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Interbank Market and Macroprudential Tools in a DSGE Model

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Interbank Market and Macroprudential Tools in a DSGE Model*

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Abstract

The interbank market helps regulate liquidity in the banking sector. Banks with outstanding resources usually lend to banks that are in needs of liquidity. Regulating the interbank market may actually benefit the policy stance of monetary policy. Introducing an interbank market in a general equilibrium model may allow better identification of the final effects of non-conventional policy tools such as reserve requirements. We introduce an interbank market in which there are two types of private banks and a central bank that has the ability to issue money into a DSGE model. Then, we use the model to analyse the effects of changes to reserve requirements (a macroprudential tool), while the central bank follows a Taylor rule to set the policy interest rate. We find that changes to reserve requirements have similar effects to interest rate hikes and that both monetary policy tools can be used jointly in order to avoid big swings in the policy rate (that could have an undesired effect on private expectations) or a zero bound (i.e. liquidity trap scenarios).

Keywords: reserve requirements, collateral, banks, interbank market, DSGE.
JEL classification: E31, O42.

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1 Introduction

In this paper, we study the effects of transitory changes in reserve requirements in a general equilibrium context. In particular, we are interested in the short-run effects of transitory shocks to reserve requirements on real and financial variables and the transmission mechanism behind those effects. We also explore the interaction of reserve requirement shocks with traditional, interest-rate based, monetary policy shocks.

We find that reserve requirement shocks are qualitatively similar to traditional monetary policy shocks. They generate a short-run fall in inflation, output and asset prices while pushing up lending and deposit rates. However, reserve requirement shocks differ from monetary policy shocks in that they expand interbank lending and contract households' deposits in the model. Additionally, we show that changes in reserve requirements can complement traditional monetary policy actions such as a hike in interest rates. Thus, our policy-maker can obtain the same desired impact on real aggregates with a smaller change in the interest rate, provided he is willing to complement his actions with a change to reserve requirements.

In our model, an increase in reserve requirements reduces the loanable funds of financial intermediaries. These institutions will react demanding more interbank lending, pushing up the interest rate charged on those operations due to higher monitoring costs (a financial friction). Thus, banks' average cost of funding will increase. This cost hike will be transmitted to lending rates and deposits rates, resulting in a slowdown of economic activity.

When used together, reserve requirements can partly substitute interest rate hikes. The reason behind this is that both variables affect banks' average cost of funding. This is particularly relevant when the policy-maker faces a situation where the desired response would be a very big shift in the policy rate,[1] or, even worse, the required policy action entails getting uncomfortably close to the zero lower bound.

Standard New-Keynesian models cannot accommodate both a reserve requirement shock and monetary policy formulated by a Taylor Rule. The reason is that changes to reserve requirements alter the monetary base but the latter becomes endogenous when the monetary policy interest rate is governed by a Taylor Rule.[2] Any reserve requirement "shocks" become undone immediately by endogenous changes to the policy rate.

In spite of the theoretical conundrum exposed above, central banks in Latin America have been using reserve requirements as a policy tool recently, usually in conjunction with traditional, interest-rate based, policy actions. Furthermore, Tovar et al. (2012) provide empirical and anecdotal evidence that *"monetary and macroprudential instruments, including*

[1] Historically, policy makers tend to be reluctant to do this, possibly as an endogenous response to uncertainty.

[2] This is the case in the models developed in Clarida et al. (1999) and Bernanke et al. (1999), for example.

reserve requirements, appear to have complemented each other in recent episodes".

Reserve requirements have also been used as a macroprudential tool in Latin America for the past decade. As Tovar et al. (2012) report,

> *"...policy makers in Latin America have adopted a number of macroprudential instruments to manage the procyclicality of bank credit dynamics to the private sector and contain systemic risk. Reserve requirements, in particular, have been actively employed."*

In order to have changes in reserve requirements interacting with traditional monetary policy (i.e.: a Taylor Rule), we propose a New-Keynesian model incorporating a financial system with frictions, particularly in the interbank market. It turns out that very little work has been done in this area: studying interbank markets and the frictions associated with them in a general equilibrium context is a relatively new subject. Thus, our work also contributes to the literature by providing a fresh take on how to model the agents that participate in this market and their interactions.

Interbank markets play an important role in the transmission process from monetary policy to economic activity because they help allocate resources between financial institutions. Financial frictions (usually involving credit) and regulation (hair-cuts, reserve requirements, and collateral constraints) constitute important features of interbank markets that have an impact on their effectiveness in amplifying or dampening the real effects of monetary policy.

The rest of the paper is organized as follows. Section 2 is devoted to short review of related literature, section 3 describes our model with an interbank market. Section 4 details the calibration procedure. Section 5 presents our results. Finally, section 6 concludes.

2 A (Short) Literature Review on Reserve Requirements and Interbank Markets

2.1 Reserve Requirements

The literature on reserve requirement shocks in general equilibrium is very scarce. Our model bears some resemblance to Edwards and Vegh (1997) and, more recently, Prada (2008). The work of Edwards and Vegh (1997) shows how foreign business cycles and shocks to the banking system affect output and employment through fluctuations in bank credit. In this context, they explore the countercyclical use of reserve requirements and find they can be used to insulate the economy from the world business cycle. In order to obtain this result, Edwards and Vegh (1997) assume the production of banking services is costly.

Costly banking services are present in Prada (2008) as well. This author elaborates on the work done by Edwards and Vegh (1997) adding New-Keynesian rigidities to the open economy (Calvo (1983) pricing, investment adjustment costs in the spirit of Christiano et al. (2005), etc.). He finds that reserve requirements do not have quantitatively significant effects.

Our model bears some resemblance to Edwards and Vegh (1997) because the financial friction we impose on the interbank market (a monitoring cost) is akin to their "costly banking services". Yet, neither Edwards and Vegh (1997) nor Prada (2008) include an interbank market in their model. More importantly, their findings with respect to reserve requirements are different from ours. Edwards and Vegh (1997) does not study short-run changes in reserve requirements and their effect on real aggregates nor how they interact with policy rates. Prada (2008) dismisses reserve requirements because his quantitative results are not significant while we find that reserve requirements impact the economy in a similar manner than policy rates and, more importantly, they can be used to complement policy rate hikes.

2.2 Interbank Markets

The financial accelerator of Bernanke et al. (1999) usually amplifies, spreads, and gives more persistence to different types of shocks in the economy, particularly shocks that directly affect financial intermediaries. After the financial crisis of 2007 - 2009, several economists use Bernanke et al. (1999) as a stepping stone for valid extensions of the original model. One of those extensions is the inclusion of an interbank market. As Walsh (2010) points out, imperfect credit markets make the policy interest rate insufficient to characterize the monetary policy stance. Moreover, credit effects may arise when frictions are present in these financial markets. Thus, one source of motivation for recent research is the nature of the transmission of monetary policy through more than one interest rate (interest rate pass-through) and the conditions of such transmission (the interbank lending market).

The recent literature reviews of Carrera (2012) and Roger and Vlcek (2012), highlight the lack of models featuring an interbank market. In that regard, the work of Gerali et al. (2010), Curdia and Woodford (2010), Dib (2010), and Hilberg and Hollmayr (2011) are among the first on this arena.

The banking sector in Gerali et al. (2010) encompasses many banks each composed of two "retail" branches and one "wholesale" unit. The first retail branch is responsible for giving out differentiated loans to households and entrepreneurs; the second for raising deposits. The wholesale unit manages the capital position of the group. In Curdia and Woodford (2010), the frictions associated with financial intermediation (intermediation requires real resources and bank lending activities create opportunities for borrowers to take out loans without being made to repay) determine both the spread between borrowing and lending rates and the resources consumed by the intermediary sector. Dib (2010) introduces the distinction between banks that only raise deposits and banks that only give out credit, and sets them up in an interbank market in which the first group of banks borrows from the second group.

Hilberg and Hollmayr (2011) take a different approach and separate the interbank market in two types of banks: commercial banks and investment banks. Hilberg and Hollmayr notice that only a few banks actually interact with the central bank, and then fund the rest of the banking system. While the capital of the banks plays an important role in Gerali et al. (2010) and Dib (2010), for Hilberg and Hollmayr (2011) it is the structure of the market and collateral that matters the most.

We partially follow on the structure of Hilberg and Hollmayr (2011) (see Figure 1). The hierarchical interbank market is a good representation of the structure in the U.S. (only Primary Dealers deal with the central bank whereas a vast group of commercial banks is not allowed to deal directly with the monetary authority) and in Europe (only 6 out of 2500 banks are allowed to participate in the bidding process in main refinancing operations of the ECB and other banks rely on interbank funding).[3]

We depart from Hilberg and Hollmayr (2011) in four dimensions: (i) retail banks are subject to required reserves,[4] (ii) narrow banks incur in monitor-credit costs, (iii) narrow banks obtain funding from households, not the central bank and (iv) the bond market is used by the central bank to implement monetary policy in the form of open market operations.

Dinger and Hagen (2009) point out that banks are particularly good at identifying the risk of other banks and present evidence of the importance of interbank transactions. We add monitoring costs in the same fashion as Curdia and Woodford (2010). In doing this, we find that reserve requirements can actually complement the effects of the interest rate, a result that helps understand the importance of this macroprudential tool.

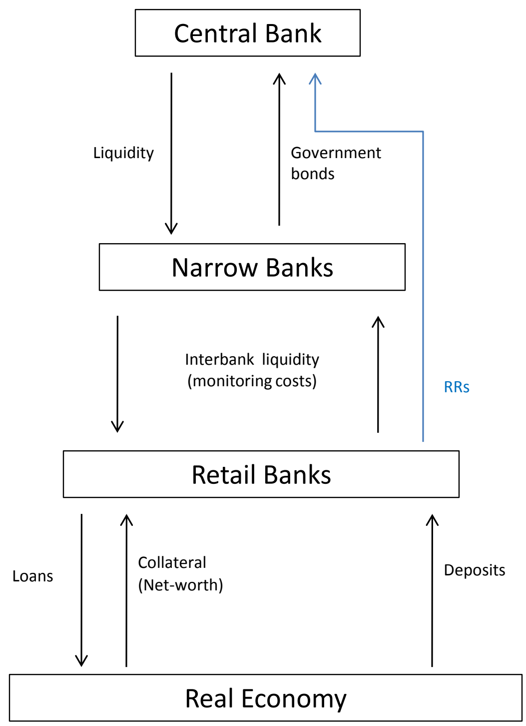
3 The Model

Our model exhibits a fairly standard real sector coupled with the financial accelerator mechanism of Bernanke et al. (1999) (taking some additional elements of Cohen-Cole and Martínez-García (2010)). On top of this, we add an interbank market structure along the lines of Hilberg and Hollmayr (2011), with bank monitoring costs in Curdia and Woodford (2010) fashion.

[3]See Walsh (2010), chapter 11, for a description of the FED's operating procedures, and http://www.ny.frb.org/markets/primarydealers.html for more information on the FED's Primary Dealers.

[4]These can also be interpreted as liquidity requirements in line with Basel III proposals.

Figure 1: Interbank market structure



Even though this model does not justify the existence of banks (or why they should be regulated), it is still flexible enough to capture the transmission of monetary policy with an interbank market operating. In that sense, banks are assumed to be essential because they provide households with the only risk-free asset in the economy (deposits) and entrepreneurs can only attract external finance from banks.[5]

There are two financial frictions in the model: one on the liability side of retail banks and one on the asset side of narrow banks. Our first friction takes the form of an adjustment cost on deposit rates (given imperfect competition in the banking sector, a la Gerali et al. (2010)). Our second friction arises from convex monitoring costs (a la Curdia and Woodford (2010)) originated by interbank loans from narrow banks to retail banks.

Finally, our model has one-period nominal loan contracts. Contracts are nominal by assumption but we consider the feature to be realistic and it has the added benefit of allowing

[5]The model abstracts from bank moral hazard, bank runs, etc. (as in, for example, Dib (2010)) in order to stress the role of the interbank market and its frictions.

us to introduce (minor) Fisherian debt deflation effects in the monetary policy transmission mechanism.

3.1 Households

We assume a continuum of households that have an identical utility function. The utility function of each household is additively separable in consumption, (C_t), real cash holdings, (CHS_t/P_t), and labor (H_t). Thus, the household's objective is to maximize:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \frac{(C_s - bC_{s-1})^{1-\sigma^{-1}}}{1-\sigma^{-1}} + \chi_M \frac{\left(\frac{CSH_s}{P_s}\right)^{1-\sigma_M^{-1}}}{1-\sigma_M^{-1}} - \chi_H \frac{H_s^{1+\varphi^{-1}}}{1+\varphi^{-1}} \right\} \tag{1}$$

where $0 < \beta < 1$ is the subjective intertemporal discount factor, b is the habit parameter in household consumption, $\sigma > 0$ and $\sigma_M > 0$ are the elasticities of intertemporal substitution of consumption and real cash holdings respectively, and $\varphi > 0$ is the Frisch elasticity of labor supply.

We include real cash holdings in the household's utility function to generate a money demand. This "money in the utility function" (MIU) approach has been studied extensively in the literature (see, for example, Walsh (2010)). It is usually rationalized arguing that money holdings provide transaction services, facilitating the acquisition of consumption goods by, for example, reducing the time needed to purchase them. It should be noted that without this assumption households would never hold cash: any asset offering a positive return (e.g.: deposits) would be a superior substitute.

Household income is derived from renting labor to wholesale producers at competitive nominal wages (W_t). Given that households own the retailers and capital goods producers, they receive their total real profits (Π_t^R and Π_t^K respectively). The unanticipated profits of retail banks are also fully rebated to households in each period (Π_t^{RB}). Turning to assets, households demand one period deposits which pay a fixed nominal interest, invest in shares which entitle them to a proportional fraction of the narrow banks' dividends (DIV_t^{NB}/S_{t-1}) and hold cash deposits between periods. Available income is used to finance aggregate consumption (C_t), open new deposits (D_t), invest in shares $(P_t^S S_t)$, hold cash (CSH_t/P_t), and pay the real (lump-sum) tax bill $(T_t^S + T_t^B)$. Therefore, the households' budget constraint is defined as:

$$C_t + T_t^S + T_t^B + D_t + P_t^S S_t + \frac{CSH_t}{P_t} = \frac{W_t}{P_t} H_t + R_{t-1}^D D_{t-1} \frac{P_{t-1}}{P_t}$$
$$+ \left(\frac{DIV_t^{NB} + P_t^S S_{t-1}}{P_{t-1}^S S_{t-1}} \right) P_{t-1}^S S_{t-1} + \frac{CSH_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t}$$
$$+ \Pi_t^R + \Pi_t^K + \Pi_t^{RB} \tag{2}$$

where R_t^D is the nominal one-period interest rate offered to depositors by retail banks, P_t^S is the narrow bank's relative price per share, and P_t is the consumer price index (CPI, defined later). As a convention, D_t denotes real deposits from time t to $t+1$. Therefore, the interest rate R_t^D paid at $t+1$ is known and determined at time t.

From the household's first order conditions we obtain,

$$(C_t - bC_{t-1})^{-\frac{1}{\sigma}} - \lambda_t = \beta b E_t \left[(C_{t+1} - bC_t)^{-\frac{1}{\sigma}} \right] \tag{3}$$

$$\chi_H H_t^{\frac{1}{\phi}} = \lambda_t \frac{W_t}{P_t} \tag{4}$$

$$\lambda_t = \beta E_t \left[\lambda_{t+1} \frac{P_t}{P_{t+1}} \right] R_t^D \tag{5}$$

$$\lambda_t = \beta E_t \left[\lambda_{t+1} \left(\frac{DIV_{t+1}^{NB} + P_{t+1}^S S_t}{P_t^S S_t} \right) \right] \tag{6}$$

$$\chi_M \left(\frac{CSH_t}{P_t} \right)^{-\frac{1}{\sigma_M}} - \lambda_t = \beta E_t \left[\lambda_{t+1} \frac{P_t}{P_{t+1}} \right] \tag{7}$$

where λ_t is the Lagrange multiplier associated to the household's budget constraint.

We solve for the Euler equation that links consumption to the deposit rate and past consumption.

$$MUC_t = \beta E_t \left[MUC_{t+1} \frac{P_t}{P_{t+1}} \right] R_t^D \tag{8}$$

where $MUC_t = (C_t - bC_{t-1})^{-\frac{1}{\sigma}} - \beta b E_t \left[(C_{t+1} - bC_t)^{-\frac{1}{\sigma}} \right]$.

Using the definition of MUC_t, we also solve for the labor supply, money demand, and the demand for shares.

$$\chi_H H_t^{\frac{1}{\phi}} = MUC_t \frac{W_t}{P_t} \tag{9}$$

$$\chi_M \left(\frac{CSH_t}{P_t}\right)^{-\frac{1}{\sigma_M}} = MUC_t \left(\frac{R_t^D - 1}{R_t^D}\right) \tag{10}$$

$$E_t \left[MUC_{t+1} \frac{P_t}{P_{t+1}}\right] R_t^D = E_t \left[MUC_{t+1} \left(\frac{DIV_{t+1}^{NB} + P_{t+1}^S S_t}{P_t^S S_t}\right)\right] \tag{11}$$

3.2 Wholesale Producers

We assume the existence of a representative wholesale producer operating under perfect competition. Our wholesale producer employs entrepreneurial (H_t^E) and household (H_t) labor combined with rented capital goods (K_t) in order to produce homogeneous wholesale goods (Y_t^W). The technology involved is Cobb-Douglas:

$$Y_t^W = e^{a_t}(K_t)^{1-\psi-\varrho}(H_t)^{\psi}(H_t^E)^{\varrho} \tag{12}$$

where a_t is a productivity shock.

In this constant returns-to-scale technology, the non-managerial and managerial labor shares in the production function are determined by the coefficients $0 < \varrho < 1$ and $0 < \psi < 1$. As in Bernanke et al. (1999), the managerial share (ϱ) is assumed to be very small. The productivity shock follows an AR(1) process of the following form:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{13}$$

where ε_t^a is normal i.i.d. (with zero mean and σ_a^2 variance) and ρ_a captures the degree of persistence of the shock.

Wholesale producers seek to maximize their nominal profits:

$$P_t \Pi_t^W = P_t^W Y_t^W - R_t^W K_t - W_t H_t - W_t^E H_t^E \tag{14}$$

where Π_t^W is the real profit of the wholesale producer, P_t^W is the nominal price of the wholesale good, R_t^W is the nominal rent paid per unit of capital to entrepreneurs, and W_t and W_t^E are the nominal wages of household and entrepreneurial labor respectively.

The first order conditions for this problem result in the usual demands for labor (household and entrepreneurial) and capital,

$$R_t^W = (1-\psi-\varrho)\frac{P_t^W Y_t^W}{K_t} \tag{15}$$

$$W_t = \psi \frac{P_t^W Y_t^W}{H_t} \tag{16}$$

$$W_t^E = \varrho \frac{P_t^W Y_t^W}{H_t^E} \tag{17}$$

Wholesale producers make zero profits. Households, who own these firms, do not receive any dividends. Entrepreneurs receive income from their supply of managerial labor and rented capital to wholesalers. Wholesale producers rent capital from the entrepreneurs and return the depreciated capital after production has taken place.

3.3 Capital Goods Producers

We assume a continuum of competitive capital goods producers who at time t purchase a bundle of retail goods that will be used as "investment" (X_t) and depreciated capital ($(1-\delta)K_t$) to manufacture new capital goods (K_{t+1}). The production of new capital is limited by technological constraints. We assume that the aggregate stock of new capital considers investment adjustment costs and evolves following the law of motion:

$$K_{t+1} = (1-\delta)K_t + \Phi\left(\frac{X_t}{X_{t-1}}\right)X_t \tag{18}$$

where $\Phi(\cdot)$ is an investment adjustment cost function. We follow Christiano et al. (2005) and describe the technology available to the capital good producer as:

$$\Phi\left(\frac{X_t}{X_{t-1}}\right) = \left[1 - 0.5\kappa\frac{\left(\frac{X_t}{X_{t-1}}-1\right)^2}{\frac{X_t}{X_{t-1}}}\right] \tag{19}$$

where $\frac{X_t}{X_{t-1}}$ is the investment growth rate and $\kappa > 0$ regulates the degree of concavity of the technological constraint. Note that given our functional choice, $\Phi(1) = 1$ and $\Phi'(1) = 0$ implying constant returns to scale in steady state only.[6]

A representative capital goods producer chooses his investment (X_t) and depreciated capital ($(1-\delta)K_t$) demand to maximize the expected discounted value of his profits, solving the following problem:

$$E_t \sum_{s=t}^{\infty} M_{s-t}^H \left\{Q_s K_{s+1} - (1-\delta)Q_s K_s - X_s\right\} \tag{20}$$

[6] This follows from the realization that the marginal product of investment in the production of new capital is $\Phi'\left(\frac{X_t}{X_{t-1}}\right)\frac{X_t}{X_{t-1}} + \Phi\left(\frac{X_t}{X_{t-1}}\right)$, an expression that equals unity only in steady state where $\frac{X_t}{X_{t-1}} = 1$.

where M^H_{s-t} is a stochastic discount factor and Q_t is the price of new capital for entrepreneurs which determines the relative cost of investment in units of consumption (Tobin's Q).

Since households own the capital goods producers, capital producers compute present value using the household's stochastic discount factor defined as:

$$M^H_\tau = \beta^\tau \frac{MUC_{t+\tau}}{MUC_t} \frac{P_t}{P_{t+\tau}} \\ = \begin{cases} 1 & \tau = 0 \\ \prod_{i=0}^{\tau-1} \frac{1}{R^D_{t+i}} & \tau > 0 \end{cases} \tag{21}$$

where the second equality can be obtained using the household's Euler equation.

Given the capital goods producer's production function, the marginal rate of transformation of depreciated capital to new capital is unity. Thus, it must be the case that, in equilibrium, depreciated and new capital share the same price Q_t. Furthermore, any quantity of depreciated capital is profit-maximizing as long as its price is the same as that of new capital: the capital good producer's demand for depreciated capital is perfectly elastic at price Q_t. Since entrepreneurs (discussed later) will supply depreciated capital inelastically, market clearing guarantees capital goods producers acquire all the depreciated capital stock from entrepreneurs at price Q_t. Thus, we take a short-cut to this result incorporating $(1-\delta)Q_tK_t$ directly in the capital goods producer objective function.

The first order conditions derived from the optimization process of the capital goods producers yield a standard link between our Tobin's Q analogue (Q_t) and investment (X_t):

$$Q_t \left[\Phi\left(\frac{X_t}{X_{t-1}}\right) + \Phi'\left(\frac{X_t}{X_{t-1}}\right) \frac{X_t}{X_{t-1}} \right] = 1 + \frac{1}{R^D_t} E_t \left[Q_{t+1} \Phi'\left(\frac{X_{t+1}}{X_t}\right) \left(\frac{X_{t+1}}{X_t}\right)^2 \right] \tag{22}$$

Aggregate profits for the capital goods producers are defined as:

$$\Pi^K_t = Q_t K_{t+1} - (1-\delta) Q_t K_t - X_t \tag{23}$$

Because our capital goods producers operate in a competitive environment, there are no profits in equilibrium. However, during the transition to steady state, these firms can generate short-term profits (or losses) because X_{t-1} is predetermined at time t implying the marginal product of investment can deviate from unity. When transitioning from one steady state equilibrium to another, the adjustment cost cannot be set to its optimal level.

3.4 Retailers

There is a continuum of retailers indexed by $z \in [0,1]$ that purchase the homogeneous good (Y_t^W) from wholesalers and differentiate it costlessly in order to sell it to households, entrepreneurs, and capital goods producers (for consumption or investment). These customers love variety and demand a CES bundle (Y_t) composed by the differentiated varieties ($Y_t(z)$) offered by retailers, aggregated with elasticity of substitution $\theta > 1$:

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \tag{24}$$

Standard optimization of a CES utility defined over the retail good varieties yields a relative demand for each variety:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\theta} Y_t \tag{25}$$

where $P_t(z)$ is the price of variety z being charged at time t and P_t is a price index given by:

$$P_t = \left[\int_0^1 P_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}} \tag{26}$$

Given the opportunity, a retailer will set its price ($P_t(z)$) to maximize the expected discounted value of its profit stream. Because of the market structure in which these firms operate (monopolistic competition), they have the power to charge a retail mark-up over the price of the homogeneous good. However, their re-optimizing processes are constrained by nominal rigidities as in Calvo (1983). At time t, an individual retailer maintains its price fixed from the previous period with probability $0 < \alpha < 1$. Thus, it is allowed to optimally reset its price with probability $(1-\alpha)$.

Our paper concentrates on the effects of reserve requirements and the incorporation of an interbank market into a relatively standard New - Keynesian DSGE framework, without delving into welfare implications. Thus, we choose to eliminate the distortion introduced by retailers' mark-up pricing assuming the government subsidizes a fraction (τ^R) of their input costs. Therefore, the retailer pays only $\left(1-\tau^R\right) P_t^W$ per unit of wholesale good acquired.[7]

A retailer z that is allowed to change its price at time t will choose it to maximize:

$$E_t \sum_{s=t}^{\infty} M_{s-t}^H \alpha^{s-t} \left\{ \left(\tilde{P}_t(z) - (1-\tau^R) P_s^W \right) \tilde{Y}_{s,t}(z) \right\} \tag{27}$$

[7] In this set up, it can be shown that $\tau^R = \frac{1}{\theta}$ is required to guarantee $P_t = P_t^W$ in equilibrium.

where $\tilde{P}_t(z)$ is the optimal price chosen at time t and $\tilde{Y}_{s,t}(z) = \left(\frac{\tilde{P}_t(z)}{P_s}\right)^{-\theta} Y_s$ is the relative demand of good z at time s given that its price remains fixed at $\tilde{P}_t(z)$.

The first order condition for this problem is:

$$E_t \sum_{s=t}^{\infty} M_{s-t}^H \alpha^{s-t} \left\{ \left(\tilde{P}_t(z) - \frac{\theta}{\theta-1}(1-\tau^R) P_s^W \right) \tilde{Y}_{s,t}(z) \right\} = 0 \tag{28}$$

where $\frac{\theta}{\theta-1}$ would be the retail mark-up without the government's subsidy.

Since all re-optimizing retailers face a symmetric problem, the aggregate CPI (P_t) can be expressed as a weighted geometric average of "old" and "new" prices:

$$P_t = \left[\alpha P_{t-1}^{1-\theta} + (1-\alpha) \tilde{P}_t^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{29}$$

where $\tilde{P}_t = \tilde{P}_t(z)$ is the symmetric optimal price.

Wholesale goods market clearing requires that aggregate retailer demand equal the total output of wholesale producers:

$$\int_0^1 Y_t(z) dz = Y_t^W \tag{30}$$

Introducing the individual retailer relative demands into this expression will result in:[8]

$$Y_t = \left(\frac{P_t^*}{P_t} \right)^{\theta} Y_t^W \tag{31}$$

where

$$P_t^* = \left[\int_0^1 P_t(z)^{-\theta} dz \right]^{-\frac{1}{\theta}} = \left[\alpha \left(P_{t-1}^* \right)^{-\theta} + (1-\alpha) \tilde{P}_t^{-\theta} \right]^{-\frac{1}{\theta}} \tag{32}$$

is an alternative price index introduced to ease notation and highlight the efficiency distortion due to sticky prices.[9]

Aggregate nominal profits transferred to the households are:

$$P_t \Pi_t^R = \int_0^1 \left(P_t(z) - \left(1-\tau^R\right) P_t^W \right) Y_t(z) dz \tag{33}$$

which implies,

[8] Recall that Y_t is a CES bundle of the individual $Y_t(z)$ and generally not equal to $\int_0^1 Y_t(z)dz$.

[9] Actually, P_t and P_t^* are identical in a first-order approximation.

$$\Pi_t^R = Y_t - (1-\tau^R)\frac{P_t^W}{P_t}Y_t^W. \tag{34}$$

3.5 Entrepreneurs and Retail Banks

The following description of the interaction between entrepreneurs and retail banks draws heavily from Bernanke et al. (1999), Christiano et al. (2010) and Gerali et al. (2010). Entrepreneurs supply one unit of managerial labor ($H_t^E = 1$) to wholesale producers inelastically. They accumulate real net worth (N_t) and take real loans (L_t) in order to buy new capital (K_{t+1}) from capital goods producers at relative price Q_t. Thus, an entrepreneur's balance sheet can be described as:

$$Q_t K_{t+1} = L_t + N_t \tag{35}$$

There is risk involved in the entrepreneurial activity: after the acquisition of new capital, entrepreneurs experience a private idiosyncratic shock ω which transforms the capital they acquired, K_{t+1}, into ωK_{t+1}. In the literature incorporating the financial accelerator mechanism described in Bernanke et al. (1999), ω is usually assumed to be log-normally distributed with parameters μ_ω and σ_ω. These parameters are then picked to be consistent with $E[\omega] = 1$ and a particular steady state default rate on the loans. We follow this convention.

At the end of period t, an individual entrepreneur receives a nominal wage, W_t^E, and earns income from capital rented to the producers of wholesale goods, $R_t^W \omega K_t$, plus the resale value of the depreciated capital which is sold back to the capital goods producers ($(1-\delta)P_t Q_t \omega K_t$). Therefore, we can express the individual entrepreneur's nominal return on capital as the ratio between income received from renting capital and selling it after depreciation divided by its nominal cost:

$$\omega R_t^E = \omega \frac{R_t^W K_t + (1-\delta)P_t Q_t K_t}{P_{t-1} Q_{t-1} K_t} \tag{36}$$

where R_t^E is defined implicitly as the gross nominal return on capital of the average entrepreneur.

At t, our representative entrepreneur signs a loan contract with a retail bank specifying a loan amount (L_t) and a nominal lending rate (R_t^L). Both the entrepreneur and the retail bank understand the loan is destined to finance part of the acquisition of new capital, K_{t+1}. The debt has to be repaid at time $t+1$. In case of default, retail banks can only appropriate the gross capital return of the entrepreneur at that time, i.e. $\omega R_{t+1}^E P_t Q_t K_{t+1}$.

Thus, we can define the cut-off $\overline{\omega}_{t+1}$ as the particular value of the idiosyncratic shock ω that allows the entrepreneur to honor his debt next period, leaving him with zero net income (individual entrepreneurs experiencing an idiosyncratic shock $\omega < \overline{\omega}_{t+1}$ default on their loans):

$$\overline{\omega}_{t+1} R^E_{t+1} P_t Q_t K_{t+1} = R^L_t P_t L_t$$

$$\overline{\omega}_{t+1} = \frac{R^L_t P_t L_t}{R^E_{t+1} P_t Q_t K_{t+1}} \tag{37}$$

where $\overline{\omega}_{t+1} R^E_{t+1}$ is the minimum return that entrepreneurs require in order to pay back to the bank, and $R^L_t P_t L_t$ is the payment amount agreed with the bank at time t. Note that the cut-off $\overline{\omega}_{t+1}$ depends positively on the lending rate (R^L_t) and negatively on the entrepreneur's leverage ($Q_t K_{t+1}/N_t$).

The loan market is competitive. There is a continuum of retail banks that offer contracts with lending rate R^L_t, obtain deposits at rate $R^D_t(j)$ in a market characterized by monopolistic competition, and take the interest rate on the (competitive) interbank market R^{IB}_t as given. On the liability side, the representative retail bank has deposits ($D_t(j)$) and interbank funds ($IB_t(j)$) that are obtained from households and narrow banks, respectively. These funds are allocated by the retail bank into loans ($L_t(j)$) to entrepreneurs and reserves at the central bank ($RR_t D_t(j)$), constituting the asset side of the retail bank's balance sheet. Reserves are compulsory due to regulation: a fraction RR_t of every unit of deposits received by the retail bank must be deposited at the central bank.

Table 1: Balance Sheet of Retail Banks

Assets	Liabilities
Loans (L_t)	Deposits (D_t)
Reserves ($RR_t D_t$)	Interbank loans (IB_t)

The balance sheet identity of the retail bank is:

$$L_t = (1 - RR_t) D_t + IB_t \tag{38}$$

When the entrepreneur's idiosyncratic shock is below the cut-off, $\omega < \overline{\omega}_{t+1}$, the bank forecloses the entrepreneur. Given the private nature of the idiosyncratic shock, the retail bank must pay a monitoring cost in order to observe ω and absorb the entrepreneur's gross capital return. Following convention in the costly state verification literature dating back to Townsend (1979), we assume that in this scenario, the retail bank keeps a fraction $(1 - \mu)$ of the entrepreneur's gross capital return after paying for monitoring costs and the entrepreneur walks out empty handed.

The retail bank's expected real profits next period are:

$$E_t\left[\Pi^{RB}_{t+1}\right] = E_t\Bigg[\Bigg(\int_{\overline{\omega}_{t+1}}^{\infty} R^L_t L_t dF(\omega) + (1-\mu)\int_0^{\overline{\omega}_{t+1}} \omega R^E_{t+1} Q_t K_{t+1} dF(\omega) +$$
$$R^{RR}_t RR_t D_t(i) - R^D_t(i) D_t(i) - \frac{\kappa^D}{2}\left(\frac{R^D_t(i)}{R^D_{t-1}(i)} - 1\right)^2 R^D_t D_t -$$
$$R^{IB}_t IB_t\Bigg)\frac{P_t}{P_{t+1}}\Bigg] \tag{39}$$

where $F(\omega)$ is the cumulative density of the idiosyncratic shock and R^{RR}_t is the gross interest the central bank pays on reserves.

The retail bank has three income sources: loan repayments from entrepreneurs that performed well (those with $\omega > \overline{\omega}_{t+1}$), the gross capital return of defaulting entrepreneurs (with $\omega < \overline{\omega}_{t+1}$) net of monitoring costs and interest on reserves deposited at the central bank. On the other hand, retail bank expenses include deposit repayment with interest $(R^D_t(i)D_t(i))$, interbank loan repayment with interest $(R^{IB}_t IB_t)$ and an adjustment cost on the deposit rate. The inclusion of an adjustment cost on the deposit rate can be justified theoretically using the classic menu costs argument of the price rigidity literature: retail banks incur in costs to market their deposit "product" in the form of advertising material. These costs increase whenever the deposit rate changes.

Departing from Gerali et al. (2010), we do not include adjustment costs related to the lending or interbank rates. The rationale behind this decision is that lending rates usually vary on a client to client basis (thus, there is no unique number to publicize) and the interbank market rate is determined on a day to day basis in a perfectly competitive market with almost perfect information.[10]

Recall that the loan market (where retail banks and entrepreneurs interact) and the interbank market (where narrow banks and retail banks meet) are competitive but the deposit market is not. Following Gerali et al. (2010), monopolistic competition in the deposit market implies that every retail bank faces a particular demand for its slightly differentiated deposit $(D_t(i))$. We assume the consumer loves variety and demands a bundle of deposits (D_t) constructed as a CES aggregate of the individual deposits with elasticity of substitution ϵ. Thus, the retail bank sets its deposit rate $R^D_t(i)$ taking into account the consumer's relative demand for its particular deposit given by:

$$D_t(i) = \left(\frac{R^D_t(i)}{R^D_t}\right)^{\epsilon} D_t \tag{40}$$

[10]From an empirical perspective, the deposit rate adjustment cost is meant to capture the stylized fact that the pass-through from interbank rates to deposit rates is small and slow while the pass-through to lending rates is much bigger and faster.

where R_t^D is a CES index of the deposit rates $R_t^D(i)$.

The retail bank's expected real profit next period can be simplified using its balance sheet and the cut-off definition in order to substitute away L_t and R_t^L which yields:

$$
\begin{aligned}
E_t\left[\Pi_{t+1}^{RB}\right] = E_t\Bigg\{ \Bigg[& g(\overline{\omega}_{t+1})R_{t+1}^{E}p_t - \Bigg(\frac{(R_t^D(i) - R_t^{RR}RR_t)}{1-RR_t} + \\
& \left(1 - \frac{(1-RR_t)}{p_t-1}\left(\frac{R_t^D(i)}{R_t^D}\right)^{\epsilon}\frac{D_t}{N_t}\right)\left(R_t^{IB} - \frac{(R_t^D(i) - R_t^{RR}RR_t)}{1-RR_t}\right)\Bigg) \\
& (p_t-1) - \frac{\kappa^D}{2}\left(\frac{R_t^D(i)}{R_{t-1}^D(i)} - 1\right)^2 R_t^D\frac{D_t}{N_t}\Bigg] N_t\left(\frac{P_t}{P_{t+1}}\right)\Bigg\}
\end{aligned}
\tag{41}
$$

where $p_t \equiv \frac{Q_t K_{t+1}}{N_t}$ is the entrepreneur's leverage and

$$
g\left(\overline{\omega}_{t+1}\right) \equiv \left[\overline{\omega}_{t+1}\Pr\left(\omega > \overline{\omega}_{t+1}\right) + (1-\mu)\,E\left(\omega \mid \omega < \overline{\omega}_{t+1}\right)\Pr\left(\omega < \overline{\omega}_{t+1}\right)\right] \tag{42}
$$

is the fraction of the gross nominal return on capital of the average entrepreneur (R_{t+1}^E) that is given to the retail bank in compensation for the loan it provided at time t. $g\left(\overline{\omega}_{t+1}\right)$ is increasing in $\overline{\omega}_{t+1}$ given a reasonably small steady state default rate and some restrictions on the parameters of $F(\omega)$ imposed by Bernanke et al. (1999).

Turning back to entrepreneurs, their aggregate profit next period would be:

$$
\left(\int_{\overline{\omega}_{t+1}}^{\infty}\omega R_{t+1}^E Q_t K_{t+1} dF(\omega) - R_t^L L_t\right)\left(\frac{P_t}{P_{t+1}}\right) \tag{43}
$$

Note that only entrepreneurs that manage to repay their loans ($\omega > \overline{\omega}_{t+1}$) make a profit. Using the cut-off ($\overline{\omega}_{t+1}$) and leverage (p_t) definitions to substitute away R_t^L and L_t again, results in a new expression for entrepreneurs' profit:

$$
\left[E\left(\omega \mid \omega > \overline{\omega}_{t+1}\right)\Pr\left(\omega > \overline{\omega}_{t+1}\right) - \overline{\omega}_{t+1}\Pr\left(\omega > \overline{\omega}_{t+1}\right)\right] R_{t+1}^E p_t N_t\left(\frac{P_t}{P_{t+1}}\right) \tag{44}
$$

Given a well behaved distribution of ω (e.g.: log-normal), it can be demonstrated that $f(\overline{\omega}_{t+1}) \equiv E(\omega \mid \omega > \overline{\omega}_{t+1})\Pr(\omega > \overline{\omega}_{t+1}) - \overline{\omega}_{t+1}\Pr(\omega > \overline{\omega}_{t+1})$, the average entrepreneur's share of R_{t+1}^E, is decreasing in $\overline{\omega}_{t+1}$.[11]

Retail banks need to offer entrepreneurs a loan contract specifying R_t^L and L_t. However, we follow common practice in the literature redefining the problem in terms of $\overline{\omega}_{t+1}$ and p_t

[11] It should be noted that the retail bank's share and the entrepreneur's share do not add up to unity: $g(\overline{\omega}_{t+1}) + f(\overline{\omega}_{t+1}) < 1$ because part of R_{t+1}^E is lost due to monitoring costs.

to facilitate exposition. Intuitively speaking, a higher lending rate is equivalent to a higher cut-off and a bigger loan can be interpreted as higher leverage. Given the competitive environment in the loans market, retail banks will offer entrepreneurs the most desirable contract possible, driving down the present discounted value of their profits to zero.

Thus, the optimal contract is determined by the retail bank choosing $\overline{\omega}_{t+1}$, p_t and $R_t^D(i)$ to maximize the expected present discounted value of the entrepreneurs' aggregate profit subject to the restriction that the expected present discounted value of their own profits is non-negative.[12]

The Lagrangian of the problem we just described would be:

$$\begin{aligned}\max_{\overline{\omega}_{t+1},p_t,R_t^D(i)} \mathcal{L} = E_t \Bigg[& \sum_{s=t}^{\infty} M_{s-t}^H f(\overline{\omega}_{t+1}) R_{s+1}^E p_s N_s \frac{P_s}{P_{s+1}} \\ & + \lambda \sum_{s=t}^{\infty} M_{s-t}^H \Bigg\{ g(\overline{\omega}_{t+1}) R_{s+1}^E p_s - R_s^{IB}(p_s - 1) \\ & + \left(\frac{R_s^D(i)}{R_s^D} \right)^{\epsilon} \frac{D_s}{N_s} \left(R_s^{IB}(1 - RR_s) - R_s^D(i) + R_s^{RR} RR_s \right) \\ & - \frac{\kappa^D}{2} \left(\frac{R_s^D(i)}{R_{s-1}^D(i)} - 1 \right)^2 R_s^D \frac{D_s}{N_s} \Bigg\} N_s \frac{P_s}{P_{s+1}} \Bigg] \end{aligned} \tag{45}$$

where λ is a (constant) lagrangian multiplier and M_{s-t}^H is the households' stochastic discount factor, previously defined.

The solution to this problem yields the financial accelerator of Bernanke et al. (1999): a positive relationship between the external finance premium ($E_t\left[R_{t+1}^E\right]/R_t^{IB}$) and entrepreneurial leverage, defined as the ratio of assets to net worth ($Q_t K_{t+1}/N_t$). The particular functional form of the relationship depends on $f(\cdot)$, $g(\cdot)$ and their derivatives. We follow common practice and approximate it by,

$$\frac{E_t\left[R_{t+1}^E\right]}{R_t^{IB}} = \left[\frac{Q_t K_{t+1}}{N_t}\right]^{\upsilon} \tag{46}$$

where υ is the (positive) elasticity of the external finance premium with respect to leverage.

This relationship constitutes the entrepreneur's demand for new capital (recall that the retail bank is maximizing entrepreneurial profit): it is intuitive that demand for K_{t+1} should be decreasing in Q_t and increasing in $E_t\left[R_{t+1}^E\right]$ and N_t. That demand for new capital should

[12] Actually, the present discounted value of retail banks' profits will have to be zero at the optimum, otherwise they could obtain a better outcome cutting the lending rate marginally to offer entrepreneurs a more desirable contract.

be decreasing in R_t^{IB} is not very intuitive but we can remedy this by pointing out that higher R_t^{IB} must translate into higher R_t^L in order to comply with the participation constraint of the retail bank (zero expected discounted present value of profits).

It is important to note that the costly-state verification framework implies that external funding is more expensive for the entrepreneur than internal funding always. Thus, the entrepreneur always uses all available net worth N_t plus some loans to fund the acquisition of new capital.

Given our assumption of monopolistic competition in the market for deposits (a la Gerali et al. (2010)), the retail bank does not find it optimal to perfectly arbitrage between its sources of funding when transitioning from one steady state to another. Adjustment costs and monopolistic competition imply the following relationship between the deposit rate (R_t^D) and the net cost of funding obtained from narrow banks in the interbank market (after imposing the condition that $R_t^D(i) = R_t^D$ for all i by symmetry).

$$\begin{aligned} &\kappa^D \left(\frac{R_t^D}{R_{t-1}^D} - 1\right) \frac{R_t^D}{R_{t-1}^D} E_t\left[\frac{P_t}{P_{t+1}}\right] = \\ &\quad \left(-1 - \epsilon + \epsilon \frac{R_t^{IB}(1 - RR_t) + R_t^{RR} RR_t}{R_t^D}\right) E_t\left[\frac{P_t}{P_{t+1}}\right] + \\ &\quad \frac{\kappa^D}{R_t^D} E_t\left[\left(\frac{D_{t+1}}{D_t}\right)\left(\frac{R_{t+1}^D}{R_t^D} - 1\right)\left(\frac{R_{t+1}^D}{R_t^D}\right)^2 \frac{P_{t+1}}{P_{t+2}}\right] \end{aligned} \tag{47}$$

Once the optimal contract between entrepreneur and retail bank has been defined, all that remains is to characterize entrepreneurial net worth and entrepreneurial consumption. We assume that each period, after settling with the retail banks, entrepreneurs make a decision whether to stay in business or "retire". In order to avoid unnecessary complications, we follow the literature and assign the value γ to the probability that a particular entrepreneur will remain in business. Entrepreneurs choosing not to retire use all their gross return on capital plus labor income to accumulate net worth for the next period:

$$N_t = \gamma f(\overline{\omega}_t) R_t^E Q_{t-1} K_t \left(\frac{P_{t-1}}{P_t}\right) + \frac{W_t^E}{P_t} \tag{48}$$

This expression provides insight on the necessity of entrepreneurial labor. In our set up, an individual entrepreneur that incurs in default might not choose to "retire". Given that the retail bank has appropriated all his assets, he needs some net worth in order to participate in the loan market next period (retail banks do not lend to entrepreneurs with zero net worth). Entrepreneurial labor provides the net worth "seed" required to start over. We implicitly assume that credit history is erased every period and the entrepreneur's past credit performance does not affect his ability to obtain a loan from a retail bank in period t in any way.

Entrepreneurs that exit the market ("retire") consume their entire gross return on capital in period t:

$$C_t^E = (1-\gamma)f(\overline{\omega}_t)R_t^E Q_{t-1}K_t\left(\frac{P_{t-1}}{P_t}\right) \tag{49}$$

3.6 Narrow banks

Our set up assumes the existence of a handful of competitive narrow banks. These financial institutions are key in our model. Just like retail banks, narrow banks require funding in period t in order to make financial investments that pay off in $(t+1)$.[13] There is a crucial difference though: retail banks promise a fixed return in exchange for funding (in the form of deposits and interbank loans) whereas narrow banks offer a variable return on their shares.

Funds obtained by issuing shares ($P_t^S S_t$) are used by narrow banks to invest in government bonds (B_t^{NB}), purchased in the open market at relative price P_t^B, and offer interbank loans (IB_t) competitively to retail banks.

Table 2: Balance Sheet of Narrow Banks

Asset	Liabilities
Government bonds ($P_t^B B_t^{NB}$)	Equity ($P_t^S S_t$)
Interbank Loans (IB_t)	

Note that narrow bank liabilities consist only of equity obtained by issuing shares ($P_t^S S_t$), these represent household investment (in a financial sense) in the narrow bank. Thus, the balance sheet a representative narrow bank is the following:

$$P_t^B B_t^{NB} + IB_t = P_t^S S_t \tag{50}$$

Participation in the open market is restricted to narrow banks only. The reason is that our stylized open market is meant to resemble a secondary bond market where the central bank will carry out open market operations (repos) buying and selling its own holdings of government bonds. Thus, participation in the market is limited due to regulatory restrictions.[14]

[13] Note that real sector firms such as retail and capital goods producers do all their business in t and thus require no funding in a financial sense.

[14] Hilberg and Hollmayr (2011) argue that a hierarchical interbank market is justified by the structure found in the U.S. where only Primary Dealers deal with the central bank whereas a vast group of commercial banks is not allowed to directly deal with the monetary authority. In Europe, only 6 out of 2500 banks are allowed to participate in the bidding process in main refinancing operations of the ECB and other banks rely on interbank funding.

Narrow banks choose optimally their supply of interbank lending (IB_t) and demand of government bond holdings ($P_t^B B_t^{NB}$) obtained through open market operations. The interest rate on interbank loans (R_t^{IB}) is the competitive outcome of the profit-maximizing behaviour of both bank types. However, the equilibrium price of government bonds (P_t^B) will be chosen by the central bank ensuring consistency with its monetary policy objectives.

In order to simplify exposition and highlight the interaction between narrow banks and the central bank, we will assume that government bonds are consols issued at some undisclosed point in the past at relative price P^B (without a subscript) and that they pay a fixed nominal interest (R^B) perpetually. Thus, buying a government bond unit at time t makes the holder eligible to receive a fixed coupon payment of $\left(R^B - 1\right) P^B$ at time $(t+1)$.[15] Furthermore, government bond supply is fixed for the duration of our shock experiments.

Narrow bank's dividends are defined as:

$$\begin{aligned} DIV_t^{NB} = R_{t-1}^{IB} IB_{t-1} \left(\frac{P_{t-1}}{P_t}\right) + \left(\frac{\left(R^B - 1\right) P^B \frac{P_{t-1}}{P_t} + P_t^B}{P_{t-1}^B}\right) P_{t-1}^B B_{t-1}^{NB} \\ - \Xi\left(IB_{t-1}\right) \left(\frac{P_{t-1}}{P_t}\right) - P_t^S S_{t-1} \qquad (51) \end{aligned}$$

where $\Xi\left(IB_{t-1}\right)$ is a convex monitoring cost incurred by the narrow bank when lending to a retail bank.

The monitoring cost captures all expenses incurred by the retail bank during the evaluation process, follow-up and monitoring that takes place for the duration of its credit relationship with a narrow bank. We argue that the central bank is ill-equipped to perform the monitoring task and, therefore, does not offer loans directly to retail banks. Later, we will show this monitoring cost constitutes an important friction in our interbank market set up.[16]

The narrow bank maximizes shareholder return:

[15] Multiplying the interest rate $\left(R^B - 1\right)$ by P^B is necessary to ensure R^B is an interest factor expressed in nominal good units and not bond units, making it comparable to other interest rates introduced before. In order to normalize P^B to unity, R^B must be equal to the steady state deposit rate.

[16] See Curdia and Woodford (2010) for more details on this particular formulation of monitoring costs in a context of households borrowing from financial intermediaries.

$$\max_{IB_t, B_t^{NB}} E_t \left[\frac{DIV_{t+1}^{NB} + P_{t+1}^S S_t}{P_t^S S_t} \right] = E_t \left[\frac{R_t^{IB} IB_t \left(\frac{P_t}{P_{t+1}} \right) - \Xi(IB_t) \left(\frac{P_t}{P_{t+1}} \right)}{P_t^B B_t^{NB} + IB_t} + \frac{\left(\frac{(R^B - 1) P^B \left(\frac{P_{t-1}}{P_t} \right) + P_{t+1}^B}{P_t^B} \right) P_t^B B_t^{NB}}{P_t^B B_t^{NB} + IB_t} \right] \tag{52}$$

First order conditions for this problem are:

$$E_t \left[R_t^{IB} \left(\frac{P_t}{P_{t+1}} \right) - \Xi'(IB_t) \left(\frac{P_t}{P_{t+1}} \right) \right] = E_t \left[\frac{DIV_{t+1}^{NB} + P_{t+1}^S S_t}{P_t^S S_t} \right] \tag{53}$$

and

$$E_t \left[\frac{\left(R^B - 1 \right) P^B \left(\frac{P_t}{P_{t+1}} \right) + P_{t+1}^B}{P_t^B} \right] = E_t \left[\frac{DIV_{t+1}^{NB} + P_{t+1}^S S_t}{P_t^S S_t} \right] \tag{54}$$

implying the expected return on interbank loans and government bond investments must equate to shareholder return. Eliminating shareholder return from the first order conditions yields:

$$E_t \left[R_t^{IB} \left(\frac{P_t}{P_{t+1}} \right) - \Xi'(IB_t) \left(\frac{P_t}{P_{t+1}} \right) \right] = E_t \left[\frac{\left(R^B - 1 \right) P^B \left(\frac{P_t}{P_{t+1}} \right) + P_{t+1}^B}{P_t^B} \right] \tag{55}$$

Thus, the interest rate being charged to retail banks (R_t^{IB}) depends positively on the volume of interbank lending (IB_t) and negatively on the price of bonds (P_t^B). The central bank will exploit this relationship when pursuing monetary policy, effectively turning the narrow bank's return on government bonds into its monetary policy instrument. In order to do this, the central bank will supply(demand) government bonds in the open market whenever it wants to contract(expand) money supply, effectively setting P_t^B.

3.7 Central bank

The central bank's liabilities correspond to the components of the monetary base: retail bank reserves ($RR_t D_t$) and household cash holdings (CSH_t/P_t). On the asset side, the central bank holds the remaining government bonds ($P_t^B B_t^{CB}$). Thus, the total government bond supply ($P_t^B B_t$) must equate to the joint demand from narrow banks and the central bank ($P_t^B B_t^{CB} + P_t^B B_t^{NB}$).

Table 3: Balance Sheet of the Central Bank

Asset	Liabilities
Government bonds ($P_t^B B_t^{CB}$)	Reserves ($RR_t D_t$) Cash holdings ($\frac{CSH_t}{P_t}$)

The balance sheet of the central bank is as follows:

$$P_t^B B_t^{CB} = RR_t D_t + \frac{CSH_t}{P_t} \tag{56}$$

The central bank obtains interest and capital gains from its bond holdings.[17] These funds are used to pay some interest on reserves (R_t^{RR}) but, given the fact that part of the central bank's funding has zero cost (cash holdings), the central bank makes profits in steady state. These profits (Π_t^{CB}) are transferred to the government.

$$\Pi_t^{CB} = \left(\frac{\left(R^B - 1\right) P^B \left(\frac{P_{t-1}}{P_t}\right) + P_t^B}{P_{t-1}^B} \right) P_{t-1}^B B_{t-1}^{CB} - R_{t-1}^{RR} RR_{t-1} D_{t-1} \left(\frac{P_{t-1}}{P_t}\right) - \frac{CSH_{t-1}}{P_{t-1}} \left(\frac{P_{t-1}}{P_t}\right) \tag{57}$$

The central bank in this model controls liquidity by conducting open market operations, buying or selling bonds to the narrow bank. We assume the central bank's interventions are guided by a pseudo Taylor rule: if contemporaneous inflation is above its target, the central bank sells government bonds in the secondary open market to the narrow banks, pushing down their price. The result is a higher return on government bonds for narrow banks and a decrease of the central bank's monetary base. Additionally, the central bank also reacts to deviations of output from its long run trend in a similar fashion. It is useful to introduce an auxiliary variable, R_t^P to help characterize traditional monetary policy:

$$\left(\frac{R_t^P}{\bar{R}^P}\right) = \left(\frac{R_{t-1}^P}{\bar{R}^P}\right)^{\rho_R} \left[\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y} \right]^{1-\rho_R} \exp\left(\varepsilon_t^R\right) \tag{58}$$

where ρ_R captures interest rate rigidity, ϕ_π is the weight of inflation in the Taylor rule, ϕ_y is the weight of the output-gap, and ε_t^R is our i.i.d. monetary policy shock.

Auxiliary variable R_t^P is useful because it characterizes clearly the central bank's monetary policy stance. Given our assumption that the central bank's actual monetary policy

[17] We could assume the central bank does not receive interest on its bond holdings and our results would not be affected. This is because the central bank will transfer its profits to the government anyway. We choose to include interest payments to the central bank from the government in order to avoid confusion and ease exposition.

instrument is the narrow bank's return on government bonds, our auxiliary "policy rate" must be "translated" into a bond price:

$$\frac{\left(R^B-1\right)P^BP_t+E_t\left[P_{t+1}P_{t+1}^B\right]}{P_tP_t^B}=R_t^P \tag{59}$$

this expression characterizes the central bank's "demand" for government bonds. The central bank will adjust its bond holdings (B_t^{CB}) until bond price (P_t^B) is consistent with (59).

In our set up, the central bank has a second, albeit unconventional, monetary policy tool: the reserve requirement rate (RR_t). Given our intent of studying the pure effects of this instrument on the short-run evolution of the model economy, we do not tie it to a particular rule:

$$\left(\frac{1+RR_t}{1+\overline{RR}}\right)=\left(\frac{1+RR_{t-1}}{1+\overline{RR}}\right)^{\rho_{RR}}\exp\left(\varepsilon_t^{RR}\right) \tag{60}$$

where ρ_{RR} captures reserve requirement rigidity and ε_t^{RR} is an i.i.d. shock.

3.8 Government

We rule out government demand for retail goods. Given that our model focuses on monetary policy and the interactions taking place in the interbank market, we try to minimize the government's role in our model economy. The government's intertemporal budget constraint is:

$$\Pi_t^{CB}+P_t^BB_t+T_t^S+T_t^B=\tau^R\frac{P_t^W}{P_t}Y_t^W+(R^B-1)P^BB_{t-1}\frac{P_{t-1}}{P_t}+P_t^BB_{t-1} \tag{61}$$

where T_t^B is a (small) lump-sum tax required to finance part of the interest payments on the stock of government bonds B_{t-1}.

We make a number of simplifying assumptions to characterize government behaviour. First, we impose that lump-sum tax T_t^S be destined to finance the retailer subsidy exclusively:

$$T_t^S=\tau^R\frac{P_t^W}{P_t}Y_t^W \tag{62}$$

Second, as explained before, we assume government bond supply to be fixed ($B_t=B_{t-1}=B$). Introducing these assumptions allows us to rewrite the government's budget constraint:

$$\Pi_t^{CB}+T_t^B=(R^B-1)P^BB\frac{P_{t-1}}{P_t} \tag{63}$$

Note then that fluctuations in central bank profit (Π_t^{BC}) or inflation (P_{t-1}/P_t) must be compensated by altering the government's lump-sum "bond tax" (T_t^B) paid by households.

3.9 Goods and bonds market equilibrium

All that is left to tie up our model is to define the resource constraint. Production of the (final) retail good is allocated to private consumption (by households and entrepreneurs), investment (by capital goods producers), deposit rate adjustment costs, and to cover monitoring costs incurred by retail banks (costly state verification) and narrow banks. The resource constraint takes the following form:

$$Y_t = C_t + C_t^E + X_t + \frac{\kappa^D}{2}\left(\frac{R_t^D}{R_{t-1}^D} - 1\right)^2 R_{t-1}^D D_{t-1}\left(\frac{P_{t-1}}{P_t}\right) + \left(1 - f(\overline{\omega}) - g(\overline{\omega})\right) R^E Q_{t-1} K_t \left(\frac{P_{t-1}}{P_t}\right) + \Xi(IB_{t-1})\left(\frac{P_{t-1}}{P_t}\right) \tag{64}$$

Finally, bonds market equilibrium requires:

$$B = B_t^{CB} + B_t^{NB} \tag{65}$$

Table 4 summarizes the model.

4 Calibration

Our calibration of the model's parameters captures the key features of the U.S. economy. In Table 5 and 6 we report the calibration values and steady state values and ratios.

Regarding the households, the steady-state gross domestic inflation rate (P_t/P_{t-1}) is set equal to 1.00. The discount factor, (β) is set to 0.99 to match the historical averages of nominal deposit and risk-free interest rates, R_t^D and R_t^P. The intertemporal substitution parameter in workers' utility functions (σ) is set to 1. Assuming that workers allocate one third of their time to market activities, we set the parameter determining the weight of leisure in utility (χ_H) and the inverse of the elasticity of intertemporal substitution of labor (φ) to 1.0 and 0.33, respectively. The habit formation parameter, (b), is set to 0.75, as estimated in Christiano et al. (2010).

The capital share in aggregate output production ($1 - \psi - \varrho$) and the capital depreciation rate (δ) are set to 0.33 and 0.025, respectively. The parameter measuring the degree of

Table 4: Flow of Resources

Agent	Inflow	Outflow
consumer	$\frac{W}{P}H + R^D_{-1}D_{-1}(\frac{P_{-1}}{P}) + \Pi^R + \Pi^K + \Pi^{RB} + \left(\frac{DIV^{NB}+P^S S_{-1}}{P^S_{-1}S_{-1}}\right) P^S_{-1}S_{-1} + \frac{CSH_{-1}}{P_{-1}}(\frac{P_{-1}}{P})$	$C + T^S + T^B + D + P^S S + \frac{CSH}{P}$
retailer	$Y + \tau^R \frac{P^w}{P} Y^w$	$\Pi^R + \frac{P^w}{P} Y^w$
capital good producer	QK_{+1}	$\Pi^K + (1-\delta)QK + X$
wholesale producer	$\frac{P^w}{P} Y^w$	$\frac{W}{P}H + \frac{R^W}{P}K + \frac{W^E}{P}$
entrepreneur BS	$L + N$	QK_{+1}
entrepreneur return	$\frac{R^W}{P}K + (1-\delta)QK$	$R^E Q_{-1} K \left(\frac{P_{-1}}{P}\right)$
entrepreneur BC	$f(\overline{\omega}) R^E Q_{-1} K \left(\frac{P_{-1}}{P}\right) + \frac{W^E}{P}$	$C^E + N$
retail bank	$g(\overline{\omega}) R^E Q_{-1} K(\frac{P_{-1}}{P}) + R^{RR}_{-1} RR_{-1} D_{-1}(\frac{P_{-1}}{P})$	$\Pi^{RB} + R^D_{-1} D_{-1}(\frac{P_{-1}}{P}) + R^{IB}_{-1} IB_{-1}(\frac{P_{-1}}{P})$
retail bank BS	$D + IB$	$L + RRD$
narrow bank	$R^{IB}_{-1} IB_{-1} \left(\frac{P_{-1}}{P}\right) + \left(\frac{(R^B-1)P^B\left(\frac{P_{-1}}{P}\right)+P^B}{P^B_{-1}}\right) P^B_{-1} B^{NB}_{-1}$	$DIV^{NB} + \Xi(IB_{-1}) \left(\frac{P_{-1}}{P}\right) + P^S S_{-1}$
narrow bank BS	$P^S S$	$P^B B^{NB} + IB$
government	T^S	$\tau^R \frac{P^w}{P} Y^w$
more government	$T^B + \Pi^{CB} + P^B B$	$(R^B - 1)P^B B_{-1} \left(\frac{P_{-1}}{P}\right) + P^B B_{-1}$
central bank	$\left(\frac{(R^B-1)P^B\left(\frac{P_{-1}}{P}\right)+P^B}{P^B_{-1}}\right) P^B_{-1} B^{CB}_{-1}$	$\Pi^{CB} + R^{RR}_{-1} RR_{-1} D_{-1}(\frac{P_{-1}}{P}) + \frac{CSH_{-1}}{P_{-1}}(\frac{P_{-1}}{P})$
central bank BS	$RRD + \frac{CSH}{P}$	$P^B B^{CB}$
resource constraint	$C + C^E + X + (1 - f(\overline{\omega}) - g(\overline{\omega})) R^E Q_{-1} K \left(\frac{P_{-1}}{P}\right) + \Xi(IB_{-1}) \left(\frac{P_{-1}}{P}\right)$	Y

Notes: Bond market requires $B = B^{NB} + B^{CB}$, adjustment costs and t subscripts have been omitted to improve presentation.

monopoly power in the retail-goods market (θ) is set to 6, which would have implied a 20 per cent mark-up.

The nominal price rigidity parameter (α) in the Calvo set up is assumed to be 0.75, implying that the average price remains unchanged for four quarters.

The probability that an entrepreneur will stay in the market the next period is 0.97. In the same line, the probability that an entrepreneur does not meet the required income to avoid default ($\Pr(\omega < \bar{\omega})$) is 0.0075. Turning to the narrow banks, monitoring costs are captured using the functional form $\Xi(IB_t) = \Xi_0(IB_t)^\eta$, as in Curdia and Woodford (2010).

Monetary policy parameter ϕ_π is set to 1.5 while ϕ_Y is set to zero (as in Bernanke et al. (1999)). These values satisfy the Taylor principle (see Taylor (1993)).

Following Bernanke et al. (1999), the steady-state leverage ratio of entrepreneurs $(1 - N/K)$, is set to 0.5, matching the historical average. The steady-state elasticity of the external finance premium (υ) is set to 0.05, the value that is used by Bernanke et al. (1999).

Table 5: Parameter Calibration

Preferences			
$\beta = 0.99$	$b = 0.75$	$\sigma = 1$	$\chi_M = 0.008$
$\sigma_M = 1$	$\chi_H = 1$	$\varphi = 0.333$	$\theta = 6$
Technologies			
$\delta = 0.025$	$\psi = 0.66$	$\varrho = 0.01$	$\kappa = 8$
Nominal rigidities			
$\alpha = 0.75$			
Financial sector			
$\mu = 0.12$	$\kappa^D = 10$	$\epsilon = 237.5$	$\upsilon = 0.0506$
$\gamma = 0.9728$	$\Xi_0 = 0.000726$	$\eta = 10$	
Monetary policy			
$\phi_\pi = 1.5$	$\rho_R = 0.7$		
Government			
$\tau^R = 0.166$			
Exogeneous processes			
$\rho_a = 0.95$	$\rho_{RR} = 0.9$		

Table 6: Steady-State Values and Ratios

Variables	Definitions	Values
π	inflation	1.0000
R	policy rate	1.0141
R^D	deposit rate	1.0097
RR	reserve requirements	0.06
R^{RR}	reserve requirements' remuneration rate	1.0092
C/Y	household's consumption to output	0.681
C^E/Y	entrepreneur's consumption to output	0.143
I/Y	investment to output	0.177
K/Y	capital stock to output	7.069
L/Y	lending to output	1.961
D/Y	deposit to output	1.699
IB/Y	interbank funding to output	0.363
CSH/Y	cash holding to output	0.551
$P^S S/Y$	shares to output	0.002

5 Results

There are important second-order effects that appear when introducing a change to reserve requirements. However, we do not pursue a proper second-order approximation of our model[18] and, instead, choose to approximate only equation (47) to second-order because it is the one that captures the multiplicative nature of the interactions between the interbank rate, deposit rate and reserve requirements. Close inspection of equation (47) shows that the marginal effect of the interbank rate depends on the level of reserve requirements and vice-versa.

Figure 2 shows the model's impulse responses to a one percent productivity shock. Most variables exhibit fairly standard behaviour. Output returns slowly to steady state thanks to habit formation, adjustment costs associated to investment, and the shock's own persistence. In addition, inflation and the policy rate decrease. Debt contracts are signed in nominal terms according to the contract being offered by retail banks. Therefore, deflation increases the real value of debt obligations, generating a negative effect on entrepreneurial net worth that eventually outweighs the productivity boost. Similar to Gerali et al. (2010),

[18] This would require us to specify functional forms we don't really have much detail on such as $f(\cdot)$ and $g(\cdot)$.

the banking sector is imperfectly competitive, mark-ups applied on loan rates eventually raise the cost of debt servicing. A given deflation leaves debtors with a higher burden of real debt obligations which weigh more on their resources and on their spending, dampening the supply shock. Given that capital moves very slowly, the demand for loans increases given that self-funding falls. Consistent with the productivity shock and the investment response, capital's relative price shoots up and then falls quickly after the shock hits.

The short term dip in the deposit rate (which follows the interbank rate) explains the dip in deposit volume. Given the increase in loans demanded, retail banks must rely on interbank lending a few quarters. Narrow banks' optimal response then is to get rid of government bonds which the central bank has to monetize, increasing money supply.

[Figure 2 about here]

The model's impulse responses to a (negative) monetary policy shock of 50 basis points are shown in Figure 3. Output decreases and returns slowly to steady state. The demand contraction has a negative impact on inflation, which will lead the monetary authority to decrease the policy rate quickly. The increase in interest rates punishes entrepreneurial net worth (and the relative price of capital), resulting in an increase in the entrepreneur's demand for funding. Thus, loans increase given that capital is fixed in the short run. There is a strong increase in deposits given the initial higher rates paid on them, prompting a decrease in retail banks' demand for interbank funds which translates into a short lived increase in their demand for government bonds. The central bank adjusts money supply to accommodate the needs of narrow banks.

[Figure 3 about here]

The reserve requirement shock depicted in Figure 4 corresponds to an increase in reserve requirements from 6% to 9%. The resulting decrease in aggregate demand pushes down output and inflation. Given nominal contracts, lower inflation leads to higher real debt (and therefore lower net worth of entrepreneurs) and a drop in the price of capital (which reinforces the decline in net worth). Loans rise to partially compensate for lower net worth (high leverage). Given the increase in loan demand coupled with the fall in deposits, retail banks are forced to demand interbank loans from narrow banks. The higher monitoring costs prompts the latter to push up the interbank rate. Thus, for a few quarters, the interbank rate and the policy rate move in opposite directions: the central bank pushes the policy rate down to fight deflation and the fall in output.

Narrow banks find themselves with more funding available and demand government bonds from the central bank which adjusts money supply accordingly.

[Figure 4 about here]

Reserve requirement shocks could have an even bigger impact on our model economy. To justify this claim, note that traditional monetary policy (i.e. the Taylor rule) is trying to undo the effects of the shock right from the outset which is counterintuitive. Thus, in Figure 5 we show the combined effects of a monetary policy (50bps) and reserve requirement (3 per cent) shock. Reserve requirements can help obtain a bigger reaction of output and inflation for a given interest rate policy shock. This result suggests that lower movements in interest rate can achieve the same desired inflation and output, if used together with a consistent reserve requirement policy.

[Figure 5 about here]

6 Conclusions

When the central bank regulates the interbank market using reserve requirements, the monetary authority also affects liquidity in the banking sector, first, and the economy, later. This way of influencing bank funding, without any use of the policy interest rate, is a macroprudential tool.

In terms of modelling, the introduction of an interbank market allows a better identification of the final effects of different type of shocks in the economy. Important conclusions such as the complementarity of a central bank's tools can be potentially answered in a model with this additional feature.

The properties of macroprudential tools, developed in this model, are combined with the traditional effect of an interest rate policy shock. The complementarity of these two tools is one of our results. Reserve requirements act as a tax to financial intermediation, increasing the cost of funding economic activity through deposits and ultimately affecting output and inflation. Thus, a central bank can achieve a similar reaction on inflation and output with a lower increase of the policy interest rate if reserve requirements are increased at the same time. This is particularly relevant when the required policy rate cut is very big and could bring the monetary authority close to the zero lower bound, a problem faced by several countries in the wake of the Lehman bankruptcy.

Our results are in line with those of Carrera (2012) and Whitesell (2006). In his review of the relevant literature, Carrera (2012) finds that complementarity of these policy tools is normally achieved using different modelling strategies, however there is room for more research to explore the mechanism by which these and other related tools operate (e.g. collaterals). In the same line, Whitesell (2006) shows that combined policies of interest rate and reserve requirements result in lower volatility of the policy interest rate.

While the research conclusion for this paper is clear enough, this model can be extended to consider the possibility of collateral from retail banks to either narrow banks or a shadow banking system. The flexibility of our model allows for questions that are directly related with the liquidity of the financial system, and that is part of our research agenda.

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A The Model - Log linear equations

A.1 Households

All uppercase variables (except first line of each mini section) represent steady state values. Lowercase variables are deviations from steady state. No subscript implies variable is in current period.

The first derivative of the instantaneous utility of consumption

$UC * (C - hab * C_{-1})^{(\frac{1}{\sigma})} = 1$

$uc + \left(\frac{1}{\sigma}\right)\left(\frac{1}{1-hab}c - \frac{hab}{1-hab}c_{-1}\right) = 0$

-

Marginal utility of consumption

$MUC = UC - \beta * hab * UC_{+1}$

$muc = \left(\frac{1}{\sigma}\right)\left(\frac{\beta * hab}{1-\beta * hab}\left(\frac{1}{1-hab}c_{+1} - \frac{hab}{1-hab}c\right) - \frac{1}{1-\beta * hab}\left(\frac{1}{1-hab}c - \frac{hab}{1-hab}c_{-1}\right)\right)$

-

Household's budget constraint

$C_t + T_t^S + T_t^B + D_t + P_t^S S_t + \frac{CSH_t}{P_t} = \frac{W_t}{P_t}H_t + R_{t-1}^D D_{t-1}\frac{P_{t-1}}{P_t} + DIV_t^{NB} + P_t^S S_{t-1} + \frac{CSH_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} + \Pi_t^R + \Pi_t^K + \Pi_t^{RB}$

$\frac{C}{Y}c + \frac{T^B}{Y}t^B + \frac{D}{Y}d + \frac{P^S S}{Y}\left(p^S + s\right) + \frac{CSH}{Y}(csh - p) = \psi y + (1-\psi)(p - p^w) + \delta\frac{K}{Y}q + \frac{P^B B^{NB}}{Y}\left(p^B - R_{SS}^B p_{-1}^B\right) + R_{SS}^B\frac{P^B B^{NB}}{Y}\left(p_{-1}^B + b_{-1}^{NB}\right) - \frac{\Xi(IB)}{Y}(\eta ib_{-1} - \pi) + \frac{CSH}{Y}(csh_{-1} - p_{-1} - \pi) + \left(\left(R^D - R^{RR}RR\right)\frac{D}{Y} + R^{IB}\frac{IB}{Y}\right)\left(\frac{g'(\overline{\omega})\overline{\omega}}{g(\overline{\omega})}\left(r_{-1}^L + l_{-1} - r^E - q_{-1} - k\right) + r^E - \pi + q_{-1} + k\right) + R^{RR}RR\frac{D}{Y}\left(r_{-1}^P - \pi - rr_{-1} - d_{-1}\right)$

-

Household's Euler equation

$\frac{MUC}{P} = \beta R^d \frac{MUC_{+1}}{P_{+1}}$

$muc = r^d + muc_{+1} - \pi_{+1}$

-

Labor supply

$W * MUCP = \chi_h * H^{(\frac{1}{\phi})}$;

$p^w - p + y + muc = (\frac{\phi+1}{\phi})h$

but $y^w = a + (1 - \psi - \varphi)k_{-1} + \psi h$ then

$p^w - p + muc = \frac{\phi(1-\psi)+1}{\phi\psi}y - \frac{\phi+1}{\phi\psi}((1 - \psi - \varphi)k_{-1} + a)$

-

Demand for shares

$E_t\left[MUC_{t+1}\frac{P_t}{P_{t+1}}\right]R_t^D = E_t\left[MUC_{t+1}\left(\frac{DIV_{t+1}^{NB}+P_{t+1}^S S_t}{P_t^S S_t}\right)\right]$

$r_t^D - \pi_{t+1} + p_t^S + s_t = \frac{DIV^{NB}}{DIV^{NB}+P^S S}div_{t+1}^{NB} + \frac{P^S S}{DIV^{NB}+P^S S}\left(p_{t+1}^S + s_t\right)$

-

Money demand

$\chi_M\left(\frac{CSH_t}{P_t}\right)^{-\frac{1}{\sigma_M}} = MUC_t\left(\frac{R_t^D-1}{R_t^D}\right)$

$-\frac{1}{\sigma_M}(csh-p) = muc + \left(\frac{1}{R^D-1}\right)r^D$

A.2 Retailer supply and aggregation

Retailer profit

$\Pi^R = Y - (1-\tau^R) * \frac{P^w}{P} * Y^w$

$\pi^R = y - \frac{(1-\tau^r)}{\tau^r}(p^w - p)$

-

retailer demand = wholesaler supply

$(P^\epsilon) * Y = (P^*)^\epsilon * Y^w$

$y = y^w$

-

domestic price evolution, alternative cpi weights

$P^* = (\alpha P^{*}_{-1}{}^{(-\theta)} + (1-\alpha)P^{z(-\theta)})^{(-1/\theta)}$

$p^* = \alpha p^*_{-1} + (1-\alpha)p^z$

-

additional variables required to characterize price setting

$MUCPVN = MUCP * VN$

$mucpvn = mucp + vn$

-

ídem

$MUCPVD = MUCP * VD$

$mucpvd = mucp + vd$

-

domestic price evolution

$$P = (\alpha {P_{-1}}^{(1-\theta)} + (1-\alpha) P^{z(1-\theta)})^{(\frac{1}{1-\theta})}$$

$$p = \alpha p_{-1} + (1-\alpha) p^z$$

thus, $p = p*$

-

optimal retail price derivation

$$P^z(\theta - 1)VD = \theta(1-\tau^R)VD$$

$$p^z + vd = vn$$

-

$$VN * MUCP = Y * (P)^\theta * P^w * MUCP + \alpha * \beta * MUCPVN_{+1}$$

$$vn = \frac{Y}{Y+\alpha\beta VN}(y + \theta p + p^w) + \frac{\alpha\beta VN}{Y+\alpha\beta VN}(vn_{+1} - r^D)$$

but $VN = Y/(1-\alpha\beta)$ then

$$vn = (1-\alpha\beta)(y + \theta p + p^w) + \alpha\beta(vn_{+1} - r^d)$$ -

$$VD * MUCP = Y * (P^\epsilon) * MUCP + \alpha * \beta * MUCPVD_{+1}$$

$$vd = \frac{Y}{Y+\alpha\beta VD}(y + \epsilon p) + \frac{\alpha\beta VD}{Y+\alpha\beta VD}(vd_{+1} - r^d)$$

but $VD = Y/(1-\alpha\beta)$ then

$$vd = (1-\alpha\beta)(y + \theta p) + \alpha\beta(vd_{+1} - r^d)$$

then, given that $p^z = vn - vd$

$$vn - vd = (1-\alpha\beta)p^w + \alpha\beta(vn_{+1} - vd_{+1})$$

$$p^z = (1-\alpha\beta)p^w + \alpha\beta p^z_{+1}$$

but $p^z = \frac{1}{1-\alpha}p - \frac{\alpha}{1-\alpha}p_{-1}$

$$\frac{1}{1-\alpha}p - \frac{\alpha}{1-\alpha}p_{-1} = (1-\alpha\beta)p^w + \alpha\beta(\frac{1}{1-\alpha}p_{+1} - \frac{\alpha}{1-\alpha}p)$$

going for Phillips curve:

$$-\alpha\beta p_{+1} + (1+\alpha^2\beta)p - \alpha p_{-1} - (1-\alpha)(1-\alpha\beta)p = (1-\alpha)(1-\alpha\beta)p^w - (1-\alpha)(1-\alpha\beta)p$$

$$-\alpha\beta(p_{+1} - p) + \alpha(p - p_{-1}) = (1-\alpha)(1-\alpha\beta)(p^w - p)$$

$$-\alpha\beta\pi_{+1} + \alpha\pi = (1-\alpha)(1-\alpha\beta)(p^w - p)$$

Phillips curve:

$$\pi = \beta\pi_{+1} + \frac{(1-\alpha)}{\alpha}(1-\alpha\beta)(p^w - p)$$

-

A.3 Capital Goods Producers

Capital accumulation

$K = (1-\delta) * K_{-1} + CPHI * X$

$k = (1-\delta)k_{-1} + \delta x$

-

Tobin's Q

$Q(1+\kappa-\kappa(\frac{X}{X_{-1}})) = 1 - \frac{0.5\kappa\beta}{MUC}(\frac{MUC_{+1}Q_{+1}X_{+1}^2}{X^2} - MUC_{+1}Q_{+1})$

$q - \kappa(x - x_{-1}) = -\kappa\beta(x_{+1} - x)$

-

A.4 Wholesale Producer

Production function

$Y^w = \exp(a) * K_{-1}{}^{(1-\psi-\varphi)} * H^{\psi} * (H^e)^{\varphi}$

$y^w = a + (1-\psi-\varphi)k_{-1} + \psi h$

-

Productivity shock

$a = \rho^a * a_{-1} + \varepsilon^a$

-

Capital demand

$R^w * K_{-1} = (1-\psi-\varphi) * P^w * Y^w$

$r^w + k_{-1} = p^w + y^w$

-

Household labor demand

$W * H = \psi * P^w * Y^w$

$w + h = p^w + y^w$

-

Entrepreneurial labor demand

$W^e * H^e = \varphi * P^w * Y^w$

$w^e = p^w + y^w$

A.5 Entrepreneurs

Net return on capital (definition)

$R^e * P_{-1} * Q_{-1} = R^w + (1-\delta) * P * Q$

$r^e - \pi + q_{-1} = \frac{R^e-(1-\delta)}{R^e}(p^w - p + y - k_{-1}) + \frac{(1-\delta)}{R^e}q$

-

Entrepreneur's balance sheet

$Q * K = B + N$

$q + k = \left(\frac{B}{K}\right) b + \left(\frac{N}{K}\right) n$

-

Entrepreneur's net worth evolution

$N_t = \gamma f\left(\overline{\omega}_{t-1}\right) R_t^e Q_{t-1} K_{t-1} \left(\frac{P_{t-1}}{P_t}\right) + \frac{W_t^e}{P_t}$

$n = \gamma f\left(\overline{\omega}\right) R^e \frac{K}{N}\left[\frac{f'(\overline{\omega})\overline{\omega}}{f(\overline{\omega})}\left(r_{-1}^L + l_{-1} - r^E - q_{-1} - k_{-1}\right) + r^e + q_{-1} + k_{-1} - \pi\right] + \varrho\frac{Y}{K}\frac{K}{N}\left(p^w - p + y\right)$

-

Entrepreneur's consumption

$C_t^E = (1-\gamma) f\left(\overline{\omega}_{t-1}\right) R_t^e Q_{t-1} K_{t-1} \left(\frac{P_{t-1}}{P_t}\right)$

$\frac{C^E}{Y}c^E = (1-\gamma) f\left(\overline{\omega}\right) R^e \frac{K}{Y}\left[\frac{f'(\overline{\omega})\overline{\omega}}{f(\overline{\omega})}\left(r_{-1}^L + l_{-1} - r^E - q_{-1} - k_{-1}\right) + r^e + q_{-1} + k_{-1} - \pi\right]$

-

A.6 Retail Bank

Retail bank balance sheet

$L = (1-RR)D + IB$

$l = (1-RR)\frac{D}{L}d - RR\frac{D}{L}rr + \frac{IB}{L}ib$

-

Retail bank profit

$\Pi_{t+1}^{RB} = \{g(\overline{\omega})R_{t+1}^e p_{t+1} - \left[\frac{\left(R_{t+1}^D(i) - R_{t+1}^{RR} RR_{t+1}\right)}{1-RR_{t+1}} + \left(1 - \frac{(1-RR_{t+1})}{p_{t+1}-1}\left(\frac{R_{t+1}^D(i)}{R_{t+1}^D}\right)^{\epsilon}\frac{D_{t+1}}{N_{t+1}}\right)\left(R_{t+1}^{ib} - \frac{\left(R_{t+1}^D(i) - R_{t+1}^{RR} RR_{t+1}\right)}{1-RR_{t+1}}\right)\right](p_{t+1}-1) -$
$\frac{\kappa^d}{2}\left(\frac{R_{t+1}^D(i)}{R_t^D(i)} - 1\right)^2 R_{t+1}^D \frac{D_{t+1}}{N_{t+1}}\} N_{t+1}\left(\frac{P_t}{P_{t+1}}\right)$

-

Determination of deposit rates (as in Gerali et al. (2010))

$\left(-1-\epsilon+\epsilon\frac{R^{ib}_{t+1}(1-RR_{t+1})+R^{RR}_{t+1}RR_{t+1}}{R^{D}_{t+1}}\right)\frac{P_t}{P_{t+1}}-\kappa^d\left(\frac{R^D_{t+1}}{R^D_t}-1\right)\frac{R^D_{t+1}}{R^D_t}\frac{P_t}{P_{t+1}}+SDF\frac{D_{t+2}}{D_{t+1}}\kappa^d\left(\frac{R^D_{t+2}}{R^D_{t+1}}-1\right)\left(\frac{R^D_{t+2}}{R^D_{t+1}}\right)^2\frac{P_{t+1}}{P_{t+2}}=0$

$r^D=\frac{\kappa^D}{1+\epsilon+(1+\beta)\kappa^D}r^D_{-1}+\frac{\beta\kappa^D}{1+\epsilon+(1+\beta)\kappa^D}E\left[r^D_{+1}\right]+\frac{R^{IB}}{R^D}\frac{\epsilon(1-RR)}{1+\epsilon+(1+\beta)\kappa^D}r^{IB}-\frac{R^{IB}-R^{RR}}{R^D}\frac{\epsilon RR}{1+\epsilon+(1+\beta)\kappa^D}rr+\frac{R^{RR}}{R^D}\frac{\epsilon RR}{1+\epsilon+(1+\beta)\kappa^D}r^P$

-

Threshold

$\overline{\omega}R^E_{+1}QK_{+1}=R^L_{+1}L_{+1}$

$\frac{d\overline{\omega}}{\overline{\omega}}+r^E_{+1}+q+k=r^L+l$

-

External finance premium

$\frac{R^E_{+1}}{R^{IB}}=\rho\left(\overline{\omega}\right)$

$r^E_{+1}-r^{IB}=\frac{\rho'(\overline{\omega})\overline{\omega}}{\rho(\overline{\omega})}\left(r^L+l-r^E_{+1}-q-k\right)$

A.7 Narrow Bank

Balance sheet

$P^BB^{NB}+IB=P^SS$

$\frac{P^BB^{NB}}{Y}\left(p^B+b^{NB}\right)+\frac{IB}{Y}ib=\left(\frac{P^BB^{NB}}{Y}+\frac{IB}{Y}\right)\left(p^S+s\right)$

-

Narrow bank benefit/dividend

$R^{IB}_{-1}IB_{-1}\left(\frac{P_{-1}}{P}\right)+\left(\frac{\left(R^B-1\right)\left(\frac{P_{-1}}{P}\right)+P^B}{P^B_{-1}}\right)P^B_{-1}B^{NB}_{-1}=DIV^{NB}+\Xi\left(IB_{-1}\right)\left(\frac{P_{-1}}{P}\right)+P^SS_{-1}$

$\frac{DIV^{NB}}{Y}div^{NB}=R^{IB}\frac{IB}{Y}\left(r^{IB}_{-1}+ib_{-1}-\pi\right)+\frac{P^BB^{NB}}{Y}\left(p^B-R^Bp^B_{-1}-\left(R^B-1\right)\pi\right)+R^B\frac{P^BB^{NB}}{Y}\left(p^B_{-1}+b^{NB}_{-1}\right)-\frac{\Xi(IB)}{Y}\left(\eta ib_{-1}-\pi\right)-\frac{P^SS}{Y}\left(p^S+s\right)$

-

Interbank supply of funds

$R^{IB}\left(\frac{P}{P_{+1}}\right)-\Xi'\left(IB\right)\left(\frac{P}{P_{+1}}\right)=\left(\frac{DIV^{NB}_{+1}+P^S_{+1}S}{P^SS}\right)$

-

Government bonds' demand

$\left(\frac{\left(R^B-1\right)\left(\frac{P}{P_{+1}}\right)+P^B_{+1}}{P^B}\right)=\left(\frac{DIV^{NB}_{+1}+P^S_{+1}S}{P^SS}\right)$

$\frac{P^S S}{Y}(p^B_{+1} - R^B p^B - (R^B - 1)\pi_{+1}) = \{R^{IB}\frac{IB}{Y}\left(r^{IB} + ib - \pi_{+1}\right) + \frac{P^B B^{NB}}{Y}\left(p^B_{+1} - R^B p^B\right) + R^B \frac{P^B B^{NB}}{Y}\left(p^B + b^{NB}\right) - \frac{\Xi(IB)}{Y}\left(\eta ib - \pi_{+1}\right) - \frac{P^S S}{Y}\left(p^S_{+1} + s_{+1}\right) + \frac{P^S S}{Y} p^S_{+1} - \left(\frac{DIV^{NB}}{Y} + \frac{P^S S}{Y}\right) p^S\}$

-

In equilibrium,

$R^{IB} - \Xi'(IB) = \left(\frac{\left(R^B - 1\right)\left(\frac{P}{P_{+1}}\right) + P^B_{+1}}{P^B}\right)\left(\frac{P_{+1}}{P}\right)$

$\frac{R^{IB}\frac{IB}{Y}}{R^{IB}\frac{IB}{Y} - \eta\frac{\Xi(IB)}{Y}} r^{IB} - \frac{\eta\frac{\Xi(IB)}{Y}}{R^{IB}\frac{IB}{Y} - \eta\frac{\Xi(IB)}{Y}} \frac{(\eta - 1)}{\widetilde{\Xi} R^{IB}} \frac{\Xi(IB)}{Y}\left(\frac{IB}{Y}\right)^{-1} ib = -\frac{1}{R^B}\pi_{+1}\frac{1}{R^B} p^B_{+1} - p^B + \pi_{+1}$

A.8 Central Bank & Government

Central bank's balance sheet

$P^B B^{CB} = RRD + \frac{CSH}{P}$

$\left(RR\frac{D}{Y} + \frac{\frac{CSH}{P}}{Y}\right)\left(p^B + b^{CB}\right) = \frac{RRD}{Y}(rr + d) + \frac{\frac{CSH}{P}}{Y}(csh - p)$

-

Central bank's profits

$\Pi^{CB} = \left(\frac{\left(R^B - 1\right)\left(\frac{P_{-1}}{P}\right) + P^B}{P^B_{-1}}\right) P^B_{-1} B^{CB}_{-1} - R^{RR}_{-1} RR_{-1} D_{-1}(\frac{P_{-1}}{P}) - \frac{CSH_{-1}}{P_{-1}}(\frac{P_{-1}}{P})$

$\left(R^B\frac{B^{CB}}{Y} - R^{RR} RR\frac{D}{Y} - \frac{\frac{CSH}{P}}{Y}\right)\pi^{CB} = P^B\frac{B^{CB}}{Y}\left(p^B + R^B b^{CB}_{-1} - \left(R^B - 1\right)\pi\right) - R^{RR} RR\frac{D}{Y}\left(r^{RR}_{-1} + rr_{-1} + d_{-1} - \pi\right) - \frac{\frac{CSH}{P}}{Y}(csh_{-1} - p_{-1} -$

-

Taylor rule

$\frac{R}{R^{SS}} = (\frac{R_{-1}}{R^{SS}})^{\rho^r} * (\Pi^{\phi_\pi} * (\frac{Y}{Y^{SS}})^{\phi_y})^{(1-\rho_r)} * \exp(\varepsilon^r)$

$r = \rho_r r_{-1} + (1 - \rho_r)(\phi_\pi \pi + \phi_y y) + \varepsilon^r$

-

Open market operations

$\left(\frac{\left(R^B - 1\right)(\frac{P}{P_{+1}}) + P^B_{+1}}{P^B}\right)\frac{P_{+1}}{P} = R$

$-\frac{1}{R^B}\pi_{+1} + \frac{1}{R^B} p^B_{+1} - p^B + \pi_{+1} = r$

-

Reserve requirement shock

$rr = \rho^{RR} rr_{-1} + \epsilon^{RR}$

-

Remuneration to reserve requirements should be a fraction of the policy rate:
$R^{RR} = \theta^{RR} R^P$ with $\theta^{RR} < 1$
$r^{RR} = r^P$
-
Tax to finance subsidy to retailers
$T^S = \tau^r \frac{P^w}{P} Y^w$
-
Government's budget constraint
$\Pi^{CB} + P^B B + T^B - G = (R^B - 1)B_{-1} + P^B B_{-1}$
$P^B B^{CB}\left(-\left(R^B - 1\right)\pi + p^B\right) + R^B P^B B^{CB} b_{-1}^{CB} - R^{RR} RRD\left(r_{-1}^{RR} + rr_{-1} + d_{-1} - \pi\right) - CSH\left(csh_{-1} - p_{-1} - \pi\right) + T^B t^b - Gg = (R^B - 1)B\left(b - \pi\right)$

A.9 Resource Constraint and Bond Market

Resource constraint
$Y = C + C^e + X + (1 - f(\overline{\omega}) - g(\overline{\omega}))R^e Q_{-1} K(\frac{P_{-1}}{P}) + \Xi\left(IB_{-1}\right)(\frac{P_{-1}}{P}) + \frac{\kappa^d}{2}\left(\frac{R^D}{R_{-1}^D} - 1\right)^2 R_{-1}^D D_{-1}(\frac{P_{-1}}{P})$
$y = \frac{C}{Y}c + \frac{C^e}{Y}c^e + \frac{X}{Y}x - R^e \frac{K}{Y}(f'(\overline{\omega}) + g'(\overline{\omega}))d\overline{\omega} + (1 - f(\overline{\omega}) - g(\overline{\omega}))R^e \frac{K}{Y}\left(r^e - \pi + q_{-1} + k\right) + \frac{\Xi(IB)}{Y}\left(\eta ib_{-1} - \pi\right)$
-
Bond market
$B = B^{CB} + B^{NB}$
$\frac{P^B B}{Y} b = \left(\frac{P^B B}{Y} - \frac{P^B B^{NB}}{Y}\right) b^{CB} + \frac{P^B B^{NB}}{Y} b^{NB}$
-
Inflation definition
$\pi = p - p_{-1}$

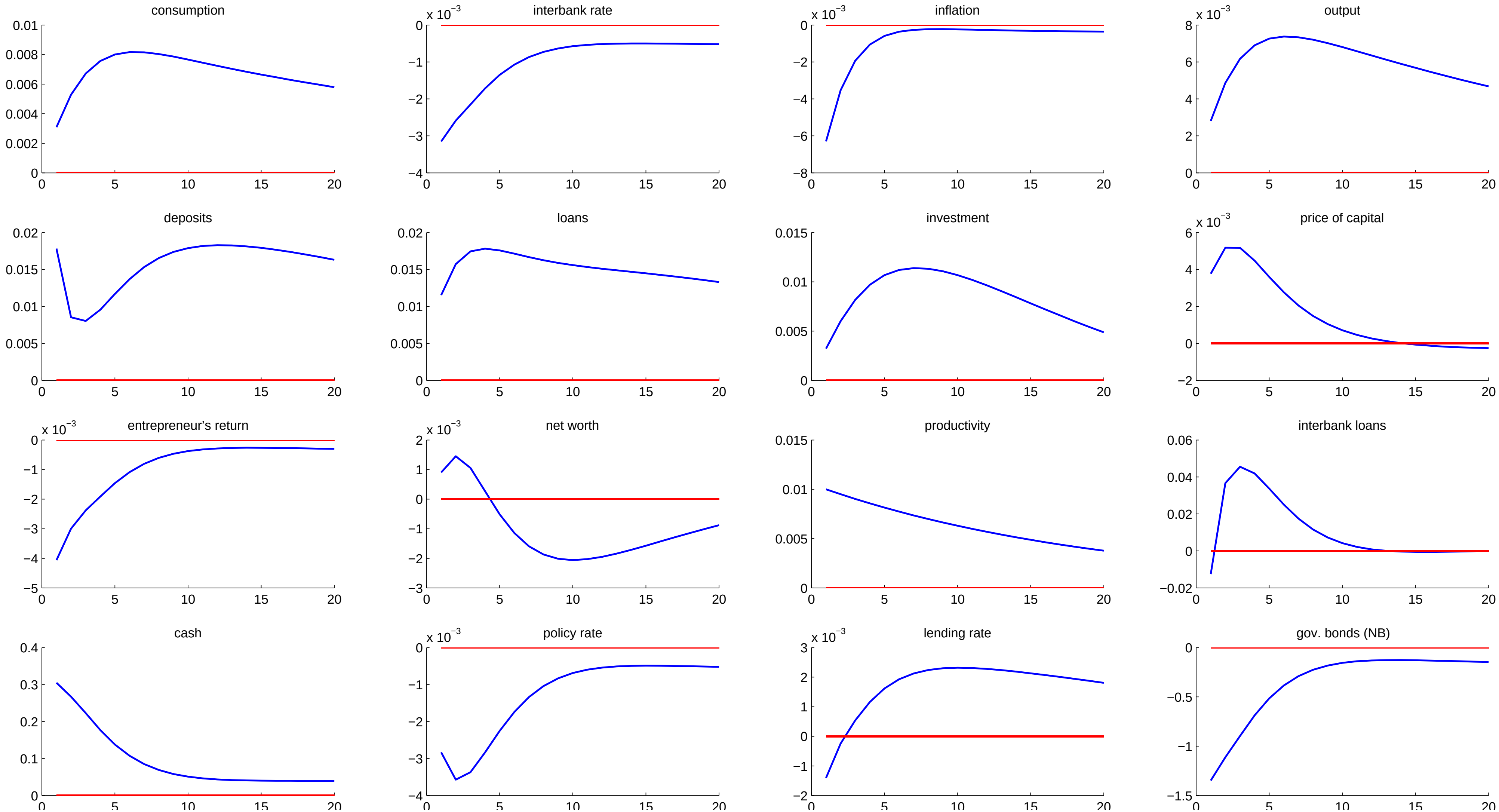

Figure 2: Productivity shock

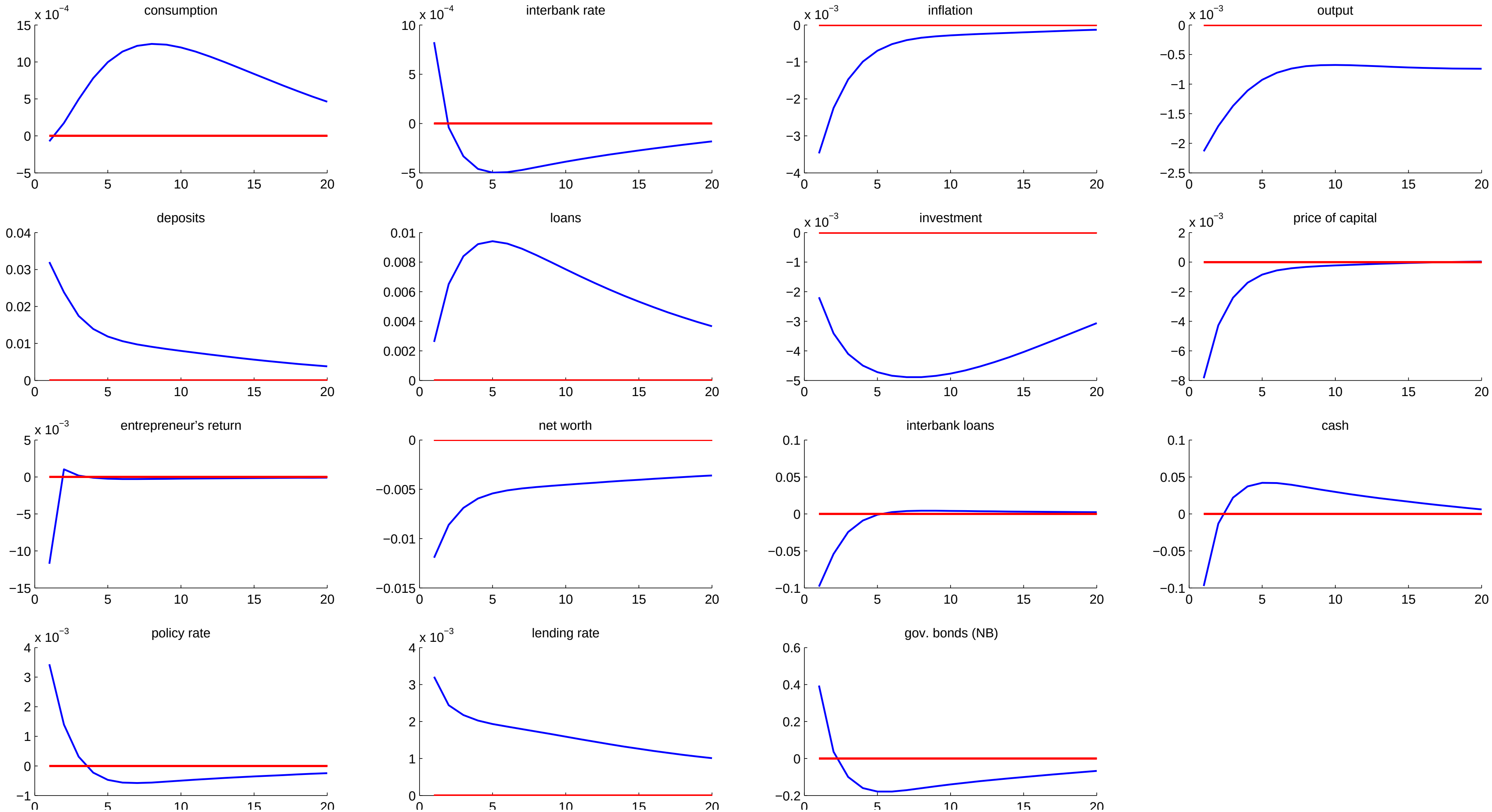

Figure 3: Monetary policy (MP) shock

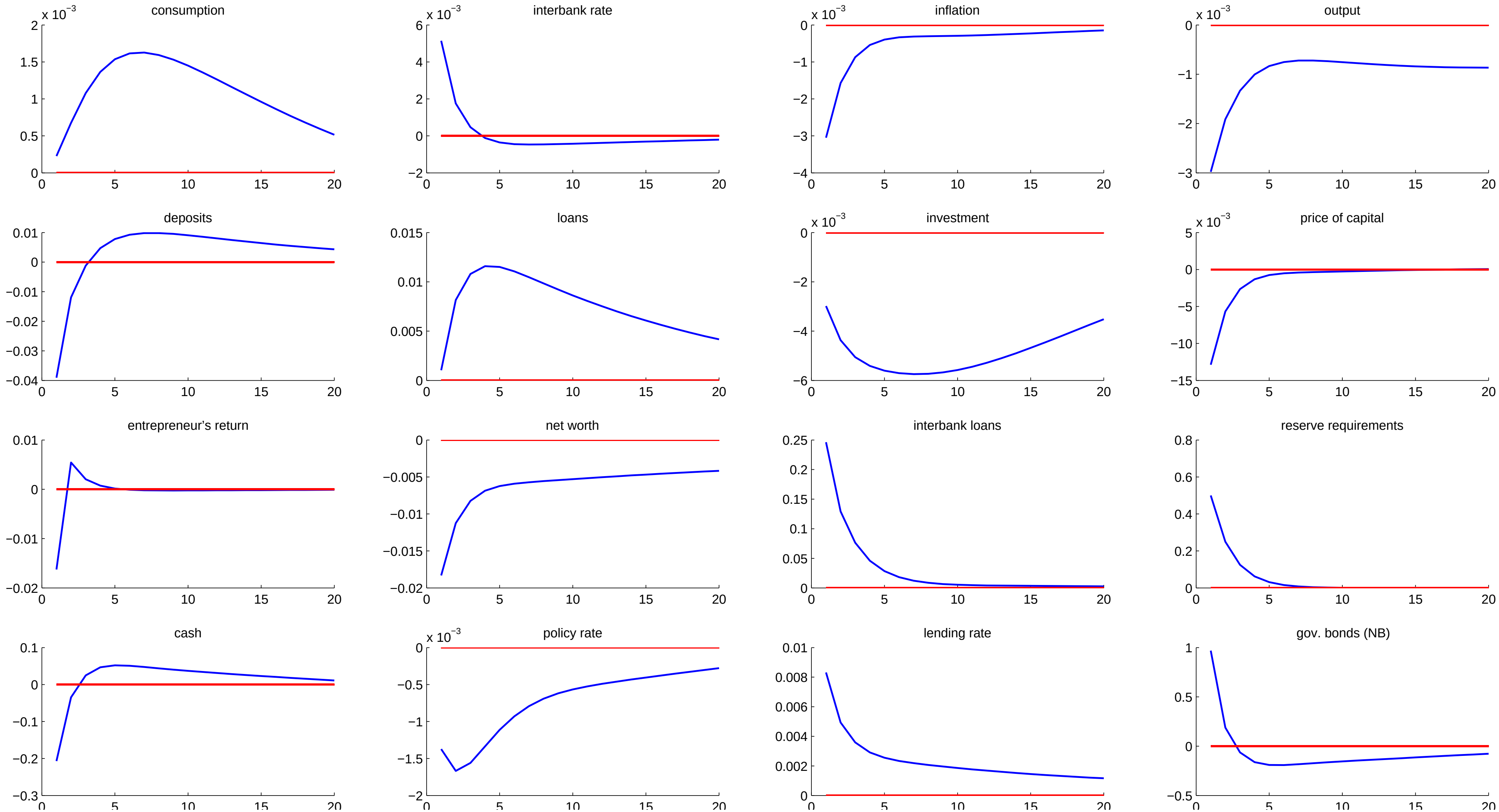

Figure 4: Reserve requirement (RR) shock

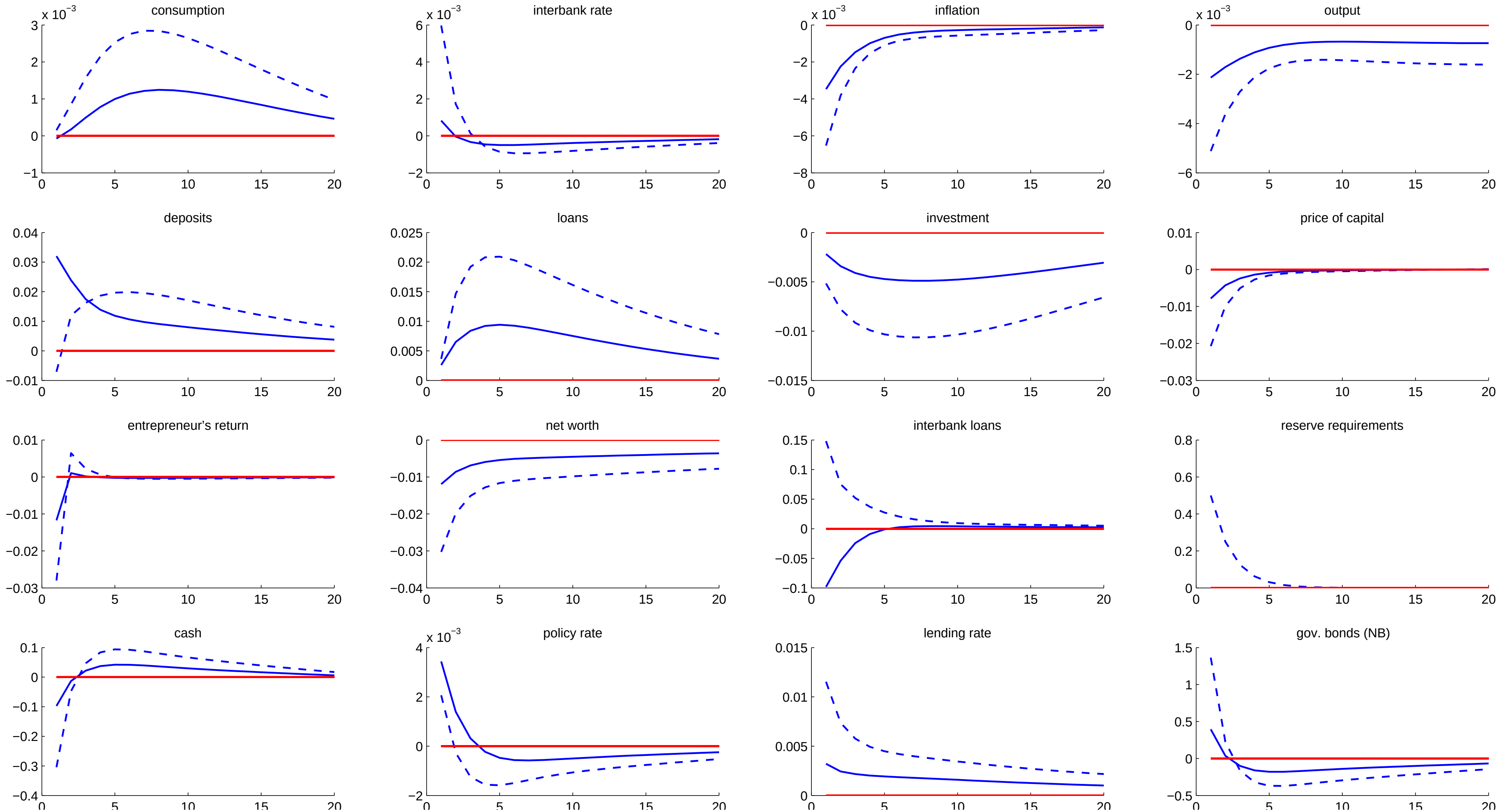

Figure 5: Combined MP and RR shock (dashed) versus MP shock only (solid)