)^{\rho_{\bar{\varepsilon}}} \exp\left(\epsilon_t^{\bar{\varepsilon}}\right) \tag{71}$$

where $\bar{\varepsilon}$ is the steady state value of the payment shock. Real reserve balances follows the following process

$$hp_t^s = \left(\frac{1+\theta_t}{1+\pi_t}\right) hp_{t-1}^s \tag{72}$$

where θ_t is the given by

$$\left(\frac{\theta_t}{\theta}\right) = \left(\frac{\theta_{t-1}}{\theta}\right)^{\rho_\theta} \exp\left(\epsilon_t^\theta\right) \tag{73}$$

3 Computation and simulations

The summary of the non-linear dynamic equations that characterize the aggregate equilibrium of the model is presented in appendix (4.1). We assume that anytime a credit contract ends, contractual parties are able to search for a new contract within the same period of time that the credit separation has occurred. The latter holds for exogenous as well as for endogenous separations. Then, the mass of firms searching for funds follows equation (13) while the credit creation and credit separation rates are given by equations (16) and (17), respectively.

The model is solved by using a standard perturbation method applied to a first order approximation around the non-stochastic steady state of the model. The steady state of the model is consistent with a zero inflation rate $\pi = 0$ which implies a steady state value of $\theta = 0$ according to equation (72) evaluated at the steady state.

3.1 Calibration

The calibration strategy is as follows: First we parametrize the following 12 parameters according to the standard literature as well as data for the U.S great moderation period.

Parameter	Description	Value
ρ	Reserve requirements	0.094
ξ_b	Haircut on U.S bills and bonds	0.99
ξ_L	Haircut on loans	0.65
s	Width of the symmetric corridor	0.0025
κ	Search cost for banks	1.58
$\overline{\eta}$	Firm's Nash bargaining share	0.32
φ	Elasticity of matching function w.r.t searching firms	0.5
z	Aggregate technology	1
$\overline{\omega}$	Upper support of idiosyncratic productivity	1
$\underline{\omega}$	Lower support of idiosyncratic productivity	0
η	Utility function parameter 1	1
σ	Utility function parameter 2	1

Table 1: Parameter values taken from the data and literature

The value for the ratio of reserve requirements is obtained from the Federal Reserve Board's regulation D and it is close to the 10% of liabilities requirement since the model does not take into account required reserves in the form of vault cash. On the other hand, the Federal Reserve currently sets the haircut on U.S. bills and bonds (ξ_b) with less than 5 years to maturity to 99% and the haircut on zero coupon, normal risk-rated commercial loans (ξ_l) of 5 year maturity to 65%.[21] We fix the width of the symmetric

[21] See the Fed's Discount Window and Payment System Risk Collateral Margins Table, available at

corridor s to be 25 basic points as recent experience for the U.S. shows. The search cost for banks κ and the bank's Nash bargaining share $\overline{\eta}$ are taken from the baseline calibration in Petrosky-Nadeau and Wasmer (2012). Both parameters are obtained from calculating the empirical financial sector's share of aggregate value added and matching it to their model counterpart. The data is taken from the industry value added tables provided by the Bureau of Economic Analysis over the period 1985-2002 and subtracting the share of GDP of household financial services and insurance from the National Income and Product Accounts tables. We assume that the Hosios condition holds at the steady state, implying that $\overline{\eta} = \varphi = 0.32$ but we also consider a value of $\overline{\eta} = 0.5$ as in Petrosky-Nadeau and Wasmer (2012). The level of aggregate technology in the steady state is normalized to be $z = 1$ while the support of the distribution associated to the idiosyncratic productivity is normalized to be $[\underline{\omega}, \overline{\omega}] = [0, 1]$. Finally, the assumption of $\eta = \sigma = 1$ is consistent with a logarithmic utility function for the representative household.

Second, we target the following 12 variables and ratios at steady state.

Variable	Description	Value
Y	GDP	1
N	Employment	1/3
$\frac{wN}{Y}$	Labor share	2/3
$\varphi(\widetilde{\omega})$	Continuation rate	0.7
cd	Credit destruction rate	0.029
$\frac{l}{d}$	Loan deposit ratio	0.63
h^*	Excess reserves as a fraction of deposits	0.015
i^b	Bond interest rate	0.015
i^d	Deposit rate	0.0147
i	Interbank rate	0.016
$\frac{\varphi(\widetilde{\omega}) f^m x^f}{Y}$	Fixed cost of production share of GDP	0.2
π	Inflation rate	0

Table 2: Steady state targets

$\frac{wN}{Y}$ and $\frac{\varphi(\widetilde{\omega}) f^m x^f}{Y}$ denote the labor share on GDP and the fixed cost of production share on GDP respectively. We assume the former to be 2/3 as it is standard in the literature and the latter to be 20% of GDP. The steady-state value for the continuation rate $\varphi(\widetilde{\omega})$ is taken from Chowdorow-Reich (2013) who estimates a probability between 70% and 80% that a previous loan is renovated by the same group of banks at the U.S. syndicated loan market. The steady-state credit destruction rate cd is calculated from its empirical counterpart from Contessi and Francis (2010) as an average during the great moderation period. We target a loan over deposit ratio $\frac{l}{d} = 0.63$ using quarterly data on

http://www.frbdiscountwindow.org/discountwindowbook.cfm?hdrID=14&dtlID=43

commercial and industrial loans and saving deposits for all commercial banks during 1985-2007. Excess reserves as a fraction of deposits h^* is set to be 1.5% by using the average of all quarterly reserve balances with federal reserve banks during the great moderation period. We target a deposit rate to be slightly lower than the bond rate in order to have the spread $1 + i^b - i^d$ as a tax on consumption and be consistent with the CIA framework used in the model.

Finally, we calibrate the following 12 parameters by solving the non linear steady-state of the model to be consistent with the above specified targets. The following table presents our calibration results:

Parameter	Description	Value
ξ	Scale parameter production function	3.8672
Θ	Labor supply parameter	8.1473
α	Production function elasticity	0.6769
μ	Scale parameter matching function	1.4108
δ	Exogenous separation rate at steady state	0.0799
ξ^{cia}	Residual parameter in CIA	0.9884
$\widehat{\xi}^{bs}$	Residual parameter in collateral constraint	-0.6458
β	Subjective discount factor	0.9852
$\overline{\varepsilon}$	Support of payment shock	0.0371
x^f	Fixed cost of production	0.4214
x^d	Cost of managing deposits	0.0009
θ	Nominal growth rate of reserve balances at steady state	0

Table 3: Calibrated parameter consistent with steady state targets

3.2 Model experiments

3.2.1 A payment shock in the interbank market

In this section we present the response of several variables of interest to a persistent increase in the support of the payment shock expressed as a fraction of deposits $\overline{\varepsilon}_t$ (See figures (1)-(5)). Recall that the payment shock as a fraction of deposits is assumed to be distributed according to $\varepsilon_t = \frac{\phi_t(j)}{D_t(j)} \sim Unif\left(-\overline{\varepsilon}_t, \overline{\varepsilon}_t\right)$ with density $f\left(\varepsilon_t\right) = \frac{1}{2\overline{\varepsilon}_t}$ and variance $var(\overline{\varepsilon}_t) = \frac{\overline{\varepsilon}_t^2}{3}$, where $\overline{\varepsilon}_t$ follows (71). Therefore, an unexpected increase in $\overline{\varepsilon}_t$ is interpreted as an unexpected increase in the volatility of the net payment shock. An increase in $\overline{\varepsilon}_t$ generates an increase in the aggregate demand for excess reserves as a fraction of deposits, h_t^* (See figure (1) and (2)). Given that the central bank keeps the nominal growth of high powered money constant, the latter effect leads to a persistent increase in the interbank rate as well as two interest rate spreads: The spread between the average loan rate and the opportunity cost of lending and the spread between the

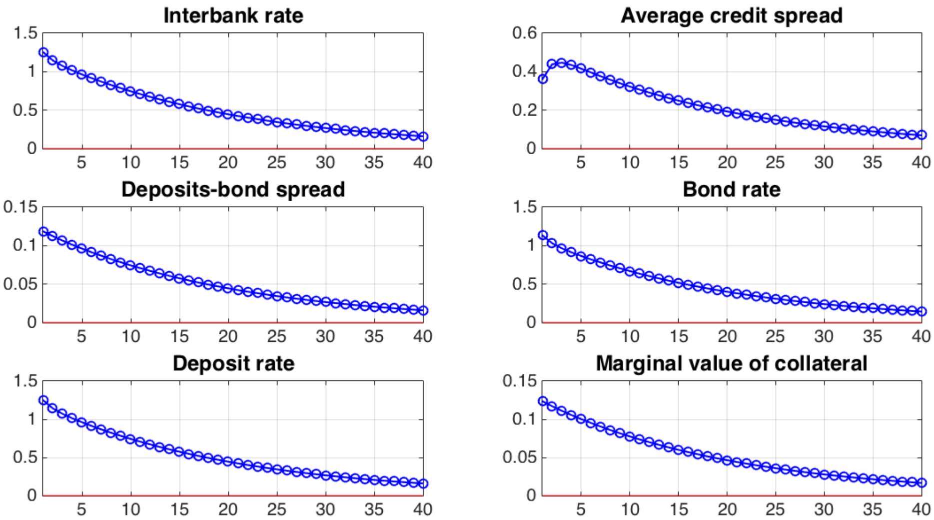

Figure 1: Model responses to a payment shock: A
Note: Source: Authors' calculations based model simulation

interbank rate and the bond rate (see figure (1)). The latter spread produces a significant increase in the marginal value of having collateral in the form of excess reserves χ_t relative to bonds and loans reinforcing the increase in h_t^*. The former spread reflects a persistent tightening in credit conditions as a consequence of the decrease in the joint surplus to a loan contract. In this new scenario, banks allocate more funds into excess reserves as a precautionary motive since the marginal value of having additional collateral in the form of excess reserves improves relative to other collateralizable assets. This effect is strong enough that banks reduce their holdings of government bonds despite the fact that the bond rate i_t^b increases. Since government bonds are assumed to be fixed in net supply, in accordance with the consolidated government budget constraint is given by (56), the representative household must increase its holdings of government bonds. The increase in the deposit and bond rates motivates households to save, which ultimately results in a persistent increase in aggregate deposits with the banking sector (see figure (2)). The response of interest rates and spreads configure a higher opportunity cost of lending, R_t. Moreover, in accordance with the aggregate balance sheet of banks, the banking sector reduces not only its government bond holdings but also its lending. This is a direct consequence of the persistent increase in the marginal value of collateral, inducing banks to hold more excess reserves at the expense of government bonds and loans (see figure (2)). The initial reduction in bank lending is further amplified in the credit market due to the presence of search and matching frictions in the form of effects on the intensive and extensive margin.

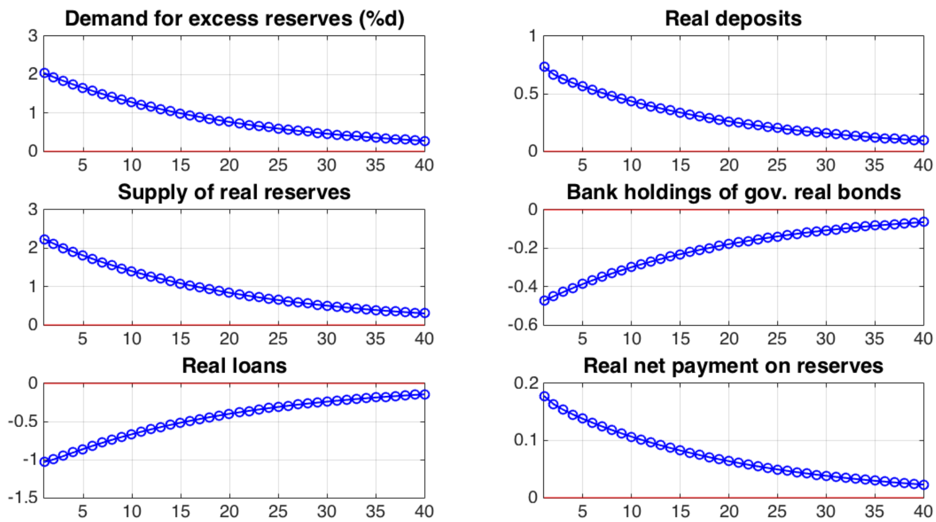

Figure 2: Model responses to a payment shock: B
Note: Source: Authors' calculations based model simulation

As a result of the persistent increase in the real marginal cost of labor $MC_t = w_t R_t$, the reservation productivity $\widetilde{\omega}_t$ rises. The Nash bargaining solution requires firms to equate their marginal product of labor to the real marginal cost of labor, and therefore a higher MC_t will induce active firms to reduce employment (intensive margin effect). The increase in $\widetilde{\omega}_t$ and in MC_t, induce a negative response on the joint surplus of a credit relationship as well as on the overall probability of continuation for credit contracts $\varphi_t(\widetilde{\omega}_t)$ (see figure (3)). A fraction of the mass of banks searching for borrowers exit the loan market, and therefore less banks are searching for borrowers (a temporary fall in b_t^u) while more firms are searching for lenders (a persistent rise in f_t). This means that the matching rate for firms falls but the corresponding matching rate for banks rises as a response to the initial shock. Therefore, the credit market becomes tighter (a persistent rise in τ_t), meaning that credit market conditions are worse from the point of view of firms, reinforcing the initial effect over $\widetilde{\omega}_t$, and reducing the mass of active firms at the end of each period f_{t-1}^m (extensive margin effect). These effects are also reflected in a higher credit destruction rate, a lower credit creation rate and an increase in the average credit spread (see figure (4)).

Finally, the payment shock affect the real side of the economy as a deep and prolonged recession: aggregate output, employment and consumption persistently fall together, concurrent with an increase in household savings and a sharp drop in bank intermediation. The initial shock is amplified through a persistent fall in the "credit" input, that is, the term F_t that appears in the aggregate production function (see figure (5)). Therefore, in

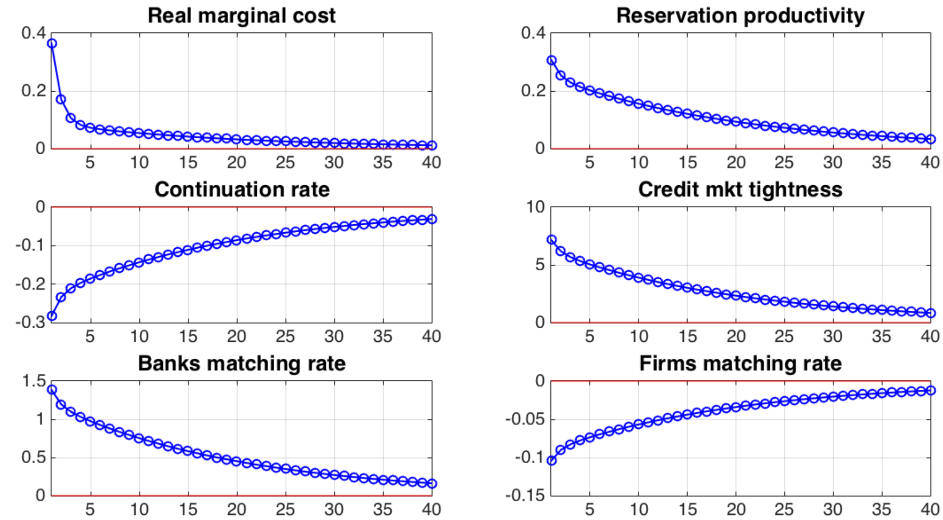

Figure 3: Model responses to a payment shock: C
Note: Source: Authors' calculations based model simulation

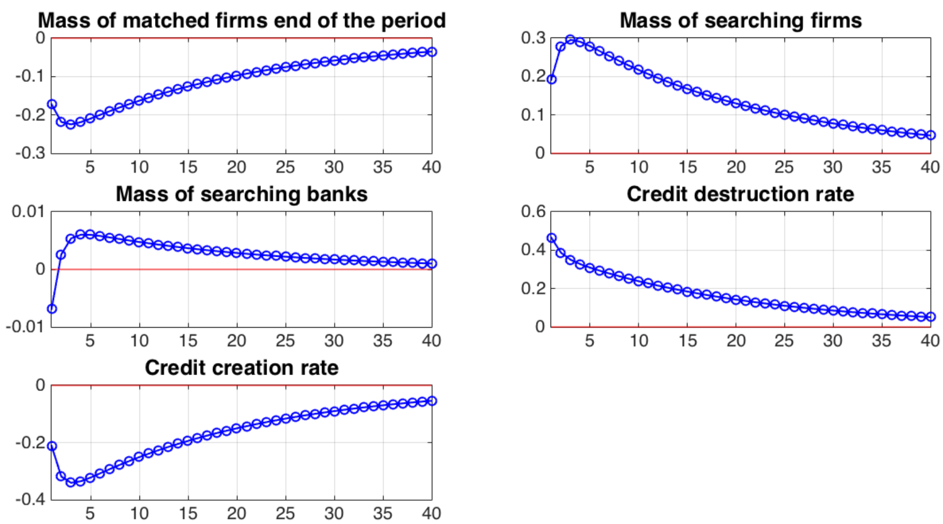

Figure 4: Model responses to a payment shock: D
Note: Source: Authors' calculations based model simulation

our setting, any shock that affects the interbank market produces inefficient fluctuations in the loan market that are propagated to the aggregate economy because of the existence of credit frictions.

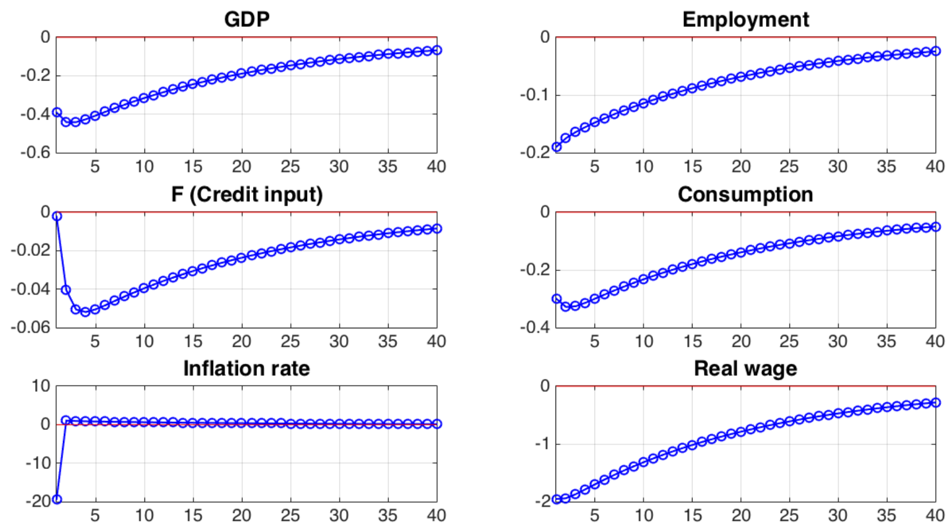

Figure 5: Model responses to a payment shock: E
Note: Source: Authors' calculations based model simulation

3.2.2 A financial shock in the loan market

Figures (6)-(10) illustrate the dynamic effects of a financial shock on a number of aggregate variables. A financial shock is defined as an unexpected persistent increase in the exogenous portion of the separation rate for credit contracts. Recall that the overall continuation rate of loan contracts (equation (62)) has both an exogenous as well as an endogenous component. We assume that the exogenous component follows a non-linear autoregressive process given by

$$\left(\frac{\delta_t}{\delta^{ss}}\right) = \left(\frac{\delta_{t-1}}{\delta^{ss}}\right)^{\rho_\delta} \exp\left(\epsilon_t^\delta\right),$$

so that equation (62) takes the following form:

$$\varphi_t\left(\widetilde{\omega}_t\right) = \left(1-\delta_t\right)\left(\frac{\overline{\omega}-\widetilde{\omega}_t}{\overline{\omega}-\underline{\omega}}\right).$$

The increase in δ_t implies that a fraction of existing credit contracts are exogenously terminated due to the decline in $\varphi(\widetilde{\omega}_t)$. There will be a larger mass of firms searching for funds f_t as well as a larger mass of banks searching for profitable projects to fund b_t^u (see figure (6)). Free entry of banks in to the loan market implies that the fraction of banks that was previously engaged in a loan contract will decide to exit the market while the other fraction will stay in the loan market and search for potential borrowers. Firms that were previously engaged in a credit contract are not able to exit the loan market implying that all firms separated, due to the initial shock, will start searching for external funding. Then, at impact, the new mass of firms searching for lenders will exceed the new mass of banks searching for borrowers, inducing a persistent increase in credit market tightness τ_t. These new credit conditions in the loan market are exhibited by a decline in the firm's matching probability p_t^f and an increase in the bank's matching probability p_t^f.

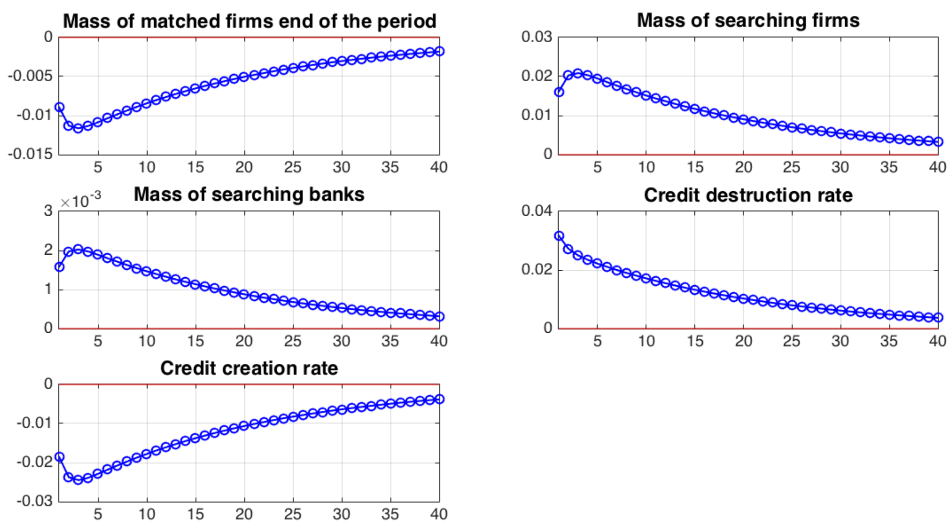

Figure 6: Model responses to a financial shock: A
Note: Source: Authors' calculations based model simulation

The financial shock is propagated through the intensive and extensive margins with opposite effects over the employment decision facing active firms. Our calibration results in a stronger employment effect from the intensive margin relative to the corresponding extensive margin. On one hand, the mass of firms and banks that start the period in a credit contract, but also survive the higher separation rate that occurs after the financial shock, will decide to raise their reservation productivity threshold $\widetilde{\omega}_t$ as a response to a decline in the joint surplus to a credit relationship and a tighter credit market (see figure (7)). This is an extensive margin effect, associated with a selection effect that reduces the subset of firms able to obtain external funds, hire workers and produce. On the other hand, the financial shock reduces the real marginal cost of labor, inducing surviving firms to hire more workers than before. Figure (10) shows that the net effect over employment is slightly positive since the negative response of MC_t overpowers the increase in τ_t (see figure (7)). We suspect that the latter occurs due to the presence of wage flexibility. Despite the small increase in employment, aggregate loans fall since tighter credit conditions reduce the joint surplus to a credit relationship, which in turn is reflected in less intermediation by banks and a higher average loan rate spread (average credit spread). In addition, the response of gross credit flows are also in line with conditions tightening in the loan market: The rate of credit creation falls while the credit destruction rate rises as a consequence of the financial shock.

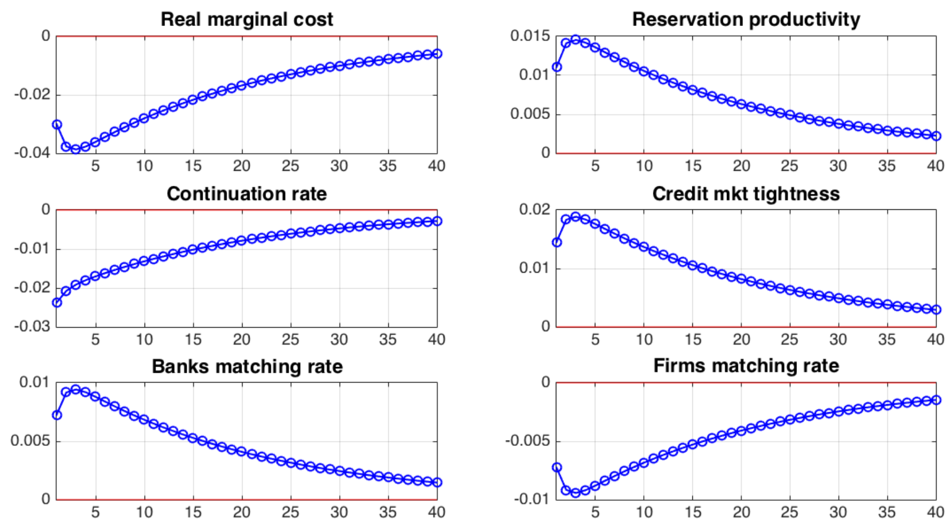

Figure 7: Model responses to a financial shock: B
Note: Source: Authors' calculations based model simulation

The shock in the loan market is transmitted to the interbank market via a portfolio reconfiguration that banks perform in response to the financial shock. Banks realize that the marginal value of collateral χ_t suddenly falls after the financial shock, inducing a higher demand for government bonds at the expense of excess reserves and loans. The lower demand for excess reserves, together with a fixed nominal supply of high powered money balances, reduces the interbank rate more than the consequent reduction in the bond rate. In summary, the financial shock that started in the loan market also affects the interbank market by inducing banks to switch their portfolios towards government bonds and less excess reserves as well as loans (see figure (8) and (9)).

Since government bonds are assumed to be in fixed net supply, households will reduce their demand for government bonds as well as their bank deposits. The latter occurs

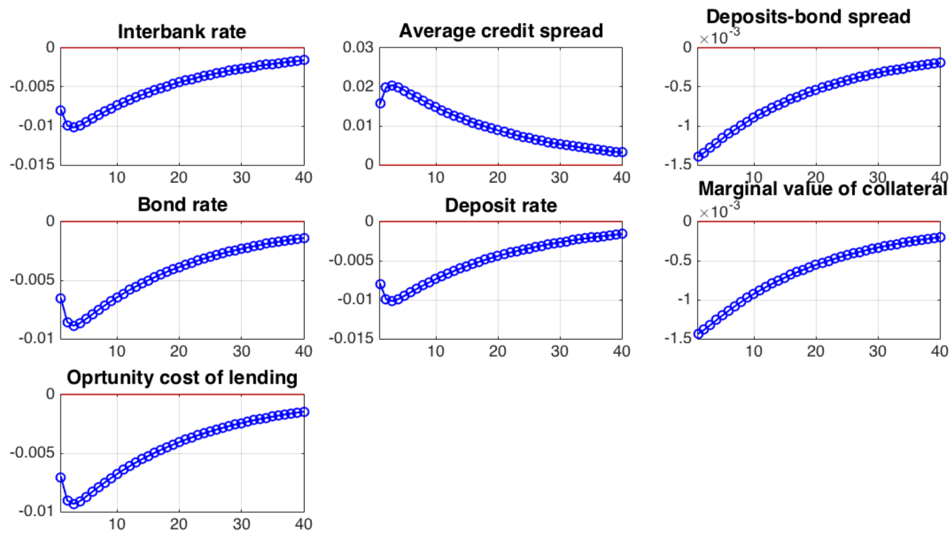

Figure 8: Model responses to a financial shock: C
Note: Source: Authors' calculations based model simulation

because the spread between the deposit rate and the bond rate falls. Therefore, banks end the period with fewer funds, which in turn reduces lending even more (see figure (8)).

At the aggregate level, a persistent adverse financial shock results in a decrease in GDP and consumption but a slight improvement in aggregate employment (see figure (10)). Such responses are a consequence of aggregating the intensive and extensive margin effects that are generated in the loan market due to the presence of lending frictions together with changes in the marginal value of collateral. A financial shock, modeled as an exogenous persistent increase in the separation rate of credit contracts, generates a deep and prolonged recession in terms of GDP and consumption together with a higher average credit spread and a lower marginal value of collateral. Therefore, the shock that was originated in the loan market affects the interbank market as a fall in aggregate excess reserves. As in the case of the payment shock, the transmission and propagation of a financial shock is through a persistent decline in the aggregate "credit" input, F_t, that affects the aggregate production function (see figure (10)).

4 Conclusions

In this paper, we study the links between the central bank's operating procedures in the interbank market, the availability of credit, and the impact of monetary policy on the real

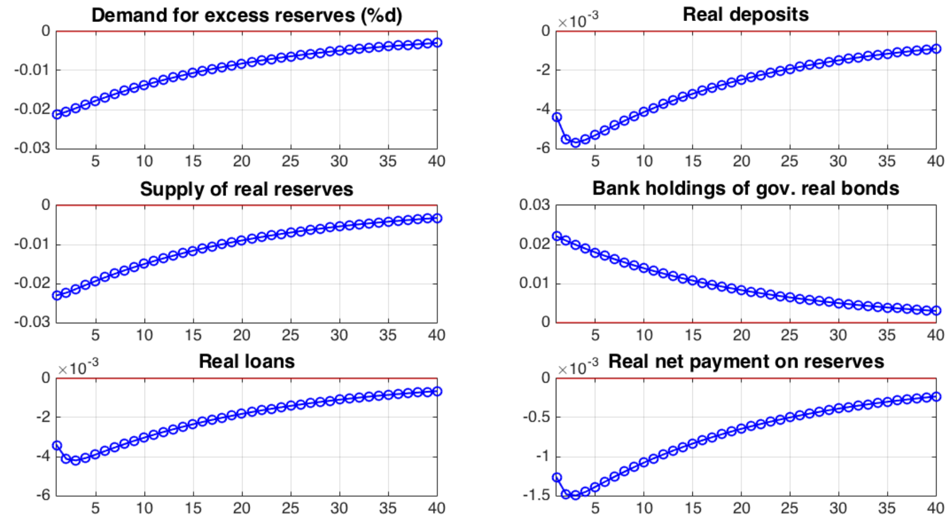

Figure 9: Model responses to a financial shock: D
Note: Source: Authors' calculations based model simulation

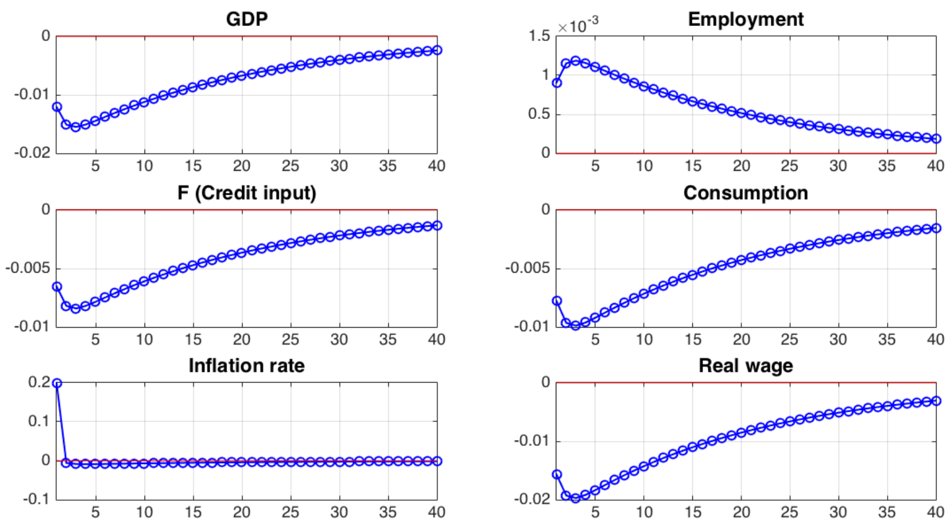

Figure 10: Model responses to a financial shock: E
Note: Source: Authors' calculations based model simulation

economy. To do so, we integrate two branches of the literature by incorporating a channel model of the reserve market with a credit market characterized by matching frictions and bilateral bargaining between lenders (banks) and borrowers (firms). The resulting general equilibrium framework was used to investigate the effects of alternative operating procedures on the economy's response to two sets of shocks; the first set disrupt the survival rate of credit matches while the second alter the volatility of the flow of repayments commercial banks face as a result of their participation in the interbank market.

The impulse responses presented show that both shocks are transmitted through the interaction of three main variables: 1) The marginal value of having collateral in the form of excess reserves relative to other collateralizable assets that are not affected by haircuts, 2) The real marginal cost of labor and 3) The reservation productivity threshold of a loan contract. The last two variables generate intensive and extensive margin effects of financial shocks over employment decisions and are a direct consequence of the way the joint surplus to a credit relationship responds. In addition, both shocks generate deep and prolonged recessions that are characterized by sharp drops in an inefficiency wedge that appears in the aggregate production function of the economy (the "credit input" term:F_t). Financial shocks are amplified by the implied dynamics of this term, producing inefficient responses on the aggregate equilibrium of the economy.In future versions of the paper, policy experiments, where the central bank responds by using different instruments will be included.

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4.1 Characterization of the aggregate non-linear equilibrium

Lower case letters denote real variables $x = \frac{X}{P}$. Variables written as $\widehat{X}$ are expressed as fraction of deposits, that is $\widehat{X} = \frac{X}{D} = \frac{x}{d}$

We assume that household's period utility function is

$$U\left(C_t, 1 - N_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \Theta \frac{N_t^{1+\eta}}{1+\eta}$$

The aggregate equilibirum of the model is characterized by the following dynamical system of non-linear equations:

1. **Nominal supply of reserves:**

$$HP_t^s = \left(1 + \theta_t\right) HP_{t-1}^s \tag{D1.1}$$

where θ_t is given by

$$\left(\frac{\theta_t}{\theta^{ss}}\right) = \left(\frac{\theta_{t-1}}{\theta^{ss}}\right)^{\rho_\theta} \exp\left(\epsilon_t^\theta\right) \tag{D1.2}$$

2. **Policy rate:**

$$i_t = i_t^* \tag{D2}$$

3. **Gross inflation:**

$$1 + \pi_t = \frac{P_t}{P_{t-1}} \tag{D3}$$

4. **Euler equation:**

$$\lambda_t = \beta \mathrm{E}_t \left(\frac{1 + i_{t+1}^b}{1 + \pi_{t+1}} \lambda_{t+1}\right) \tag{D4}$$

5. **Marginal utility of income:**

$$\lambda_t = \frac{C_t^{-\sigma}}{1 + i_t^b - i_t^d} \tag{D5}$$

6. **The marginal value of collateral:**

$$\chi_t = \frac{i_t - i_t^b}{\xi_b} \tag{D6}$$

7. **Interest rate on deposits:**

$$i_t^d = i_t\left(1 - \rho\right) + \left(i_t^* - s_t\right)\rho - x^d - \chi_t \bar{\varepsilon}_t + \frac{1}{2}\frac{\bar{\varepsilon}_t}{s_t}\left(i_t^* - i_t + \chi_t\right)^2 - \frac{s_t \bar{\varepsilon}_t}{2}$$

using the definition of h^* we have

$$i_t^d = i_t\left(1-\rho\right) + \left(i_t^* - s_t\right)\rho - x^d - \chi_t\bar{\varepsilon}_t + \frac{1}{2}\frac{s_t}{\bar{\varepsilon}_t}\left(h^*\right)^2 - \frac{s_t\bar{\varepsilon}_t}{2} \tag{D7}$$

8. **Interbank market equilibrium:** In real terms

$$hp_t^s = \left(h_t^* + \rho\right)d_t \tag{D8}$$

where

$$hp_t^s = \frac{HP_t^s}{P_t} \tag{D8.1}$$

as a fraction of deposits:

$$\widehat{hp}_t^s = h_t^* + \rho$$

where

$$\widehat{hp}_t^s = \frac{hp_t^s}{d_t}$$

or in nominal terms:

$$HP_t^s = h_t^* D_t + \rho D_t$$

9. **The demand for excess reserves:**

$$h_t^* = \frac{\overline{\varepsilon_t}}{s}\left(i_t^* - i_t + \chi_t\right) \tag{D9}$$

notice that h_t^* is expressed as a fraction of deposits and that $\overline{\varepsilon_t}$ follows the following process:

$$\left(\frac{\bar{\varepsilon}_t}{\bar{\varepsilon}}\right) = \left(\frac{\bar{\varepsilon}_{t-1}}{\bar{\varepsilon}}\right)^{\rho_{\bar{\varepsilon}}} \exp\left(\epsilon_t^{\bar{\varepsilon}}\right)$$

10. **Consolidated gov. budget constraint:** In real terms

$$T_t + \left(hp_t^s - \left(\frac{1}{1+\pi_t}\right)hp_{t-1}^s\right) = x_t + f_t \tag{D10}$$

where f_t^g is treated as exogeonus and given by

$$f_t = G_t + \left(b_t^{cb} - \left(\frac{1}{1+\pi_t}\right)b_{t-1}^{cb} - i_t^b b_t^{cb}\right) - \left(b_t^T - \left(\frac{1}{1+\pi_t}\right)b_{t-1}^T - i_t^b b_t^T\right)$$

As a fraction of deposits:

$$\frac{T_t}{d_t} + \left(\frac{hp_t^s}{d_t} - \left(\frac{1}{1+\pi_t}\right)\frac{hp_{t-1}^s}{d_{t-1}}\frac{d_{t-1}}{d_t}\right) = \frac{x_t}{d_t} + \frac{f_t}{d_t}$$

or

$$\widehat{T}_t + \left(\widehat{hp}_t^s - \left(\frac{1}{1+\pi_t}\right)\widehat{hp}_t^s \frac{d_{t-1}}{d_t}\right) = \widehat{x}_t + \widehat{f}_t^g$$

11. **Central bank's net payment of interest on reserves:**

$$x_t = (i_t^* - s_t)(\rho d_t + er_t) + (i_t^* + s_t)\, br_t \tag{D11}$$

as a fraction of deposits

$$\widehat{x}_t = (i_t^* - s)(\rho + \widehat{er}_t) + (i_t^* + s)\,\widehat{br}_t$$

12. **Aggregate excess reserves in the bankong system:**

$$er_t = \left(\frac{(h_t^*)^2}{4\overline{\varepsilon}_t} + \frac{h_t^*}{2} + \frac{\overline{\varepsilon}_t}{4}\right) d_t \tag{D12}$$

as a fraction of deposits

$$\widehat{er}_t = \frac{(h_t^*)^2}{4\overline{\varepsilon}} + \frac{h_t^*}{2} + \frac{\overline{\varepsilon}}{4}$$

13. **Aggregate borrowed reserves in the banking system:**

$$br_t = \left(-\frac{(h_t^*)^2}{4\overline{\varepsilon}_t} + \frac{h_t^*}{2} - \frac{\overline{\varepsilon}_t}{4}\right) d_t \tag{D13}$$

as a fraction of deposits

$$\widehat{br}_t = -\frac{(h_t^*)^2}{4\overline{\varepsilon}} + \frac{h_t^*}{2} - \frac{\overline{\varepsilon}}{4}$$

14. **Aggregate collateral constraint:**

$$h_t^* d_t + \xi_b b_t^b + \xi_L l_t = \overline{\varepsilon}_t d_t + \xi^{bs} \tag{D14}$$

as a fraction of deposits

$$h_t^* + \xi_b \widehat{b}_t^b + \xi_L \widehat{l}_t = \overline{\varepsilon} + \widehat{\xi}^{bs}$$

15. **Aggregate banks balance sheet:**

$$l_t + b_t^b + h_t^* d_t = (1-\rho)\, d_t \tag{D15}$$

as a fraction of deposits

$$\widehat{l}_t + \widehat{b}_t^b + h_t^* = (1-\rho)$$

16. **The CIA constraint:**

$$C_t = d_t + w_t N_t + \xi^{cia} \tag{D16}$$

17. **Labor supply:**

$$\Theta N_t^{\eta} C_t^{\sigma} = w_t \tag{D17}$$

18. **Aggregate loans:**

$$l_t = w_t N_t \tag{D18.1}$$

if interbank market is written as a fraction of deposits then we need the following equation

$$l_t = \widehat{l}_t d_t \tag{D18.2}$$

19. **Aggregate resource constraint of the economy:**

$$Y_t = C_t + G_t + \varphi_t(\widetilde{\omega}_t) f_{t-1}^m x^f + \kappa b_t^u + x^d d_t \tag{D19}$$

where G_t is exogenous.

20. **Oportunity cost of lending:**

$$R_t = 1 + i_t - \chi_t \xi_L \tag{D20}$$

21. **Aggregate employment:**

$$N_t = \left(\frac{\alpha z_t \xi}{w_t R_t}\right)^{\frac{1}{1-\alpha}} F_t \tag{D21}$$

22. **Aggregate output:**

$$Y_t = z_t \xi (F_t)^{1-\alpha} (N_t)^{\alpha} \tag{D22}$$

23. **Credit market tightness:**

$$\tau_t = \frac{f_t}{b_t^u} \tag{D23}$$

24. **Number of firms in a credit relationship:**

$$f_t^m = \varphi_t(\widetilde{\omega}_t) f_{t-1}^m + p_t^f f_t \tag{D24}$$

25. **Number of firms searching for workers:**

$$f_t = 1 - \varphi_t(\widetilde{\omega}_t) f_{t-1}^m \tag{D25}$$

26. **Continuation rate:**

$$\varphi_t\left(\widetilde{\omega}_t\right) = \left(1-\delta_t\right)\left(\frac{\overline{\omega}-\widetilde{\omega}_t}{\overline{\omega}-\underline{\omega}}\right) \tag{D26}$$

where δ_t follows an AR(1) process given by

$$\left(\frac{\delta_t}{\delta^{ss}}\right) = \left(\frac{\delta_{t-1}}{\delta^{ss}}\right)^{\rho_\delta} \exp\left(\epsilon_t^\delta\right)$$

27. **Credit input:** F_t :

$$F_t = \left(1-\delta_t\right)\left(\frac{\left(\overline{\omega}\right)^k - \left(\widetilde{\omega}_t\right)^k}{k\left(\overline{\omega}-\underline{\omega}\right)}\right) f_{t-1}^m \tag{D27}$$

28. **Cut-off productivity level:**

$$\left[\alpha^\alpha\left(1-\alpha\right)^{1-\alpha}\xi z_t\widetilde{\omega}_t\right]^{\frac{1}{1-\alpha}} = \left(MC_t\right)^{\frac{\alpha}{1-\alpha}}\left[x^f - \left(\frac{1-\overline{\eta}p_t^f}{1-\overline{\eta}}\right)\frac{\kappa}{p_t^b}\right] \tag{D28}$$

29. **Credit market tightness:**In terms of τ_t:

$$\begin{aligned}&\frac{\kappa}{\mu\left(\tau_t\right)^\varphi} - \mathrm{E}_t\Delta_{t,t+1}\varphi_{t+1}\left(\tilde{\omega}_{t+1}\right)\left(1-\overline{\eta}\mu\left(\tau_{t+1}\right)^{\varphi-1}\right)\frac{\kappa}{\mu\left(\tau_{t+1}\right)^\varphi}\\ &= \left(1-\overline{\eta}\right)\mathrm{E}_t\Delta_{t,t+1}\left(\left(1-\alpha\right)\frac{Y_{t+1}}{f_t^m} - \varphi_{t+1}\left(\tilde{\omega}_{t+1}\right)x^f\right)\end{aligned}$$

30. **Stochastic discount factor:**

$$\Delta_{t,t+1} = \beta\frac{\lambda_{t+1}}{\lambda_t} \tag{D30}$$

31. **Credit destruction rate:**

$$cd_t = 1 - \varphi_t\left(\widetilde{\omega}_t\right) - p_t^f\varphi_t\left(\widetilde{\omega}_t\right) \tag{D31}$$

32. **Credit creation rate:**

$$cc_t = \frac{m_t}{f_{t-1}^m} - p_t^f\varphi_t\left(\widetilde{\omega}_t\right) \tag{D32}$$

where the flow of new matches m_t is

$$m_t = \mu_t f_t^\varphi\left(b_t^u\right)^{1-\varphi}$$

then cc_t can be written as

$$cc_t = \frac{\mu_t \tau_t^{\varphi} b_t^u}{f_{t-1}^m} - p_t^f \varphi_t \left(\widetilde{\omega}_t\right)$$

33. **Matching rate for firms:**

$$p_t^f = \mu \left(\tau_t\right)^{\varphi - 1} \tag{D33}$$

34. **Matching rate for banks:**

$$p_t^b = \mu \left(\tau_t\right)^{\varphi} \tag{D34}$$

35. **Gross real interest rate:**

$$1 + r_t = \frac{1 + i_{t+1}^b}{1 + \pi_{t+1}} \tag{D35}$$

36. **Average credit spread:**

$$\frac{R_t^l l_t - R_t l_t}{l_t} = \left(1 - \alpha\right) \left(1 - \overline{\eta}\right) \frac{Y_t}{\varphi_t \left(\tilde{\omega}_t\right) f_{t-1}^m l_t} - \left(\frac{\left(1 - \overline{\eta}\right) x^f + \overline{\eta} \frac{\kappa}{\tau_t}}{l_t}\right) \tag{D36}$$

37. **Real marginal cost:**

$$MC_t = w_t R_t \tag{D37}$$