BANCO CENTRAL DE RESERVA DEL PERÚ

Monetary Policy in the presence of Informal Labour Markets

Paul Castillo* and Carlos Montoro**

* Banco Central de Reserva del Perú
** Banco Central de Reserva del Perú and Bank for International Settlements

DT. N° 2010-009
Serie de Documentos de Trabajo
Working Paper series
Julio 2010

Monetary Policy in the Presence of Informal Labour Markets*

Paul Castillo†
Banco Central de Reserva del Perú

Carlos Montoro‡
Banco Central de Reserva del Perú and Bank for International Settlements

First version: September 2007
This version: March 2010

Abstract

In this paper we analyse the effects of informal labour markets on the dynamics of inflation and on the transmission of aggregate demand and supply shocks. In doing so, we incorporate the informal sector in a modified New Keynesian model with labour market frictions as in the Diamond-Mortensen-Pissarides model. Our main results show that the informal economy generates a "buffer" effect that diminishes the pressure of demand shocks on aggregate wages and inflation. Finding that is consistent with the empirical literature on the effects of informal labour markets in business cycle fluctuations. This result implies that in economies with large informal labour markets the interest rate channel of monetary policy is relatively weaker. Furthermore, the model produces cyclical flows from informal to formal employment consistent with the data.

JEL Classification: E32, E50, J64, O17

Keywords: Monetary Policy, New Keynesian Model, Informal Economy, Labour Market Frictions.

*We would like to thank Lars Ljungqvist, Randal Wright, Paul Levine, Gabriel Rodriguez, Hugo Vega, participants at the "Third Monetary Policy Research Workshop in Latin America and the Caribbean" organised by Banco de la Republica and CCBS-Bank of England, participants at the 2007 Research Meeting organized by Banco Central de Reserva del Peru, participants at the University of Surrey Economic Workshop 2008, and participants at the 2008 LACEA Meeting held in Rio de Janeiro for useful suggestions and comments. The views expressed herein are those of the authors and do not necessarily reflect those of the Banco Central de Reserva del Perú nor those of the Bank for International Settlements. Any errors are our own responsibility.

†E-mail: paul.castillo@bcrp.gob.pe.

‡Address correspondence to: Carlos Montoro, Office for the Americas, Bank for International Settlements, Torre Chapultepec - Rubén Darío 281 - 1703, Col. Bosque de Chapultepec - 11580, México DF – México; tel: +52 55 9138 0294; fax: +52 55 9138 0299; e-mail: carlos.montoro@bis.org.

1 Introduction

The New Keynesian model has become a useful tool for both academics and policy makers to analyse monetary policy design. However, this strand of the literature typically ignores labour market frictions. In particular, it assumes that labour markets are perfectly competitive and consequently aggregate fluctuations only adjust at the intensive labour margin. Nevertheless, empirical studies show that at business cycle frequencies labour usage adjusts not only at the intensive margin but also at the extensive margin, which generates fluctuations in unemployment. Thereby, this model is not suited to study the link between inflation and unemployment and its limited on explaining some stylised facts of the data[1] .

Recently, some authors have extended the New Keynesian model including labour market frictions and unemployment in the line of the Diamond-Mortensen-Pissarides (DMP) model[2]. The DMP framework includes labour market frictions, such as costs of matching vacancies and workers searching for a job. These kind of frictions generate dynamics in the unemployment rate that are closer to the data and have implications for monetary policy.

The study of the flows between employment and unemployment is important for developed economies, since they capture most of the labour market fluctuations. However, in developing economies, where labour markets are characterised for having a large proportion of the labour force employed in semi-illegal irregular jobs -the so called *informal* employment-[3], the study of the flows between the *formal* and *informal* sectors becomes more relevant.

There exists empirical evidence that shows that the presence of informal labour markets affects the business cycle dynamics of an economy. More precisely, this evidence shows that informal labour markets act as a buffer stock for the regulated formal employment, increasing labour market flexibility and affecting the transmission mechanisms of shocks to the economy. For instance, Bovi (2007), using labour market data for Italy, finds that informal employment is pro-cyclical, whereas formal employment is almost acyclical. Other authors have also found similar evidence, Carrillo and Pugno (2004) and Bowler and Morisi (2006) report a cyclical pattern for informal employment in a set of emerging economies.

Given the importance of the informal economy for developing countries, the design of monetary policy should carefully consider its effects on the labour market and inflation dynamics. In particular, from the monetary policy point of view it is important to answer the following questions: how does the presence of the informal sector affect inflation dynamics and the

[1]For instance, the basic New Keynesian model is not suitable for explaining the procyclicality of the job destruction rate and the well-documented negative correlation between unemployment and inflation.

[2]Among those authors are Walsh (2003, 2005), Alexopoulus (2004), Trigari (2004), Blanchard and Gali (2006), Krause and Lubik (2005), Thomas (2008), Gertler and Trigari (2006), and Ravenna and Walsh (2007).

[3]Djankov, et al. (2002) and Schneider (2007) estimate that informal employment is between 40% and 80% of the total labor force in developing economies.

transmission mechanism of monetary policy?, how should be the optimal design of monetary policy?, what determines de flows between formal and informal employment?

To address these questions, in this paper we extend a standard closed economy New Keynesian model adding labour market frictions as Blanchard and Galí (2006). Differently from them, however, we model a dual labour market economy considering the existence of formal and informal labour contracts. The model economy is composed of households, retailers, firms and the central bank. Households receive utility from the consumption of a continuum of differentiated goods and supply labour in a descentralised labour market subject to search and hiring costs. Retailers, on the other hand, produce under monopolistic competition differentiated consumption goods and set prices according to a Calvo type price setting rule. Retailers use as production input a wholesale good, which is produced by firms under perfect competition using labour. Finally, the central bank implements monetary policy by setting the short-term interest rate according to a Taylor-type feedback policy rule.

To the best of our knowledge this is the first paper that analyses the implications for monetary policy of the presence of an informal labour market. Previous papers have studied how the informal jobs in the labour market are generated, see for example Bosch (2004, 2006), Fugaza and Jaques (2002), Kolm and Larsen (2004), and Boeri and Garibaldi (2006). However, those models focus in the real economy and haven't analysed the interaction between the informal sector and monetary policy.

We introduce labour market frictions considering that firms face hiring costs, which depend on the degree of labour market tightness, defined by the ratio of vacancies to unemployment. This hiring cost generates a friction in the labour market similar to the cost of posting a vacancy in the standard DMP model. Furthermore, we introduce informality within the model by assuming firms in the wholesale sector can choose between two types of production processes: formal and informal. The process labeled as formal has higher productivity and larger hiring costs. In contrast, in the informal process workers are less productive but hiring costs are smaller. We focus on an equilibrium where firms use both production technologies, thereby informal and formal workers coexist.

The key implication of this dual-production technology is that firms' marginal costs would depend not only on wages, productivity and unemployment levels, but also on the level of informality measured by the proportion of informal employment on the total labour force. During periods of high aggregate demand, firms find optimal to use more intensively the informal technology because, marginal costs associated to this technology are lower than those of the formal one. Accordingly, firms' behavior optimally lessens the impact of aggregate demand on their marginal costs. On the contrary, when demand is low and therefore hiring costs are lower, firms optimally increase their relative use of formal labour.

Furthermore, informality also reduces the impact of demand shocks on wages of the formal sector. When a worker receives an offer to sign a formal labour contract, he has two options: either to accept the offer and receive the corresponding wage rate or wait for another one expecting to obtain a larger wage rate. When in the economy there are informal labour markets, the cost of waiting is larger since the probability of receiving a new offer of a formal labour contract is much lower in this case. This possibility of waiting for a longer period induces workers to accept lower wages. Hence, firms in economies with informal labour markets are more flexible to expand output, thus demand shocks generate lower inflation and larger output expansions. Thus, the positive response of informal employment to demand shocks is larger than the one observed in the formal sector.

At the aggregate level, the model shows that informality affects the dynamics of domestic inflation on several dimensions. First, it generates a link between unemployment flows and inflation dynamics. Second, through its relationship with firms' marginal costs, it reduces the impact of aggregate demand on domestic inflation. Finally, it makes inflation response to shocks more persistent.

The paper is organised as follows: the next section presents the model of an economy with monopolistic competition, nominal rigidities and dual labour market rigidites. Section 3 shows the model in log-linear form. Section 4 presents the results of the model in terms of the effects of the informal economy in the transmission mechanism of monetary policy. The last section concludes.

2 The model

The economy is populated by a continuum of households that consume final goods and supply labour in a descentralised labour market subject to search and hiring costs. Firms produce a wholesale good, which is used as input to produce differentiated final consumption goods by retailers and the central bank that sets the nominal interest rate through a Taylor rule. Retailers operate in monopolistic competitive markets, where prices are sticky.

2.1 Preferences

The representative household is made up of a continuum of members represented by the unit interval. Each household maximises the following utility function,

$$\mathcal{U}_t = E_o \left[\sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t\right) - \chi \frac{N_t^{1+\phi}}{1+\phi} \right] \right],$$

where C_t is a composite basket defined over a continuum of differentiated goods that have an elasticity of substitution $\varepsilon > 1$,

$$C_t = \left[\int_0^1 C_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

and N_t stands for the fraction of household members that are employed, that satisfies the constraint $0 \leq N_t \leq 1$. At the begining of each period a fraction u_t of the family members are unemployed and a fraction N_{t-1} is employed. From this pool of employed household members, each period a fraction δ loose their jobs and a fraction H_t is randomly hired, thus, employment evolves according the following condition,

$$N_t = (1-\delta) N_{t-1} + H_t. \tag{2.1}$$

Household members, when unemployed, receive a constant income associated to home production, W^u, whereas when they are employed they can either work under a formal contract and receive a wage rate W_t^F, or they can work under an informal contract, where the wage rate is W_t^I. Informal contracts differ from formal ones mainly because firms face lower hiring costs under informal contracts. Total employment in the economy is defined as follows,

$$N_t = N_t^F + N_t^I, \tag{2.2}$$

where N_t^F and N_t^I, represent the stock of employed workers under formal and informal contracts. We introduce an index that measures the tightness of the labour market, denoted by X_t. Alternatively, labour market tightness can be interpreted as the probability that a worker has of being hired, thus it is defined as the ratio of hirings to the level of unemployment before the hiring decision has taken place, that is $X_t = \frac{H_t}{U_t}$ where $U_t = 1 - (1-\delta) N_{t-1}$. We further assume that the job finding rate is different for formal and informal contracts, in particular we define, $X_t^F = \frac{H_t^F}{U_t}$, as the job finding rate in the formal labour market and as $X_t^I = \frac{H_t^I}{U_t}$, the corresponding rate in the informal market. It follows that:

$$X_t = X_t^F + X_t^I. \tag{2.3}$$

Households can smooth consumption using a nominal one-period discount bond, B_t which pays a nominal interest rate, i_t every period. Therefore, the households' budget constraint is given by:

$$P_t C_t + B_t = \left[W_t^F N_t^F + W_t^I N_t^I + P_t W^u (1-N_t)\right] + B_{t-1}(1+i_t) + P_t \Gamma_t^R,$$

where Γ_t^R stands for firm's profits in the retail sector and P_t is the consumer price index. The first order condition that determines the optimum level of consumption and savings is given by the following Euler equation that equalises the cost of postponing consumption with its expected marginal benefit,

$$1 = \beta E_t \left(\frac{P_t C_t}{P_{t+1} C_{t+1}} (1 + i_t) \right). \tag{2.4}$$

Optimal intratemporal consumption allocation determines the demand for each variety of consumption good as follows,

$$C_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} C_t. \tag{2.5}$$

2.2 Technology and Labour Market Dynamics

2.2.1 Wholesale Producers

Production of the wholesale good, Y_t^W uses two different constant returns to scale technologies, $Y_t^F(i)$ and $Y_t^I(i)$, such that,

$$Y_t^W(i) = Y_t^F(i) + Y_t^I(i).$$

The first of these technologies, $Y_t^F(i)$ uses formal labour for production whereas, $Y_t^I(i)$ uses workers hired under informal contracts. Formal labour contracts are only offered to the most productive workers, since only in this case it becomes profitable to pay the hiring costs that signing formal contracts involves. These two production functions are presented next,

$$Y_t^F(i) = A_t N_t^F, \tag{2.6}$$

$$Y_t^I(i) = \gamma A_t N_t^I, \tag{2.7}$$

where, $\gamma \leq 1$ and A_t stands for the level of productivity. Hiring costs capture the fact that formal and informal jobs are subject to different regulation costs. Formal jobs usually require that firms pay benefits to workers, which is not usually the case for informal jobs. Following Blanchard and Gali (2006) we assume that hiring costs are increasing on each type of labour market tightness, as follows,

$$G_t^F = B^F A_t \left(X_t^F \right)^{\alpha_F}, \quad G_t^I = B^I A_t \left(X_t^I \right)^{\alpha_I},$$

where $B^F > B^I$. Also, we restrict, $\alpha_F > \alpha_I$. This assumption captures the fact that for formal jobs, given the same level of market tighteness, hiring costs are larger due to regulation. Firms hire $H_t^F(i)$ and $H_t^I(i)$ workers of each type every period. Therefore, the laws of motion of both types of labour are determined by,

$$N_t^F(i) = (1-\delta)\, N_{t-1}^F(i) + H_t^F(i), \quad N_t^I(i) = (1-\delta)\, N_{t-1}^I(i) + H_t^I(i). \tag{2.8}$$

Under these conditions, the firms' problem consists in choosing sequences of $\left\{N_t^I(i)\right\}$, $\left\{H_t^I(i)\right\}$, $\left\{N_t^F(i)\right\}$, $\left\{H_t^F(i)\right\}$ to maximise the following expected discounted profit function,

$$E_t \left[\sum_{j=0}^{\infty} Q_{t+j,t}\Gamma_{t+j}\right],$$

where $Q_{t+j,t} \equiv \beta^j \frac{U_{c,t+j}}{U_{c,t}}$ and

$$\Gamma_t = \frac{P_t^w}{P_t}(A_t N_t^F(i) + \gamma A_t N_t^I(i)) - W_t^I N_t^I(i) - W_t^F N_t^F(i) - G_t^I H_t^I(i) - G_t^F H_t^F(i). \tag{2.9}$$

Subject to equations in (2.9). The corresponding first order conditions are given by:

$$N_t^F(i) : \frac{P_t^w}{P_t} A_t - W_t^F - G_t^F + (1-\delta)\,\beta E_t \left(Q_{t+1,t} G_{t+1}^I\right) = 0, \tag{2.10}$$

$$N_t^I(i) : \gamma \frac{P_t^w}{P_t} A_t - W_t^I - G_t^I + (1-\delta)\,\beta E_t \left(Q_{t+1,t} G_{t+1}^I\right) = 0. \tag{2.11}$$

The intuition of the previous two equations is simple. Optimal demand for each type of labour requires to equalise the value of their marginal productivity to their corresponding marginal costs. In this case, marginal costs are not given only by real wages as in the case of perfectly competitive labour markets, but also by the costs generated by hiring. Also, from the previous problem, it holds that,

$$\frac{P_t^w}{P_t} = MC_t,$$

where

$$\begin{aligned} MC_t &= \frac{W_t^F + G_t^F - (1-\delta)\,\beta E_t \left(Q_{t+1,t} G_{t+1}^F\right)}{A_t} \\ &= \frac{W_t^I + G_t^I - (1-\delta)\,\beta E_t \left(Q_{t+1,t} G_{t+1}^I\right)}{\gamma A_t}. \end{aligned} \tag{2.12}$$

According to this expression, in equilibrium labour moves from one sector to the other (and from or to unemployement) in such a way that marginal costs equalise in each sector.

2.2.2 Wage determination

We asume that wages are set in a Nash bargaining process. Let's denote by λ the bargaining power of workers and by V_t^F, V_t^I, V_t^U the value functions of a representative household that has a marginal member employed in the formal and informal sector, respectively.

$$V_t^F = W_t^F - \chi C_t N_t^{\phi} + \beta E_t \left(Q_{t,t+1} \left[\left(1 - \delta + \delta X_{t+1}^F\right) V_{t+1}^F + \delta X_{t+1}^I V_{t+1}^I + \delta \left(1 - X_{t+1}\right) V_{t+1}^U \right] \right), \tag{2.13}$$

$$V_t^I = W_t^I - \chi C_t N_t^{\phi} + \beta E_t \left(Q_{t,t+1} \left[\left(1 - \delta + \delta X_{t+1}^I\right) V_{t+1}^I + \delta X_{t+1}^F V_{t+1}^F + \delta \left(1 - X_{t+1}\right) V_{t+1}^U \right] \right). \tag{2.14}$$

A worker that signs a formal contract enjoys in period t his wage net of the marginal rate of substitution. Also, he faces the probability δ of loosing his job at the end of period t and a probability $(1 - \delta)$ of maintaining his formal job in $t + 1$ and enjoy V_{t+1}^F. Given that he looses his job, he can enjoy V_{t+1}^F, V_{t+1}^I and V_{t+1}^U with probability X_{t+1}^F, X_{t+1}^I and $(1 - X_{t+1})$, respectively. A similar interpretation applies for the value function of informal workers.

Similarly for the case of unemployed household members, the corresponding value function is determined by,

$$V_t^U = W^u + \beta E_t \left(Q_{t,t+1} \left[X_{t+1}^F V_{t+1}^F + X_{t+1}^I V_{t+1}^I + \left(1 - X_{t+1}\right) V_{t+1}^U \right] \right). \tag{2.15}$$

An unemployed worker receives the current payoff of W^u from home production and in the next period he can become either formally employed, informally employed or stay unemployed with probability X_{t+1}^F, X_{t+1}^I and $(1 - X_{t+1})$, respectively.

From the Nash bargain, we have that the workers' surplus has to be determined by:

$$V_t^F - V_t^U = \lambda G_t^F,$$

$$V_t^I - V_t^U = \lambda G_t^I.$$

Using this condition, we can transform equations (2.13) and (2.14), such that wages in the formal and informal sector are determined,

$$\lambda G_t^F = W_t^F - \left(W^u + \chi C_t N_t^{\phi} \right) + \beta \left(1 - \delta\right) \lambda E_t Q_{t,t+1} \left[\left(1 - X_{t+1}^F\right) G_{t+1}^F - X_{t+1}^I G_{t+1}^I \right], \tag{2.16}$$

$$\lambda G_t^I = W_t^F - \left(W^u + \chi C_t N_t^{\phi} \right) + \beta \left(1 - \delta\right) \lambda E_t Q_{t,t+1} \left[\left(1 - X_{t+1}^I\right) G^I{}_{t+1} - X_{t+1}^F G_{t+1}^F \right]. \tag{2.17}$$

These two conditions together with (2.12) characterise the labour market equilibrium.

2.3 Retail Firms

Each retail firm uses wholesale goods to produce differentiated final consumption goods using a one to one techology. This in turn implies that the marginal cost retailers face is exactly equal to the price of the wholesale good,

$$MC_t^R = \frac{P_t^W}{P_t} = MC_t.$$

As we can see from (2.12), marginal costs depend on real wages from both the formal and the informal labour market. Furthermore, we assume that each retailer sets prices following a staggered pricing mechanism *a la* Calvo. Each firm faces an exogenous probability of changing prices given by $(1-\theta)$. The optimal price that solves the firm's problem is given by

$$\left(\frac{P_t^*(z)}{P_t}\right) = \frac{\mu E_t \left[\sum_{k=0}^{\infty} \theta^k Q_{t+k,t} MC_{t+k} F_{t,t+k}^{\varepsilon} Y_{t+k}\right]}{E_t \left[\sum_{k=0}^{\infty} \theta^k \zeta_{t,t+k} F_{t,t+k}^{\varepsilon-1} Y_{t+k}\right]}, \tag{2.18}$$

where $\mu = \frac{\varepsilon}{\varepsilon-1}$ is the price markup, $Q_{t+k,t}$ is the stochastic discount factor, $P_t^*(z)$ is the optimal price level chosen by the firm, $F_{t,t+k} = \frac{P_{t+k}}{P_t}$ is the cumulative level of inflation and Y_{t+k} is the aggregate level of output.

Since only a fraction $(1-\theta)$ of firms changes prices every period and the remaining fraction keeps its price fixed, the aggregate price level, the price of the final good that minimises the cost of the final goods producers, is given by the following equation:

$$P_t^{1-\varepsilon} = \theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P_t^*(z)\right)^{1-\varepsilon} \tag{2.19}$$

Following Benigno and Woodford (2005), equations (2.18) and (2.19) can be written recursively introducing the auxiliary variables NN_t and DD_t:

$$\theta\left(\Pi_t\right)^{\varepsilon-1} = 1 - (1-\theta)\left(\frac{NN_t}{DD_t}\right)^{1-\epsilon} \tag{2.20}$$

$$DD_t = Y_t\left(C_t\right)^{-1} + \theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\epsilon-1} DD_{t+1}\right] \tag{2.21}$$

$$NN_t = \mu Y_t\left(C_t\right)^{-1} MC_t + \theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\epsilon} NN_{t+1}\right] \tag{2.22}$$

Equation (2.20) comes from the aggregation of individual firms' prices. The ratio NN_t/DD_t represents the optimal relative price $P_t^*(z)/P_t$. Equations (2.20), (2.21) and (2.22) summarise the recursive representation of the non- linear Phillips curve.

2.4 Market Clearing

Aggregating the budget constraint over all households we obtain,

$$C_t = W_t^I N_t^I + W_t^F N_t^F + W^u (1 - N_t) + \Gamma_t^R$$

Since the wholesale sector is in perfect competition, profits are zero for each firm, thus we have that,

$$\frac{P_t^w}{P_t} Y_t^w = W_t^I N_t^I + W_t^F N_t^F + G_t^I H_t^I + G_t^F H_t^F$$

and also since $\Pi_t^r = Y_t - \frac{P_t^w}{P_t} Y_t^w$, we have that,

$$Y_t = C_t + G_t^I H_t^I + G_t^F H_t^F - W^u (1 - N_t) \tag{2.23}$$

and

$$Y_t^w = Y_t \Delta_t$$

where $\Delta_t = \int_0^1 \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} dz$ is a measure of price dispersion.

2.5 Monetary Policy

The central bank conducts monetary policy by targeting the nominal interest rate in the following way:

$$(1 + i_t) = (1 + i) \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y} \tag{2.24}$$

where, $\phi_\pi > 1$ and $\phi_y > 0$ measure the response of the nominal interest rate to expected future inflation and output, respectively. The steady state values are expressed without time subscript.

2.6 The steady state

We can analyse the steady state of the model as the intersection of labour demand with labour supply for each sector. The complete system of equations is shown in appendix B. The labour demand for each sector equalises the real wage with its respective marginal rate of transformation, that is:

$$W^F = A\frac{1}{\mu} - G^F\left(1 - \beta\left(1 - \delta\right)\right) \quad (2.25)$$

$$W^I = \gamma A\frac{1}{\mu} - G^I\left(1 - \beta\left(1 - \delta\right)\right) \quad (2.26)$$

where G^F and G^I are both functions of N^F and N^I.

The labour supply consists on the wage curve for each sector:

$$W^F = \chi C N^{\phi} + W^U + \lambda\left[G^F - \beta\left(1 - \delta\right)\left(\left(1 - X^F\right)G^F - X^I G^I\right)\right] \quad (2.27)$$

$$W^I = \chi C N^{\phi} + W^U + \lambda\left[G^I - \beta\left(1 - \delta\right)\left(\left(1 - X^I\right)G^I - X^F G^F\right)\right] \quad (2.28)$$

where C, N, X^F and X^I are also functions of of N^F and N^I. The intersection of these two sets of equations gives the solution for real wages and labour in each sector.

In figure 2.1 we show graphically the labour market equilibrium in steady state. In the case without labour market frictions, labour demand is given by a horizontal line at A/μ and the wage curve is an upward sloping curve with intercept at W^u when $N < 1$ and a vertical line at the value of full employment. When introducing labour market frictions in a dual market, labour demand in the formal sector is a downward sloping curve that starts from the intercept at A/μ and the wage curve is an upward sloping curve that also starts in W^u, but is steeper than in the case without labour market frictions. The intersection of these two curves defines N^F. For the case of the informal economy, labour demand is a downward curve that starts at $\gamma A/\mu$ and the wage curve is an upward curve that starts at W^u. Both curves for the informal economy are less steep than those of the formal economy, which indicates that labour in the informal economy is more elastic.

Let's analyse for example the effects in steady state of an increase in the parameter of rigidity in the formal sector. In figure 2.2 we show that an increase in B^F generates in the formal sector a downward movement of labour demand curve and an upward shift of the wage setting curve, which reduces formal labour. As unemployment increases, this reduces tightness in the informal sector, moving the labour demand curve upwards and the wage setting curve downwards, increasing employment in the informal sector.

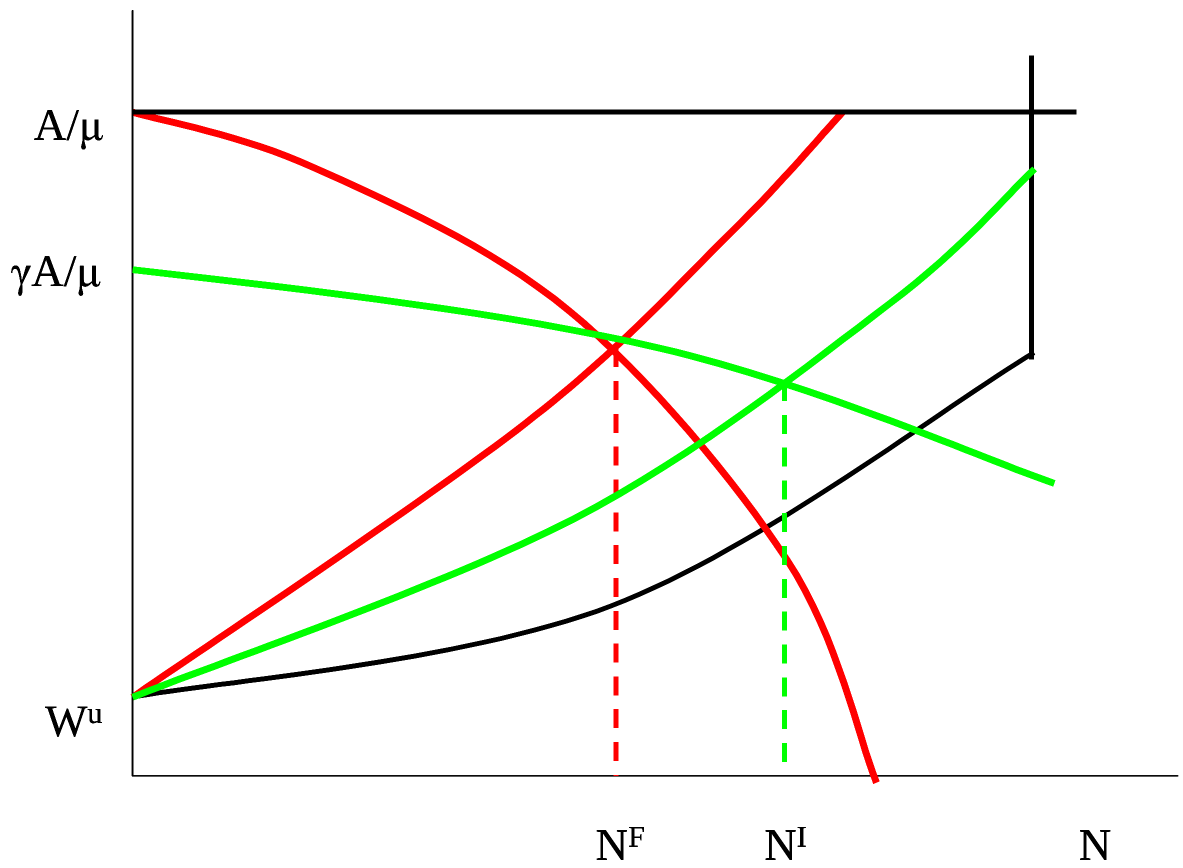

Figure 2.1: Labor martket equilibrium in steady state.

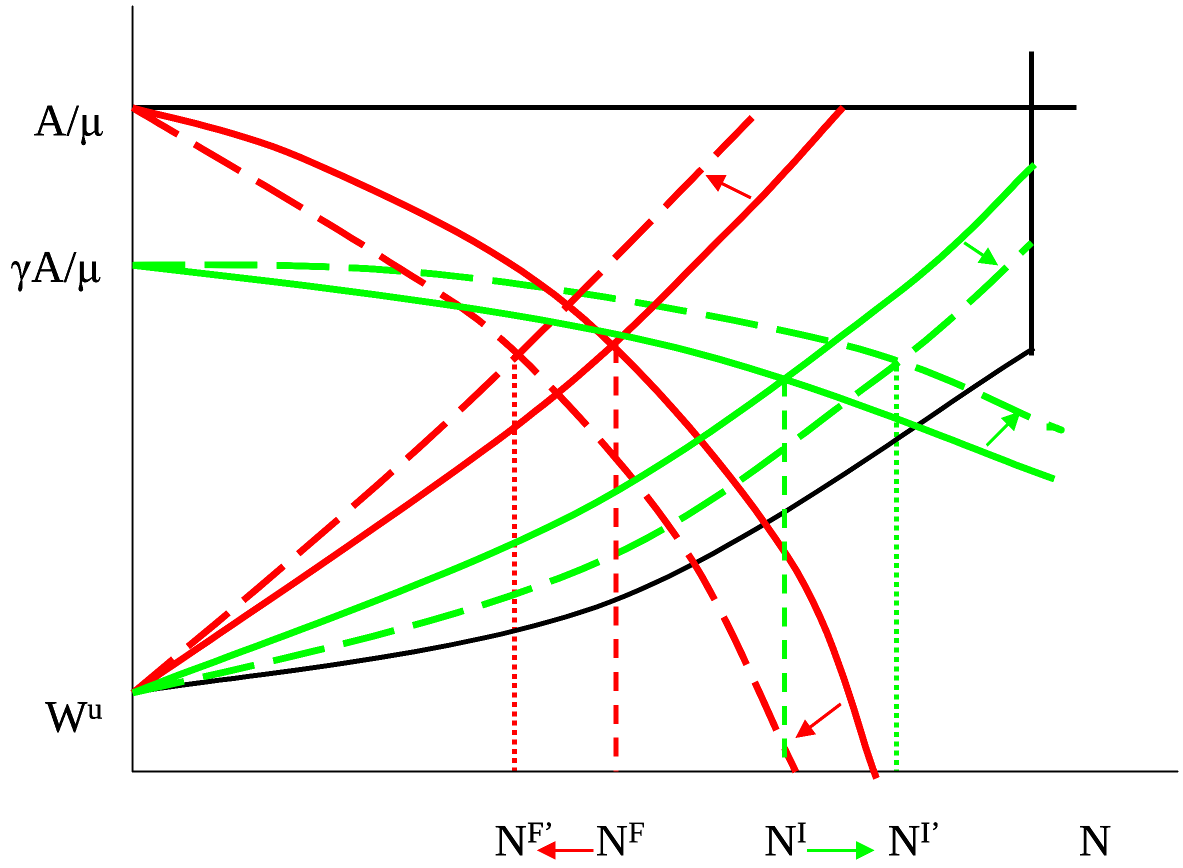

Figure 2.2: Labor market equilibrium in steady state. The effects of an increase in hiring costs of the formal sector (B^F)

3 The dynamics of the model

3.1 The log-linear system of equations

The dynamics of the model are given by the set of equations for 19 endogenous variables $\{c_t, i_t, \pi_t, mc_t, y_t^F, y_t^I, w_t^F, w_t^I, n_t^F, n_t^I, q_t, g_t^F, g_t^I, x_t^F, x_t^I, h_t^F, h_t^I, y_t, n_t\}$ and 2 exogenous variables $\{d_t, a_t\}$.

Aggregate demand is determined from the aggregate resources constraint:

$$y_t = \frac{C}{Y}c_t + \frac{G^F H^F}{Y}\left(g_t^F + h_t^F\right) + \frac{G^I H^I}{Y}\left(g_t^I + h_t^I\right) + \frac{W^u N}{Y}n_t + d_t \tag{3.1}$$

where we have included an exogenous demand shock, d_t,which follows an AR(1) process. This exogenous demand shock can be interpreted as a shock in government expenditures, when including the public sector into the model. In this model aggregate demand equals the sum of consumption, total hiring costs and demand shocks. Consumption is determined by the Euler equation:

$$c_t = E_t c_{t+1} - (i_t - E_t \pi_{t+1}) \tag{3.2}$$

and hiring costs are equal to $g_t^j = a_t + \alpha_j x_t$ for $j = \{F, I\}$ and the measure of workers hired is determined from the evolution of labour in each sector, $n_t^j = (1-\delta)\, n_{t-1}^j + \delta h_t^j$ for $j = \{F, I\}$. The labour market tightness is defined by: $x_t^j = h_t^j + \frac{(1-\delta)N}{1-(1-\delta)N}n_{t-1}$ for $j = \{F, I\}$.

Aggregate supply in this model with nominal rigidities and dual labour market rigidities is equal to tradditional New-Keynesian Phillips curve:

$$\pi_t = \kappa mc_t + E_t \pi_{t+1} \tag{3.3}$$

The informal economy affects inflation through the effects on marginal costs. Since the economy produces using two different types of tecnology, total production is $y_t = \frac{Y^F}{Y}y_t^F + \frac{Y^I}{Y}y_t^I$, where the production of each sector is given by: $y_t^j = a_t + n_t^j$ for $j = \{F, I\}$, where the technology shock (a_t) is also assumed to follow an AR(1) process.

Labour demand in each sector is equal to

$$w_t^j = \Phi^j (a_t + mc_t) + \frac{(1-\Phi^j)}{1-(1-\delta)\beta} \left[g_t^j - (1-\delta)\beta E_t \left(q_{t+1} + g_{t+1}^j \right) \right] \tag{3.4}$$

for $j = \{F, I\}$ and $E_t q_{t+1} = c_t - E_t c_{t+1}$ is the stochastic discount factor. These relative weight in the labour demand of productivity and marginal costs depends on $\Phi^F \equiv \frac{A}{W^F \mu}$ and $\Phi^I \equiv \gamma \frac{A}{W^I \mu}$.

On the other hand, the labour suply of each sector is the wage curve

$$\begin{aligned} w_t^j &= \Omega^j (c_t + \eta n_t) \\ &\quad + \Psi^j \left[G^j g_t^j - (1-\delta)\beta E_t \left(\begin{array}{c} (1-X^j) G^j \left(q_{t+1} + g_{t+1}^j \right) - X^j G^j x_{t+1}^j \\ -X^{\tilde{j}} G^{\tilde{j}} \left(q_{t+1} + g_{t+1}^{\tilde{j}} + x_{t+1}^{\tilde{j}} \right) \end{array} \right) \right] \end{aligned} \tag{3.5}$$

for $j = \{F, I\}$ and $\tilde{j}$ stands for the other sector different from j. The weights are given respectively by $\Omega^j \equiv \chi C N^\eta / W^j$ and $\Psi^j \equiv \frac{\left(1-\Omega^j - \frac{W^U}{W^j}\right)}{1-(1-\delta)\beta[(1-X^j)G^j - X^{\tilde{j}} G^{\tilde{j}}]}$. Total labour equals: $n_t = \frac{N^I}{N} n_t^I + \frac{N^F}{N} n_t^F$.

Finally, monetary policy is determined under a standard Taylor rule:

$$i_t = \phi_\pi \pi_t + \phi_y y_t \tag{3.6}$$

3.2 Benchmark Parameters

We calibrate the standard parameters of the model similar to the traditional parameters used in the New-Neynesian literature:

Table 1: Standard Parameters of the Model

$\beta = 0.99$	$\phi_\pi = 1.5$	$\phi_y = 0.5$
$\kappa = 0.5$	$\eta = 2$	$\mu = 1.2$
$\rho_A = 0.9$	$\sigma_A = 1$	$\chi = 0.2$
$\rho_D = 0.5$	$\sigma_D = 1$	$\psi = 0.75$

We consider the reservation wage as a proportion of the value added of the informal sector in steady state, that is: $W^u = \psi \left(\frac{\gamma A}{\mu} \right)$ for $\psi = 0.75$. For the tecnology parameters we take $A = 1$ and $\gamma = 0.95$. For the hiring costs functions we use the following: $\alpha_F = 1.5 > \alpha_I = 0.75$ and

$B_F = 2 > B_I = 0.5$ to characterise the flexibility of the informal labour market in comparison with the formal one. The separation rate $\delta = 0.12$ is calibrated as in Blanchard and Gali (2006). The workers' bargaining power is calibrated as $\lambda = 0.5$.

Given this calibration, we show in Table 2 the implied steady state of the model for the case when no labour market rigidities are present ($B^F = B^I = 0$), the case with informal economy and the case when informality is not present ($\gamma = 0$).

Table 2: Implied steady state of the model

	Without labour market rigidities	With informality	Without Informality
Y	1	0.861	0.825
N	1	0.880	0.801
N^F/N	1	0.507	0.801
N^I/N	0	0.373	0.000

In the case where labour market frictions are absent, labour is at full employment and output is normalised at 1, labour is hired completely in the formal sector because of the lower productivity of the informal sector. When introducing hiring costs in both sectors, informal production arises. However, it is important to note that total production is higher in the economy with informality than in the case without it, because the informal sector becomes an optimal second best alternative to larger hiring costs in the formal sector. Moreover, total employment is higher in the economy with an informal sector.

4 The buffer effect of informal labour markets

The empirical evidence reported in the introduction shows that informal labour markets act as a buffer stock of labour, increasing the flexibility of the labour market and affecting the transmission mechanism of shocks to the economy. The micro-founded model developed in this paper delivers this result and shows how the presence of an informal economy affects the transmission mechanism of monetary policy.

As figure 4.1 shows, inflation response to a demand shock is almost 42 percent larger in an economy where all labour contracts are formal than in an economy where informal employment exists. Consistently, output increases more in this latter case, since informal employment helps to reduce the pressure on wages in formal labour markets, generating a larger incentive for firms to increase production.

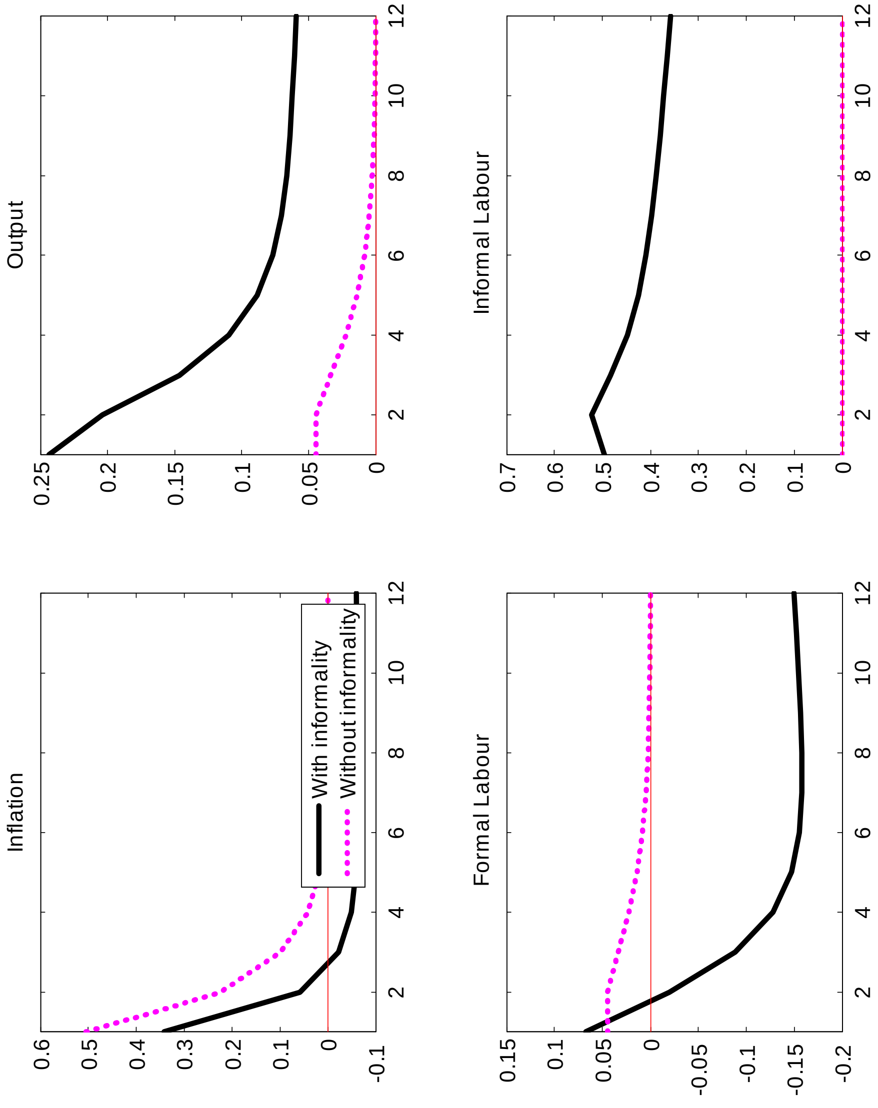

Figure 4.1: Impulse responses to a demand shock.

When a worker receives an offer to sign a formal labour contract, he has two options: either to accept the offer and receive the corresponding wage rate or wait for another one hoping to obtain a larger wage rate. When in the economy there are informal labour markets, the cost of waiting is larger since the probability of receiving a new offer of a formal labour contract is much lower in this case. This possibility of waiting for a longer period induces workers to accept lower wages. Hence, firms in economies with informal labour markets have more flexibility to expand output, thus demand shocks generate lower inflation and larger output expansions. The impulse response functions depicted on the two panels at the bottom of figure 4.1 show this buffer effect in terms of employment flows. As these pictures shows, informal employment increases in response to demand shocks more than the increase of employment in the formal labour market sector.

The buffer effect also works in the case of productivity shocks. In this case, informal labour markets amplify the effects of these shocks on inflation and output. As figure 4.2 shows, output and inflation responses to productivity shocks are larger in economies where informal labour markets exist. Informal labour markets in this case also allow firms more flexibility when hiring workers. Although, at the margin the improvement in productivity is larger in formal labour contracts, firms still have incentives to hire workers under informal labour contracts since this type of contracts are relatively cheaper than formal ones. Similarly to the case of demand shocks, the buffer effect generates flows of employment from the formal to the informal sector in response to productivity shocks.

There are some key parameters that determine the magnitude of the buffer effect; particularly important are those that define the hiring cost function of both formal and informal labour markets. As figure 4.2 shows, the buffer effect is larger when for the same level of labour market tightness; hiring costs in the formal sector are larger than in the informal sector. In this case the incentives that firms face to substitute formal for informal labour are larger since marginal costs with formal labour increase much more than with informal labour.

The key implication for inflation dynamics that informal labour markets generate is that the Phillips curve depends, not only on the level of aggregate unemployment, but also on the flows of unemployment in the formal and informal labour markets. Furthermore, this result implies that in economies with large informal labour markets, the correlation between inflation and the output gap conditional on demand shocks is lower, thus the interest rate channel of monetary policy is relatively weaker.

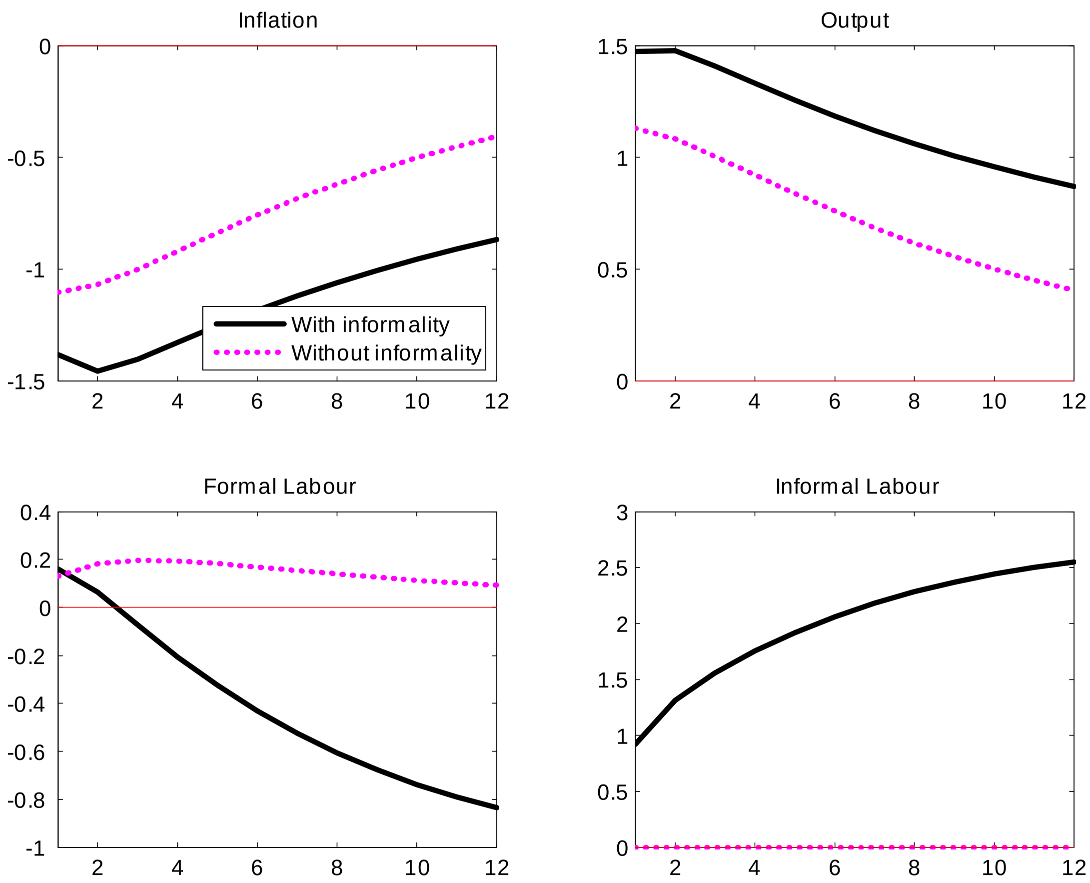

Figure 4.2: Impulse responses to a productivity shock.

5 Concluding Remarks

Informal labour markets are widespread in emerging economies. This paper shows that this feature of labour markets has profound impact on the dynamics of inflation and the transmission mechanism of monetary policy. A large pool of informal workers is a buffer stock of labour that allows firms to expand output in a more flexible manner without putting pressure on wages. In particular, firms at the margin can substitute formal jobs with informal ones and expand output without raising their marginal costs. In this case, inflation depends not only on the level of unemployment but also on the flows of unemployment from formal to informal labour markets. Consequently, inflation also becomes less responsive to demand shocks.

Furthermore, the buffer stock effect on labour markets that this model generates is consistent with empirical evidence that shows that formal employment is less procyclical than informal employment. This result has important implications for the costs of stabilisation policies. In particular, since inflation is less responsive to demand shocks, larger contractions on output would be required to stabilise inflation. Therefore, in this type of economies it becomes even more important to act preemptively to avoid deviations of inflation expectations.

The model presented in this paper is highly stylised, mainly to keep tractability. However, it can be extended in many directions; for instance, alternative frictions to generate informal labour markets in equilibrium can be considered besides hiring costs to discuss the interaction between monetary policy and labour market policies . Also, this framework can used to analyse optimal monetary policy, following the work of Thomas (2008).

References

[1] Alexopoulus, M. (2004) "Unemployment and the business cycle", Journal of Monetary Economics, 51 (2), 257-298.

[2] Blanchard, Oliver and Jordi Gali (2006), "A new Keynesian model with Unemployment", mimeo.

[3] Benigno, P. y M. Woodford. (2005). "Inflation Stabilization and Welfare: The Case of a Distorted Steady State." *Journal of the European Economic Association* 3(6), 1-52.

[4] Boeri T, and P. Garibaldi (2006), "Shadow Sorting", CEPR discussion paper DP5487.

[5] Bosch, Mariano (2004), "Start up costs, Informality and Policy Complentarities", mimeo LSE.

[6] Bosch, Mariano (2006), "Job Creation and Job Destruction in the Presence of Informal Labour Markets", mimeo LSE.

[7] Bovi, Maurizio (2007), "Shadow employment and the labour productivity dynamics", mimeo.

[8] Bowler Mary and Morisi Teresa. 2006. "Understanding the employment measures from the CPS and CES survey". Monthly Labor Review, Bureau of Labor Statistics (U.S. Department of Labor), February.

[9] Carrillo, Maria Rosa and Maurizio Pugno. 2004. "Underground Economy and Underdevelopment", Economic Systems No. 28 pp. 257-279.

[10] Clarida, Richard, Jordi Gali and Mark Gertler (1999), "The Science of Monetary Policy: a New Keynesian Perspective, "Journal of Economic Literature, 37(4).

[11] Djankov, S., R. La Porta, F. Lopez-de-Silanes and A. Shleifer (2002), "The Regulation of Entry", Quarterly Journal of Economics, Vol. 117, N° 1, 1-37.

[12] Fugazza, M and J. Jaques (2002), "Labour Market institutions, Taxations and the Underground Economy", Journal of Public Economics. Vol. 88, N° 1-2, 395-418.

[13] Gertler, Mark and Antonella Trigari (2006), "Unemployment Fluctuations with Staggered Nash Wage Bargaining", mimeo.

[14] Kolm, A.S. and B. Larsen (2004), "Does tax evasion affect unemployment and educational choice?". Working paper 4-2004. Department of Economics, Copenhagen Business School.

[15] Krause, Michael and Thomas Lubik (2005), "The (Ir)relevance of Real Wage Rigidity in the New Keynesian Model with Search Frictions", Journal of Monetary Economics. In Press.

[16] Pissarides, Christopher (2000), Equilibrium Unemployment Theory, MIT Press.

[17] Ravenna, Federico and Carl Walsh (2007), "Vacancies, Unemployment and the Phillips Curve", mimeo

[18] Schneider, F (2007), "The Size of the Shadow Economies of 145 Countries all over the World: First Results over the Period 1999 to 2003", Journal of Population Economics, 20 (3), 495 - 526.

[19] Thomas, Carlos (2008), "Search and matching frictions and optimal monetary policy", Journal of Monetary Economics.

[20] Trigari, A. (2004), "Equilibrium unemployment, job flows and inflation dynamics", European Central Bank, Working Paper Series N° 304.

[21] Walsh, Carl (2003), "Labor market search and monetary shocks", in Elements of Dynamic Macroeconomic Analysis, S. Altuĝ, J. Chadha, and C. Nolan (eds.), Cambridge: Cambridge university Press, 451-486.

[22] Walsh, Carl (2005), "Labor Market search, Sticky Prices, and Interest Rate Rules, Review of Economic Dynamics, 8, 829-849.

[23] Woodford, Michael. Interest and Prices: Foundations of a Theory of Monetary Policy, Princeton University Press, 2003.

A The non-linear system of equations

The dynamic equilibrium of this economy is given by the following set of 19 equations with 19 endogenous variables:

A.1 Aggregate demand

$$1 = \beta E_t \left(\frac{C_t}{C_{t+1}} \frac{(1+i_t)}{1+\pi_{t+1}} \right) \tag{A.1}$$

$$(1+i_t) = (1+i_{t-1})^{\phi_r} \left[(1+i) \left(\frac{E_t \Pi_{t+1}}{\overline{\Pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{\overline{Y}} \right)^{\phi_y} \right]^{1-\phi_r} \tag{A.2}$$

A.2 Aggregate Supply

Price setting in the retail sector gives the Phillips curve:

$$\theta \left(\Pi_t\right)^{\varepsilon-1} = 1 - (1-\theta) \left(\frac{NN_t}{DD_t} \right)^{1-\epsilon} \tag{A.3}$$

$$DD_t = Y_t \left(C_t\right)^{-1} + \theta \beta E_t \left[\left(\Pi_{t+1}\right)^{\epsilon-1} DD_{t+1} \right]$$

$$NN_t = \mu Y_t \left(C_t\right)^{-1} MC_t + \theta \beta E_t \left[\left(\Pi_{t+1}\right)^{\epsilon} NN_{t+1} \right]$$

The production function, which determines marginal costs:

$$Y_t^W = Y^F + Y^I \tag{A.4}$$

$$Y_t^F = A_t N_t^F \tag{A.5}$$

$$Y_t^I = \gamma A_t N_t^I \tag{A.6}$$

A.3 Labour Market

Labour demands:

$$W_t^F = A_t MC_t - \left[G_t^F - (1-\delta) E_t Q_{t,t+1} G_{t+1}^F \right] \tag{A.7}$$

$$W_t^I = \gamma A_t MC_t - \left[G_t^I - (1-\delta) E_t Q_{t,t+1} G_{t+1}^I \right] \tag{A.8}$$

The wage setting curves are:

$$W_t^F = \frac{V_{N,t}}{U_{C,t}} + W^U + \lambda \left[G_t^F - (1-\delta) E_t Q_{t,t+1} \left(G_{t+1}^F \left(1 - X_{t+1}^F\right) - G_{t+1}^I X_{t+1}^I \right) \right] \quad (A.9)$$

$$W_t^I = \frac{V_{N,t}}{U_{C,t}} + W^U + \lambda \left[G_t^I - (1-\delta) E_t Q_{t,t+1} \left(G_{t+1}^I \left(1 - X_{t+1}^I\right) - G_{t+1}^F X_{t+1}^F \right) \right] \quad (A.10)$$

where:

$$Q_{t,t+1} = \beta \frac{C_t}{C_{t+1}} \quad (A.11)$$

Hiring costs are given by:

$$G_t^F = B^F A_t \left(X_t^F\right)^{\alpha_F} \quad (A.12)$$

$$G_t^I = B^I A_t \left(X_t^F\right)^{\alpha_I} \quad (A.13)$$

Labour market tighness evolves as:

$$X_t^F = \frac{H_t^F}{1 - (1-\delta)\left(N_{t-1}\right)} \quad (A.14)$$

$$X_t^I = \frac{H_t^I}{1 - (1-\delta)\left(N_{t-1}\right)} \quad (A.15)$$

The evolution of labour in the formal and informal sector:

$$N_t^F = (1-\delta) N_{t-1}^F + H_t^F \quad (A.16)$$

$$N_t^I = (1-\delta) N_{t-1}^I + H_t^I \quad (A.17)$$

A.4 Aggregation

The aggregate resource constraint:

$$Y_t = C_t + G_t^F H_t^F + G_t^I H_t^I - W^u (1 - N_t) \quad (A.18)$$

Aggregate production for wholesale goods:

$$Y_t^W = Y_t \Delta_t$$

where:

$$\Delta_t = \int_0^1 \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} dz$$

Also, we have total labour as:

$$N_t = N_t^I + N_t^F \tag{A.19}$$

B Solving the steady-state

When solving the steady state, we have 13 equations for the same number of variables $\left\{Y, N, W^F, W^I, N^F, N^I, C, Y^F, Y^I, X^F, X^I, G^F, G^I\right\}$:

Aggregate conditions:

$$\begin{aligned} Y &= Y^F + Y^I && \text{(B.1)} \\ N &= N^F + N^I && \text{(B.2)} \end{aligned}$$

Consider each labour demand:

$$\begin{aligned} W^F &= A\frac{1}{\mu} - G^F\left(1 - \beta\left(1-\delta\right)\right) && \text{(B.3)} \\ W^I &= \gamma A\frac{1}{\mu} - G^I\left(1 - \beta\left(1-\delta\right)\right) && \text{(B.4)} \end{aligned}$$

Labour supply:

$$\begin{aligned} W^F &= \chi C N^{\phi} + W^U + \lambda\left[G^F - \beta\left(1-\delta\right)\left(\left(1-X^F\right)G^F - X^I G^I\right)\right] && \text{(B.5)} \\ W^I &= \chi C N^{\phi} + W^U + \lambda\left[G^I - \beta\left(1-\delta\right)\left(\left(1-X^I\right)G^I - X^F G^F\right)\right] && \text{(B.6)} \end{aligned}$$

The aggregate budget constraint:

$$Y = C + \delta G^F N^F + \delta G^I N^I - W^u\left(1-N\right) \tag{B.7}$$

The production function:

$$\begin{aligned} Y^F &= A N^F && \text{(B.8)} \\ Y^I &= \gamma A N^I && \text{(B.9)} \end{aligned}$$

Labour tightness:

$$\begin{aligned} X^F &= \frac{\delta N^F}{1-\left(1-\delta\right)N} && \text{(B.10)} \\ X^I &= \frac{\delta N^I}{1-\left(1-\delta\right)N} && \text{(B.11)} \end{aligned}$$

Hiring costs:

$$G^F = B^I A(X)^{\alpha_F} \tag{B.12}$$

$$G^I = B^F A(X)^{\alpha_I} \tag{B.13}$$

We can replace the aggregate production function and labour equation (equations B.1 and B.2), the aggregate budget constraint (equation B.7), the production function for each sector (equations B.8 and B.9), the definition of labour tightness (equations B.10 and B.11) and the hiring costs functions (equations B.12 and B.13) in the labour demand and supply curve equations, to obtain a system of 4 equations for the real wage and labour in each sector.