BANCO CENTRAL DE RESERVA DEL PERÚ

Income Distribution and Endogenous Dollarisation

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Income Distribution and Endogenous Dollarisation.*

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Abstract

In this paper we develop a monetary economy model where dollarisation emerges endogenously as an optimal decision of individuals and firms in an environment where the exchange rate is uncertain and individuals are heterogenous in their asset holdings and in their consumption baskets. We show that in this environment income distribution plays a key role in explaining the pattern of price dollarisation and its links with asset dollarisation. The model shows that for economies with relatively high income inequality, price dollarisation is not important at the aggregate level, even when asset dollarisation is high. In this case, only luxury goods, those associated to the consumption basket of high-income customers, are endogenously priced in foreign currency, whilst necessity goods, those associated to the consumption basket of low-income customers, are priced in domestic currency. This result may explain why in countries with remarkably high levels of asset dollarisation, countries like Argentina, Bolivia and Peru, the levels of transaction and price dollarisation are relatively low. We also show that asset dollarisation causes price dollarisation and that the relationship depends on income distribution.

Keywords: Dollarisation, Income Inequality, non-homothetic preferences, monetary policy.

JEL Classification: D11, D31, D42, D50, E40.

1 Introduction

A history of monetary mismanagement and episodes of hyperinflation, especially during the eighties and in some cases during the nineties, transformed the monetary systems of many

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emerging economies into defacto bimonetary systems. Argentina, Bolivia, Peru, Uruguay, Turkey, and more recently Russia are amongst those economies where the domestic currency have been partially replaced in their functions of reserve of value, medium of payment and unit of account by a foreign currency, usually the US Dollar, a phenomenon known in the literature as dollarisation[1].The process of dollarisation has a well documented pattern: usually a foreign currency is used first as reserve of value, then as a medium of payment in some transactions, particularly large transactions, and finally as unit of account.

Nowadays, even after several years of low and stable inflation, the dollarisation levels remain high in most of these countries. However, the levels of asset dollarisation, measured by the proportion of deposits or bank loans in dollars, tend to be much higher that the levels of transaction dollarisation, usually measured by the most liquid component of deposits[2]. Not only dollarisation is different across assets and transactions, but also amongst types of transactions. It is observed that the dollar seems to dominate transactions associated to consumption of high income customers, whilst transactions and prices of goods associated to consumption of low income customers, like necessity goods, tend to be in domestic currency. This is independent on whether the goods are tradable or not tradable, durable or nor durables or on the size of the transaction[3]

The distinction between different types of dollarisation and the links between them have crucial implications for monetary policy and macroeconomic performance. As Ize and Yeyati (2002) point out, whilst asset dollarisation could affect seriously the transmission mechanism of monetary policy and make the financial system more vulnerable to exchange rate fluctuations, it is price dollarisation what ultimately determines the effectiveness of monetary policy. In an economy where most of the prices are set in foreign currency, prices become perfectly indexed to the exchange rate eliminating the short-run effects of monetary policy. Moreover, understanding the pattern of price and transaction dollarisation and their links with asset dollarisation can be useful to guide policy makers in their attempts to implement policies

[1] Through this paper we distinguish among three different concepts of dollarisation: Transaction Dollarisation - TD, the substitution of domestic currency as medium of payment, asset dollarisation - AD, the substitution of domestic currency as reserve of value, and price dollarisation-PD, the substitution of domestic currency as unit of account.

[2]Honanhan and Shi (2002) provide indirect evidence of low levels of price dollarisation in countries with high levels of asset dollarisation. They measure price dollarisation by the short-run level of pass-through of the exchange rate. Also, see , Armas et al, (2001), Miller (2003) and Winkelried (2002) for estimations of pass through for Peru.

[3]For instance in Peru, firms offering education services set prices in different currencies depending on the location of the institution, in rich neighbourhoods prices are in dollars, whilst in poor ones prices are in pesos. Moreover, small transactions like haircuts are charged in dollars in some beauty shops located in rich neighbourhoods, and big transactions, like real states, are priced in pesos in poor areas.

aimed at reducing dollarisation.

In this paper, we provide a theory of endogenous dollarisation of assets and prices that explains the pattern of dollarisation across types of goods and the links between them. The model combines dollarisation decisions of individuals and invoicing decisions of firms into a general equilibrium cash-in-advance monetary model where the only source of uncertainty is the value of the exchange rate. Individuals and firms take decisions before observing the realisation of the exchange rate. In modeling individuals' dollarisation decisions we follow Chatterjee and Corbae (1992) in that a fixed cost of accessing to financial markets determines endogenously the market participation of agents. In our setting, individuals have to pay a fixed cost to dollarise their assets, therefore only those agents with levels of income high enough to pay the cost dollarise their assets. This simple assumption generates the result that not all agents in the economy dollarise their assets, but only those who can afford it, thus income distribution plays an important role in explaining the extent of asset dollarisation.

On the other hand, firms decide the currency in which to set their prices, maximizing the expected value of their profits. The choice of the price denomination, the invoicing problem, is not trivial under uncertainty, since in this case the expected value of profits depends on the currency denomination[4] . Furthermore, we choose to deviate from a standard formulation of preferences, introducing non-homotheticity[5] which allows us to generate endogenous heterogeneity in the demand for goods, where demand and price elasticity depend on the income distribution[6].

A key feature of the model is that individuals consume different number of goods, and consequently each firm sells its good not to every individual in the economy, but only to those who can afford it. We show that with non-homothetic preferences and some degree of asset dollarisation, a group of firms is willing to set prices in foreign currency[7]. The decision

[4] One of the first works in invoicing decision theory is Klemperer and Meyer (1986), who discuss the decision between Cournot and Bertrand oligopoly competition. Other papers, such as: Giovannini (1988), Donnafeld and Zilcha (1991), Friberg (1998), Johnson and Pick (1997) and Bacchetta and van Wincoop (2001), study the decision of pricing in the exporter's or the importer's currency under international trade.

[5] A set of preferences is said to be non-homothetic if it exhibits non linear Engel's curves, i.e. the expenditure in good i increases non linearly with income. With homothetic preferences, for some normalization of the utility function, doubling quantities doubles utility therefore, Engel's curves are lines that go through the origin, thus expenditure in good i increases linearly with income. We follow the preference-setting of Matsuyama (2002), in which individuals can consume just one unit of each good and goods are not substitutes. This setup allows to relate explicitly the demand for each good to the income distribution, which simplifies the analysis.

[6] This feature is in contrast to the case of homothetic preferences, where only the average level of income in the economy determines the demand for goods, and other moments of income distribution do not play any role.

[7] With homothetic preferences, only in the case of increasing marginal costs some firms find optimal to set prices in foreign currency. With constant marginal costs, firms always choose set prices in domestic currency

of some individuals to dollarise their income generates a correlation between the demand of some goods[8], those whose demand is concentrated in individuals with assets in dollars, and the exchange rate. Moreover, this correlation is what gives the incentives to set prices in foreign currency: for those goods, when prices are set in pesos, fluctuations in the exchange rate generate volatility of demand. The firm can stabilize its demand by setting its price in dollars, increasing its expected profits. When the demand of the firm's good is concentrated in individuals with assets only in pesos the correlation between the exchange rate and demand for goods is close to zero, therefore the firm doesn't have any incentive to set prices in dollars.

The general equilibrium shows that asset dollarisation causes price dollarisation and that income distribution plays an important role in explaining the pattern of price dollarisation across type of goods. In particular, we find that for income distributions that show some degree of inequality, necessity goods, those associated with the consumption of low-income customers are endogenously priced in pesos[9], whereas luxury goods have prices in dollars. Moreover, the model shows that asset dollarisation is bigger than price dollarisation and the gap between them is increasing in the degree of inequality.

Our model is related to the work of Sturzeneger (1997) and Ize and Parrado (2002)[10]. In these two papers, endogenous dollarisation decisions are analysed but in different frameworks. Sturzeneger (1997) uses an endogenous cash-in-advance model to analyse the welfare implications of endogenous currency substitution. In his framework, the size of the transaction is the key feature in explaining the pattern of dollarisation. Agents decide the currency in which to trade comparing the fixed cost that implies trading in dollars with the cost of trading in domestic currency, the inflation tax. As the inflation tax is proportional to the value of the transaction, they show that expensive goods are endogenously traded in foreign currency since the benefit of trading with this currency (avoiding the inflation tax) exceeds its cost. On the contrary, with cheap goods the cost of trading in dollars is higher than the inflation tax, therefore the transaction is made using domestic currency.

This approach, however, does not explain why small transactions associated with high-income customers are made in foreign currency. We instead consider that the most important

(see Bacheta and Wincoop, 2001). This, however is not the case with non homothetic preferences, in this case firms may set prices in dollars independently of the nature of marginal costs.

[8] Those goods whose demand come mainly from individuals that have dollarised their assets.

[9] For the sake of simplicity, we name the domestic currency as "Pesos" and the foreign currency as "Dollars".

[10] Other recent papers on dollarisation but not directly related to our paper are Calvo (2002), Chang and Velasco(2001), which focus more on the effects of liabilities dollarisation on the transmission mechanism of monetary policy and in exchange rates regimes.

element in determining dollarisation patterns is not the size of the transaction, but the interaction between the level of income of customers and the optimal strategies of firms in setting prices[11]. This interaction implies that price dollarisation is not independent of asset dollarisation as in the model of Sturzeneger (1997) but that asset dollarisation causes price dollarisation.

On the other hand, Ize and Parrado (2002) use a representative agent general equilibrium model to analyse the interaction between price dollarisation, asset dollarisation and monetary policy. In their model, asset and price dollarisation are endogenous decisions based on minimum variance portfolios. They find that both asset and price dollarisation respond to the variance of real exchange rate and inflation, but price dollarisation also responds to monetary policy and to the nature of the shocks. In the same direction, more recently Chang and Velasco (2004) propose a model where optimal portfolio decisions of agents interact with optimal monetary policy, asset dollarisation emerges endogenously when policy is expected to be committed to fix the exchange rate. However, also multiple equilibriums are possible, in particular equilibriums where there is no asset dollarisation. Although these papers have very interesting insights on the interaction between dollarisation and optimal monetary policy, since they are representative agent models, they are not able to explain the pattern of dollarisation across types of goods.

We consider that this paper fills some of the gaps left by the previous literature. In particular, the model explains, in a simple way, the pattern of price and asset dollarisation across type of goods and agents, and also provides a theoretical link between asset and price dollarisation. Moreover, our results suggest that policy makers who are looking for policies aiming at reducing dollarisation should focus mainly on reducing asset dollarisation[12], since price and transactions dollarisation will endogenously follow the pattern of asset dollarisation. Furthermore, our model is able to explain why high levels of asset dollarisation may coexist with low levels of price dollarisation.

The paper is organized as follows: in section 2 we present the general equilibrium cash-in-advance model without considering dollarisation decisions. In section 3, we extend this framework to include the dollarisation decision of individuals and firms, and the general equilibrium with dollarisation is analysed. In section 4 we discuss the link between asset and price

[11] An example of this type of interaction is the pricing strategy for mobile phones used by the biggest telephone company in Peru during the nineties. When mobile phones were introduced into the market the service was priced in dollars, at that time mobile phones were considered a luxury good. As competition increased and mobile phones become cheaper, therefore less of a luxury good, a competitor company started pricing mobiles phones in domestic currency.

[12] However is worth to mention that we are not suggesting that reducing price dollarisation is an optimal policy to pursue. In the paper we do not take the issue of welfare implications of price dollarisation, we left that issue for future research.

dollarisation. Section 5 concludes. The proofs of the propositions are detailed in the appendix.

2 Basic Environment

2.1 General framework

The economy is populated by a continuum of infinitely lived agents that enjoy utility from consuming a set of differentiated consumption goods. There are no savings decisions in the economy, that is in every period agents consume all their income[13]. Agents are heterogeneous in their asset holdings. There are two types of assets in the economy, currency and participations in a mutual fund (shares); and one production factor, land, that exists in a fixed amount and does not depreciate. The mutual fund owns all the firms in the economy and the stock of land; it acts as a implicit insurance mechanism, pooling the profits generated by the firms and the flow of income generated by the stock of land.

The distribution of shares determines the distribution of income across agents, which is time invariant. Therefore there is no social mobility: an individual born with certain amount of shares would always consume according to the income associated to those shares. The demand for money in the economy is determined by a cash-in-advance constraint that limits the amount of goods that individuals can purchase to the amount of their money holdings. The central bank can change the amount of money in the economy through transfers of currency to individuals.

Moreover, firms transform land into a variety of consumption goods using a linear technology. Each firm produces only one type of consumption good and sets prices to maximise monopolistic rents. In this general framework there exists only one currency in which income and prices are denominated, the "peso", and there is no uncertainty. In the next section we introduce a second currency, the "dollar", and uncertainty in the exchange rate. All the dividends are distributed in pesos. In this basic set up, the timing is as follows: at the beginning of every period agents receive income distributed from the mutual fund that corresponds to the profits generated by firms and the rent of land from the previous period. Then, the central bank makes a transfer of money to households, firms set prices; and production and consumption takes place. Finally, profits and the rent of land are transferred to the mutual fund.

[13] We abstract from saving decisions because we want to highlight the cross section differences in dollarisation decisions amongst individuals and firms. For this analysis intertemporal effects are not necessary.

In the remaining of the text we adopt the convention of representing nominal variables with capital letters and real variables with lower case letters, indices i and j correspond to individual's and firm's variables, respectively. Variables without index are aggregate variables. Also, variables with superscript index prime ($'$) denote next period variables.

2.2 Goods and preferences

There is a discrete number $J+1$ of goods, indexed by $j = \{0..J\}$. In this economy, the number of goods produced by firms, $J+1$, is endogenously determined by the income distribution and the structure of preferences. Preferences are non-homothetic, i.e: income changes the marginal utility over goods. As a result, individuals will have different consumption baskets in equilibrium, being the richer agents the ones who consume higher number of types of goods. All the individuals have the same preferences, given the following utility function:

$$U_i = \sum_{j=1}^{J} \left(\prod_{r=1}^{j} x_{r,i} \right) + \varepsilon x_{o,i} \tag{1}$$

Where $x_{r,i}$ is an indicator function, with $x_{r,i} = 1$ if good $r > 1$ is consumed and $x_{r,i} = 0$ if it is not. $x_{o,i}$ is a leisure good, which in our setup is just a residual good, i.e the amount spent in leisure services is just the income that individuals do not spend in consumption goods. These preferences have the property that the individuals benefit nothing from consuming good h, if $x_{r,i} = 0$ for some $r < h$. This implies that the individuals consume good h, only if they can also consume all the other goods with indexes lower than h. In other words, individuals have a well-defined priority over the set of goods in their shopping list, goods with a lower index are necessity goods, whilst those with higher index are luxury goods. Individuals consume only one unit of each type of good. Also, it is assumed that ε is small enough such that $\varepsilon P_j / P_o < 1$ for every j. This condition guarantees that the consumption of any affordable good would be always preferable to the consumption of the leisure good

The budget constraint of an individual i is given by:

$$M_i' + \sum\nolimits_{j=0}^{J} P_j x_{j,i} \leq M_i + P y_i + T_i \tag{2}$$

Where M_i represents the beginning of period money holdings, M_i' money holdings at the beginning of next period, $P y_i$ the income transfer from the mutual fund, T_i transfers from the central bank, P_j is the nominal price of consumption good j and P the price deflator of aggregate output. Individuals also face a cash-in-advance (CIA) constraint that generates their demand for money. The CIA constraint is given by:

$$\sum_{j=0}^{J} P_j x_{j,i} \leq M_i + T_i \tag{3}$$

The CIA constraint limits the amount of consumption of individuals to their money holdings: initial money balances plus the transfer from the central bank. They can not use their current income to purchase consumption goods. Notice that because the utility of future consumption is zero, the CIA constraint is always binding, individuals find optimal to spend all their cash holdings at every period[14].

Because of the well-defined priority over the goods, the individual's consumption problem can be simplified as: choosing q, the number of consumption goods, and x_o ,the amount of leisure good to consume,[15] to maximize:

$$U_i = q_i + \varepsilon x_{o,i} \tag{4}$$

Therefore, the consumer problem can be stated as individuals purchasing as many goods as possible from the top of their shopping list and spending the remaining of their cash holdings in the leisure good. Then, the consumption demand of individual i takes the following form:

$$I_q \leq M_i + T_i < I_{q+1}$$
$$x_{i,0} = (M_i + T_i - I_q)\,/P_0$$

where $I_q = \sum_{j=1}^{q} P_j$ can be interpreted as the minimum level of cash holdings that allows individual i to consume q goods. An important feature of these preferences is that additional

[14] Thus, the implicit demand for money of individual i is given by:

$$M_i^d = P y_i$$

Aggregating the individuals money demand functions, we can express the equilibrium condition in the money market as:

$$Py = M'$$

where, M' is the money supply defined as:

$$M' = M + T$$

[15] Note that q is a discrete variable, whilst x_0 is continuous.

cash holdings translates into an additional demand for the next good in the shopping list, but only when it passes a threshold, otherwise the leisure good is consumed. Then, the indirect utility can be expressed as:

$$V_i = q_i + \bar{\varepsilon}\left(M_i + T_i - I_q\right) \tag{5}$$

where $\bar{\varepsilon} = \varepsilon / P_o$.

2.3 Income distribution and aggregate demand

In this economy there is a mutual fund that aggregates the profits of the monopolistic firms, the stock of land and the sales of the leisure good. Individuals own shares, θ, in this mutual fund. At the end of every period the mutual fund transfers to the individuals the income obtained from these three different sources. The income distribution is described by the cumulative density function of the shares $F(\theta)$ and it has support over the interval $\left[\underleftarrow{\theta}, \overrightarrow{\theta}\right]$, with $0 < \underleftarrow{\theta} < \overrightarrow{\theta} < \infty$ and $\int_{\underleftarrow{\theta}}^{\overrightarrow{\theta}} \theta dF(\theta) = 1$. Income of individual i at the end of the period is given by:

$$Py_i = \theta_i \left(\Pi + Rl + P_o x_o\right) \tag{6}$$

where Π, l and R are the total nominal profits, the land endowment and the rental price of land, respectively. From the cash in advance constraint, the implicit demand for money of individual i is given by:

$$M_i^{d\prime} = Py_i = \theta_i Py$$

where $y = \int_{\underleftarrow{\theta}}^{\overrightarrow{\theta}} y(\theta)\, dF(\theta)$. The individual cash holdings during the period are equal to $M_i + T_i$, the initial cash holdings plus the transfer from the central bank. Then, $F\left(\frac{Z}{M}\right)$ is the fraction of individuals whose cash holdings are lower than or equal to Z.

The share θ is the only source of heterogeneity across individuals. Since only the individuals with cash holdings higher than $I_j = \sum_{h=1}^{j} P_h$ purchase the good j, and no individual purchases more than one unit of each good, the aggregate demand for good j is equal to the mass of individuals whose cash holdings are higher than $I_j = I_{j-1} + P_j$:

$$x_j^d = 1 - F\left(\frac{I_j}{M}\right) \tag{7}$$

The non-homothecity of the preferences gives some special features to this demand function.

As in Matsuyama (2002), the demand is bounded from above by one and it depends on the income distribution. Moreover, because the marginal propensity to spend on a good varies with the individual income, higher index goods will be purchased only by high income customers whilst lower index goods will be purchased by almost all of them. Moreover, a decline in the price of good h does not affect the demand for good $j < h$ $(\partial x_j^d / \partial P_h = 0)$, whilst it generally increases the demand for good $j > h$ $(\partial x_j^d / \partial P_h > 0)$, therefore it exists demand complementarity from a lower indexed good to a higher indexed good, but not the other way around.

2.4 Firms

There is a discrete number of firms J, each one producing monopolistically a variety of good $j = 1..J$. All of them have the same linear technology in land: $x_j = l_j$. Firms choose prices optimally to maximise profits: $\Pi_j = P_j x_j^d - R l_j = (P_j - R)\left(1 - F\left(\frac{I_j}{M}\right)\right)$. From the first order condition, prices must satisfy[16]:

$$\frac{P_j}{M} = \frac{1 - F\left(\frac{I_j}{M}\right)}{F'\left(\frac{I_j}{M}\right)} + \frac{R}{M} \tag{8}$$

The price of land R is determined from the clearing market condition: $\sum_{j=1}^{J} l_j^d \leq l$, where $l_j^d(R) = x_j = \left[1 - F\left(\frac{I_j(R)}{M}\right)\right]$ is the demand of land of the firm j. From this clearing market and the profit maximisation conditions it is possible to see that R is proportional to M and that the firm's land demands depend negatively on R. We further assume that the stock of land is high enough such that $\sum_{j=1}^{J} l_j^d(0) < l$, in this case the price of land will be $R = 0$. This assumption simplifies greatly the algebra without changing the results. Firms do not use all the land stock even with zero cost, because under monopolistic competition they find optimal to limit the quantity produced below the maximum capacity.

The leisure good, good 0, is sold directly by the mutual fund and it has zero production costs. We assume that its price is proportional to the average price of monopolistic goods, $P_o = \kappa \sum_{j=1}^{J} P_j / J$. Since the price charged by monopolistic firms is proportional to the

[16] This condition is equivalent to: $\frac{P_j - R}{P_j} = \frac{1}{\eta(P_j)}$, where $\eta(P_j) = \frac{P_j/M}{1 - F\left(\frac{I_j}{M}\right)} F'\left(\frac{I_j}{M}\right)$ is the price elasticity of demand. This condition states that the "Lerner index" , the relation between the profit margin (price minus marginal cost) and the price, is equal to the inverse of the price elasticity of demand. When marginal costs are zero ($R = 0$), the price that satisfies this condition is such that the price elasticity of demand is equal to 1.

money supply, P_o is proportional as well [17].

All firms choose their prices simultaneously taking as give the distribution of income. Notice that the distribution of income is the only information relevant that the firm requires to set its price because each monopolist knows the optimal price that other firms will choose given the distribution of income[18].

Let's define the following functions $G(z) = \frac{1-F(z)}{F'(z)} + R/M$ and $\gamma(z) = -F''(z)\frac{1-F(z)}{(F'(z))^2}$ valid for any $z \in \left[\underleftarrow{\theta}, \overrightarrow{\theta}\right]$. The former function, $G(z)$, is useful to determinate the optimal price for good j that satisfies $\frac{P_j}{M} = G\left(\frac{I_{j-1}+P_j}{M}\right)$ and the latter, $\gamma(z)$, represents a local measure of the concavity of the income distribution[19].

The sequence of prices $\{P_j\}$ is defined by the following proposition:

Proposition 1 *The second order profit maximizing condition implies that, to have bounded prices, it is necessary that $\gamma(z) < 2$. Moreover, for $z = I_j/M$ prices would be locally decreasing if $\gamma(z) \in (-\infty, 1)$, locally increasing for $\gamma(z) \in (1, 2)$ and constant for $\gamma(z) = 1$.*

Proposition 1 shows that the sequence of prices is shaped by the income distribution. For distributions that are convex, prices are locally increasing , whilst for concave distributions, prices are decreasing for relatively low concavity $(0 < \gamma(z) < 1)$, increasing when concavity is high $(1 < \gamma(z) < 2)$ and constant for $\gamma(z) = 1$. Additionally, the second order condition implies that $G' < 1$, which guarantees that $\frac{P_j}{M} = G\left(\frac{I_{j-1}+P_j}{M}\right)$ has a solution for any $j < J$.

When income distribution is more concave the demand curve becomes more inelastic when moving from the top to the bottom of the shopping basket, increasing the monopolistic power of firms. When $\gamma(z) > 1$, the goods with higher indexes become more inelastic than those with lower indexes, and the monopolistic firms can charge a higher price for those goods.

[17] Note that we introduce the leisure good in the model to avoid the accumulation of resources from one period to the other, i.e. the remaining income from the consumption in monopolistic goods is expended in the leisure good. This assumption guarantees that individuals spend all their cash holdings every period. This is a useful assumption because the objective of the paper is not to analyse intertemporal decisions.

[18] Notice that our assumption of preferences imply that there are no strategic interaction in price setting.

[19] Notice that $\gamma(z)$ can also be related to the income inequality. When $\gamma(z) > (<) 0$, the income distribution is concave (convex) around z. Given two distributions of θ with the same support $\left[\underleftarrow{\theta}, \overrightarrow{\theta}\right]$, named A and B, if A has higher $\gamma(z)$ than B for every z , then B first-order stochastically dominates A and because of this the income distribution of B is less unequal than that of A. We can interpret this result as a "lottery of life": if a risk adverse individual has to choose between country A and B to live without knowing ex-ante her income, she will choose country B because it gives her higher expected utility, we are thankful to Andrei Sarichev who suggest us this interpretation.

On the other hand, when $\gamma(z) < 1$ the higher indexed goods become more elastic, and the monopolistic firms charge a decreasing sequence of prices.

P_J, the last good's price, is a special case. It must satisfy that: $R < P_J = \overrightarrow{M} - I_{J-1} \leq MG\left(\frac{I_{J-1}+P_J}{M}\right)$, where $\overrightarrow{M}$ is the cash holdings associated to the upper bound $\overrightarrow{\theta}$. The last firm J charges a price lower or equal than the optimum, such that only the richest individuals can buy this good. Therefore, this condition determines the number of firms J, which depends on the shape of the income distribution. Moreover, the number of firms is bounded because individual income is bounded[20].

2.5 Basic equilibrium

The equilibrium in this economy is defined as a number of firms and goods J, a set of consumption bundles $\{x(\theta)_j\}_{j\in\{0..J\}}^{\theta\in\left[\underleftarrow{\theta},\overrightarrow{\theta}\right]}$ and a set of prices $\{P_j\}_{j\in\{0..J\}}$,R, such that all individuals maximise utility subject to their budget constraints, all firms maximise profits and the goods, factors and money markets clear. Given that the preferences are non-homothetic, the distribution of income shapes the general equilibrium of the real variables, i.e. quantities and relative prices.

We define the steady state in this economy as an equilibrium where the cash holdings distribution is invariant, thus real variables will be constant and the nominal variables will grow at a constant rate. The economy deviates from the steady state if the central bank implements monetary policy through transfers that temporarily change the distribution of cash holdings.

[20] The relationship between J and income inequality has an inverted U-shape form: the number of goods increases for low levels of inequality and decreases for high inequality. This result is consistent with Matsuyama (2002). In contrast, Foellmi and Zwimuller (2003) have that higher income inequality increases the number of goods. This is because in their model they have perfect competition, then higher income inequality only increases the diversity of the goods. In our model, there is a second effect of income inequality, more income inequality increases the monopolistic power of firms reducing the number of goods.

Income inequality has two effects in J. On one hand, higher income inequality increases the monopolistic power of the firms, then prices are higher and the number of goods is smaller. On the other hand, income inequality increases the dispersion of income, which increases the number of goods because demand become more heterogeneous. For example, consider a perfectly egalitarian distribution, such that all the individuals have the same income. In this case, the first firm would charge a price equal to the total individual income and the number of goods produced would be one. On the other hand, consider a very unequal income distribution such a small mass of the population has a extremely high income, in this case the first firm would charge a price equal to the total individual income of the rich, and the poor would consume only the leisure good. In these two extreme cases the number of goods produced is one, cases in between would have higher J. Therefore, for low (high) levels of inequality the latter (former) effect dominates and the number of goods is increasing (decreasing).

The effects of monetary policy are summarised by the following proposition:

Proposition 2 *In this economy, money is neutral only if monetary injections are made through transfers proportional to the initial money holdings.*

As the demand functions depend on the distribution of cash holdings, monetary policy does not affect real variables when transfers are proportional to the initial money holdings. Any other form of transfer will change the distribution of cash holdings, affecting the demand of some goods, and therefore the equilibrium of real variables. This result would be different if preferences were homothetic, since in that case changes in the income distribution would not affect aggregate real variables in equilibrium[21].

3 Dollarisation decisions

In this section we extend the basic model introducing a second currency, the "dollar", that circulates simultaneously with the peso. Thus, individuals and firms have an extra decision to make: choose the currency denomination of their assets[22] and prices, respectively. The price of the dollar in terms of pesos, the exchange rate, S, is an exogenous random variable and represents the only source of uncertainty in the model. The percentage change of the exchange rate, s, is distributed with a cumulative density function $\Gamma(s)$. The distribution has support over the interval $\left[\underleftarrow{s}, \overrightarrow{s}\right]$ with $-1 < \underleftarrow{s} \leq 0 \leq \overrightarrow{s} < \infty$ and $\overline{s} = \int_{\underleftarrow{s}}^{\overrightarrow{s}} sd\,\Gamma(s)$ is the expected value of s, assumed to be positive.[23]

In our model, dollarisation, choosing the currency denomination, is costless only for prices, but not for assets. In order to dollarise their assets, individuals have to sign a stage contingent

[21] Notice also that given the non-homothecity of the preferences, the way that transfers are implemented affects differently relative prices amongst goods. When transfers are more than proportional to initial money holdings for the lower income individuals, the demand and the price for low-index goods increase, and because of the asymmetric demand complementarity, the demand of high-index goods decreases. In other words, this form of monetary policy expands the demand for low-index goods but contracts the demand for high-index goods. On the other hand, transfers that are more than proportional for the higher income individuals change the prices of high-index goods, however, this does not affect the demand of low-index goods. These results contrast with the case of homothetic preferences, in which all the individuals have the same consumption basket, thus monetary transfers affect all prices in the economy in the same way, independently of the way these transfers are implemented.

[22] More precisely, in this model the decision of dollarisation is referred to denominate in dollars the flow of cash generated by shares. Which in this model coincides with individuals cash holdings.

[23] As we have defined the exchange rate S as the price of foreign currency in terms of domestic currency, a positive realization of s represents a depreciation of domestic currency and a negative realization of s an appreciation of the exchange rate.

contract with the central bank at a fixed real cost c [24]. In this contract, the central bank commits to transfer an amount of pesos, T_i, contingent on the realization of the depreciation of the exchange rate s :

$$T_i = s\,(M_i - Pc) \tag{9}$$

The transfer is proportional to the nominal value of the income flow generated by individual's assets M_i, net of the contract cost [25], Pc. Firms and individuals make dollarisation decisions before observing the realization of the depreciation rate, taking as given the income and depreciation rate distributions. For simplicity, we assume that in this equilibrium, monetary policy takes place only through dollarisation contracts with individuals. Thus, money supply changes only through T_i.

The timing in the model with dollarisation is as follows: at the beginning of every period, agents receive income transfers from the mutual fund that corresponds to the firms' profits and the land rents generated during the previous period. After agents have received their income, simultaneously individuals decide whether or not to dollarise their assets and firms decide to set prices either in pesos or in dollars given the income and exchange rate distributions. Then, nature draws a realization of the exchange rate, production and consumption take place, given the set of prices, the realization of the exchange rate and the income distribution. Finally, profits and the rental payments of land are transferred to the mutual fund.

3.1 Individuals dollarisation decisions

This section analyses the portfolio decision of the individuals. For tractability we limit the analysis to the case of a small variance for the depreciation rate. Individuals decide to dollarise their income comparing their expected utility levels with and without dollarisation. Since the exchange rate is expected to depreciate, individuals have the incentive to dollarise their assets to take advantage of the expected capital gain of holding foreign currency, however not every one can afford it, since individuals have to pay a fixed real cost c to dollarise their income. [26]

[24] We consider this cost as fixed, but we are aware that there may exist important links with the level of dollarisation observed in the economy. Economies with a history of dollarisation may develop cheaper ways to dollarise.

[25] We assume that the revenues generated by the central bank through the dollarisation contracts are transferred to the mutual fund at the end of every period. This assumption avoids that the central bank accumulates real resources through time.

[26] The assumption of the fixed cost tries to capture the fact that not every individual in a society have access to financial instruments to protect financial wealth against inflation or devaluations. In Peru, for instance, only 25 percent of the population possesses a saving account.

It is easier to analyse the portfolio problem of individuals considering their indirect utility function. Notice that the indirect utility function of individual i can be written as the sum of the number of goods she can afford to consume, q_i, plus the amount spent in the leisure good, $M_i - \overline{I}_q$, weighted by $\bar{\varepsilon}$. Let's denote $\overline{I}_q$ as the ex-post[27] expenditure in domestic currency of consuming q goods. Notice that, $\overline{I}_q$ is a contingent variable, its value depends on the realization of the depreciation rate since $q-m$ goods have prices in dollars: $\overline{I}_q = I_q + (1+s)(I_q - I_m)$ for $q > m$ and $\overline{I}_q = I_q$, otherwise. Where m, is the number of goods with prices in pesos. Thus, the indirect utility function can be expressed as:

$$V_i = q_i + \bar{\varepsilon}\left(M_i - \overline{I}_{q_i}\right)$$

Using the previous utility function, the corresponding levels of utility with and without dollarisation are given by:

$$V_i^D = q_i^D + \bar{\varepsilon}\left((M_i - Pc) + T_i - \overline{I}_{q_i^D}\right) \tag{10}$$

$$V_i^{ND} = q_i^{ND} + \bar{\varepsilon}\left(M_i - \overline{I}_{q_i^{ND}}\right) \tag{11}$$

Note that with dollarisation, the level of cash holdings decreases with the payment of the fixed cost, Pc, but increases (decreases) with the transfer from the central bank, T_i, in states of the nature where the exchange rate depreciates (appreciates). Without dollarisation cash-holdings are not affected by the exchange rate, but since some goods have prices in dollars the exchange rate affects the number of goods that individuals can afford. Therefore, we consider utility under dollarisation and non dollarisation as state contingent variables. Thus, dollarisation will take place only when:

$$E\left(V_i^D\right) - E\left(V_i^{ND}\right) \geq 0$$

Proposition 3 *Let $\sigma^2 = var(s)$, then for $\sigma^2 \to 0$, only those individuals with cash holdings , M_i, higher than $M_n = \frac{(1+\bar{s})Pc}{\bar{s}}$ and that can afford to pay the fixed cost c, choose to dollarise their income.*

Proof. see appendix ■

From proposition 3, the individuals with cash-holdings higher than $\frac{(1+\bar{s})Pc}{\bar{s}}$ will dollarise their income. The mass of individuals who do not choose dollarisation is given by those with

[27] After the realization of the exchange rate is observed.

cash-holdings : $M_i < \frac{(1+\overline{s})Pc}{\overline{s}}$. Therefore, we can define as n the mass of individuals who choose not to dollarise as[28]:

$$n = F\left(\frac{(1+\overline{s})\,Pc}{\overline{s}M}\right) \tag{12}$$

$$M_n = \frac{(1+\overline{s})\,Pc}{\overline{s}} \tag{13}$$

The dollarisation decision is independent of m, the number of goods with prices in domestic currency. This result holds for small levels of risk[29] . Since the indirect utility function is piecewise linear in income[30] and the variance of the depreciation of the exchange rate is small, the exchange doesn't affect the number of goods that the individual can afford to consume but only the level of leisure, therefore, the difference between the expected utility under dollarisation and non dollarisation is only a function of expected income. Consequently, under these assumptions, the only moment of the distribution of s that is relevant for dollarisation decisions of the individuals is its mean, $\overline{s}$.

3.1.1 Dollarisation and income distribution

The dollarisation decisions of individuals change the ex-post distribution of money holdings.[31] When the exchange rate depreciates (appreciates) money holdings will increase (decrease) for those individuals who decide to dollarise, whilst it will be unchanged for those individuals who don't dollarise. For instance, an individual with money holdings $M_i \geq M_n$ has ex-post income of $(1+s)(M_i - Pc)$ that is higher (lower) than her initial money holdings if $s > (<)\ \overline{s}$. The money holdings of an individual who don´t dollarise is not affected by the exchange rate fluctuations.

Denoting as M_i the money holdings of individual i, the ex-post money holdings distribution, conditional on a mass of individuals with assets in pesos and on a realisation of the exchange

[28] The individual dollarisation threshold can also be expressed in terms of shares holdings, θ_i, as

$$\theta_n = \frac{c/y}{\overline{s}/(1+\overline{s})}$$

Where c/y is the cost of dollarising assets as a proportion of the mean income and $\overline{s}/(1+\overline{s})$ is an index of the expected depreciation rate. Notice that θ_n is increasing in c/y and decreasing in $\overline{s}$.

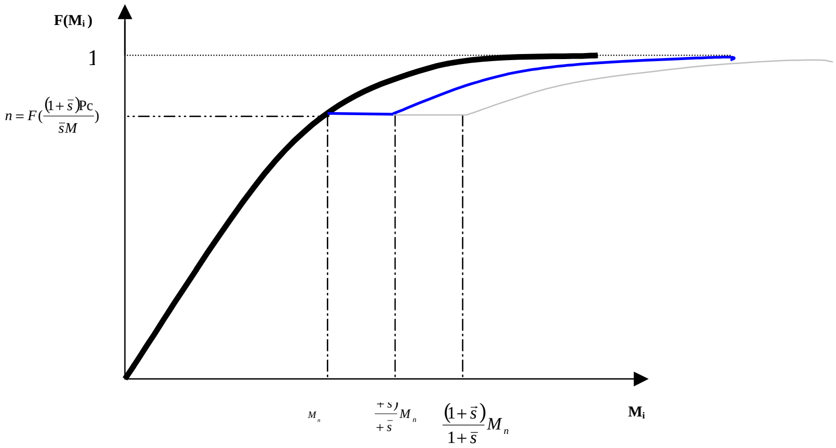
[29] Numerical simulations shows that this is true for standard deviation of the depreciation of the exchange rate of up to 20 percent the mean expected depreciation.

[30] Higher income allows either to consume more type of goods or enjoy higher level of leisure

[31] The money holdings after the depreciation has taken place.

rate above its mean ($s > \overline{s}$), can be written as:

$$F\left(\frac{M_i}{M}|n, s > \overline{s}\right) = \begin{cases} F\left(\frac{M_i}{M}\right) & \text{if} \quad M_i < M_n \\ F\left(\frac{M_n}{M}\right) & \text{if} \quad M_n \leq M_i \leq \frac{(1+s)}{1+\overline{s}} M_n \\ F\left(\frac{M_i}{(1+s)M} + \frac{Pc}{M}\right) & \text{Otherwise} \end{cases} \tag{14}$$

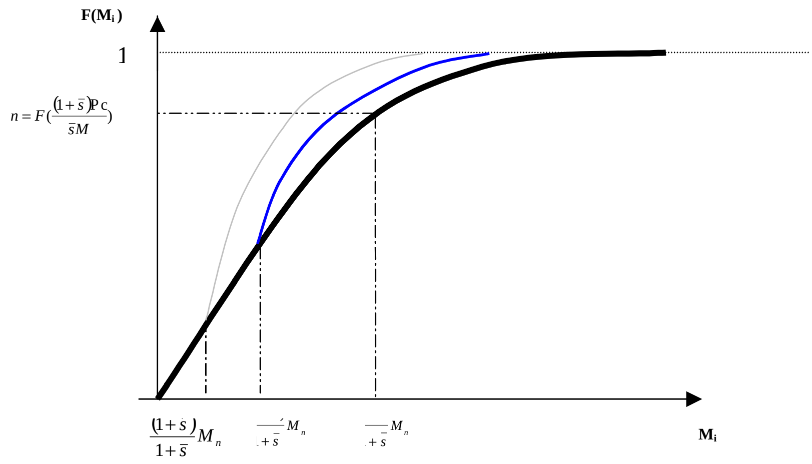


Graph 1. Cash-holding Distribution for $s > \overline{s}$

The conditional distribution of cash holdings is contingent on the realisation of the exchange rate. When $s > \overline{s}$ the ex-post money holdings distribution is the same for $M_i < M_n$, but it shifts to the right from that threshold. This function has a piecewise form with a flat segment on $n = F\left(\frac{M_n}{M}\right)$ and is flatter than the initial distribution for $M_i > M_n$ because when the exchange rate depreciates individuals with dollarised assets increase their levels of money holdings.

Similarly, the ex-post money holdings distribution conditional on $s < \overline{s}$ has the following form:

$$F\left(\frac{M_i}{M}|n, s < \overline{s}\right) = \begin{cases} F\left(\frac{M_i}{M}\right) & \text{if} \quad M_i \leq M_n \frac{(1+s)}{(1+\overline{s})} \\ F\left(\frac{M_i}{(1+s)M} + \frac{Pc}{M}\right) + F\left(\frac{M_i}{M}\right) - F\left(\frac{M_n}{M}\right) & \text{if } M_n \frac{(1+s)}{(1+\overline{s})} < M_i \leq M_n \\ F\left(\frac{M_i}{(1+s)M} + \frac{Pc}{M}\right) & \text{Otherwise} \end{cases} \tag{15}$$



Graph 2: Cash-holdings Distribution for $s < \overline{s}$

In this case, the ex-post money holdings distribution is the same for $M_i < M_n \frac{(1+s)}{(1+\overline{s})}$ and steeper than the initial one on the right of that threshold. The distribution is steeper in this range because when the exchange rate appreciates, individuals with dollarised assets reduce their levels of money holdings[32], therefore the mass of agents with lower money holdings increases.

3.2 Firms dollarisation decisions

When we introduce the second currency, firms have another decision to make besides setting the profit maximizing price, they have to choose in which currency to set the price, pesos or dollars. On the individual side, income inequality and the fixed cost make that richer individuals are the only agents that dollarise their assets. Therefore, fluctuations of the exchange rate affect the income of those individuals rich enough to dollarise their assets. Furthermore, the non-homotheticity of the preferences allows fluctuations in the exchange to feed into the demand elasticity of those products whose demand is concentrated in customers with dollarised assets. Consequently, good's demand becomes uncertain. With uncertainty in demand the problem of choosing the currency denomination of prices is not trivial.

[32] Note, that in the model when the exchange rate appreciates, agents who decide to dollarise tranfer money holdings to the central bank, T_i is negative for those agents.

Under these conditions the demand of a particular good j and the profits of the firm producing this good will depend not only on the price of the good, but also on the exchange rate, i.e. $x_j^d(P_j, s)$ and $\Pi_j(P_j, s)$. Let's denote P_j^* the price of the goods in dollars expressed in domestic currency at the initial exchange rate, then $\Pi_j^D\left((1+s)P_j^*, s\right)$ and $\Pi_j^{ND}(P_j, s)$ are the nominal profits expressed in domestic currency when the price is in dollars and in pesos, respectively. It is important to note that under perfect certainty about the exchange rate, the currency price-setting problem becomes irrelevant, because $P_j = (1+\overline{s})P_j^*$ for $\overline{s} = Es$ and $\Pi_j^D\left((1+\overline{s})P_j^*, \overline{s}\right) = \Pi_j^{ND}(P_j, \overline{s})$.

The firm chooses the currency to denominate its price comparing the expected profits under the two price setting options: $E\Pi_j^D - E\Pi_j^{ND}$. This difference in the expected value of profits depends no linearly on the exchange rate, therefore the expected value of profits does not coincide with profits evaluated at the expected exchange rate. To characterize the problem, we follow a methodology similar to Bacchetta and van Wincoop (2001)[33]. As in their work, in order to have a tractable problem, we focus on uncertainty near $s = Es = \overline{s}$, the expected exchange rate, and on "small" levels of risk where the variance of s, σ^2, tends to zero.

A firm will set the price in dollars if $E\left(\Pi_j^D - \Pi_j^{ND}\right) > 0$, Since firms are heterogenous and the demand they face depends on the type of good they sell[34] the exchange rate will affect differently the firm's profits. In order to establish conditions under which firms set prices in dollars we evaluate the impact of a small amount of risk on the optimal pricing strategy. We take the total derivative of $E\left[\Pi_j^D\left((1+s)P_j^*, s\right) - \Pi_j^{ND}(P_j, s)\right]$ with respect to the variance σ^2 of the nominal exchange rate, evaluated at $\sigma^2 = 0$ and $s = \overline{s}$:

Lemma 1 *Let $\Pi_j^D\left((1+s)P_j^*, s\right)$ and $\Pi_j^{ND}(P_j, s)$ be the profit functions for firm j expressed in domestic currency with prices in dollars and pesos, respectively. For any twice differentiable Π function we have that:*

$$\frac{\partial E\left[\Pi^D\left((1+\overline{s})P_j^*, \overline{s}\right) - \Pi^{ND}(P_j, \overline{s})\right]}{\partial\sigma^2} \simeq \left(\frac{P_j}{1+\overline{s}}\right)\left[\Pi_{12}(P_j, \overline{s}) + \frac{1}{2}\frac{P_j}{1+\overline{s}}\Pi_{11}(P_j, \overline{s})\right]$$

evaluated at $s = \overline{s}$ and $\sigma^2 = 0$

The proof of lemma 1 is in the appendix. This lemma establishes that the decision to set prices in dollars depends on the value of the second order derivatives of the profit function

[33] Bacchetta and Wincoop (2001) study the currency denomination of international trade. They found that the higher the market share of an exporting country in an industry, and the more differentiated its goods, the more likely its exporters will price in the exporter' currency.

[34] A necessity goods is sold to almost every individual in the economy, whilst a luxury good is sold just to the richiest individuals.

evaluated at the price in pesos and the exchange rate under perfect certainty, and it does not depend on the price in dollars. The functions Π_{12} and Π_{11} are the derivatives of the marginal profits with respect to the exchange rate and the price, respectively. The former is related to the marginal benefits of setting the price in dollars (the increase in marginal profits due to an increase in the exchange rate), whilst the latter is related to the marginal cost (the decrease in marginal profits due to an increase in the price).

The currency price setting depends on the sign of the expression in lemma 1[35]. Prices are set in dollars when the difference of expected profits, $E\left(\Pi^{D}-\Pi^{ND}\right)$, is a convex function of the exchange rate, i.e. when $\Pi_{12}\left(P,\overline{s}\right) > -\frac{1}{2}\frac{P}{1+\overline{s}}\Pi_{11}\left(P,\overline{s}\right)$. From the profit maximisation second order condition we have that $\Pi_{11} < 0$, then the right hand side of the inequality is positive. This implies that for setting the price in dollars it is necessary that the marginal benefits of dollarisation (Π_{12}) to be positive and higher than the marginal costs ($-\frac{1}{2}\frac{P}{1+\overline{s}}\Pi_{11}$). Therefore, for any case where $\Pi_{12} \leq 0$ it will not be optimal to set the price in dollars, such is the case when a depreciation of the exchange rate reduces the demand[36] of the good or does not affect it at all.

We use the result from lemma 1 to analyse the case with non-homothetic preferences and income inequality[37]. In order to have a tractable problem, we focus on small levels of exchange rate volatility, $\sigma^2 \to 0$. Under this assumption, the profit function becomes a two-zone piecewise function in I_{j-1}. Given the fraction n of the population with assets in pesos and the other $1-n$ with assets in dollars, firms profits have the following form:

$$\Pi_j\left(P_j,s,I_{j-1}\right)=\begin{cases}\left(P_j-R\right)\left(1-F\left(\frac{I_{j-1}+P_j}{M}\right)\right) & \text{if } I_{j-1}+P_j<\theta_n M\\ \left(P_j-R\right)\left(1-F\left(\frac{1}{1+s}\frac{I_{j-1}+P_j}{M}+\frac{c}{y}\right)\right) & \text{otherwise}\end{cases} \tag{16}$$

where $\frac{c}{y}=\frac{\overline{s}}{1+\overline{s}}\theta_n$ is the proportion of the fixed cost to the average income.

The demand of the good j depends on its price (P_j), the depreciation rate (s) and the price of the goods with index lower than j (I_{j-1}). The depreciation rate affects the demand

[35] These results assume that the objective function of the firms are the profits expressed in domestic currency. However, the results are robust to expressing the profit function in foreign currency, because the analysis is done for cases of small variance of s.

[36] For instance, when the price is set in dollars and customers have income in pesos.

[37] Lemma 1 is a general result for any profit function affected by the exchange rate. We use income inequality with non-homothetic preferences to show the links between AD and PD. The general setup could be used as well to analyse the case when the production cost of some firms is denominated in dollars. Intuitively, in this alternative setup, the firms that set their price in dollars would be those with costs in dollars and relatively more inelastic demands.

of goods only when the total expenditure in goods (I_j) passes the threshold $\theta_n M$. This is the case when all the individuals that demand the good have their assets in dollars.

Proposition 4 *Given that a mass n of individuals maintain their assets in pesos and a distribution of s, such that* $Es = \overline{s} > 0$ *and* $Vs = \sigma^2 \to 0$, *there exists a threshold level of expenditure* $I_{\overline{m}} = M\left(\theta_n - G\left(\theta_n\right)\right)$ *such that* $E\Pi^{ND}\left(I_{j-1}\right) > \Pi^{D}\left(I_{j-1}\right)$ *for* $I_{j-1} < I_{\overline{m}}$. *Additionally, if* $\gamma\left(\frac{I_{j-1}+P_j}{M}\right) < \overline{\gamma} = 1/\left(1 - \frac{1}{2}\frac{P_j}{I_{j-1}+P_j}\right)$, *then* $E\Pi^{ND}\left(I_{j-1}\right) < \Pi^{D}\left(I_{j-1}\right)$ *for* $I_{j-1} > I_{\overline{m}}$.

This proposition characterize the optimal currency denomination of firms. Firms are divided into two zones according to their optimal pricing strategy, firms in zone I find optimal to set prices in pesos, whilst firms in zone II in dollars. The threshold in proposition 4 that describes the two zones depends on the fraction of the population that dollarise their assets n. Furthermore, the income threshold $I_{\overline{m}}$ fully characterize the dollarisation decision for firms:

Corollary 1 *the number of goods that are priced in domestic currency, m, is defined by:* $I_{m-1} \leq I_{\overline{m}} < I_m$. *Goods with index* $j \leq m$ *are priced in pesos, and* $j > m$ *are priced in dollars.*

Proposition 4 and corollary 1 establish the main result in this paper. Given the fixed cost for asset dollarisation, the distribution of income and the distribution of the exchange rate depreciation, there exists a threshold good, m, such that firms producing goods with indexes $j > m$ find optimal to set prices in dollars, and firms producing goods with indexes $j \leq m$ set prices in pesos.

Firms in zone I with index $j \leq m$, produce goods that are demanded by lower and higher income individuals, i.e. necessity goods. As the higher income individuals will always consume these goods independently of the currency in which the price is set, firms consider only what happens with the demand of the low-income individuals. Because low income individuals have their assets in pesos, fluctuations in the exchange rate do not affect their income and pricing in dollars would introduce uncertainty to their demand, reducing expected profits. Therefore, for goods in zone I, $\Pi_{12} = 0$ and the optimal pricing solution is in pesos.

On the other hand, goods in zone II are only consumed by those individuals rich enough to dollarise their assets. For firms producing those goods, setting prices in pesos introduces volatility in their demand. Therefore by setting prices in dollars they can stabilize their demand, this is $\Pi_{12} > 0$. There is however a second effect in profits when setting prices in dollars, the effect of the exchange rate on the price expressed in pesos. Since profits are evaluated in pesos, the volatility of exchange rate feeds into profits through its effect on prices, under some regularity conditions the first effect dominates. These conditions are given by the value of γ. Recall that γ is a local measure of the concavity of the income distribution function, related to income inequality, and in our setting is also related to the demand price elasticity.

A sufficient condition to have the firms set prices in dollars is that $\gamma\left(\frac{I_{j-1}+P_j}{M}\right) < \overline{\gamma}$ where $1 \leq \overline{\gamma} \leq 2$[38], this is the case when the concavity (income inequality) on the income distribution is moderate, and when demand price elasticity is not to low. When price elasticity is very low the benefits of stabilizing demand are less significant, since in this case exchange rate fluctuations have a minor effect of demand volatility. Therefore, when $\gamma\left(\frac{I_{j-1}+P_j}{M}\right) > \overline{\gamma}$ the effect on profits through fluctuations in price dominate the effect on demand stability [39].

Proposition 5 *The function $I_{\overline{m}}(\theta_n) = M\left(\theta_n - G\left(\theta_n\right)\right)$ that links $I_{\overline{m}}$ (the threshold of expenditure in goods priced in pesos) with θ_n (the threshold of individuals with assets in pesos) is increasing ($\partial I_{\overline{m}}/\partial\theta_n > 0$) and convex in θ_n ($\partial^2 I_{\overline{m}}/\partial\theta_n^2 > 0$) for most income distributions.*

Proof. see appendix ■

This proposition establishes formally the relationship between the threshold of individuals with assets in pesos, θ_n, and the threshold of expenditure in goods priced in pesos, $I_{\overline{m}}$. It shows that there is a causality relationship from θ_n to $I_{\overline{m}}$. When no individual finds profitable to dollarise assets, then no firm finds optimal to dollarise its price, because it would increase the uncertainty in the demand given that the elasticity of demand is higher when the price is in dollars. On the other hand, when all individuals dollarise their incomes, the income of all individuals changes proportionally to the depreciation rate, thus the demand faced by each firm becomes less elastic to the pricing in dollars, making demand more stable. When dollarisation of assets is partial only some firms, those with sales concentrated on high-income customers with dollarised assets, set prices in dollars. The shape of the relationship between θ_n and $I_{\overline{m}}$ depends on the form of the income distribution. More precisely, for most income distributions that show some degree of inequality, we establish that $I_{\overline{m}}$ is an increasing and convex function of θ_n.

[38] Recall that from the sufficient condition for profit maximization $\gamma < 2$

[39] Note that with some firms facing costs in dollars and homothetic preferences, the price setting decision is similar to the invoicing decision facing by an exporting firm, as Bacchetta and Wincoop (2001) show, in this case, firms will set prices in dollars when the demand price elasticity is low, the higher their market share and the more differentiated their goods. The difference is that with non-homothetic preferences, this result holds for more general production technology, more precisely, it also holds for constant returns to scale technology and not only for decreasing returns to scale as in Bacchetta and Wincoop

3.3 Equilibrium with dollarisation

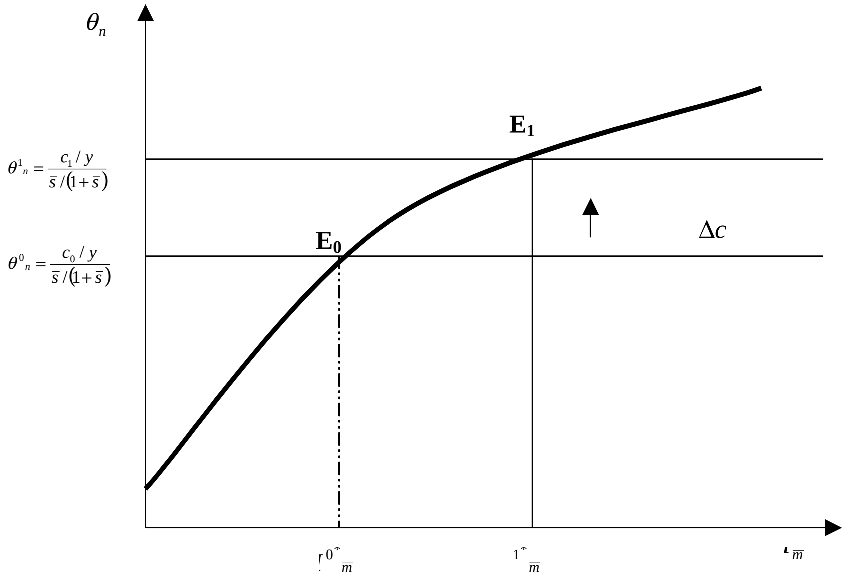
When we introduce a second currency in the model, the equilibrium in this economy is defined as the number of firms and goods J, the number of firms that set the price in pesos m, the mass of individuals that maintain their assets in pesos n, the set of consumption bundles $\{x(\theta)_j\}_{j\in\{0,..,J\}}^{\theta\in[\underleftarrow{\theta},\overrightarrow{\theta}]}$ and the set of prices $\{P_j\}_{j\in\{0,..,m\}}, \left\{P_j^*\right\}_{j\in\{m+1,..,J\}}, R$ such that all the individuals maximise expected utility subject to their budget constraints, all the firms maximise expected profits and the goods, factors and money market clear.

3.4 Comparative statics

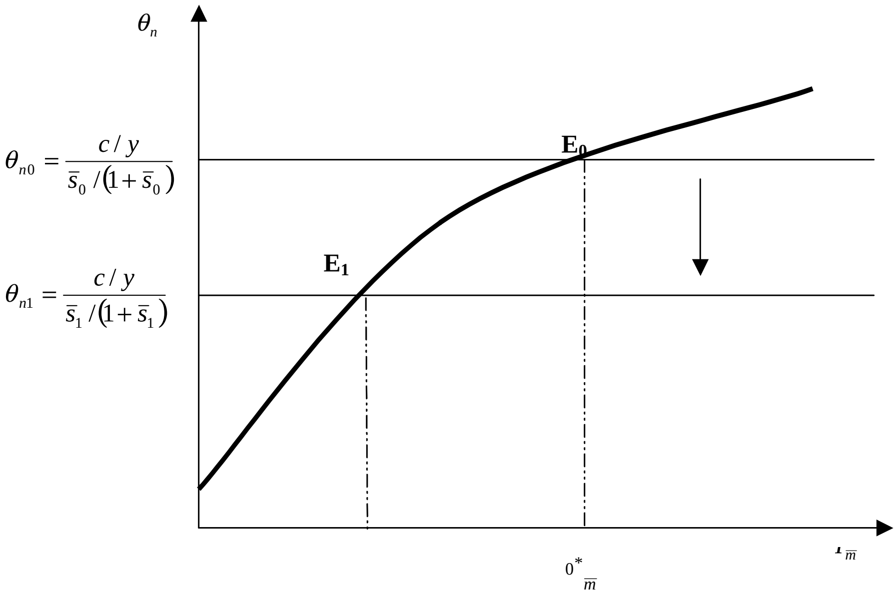
Given the fixed cost for asset dollarisation c, the income distribution $F(\theta)$ and the exchange rate distribution $\Gamma(s)$, the nash equilibrium that determines the levels of dollarisation is given by the intersection of the schedules: $\theta_n = \frac{c/y}{\overline{s}/(1+\overline{s})}$ and $I_{\overline{m}} = M(\theta_n - G(\theta_n))$[40]. This intersection match $n^* = F\left(\frac{c/y}{\overline{s}/(1+\overline{s})}\right)$, the mass of individuals with assets in pesos, with $m^*(n^*)$, the number of goods priced in pesos. Therefore, the mass $1-n^*$ of individuals, those with cash holdings $M_i > \theta_n M$, and the firms selling a good with index $j > m^*$ find optimal to dollarise.

An increase in the cost c shifts up the θ_n schedule and increases in equilibrium both θ_n and $I_{\overline{m}}$ (Graph 3), reducing the mass of individuals and the number of goods dollarised. On the other hand, an increase in the expected depreciation rate, $\overline{s}$, shifts down the θ_n schedule (Graph 4) and reduces the values in equilibrium of θ_n and $I_{\overline{m}}$, increasing the dollarisation for individuals and firms. Is it important to notice that the expected exchange rate only affects $I_{\overline{m}}$ through the effect on θ_n, the $I_{\overline{m}}$ schedule does not shift because for the firms the only relevant moment of the distribution of s is the variance. Furthermore, numerical simulations show that higher variance of s shifts the $I_{\overline{m}}$ schedule to the left, increasing m for given value of $\overline{s}$.

[40] Proposition 5 states that for most income distributions $I_{\overline{m}}$ is a convex function in θ_n. However, note that when $I_{\overline{m}}(\theta_n)$ is convex, as we plot it as the inverse function $\theta_n = I_{\overline{m}}^{-1}$, the figure is concave.



Graph 3: Comparatitive Statics Increase in c



Graph 4: Comparative Statics: An increase in $\bar{s}$

4 Links between asset dollarisation and price dollarisation

The equilibrium under dollarisation establishes a link between the mass of individuals with assets in dollars and the number of firms that set their prices in dollars. Aggregating individual decisions of firms and individuals we can establish a link between asset and transaction dollarisation. Let's first define asset and transaction dollarisation:

Definition 1 *Asset Dollarisation (AD): Is defined as the ratio between the sum of income of those individuals who dollarise respect to the sum of income of the total population.*

$$AD = \frac{\int_{\theta_n}^{\overrightarrow{\theta}} PY(\theta_i)\, dF(\theta)}{\int_{\overleftarrow{\theta}}^{\overrightarrow{\theta}} PY(\theta_i)\, dF(\theta)} = \int_{\theta_n}^{\overrightarrow{\theta}} \theta dF(\theta)$$

It is important to mention that our measure of dollarisation is directly comparable with measures of dollarisation associated with the financial system only under the assumption that the cost of participating in the financial system is the same to the cost of participating in the exchange market. In countries with a history of dollarisation, the cost of participating in the exchange market is usually much lower than the cost of participating in the financial system, therefore, in those cases, our measure of dollarisation will be systematically higher than those associated to the financial system.

Furthermore, note that asset dollarisation is a decreasing function of n , the higher the mass of individuals who have chosen not to dollarise, the lower the ratio of asset dollarisation in the economy[41].

Definition 2 *Price Dollarisation (PD): Is defined as the ratio of the sum of sales of those firms with prices in foreign currency respect to the sum of sales over the whole spectrum of goods.*

$$PD = \frac{\sum_{j=m+1}^{J} P_j^* x_j^d}{\sum_{j=1}^{m} P_j x_j^d + \sum_{j=m+1}^{J} P_j^* x_j^d}$$

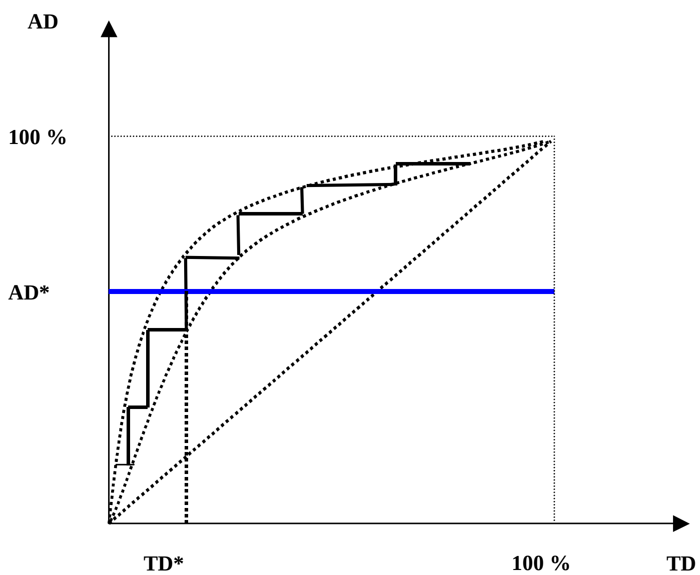
[41] Note that in the definitions of asset and price dollarisation we are using the income distribution ex-ante the depreciation of the currency, in order to abstract it from the income effects that occurs with the depreciation.

Notice that PD is a strictly decreasing function of m, the number of goods that set prices in domestic currency. Also AD is a decreasing function of n, the mass of individuals who dollarise their assets. By proposition 5 we know that m is an increasing function of n, thus, we can establish that PD is also an increasing function of AD and that $AD \geq PD$ as we can see in the following proposition:

Proposition 6 *AD is always higher than or equal to PD.*

Proof. see appendix ■

AD is higher than PD because individuals with assets in dollars consume both goods in pesos and in dollars, thus the proportion of goods traded in dollars would be smaller than the proportion of assets hold in dollars. This is true even when some individuals with assets in pesos consume goods in dollars, because those goods are a small part in their consumption basket. The exact shape of the relationship between AD and PD depends on the income distribution, more precisely, when the proportion of total income in hands of the higher income individuals is higher, the difference between AD and PD would be higher. Notice, that because of the discrete number of goods PD is a step function of AD, and as the number of goods become larger the steps in the function become smaller.



Graph 5: Asset Dollarisation and Transaction Dollarisation

In our model AD causes PD, but not the other way around: when no individual finds optimal to dollarise her assets, no firm has the incentives to set prices in dollars. Moreover, AD is independent of PD because of the linearity of individuals preferences in the number of goods consumed. With a more general preferences specification, the individual portfolio decision will depend on m, the number of goods with peso prices. Therefore, m and n will be determined simultaneously, but the equilibrium value for m, described in this paper, is a lower bound for the dollarisation decisions of firms in this more general case.

The model explains why in economies with very high levels of inflation domestic currency remains in circulation, and it is not fully replaced by a foreign currency as medium of exchange and unit of account. In the model, if there exist some degree of income inequality, the use dollars is not an option for the segment of the population with lower income. Therefore, firms producing goods whose demand is concentrated in low income customers will find optimal setting prices in pesos. In a more egalitarian society, everything else equal, the model predicts that both asset and price dollarisation will be higher. The model also explains why in countries with remarkably high levels of asset dollarisation, countries like, Argentina, Bolivia and Peru, the levels of transaction and price dollarisation are relatively low. Moreover, our results suggest that policy makers that are looking for policies aiming at reducing dollarisation should focus mainly on reducing asset dollarisation, because price and transactions dollarisation will endogenously follow the pattern of asset dollarisation

Although, the main objective of the paper was not to explain the persistence of dollarisation, the model shed some light on the issue. In the model, persistence in asset dollarisation can be generated if the cost of participating in the exchange market falls in parallel with expected level of devaluation. If this cost is small enough even for very small levels of expected depreciation, the levels of asset dollarisation may remain high[42].

Proposition 7 *The degree of pass-through in the short-run and the long-run are approximately equal to PD and AD, respectively.*

The model also has implications regarding the pass-trough, the effect on prices of a depreciation of the exchange rate. In the short-run, the pass-trough is approximately equal to PD because transactions in dollars increase proportionally to the depreciation rate[43], i.e.$\dot{P} \simeq sPD$. However, one period after, in the long-run the pass-trough is equal to AD, i.e. $\dot{P}' = sAD$.

[42] For a general equilibrium model of persistence in transaction dollarisation see Uribe (1997)

[43] The short run pass-through is approximately proportional to the TD index, because we use the GDP deflator as the price index, P, which adjust the weights of the goods to the changes in demand. It would be exactly

This is so because the income gained from a depreciation is distributed across all the individuals in the next period through the mutual fund, therefore the general level of prices P increases proportionally. If asset dollarisation were equal to 1, then the nominal increase in profits and transactions would be equal to the depreciation rate. For levels of AD less than 1, both the short-run and long-run pass-through would be lower than 1. Moreover, for a given c, higher expected depreciation rate would imply higher degree of pass-through in the short and long-run.

equal to the TD index if we consider a Laspeyre's price index, which maintains the weights of the goods fixed to the ex-ante demand. On the other hand, the long-run pass-through is exactly equal to the AD index, because all the prices change by the same proportion.

5 Conclusions

In this paper we have presented a simple model that explains the pattern of dollarisation across types of goods and provides a theoretical link between asset and transaction dollarisation.The model shows that asset dollarisation causes price dollarisation and that income distribution plays an important role in explaining the pattern of price dollarisation across types of goods. In particular, we find that for income distributions that show some degree of inequality, necessity goods, those associated with the consumption of low-income customers, have prices in pesos, whereas luxury goods have prices in dollars. This result comes from the interaction between portfolio decisions of individuals and pricing decisions of firms: a firm that sells its product to consumers with assets mostly in dollars prefers also to put prices in dollars to stabilize its demand.

Furthermore, the model shows that the dollarisation of assets is bigger than the dollarisation of prices and the gap between them is increasing in the degree of inequality. That is, the more income in hands of the individuals with assets in dollars, the higher the difference between these two measures of dollarisation. Moreover, the model explains why in economies with high levels of inflation domestic currency remains in circulation, and it is not fully replaced by a foreign currency as medium of exchange and unit of account. In the model, if there exist some degree of income inequality, the use dollars is not an option for the segment of the population with lower incomes. Therefore, firms producing goods whose demand is concentrated in low-income customers will find optimal setting prices in pesos.

The model also explains why in countries with remarkably high levels of asset dollarisation, countries like Argentina, Bolivia and Peru, the levels of transaction and price dollarisation are relatively low. Moreover, our results suggest that policy makers that are looking for policies aiming at reducing dollarisation should focus mainly on reducing asset dollarisation, because price and transactions dollarisation will endogenously follow the pattern of asset dollarisation

Although the model is highly stylised, it captures reasonable well the main stylised facts we intended to explain. However, we aim to explore some extensions to the model, in particular, we would like to generalize the preferences setting to introduce some degree of substitution amongst goods. Also, a multiperiod decision-making is considered in our future research agenda.

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A Proofs

Proposition 1: *The second order profit maximisation condition implies that to have bounded prices it is necessary that $\gamma(z) < 2$. Moreover, for $z = I_j/M$ prices would be locally decreasing if $\gamma(z) \in (-\infty, 1)$, locally increasing for $\gamma(z) \in (1,2)$ and constant for $\gamma(z) = 1$.*

Proof. proposition 1:

The second order profit maximisation condition implies

$$\frac{\partial^2 \Pi_j}{\partial P_j^2} = -\left(2F'(z) + \left(\frac{P_j - \frac{R}{\lambda_k}}{M}\right) F''(z)\right) < 0$$

where $z = \frac{I_j}{M}$.

This can be written as $-\left(\frac{P_j - \frac{R}{\lambda_k}}{M}\right) F''(z)/F'(z) = \gamma(z) < 2$.

The optimal price for good j satisfies $\frac{P_j}{M} = G\left(\frac{I_{j-1}+P_j}{M}\right)$, given the definition of $G(z) = \frac{1-F(z)}{F'(z)} + \frac{R}{M}$ we have that $G'(z) = -F''(z)\frac{1-F(z)}{(F'(z))^2} - 1 = \gamma(z) - 1$. The price has a finite solution if $\gamma(z) < 2$ or $G'(z) < 1$. Moreover, prices would be locally decreasing (increasing) if $G' < (>) 0$, and this is given by $\gamma(Z) < 1$ $(2 > \gamma(Z) > 1)$. ■

Proposition 2: *In this economy, money is neutral only if monetary injections are made through transfers proportional to the initial money holdings.*

Proof. proposition 2: In text ■

Proposition 3: For $\sigma^2 \to 0$, from those individuals who can afford to pay the fixed cost c, only those with cash holdings , M_i, higher than $M_n = \frac{(1+\overline{s})Pc}{\overline{s}}$ choose to dollarise their income.

Proof. proposition 3: Notice that for small levels of risk, this is for $\sigma_s^2 \longrightarrow 0$ we have that $q^{ND} = q^D$.Therefore $\overline{I}_{q_i^D} = \overline{I}_{q_i^{ND}}$. Using these two conditions we can express the differences in utilities functions with dollarisation and without it as follows; $V_i^D - V_i^{ND} = \overline{\varepsilon}(T_i - Pc) = \overline{\varepsilon}(s(M_i - Pc) - Pc)$. Hence, $E\left(U_i^D\right) > E\left(U_i^{ND}\right) \iff \overline{\epsilon}(M_i - Pc)(1+\overline{s}) > \overline{\epsilon} M_i$, therefore, only individuals with money holdings higher than the following threshold.$M_n = \frac{(1+\overline{s})Pc}{\widehat{s}}$ dollarise their income. ■

Lemma1: Let $\Pi_j^D\left((1+s)P_j^*, s\right)$ and $\Pi_j^{ND}(P_j, s)$ be the profit functions with prices in dollars and pesos, respectively. For any twice differentiable Π function we have that:

$$\frac{\partial E\left[\Pi_j^D\left((1+\overline{s})P_j^*, \overline{s}\right) - \Pi_j^{ND}(P_j, \overline{s})\right]}{\partial \sigma^2} \simeq \left(\frac{P_j}{1+\overline{s}}\right)\left[\Pi_{12}(P_j, \overline{s}) + \frac{1}{2}\frac{P_j}{1+\overline{s}}\Pi_{11}(P_j, \overline{s})\right]$$

evaluated at $s = \overline{s}$ and $\sigma^2 = 0$

Proof. lemma 1: Under perfect certainty of the exchange rate, the currency setting problem of the price is irrelevant. We have that $P_j = (1+\overline{s})\, P_j^*$ and $\Pi^D\left((1+\overline{s})\, P_j^*, \overline{s}\right) = \Pi^{ND}(P_j, \overline{s})$. Under uncertainty, the decission would depend on how s change the elasticity of demand of each good (i.e. x_{12}^d):

A second order taylor expansion of $\Pi^D\left((1+s)\, P_j^*, s\right) - \Pi^{ND}(P_j, s)$ on the expected exchange rate gives us:

$$\begin{aligned}
\Pi^D\left((1+s)\, P_j^*, s\right) - \Pi^{ND}(P_j, s) &= \Pi^D\left((1+\overline{s})\, P_j^*, \overline{s}\right) - \Pi^{ND}(P_j, \overline{s}) \\
&\quad + (s-\overline{s})\left[\frac{\partial \Pi^D\left((1+\overline{s})\, P_j^*, \overline{s}\right)}{\partial s} - \frac{\partial \Pi^{ND}(P_j, \overline{s})}{\partial s}\right] \\
&\quad + \frac{(s-\overline{s})^2}{2}\left[\frac{\partial^2 \Pi^D\left((1+\overline{s})\, P_j^*, \overline{s}\right)}{\partial s^2} - \frac{\partial^2 \Pi^{ND}(P_j, \overline{s})}{\partial s^2}\right] + \mathcal{O}\left(\left\|s^3\right\|\right)
\end{aligned}$$

For this condition to hold, we need the profit function to be continuous and twice differentiable on s.

The expected value of this gives us:

$$\begin{aligned}
E\left[\Pi^D\left((1+s)\, P_j^*, s\right) - \Pi^{ND}(P_j, s)\right] &= E\left[\Pi^D\left((1+\overline{s})\, P_j^*, \overline{s}\right) - \Pi^{ND}(P_j, \overline{s})\right] \\
&\quad + \frac{\sigma^2}{2} E\left[\frac{\partial^2 \Pi^D\left((1+\overline{s})\, P_j^*, \overline{s}\right)}{\partial s^2} - \frac{\partial^2 \Pi^{ND}(P_j, \overline{s})}{\partial s^2}\right] + \mathcal{O}\left(\left\|s^3\right\|\right)
\end{aligned}$$

To evaluate the impact of a small amount of risk on the optimal pricing strategy, we take the marginal derivative of $E\left[\Pi^D\left((1+s)\, P_j^*, s\right) - \Pi^{ND}(P_j, s)\right]$ with respect to the variance σ^2 of the nominal exchange rate, evaluated at $\sigma^2 = 0$:

$$\frac{\partial E\left[\Pi^D\left((1+s)\, P_j^*, s\right) - \Pi^{ND}(P_j, s)\right]}{\partial \sigma^2} = \frac{1}{2}\left[\frac{\partial^2 \Pi^D\left((1+\overline{s})\, P_j^*, \overline{s}\right)}{\partial s^2} - \frac{\partial^2 \Pi^{ND}(P_j, \overline{s})}{\partial s^2}\right]$$

The first term disappears, because $P_j = (1+\overline{s})\, P_j^*$ when $\sigma^2 = 0$.The decision of the price in "pesos" or "dollars" depends only on the second moments of the profit function.

Note that $\lim_{\sigma^2 \to 0} \frac{\partial^2 \Pi^{ND}(P_j, \overline{s})}{\partial s^2} = \Pi_{22}$ and $\lim_{\sigma^2 \to 0} \frac{\partial^2 \Pi^D\left((1+\overline{s})P_j^*, \overline{s}\right)}{\partial s^2} = \lim_{\sigma^2 \to 0}\left(\Pi_{22}^D + 2P^*\Pi_{12}^D + (P^*)^2\, \Pi_{11}^D\right) = \Pi_{22} + 2\left(\frac{P}{1+\overline{s}}\right)\Pi_{12} +$

$\left(\frac{P}{1+\overline{s}}\right)^2 \Pi_{11}$

Then:

$$\frac{\partial E\left[\Pi^D\left((1+s)\,P_j^*, s\right) - \Pi^{ND}\left(P_j, s\right)\right]}{\partial \sigma^2} = \frac{1}{2}\left[2\left(\frac{P_j}{1+\overline{s}}\right)\Pi_{12}\left(P_j, \overline{s}\right) + \left(\frac{P_j}{1+\overline{s}}\right)^2 \Pi_{11}\left(P_j, \overline{s}\right)\right]$$

Note that this last expression uses only the prices in "pesos" , the "D" subscripts are gone.

The price would be in dollars if $\left[2\Pi_{12}\left(P_j, \overline{s}\right) + \frac{P}{1+\overline{s}}\Pi_{11}\left(P_j, \overline{s}\right)\right] > 0$ and in pesos otherwise. We have that $\Pi_{11}\left(P_j, \overline{s}\right) \leq 0$ from the second order condition, then the pricing decision depends on the size and magnitude of $\Pi_{12}\left(P_j, \overline{s}\right)$. When $\Pi_{12}\left(P_j, \overline{s}\right) \leq 0$ the optimal price is in domestic currency, this case includes when the elasticity of demand is independent of the exchange rate $(\Pi_{12} = 0)$.The condition for pricing in dollars is $\Pi_{12}\left(P_j, \overline{s}\right) > -\frac{1}{2}\frac{P}{1+\overline{s}}\Pi_{11}\left(P_j, \overline{s}\right) > 0$. ■

Proposition 4: Given that a mass n of individuals maintain their assets in pesos and a distribution of s, such that $Es = \overline{s} > 0$ and $Vs = \sigma^2 \rightarrow 0$, there exists a threshold level of expenditure $I_{\overline{m}} = M\left(\theta_n - G\left(\theta_n\right)\right)$ such that $E\Pi^{ND}\left(I_{j-1}\right) > \Pi^D\left(I_{j-1}\right)$ for $I_{j-1} < I_{\overline{m}}$. Additionally, if $\gamma\left(\frac{I_{j-1}+P_j}{M}\right) < \overline{\gamma} = 1/\left(1 - \frac{1}{2}\frac{P_j}{I_{j-1}+P_j}\right)$, then $E\Pi^{ND}\left(I_{j-1}\right) < \Pi^D\left(I_{j-1}\right)$ for $I_{j-1} > I_{\overline{m}}$.

Proof. proposition 4: The general case for the demand functions in the appendix 1 has six different thresholds for I_{j-1}, three for the price in pesos and other three for the price in dollars. Therefore, the profits is a seven-zone piecewise function on I_{j-1}, and from those zones in only two the profits function is differentiable on s. At first sight, lemma 1 can be used only in those two out seven zones where the profit function is continuous and twice differentiable in s, but as we are taking approximations of the profit function for small values of σ^2, the six thresholds converge to one, given by $I_{\overline{m}} = M\left(\theta_n - G\left(\theta_n\right)\right)$ when $\sigma^2 \rightarrow 0$, and we end with only two zones in which the profit function is twice differentiable on s. In the proof of this proposition, we apply lemma 1 to these two zones.

From lemma 1 the condition for dollarisation for the firms is:

$$2\Pi_{12}\left(P, \overline{s}, I_{j-1}\right) + \frac{P}{1+\overline{s}}\Pi_{11}\left(P, \overline{s}, I_{j-1}\right)$$

Given the profit function in equation 16, the dollarisation condition becomes:

$$-\frac{P_j}{1+\overline{s}}\left(2F'(\tfrac{I_{j-1}+P_j}{M}) + P_j F''(\tfrac{I_{j-1}+P_j}{M})\right) < 0 \qquad I_{j-1} < I_{\overline{m}}$$

$$\left(\frac{1}{1+\overline{s}}\right)^2\left(\frac{I_{j-1}+P_j}{M}\right)\left(2F'(..) + \left(2 - \frac{P_j}{I_{j-1}+P_j}\right)\frac{P_j/M}{1+\overline{s}}F''(..)\right) \gtrless 0 \quad \text{otherwise}$$

where the value $(..) = (\frac{1}{1+\overline{s}}\frac{I_{j-1}+P_j}{M} + \frac{c}{y})$

The dollarisation condition for firms is never satisfied for $I_{j-1} < I_{\overline{m}}$, and it would be satisfied for $I_{j-1} > I_{\overline{m}}$ if $-\frac{P_j/M}{1+\overline{s}}F''(..)/F'(..) = \gamma\left(..\right) < 1/\left(1 - \frac{1}{2}\frac{P_j}{I_{j-1}+P_j}\right)$.

Moreover, we have that the second order condition implies that $\gamma(..) < 2$, this is a limit to the concavity of the income distribution in order to have bounded profits, but it is not enough to satisfy the condition for dollarisation for the firms. A sufficient condition to have that the firms set prices in dollars is that $\gamma(..) < \overline{\gamma} = 1/\left(1 - \frac{1}{2}\frac{P_j}{I_{j-1}+P_j}\right)$, where $1 \leq \overline{\gamma} \leq 2$. ∎

.

Corollary 1: *the number of goods that are priced in domestic currency, m, is defined by: $I_{m-1} \leq I_{\overline{m}} < I_m$. Goods with index $j \leq m$ are priced in pesos, and $j > m$ are priced in dollars.*

Proposition 5: *The function $I_{\overline{m}}(\theta_n) = M(\theta_n - G(\theta_n))$ that links $I_{\overline{m}}$ (the threshold of expenditure in goods priced in pesos) with θ_n (the threshold of individuals with assets in pesos) is increasing ($\partial I_{\overline{m}}/\partial\theta_n > 0$) and convex in θ_n ($\partial^2 I_{\overline{m}}/\partial\theta_n^2 > 0$) for most income distributions.*

Proof. proposition 5: Given that $I_{\overline{m}} = M(\theta_n - G(\theta_n))$, we have that $\partial I_{\overline{m}}/\partial\theta_n = M(1 - G'(\theta_n)) > 0$ because $G'(\theta_n) < 1$, as is satisfied by the soc. The convexity is given by $\partial^2 I_{\overline{m}}/\partial\theta_n^2 = MG''(\theta_n) = M\frac{F''}{F'}\left(1 + 2\frac{1-F}{(F')^2}F'' - \frac{1-F}{F'}\frac{F'''}{F''}\right) > 0$. We can write this expression as: $\partial^2 I_{\overline{m}}/\partial\theta_n^2 = M\frac{\gamma(\theta_n)}{G(\theta_n)}\left(1 - 2\gamma(\theta_n) + \gamma(\theta_n)\frac{F'}{(F'')^2}F'''\right)$.

For any concave income distribution ($F'' < 0$), we have that $\partial^2 I_{\overline{m}}/\partial\theta_n^2 > 0$ for

$$\begin{array}{ll} \text{any } \gamma(\theta_n) > 0 & \text{when } \frac{F'}{(F'')^2}F''' \geq 2 \\ \text{for } \gamma(\theta_n) \in \left(0, \frac{1}{2 - \frac{F'}{(F'')^2}F'''}\right) & \text{when } \frac{F'}{(F'')^2}F''' < 2 \end{array}$$

Note that when $F''' > 0$ the density of the income distribution is convex, and this is the case for most income distributions. Additionally, when γ is low (high), $\frac{F'}{(F'')^2}F'''$ tends to be higher (smaller) than 2. The condition for the convexity would not be satisfied for distributions with high values of γ, such is the case of the exponential income distribution where $\gamma(z) = \frac{F'}{(F'')^2}F''' = 1$ for every z.

However, note that when $I_{\overline{m}}(\theta_n)$ is convex, as we plot it as the inverse function $\theta_n = I_{\overline{m}}^{-1}$, the figure is concave. ∎

Proposition : 6 *AD is always higher than or equal to PD.*

Proof. 6 Asset dollarisation and price dollarisation are defined as follows:

$$
\begin{aligned}
AD &= \frac{\int_{\theta_n}^{\overrightarrow{\theta}} Py\left(\theta_i\right) dF(\theta)}{\int_{\underleftarrow{\theta}}^{\overrightarrow{\theta}} Py\left(\theta_i\right) dF(\theta)} \\
PD &= \frac{\sum_{j=m+1}^{J} P_j^* x_j^d}{\sum_{j=1}^{m} P_j x_j^d + \sum_{j=m+1}^{J} P_j^* x_j^d}..
\end{aligned}
$$

Note that it is possible to show from the individual budget constraint that:
$\int_{\underleftarrow{\theta}}^{\overrightarrow{\theta}} Py\left(\theta_i\right) dF(\theta) = \sum_{j=1}^{m} P_j x_j^d + \sum_{j=m+1}^{J} P_j^* x_j^d + P_o \int_{\underleftarrow{\theta}}^{\overrightarrow{\theta}} x_0\left(\theta_i\right) dF(\theta)$,
total income is equal to total expenditure in goods (both in dollars and pesos) and in leisure leisure good.
On the other hand, for those individuals that dollarise their assets:
$\int_{\theta_n}^{\overrightarrow{\theta}} Py\left(\theta_i\right) dF(\theta) = \sum_{j=m+1}^{J} P_j^* x_j^d + I_m\left(1 - F\left(\theta_n\right)\right) + P_o \int_{\theta_n}^{\overrightarrow{\theta}} x_0\left(\theta_i\right) dF(\theta)$,
their total income is equal to the total expenditure of goods in dollars, $\sum_{j=m+1}^{J} P_j^* x_j^d$, one part of the expenditure in pesos in the economy, $I_m\left(1 - F\left(\theta_n\right)\right)$, and the expenditure in the leisure good.

Then AD becomes:

$$
\begin{aligned}
AD &= \frac{\sum_{j=m+1}^{J} P_j^* x_j^d + I_m\left(1 - F\left(\theta_n\right)\right) + P_o \int_{\theta_n}^{\overrightarrow{\theta}} x_0\left(\theta_i\right) dF(\theta)}{\sum_{j=1}^{m} P_j x_j^d + \sum_{j=m+1}^{J} P_j^* x_j^d + P_o \int_{\underleftarrow{\theta}}^{\overrightarrow{\theta}} x_0\left(\theta_i\right) dF(\theta)} \\
&> PD = \frac{\sum_{j=m+1}^{J} P_j^* x_j^d}{\sum_{j=1}^{m} P_j x_j^d + \sum_{j=m+1}^{J} P_j^* x_j^d}.
\end{aligned}
$$

for $\theta_n \in \left(\underleftarrow{\theta}, \overrightarrow{\theta}\right)$ given that $I_m\left(1 - F\left(\theta_n\right)\right) + P_o \int_{\theta_n}^{\overrightarrow{\theta}} x_0\left(\theta_i\right) dF(\theta) > P_o \int_{\underleftarrow{\theta}}^{\overrightarrow{\theta}} x_0\left(\theta_i\right) dF(\theta)$

Be can show that the last expression is true from the individual budget constraint: $P_o x_0\left(\theta_i\right) + I_q\left(\theta_i\right) = Py\left(\theta_i\right)$. Aggregating for $\theta_i \leq \theta_n$ we have: $P_o \int_{\underleftarrow{\theta}}^{\theta_n} x_0\left(\theta_i\right) dF(\theta) + \int_{\underleftarrow{\theta}}^{\theta_n} I_q\left(\theta_i\right) dF(\theta) = \int_{\underleftarrow{\theta}}^{\theta_n} Py\left(\theta_i\right) dF(\theta) \leq I_m\left(1 - F\left(\theta_n\right)\right)$ because the expenditure of the individuals with $\theta_i \leq \theta_n$ is lower or equal to I_m, the cost of the total basket in pesos. Therefore $P_o \int_{\underleftarrow{\theta}}^{\theta_n} x_0\left(\theta_i\right) dF(\theta) < I_m\left(1 - F\left(\theta_n\right)\right)$ that is what we need to show the inequality.

From this, it is also possible to see that in the extreme values when $\theta_n = \{\underleftarrow{\theta}, \overrightarrow{\theta}\}$, we have that asset dollarisation is either null or complete (i.e. $AD = \{0, 1\}$) and equal to price dollarisation ($AD = PD$). ■

Proposition: 7 *The degree of pass-through in the short-run and the long-run are equal to PD and AD, respectively.*