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Wavelet-based Core Inflation Measures: Evidence from Peru

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Abstract

Under inflation targeting and other related monetary policy regimes, the identification of non-transitory inflation and forecasts about future inflation constitute key ingredients for monetary policy decisions. In practice, central banks perform these tasks using so-called "core inflation measures". In this paper we construct alternative core inflation measures using wavelet functions and multiresolution analysis (MRA), and then evaluate their relevance for monetary policy. The construction of wavelet-based core inflation measures (WIMs) is relatively new in the literature and their assessment has not been addressed formally, this paper being the first attempt to perform both tasks for the case of Peru. Another main contribution of this paper is that it proposes a VAR-based long-run criterion as an alternative criteria for evaluating core inflation measures. Evidence from Peru shows that WIMs are superior to official core inflation in terms of both the proposed criterion and forecast-based criteria.

Key words	:	Core inflation, wavelets, forecast, structural VAR
JEL Clasification	:	C45, E31, E37, E52

1 Introduction

Under inflation targeting and other related monetary policy regimes, the identification of non-transitory inflation and forecasts about future inflation constitute key ingredients for monetary policy decisions. Thus, when central bankers look at inflation data they try to convey which part of observed inflation is not affected by transitory shocks, and thus could help forecast medium-to-long term level of inflation. Alan Blinder confirms this statement when he recalls his period at the Federal Reserve Bank (FED):

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> *The name of the game was distinguishing the signal from the noise, which was often difficult. What part of each monthly observation on inflation is durable and which part is fleeting?*[1]

In practice, central banks perform these tasks using so-called "core inflation measures"; however, there is not a single and widely accepted core inflation measure but a myriad of them. In this paper we construct alternative core inflation measures using wavelet functions and multiresolution analysis (MRA), and then evaluate their relevance for monetary policy. The construction of wavelet-based core inflation measures (WIMs) is relatively new in the literature and their assessment has not been addressed formally, this paper being the first attempt to perform both tasks for the case of Peru. [2] Another main contribution of this paper is that it proposes a VAR-based long-run criterion as an alternative criteria for evaluating core inflation measures. Evidence from Peru shows that WIMs are superior to official core inflation in terms of both the proposed criterion and forecast-based criteria.

Measures of core inflation can be classified into two broad categories: (i) exclusion-based measures, which are obtained subtracting some specific prices from the general index of prices, and (ii) statistical-based measures, which are obtained using statistical methods to extract trend inflation from headline inflation. Exclusion-based core inflation measures are are generally preferred by monetary policy makers because (i) they are simple to construct and thus easier to communicate to both policy makers and the public, and (ii) they are not revised backwards unless the criteria for exclusion changes, which occur rarely if at all.

In the particular case of Peru, Valdivia y Vallejos (2000), BCRP (2006) and Armas et.al (2011) study a number of core inflation measures available to the central bank. One of the measures studied in this literature is published monthly by the Central Bank of Peru and has become the official core inflation measure. BCRP (2006) and Armas et.al (2011) compare the official core inflation with other core inflation indicators. The results obtained are threefold: (i) the official core inflation behaves as well as a number of statistical core measures, (ii) the official core inflation is better than other exclusion-based measures and (iii) the choice of the official core inflation by the Central Bank of Peru is reasonable given that it is an exclusion-based method that behaves as well as statistical-based indicators.

This paper explores another set of core inflation measures that can potentially improve the performance of the official core inflation. Using wavelet

[1] See Blinder (1997), page 157.

[2] Lahura (2004) is the first attempt to construct wavelet-based core inflation measures; however, no assessment is performed.

functions and multiresolution analysis (MRA) - which are well-known, signal processing tools - we exploit information from both the frequency and time domain contained in headline inflation and build alternative core inflation measures that capture medium-to-long term movements in inflation, i.e. movements that occur over long periods of time and contain low frequency information. The results show that wavelet-based core inflation measures (WIMs) are superior in terms of a VAR-based long-run criteria, and that these new measures can be considered as a complement to Peru's official core inflation as they improve short-run (up-to-six-months) forecasts.

The paper is organised as follows. Section 2 presents a brief introduction to wavelets emphasizing its usefulness for time series analysis. Section 3 describes the data and the procedure to construct the alternative measures of core inflation using wavelets (WIMs). The performance of WIMs and the official core inflation is analysed in section 4, using the proposed two criteria: (i) VAR-based long-run behavior , and (ii) ability to forecast future inflation. Section 5 presents the main conclusions.

2 An introduction to wavelets

Wavelets are mathematical functions that have recently been used to analyze images and time series. Although wavelet functions appear in Haar (1910), a formal mathematical theory of wavelets started with Grossmann and Morlet (1984) and Mallat (1989).[3].

A key wavelet-based tool is the Wavelet Transform (WT), which is able to describe features of the data that are local in time and in frequency; thus WT has been considered superior to important frequency-domain tools usually applied to time series analysis, such as the well-known Fourier transform (FT) - which describes the data as a function of frequency only - and the Short-Time Fourier transform (STFT) - which is a FT applied to a sliding window across time.[4] As stated by Gencay et.al (2002), "*the wavelet transform intelligently adapts itself to capture features across a wide range of frequencies and thus has the ability to capture events that are local in time. This makes the wavelet transform and ideal tool for studying non-stationary or transient time series*". Overall, wavelet-based analysis of a signal can be

[3]As stated in Misiti et.al (2010), the concept of wavelets -as it is known in the present- was first proposed by Jean Morlet and the team at the Marseille Theoretical Physics Center (France) while working under Alex Grossmann. The main algorithm dates back to Mallat (1998)

[4]The STFT is also known as the Windowed Fourier Transform. A good reference is Kaiser (1994).

compared to a camera with sophisticated lenses: it provides both a panoramic view of a city (i.e., buildings, avenues), and a detailed view[5] (i.e., trees, cars, windows).

In this paper we use one of the many applications of wavelet functions documented in the literature,[6] the so-called multiresolution analysis (MRA), which decomposes a time series into components that contain information at different timescales (where the scale is related to frequency). The next subsections presents the key ingredients of wavelet functions and multiresolution analysis.

2.1 Definition of wavelet

A ***wavelet*** $\psi(t)$ is a function that satisfies two properties: (1) it integrates to zero, $\int_{-\infty}^{\infty} \psi(s)ds = 0$, and (2) it is square integrable, $\int_{-\infty}^{\infty} \psi^2(s)ds = 1$. Property (2), is usually referred to as unit energy and states that $\psi(t)$ is not always zero, whereas property (1) tells us that $\psi(t)$ is such that all positive values are canceled out by negative values. Thus, (1) and (2) describe $\psi(t)$ as a function that fluctuates around zero but its fluctuation, unlike sine or cosine functions, is limited to a finite interval[7]. Therefore, a function $\psi(t)$ that satisfies (1) and (2) is referred to as a "small wave" or "wavelet".

The most basic wavelet is the Haar wavelet (Haar, 1910), which is defined as:

$$\psi^H(t) = \begin{cases} -\frac{1}{\sqrt{2}} & , \quad -1 < t \leq 0 \\ \frac{1}{\sqrt{2}} & , \quad 0 < t \leq 1 \\ 0 & , \quad other \end{cases}$$

It is easily seen that the function ψ^H satisfies properties (1) and (2) and thus it is a wavelet. Other well-known wavelet functions are Morlet and Mexican Hat, which are shown in Figure 1 together with the Haar wavelet.

2.2 Information content of wavelets

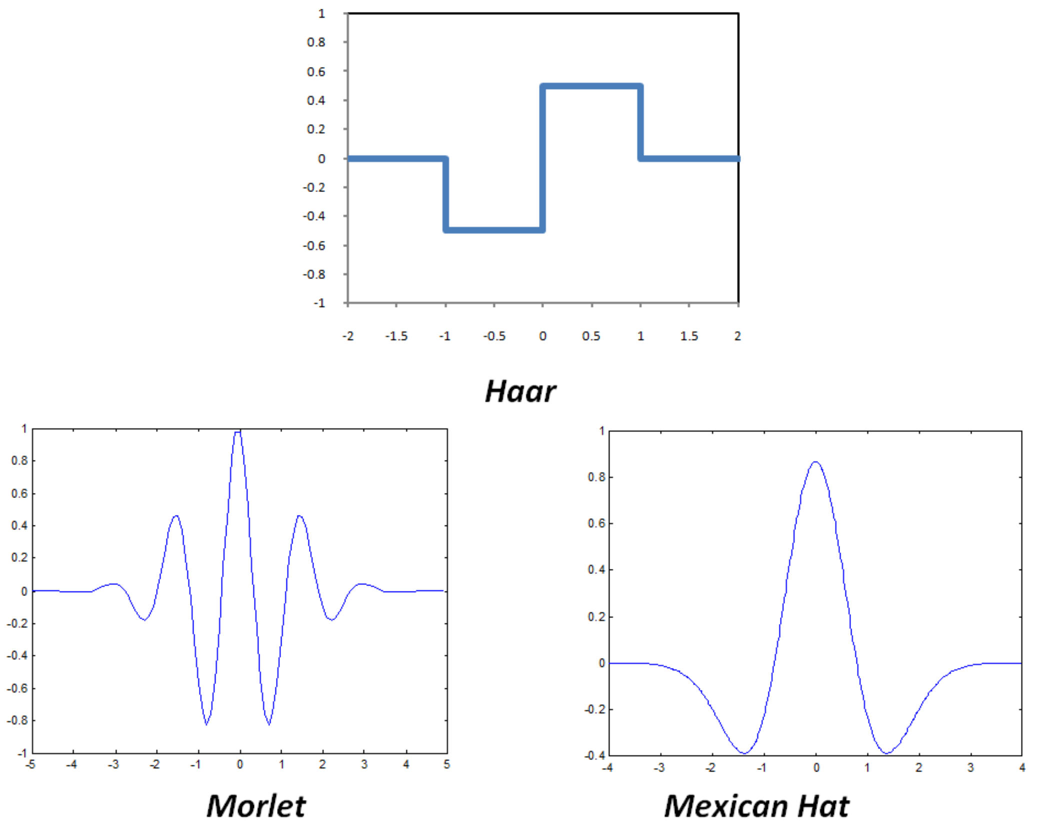
Wavelet functions tell us how weighted averages of a signal change from one averaging period to the next. Following Percival and Walden (2000), let $x(.)$

[5] This analogy follows Schleicher (2010).

[6] See for example Gencay et.al (2002), Percival and Walden (2000).

[7] Property (2) implies that for any $\epsilon > 0$ there exists a finite interval $[a, a]$ such that $\int_{-a}^{+a} \psi^2(s)ds > 1 - \varepsilon$. Therefore, when ε is very close to $\psi(s)$ takes non-zero values only inside the finite interval $[a, a]$. See Percival and Walden (2000).

Figure 1. *Wavelet Functions*



be a real-valued function of time t, usually called the "signal". The average value of $x(.)$ over $[a, b]$ is given by:

$$\alpha(a,b) \equiv \frac{1}{b-a}\int_a^b x(u)du, \;\; a < b$$

This function can be re-written in terms of the length of the interval $\lambda \equiv b - a$ and the center time of the interval, $t = \frac{a+b}{2}$, as follows:

$$A(\lambda,t) \equiv \alpha(t-\frac{\lambda}{2}, t+\frac{\lambda}{2}) = \frac{1}{\lambda}\int_{t-\frac{\lambda}{2}}^{t+\frac{\lambda}{2}} x(u)du$$

In wavelets jargon, λ is referred to as the "scale" associated with the average. Thus, $A(\lambda, t)$ is called the average value of the signal $x(.)$ over a scale of λ centered at time t. The change of this average from one period to another is defined as:

$$D(\lambda,t) \equiv A(\lambda, t+\frac{\lambda}{2}) - A(\lambda, t-\frac{\lambda}{2}) = \frac{1}{\lambda}\int_t^{t+\lambda} x(u)du - \frac{1}{\lambda}\int_{t-\lambda}^{t} x(u)du$$

If, for example $x(.)$ represents monthly inflation, a plot of $D(3,t)$ against time would tell us how quickly the average inflation over a quarter changes from one quarter to the next; increasing the scale λ up to a year, a plot of $D(\lambda,t)$ would tell us how much the average inflation over a year changes from one year to the next.

Given that $D(\lambda,t)$ involves two adjacent non overlapping intervals, $[t-\lambda,t]$ and $[t,t+\lambda]$, the integrals can be combined into a single one:

$$D(\lambda,t)=\int_{-\infty}^{\infty} x(u)\tilde{\Psi}_{\lambda,t}(u)du$$

where

$$\tilde{\Psi}_{\lambda,t}(u)\equiv\begin{cases}-\frac{1}{\lambda} & , \quad t-\lambda<u\leq t\\ \frac{1}{\lambda} & , \quad t<u\leq t+\lambda\\ 0 & , \quad otherwise\end{cases}$$

When $\lambda=1$ and $t=0$, $D(1,0)$ becomes:

$$D(1,0)=\int_{-\infty}^{\infty} x(u)\tilde{\Psi}_{1,0}(u)du \tag{1}$$

with

$$\tilde{\Psi}_{1,0}(u)\equiv\begin{cases}-1 & , \quad -1<u\leq 0\\ 1 & , \quad 0<u\leq 1\\ 0 & , \quad otherwise\end{cases} \tag{2}$$

It is straightforward to see that the Haar wavelet $\Psi^H(.)$ and $\tilde{\Psi}_{1,0}(u)$ are related according to $\tilde{\Psi}_{1,0}(u)=\sqrt{2}\Psi^H(u)$ Therefore, looking at differences of averages on unit scale at time $t=0$ is equivalent to

$$W^H(1,0)=\int_{-\infty}^{\infty} x(u)\Psi^H(u)du$$

thus, the Haar wavelet extracts, through this formula, information about how much difference there is between the two unit scale averages of $x(.)$ bordering time $t=0$. In the same fashion, to extract similar information about other scales λ and times t the Haar wavelet can be used through the following formula:

$$W^H_{(\lambda,t)}=\int_{-\infty}^{\infty} x(u)\Psi^H_{\lambda,t}(u)du \ \propto \ D(\lambda,t)$$

where,

$$\Psi^H_{\lambda,t}(u) = \frac{1}{\sqrt{\lambda}}\psi^H\left(\frac{u-t}{\lambda}\right) = \begin{cases} -\frac{1}{\sqrt{2\lambda}} & , \quad t-\lambda < u \leq t \\ \frac{1}{\sqrt{2\lambda}} & , \quad t < u \leq t+\lambda \\ 0 & , \quad otherwise \end{cases}$$

By varying λ it can be analyzed how averages of $x(.)$ over many different scales are changing from one period (of length λ) to the next.

2.3 Continuous and discrete wavelet transforms

The collection of variables $\left\{W^H(\lambda,t) : \lambda > 0, -\infty < t < \infty\right\}$ is known as the Haar Continuous Wavelet Transform (Haar CWT) of $x(.)$. The interpretation of $W^H_{(\lambda,t)}$ is that it is proportional to the difference between two adjacent averages of scale λ, the first average beginning at time t and the second, ending at time t. In general, it is possible to construct a Continuous Wavelet Transform (CWT) using any wavelet $\psi(.)$ that satisfies properties (1) and (2). For every particular wavelet a different interpretation will emerge, but in all cases the idea is the analysis of the difference between averages[8].

Formally, the Continuous Wavelet Transform (CWT) can be defined as a collection of elements $W(\lambda,t)$ which are obtained as a projection of a signal or function $x(.)$ onto a particular wavelet function $\psi_{(\lambda,t)}(u)$:

$$W(\lambda,t) = \int_{-\infty}^{\infty} x(u)\psi_{\lambda,t}(u)du \tag{3}$$

A key feature of the CWT is that every coefficient is obtained dilating and translating a particular wavelet function $\psi(.)$, called "mother wavelet", according to:

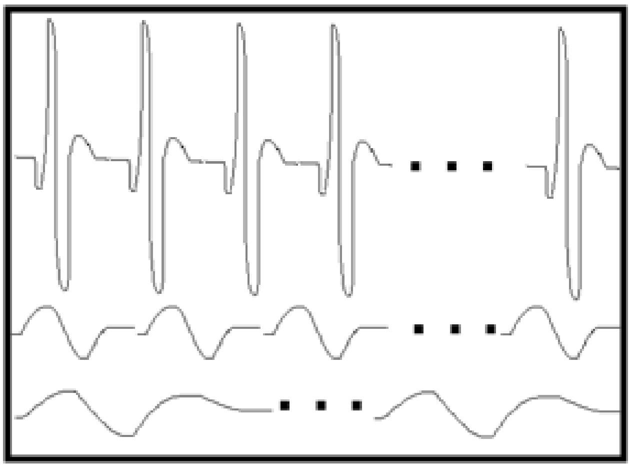
$$\psi_{\lambda,t}(u) \equiv \frac{1}{\sqrt{\lambda}}\psi\left(\frac{u-t}{\lambda}\right) \tag{4}$$

where λ is the dilation parameter, t is the translation parameter, and $\psi_{\lambda,t}(u)$ is referred to as a family of wavelets. Thus every member of the family $\psi_{\lambda,t}(u)$ is associated to a specific scale and temporal location, also called timescale. The scale parameter λ allows to expand the range of a wavelet, so that when λ is high, $\psi(.)$ completes its movement along a wider range than when λ is

[8]The Mexican hat $W(\lambda,t)$ yields a difference between a weighted average on unit scale and an average of two weighted averages surrounding it.

low; on the other hand, the translation parameter t allows moving the range of $\psi(.)$. Thus, the CWT is a function of a scale λ and a location t even though the original function only depends on time, and therefore provides a richer description of $x(.)$ by translating and dilating a mother wavelet function. In particular, the process of dilating and translating a wavelet in order to construct the CWT makes wavelets capable of capturing events that are local in time and in frequency. Figure 2 shows examples of dilating and translating wavelets: the first row contains wavelets associated to a lower λ compared to the second and last row, which needs to be translated more often in order to cover the same range.

Figure 2. *Dilation and translation of a wavelet*



Usually, it is necessary to impose further conditions on wavelet functions in addition to properties (1) and (2), so that wavelets are useful in practice. One important condition is called admissibility condition, which we refer to as property (3). Let $C_\Psi \equiv \int_0^\infty \frac{|\Psi(f)|^2}{f} df$, where $\Psi(f)$ denotes the Fourier transform of a wavelet $\psi(.)$:

$$\Psi(f) \equiv \int_{-\infty}^{\infty} \psi(u) e^{-i2\pi f u} du$$

where f denotes frequency. Then, the wavelet $\psi(.)$ is admissible if $0 < C_\Psi < \infty$. As stated in several papers related to wavelets[9], if a wavelet function satisfies properties (1), (2) and (3), then the corresponding CWT of $x(.)$ preserves all the information in $x(.)$. Furthermore, if this function or signal

[9]Calderón (1964), Grossmann and Morlet (1984), Mallat (1998), among others.

$x(.)$ is such that $\int_{-\infty}^{\infty} x^2(t)dt < \infty$, then $x(.)$ can be recovered from its CWT using:

$$x(t) = \frac{1}{C_\Psi} \int_0^\infty \left[\int_{-\infty}^{\infty} W(\lambda, u) \psi_{\lambda,t}(u) du \right] \frac{d\lambda}{\lambda^2}$$

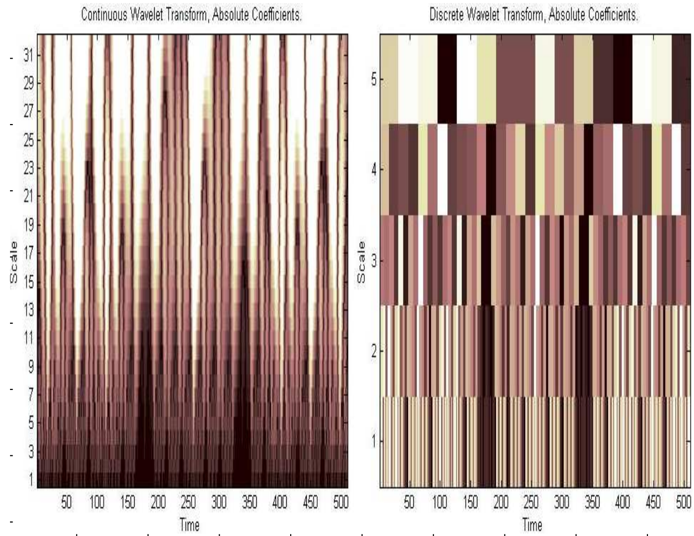
where

$$\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{C_\psi} \int_0^\infty \left[\int_{-\infty}^{\infty} W^2(\lambda, u) dt \right] \frac{d\lambda}{\lambda^2}$$

Thus, a signal $x(.)$ and its corresponding CWT are two equivalent representations of the same mathematical entity.

The analysis of a signal based on CWT yields a lot of information. The leftmost graph of Figure 3 shows a plot of all the CWT coefficients for a signal along 512 time periods, considering 32 scales; the darkness (lightness) is associated to small changes (large changes) in averages. However, it is difficult to analyze accurately this information, especially because of the redundancy in CWT as it is a two dimensional representation of a one dimensional signal.

Figure 3. *Continuous and Discrete Wavelet Transforms*



In this context, a more simplified version of the CWT can be more informative, and it is given by the so-called Discrete Wavelet Transform (DWT). A DWT is obtained using dyadic scales, which are defined by $\lambda's$ that take discrete values in the form of 2^j, for $j = 1, 2, 3, , J$. Thus, a DWT can be considered as a subsampling of the CWT elements, $W(\lambda, t)$, in which only dyadic scales are considered and within a given dyadic scale 2^j time points t

are chosen such that they are separated by multiples[10] of 2^{j-1}. A particular DWT that is obtained by translating and dilating a wavelet function using $t = k2^{-j}$ and $\lambda = 2^{-j}$,which implies $\psi_{\lambda,t}(u) = 2^{j/2}\psi(2^j u - k)$, is called a critical sampling of the CWT.

In time series analysis, this DWT is still not desirable because time periods are lost at higher scales. As can be seen in Figure 3, there are only 16 coefficients associated to scale 5 ($2^5 = 32$) which are separated by 16 time periods each ($2^{5-1} = 16$). Instead, it is more useful to consider subsamples of the CWT that preserve some of its redundancy (i.e. more coefficients). A very useful variation of DWT is called the maximal overlap DWT (MODWT). The MODWT can be thought of as a subsampling of the CWT at dyadic scales, but, in contrast to the DWT, we now deal with all times t and not just those that are multiples of 2^j. In particular, the MODWT is obtained using $t = k$ and $\lambda = 2^{-j}$. Retaining all possible times can lead to a more appropriate summary of the CWT because this can eliminate certain “alignment” artifacts attributable to the way the DWT subsamples the CWT across time.

2.4 Multiresolution analysis (MRA).

Multiresolution analysis is the mathematical formalization of a simple idea: to obtain successive approximations of a signal, so that each new approximation is better than the last one. If $\{\cdots, S_J, S_{J-1}, S_{J-2}, \cdots\}$ represents a MRA, then S_{J-1} is a better approximation than S_J, i.e. with a better resolution. The differences between the various successive approximations are called details and can be denoted as $D_J \equiv S_{J-1} - S_J$; thus, an approximation can be expressed as the sum of a lower-resolution approximation plus a detail, $S_{J-1} = D_J + S_J$. In general, if S_1 denotes the best approximation (the one with the highest resolution) of a signal $f(t)$, then $f(t) = S_1 + D_1$. If a MRA for a signal exists, then it is possible to obtain different approximations of the signal expressed as the sum of an approximation of lower resolution and a detail. In particular, the sequence $S_1 = D_2 + S_2, S_2 = D_3 + S_3, ..., S_{j-2} = D_{j-1} + S_{j-1}, S_{j-1} = D_j + S_j$. In this way, multiresolution analysis is able to express a signal $f(t)$ as the (orthogonal) sum of an approximation S_j and different details D_j :

$$f(t) = S_j + D_j + D_{J-1} + \cdots D_j + \cdots + D_1$$

MRA can be performed using wavelets. One of the most important results of wavelet theory is the existence of a correspondence between multiresolution

[10] In general, although DWT can be motivated as a subsampling of the CWT, it can be justified independently of it.

analysis of a signal and a wavelet family. In particular, Daubechies (1992) shows that if there exists a MRA for a signal in the $L^2(\Re)$ space[11] (or square integrable signal), then there exists an associated orthonormal wavelet basis for $L^2(\Re)$, such that it allows decomposing a signal into orthonormal components S_J and D_j given by:

$$D_j = \sum_t d_{j,t}\psi_{j,t}(u) \quad j = 1, 2, 3 \cdots, J \tag{5}$$

$$S_j = \sum_t s_{j,t}\phi_{J,t}(u) \tag{6}$$

The details D_j are associated to scales $j = 1, 2, 3, \cdots, J$. Formally, these details are obtained from discrete wavelet transforms based on a family of wavelets $\psi_{j,t}(u)$, which is generated by translation and dilation of a mother wavelet ψ, using as the translation factor $t = k2^j$ and $\lambda = 2^j$ as the dilation factor, with $j = 1, 2, 3, \cdots, J$. On the other hand, the approximation S_J is the component associated to the highest scale J of the signal, which is obtained as the projection of the signal onto a wavelet family $\phi_{J,t}(u)$, which is generated by the translation of a wavelet ϕ with scale $\lambda = 2^j$ using the factor t. The wavelet function ϕ (given that its integral is equal to one) is called *father wavelet*, and is used to capture trend components usually associated to low frequencies. A mother wavelet is used to capture components associated to lower scales, which correspond usually to higher frequencies. In other words, S_j represents the trend components of the signal as long as it is associated to longer scales, while the details $D_j, D_{j-1}, \cdots, Dj, \cdots, D_1$ represent low scale (high frequency) movements (deviations from S_j). In this way, a signal can be expressed as:

$$f(t) = \sum_t s_{j,t}\phi_{J,t}(u) + \sum_t d_{j,t}\psi_{J,t}(u) + \sum_t d_{j-1,t}\psi_{J-1,t}(u) + \cdots + \sum_t d_{1,t}\psi_{1,t}(u)$$

where J denotes the wavelet scale. The decomposition of the signal $f(t)$ into different time scales (associated to different frequencies) is referred to as time scale decomposition, and is represented by $S_J, D_J, D_{J-1}, \cdots, D_1$. Detail 1 (scale 1) contains information of the signal that take place in the interval of time $[2^1, 2^2]$, which are short term movements that can be linked to high-frequency movements. In general, detail j contains information of the signal that corresponds to movements that take place inside the interval

[11]A function f belongs to the $L^2(\Re)$ space if the integral of $|f|^2$ is finite. For further details, see for example Kaiser (1994)

$[2^j, 2^{j+1}]$. In this fashion, higher (lower) details or scales contain information about long-term (short-term) movements, which are usually associated to low-frequency (high-frequency) changes.

The MRA can also be performed using the MODWT instead of DWT (see Gencay et.al, 2002). Two important advantages of using MODWT and not DWT for MRA are the following: (i) the size of the sample is not restricted to be a multiple of 2^j, and (ii) the details and smooth terms are associated with zero-phase filters[12]. An important implication of this association with zero-phase filters is that any important feature observed in the details or the smooth terms may be perfectly matched with the original time series (Percival and Walden (2000), Gencay et.al (2002)).

3 Construction of wavelet-based inflation indicators

The use of wavelets and MRA in the empirical analysis of time series requires the choice of three important ingredients, which will determine the main features of the analysis: (i) a specific wavelet function, which implies the choice of a wavelet family and the length of the wavelet, (ii) a wavelet transform, and (iii) the level J for MRA.

The choice of an appropriate wavelet function usually depends on the nature of the time series. The Haar wavelet would be appropriate to analyze a series with flat segments, whereas longer wavelets like sym(12) would provide better results when analyzing smooth series. However, the choice of a particular wavelet is not so crucial if the MODWT is used, as documented in Percival and Walden (2000), and Gencay et.al (2002).

The use of the MODWT in time series analysis has several advantages compared to DWT. First, a main advantage is that sample size is not restricted to being a power of 2; as shown in Percival and Walden (2000), MRA based on MODWT will not be affected by the use of sample sizes that are not powers of 2. Second, MRA will be associated with zero-phase filters (which preserve the phase properties of the series) and thus will allow the alignment between the signal and its MRA. Third, given that MODWT oversamples data (i.e. more information is captured) it will provide higher resolution at lower scales. Finally, the MODWT is not affected by the inclusion of new observations, except on the boundaries (as happens with most filters); this problem is overcome by the use of standard procedures like the

[12] A zero-phase filter is a filter that preserves the phase properties of the series being analysed. See Gencay et.al (2002), p. 36-47 for a discussion about zero-phase filters.

circularly shifting method as established in Percival and Walden (2000) or by using fewer scales as in Baqaee (2010).[13]

In this paper, we choose eighteen wavelets from two wavelet families, Daubechies (db) and Symmlets (sym), and use them to perform level $J = 5$ MRA based on MODWT. The specific wavelets (length or vanishing moments in parenthesis) are: db(4), db(5), db(6), db(7), db(8), db(9) , db(10), db(11), db(12), sym(4), sym(5), sym(6), sym(7), sym(8), sym(9), sym(10), sym(11), sym(12). In this context, we construct wavelet-based core inflation measures by filtering headline inflation using a wavelet function. In particular, we perform level $J = 5$ MRA of headline inflation using MODWT; the corresponding core inflation measure is obtained by subtracting detail 1 (D1), detail 2 (D2), and detail 3 (D3), from headline inflation. The idea is that leaving out D1, D2 and D3 (which corresponds to movements between 2 and 4 months, between 4 and 8 months, and between 8 and 16 months, respectively) we are removing most of the transitory component of inflation, which is a desirable property of a core inflation measure. This procedure can be repeated using each of the 18 wavelet functions chosen, so that we end up with 18 alternative measures of core inflation. Appendix A shows an example of J=5 MRA of headline inflation using sym(12) wavelet and MODWT, and a resulting wavelet-based core inflation measure.

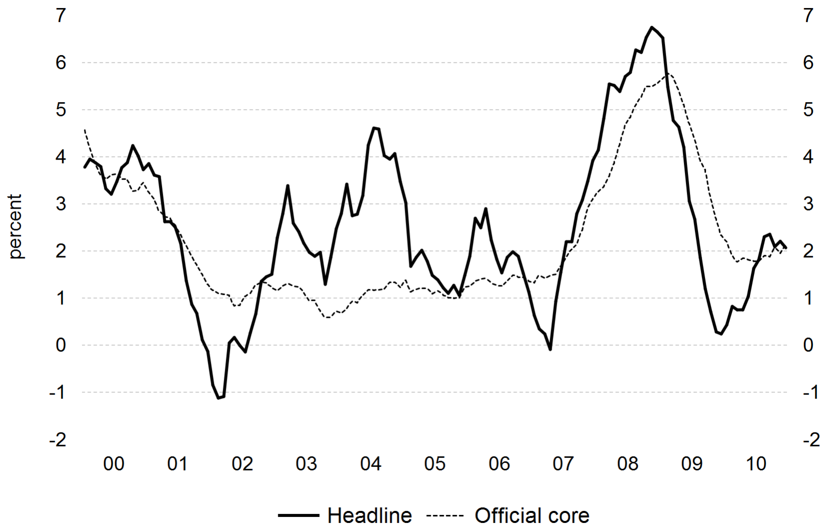
We use the year-on-year headline inflation rate as the original signal as Baqaee (2010) and Dowd et.al (2010), which will allow the comparison with the core inflation measures used for Peru in Armas et.al (2011). When recursive evaluation is performed, the minimum sample size used is $N = 72$, which is compatible with a MRA based on 5 levels (this helps in terms of the boundary problems because the full sample size allows the use of up to 6 levels for MRA).[14]

In figure 4 we depict the year-on-year headline inflation rate together with the official core inflation. In figure 5 and 6 we show the sets of WIMs we use in this paper. Due to sample size restrictions, all WIMs correspond to a level

[13]As other filtering methods, wavelet filtering suffers from boundary effects which deliver poor smoothing at the end of the sample. Standard DWT and MODWT treat time series as if they were circular (periodic). Given a series X, circularity means that X_{N-1} is useful for X_{-1}.

[14]The way we obtain wavelet-based core inflation measures is equivalent to using "wavelet denoising" techniques using a simple linear thresholding. Furthermore, as shown in Baqaee (2010), the best performing thresholding algorithms provides similar results to simple linear thresholding: "Linear thresholding is where we simply discard noisy daughter wavelets, leaving behind a smoothed trend line. This is justifiable theoretically because we have defined our noise to be short-term fluctuations in headline inflation that do not last into the medium term. Since the last two daughter wavelets are picking up exactly these fluctuations in the data, we can safely discard them."

Figure 4. *Headline and Official Core*



5 MRA[15]. We consider two sets of wavelet-based core inflation measures that can sensibly be constructed with the available data: the first set leave out details 1 to 3, and the second set discards details 1 to 4. Note that leaving out more details softens the WIMs.

4 Assessment of core inflation measures

The literature has identified certain desirable features that a core inflation indicator should possess. Smith (2004), Cogley (2002), Hansson et.al (2008), among others, illustrate how to assess statistically core inflation measures in terms of these desirable characteristics. For the case of Peru, previous work by Armas et.al (2011) finds that the official core inflation, which is constructed using exclusion methods,[16] has good properties in terms of two criteria suggested by the literature: (i) a good indicator of future inflation, and (ii) same mean as headline inflation over a large sample. However, some of the statistical measures of core inflation analysed by Armas et.al (2011) performed better than other indicators constructed by exclusion methods.

In this paper, we try to establish whether wavelet-based core inflation

[15]A small sample size limits the number of level of the MRA

[16]The official core inflation measure is well described in Armas et.al (2011)

Figure 5. *Headline and WIMs at level 5 and up to 3 details to leave out*

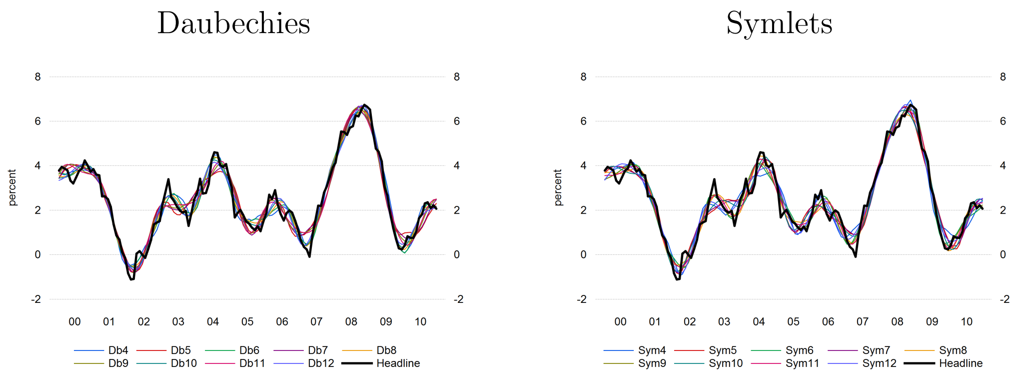
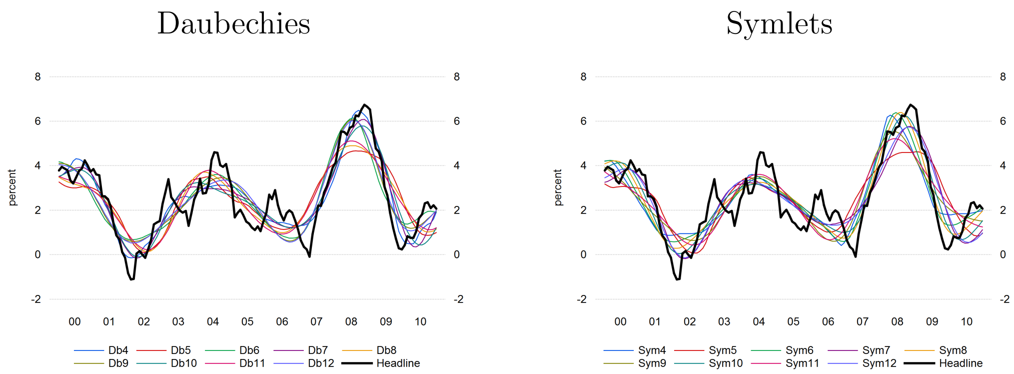


Figure 6. *Headline and WIMs at level 5 and up to 4 details to leave out*



measures (WIMs) behave as good as the official core inflation measure and other existing measures for Peru (both statistical and exclusion-based indicators), using two sets of criteria: (i) desirable long-run properties, and (ii) the ability to forecast future inflation.

4.1 Criterion 1: Long-run properties

This criterion relies on the dynamic behavior of core inflation measures, which is analysed using vector autoregression (VAR) models. In particular, using impulse response functions and variance decomposition it is possible to evaluate empirically to which extent a core inflation measure reflects medium-to-long term movements inflation.[17]

Let P denote the price index, $\pi_t \equiv log(P_t) - log(P_{t-1})$ the inflation rate and π_t^c a core inflation measure. Under the assumption that π^c is stationary, $log(P_t)$ is non-stationary, and π is stationary, the vector moving average (VMA) representation for π and π^c in terms of fundamental innovations is given by:

$$\begin{bmatrix} \pi_t \\ \pi_t^c \end{bmatrix} = \begin{bmatrix} \overline{\pi} \\ \overline{\pi}^c \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} \phi_{11}(i) & \phi_{12}(i) \\ \phi_{21}(i) & \phi_{22}(i) \end{bmatrix} \begin{bmatrix} \varepsilon_{t-i}^T \\ \varepsilon_{t-i}^P \end{bmatrix}$$

where ε_t^T and ε_t^P represent two disturbances affecting core inflation and the price index. Given that both core inflation and headline inflation are assumed to be stationary, then none of these disturbances will have permanent or long-run effects on them. However, given that the price index is non stationary, then these disturbances might have long-run effects on it.[18]

In order to identify this VAR model, we follow the strategy suggested by Blanchard and Quah (1989) which is based on a long-run restriction. In particular, we assume that the disturbance term ε_t^T has no long-run effect on the price index, which requires that $\sum_{i=0}^{\infty} \phi_{11}(i) = 0$. The impulse response functions (IRFs) and variance decomposition (VD) analysis from this identified VAR model can be helpful to assess alternative core inflation measures, using the idea that a policy maker expects to observe medium-to-long run movements rather than transitory ones when looking at core inflation indicator. In terms of the identified VAR model, this is equivalent to say that: (i) the response of core inflation to ε_t^T shocks - measured by cumulative impulse

[17] Ribba (2003) is one example, but the assumptions that underlie the analysis are different to the ones in this paper.

[18] If inflation were integrated of order 1, then the VAR model must be written using the first-difference of headline and core inflation.

response functions - is small, and (ii) the contribution of ε_t^T shocks to fluctuations in core inflation is small. Thus, we can say that core inflation A is better than B if (i) $\{\sum_{i=0}^{\infty} \phi_{21}(i)\}\,|_A < \{\sum_{i=0}^{\infty} \phi_{21}(i)\}\,|_B$, i.e. A's accumulated response to a ε_t^T shocks is smaller than B's accumulated response, and (ii) the portion of core inflation A's fluctuations explained by ε_t^T shocks is smaller than the portion of core inflation B's explained by the same shocks. Up to now, it should be clear for the reader that, in contrast to Quah and Vahey (1995), we do not use Blanchard and Quah (1989) approach to construct core inflation measures but to assess them.

We estimate several structural VAR (SVAR) models, each containing both headline inflation and a measure of core inflation. Table 1 shows the estimates of the term $\sum_{i=0}^{\infty} \hat{\phi}_{21}(i)$ for each SVAR model. We consider 6 exclusion-based indicators (including the official core inflation measure), and 22 statistical measures grouped into three sets: (i) standard measures, and (ii) wavelet-based measures (Daubechies and Symmetric wavelets). For the case of the official core inflation, the term $\sum_{i=0}^{\infty} \hat{\phi}_{21}(i)$ is equal to 3.46 and it is statistically significant, meaning that a 1 percent shock to ε_t^T will increase official core CPI permanently by 3.7. However, all wavelet-based core inflation measures show a much lower sensibility to ε_t^T shocks; in particular, a 1 percent shock to ε_t^T will increase any implicit wavelet-based core CPI by 0.02 percent on average. Thus, wavelet-based core inflation measures are superior to the official core inflation because they react relatively much less than the official indicator.

Table 1. *Estimated long-run effect of a transitory shock over core inflation measures.*

Exclusion Measures		**Statistical measures**					
		Standard measures		Daubechies wavelet		Symmetric wavelet	
Official Core	**3.7**	Coretrim50	2.0	db4	0.03	sym4	0.03
Coresa	3.7	Kernel	0.4	db5	0.02	sym5	0.02
Coresab	4.7	Mean	1.9	db6	0.02	sym6	0.02
Coresace	4.4	Perc63	2.8	db7	0.02	sym7	0.02
Coresaceh	3.2	Repond	2.7	db8	0.02	sym8	0.02
				db9	0.02	sym9	0.02
				db10	0.02	sym10	0.02
				db11	0.02	sym11	0.02
				db12	0.02	sym12	0.02

Note: All estimates are statistically different from zero at 1% level of significance.

Tables 2 and 3 show the corresponding variance decompositions for each model, which confirms that wavelet-based core inflation measures are superior

Table 2. *Variance Decomposition of Core inflation Measures: Contribution of Transitory Shocks(in percentage terms).*

Exclusion Measures Quarters	Official	Coresa	Coresab	Coresace	Coresaceh
1	38.3	35.9	45.7	64.5	49.6
2	29.1	29.6	44.3	52.6	36.1
4	15.6	15.5	31.0	29.6	18.0
8	10.2	9.2	16.7	18.6	10.6
15	10.0	11.2	13.7	15.4	9.4
40	10.0	11.2	13.8	15.1	9.3
80	10.0	11.2	14.0	15.1	9.3
150	**10.0**	**11.3**	**14.0**	**15.1**	**9.3**

to both exclusion-based and statistical core inflation indicators. As it is evident from the tables, the contribution of transitory shocks to the variation of wavelet-based core inflation is almost null compared at all horizons to the case of the official core inflation measure. In particular, after one quarter 38.3 percent of the variation in official core inflation is initially explained by shocks to ε_t^T, diminishing to 15.6 percent after one year (4 quarters), to 10.2 percent after 2 years (8 quarters) and reaching a long-run level of 10 percent, whereas for db12 the proportion goes from 2.4 percent (after one quarter) to 0.23 percent (after one year), reaching a long-run level of 0.3 percent. Furthermore, all the 18 WIMs considered are such that shocks to ε_t^T explain less than 4 percent of their variances, less than a half of the proportion of the official indicator that is explained by the same shocks. Following the same reasoning, Tables 2 and 3 show that all WIMs are superior to both exclusion-based and statistical indicators of core inflation.

It is interesting to note that -according to the proposed criterion- some exclusion-based indicators and all statistical indicators seem superior to the official core inflation measure. In terms of the variance decomposition analysis, one exclusion-based indicator ("Coresaceh") and all the statistical measures with the exception of one ("kernel") are superior to the official indicator when comparing the corresponding long-run variance. In terms of the accumulated impulse-response functions, one exclusion-based indicator (again "Coresaceh") and all the statistical indicators are superior to the official indicator when comparing the long-run effects of a transitory shock.

Table 3. *Variance Decomposition of Core inflation Measures: Contribution of Transitory Shocks(in percentage terms).*

Statistical Measures

Quarters	Coretrim50	Kernel	Mean38	Per63	Repond
1	19.2	6.0	16.2	24.3	8.9
2	15.1	8.9	14.2	18.7	7.5
4	8.6	13.7	8.3	9.5	7.5
8	6.6	12.8	6.6	7.2	8.7
15	7.1	17.0	7.0	7.6	8.3
40	7.2	17.2	7.0	7.6	8.3
80	7.2	17.2	7.0	7.6	8.3
150	**7.2**	**17.2**	**7.0**	**7.6**	**8.3**

Quarters	db4	db5	db6	db7	db8	db9	db10	db11	db12
1	2.7	2.7	0.8	1.9	0.7	0.4	0.4	0.1	0.6
2	1.9	0.9	2.2	1.0	0.7	0.1	0.3	0.1	0.2
4	1.8	0.2	1.2	0.3	0.2	0.0	0.0	0.1	0.0
8	3.3	0.3	2.4	0.3	0.7	0.1	0.1	0.3	0.1
15	3.6	0.3	2.8	0.3	0.9	0.3	0.1	0.9	0.1
40	3.7	0.3	3.0	0.3	0.9	0.3	0.2	0.9	0.1
80	3.8	0.3	3.1	0.3	1.0	0.3	0.2	0.9	0.1
150	**3.8**	**0.3**	**3.1**	**0.3**	**1.0**	**0.3**	**0.2**	**0.9**	**0.1**

Quarters	sym4	sym5	sym6	sym7	sym8	sym9	sym10	sym11	sym12
1	7.3	9.4	0.0	0.1	1.4	0.1	0.0	1.1	2.4
2	2.8	3.6	0.2	0.0	1.3	0.1	0.0	0.5	1.5
4	0.9	1.0	0.1	0.0	0.4	0.0	0.1	0.0	0.3
8	1.0	1.3	0.1	0.3	1.4	0.1	0.7	0.0	0.2
15	1.0	1.4	0.1	0.6	2.1	0.1	1.5	0.1	0.3
40	1.0	1.4	0.1	0.6	2.1	0.2	1.5	0.1	0.3
80	1.0	1.4	0.1	0.6	2.1	0.2	1.5	0.1	0.3
150	**1.0**	**1.4**	**0.1**	**0.6**	**2.1**	**0.2**	**1.5**	**0.1**	**0.3**

4.2 Criterion 2: Core inflation as an indicator of future inflation

Core inflation indicators may contain systematic signals about future inflationary pressures. Therefore, we can think of the various core inflation measures as forecasts of headline inflation h-periods ahead. Under this view we can resort to the forecast evaluation literature to test if a certain group of indicators is systematically better than the official core inflation indicator (OCI) to forecast headline inflation over various time horizons. We measure two statistics, the Diebold-Mariano test outlined in Diebold and Mariano (1995) and the success ratio statistic described in Pesaran and Timmermann (1992).

In order to compute the Diebold-Mariano (DM henceforth) statistic, we use the mean squared projection error

$$mspe_h = \frac{1}{T}\sum_t \left(\pi_{t+h}^{cpi} - \pi_t^{core}\right)^2 \tag{7}$$

and the absolute-valued projector error as loss criteria

$$mape_h = \frac{1}{T}\sum_t |\pi_{t+h}^{cpi} - \pi_t^{core}| \tag{8}$$

where π_{t+h}^{cpi} is year-on-year headline inflation in period $t+h$, h is the forecast horizon which takes values $h = 1, 2, ...36$, π_t^{core} is any relevant year-on-year core inflation measured at month t.

To perform this exercise we use monthly data between January 1996 and December 2010 (180 data points). We use the first 132 points as a starting sample to produce out-of-sample forecasts and compare them to the forecasts produced by the official core inflation. We compute the overall DM statistic as well as its dynamic updating as we increase the sample size. The DM statistic is calculated for both quadratic and absolute-value losses based on equations (7) and (8). Tables B-1 to B-7 in Appendix B show the DM statistic for various horizons for both the quadratic (MSPE) and mean absolute-value (MAPE) losses. MSPE stands for the difference between the mspe of the official core inflation and the mspe of the WIMs so that positive values of MSPE mean that the WIMs perform better than the official core. The column to the right of MSPE refers to the corresponding DM statistic. Likewise we report the MAPE values and their DM statistic, as well as the forecast bias of each WIM and their success ratio statistic.

We observe that up to forecast horizon $h = 3$, the WIMs perform better than the official core in terms of forecast ability. For slightly longer horizons, there is no strong evidence of superiority of WIMs over core inflation and viceversa. It is only for horizons of more than a year that the official core inflation becomes significantly better than the WIMs.

In terms of forecasting the direction of change, WIMs also perform better for short horizons whereas the official core is superior for horizons of h=18 or more.

These results point to the fact that WIMS should be followed closely to asses short-term inflationary forecasts while the official core remains a valid measure for long horizons.

We also compute the stability of the DM statistics as calculated over a window that covers the last two years of the data. We depict this behaviour in figures C-5 to C-8. We can see that the Daubechies family has ***db4*** and

db6 as the best performers whereas the Symlet family has ***sym4*** and ***sym5*** as the indicators with better predictive ability.

Another way to assess a core inflation measure in terms of its ability to forecast future inflation is proposed by Cogley (2002). This test is based on the value of the β parameter in the following regression equation:

$$\pi_{t+h}^{cpi} - \pi_t^{cpi} = \alpha_H + \beta_H \left(\pi_t^{cpi} - \pi_t^{core}\right) \tag{9}$$

When current headline inflation is high relative to core inflation $\left(\pi_t^{cpi} - \pi_t^{core}\right\rangle 0)$ and if current core inflation represents true inflationary pressures, then we should expect future headline inflation to revert to its core, namely $\pi_{t+h}^{cpi} - \pi_t^{cpi} < 0$. Equation (9) tests the ability of any measure of core inflation to induce future inflationary reversals at horizon h. The closer the β_H parameter is to -1, the more useful core inflation is for respective horizon h. Figures C-9 and C-10 in Appendix C provide an overview of the behaviour of β over different horizons. We observe that the β_H's associated with the WIMs quickly move from cero to below -1 whereas the official core takes time to achieve this if at all. This means that the information content of the gap between headline inflation and the WIMs supports the idea we found in the previous exercise: WIMs behave better than the official core inflation over shorter horizons.

5 Conclusions

The main purpose of this paper was the construction of alternative measures of core inflation that could improve the performance of the official core inflation in Peru. Using wavelet functions and multiresolution analysis (MRA), we constructed core inflation measures that capture medium-to-long term movements in inflation, i.e. movements that occur over longer periods of time and contain low frequency information. The construction of WIMs is relatively new in the literature and their assessment has not been addressed formally, this paper being the first attempt to perform both tasks. Another main contribution of the paper is that it proposes a VAR-based long-run criterion as an alternative criteria for evaluating core inflation measures. The results show that wavelet-based core inflation measures (WIMs) are superior to Peru's official core inflation and other existing measures (both exclusion-based and statistical indicators). In particular, compared to the official core inflation, WIMs react much less to transitory shocks. Finally, the results also show that WIMs could improve short-term (up-to-6-months)inflation forecasts.

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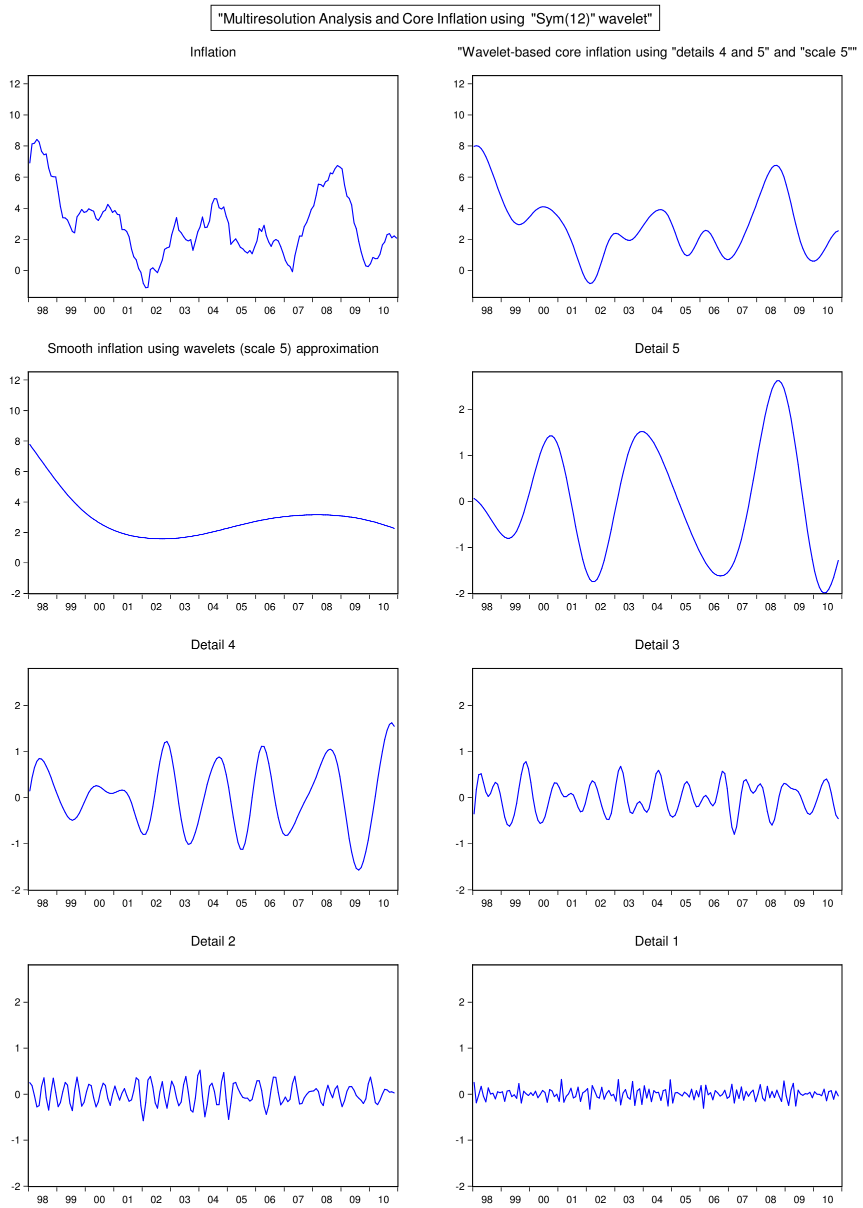
APPENDICES

A An example of MRA of headline inflation

Figure A-1. *Multiresolution Analysis of Headline Inflation using Wavelets*



B Tables with results

Table B-1. *One-month horizon*

Indicator	MSPE	DM	MAPE	DM	Bias	Success Ratio
db4	1.96	3.20	0.75	4.12	3.20	0.92 (0.00)
db5	1.96	3.23	0.75	4.16	3.23	0.90 (0.00)
db6	1.93	3.16	0.72	4.01	3.16	0.90 (0.00)
db7	1.92	3.13	0.72	3.95	3.13	0.89 (0.00)
db8	1.90	3.15	0.71	3.99	3.15	0.90 (0.00)
db9	1.88	3.11	0.67	3.80	3.11	0.89 (0.00)
db10	1.86	3.04	0.67	3.69	3.04	0.88 (0.00)
db11	1.86	3.08	0.68	3.78	3.08	0.89 (0.00)
db12	1.85	3.08	0.66	3.69	3.08	0.88 (0.00)
sym4	1.82	2.99	0.67	3.66	2.99	0.92 (0.00)
sym5	1.91	3.13	0.72	3.89	3.13	0.92 (0.00)
sym6	1.91	3.15	0.70	3.87	3.15	0.89 (0.00)
sym7	1.89	3.10	0.69	3.86	3.10	0.90 (0.00)
sym8	1.87	3.04	0.69	3.74	3.04	0.90 (0.00)
sym9	1.90	3.10	0.70	3.82	3.10	0.89 (0.00)
sym10	1.92	3.17	0.70	3.89	3.17	0.89 (0.00)
sym11	1.90	3.14	0.69	3.88	3.14	0.89 (0.00)
sym12	1.88	3.06	0.69	3.79	3.06	0.90 (0.00)
CORE					-3.37	0.45 (0.88)

Table B-2. *Two-month horizon*

Indicator	MSPE	DM	MAPE	DM	Bias	Success Ratio
db4	1.89	2.77	0.59	3.02	2.77	0.86 (0.00)
db5	1.87	2.80	0.59	3.03	2.80	0.84 (0.00)
db6	1.85	2.77	0.56	2.92	2.77	0.84 (0.00)
db7	1.83	2.70	0.56	2.86	2.70	0.83 (0.00)
db8	1.81	2.70	0.55	2.85	2.70	0.84 (0.00)
db9	1.78	2.68	0.53	2.77	2.68	0.83 (0.00)
db10	1.75	2.59	0.52	2.63	2.59	0.82 (0.00)
db11	1.75	2.62	0.52	2.68	2.62	0.83 (0.00)
db12	1.74	2.63	0.52	2.68	2.63	0.82 (0.00)
sym4	1.72	2.56	0.49	2.55	2.56	0.86 (0.00)
sym5	1.81	2.67	0.54	2.79	2.67	0.86 (0.00)
sym6	1.83	2.71	0.56	2.84	2.71	0.83 (0.00)
sym7	1.81	2.68	0.54	2.75	2.68	0.84 (0.00)
sym8	1.78	2.61	0.53	2.66	2.61	0.84 (0.00)
sym9	1.80	2.65	0.54	2.74	2.65	0.83 (0.00)
sym10	1.84	2.73	0.56	2.85	2.73	0.83 (0.00)
sym11	1.83	2.72	0.55	2.82	2.72	0.83 (0.00)
sym12	1.79	2.64	0.53	2.71	2.64	0.84 (0.00)
CORE					-3.03	0.46 (0.76)

Table B-3. *Three-month horizon*

Indicator	MSPE	DM	MAPE	DM	Bias	Success Ratio
db4	1.74	2.30	0.43	2.03	2.30	0.82 (0.00)
db5	1.70	2.29	0.43	2.04	2.29	0.80 (0.00)
db6	1.69	2.27	0.43	2.02	2.27	0.80 (0.00)
db7	1.66	2.21	0.41	1.92	2.21	0.79 (0.00)
db8	1.63	2.20	0.40	1.89	2.20	0.80 (0.00)
db9	1.60	2.18	0.40	1.87	2.18	0.79 (0.00)
db10	1.57	2.10	0.38	1.78	2.10	0.78 (0.00)
db11	1.57	2.12	0.37	1.77	2.12	0.79 (0.00)
db12	1.56	2.12	0.38	1.77	2.12	0.78 (0.00)
sym4	1.57	2.10	0.36	1.76	2.10	0.82 (0.00)
sym5	1.65	2.19	0.40	1.88	2.19	0.82 (0.00)
sym6	1.66	2.19	0.41	1.91	2.19	0.79 (0.00)
sym7	1.66	2.18	0.41	1.92	2.18	0.80 (0.00)
sym8	1.63	2.15	0.39	1.82	2.15	0.80 (0.00)
sym9	1.65	2.17	0.40	1.85	2.17	0.79 (0.00)
sym10	1.67	2.21	0.42	1.94	2.21	0.79 (0.00)
sym11	1.67	2.22	0.41	1.95	2.22	0.79 (0.00)
sym12	1.65	2.17	0.40	1.88	2.17	0.80 (0.00)
CORE					-2.54	0.52 (0.31)

Table B-4. *Four-month horizon*

Indicator	MSPE	DM	MAPE	DM	Bias	Success Ratio
db4	1.58	1.89	0.31	1.36	1.89	0.74 (0.00)
db5	1.54	1.85	0.30	1.30	1.85	0.73 (0.00)
db6	1.50	1.83	0.29	1.27	1.83	0.73 (0.00)
db7	1.48	1.80	0.28	1.24	1.80	0.72 (0.00)
db8	1.45	1.77	0.26	1.17	1.77	0.73 (0.00)
db9	1.40	1.72	0.25	1.13	1.72	0.72 (0.00)
db10	1.38	1.69	0.25	1.09	1.69	0.70 (0.00)
db11	1.38	1.68	0.24	1.07	1.68	0.72 (0.00)
db12	1.35	1.65	0.24	1.06	1.65	0.70 (0.00)
sym4	1.38	1.68	0.24	1.11	1.68	0.74 (0.00)
sym5	1.47	1.77	0.27	1.21	1.77	0.74 (0.00)
sym6	1.45	1.73	0.27	1.17	1.73	0.72 (0.00)
sym7	1.46	1.74	0.27	1.19	1.74	0.73 (0.00)
sym8	1.44	1.71	0.26	1.16	1.71	0.73 (0.00)
sym9	1.46	1.74	0.27	1.18	1.74	0.72 (0.00)
sym10	1.48	1.77	0.28	1.23	1.77	0.72 (0.00)
sym11	1.48	1.77	0.28	1.24	1.77	0.72 (0.00)
sym12	1.47	1.74	0.27	1.22	1.74	0.73 (0.00)
CORE					-2.13	0.53 (0.17)

Table B-5. *One-year horizon*

Indicator	MSPE	DM	MAPE	DM	Bias	Success Ratio
db4	-1.76	-1.59	-0.32	-1.34	-1.59	0.67 (0.01)
db5	-1.77	-1.62	-0.31	-1.29	-1.62	0.63 (0.06)
db6	-1.77	-1.58	-0.31	-1.30	-1.58	0.63 (0.06)
db7	-1.78	-1.63	-0.32	-1.35	-1.63	0.64 (0.03)
db8	-1.82	-1.69	-0.32	-1.36	-1.69	0.63 (0.06)
db9	-1.82	-1.68	-0.31	-1.30	-1.68	0.62 (0.10)
db10	-1.83	-1.70	-0.32	-1.36	-1.70	0.66 (0.02)
db11	-1.86	-1.74	-0.33	-1.39	-1.74	0.64 (0.03)
db12	-1.84	-1.74	-0.30	-1.30	-1.74	0.63 (0.06)
sym4	-1.89	-1.70	-0.34	-1.42	-1.70	0.66 (0.02)
sym5	-1.85	-1.68	-0.33	-1.41	-1.68	0.66 (0.02)
sym6	-1.88	-1.68	-0.33	-1.38	-1.68	0.64 (0.03)
sym7	-1.91	-1.65	-0.34	-1.41	-1.65	0.63 (0.06)
sym8	-1.89	-1.66	-0.34	-1.44	-1.66	0.66 (0.02)
sym9	-1.85	-1.65	-0.33	-1.41	-1.65	0.67 (0.01)
sym10	-1.81	-1.63	-0.32	-1.32	-1.63	0.64 (0.03)
sym11	-1.80	-1.61	-0.32	-1.32	-1.61	0.64 (0.03)
sym12	-1.84	-1.61	-0.33	-1.39	-1.61	0.66 (0.02)
CORE					1.36	0.58 (0.06)

Table B-6. *Eighteen-months horizon*

Indicator	MSPE	DM	MAPE	DM	Bias	Success Ratio
db4	-2.54	-2.11	-0.35	-1.79	-2.11	0.61 (0.27)
db5	-2.53	-2.14	-0.35	-1.80	-2.14	0.57 (0.62)
db6	-2.57	-2.14	-0.34	-1.72	-2.14	0.57 (0.62)
db7	-2.52	-2.12	-0.35	-1.80	-2.12	0.58 (0.53)
db8	-2.53	-2.17	-0.35	-1.86	-2.17	0.57 (0.62)
db9	-2.54	-2.18	-0.34	-1.80	-2.18	0.55 (0.78)
db10	-2.54	-2.16	-0.35	-1.83	-2.16	0.57 (0.62)
db11	-2.53	-2.20	-0.35	-1.88	-2.20	0.58 (0.53)
db12	-2.51	-2.19	-0.34	-1.83	-2.19	0.57 (0.62)
sym4	-2.61	-2.19	-0.36	-1.90	-2.19	0.60 (0.42)
sym5	-2.58	-2.15	-0.35	-1.81	-2.15	0.60 (0.42)
sym6	-2.63	-2.17	-0.35	-1.77	-2.17	0.58 (0.53)
sym7	-2.70	-2.17	-0.36	-1.78	-2.17	0.57 (0.62)
sym8	-2.65	-2.15	-0.35	-1.76	-2.15	0.60 (0.42)
sym9	-2.59	-2.13	-0.35	-1.78	-2.13	0.58 (0.53)
sym10	-2.56	-2.15	-0.34	-1.73	-2.15	0.58 (0.53)
sym11	-2.56	-2.14	-0.34	-1.71	-2.14	0.58 (0.53)
sym12	-2.61	-2.13	-0.34	-1.71	-2.13	0.60 (0.42)
CORE					2.01	0.66 (0.05)

Table B-7. *Two-years horizon*

Indicator	MSPE	DM	MAPE	DM	Bias	Success Ratio
db4	-2.19	-1.63	-0.40	-1.66	-1.63	0.52 (0.98)
db5	-2.17	-1.63	-0.42	-1.71	-1.63	0.49 (0.99)
db6	-2.24	-1.68	-0.44	-1.76	-1.68	0.49 (0.99)
db7	-2.11	-1.59	-0.40	-1.63	-1.59	0.51 (0.99)
db8	-2.10	-1.60	-0.41	-1.64	-1.60	0.49 (0.99)
db9	-2.09	-1.61	-0.42	-1.70	-1.61	0.49 (0.99)
db10	-2.04	-1.56	-0.40	-1.61	-1.56	0.49 (0.99)
db11	-2.05	-1.57	-0.40	-1.59	-1.57	0.51 (0.99)
db12	-1.99	-1.55	-0.41	-1.63	-1.55	0.51 (0.99)
sym4	-2.20	-1.65	-0.40	-1.60	-1.65	0.52 (0.98)
sym5	-2.16	-1.61	-0.41	-1.64	-1.61	0.52 (0.98)
sym6	-2.22	-1.64	-0.43	-1.71	-1.64	0.51 (0.99)
sym7	-2.34	-1.71	-0.45	-1.75	-1.71	0.49 (0.99)
sym8	-2.22	-1.65	-0.41	-1.64	-1.65	0.52 (0.98)
sym9	-2.16	-1.62	-0.41	-1.63	-1.62	0.51 (0.99)
sym10	-2.14	-1.62	-0.42	-1.69	-1.62	0.51 (0.99)
sym11	-2.15	-1.63	-0.42	-1.69	-1.63	0.51 (0.99)
sym12	-2.21	-1.66	-0.42	-1.67	-1.66	0.52 (0.98)
CORE					1.76	0.67 (0.00)

C Figures with results

Figure C-1. *Impulse Response Functions to a transitory shock*

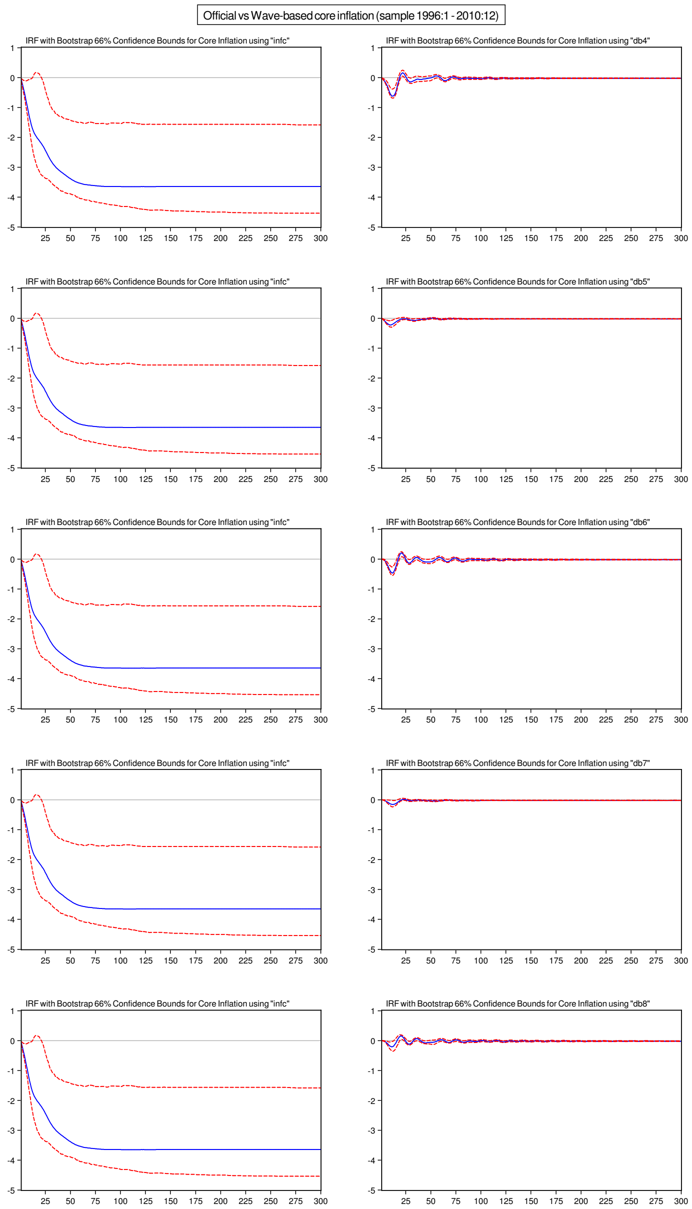


Figure C-2. *Impulse Response Functions to a transitory shock*

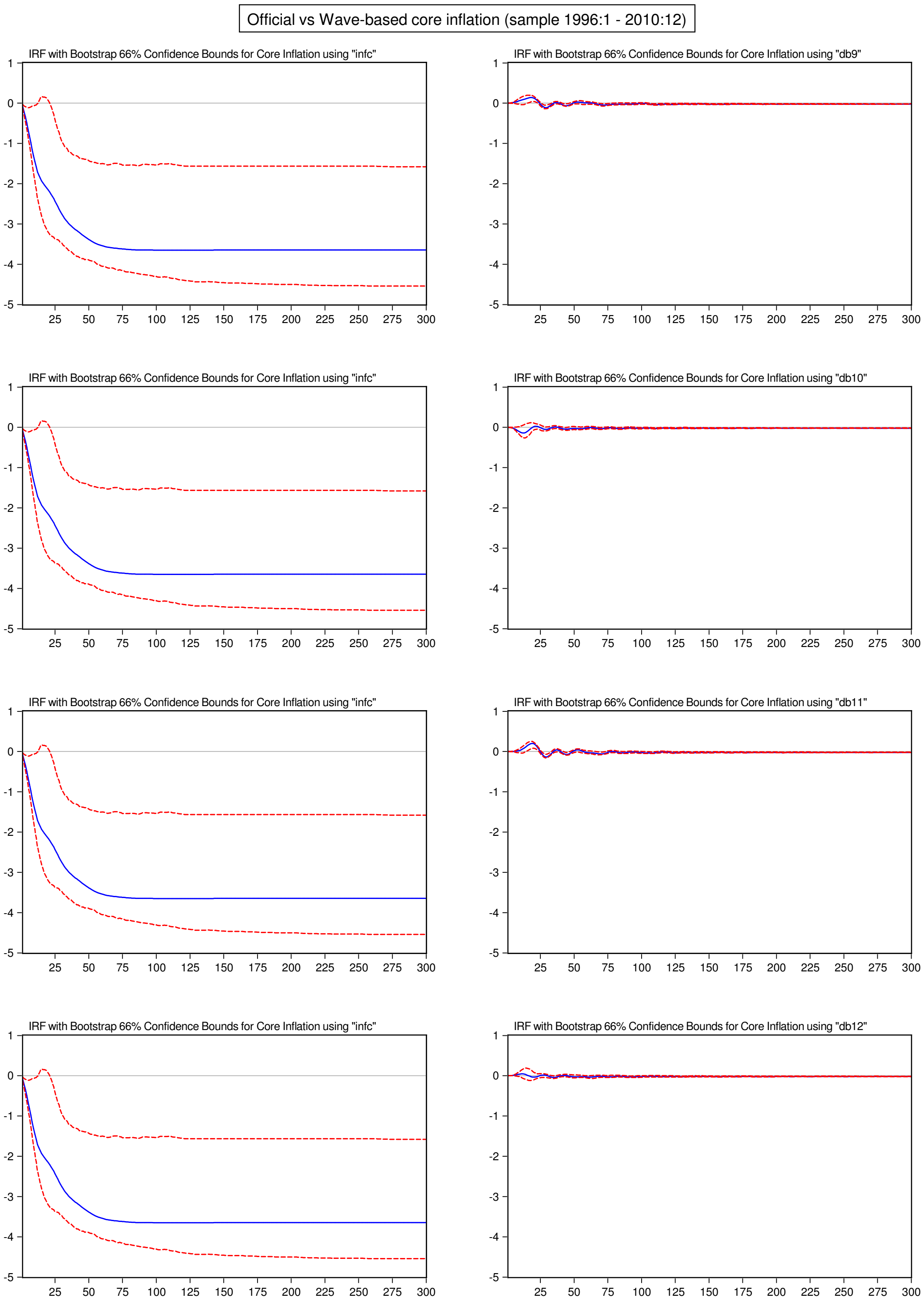


Figure C-3. *Impulse Response Functions to a transitory shock*

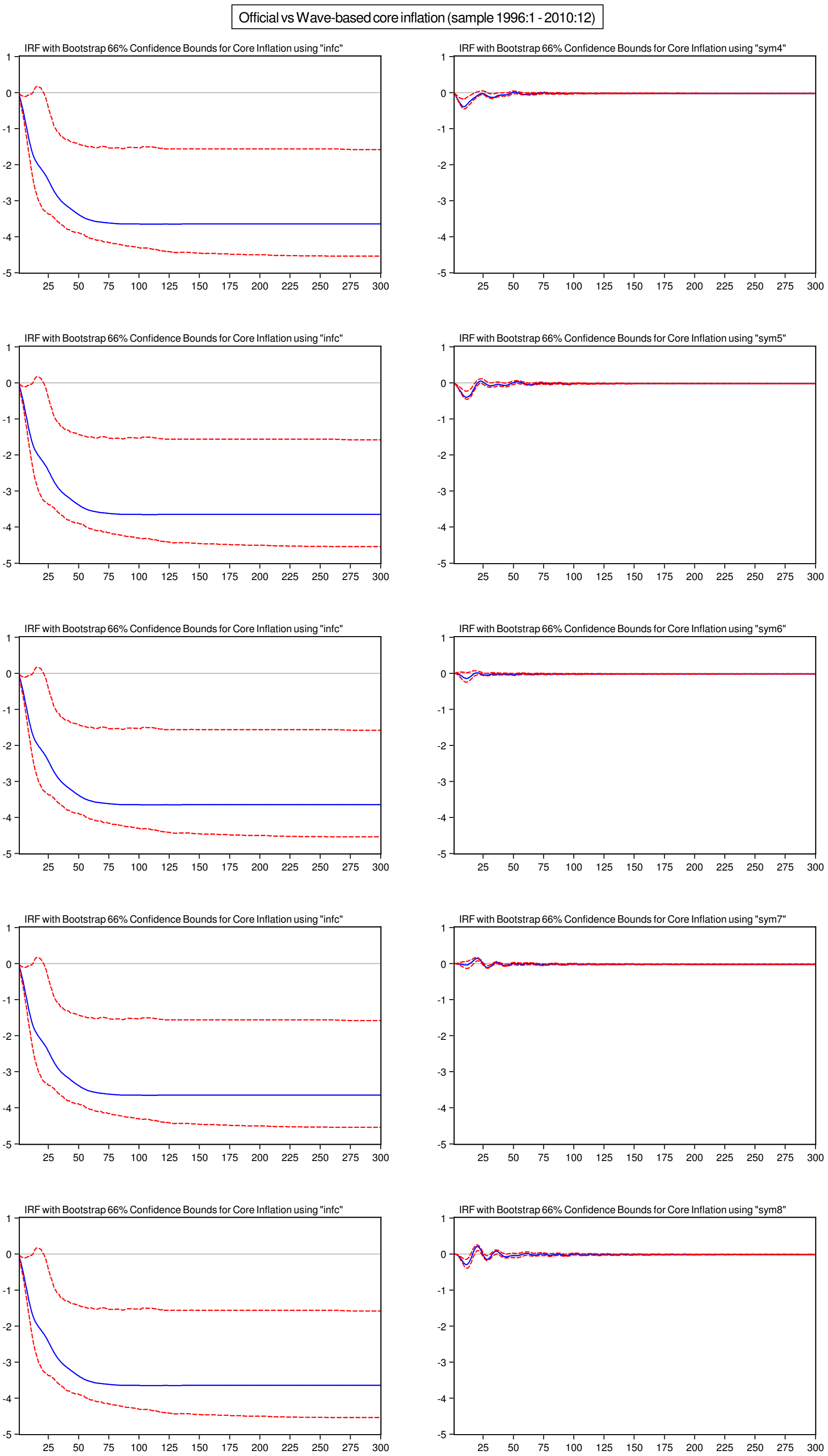


Figure C-4. *Impulse Response Functions to a transitory shock*

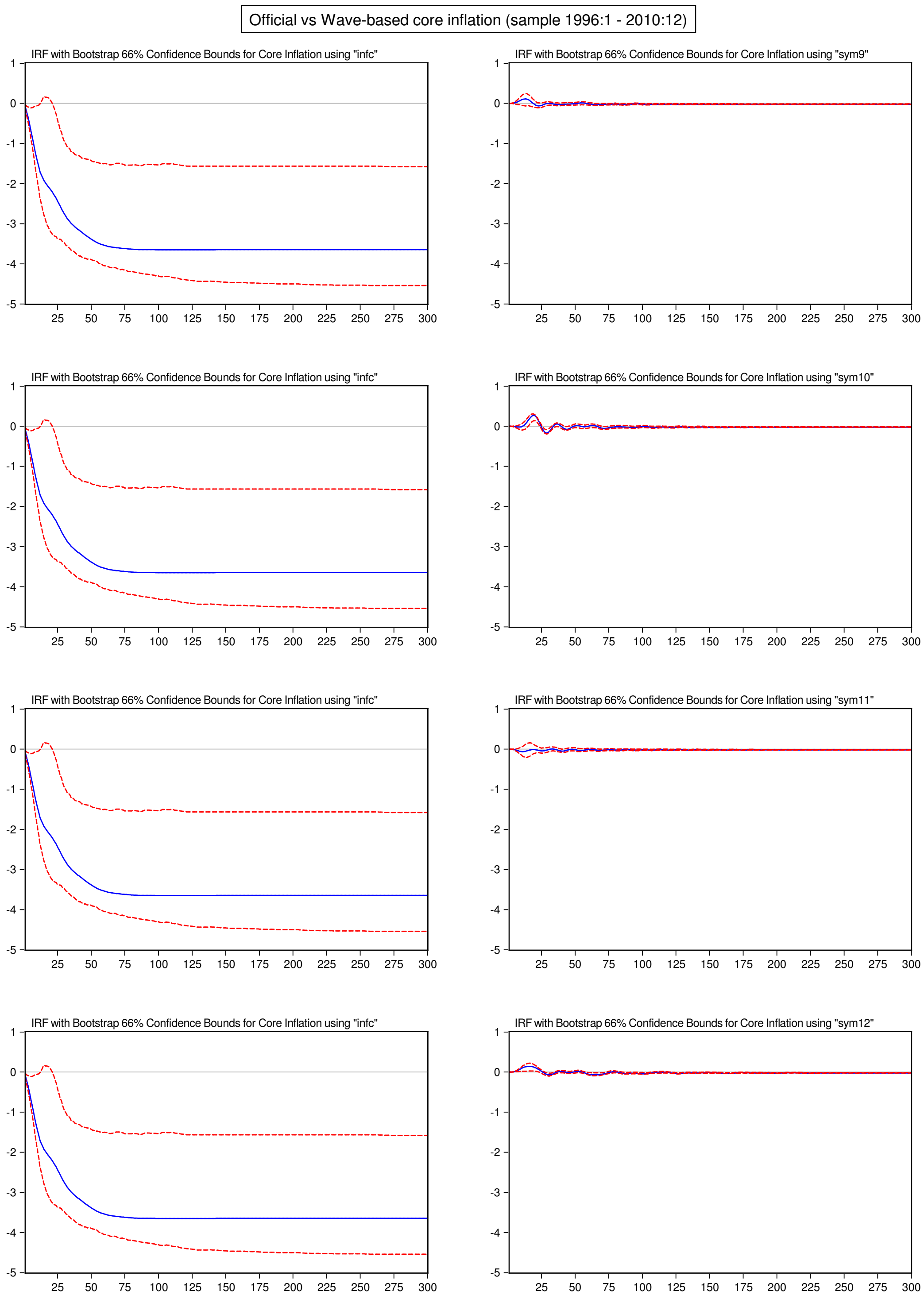


Figure C-5. *DM statistics over recent samples*

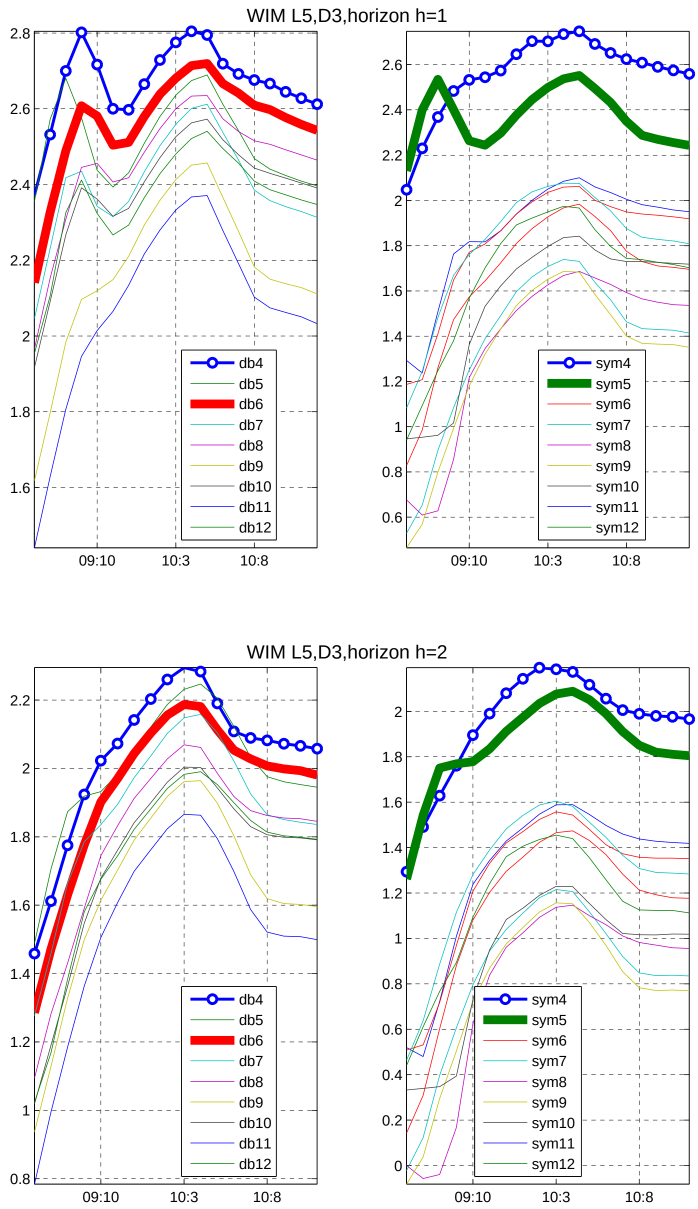


Figure C-6. *DM statistics over recent samples*

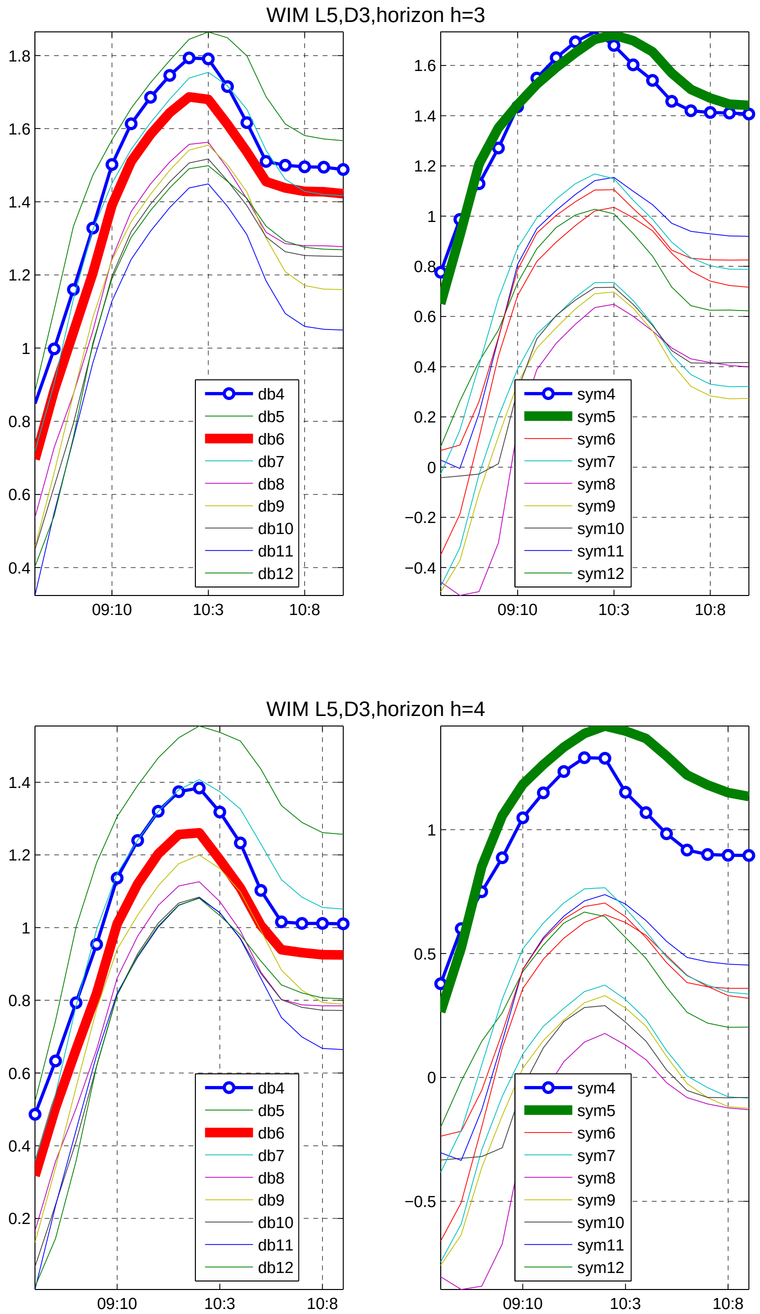


Figure C-7. *DM statistics over recent samples*

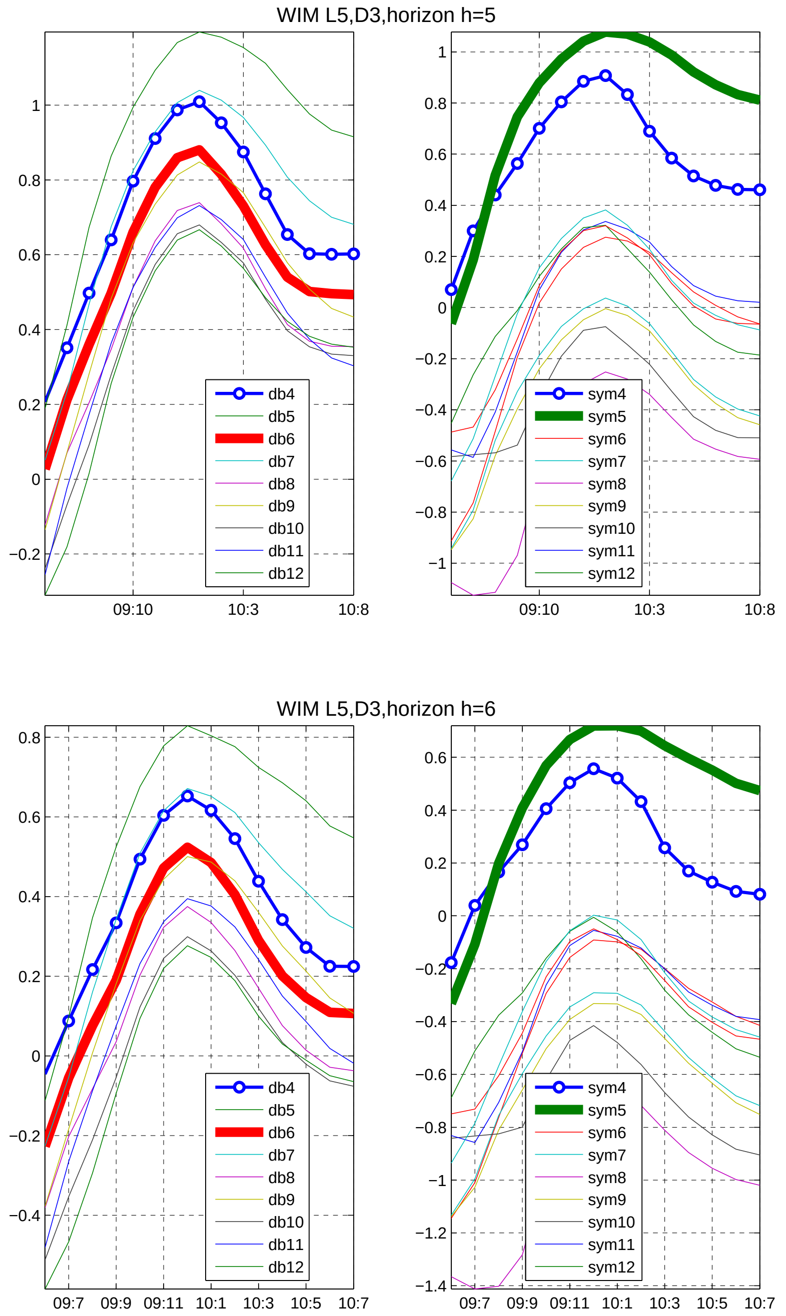


Figure C-8. *DM statistics over recent samples*

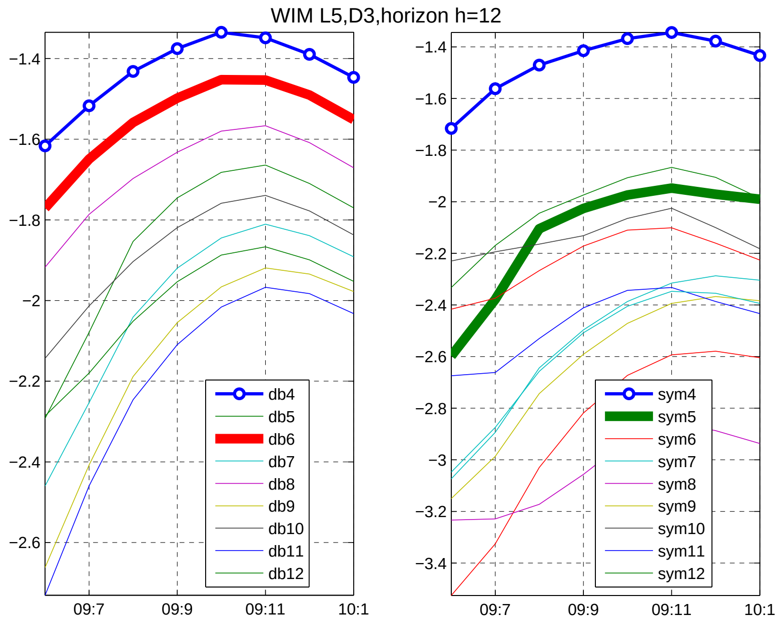


Figure C-9. *Estimation of β parameter: Official core against WIMs based on Daubechies*

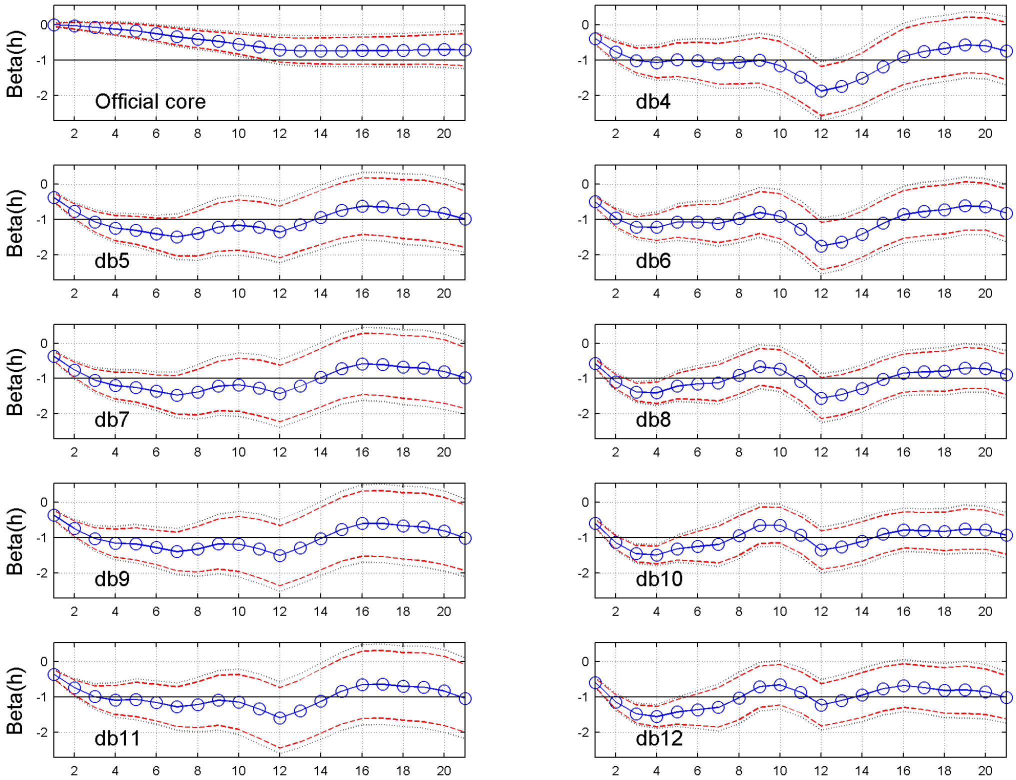


Figure C-10. *Estimation of β parameter: Official core against WIMs based on Symlets*