BANCO CENTRAL DE RESERVA DEL PERÚ

Política óptima de estabilización del tipo de cambio en una economía dolarizada con metas de inflación

Nicoletta Batini*
Paul Levine**
Joseph Pearlman***

* Fondo Monetario Internacional
** Universidad de Survey
*** Universidad Metropolitana de Londres

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Optimal Exchange Rate Stabilization in a Dollarized Economy with Inflation Targets*

Nicoletta Batini
International Monetary Fund

Paul Levine
University of Surrey

Joseph Pearlman
London Metropolitan University

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Abstract

We build a small open-economy model with partial dollarization–households hold wealth in domestic currency and a foreign currency as in Felices and Tuesta (2006). The degree of dollarization is endogenous to the extent of exchange rate stabilization by the central bank. We identify the optimal monetary policy response under commitment and discretion and assess the optimal degree of exchange rate stabilization in this set up, drawing policy implications for countries that target inflation in economies of this kind.

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Contents

1 Introduction

Dollarization is common in many emerging market economies.[1] Under dollarization, household, firms and governments make extensive use of a second currency–dollars–to either pay for transactions, borrow, lend or store their wealth. In addition, dollars may serve as a reference value for contracting prices, wages, and financial assets. One question that has received considerable attention in recent years is what does dollarization imply for monetary policy? In particular, is it possible to achieve an explicit inflation target when an economy is dollarized? Among highly-dollarized economies, so far only Peru pursues a numerical objective for inflation through an independent monetary policy. However, many dollarized economies are thinking to adopt inflation targets, hence answers to these questions have great practical relevance.

This paper sheds light on these issues by evaluating the optimal monetary policy rule for a central bank that targets inflation in an economy that uses both domestic currency and dollars as a medium of exchange, a phenomenon also known as "currency substitution" or "transaction dollarization". To this end it analyzes welfare implications under different rules using a two-country DSGE model as in Felices and Tuesta (2006), where households hold two currencies for transaction purposes, and where the amount of dollars held relative to the domestic currency depends on the extent by which the central bank smooths the exchange rate between such two currencies. The paper has six sections. Section 2 provides a brief survey of the related literature. Section 3 sets out the model that we use. Section 4 provides analytical results on the stability and determinacy of various simple interest rate rules. Section 5 provides numerical results for optimal commitment and discretionary policy and for the optimized simple rules analyzed in section 4. Section 6 concludes by summarizing the main findings, drawing out with some important policy implications for dollarized economies aiming to implement inflation targeting, and suggesting avenues for future research.

[1] See Reinhart *et al.* (2003), and Batini and Laxton (2005).

2 Related Literature

In the literature the term "dollarization" has often been used interchangeably to indicate that dollars (or a foreign currency more generally) serve as a unit of account ("real dollarization" or "price dollarization"), as a store of value ("financial dollarization"), or as a medium of exchange ("transaction dollarization"). In this paper we focus on the last kind of dollarization, also known as "currency substitution". In this case dollars are accepted as a mean of payment alongside the domestic currency. This type of dollarization is typical in economies with high levels of inflation, where there is a great opportunity cost of holding domestic currency. In these economies dollars can remain the preferred mean of payment–particularly for real estate, cars and other durable goods–even when inflation has been stabilized, due to hysteresis, and to the fact that once dollars become the most used currency, they also develop into the most convenient currency to carry around for transaction purposes–in turn promoting the use of dollars for all payments.

The conventional wisdom is that a central bank can conduct an independent monetary policy (ether through money or inflation targeting) whenever dollarization is either "real" or "financial" or both–although in both cases, monetary transmission is different than in the case of no dollarization, in the sense that changes in the exchange rate have a stronger impact either on inflation expectations or on real activity or on both. This has to be taken into account when setting monetary conditions in response to shocks, although it does not require intrinsic changes to the operational strategy for achieving price stability used in non-dollarized economies (see Ize, 2005; Reinhart et al, 2003; and Armas and Grippa, 2006). In this sense, the question whether a central bank should pay more attention to exchange rates in a dollarized economy, is nothing but a variation to the familiar question of whether central banks pursuing (general) price stability should respond or not to asset prices-of which the exchange rate is one.

It is considerably more difficult to target inflation successfully with transaction dollarization than with other forms of dollarization. If dollars are the only accepted means of payment, agents earn in dollars and spend in dollars. In this case, the relevant interest rate for decisions on intertemporal consumption (and thus aggregate demand) becomes the interest rates paid on dollar saving. Although monetary policy can still determine the interest rate on saving in domestic currency-because it remains the monopoly supplier of base money in domestic currency-it cannot affect dollar interest rates. These largely depend on the existing stock of dollars in the economy-that, under no capital controls, are a function of the country's overall net foreign asset position. Additionally, in this scenario, inflation expectations are disconnected from changes in domestic monetary conditions, depending rather on expectations of exogenously-driven changes in dollar liquidity. Likewise-although monetary policy can still affect the exchange rate by opening interest rate differentials between domestic and dollar denominated assets-changes in the exchange rate no longer have material effects on domestic inflation since all nominal variables are already expressed in dollars. It follows that the only monetary policy regime compatible in the short run with high or full transaction dollarization is fixed exchange rates.

There is now a body of empirical literature on transaction dollarization, analyzing its causes and its consequences for policy management. Key findings are that transaction dollarization contributes to exchange rate volatility and it undermines the basis for targeting domestic nominal anchors because it pushes monetary policymakers to intervene more in the foreign exchange market (see for example Artis and Gazioglu,1986; Calvo and Vegh, 1992; Savastano, 1992; and Lubo and Meleck, 2001, among others). From a theoretical point of view, the literature on dollarization has tended to look at dollarization as a policy choice, trying to answer the question whether fully dollarizing brings any benefits. One common finding is that dollarization is a bad idea, unless the central bank lacks credibility. For example, Schmidt-Grohe and Uribe (2000) compare the welfare costs of business cycles in a dollarized economy to those arising in economies with different monetary arrangements, and find that dollarization is the least successful of the monetary policy rules considered. Likewise, Chang and Velasco (2003) develop a simple model to show that, under full credibility, dollarization implies the loss of independent monetary policy and of seigniorage-which as Chang (2000) shows can be quite high for large Latin American countries-although results are ambiguous if credibility is absent Simulations of DSGE models of dollarized economy by Duncan (2003) indicate that full dollarization typically adds to real volatility-especially output and investment-and the volatility of the fiscal deficit, raising country risk. Along these lines, Sims (2001) finds that dollarization is likely to raise the interest costs of public borrowing, it may create incentives to greater fiscal expenditure and has ambiguous implications for the stability of the financial system, in part because it reduces the range of assets available to the private sector in trading risk. Importantly, dollarization strips the converting nation's central bank of its capacity to provide emergency liquidity in the event of a domestic banking crisis (Chang, 2000).

In practice, however, in many countries transaction dollarization (and often the other forms of dollarization as well) is an uncomfortable reality rather than a policy choice. What is the optimal monetary policy in this case? This paper addresses this question by examining whether – once the central bank has opted for explicit inflation targets – for an economy which uses multiple currencies as media of exchange, it is welfare-superior to use rules that, in addition to responding to expected inflation, smooth the exchange rate as is done in countries with currency substitution and inflation targets like Peru.

3 The Model

The model is fairly standard apart from preferences for real money balances and follows Felices and Tuesta (2006) closely in this respect. There are two asymmetric unequally-sized blocs with the different household preferences and technologies. The single small open economy then emerges as the limit when the relative size of the larger bloc tends to infinity. There are two departures from the standard open-economy model that lead to interesting results. First money enters utility in a non-separable way. Second, in the developing bloc there is a dollarized component in utility from money holdings. There are complete asset markets. The consumption index in each bloc is of Dixit-Stiglitz nested CES form with domestic and foreign components consisting of a basket of differentiated goods produced in each bloc. Goods producers and household suppliers of labor have monopolistic power. Producers set prices in their own currency and these prices are sticky.

3.1 Households

Normalizing the total population to be unity, there are ν households in the 'home' bloc and $(1-\nu)$ households in the 'foreign' bloc. A representative household h in the home bloc maximizes

$$E_t \sum_{t=0}^{\infty} \beta^t U\left(C_t(h), H_t, \frac{M_{H,t}(h)}{P_t}, \frac{M_{F,t}(h)S_t}{P_t}, N_t(h), \varepsilon_{C,t}, \varepsilon_{M_H,t}, \varepsilon_{M_F,t}, \varepsilon_{N,t}\right) \qquad (1)$$

where E_t is the expectations operator indicating expectations formed at time t, β is the household's discount factor, $C_t(h)$ is a Dixit-Stiglitz index of consumption defined below in (4), $H_t = hC_{t-1}$ is external habit, $M_{H,t}(h)$ and $M_{F,t}(h)$ are end-of-period nominal domestic and foreign currency balances respectively, P_t is a Dixit-Stiglitz price index defined in (13) below, S_t is the nominal exchange rate and $N_t(h)$ are hours worked. $\varepsilon_{C,t}$ is a preference shock to the marginal utility of consumption and $\varepsilon_{M_H,t}$, $\varepsilon_{M_F,t}$ and $\varepsilon_{N,t}$ are shocks to demand for domestic currency, demand for foreign and labour supply respectively.

An analogous symmetric intertemporal utility is defined for the 'foreign' representative household and the corresponding variables (such as consumption) are denoted by $C_t^*(h)$, etc.

The representative household r must obey a budget constraint:

$$\begin{aligned} & P_t C_t(h) + E_t(Q_{t,t+1} D_{t+1}(h)) + M_{H,t}(h) + M_{F,t}(h) S_t \\ = \; & W_t(h) N_t(h) + D_t(h) + M_{H,t-1}(h) + M_{F,t-1}(h) S_t + \Gamma_t(h) - T_t \end{aligned} \tag{2}$$

$D_{t+1}(h)$ is a random variable denoting the payoff of the portfolio purchased at time t and $Q_{t,t+1}$ is the period-t price of an asset that pays one unit of domestic currency in a particular state of period $t+1$ divided by the probability of an occurrence of that state given information available in period t. $W_t(h)$ is the wage rate, T_t are flat rate taxes and $\Gamma_t(h)$ are dividends from ownership of firms.

Assume the existence of nominal one-period riskless bonds denominated in domestic currency with nominal interest rate R_t over the interval $[t, t+1]$. Then arbitrage considerations imply that $E_t Q_{t,t+1} = \frac{1}{1+I_t}$. In addition, if we assume that households' labour supply is differentiated with elasticity of supply η, then (as we shall see below) the demand for each consumer's labor supplied by ν_H identical households is given by

$$N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\eta} N_t \tag{3}$$

where $W_t = \left[\frac{1}{\nu}\sum_{r=1}^{\nu} W_t(h)^{1-\eta}\right]^{\frac{1}{1-\eta}}$ and $N_t = \left[\left(\frac{1}{(1-\nu)}\right)\sum_{r=1}^{\nu} N_t(h)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$ are the average wage index and average employment respectively.

Let the number of differentiated goods produced in the home and foreign blocs be n and $(1-n)$ respectively, again normalizing the total number of goods in the world at unity. We also assume that the the ratio of households to firms are the same in each bloc.

It follows that n and $(1-n)$ (or ν and $(1-\nu)$) are measures of size. The per capita consumption index in the home bloc is given by

$$C_t(h) = \left[\mathrm{w}_H^{\frac{1}{\mu}} C_{H,t}(h)^{\frac{\mu-1}{\mu}} + (1-\mathrm{w}_H)^{\frac{1}{\mu}} C_{F,t}(h)^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}} \tag{4}$$

where μ is the elasticity of substitution between home and foreign goods,

$$C_{H,t}(h) = \left[\left(\frac{1}{n}\right)^{\frac{1}{\zeta}} \sum_{f=1}^{n} C_{H,t}(f,h)^{(\zeta-1)/\zeta} \right]^{\zeta/(\zeta-1)} \tag{5}$$

$$C_{F,t}(h) = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\zeta}} \left(\sum_{f=1}^{1-n} C_{F,t}(f,h)^{(\zeta-1)/\zeta} \right) \right]^{\zeta/(\zeta-1)} \tag{6}$$

where $C_{H,t}(f,h)$ and $C_{F,t}(f,h)$ denote the home consumption of household h of variety f produced in blocs H and F respectively, ζ is the elasticity of substitution between varieties in each bloc (note that we impose equality between blocs for this elasticity). Analogous expressions hold for the foreign bloc. Weights in the consumption baskets in the two blocs are defined by

$$\mathrm{w}_H = 1-(1-n)(1-\omega_H) \tag{7}$$

$$\mathrm{w}_F = 1-n(1-\omega_F) \tag{8}$$

In (7) and (8), ω_H, $\omega_F \in [0,1]$ are a parameters that captures the degree of 'bias' in the two blocs. If $\omega_H = \omega_F = 1$ we have autarky, while $\omega_H = \omega_F = 0$ gives us the case of perfect integration. As $\mu \to 1$ we approach a Cobb-Douglas utility function $C_t(h) = \mathrm{w}_H^{-\mathrm{w}_H}(1-\mathrm{w}_H)^{-(1-\mathrm{w}_H)} C_{H,t}(h)^{\mathrm{w}_H} C_{F,t}(h)^{1-\mathrm{w}_H}$ as in Clarida *et al.* (2002).

In the limit as the home country becomes small $n \to 0$ and $\nu \to 0$. Hence $\mathrm{w}_H \to \omega_H$ and $\mathrm{w}_F \to 1$. Thus the foreign bloc becomes closed but as long as there is a degree of home bias and $\omega_H > 0$, the home bloc continues to consume domestically produced consumption goods.

If $P_{H,t}(f)$, $P_{F,t}(f)$ are the prices in domestic currency of the good produced by firm f in the relevant bloc, then the optimal intra-temporal decisions are given by standard results:

$$C_{H,t}(r,f) = \left(\frac{P_{H,t}(f)}{P_{H,t}}\right)^{-\zeta} C_{H,t}(h)\,; \;\; C_{F,t}(r,f) = \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\zeta} C_{F,t}(h) \tag{9}$$

$$C_{H,t}(h) \;=\; \mathrm{w}_H \left(\frac{P_{H,t}}{P_t}\right)^{-\mu} C_t(h)\,;\;\; C_{F,t}(h) = (1-\mathrm{w}_H)\left(\frac{P_{F,t}}{P_t}\right)^{-\mu} C_t(h) \tag{10}$$

where aggregate price indices for domestic and foreign consumption bundles are given by

$$P_{H,t} \;=\; \left[\frac{1}{n}\sum_{f=1}^{n} P_{H,t}(f)^{1-\zeta}\right]^{\frac{1}{1-\zeta}} \tag{11}$$

$$P_{F,t} \;=\; \left[\frac{1}{1-n}\sum_{f=1}^{1-n} P_{F,t}(f)^{1-\zeta}\right]^{\frac{1}{1-\zeta}} \tag{12}$$

and the domestic consumer price index P_t given by

$$P_t = \left[\mathrm{w}_H (P_{H,t})^{1-\mu} + (1-\mathrm{w}_H)(P_{F,t})^{1-\mu}\right]^{\frac{1}{1-\mu}} \tag{13}$$

Similarly for the foreign bloc we have

$$P_t^* = \left[\mathrm{w}_F (P_{F,t}^*)^{1-\mu^*} + (1-\mathrm{w}_F)(P_{H,t}^*)^{1-\mu^*}\right]^{\frac{1}{1-\mu^*}} \tag{14}$$

Let S_t be the nominal exchange rate. The law of one price applies to differentiated goods so that $\frac{S_t P_{F,t}^*}{P_{F,t}} = \frac{S_t P_{H,t}^*}{P_{H,t}}$. Then it follows that the real exchange rate $RER_t = \frac{S_t P_t^*}{P_t}$ and the terms of trade, defined as the domestic currency relative price of imports to exports $\mathcal{T}_t = \frac{P_{F,t}}{P_{H,t}}$, are related by the relationship

$$RER_t \equiv \frac{S_t P_t^*}{P_t} = \frac{\left[\mathrm{w}_F + (1-\mathrm{w}_F)\mathcal{T}_t^{\mu^*-1}\right]^{\frac{1}{1-\mu^*}}}{\left[1-\mathrm{w}_H + \mathrm{w}_H \mathcal{T}_t^{\mu-1}\right]^{\frac{1}{1-\mu}}} \tag{15}$$

Thus if $\mu = \mu^*$, then $RER_t = 1$ and the law of one price applies to the aggregate price indices iff $\mathrm{w}_F = 1 - \mathrm{w}_H$. The latter condition holds if there is no home bias. If there is home bias, the real exchange rate appreciates (E_t falls) as the terms of trade deteriorates.

We assume flexible wages. Then maximizing (1) subject to (2) and (3), treating habit as exogenous, and imposing symmetry on households (so that $C_t(h) = C_t$, etc) yields standard results:

$$Q_{t,t+1} \;=\; \beta \frac{U_{C,t+1}}{U_{C,t}} \frac{P_t}{P_{t+1}} \tag{16}$$

$$U_{M_H,t} \;=\; U_{C,t}\left[\frac{I_t}{1+I_t}\right] \tag{17}$$

$$U_{M_F,t} \;=\; U_{C,t}\left[\frac{I_t^*}{1+I_t^*}\right] \tag{18}$$

$$\frac{W_t}{P_t} \;=\; -\frac{\eta}{(\eta-1)}\frac{U_{N,t}}{U_{C,t}} \tag{19}$$

where $U_{C,t}$, $U_{M_H,t}$, $U_{M_F,t}$ and $-U_{N,t}$ are the marginal utility of consumption, money holdins in the two currencies and the marginal disutility of work respectively. Taking expectations of (16) we arrive at the following familiar Keynes-Ramsey rule:

$$1 = \beta(1 + I_t)E_t \left(\frac{U_{C,t+1}}{U_{C,t}} \frac{P_t}{P_{t+1}} \right) \tag{20}$$

In (17), the demand for money balances depends positively on consumption relative to habit and negatively on the nominal interest rate. If, as is common in the literature, one adopts a utility function that is separable in money holdings, then given the central bank's setting of the latter and ignoring seignorage in the government budget constraint money demand is completely recursive to the rest of the system describing our macro-model. However separable utility functions are implausible (see Woodford (2003),chapter 3, section 3.4) and following Felices and Tuesta (2006) we will not go down this route. In (19) the real disposable wage is proportional to the marginal rate of substitution between consumption and leisure, $-\frac{U_{N,t}}{U_{C,t}}$, this constant of proportionality reflecting the market power of households that arises from their monopolistic supply of a differentiated factor input with elasticity η.

3.2 Domestic Producers

In the domestic goods sector each good differentiated good f is produced by a single firm f using only differentiated labour with another constant returns CES technology:

$$Y_t(f) = A_t \left[\left(\frac{1}{\nu} \right)^{\frac{1}{\eta}} \sum_{r=1}^{\nu} N_t(f,h)^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} \equiv A_t N_t(f) \tag{21}$$

where $N_t(f,h)$ is the labour input of type r by firm f and A_t is an exogenous shock capturing shifts to trend total factor productivity in this sector. Minimizing costs $\sum_{f=1}^{\nu_H} W_t(h) N_t(f,h)$ gives the demand for each household's labour by firm f as

$$N_t(f,h) = \left(\frac{W_t(h)}{W_t} \right)^{-\eta} N_t(f) \tag{22}$$

and aggregating over firms leads to the demand for labor as shown in (3).[2] Per capita output in the home bloc is given by

$$Y_t = A_t N_t \tag{23}$$

In a equilibrium of equal households, all wages adjust to the same level W_t. For later analysis it is useful to define the real marginal cost (MC) as the wage relative to domestic producer price. Using (19) and (23) this can be written as

$$\mathrm{MC}_t \equiv \frac{W_t}{A_t P_{H,t}} \quad = \quad -\frac{\eta}{(\eta-1)} \frac{U_{N,t}}{U_{C,t}} \left(\frac{P_t}{P_{H,t}} \right) \tag{24}$$

Turning to price-setting, we assume that there is a probability of $1-\xi_H$ at each period that the price of each good f is set optimally to $\hat{P}_{H,t}(f)$. If the price is not re-optimized, then it is held constant.[3] For each producer f the objective is at time t to choose $\hat{P}_{H,t}(f)$ to maximize discounted profits

$$E_t \sum_{k=0}^{\infty} \xi_H^k Q_{t,t+k} Y_{t+k}(f) \left[\hat{P}_{H,t}(f) - P_{H,t+k} \mathrm{MC}_{t+k} \right] \tag{25}$$

where $Q_{t,t+k}$ is the discount factor over the interval $[t, t+k]$, subject to a common[4] downward sloping demand from domestic consumers and foreign importers of elasticity ζ as in (9). The solution to this is

$$E_t \sum_{k=0}^{\infty} \xi_H^k Q_{t,t+k} Y_{t+k}(f) \left[\hat{P}_{Ht}(f) - \frac{\zeta}{(\zeta-1)} P_{H,t+k} \mathrm{MC}_{t+k} \right] = 0 \tag{26}$$

and by the law of large numbers the evolution of the price index is given by

$$P_{H,t+1}^{1-\zeta} = \xi_H \left(P_{H,t} \right)^{1-\zeta} + (1-\xi_H)(\hat{P}_{H,t+1}(f))^{1-\zeta} \tag{27}$$

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[2]Note that $N_t = \frac{1}{\nu} \sum_{f=1}^{n} N_f(f) = \left[\left(\frac{1}{\nu} \right) \sum_{r=1}^{\nu} N_t(h)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$ so in a symmetric equilibrium of identical firms and households $nN_t(f) = \nu N_t(h)$. Such a symmetric equilibrium applies to the flexi-price case of our model, but not to the sticky-price case where at each point in time some firms are locked into price and wage contracts, but others are re-optimizing these contracts.

[3]Thus we can interpret $\frac{1}{1-\xi_H}$ as the average duration for which prices are left unchanged.

[4]Recall that we have imposed a symmetry condition $\zeta = \zeta^*$ at this point; i.e., the elasticity of substitution between differentiated goods produced in any one bloc is the same for consumers in both blocs.

3.3 The Equilibrium

In equilibrium, goods markets, money markets and the bond market all clear. Equating the supply and demand of the home consumer good and assuming that government expenditure goes exclusively on home goods we obtain

$$Y_t = C_{H,t} + \frac{1-\nu}{\nu_H} C^*_{H,t} + G_t \tag{28}$$

where per capita government spending (G_t) is exogenous. In this first model fiscal policy is rudimentary: a balanced government budget constraint

$$P_{H,t} G_t = M_{H,t} - M_{H,t-1} + T_t \tag{29}$$

completes the model.

Given nominal interest rates I_t, I^*_t the money supply is fixed by the central banks to accommodate money demand. By Walras' Law we can dispense with the bond market equilibrium condition. Then the equilibrium is defined at $t = 0$ as stochastic sequences C_t, C_{Ht}, C_{Ft}, P_{Ht}, P_{Ft}, P_t, $M_{H,t}$, $M_{F,t}$, W_t, Y_t, N_t, P^0_{Ht}, 12 foreign counterparts C^*_t, etc, E_t, and S_t, given past price indices and exogenous processes ε_t, $\varepsilon_{M,t}$, $\varepsilon_{M^*,t}$, $\varepsilon_{N,t}$, A_t, G_t and foreign counterparts.

From (16) and its foreign counterpart we have

$$Q_{t,t+1} = \beta \frac{U_{C,t+1}}{U_{C,t}} \frac{P_t}{P_{t+1}} = \beta \frac{(U_{C,t+1})^*}{(U_{C,t})^*} \frac{P^*_t S_t}{P^*_{t+1} S_{t+1}} \tag{30}$$

Let $z_t = \frac{S_t P^*_t}{P_t} \frac{U_{C,t}}{(U_{C,t})^*}$. Then assuming identical holdings of initial wealth in the two blocs, (30) implies that $z_{t+1} = z_t = z_0$ where initial relative consumption in prices denominated in the home currency reflects different initial wealth in the two blocs. Therefore[5]

$$\frac{U_{C,t}}{(U_{C,t})^*} = \frac{z_0 P_t}{S_t P^*_t} = \frac{z_0}{RER_t} \tag{31}$$

[5](31) is the risk-sharing condition for consumption, because it equates marginal rate of substitution to relative price, as would be obtained if utility were being jointly maximized by a social planner (see Sutherland (2002)). Note that (20) and (31) together imply the stochastic UIP condition (see Benigno and Benigno (2001)).

3.4 Specialization of The Household's Utility Function

The single period utility in (1), $U\left(C_t(h) - hC_{t-1}, \frac{M_{H,t}(h)}{P_t}, \frac{M_{F,t}(h)S_t}{P_t}, N_t(h), \varepsilon_t, \varepsilon_{M_H,t}, \varepsilon_{M_F,t}, \varepsilon_{N,t}\right)$ in (1) is now specialized to:

$$U \equiv (\varepsilon_t + 1)\left[\frac{\Phi(h)^{1-\sigma}}{1-\sigma} + (\varepsilon_{N,t} + \kappa)\frac{N_t(h)^{1+\phi}}{1+\phi}\right] \tag{32}$$

where

$$\Phi(h) \equiv \left[b(C_t(h) - hC_{t-1})^{\frac{\theta-1}{\theta}} + (1-b)Z_t(h)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{33}$$

$$Z_t(h) \equiv \left[a\left(\frac{(\varepsilon_{M_H,t}+1)M_{H,t}(h)}{P_t}\right)^{\frac{\chi-1}{\chi}} + (1-a)\left(\frac{(\varepsilon_{M_F,t}+1)S_t M_{F,t}(h)}{P_t}\right)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}} \tag{34}$$

Then from (17) and (18) the relative demand for foreign and home currencies is

$$\mathrm{RF}_t \equiv \frac{M_{F,t}S_t}{M_{H,t}} = \left(\frac{I_t^*(1+I_t)a}{(1+I_t^*)I_t(1-a)}\right)^{-\chi} \tag{35}$$

The important feature of the utility function (1) is that $U_{C,M_H} > 0$ and $U_{C,M_F} > 0$ iff $\sigma\theta > 1$ in which case money holdings and consumption are *complements*. From (35) the relative nominal holdings of dollars changes *endogenously* with monetary policy in the two blocs rising as the exchange rate depreciates (S_t falls) and the relative domestic to foreign interest rate $\frac{I_t}{I_t^*}$ rises.

3.5 The Steady State

A deterministic zero-inflation steady state, denoted by variables without the time subscripts, is given by

$$C_H = \mathrm{w}_H \left(\frac{P_H}{P}\right)^{-\mu} C \tag{36}$$

$$C_F = (1-\mathrm{w}_H) \left(\frac{P_F}{P}\right)^{-\mu} C \tag{37}$$

$$P = \left[\mathrm{w}_H P_H^{1-\mu} + (1-\mathrm{w}_H) P_F^{1-\mu}\right]^{\frac{1}{1-\mu}} \tag{38}$$

$$\frac{W}{P} = \frac{\kappa N^{\phi}}{U_C\left(1-\frac{1}{\eta}\right)} \tag{39}$$

$$1 = \beta(1+I) \tag{40}$$

$$Y = AN \tag{41}$$

$$P_H = \hat{P}_H = \frac{W}{A\left(1-\frac{1}{\zeta}\right)} \tag{42}$$

$$Y = C_H + \frac{1-\nu}{\nu} C_H^* + G = C + G \tag{43}$$

$$T = G \tag{44}$$

$$U_{M_H} = U_C \frac{I}{1+I} \tag{45}$$

$$U_{M_F} = U_C \frac{I^*}{1+I^*} \tag{46}$$

$$U_C = b\Phi^{\frac{1}{\theta}-\sigma}(C(1-h))^{-\frac{1}{\theta}} \tag{47}$$

where

$$\Phi \equiv \left[b(C(1-h))^{\frac{\theta-1}{\theta}} + (1-b)Z^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{48}$$

$$Z \equiv \left[a\left(\frac{M_H}{P}\right)^{\frac{\chi-1}{\chi}} + (1-a)\left(\frac{M_F}{P}\right)^{\frac{\chi-1}{\chi}}\right] \tag{49}$$

plus the foreign counterparts. These are identical except the foreign bloc only hold money balances in their own currency so $a^* = 1$. The steady steady is completed with

$$\mathcal{T} = \frac{P_F}{P_H} \tag{50}$$

$$RER = \frac{SP^*}{P} \tag{51}$$

$$U_C = U_C^* \frac{z_0}{RER} \tag{52}$$

Units of output are chosen so that $P_H = P_F = 1$. Hence $\mathcal{T} = P = 1$. We also normalize $S = 1$ in the steady state so that $P_F^* = P_H^* = P^* = 1$ as well. Then the steady state of the risk-sharing condition (52) becomes $C = kC^*$ where k is a constant.

3.6 State Space Representation

We can write the two-bloc model in state space form as

$$\begin{bmatrix} \mathsf{z}_{t+1} \\ E_t \mathsf{x}_{t+1} \end{bmatrix} = A \begin{bmatrix} \mathsf{z}_t \\ \mathsf{x}_t \end{bmatrix} + B\mathsf{o}_t + C \begin{bmatrix} i_t \\ i_t^* \end{bmatrix} + D\mathsf{v}_{t+1} \tag{53}$$

$$F\mathsf{o_t} = H \begin{bmatrix} \mathsf{z}_t \\ \mathsf{x}_t \end{bmatrix} + J \begin{bmatrix} i_t \\ i_t^* \end{bmatrix} \tag{54}$$

where $\mathsf{z}_t = [a_t, a_t^*, g_t, g_t^*, \varepsilon_{C,t}, \varepsilon_{C,t}^*, \varepsilon_{N,t}, \varepsilon_{N,t}^*]$ is a vector of predetermined exogenous variables, $\mathsf{x}_t = [u_{c,t}, u_{c,t}^*, \pi_{H,t}, \pi_{F,t}^*, \hat{u}_{c,t}, \hat{c}_t^*]$ are non-predetermined variables, and $\mathsf{o}_t = [mc_t, mc_t^*, c_t, y_t, y_t^*, rer_t, \tau_t, \hat{c}_t, \hat{y}_t, \hat{y}_t^*, r\hat{e}r_t, \hat{\tau}_t]$ is a vector of outputs. Matrices A, B, etc are functions of model parameters. Rational expectations are formed assuming an information set $\{z_{1,s}, z_{2,s}, x_s\}$, $s \leq t$, the model and the monetary rule.

3.7 The Small Open Economy

Following Felices and Tuesta (2006) take i_t^* as an exogenous process given by $i_{t+1}^* = \rho_i^* i_t^* + v_{i,t}^*$ and assume $\pi_{F,t}^* = 0$. Let $n \to 0$ and $\mathrm{w}_H \to \omega_H$, $\mathrm{w}_F \to 1$, $\alpha_H \to \omega_H \frac{C}{Y}$ and $\alpha_F \to (1 - \omega_F)\frac{C^*}{Y^*}$.

Then $u_{c,t+1}^* = u_{ct}^* - i_t^*$ where $u_{c,t}^* = -\sigma c_t^* + \varepsilon_{C,t}^*$ and the state space representation follows as before with $\mathsf{z}_t = [a_t, g_t, \varepsilon_{C,t}, \varepsilon_{C,t}^*, \varepsilon_{N,t}, i_t^*]$ a vector of predetermined exogenous variables, $\mathsf{x}_t = [u_{c,t}, u_{c,t}^*, \pi_{H,t},\ \hat{u}_{c,t}]$ non-predetermined variables, and $\mathsf{o}_t = [mc_t, c_t, y_t, rer_t, \tau_t, \hat{c}_t, \hat{y}_t, \hat{i}_t, r\hat{e}r_t, \hat{\tau}_t]$.

In order to proceed with the analysis in the next section we rewrite the system to its reduced form corresponding to (67). Ignoring exogenous processes except the foreign interest rate, i_t^*, (so that $a_t = g_t = \varepsilon_{C,t} = \varepsilon_{C,t}^* = \varepsilon_{N,t} = 0$)) and the equations defining the flexi-price economy, the smallest number of state-space variables possible turns out to be $u_{c,t}$ and π_{Ht}, plus i_{t-1} needed to implement the rule, defined in the next section. After some effort we can express the deterministic system given interest rates as

$$
\begin{aligned}
E_t u_{c,t+1} &= u_{c,t} - \omega_H(i_t - E_t\pi_{H,t+1}) - (1-\omega_H)i_t^* && (55)\\
\beta E_t\pi_{H,t+1} &= \pi_{H,t} + \gamma u_{c,t} - \kappa i_t && (56)
\end{aligned}
$$

where without habit ($h = 0$)

$$
\begin{aligned}
u_{c,t} &= -\sigma c_t + \delta[\bar{a} i_t + (1-\bar{a})i_t^*] + \varepsilon_{C,t} && (57)\\
\delta &= \beta(\sigma\theta - 1)(1 - b_1) && (58)\\
b_1 &= \frac{b}{\left(b + (1-b)\alpha^{\frac{\theta-1}{\theta}}\right)} && (59)\\
\bar{a} &= a^\chi/(a^\chi + (1-a)^\chi) && (60)\\
\alpha &= \left(a + a^{1-\chi}(1-a)^\chi\right)^{\frac{\theta}{\chi-1}}\left(\frac{(1-b)a}{b(1-\beta)}\right)^\theta && (61)\\
\gamma &= \lambda_H\left(\frac{1}{\omega_H} + \frac{\phi\alpha_H}{\sigma} + \frac{\phi\mu}{\omega_H}(\alpha_H(1-\omega_H) + \alpha_F)\right) && (62)\\
\kappa &= \frac{\lambda_H\phi\alpha_H\delta\bar{a}}{\sigma} && (63)
\end{aligned}
$$

Equations (55) and (56) form the basis for the analysis of the next section. The important feature of the modified Phillips curve, (56) with a non-separable utility function in money and consumption is the manner in which the domestic interest rate impacts on domestic inflation. From (56) and (57) the elasticity of domestic inflation with respect to i_t is given by $\left(\gamma - \frac{\lambda_H\phi\alpha_H}{\sigma}\right)\bar{a} > 0$, so as long as dollarization is only partial ($a > 0$), and there is a direct negative effect of increasing the interest rate on inflation alongside the indirect effect through reducing consumption. This direct effect diminishes as the degree of dollarization increases and disappears altogether in the limit as dollarization is complete ($a = 0$). Similarly the higher the degree if dollarization the less is the effect if the domestic interest rate on aggregate demand and therefore output. Thus the ability of the central to stabilize both output and inflation using the domestic interest rate diminishes with dollarization. Note that without dollarization ($a = 1$)), $\alpha > 0$ and therefore $b_a < 1$. The domestic interest rate then impacts on the marginal utility of consumption only through the effect of the non-separability of money holdings and consumption. However with complete dollarization, ($a = 0$),$\alpha = 0$ and $b_1 = 1$. Then $\delta = 0$, $u_{C,t} = -\sigma c_t$ and the model *is isomorphic to the standard open-economy model with a separable utility function* as studied, for example, in Clarida *et al.* (2002).

4 Stability and Determinacy Analysis of Interest Rate Rules

The interest rate rules of interest for the open economy found in the literature (see, for example, Benigno and Benigno (2004)) are:

Fixed Exchange Rate Regime. As we show below this is implemented by $i_t = i_t^* + \theta_s s_t$ where $\theta_s > 0$ is sufficient to both maintain the regime and stabilize the economy. For this regime and the managed exchange rate below we need to augment the system with an exchange rate equation.

Free Floating Regime. This in general would be an inflation-forecast-based rule (IFB)-Taylor type rule:

$$i_t = \rho i_{t-1} + (1-\rho)[\theta_\pi E_t \pi_{H,t+j} + \theta_y (y_t - \hat{y}_t)] \tag{64}$$

or alternatively with CPI inflation (in which case we need to add π_t to the outputs).

Managed Floating Regime. Again this in general would be an IFB-Taylor type rule plus an exchange rate target:

$$i_t = \rho i_{t-1} + (1-\rho)[\theta_\pi E_t \pi_{H,t+j} + \theta_y (y_t - \hat{y}_t) + \theta_s (s_t - s_t^T)] \tag{65}$$

or alternatively with CPI inflation. s_t^T is an exchange rate target and can follow an exogenous process.

In the rest of this section we focus on inflation-targeting interest rate rules that respond only to inflation and not the output gap, or to the nominal exchange rate or to both. This makes the analysis tractable but there are other reasons for examining such rules. First the output gap is not directly observed so the Taylor-type rule is difficult to implement in practice. Second, pure inflation-targeting or inflation-targeting with a managed exchange rate corresponds to the objectives of many modern central banks. Finally, it is of intrinsic interest to see to what extent an economy can be stabilized with the simplest possible form of rule that only tracks one or at most two nominal variables.

4.1 Conditions for the Uniqueness and Stability

The interest rate rules set out above in this paper take the general form

$$i_t = G \begin{bmatrix} \mathsf{z}_t \\ \mathsf{x}_t \end{bmatrix} \tag{66}$$

From (53), (54) and (66) we arrive at the deterministic system under control as

$$\begin{bmatrix} \mathsf{z}_{t+1} \\ E_t \mathsf{x}_{t+1} \end{bmatrix} = K(G) \begin{bmatrix} \mathsf{z}_t \\ \mathsf{x}_t \end{bmatrix} \tag{67}$$

where $K(G)$ is a matrix that is a function of the feedback parameters defining the matrix G. The condition for a stable and unique equilibrium depends on the magnitude of the eigenvalues of the matrix $K(G)$. If the number of eigenvalues outside the unit circle is equal to the number of non-predetermined variables, the system has a unique equilibrium which is also stable with saddle-path $x_t = -N z_t$ where $N = N(G)$. (See Blanchard and Kahn (1980); Currie and Levine (1993)). Instability occurs when the number of eigenvalues of $K(G)$ outside the unit circle is larger than the number of non-predetermined variables. This implies that when the economy is pushed off its steady state following a shock, it cannot ever converge back to it, but rather finishes up with explosive inflation dynamics (hyperinflation or hyperdeflation). By contrast, indeterminacy occurs when the number of eigenvalues of $K(G)$ outside the unit circle is smaller than the number of non-predetermined variables. This implies that when a shock displaces the economy from its steady state, there are many possible paths leading back to equilibrium, i.e. there are multiple well-behaved rational expectations solutions to the model economy. With forward-looking rules this can happen when policymakers respond to private sector's inflation expectations and these in turn are driven by non-fundamental exogenous random shocks (i.e. not based on preferences or technology), usually referred to as 'sunspots'. If policymakers set the coefficients of the rule so that this accommodates such expectations, the latter become self-fulfilling. Then the rule is unable to uniquely pin down the behavior of one or more real and/or nominal variables, making many different paths compatible with equilibrium (see Kerr and King (1996); Chari *et al.* (1998); CGG (2000); Carlstrom and Fuerst (1999) and Carlstrom and Fuerst (2000); Svensson and Woodford (1999); and Woodford (2000)).

4.2 Previous Results for a Standard Open Economy Model

Thus far, the feature of dollarization has been introduced by assuming a utility function for consumers that is non-separable in consumption and real balances, where the latter incorporates both foreign and domestic money holdings. This leads in turn to a price-setting behaviour by firms that responds to the current and future levels of the interest rate, as well as to current and future output and consumption. In order to set our new results in context, we first review some results for pure inflation targeting obtained in Batini *et al.* (2004) already obtained in a more standard open economy model with a separable utility function, in which case dollarization has no impact on the economy. Such a model is a special case of the one above with $\delta = 0$. The same results apply to a fully dollarized economy which is again the a special case of the model of this paper with $a = 0$ and i_t^* an exogenous process playing no role in the stability properties.

Consider either an economy with a utility function that is separable in consumption or money or a fully dollarized economy. From Batini *et al.* (2004)[6]these results are for a free-floating regime[7] with no response to the output gap ($\theta_y = 0$): (a) the system is determinate and stable for an interest rate rule that feeds back on current domestic (producer price) or CPI inflation; (b) There exists a horizon J beyond which the system is indeterminate for all feedback parameters on domestic or CPI inflation dated at time $j > J$. A similar result holds when the feedback is on average expected inflation over a horizon, with the critical value being approximately $2J$. The critical value J is primarily dependent on the smoothing parameter ρ of the interest rate rule, defined below; (c) The potential indeterminacy of IFB rules is worse when based on CPI rather than producer price inflation as becomes increasingly worse as the degree of home bias decreases.

4.3 Fixed Exchange Rate Regime

Now we need to augment the system with a definitional equation relating the change in the nominal interest rate to the change in the terms of trade and inflation:

$$s_t = s_{t-1} + \tau_t - \tau_{t-1} + \pi_{Ht} \tag{68}$$

Note that the implication of this equation is that feedback on the nominal exchange rate via (65) is a form of 'integral control' (i.e. a sum of all past values) on inflation. It is known that integral control rules are very robust in terms of their stabilization properties.

[6]This previous work of ours examines a symmetric two-bloc model, but the stability properties of the difference form of that model are identical to those of the small open economy analyzed here.

[7]The results for the managed exchange rate are presented later.

Putting $i_t = i_t^* + \theta_s s_t$ discussed above and taking z-transforms of (55), (56) and (68), it is easy to show that the characteristic equation becomes

$$[(z-1)(z-\rho) - (1-\rho)\theta_s z][(z-1)(\beta z - 1) - \gamma\omega_H z] = 0 \tag{69}$$

where z is the forward operator. We can now show:

Proposition 1

(a) The system is stable and determinate for all values of $\theta_s > 0$.

(b) The nominal exchange rate is fixed.

Proof All proofs are given in Appendix B.

As Benigno and Benigno (2004) stress the feedback from the interest rate to the interest rate is not operative in the equilibrium as $s_t = 0$ at all times. Rather it is the *belief* that the monetary authority responds in this way even for very small θ_s that maintains a fixed exchange rate. With such a regime the domestic interest rate that enters the Phillips curve in (56) remains fixed too so neither the non-separable form of the utility function, nor the existence of dollarization has an impact on the characteristic equation (68) and the stability properties.

4.4 Free Floating Regime

Now consider the rule (64). Focusing on rules involving inflation only straightforward algebra, which again ignores all exogenous and stochastic variables, yields a characteristic equation: (64), (55) is given by

$$(z-\rho)[(z-1)(\beta z - 1) - \gamma\omega_H z] + (1-\rho)\theta_\pi z^{j+1}[\kappa(z-1) + \omega_H\gamma] = 0 \tag{70}$$

The effects of dollarization can be assessed through the variation in κ, which is proportional to a, where $1-a$ is the degree of dollarization.

As pointed out in the previous section, the case of no dollarization is easily seen to be equivalent to that of a separable utility function. Indeed, for the case $\omega_H = 1$, this is equivalent to the case of a closed economy. This underlies the result of the previous section.

For the case of a partially dollarized economy, $\kappa > 0$ and it turns out that the corresponding results are heavily dependent on the ratio $\omega_H \gamma / \kappa$. Algebraically, the reason for this is that as the feedback coefficient θ_π on inflation increases, one of the roots of (70) tends to $z = 1 - \omega_H \gamma / \kappa$; this can in principle take any value less than 1.

Proposition 2

If $2 < \omega_H \gamma / \kappa$ (i.e. $1 - \omega_H \gamma / \kappa < -1$), then any feedback on current inflation ($j = 0$) leads to stability and determinacy.

Proof: See Appendix.

Thus as the degree of dollarization decreases (i.e. as a increases), the value of κ increases, and $\omega_H \gamma / \kappa$ decreases. Hence we can deduce that there is a smaller likelihood of stability and determinacy as dollarization decreases. The distinction between high and low values of a is most apparent when ω_H is small.[8]

However for $j > 0$ there is no guarantee that the above condition ensures a unique path for the system. It is possible that for various values of j there may be two separated ranges of θ_π which guarantee this. We can however provide the following sufficient condition, which is similar to that of the previous section.

Proposition 3

When $j > J$ where

$$J = 1/(1 - \rho) + (1 - \beta - \kappa)/\gamma \omega_H \tag{71}$$

the feedback rule on expectations of inflation is indeterminate for all values of θ_π.[9]

The latter is a very strong result, and imposes an upper bound on the horizon for IFB rules. Furthermore, as dollarization decreases, κ increases, so that the horizon for which indeterminacy is always present is lower. Once again therefore, the system is more prone to indeterminacy as dollarization decreases.

[8] Recall that $\omega_H = 0$ corresponds to no home bias for consumption goods.

[9] Strictly, there are some easily satisfied sufficient conditions on the parameters that are required in addition to this.

4.5 Managed Floating Regime

Now consider the rule (65). Ignoring the feedback on output deviations, it is easy to show that the characteristic equation now becomes

$$[(z-1)(z-\rho)-(1-\rho)\theta_s z][(z-1)(\beta z-1)-\gamma\omega_H z]+(1-\rho)\theta_\pi z^{j+1}(z-1)[\kappa(z-1)+\omega_H\gamma]=0 \tag{72}$$

When we introduce additional feedback terms on inflation, the results of the previous section are to some extent ameliorated, and the combination of the two feedback terms leads to a smaller range over which there is indeterminacy.

Proposition 4

For any value of feedback on the exchange rate, $\theta_s > 0$ and for any j, there is a range of values of feedback on expected inflation, with the range beginning at $\theta_\pi = 0$ such that there is no indeterminacy. This result holds for parameter values outside the range $2 < \omega_H\gamma/\kappa$, which was critical for some of the results of the previous section.

4.6 The Determinacy Boundary

We now turn to numerical results based on the following calibration and estimation in Felices and Tuesta (2006): $\beta = 0.99$, $\sigma = 4$, $\phi = 0.47$, $\xi_H = 0.75$, $\omega_H = \omega_F = 0.6$, $\chi = 4.1$, $\zeta = 7.66$, $\eta = 3$, $\mu = 1$, $\rho_i = 0.96$, $\text{sd}(v_i^*) = 1\%$, $\theta = 2$, $b = 0.83$. We choose three degrees of dollarization: $a = 1$, (no dollarization), $a = 0$ (complete dollarization) and the intermediate estimated value $a = 0.5$. We further choose $c_y = 0.7$ and $\rho_a = \rho_g = \rho_C = \rho_N = \rho_C^* = 0.85$ and the standard deviations of these other shocks to be 1%.

a	θ_s	$\bar{\theta}_\pi(0)$	$\bar{\theta}_\pi(1)$	$\bar{\theta}_\pi(2)$	$\bar{\theta}_\pi(3)$	$\bar{\theta}_\pi(4)$	$\bar{\theta}_\pi(5)$	$\bar{\theta}_\pi(6)$	$\bar{\theta}_\pi(7)$	$\bar{\theta}_\pi(8)$	J
1.0	0.0	∞	393	10	1.4	indet	indet	indet	indet	indet	3.24
0.5	0.0	∞	268	13	1.8	indet	indet	indet	indet	indet	3.35
0.0	0.0	∞	218	16	2.1	indet	indet	indet	indet	indet	3.43
0.5	0.1	∞	271	16	4.0	2.4	1.9	1.8	1.7	1.7	∞

Table 1 The Determinacy Boundary as the Forward Horizon j increases

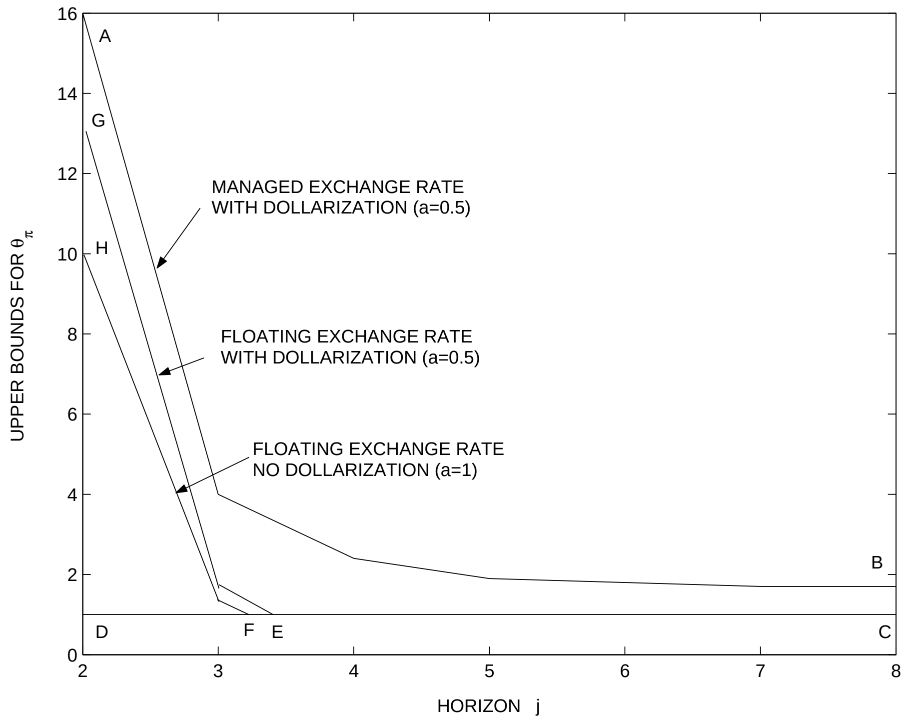

Figure 1: **Critical Upper Bounds for θ_π with $\theta_s = 0$ and $\theta_s = 0.1$**

If $\theta_\pi \leq 1$ then by the 'Taylor principle' the interest rate rule does not lead to saddlepath stability. In addition, the analytical results above indicate an upper bound for $\theta_\pi > 1$ beyond which the interest rate rule leads to indeterminacy. For each value of the forward horizon j for the expected inflation rate target there exists a boundary $\bar{\theta}_\pi(j)$, say, and by proposition 3 there is some upper bound for j, J given by (71), beyond which there is no value of θ_π that yields determinacy. Table 1 sets out values of $\bar{\theta}_\pi(j)$ for no, medium and complete degrees of dollarization ($a = 1, 0.5, 0$). For the medium degree of dollarization, $a = 0.5$, the final row of the table provides the boundary when, in addition to the feedback from expected inflation, there is also a response to the nominal exchange rate with $\theta_s = 0.1$. Figure 1 corresponds to table 1 but is confined to $a = 1, 0.5$ only. In the absence of exchange rate management ($\theta_s = 0$), with no dollarization the determinacy region is HFD. With medium dollarization and in accordance with proposition 3 this region of determinacy *increases* to GED. With some degree of exchange rate management and in accordance with proposition 4, this region increases to ABD and for this rule there is always some value of $\theta_p i$, albeit close to unity, that results in determinacy.

5 Optimal Policy and Optimized Simple Rules

As with much of the optimal monetary policy literature we adopt an ad hoc loss function of the form

$$\Omega_0 = \frac{1}{2} E_0 \left[(1-\beta) \sum_{t=0}^{\infty} \beta^t \left[w_y (y_t - \hat{y}_t)^2 + w_\pi \pi_t^2 + w_i i_t^2 \right] \right] \tag{73}$$

Indeed Clarida *et al.* (1999) provide a stout defence of a hybrid research strategy that combines a loss function based on the stated objectives of central banks with a micro-founded macro-model. The inclusion of a term that penalizes the variability of the nominal interest rate merits some discussion. In the absence of this extra term, if shocks are their variances are sufficiently large this will lead to a large nominal interest rate variability and the possibility of the interest rate becoming negative. To rule out this possibility and to remain within the convenient LQ framework of this paper, we follow Woodford (2003), chapter 6, and approximate the interest rate lower bound effect by introducing constraints that place an upper bound on the discounted of sum of the variance var(i_t). Woodford then shows this is equivalent to introducing the term $w_i i_t^2$ into the single-period utility function as in (73).

Weights w_y and w_π are the welfare-based expressions valid for the closed-economy version of the model given by the

$$w_y \;\; = \;\; \sigma + \phi; \;\; w_\pi = \frac{\zeta \xi_H}{(1-\xi_H)(1-\beta\xi_H)} \tag{74}$$

For our parameter values this gives $\frac{w_\pi}{w_y} = 13.1$ in our quarterly model, and $\frac{13.1}{16} = 0.83$ if inflation is annual, values well within those found in the optimal monetary policy literature.

5.1 Dollarization and Optimal Policy

We first calibrate the weight w_i *for each of our policy rules* so that $2sd(i_t) < I$ where $I = \frac{1}{\beta} - 1 + \pi$ is the steady state nominal interest rate. For the commitment rules the steady state inflation rate is $\pi = 0$ about which we have linearized the model, so for a normal distribution this would give a probability of hitting the interest rate lower bound of 2.5%. With $\beta = 0.99$ imposed this condition becomes $var(i_t) < 0.25(\%)^2$. For the discretionary rule we must take into account an inflationary bias pushing π above zero. We choose an quarterly inflationary bias of 0.01 or 4% per annum. Then the upper bound on $var(i_t)$ becomes $1.0(\%)^2$.

Tables 2 shows the effect on $var(i_t)$ of increasing the weight w_i under commitment.[10] Given w_i, denote the expected intertemporal loss at time $t = 0$ by $\Omega(w_i)$. But this includes a term penalizing the variance of the interest rate which does not contribute to utility loss as such but rather represents the interest rate lower bound constraint. Actual utility, found by subtracting the interest rate term, is given by $\Omega(0)$. We report the *minimum cost of fluctuations* in output gap and inflation equivalent terms obtained under the optimal commitment rule given by

$$y_e = \sqrt{\frac{2\Omega(0)}{w_y}}; \quad \pi_e = \sqrt{\frac{2\Omega(0)}{w_\pi}} \tag{75}$$

From table 2 a weight of $w_i \geq 1.5$ is required to make $\text{var}(i_t) \leq 0.25(\%)^2$. For this value of w_i, the minimal fluctuation costs are equivalent to a permanent decrease in the output gap of $y_e = 0.42\%$ and a permanent decrease in quarterly inflation of $\pi_e = 0.11\%$ or 0.44% on an annual basis. This figures are much larger than the welfare cost reported by Lucas (1987) which were of the order a permanent increase in consumption of 0.05%. The reason why they are much larger is down to the welfare costs of inflation not included in the Lucas calculations, the lower bound constraint, the non-separability of consumption and real balances and dollarization. The last three of these factors mean that inflation and the output gap cannot be perfectly stabilized.

Weight w_i	$\text{var}(i_t)$	$\Omega_0(w_i)$	$\Omega_0(0)$	y_e	π_e
0	0.47	0.36	0.36	0.40	0.11
1	0.27	0.51	0.38	0.41	0.11
1.5	0.25	0.58	0.39	0.42	0.11
2.0	0.23	0.63	0.40	0.42	0.12

Table 2. Optimal Commitment with $a = 0.5$: Imposing the Interest Rate Zero Lower Bound

[10] The solution procedures set out in Appendix A actually require a very small weight on the instrument. One can get round this without significantly changing the result by letting inflation be the instrument and then setting the interest rate at a second stage of the optimization to achieve the optimal path for inflation.

a	$\text{var}(i_t)$	$\Omega_0(1.5)$	$\Omega_0(0)$	y_e	π_e
0.0	0.37	1.69	1.41	0.79	0.22
0.25	0.34	1.44	1.19	0.73	0.20
0.5	0.25	0.58	0.39	0.42	0.11
0.75	0.20	0.27	0.12	0.23	0.06
1.0	0.20	0.28	0.13	0.24	0.07

Table 3. Optimal Commitment: The Cost of Dollarization.

a	$\text{var}(i_t)$	$\Omega_0(1.5)$	$\Omega_0(0)$	y_e	π_e
0	0.33	3.15	2.91	1.11	0.31
0.25	0.34	2.64	2.39	1.01	0.28
0.5	0.39	1.113	0.821	0.59	0.17
0.51	0.40	1.108	0.808	0.59	0.16
0.52	0.40	1.116	0.816	0.59	0.17
0.6	0.45	1.50	1.16	0.70	0.20
0.75	0.51	2.35	1.97	0.92	0.26
1.0	0.55	3.21	2.80	1.09	0.31

Table 4. Optimal Dollarization under Discretion.

We are now in a position to assess the costs of partial dollarization. This must depend on whether the central bank can commit or not. In the latter case it formulates optimal policy with discretion that it anticipated by the private sector. Intuitively under commitment there should be no benefits from loosing some control over monetary policy as a result of dollarization. Table 3 supports this intuition. As we proceed from complete dollarization ($a = 0$) to no dollarization ($a = 1$) the loss from fluctuations falls from an output gap equivalent of $y_e = 0.79\%$ to $y_e = 0.24\%$. Complete dollarization then imposes a cost of A permanent increase in the output gap of 0.55%. In terms of annual inflation the equivalent cost is a permanent increase of 0.6%.[11]

[11] Actually there is a slight drop in the loss as a falls from $a = 1$ (no dollarization) to $a = 0.75$. This needs further investigation to understand.

Turning to optimal policy with discretion, table 4 demonstrates an important result: *there exists an optimal degree of dollarization in the range* $a \in (0,1)$. For our chosen parameter values the optimum is at $a = 0.51$. The intuition behind is result is that dollarization 'ties the hands' of the central in a similar manner to that of appointing a 'conservative banker as in Rogoff (1985). We have seen from section 4.7 that the ability of the central to stabilize both output and inflation using the domestic interest rate diminishes with dollarization. Under discretion (but not under commitment) the constraint imposed by dollarization can, up to an optimal degree of this constraint, reduce the inflationary impact of discretion and lower the costs of fluctuations.

5.2 Stabilization Gains with Simple Rules

We now report results for simple commitment rules and discretionary policy. The general form of simple rule examined is

$$i_t = \rho i_{t-1} + \Theta_\pi E_t \pi_{t+j} + \Theta_s s_t\,; \;\; \rho \in [0,1],\ \Theta_\pi, \Theta_y, \Theta_s, j \geq 0 \tag{76}$$

Putting $\Theta_s = j = 0$ gives a Taylor-type rule where the interest rate only to current price inflation, $\Theta_s = 0$, $j > 0$ gives a forward-looking inflation (IFB) rule, $\Theta_s > 0$ gives a managed exchange rate.

Table 5 mirrors table 2 in seeking a weight w_i that will achieve the condition $\text{var}(i_t) \leq 0.25$ for a current inflation rule with partial dollarization, $a = 0.5$. A weight of $w_i = 3$ was sufficient for this purpose. We retain this choice of weight for the other rules examined as long as $\text{var}(i_t) \leq 0.25$, which in fact turns out to be the case.

Weight w_i	ρ	θ_π	$\text{var}(i_t)$	$\Omega_0(w_i)$	$\Omega_0(0)$	y_e
0	1	50	0.30	0.45	0.45	0.45
1.5	1	33	0.29	0.67	0.45	0.45
3	1	12	0.25	0.87	0.49	0.47
4	1	7	0.23	0.98	0.52	0.48

Table 5. The Optimized Current Inflation Rule: Imposing the Interest Rate Lower Bound.

Denote an inflation forecast-based targeting rule with horizon j with or without a managed exchange rate by $IFBj$. Results for optimized $IFBj$ rules simple are summarized in tables 6 and 7 for no dollarization ($a = 1$) and partial dollarization ($a = 0.5$) respectively. There are a number of notable results that emerge from the table and the figures. First we assess the effect of using an arbitrary rather than an optimized simple commitment rule by examining the outcome when a 'minimal rule' $i_i = 1.0001\pi_t$ that just produces saddle-path stability. This is the worse case and we see that the costs are substantial: $y_e = 2.1\%$ without dollarization, rising to $y_e = 2.231\%$ with partial dollarization. Second, an optimized simple current inflation rules perform well in that they achieve over 90% of the gain achieved by the optimal rule. Third, as we have found in previous work, the stabilization effectiveness of $IFBj$ deteriorates as the horizon j rises and does so sharply above $j = 2$ quarters. Fourth, managing the exchange rate with an interest rate response to the exchange rate improves the performance of $IFBj$ as j rises and more so with dollarization. Indeed *if* (for some reason) the central bank has a forward-looking inflation target with $j = 4$, it is optimal to respond only to the exchange rate and compared with an optimal floating IFBj rule, this lowers the fluctuations cost by $y_e = 0.21$ with no dollarization and by $y_e = 0.31$ with partial dollarization.

Rule	ρ	Θ_π	Θ_s	$\Omega(w_i)$	$\Omega(0)$	var(i_t)	y_e
Minimal Rule	0	10^{-4}	0	9.60	9.80	1.6×10^{-4}	2.1
IFB0 (floating)	1	4.76	0	0.63	0.32	0.21	0.38
IFB0 (managed)	1	25.0	0.02	0.54	0.18	0.24	0.28
IFB1 (floating)	1	16.9	0	0.66	0.32	0.23	0.39
IFB1 (managed)	1	25.0	0.04	0.55	0.18	0.23	0.28
IFB2 (floating)	1	8.10	0	0.86	0.59	0.18	0.51
IFB2 (managed)	1	8.41	0.03	0.76	0.49	0.18	0.47
IFB3 (floating)	1	2.39	0	2.69	2.53	0.11	1.06
IFB3 (managed)	1	2.41	0.02	2.52	2.37	0.10	1.03
IFB4 (floating)	1	1.13	0	8.87	8.77	0.07	1.98
IFB4 (managed)	0.41	0	0.02	7.22	7.13	0.06	1.77
Optimal Commitment	n.a.	n.a.	n.a	0.28	0.13	0.20	0.24

Table 6. Optimal Rules and Optimized Simple Rules: $a = 1.0$**.**

Rule	ρ	Θ_π	Θ_s	$\Omega(w_i)$	$\Omega(0)$	var(i_t)	y_e
Minimal Rule	0	10^{-4}	0	11.1	11.1	1.5×10^{-4}	2.23
IFB0 (floating)	1	12.1	0	0.86	0.49	0.25	0.47
IFB0 (managed)	1	12.1	0.02	0.83	0.46	0.25	0.45
IFB1 (floating)	1	14.4	0	0.91	0.58	0.22	0.51
IFB1 (managed)	1	14.4	0.06	0.87	0.54	0.22	0.49
IFB2 (floating)	1	9.5	0	1.16	0.89	0.18	0.69
IFB2 (managed)	1	10.0	0.05	1.07	0.8	0.18	0.60
IFB3 (floating)	1	2.69	0	3.12	2.96	0.11	1.15
IFB3 (managed)	0.96	2.67	0.02	2.64	2.48	0.11	1.05
IFB4 (floating)	1	1.27	0	10.3	10.2	0.07	2.14
IFB4 (managed)	0.14	0	0.03	7.60	7.48	0.08	1.83
Optimal Commitment	n.a.	n.a.	n.a	0.58	0.39	0.25	0.42

Table 7. Optimal Rules and Optimized Simple Rules: $a = 0.5$.

6 Conclusions

The main findings of the paper are as follows:

1. Under dollarization, the ability of the central to stabilize *both* output and inflation using the domestic interest rate diminishes.

2. Partially stabilizing the exchange rate to achieve the inflation target under dollarization is optimal. For our analytical set up, for example, augmenting an optimized simple forward-looking rule significantly improves the performance of the rule (delivering faster and less costly convergence–in output gap variability terms–of inflation toward its target), and more so with dollarization. This suggests that, for dollarized economies at least, the exchange-rate-smoothing behavior documented by Calvo and Reinhart may not correspond to an irrational fear of floating, but rather to efficient policy actions.

3. The costs of dollarization depend crucially on whether the central bank can commit or not. Under optimal commitment our calibrated model gave fluctuation costs with

an output gap equivalent of $y_e = 0.24\%$ without dollarization rising to a $y_e = 0.79\%$ with complete dollarization.

4. Under discretion the costs of little variations in the exchange rate are much higher and there exists an optimal degree of dollarization in the range $a \in (0, 1)$. For our chosen parameter values the optimum is at $a = 0.51$ (for which $y_e = 0.59$ compared with $y_e = 0.42$ with optimal commitment).The intuition behind is result is that dollarization 'ties the hands' of the central in a similar manner to that of appointing a 'conservative banker' as in Rogoff (1985), since the ability of the central to stabilize both output and inflation using the domestic interest rate diminishes with dollarization. Under discretion (but not under commitment) the constraint imposed by dollarization can, up to an optimal degree of this constraint, reduce the inflationary impact of discretion and lower variability costs. In practice, however, dollarization bears several other costs not contemplated in this set up. Thus this finding needs not imply that central banks should stop their campaign against dollarization.

5. With or without dollarization, optimized simple current inflation rules perform well in that they achieve over 90% of the gain achieved by the optimal rule.

6. The stabilization effectiveness of forward-looking inflation targeting rules deteriorate as the forward horizon, j, rises and does so sharply above $j = 2$ quarters.

These findings point to three key policy lessons. First, dollarization complicates the conduct of monetary policy; however monetary policy can still be carried out successfully and with low costs in terms of real activity under dollarization if the central bank commits to an inflation target. Thus, introducing an inflation target in partially dollarized economies can reduce the cost of price stabilization. Second, even if the degree of dollarization depends endogenously on the response of monetary policy from the exchange rate, it is still desirable to 'smooth' the exchange rate, in addition to correcting deviations of expected inflation from target. In this sense, an optimal simple rule for a partially dollarized is different from that of a non-dollarized economy, in that in the former economy there are substantial gains from including an exchange rate term in the rule, contrary to common findings on similar rules for non dollarized economies (see Batini *et al.* (2003). Abstracting from the many other adverse consequences of dollarization, our findings show that countries with no credibility may benefit from partial dollarization in that it constrains monetary policy to be conservative. Third, exchange rate smoothing reduces the chances of multiple equilibria under dollarization.

In future research, we plan to repeat the analysis using CPI inflation and a welfare-based loss function. The model could be fruitfully extended to incorporate imperfect financial markets and to contemplate financial dollarization as in Cespedes *et al.* (2004).

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A Linearization

We linearize the two-bloc model around the baseline and, in general, asymmetric, steady state of section 4.6 in which consumption, output, employment and prices in the two blocs are constant.[12] Then inflation is zero. Output is then at its inefficient natural rate studied in the previous section and the nominal rate of interest is given by (40). Now define all lower case level variables, such as C_t, Y_t, as proportional deviations from this baseline steady state. Rates of change, inflation and interest rates are expressed as absolute deviations.[13] Home producer and consumer inflation are defined as $\pi_{Ht} \equiv \frac{P_{Ht}-P_{H,t-1}}{P_{H,t-1}} \simeq p_{Ht} - p_{H,t-1}$ and $\pi_t \equiv \frac{P_t - P_{t-1}}{P_{t-1}} \simeq p_t - p_{t-1}$ respectively. Similarly, define foreign producer inflation and consumer price inflation. The linearized system is then:

$$
\begin{aligned}
E_t u_{c,t+1} &= u_{c,t} - (i_t - E_t \pi_{t+1}) && \text{(A.1)}\\
E_t u^*_{c,t+1} &= u^*_{c,t} - (i^*_t - E_t \pi^*_{t+1}) && \text{(A.2)}\\
\beta E_t \pi_{H,t+1} &= \pi_{H,t} - \lambda_H mc_t && \text{(A.3)}\\
\beta E_t \pi^*_{F,t+1} &= \pi^*_{F,t} - \lambda^*_F mc^*_t && \text{(A.4)}
\end{aligned}
$$

where

$$
\begin{aligned}
mc_t &= -(1+\phi)a_t - u_{c,t} + \phi y_t + p_t - p_{H,t} + \varepsilon_{N,t} && \text{(A.5)}\\
mc^*_t &= -(1+\phi)a^*_t + \sigma c^*_t + \phi y^*_t + p^*_t - p^*_{F,t} + \varepsilon^*_{N,t} && \text{(A.6)}
\end{aligned}
$$

and $\lambda_H = \frac{(1-\beta\xi_H)(1-\xi_H)}{\xi_H}$, λ^*_F similarly.

$$
\begin{aligned}
s_t - s_{t-1} + \pi^*_{H,t} &= \pi_{H,t} && \text{(A.7)}\\
s_t - s_{t-1} + \pi^*_{F,t} &= \pi_{F,t} && \text{(A.8)}
\end{aligned}
$$

which can be written

$$
\begin{aligned}
E_t s_{t+1} - s_t + E_t \pi^*_{H,t+1} &= E_t \pi_{H,t+1} && \text{(A.9)}\\
E_t s_{t+1} - s_t + E_t \pi^*_{F,t+1} &= E_t \pi_{F,t+1} && \text{(A.10)}
\end{aligned}
$$

where the linearized UIP condition is

$$
E_t s_{t+1} - s_t = i_t - i^*_t \qquad \text{(A.11)}
$$

[12]Note if $\mu = \mu^* = 1$, $b = 1$ and we introduce imperfect exchange rate pass-through, then this model is the special case in section 4 of the 'Forum' paper.

[13]That is, for a typical variable level X_t, $x_t = \frac{X_t - \bar{X}}{\bar{X}} \simeq \log\left(\frac{X_t}{\bar{X}}\right)$ where $\bar{X}$ is the baseline steady state. Rate variables, the interest and inflation rate however are expressed as an absolute deviation; i.e., $i_t = I_t - I$.

Using

$$\begin{aligned}\pi_t &= \mathrm{w}_H \left(\frac{P_H}{P}\right)^{1-\mu} \pi_{H,t} + (1-\mathrm{w}_H)\left(\frac{P_F}{P}\right)^{1-\mu} \pi_{F,t} \\ &= \mathrm{w}_H \pi_{H,t} + (1-\mathrm{w}_H)\pi_{F,t} \qquad \text{(A.12)} \\ \pi_t^* &= \mathrm{w}_F \left(\frac{P_F^*}{P^*}\right)^{1-\mu^*} \pi_{F,t}^* + (1-\mathrm{w}_F)\left(\frac{P_H^*}{P^*}\right)^{1-\mu^*} \pi_{H,t}^* \\ &= \mathrm{w}_F \pi_{F,t}^* + (1-\mathrm{w}_F)\pi_{H,t}^* \qquad \text{(A.13)}\end{aligned}$$

From the definition of the terms of trade $\Delta\tau_t = \pi_{F,t} - \pi_{H,t} = \pi_{F,t}^* - \pi_{H,t}^*$ since purchasing power parity applies to each differentiated good. Hence CPI inflation is given by

$$\begin{aligned}E_t\pi_{t+1} &= \mathrm{w}_H E_t \pi_{H,t+1} + (1-\mathrm{w}_H)E_t\pi_{F,t+1} = \mathrm{w}_H E_t\pi_{H,t+1} + (1-\mathrm{w}_H)E_t(\pi_{F,t+1}^* + s_{t+1} - s_t) \\ &= \mathrm{w}_H E_t\pi_{H,t+1} + (1-\mathrm{w}_H)E_t(\pi_{F,t+1}^* + i_t - i_t^*) \qquad \text{(A.14)}\end{aligned}$$

using the UIP condition (A.11).

We can now now write the Euler equations as

$$E_t u_{c,t+1} = u_{c,t} - \mathrm{w}_H(i_t - E_t\pi_{H,t+1}) - (1-\mathrm{w}_H)(i_t^* - E_t\pi_{F,t+1}^*) \qquad \text{(A.15)}$$

$$E_t u_{c,t+1}^* = u_{ct}^* - \mathrm{w}_F(i_t^* - E_t\pi_{F,t+1}^*) - (1-\mathrm{w}_F)(i_t - E_t\pi_{H,t+1}) \qquad \text{(A.16)}$$

where with habit

$$u_{c,t} = -\frac{\sigma}{1-h}c_t + \frac{\sigma h}{1-h}c_{t-1} + \delta[a i_t + (1-a)i_t^*] + \varepsilon_{C,t} \qquad \text{(A.17)}$$

$$\delta = \beta(\sigma\theta - 1)(1-b_1) \qquad \text{(A.18)}$$

$$b_1 = \frac{b}{\left(b + (1-b)\alpha^{\frac{\theta-1}{\theta}}\right)} \qquad \text{(A.19)}$$

$$\alpha = \left(a + a^{1-\chi}(1-a)^{\chi}\right)^{\frac{\theta}{\chi-1}} \left(\frac{(1-b)a}{b(1-\beta)}\right)^{\theta} \qquad \text{(A.20)}$$

$$u_{c,t}^* = -\sigma c_t^* + \delta i_t^* + \varepsilon_{C,t}^* \qquad \text{(A.21)}$$

The risk-sharing condition is

$$rer_t = u_{c,t}^* - u_{c,t} \qquad \text{(A.22)}$$

and output equilibrium is

$$y_t = \alpha_H[c_t - \mu(p_{H,t} - p_t)] + \alpha_F[c_t^* - \mu(p_{H,t}^* - p_t^*)] + \alpha_G g_t \qquad \text{(A.23)}$$

$$y_t^* = \alpha_F^*[c_t^* - \mu^*(p_{F,t}^* - p_t^*)] + \alpha_H^*[c_t - \mu(p_{F,t} - p_t)] + \alpha_G^* g_t^* \qquad \text{(A.24)}$$

where

$$\alpha_H = \mathrm{w}_H \frac{C}{Y}\left(\frac{P_H}{P}\right)^{-\mu} = (1-(1-n)(1-\omega_H))\frac{C}{Y} \qquad \text{(A.25)}$$

$$\alpha_F = \frac{1-n}{n}(1-\mathrm{w}_F)\frac{C^*}{Y^*}\left(\frac{P_H^*}{P^*}\right)^{-\mu^*} = (1-n)(1-\omega_F)\frac{C^*}{Y^*} \qquad \text{(A.26)}$$

$$\alpha_G = 1 - \alpha_H - \alpha_F \qquad \text{(A.27)}$$

and α_F^* etc defined similarly.

Putting

$$
\begin{aligned}
p_t - p_{H,t} &= (1-\mathrm{w}_H)\left(\frac{P_F}{P}\right)^{1-\mu}\tau_t = (1-\mathrm{w}_H)\tau && \text{(A.28)}\\
p_t - p_{F,t} &= -\mathrm{w}_H\left(\frac{P_H}{P}\right)^{1-\mu}\tau_t = -\mathrm{w}_H\tau && \text{(A.29)}\\
p_t^* - p_{F,t}^* &= (1-\mathrm{w}_F)\left(\frac{P_H^*}{P^*}\right)^{1-\mu^*}\tau_t^* = (1-\mathrm{w}_F)\tau^* && \text{(A.30)}\\
p_t^* - p_{H,t}^* &= -\mathrm{w}_F\left(\frac{P_F^*}{P^*}\right)^{1-\mu^*}\tau_t^* = -\mathrm{w}_F\tau^* && \text{(A.31)}\\
\tau_t^* &= -\tau_t && \text{(A.32)}
\end{aligned}
$$

we can write (A.23) and (A.24) as

$$
\begin{aligned}
y_t &= \alpha_H c_t + \alpha_F c_t^* + \alpha_G g_t + \mu(\alpha_H(1-\mathrm{w}_H) + \alpha_F \mathrm{w}_F)\tau_t && \text{(A.33)}\\
y_t^* &= \alpha_F^* c_t^* + \alpha_H^* c_t + \alpha_G^* g_t^* - \mu(\alpha_F(1-\mathrm{w}_F) + \alpha_H \mathrm{w}_H)\tau_t && \text{(A.34)}
\end{aligned}
$$

and (A.5) and (A.6) as

$$
\begin{aligned}
mc_t &= -(1+\phi)a_t - u_{c,t} + \varepsilon_{C,t} + \phi y_t + (1-\mathrm{w}_H)\tau + \varepsilon_{N,t} && \text{(A.35)}\\
mc_t^* &= -(1+\phi)a_t^* + \sigma c_t^* + \phi y_t^* - (1-\mathrm{w}_F)\tau + \varepsilon_{N,t}^* && \text{(A.36)}
\end{aligned}
$$

Linearizing (15) we have

$$
rer_t = -(1-\mathrm{w}_F-\mathrm{w}_H)\tau_t \qquad \text{(A.37)}
$$

and exogenous processes are for the home bloc

$$
\begin{aligned}
a_{t+1} &= \rho_a a_t + v_{a,t+1} && \text{(A.38)}\\
g_{t+1} &= \rho_g g_t + v_{g,t+1} && \text{(A.39)}\\
\varepsilon_{C,t+1} &= \rho_C \varepsilon_{C,t} + v_{C,t+1} && \text{(A.40)}\\
\varepsilon_{N,t+1} &= \rho_C \varepsilon_{N,t} + v_{N,t+1} && \text{(A.41)}
\end{aligned}
$$

with analogous processes for the foreign bloc.

For Taylor-type rules we require the *output gap* the difference between output for the sticky price-wage model obtained above and output when prices are flexible and expected inflation is zero. It is convenient and plausible to define the flexi-price economy as non-dollarized. The latter is then given as follows by putting $a = 0$, $mc_t = mc_t^* = 0$ and all expected inflation rates are zero:

$$
\begin{aligned}
E_t\hat{u}_{c,t+1} &= \hat{u}_{c,t} - \mathrm{w}_H\hat{i}_t - (1-\mathrm{w}_H)i_t^* && \text{(A.42)}\\
E_t\hat{u}^*_{c,t+1} &= \hat{u}^*_{c,t} - \hat{i}_t^* && \text{(A.43)}\\
\hat{mc}_t = 0 &= -(1+\phi)a_t - \hat{u}_{c,t} + \varepsilon_{C,t} + \phi\hat{y}_t + (1-\mathrm{w}_H)\hat{\tau} + \varepsilon_{N,t} && \text{(A.44)}\\
mc_t^* = 0 &= -(1+\phi)a_t^* + \sigma\hat{c}_t^* + \phi\hat{y}_t^* - (1-\mathrm{w}_F)\hat{\tau} + \varepsilon^*_{N,t} && \text{(A.45)}\\
\hat{y}_t &= \alpha_H\hat{c}_t + \alpha_F\hat{c}_t^* + \alpha_G g_t + \mu(\alpha_H(1-\mathrm{w}_H) + \alpha_F\mathrm{w}_F)\hat{\tau}_t && \text{(A.46)}\\
\hat{y}_t^* &= \alpha_F^*\hat{c}_t^* + \alpha_H^*\hat{c}_t + \alpha_G^* g_t^* - \mu(\alpha_F(1-\mathrm{w}_F) + \alpha_H\mathrm{w}_H)\hat{\tau}_t && \text{(A.47)}\\
\hat{u}_{c,t} &= -\sigma\hat{c}_t + \delta[a\hat{i}_t + (1-a)\hat{i}_t^*] + \varepsilon_{C,t} && \text{(A.48)}\\
\hat{u}^*_{c,t} &= -\sigma\hat{c}_t^* + \delta\hat{i}_t^* + \varepsilon^*_{C,t} && \text{(A.49)}\\
\hat{rer}_t &= \hat{u}^*_{c,t} - \hat{u}_{c,t} = -(1-\mathrm{w}_F-\mathrm{w}_H)\hat{\tau}_t && \text{(A.50)}
\end{aligned}
$$

B Proofs of Propositions

Proof of Proposition 1

(a) It is easy to establish that each of $(z-1)(z-\rho)-(1-\rho)\theta_s z$ and $(z-1)(\beta z-1)-\gamma\omega_H z$ from (72) have one root inside and one root outside the unit circle. Hence the system under the interest rate rule has exactly two unstable roots, which matches the two non-predetermined variables $\pi_{Ht},\ u_{c,t}$.

(b) Consider the linearized UIP condition (A.11) with $i_t = i_t^* + \theta_s s_t$:

$$E_t s_{t+1} = (1+\theta_s)s_t \tag{B.51}$$

Then solving forwards for s_t we have that $s_t = 0$. Thus the nominal exchange rate in deviation form about the steady state is zero, so in level form it is fixed at that steady state.

Proof of Proposition 2: The inequality in the proposition ensures that as $\theta_\pi \to \infty$, the root at $z = 1 - \omega_H\gamma/\kappa < -1$ is to the left of the unit circle. Thus the root locus diagram for the system is as shown in Figure 2.[14] This diagram gives the general picture for all combinations of parameters, but its position relative to the unit circle needs to be established algebraically. As depicted in Figure 2, there is only one root inside the unit

[14]The root locus method is a standard method for analyzing the stability of dynamic linear systems found in the engineering literature. (See Evans (1954)).[15] It was used for the first time to study the indeterminacy of forward-looking interest rate rules by Batini and Pearlman (2002). The method provides an elegant way of locating the position in the complex plane of all the roots of the characteristic equation as one of the parameters changes. In our application the parameter in question is the feedback parameter from future inflation, θ_π.

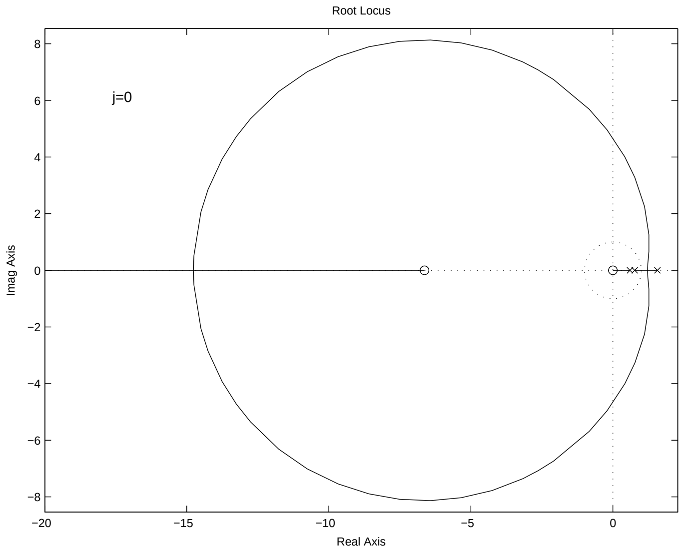

Figure 2: **Position of zeroes as θ changes: current inflation**

disc for $\theta_\pi > 1$, so that the system is stable and determinate. However, there are two things that we need to check. Firstly that the branch point on the positive real line is at a value of $z > 1$; we verify this as part of the proof of Proposition 3. Secondly, that the root locus never passes through the unit disc in the complex part of the plane; in particular, if it does so, it must pass through it twice. We now show that it can in principle pass through the unit disc only once i.e. the root locus can never enter and then leave the unit disc.

Any point on the unit circle can be represented as $z = e^{i\psi} = \cos\psi + i\sin\psi$, where ψ is the angle made with the the positive real axis. Substituting this into the characteristic equation (with $j = 0$) yields two equations (the real and the imaginary parts of the expression) in two unknowns $\theta_\pi,\ \psi$. Now multiply through by $e^{-i\psi}(\kappa(e^{-i\psi} - 1) + \omega_H\gamma)$. The term containing θ_π is then real, with no imaginary part, so that the solution for ψ is obtained by solving

$$Im\left[(\kappa(e^{-i\psi} - 1) + \omega_H\gamma)(1 - \rho e^{-i\psi})[(e^{i\psi} - 1)(\beta e^{i\psi} - 1) - \omega_H\gamma e^{i\psi}]\right] = 0 \qquad \text{(B.52)}$$

By inspection, we see that this can be rewritten in the form $A\sin\psi + B\sin 2\psi = 0$, where A, B are functions of the parameters. Using the the identity $\sin 2\psi = 2\sin\psi\cos\psi$, it follows that one of the solutions is $\sin\psi = 0$, which corresponds to $z = 1$, while the other solution is given by $A + 2B\cos\psi = 0$. But there is at most one solution of this for $0 < \psi < \pi$ i.e. the root locus cannot enter and then leave the unit disc.

Proof of Proposition 3: Figures 3, 4 and 5 demonstrate that as θ_π increases, a value is always reached where there is more than one root of the system inside the unit disc.

The implication is that there is indeterminacy when the feedback on future inflation is 'too high'. Furthermore, Figures 4 and 5 suggest that if the branch-point of the root locus near to $z = 1$ is inside the unit disc, then it may well be that there is indeterminacy for *all* values of θ_π. A necessary and sufficient condition for the branch-point to be at a value of $z < 1$ is that the the root locus passes through the point $z = 1$ from the right; equivalently $\frac{\partial z}{\partial \theta_\pi}$ is negative at $z = 1$. We evaluate this using implicit differentiation of (70):

$$[-\omega_H\gamma+(1-\rho)(\beta-1)-(1-\rho)\omega_H\gamma+(1-\rho)(\kappa+(j+1)\omega_H\gamma)]\left.\frac{\partial z}{\partial \theta_\pi}\right|_{z=1}+(1-\rho)\omega_H\gamma = 0 \quad \text{(B.53)}$$

Note that for $j = 0$, the case of Proposition 2, it is easy to see that the coefficient of $\left.\frac{\partial z}{\partial \theta_\pi}\right|_{z=1}$ is negative, so that $\left.\frac{\partial z}{\partial \theta_\pi}\right|_{z=1} > 0$, and the root locus branch-point is therefore to the right of $z = 1$. We can also see that this coefficient is increasing in j. Thus the critical value of Proposition 2 is realized for the minimum value of j such that this coefficient is positive.

Thus we have shown so far that if j is beyond its critical value J , then there is a set of values of θ_π greater than 1 for which there is indeterminacy. However this does not guarantee indeterminacy for all $\theta_\pi > 1$ beyond the critical J. It can happen that there may be a range of values of $\theta_\pi > 1$ for which this branch of the root locus leaves and then re-enters the unit disc. Relatively mild conditions (still to be derived) on the parameters ensures that this cannot happen.

Proof of Proposition 4: This is a corollary of Proposition 1. Given feedback θ_s on the exchange rate, and no feedback on inflation, we have seen that the system is determinate. It follows that sufficiently small feedback θ_π on inflation will barely shift the roots of the system for any j. Hence the result. The root locus diagram in Figure 6 demonstrates how a small feedback from the exchange rate can transform the indeterminacy in Figure 5 for all values of θ_π into a rule where determinacy exists for values of θ_π close to unity (see also Figure 1 in the main text).

j=1

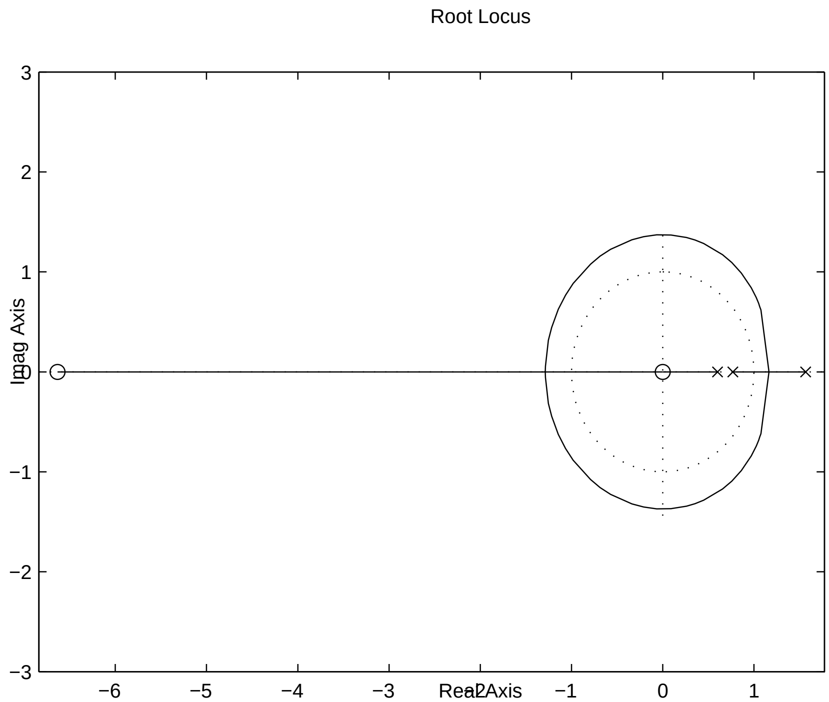

Figure 3: **Position of roots as θ_π changes: 1-period ahead expected inflation**

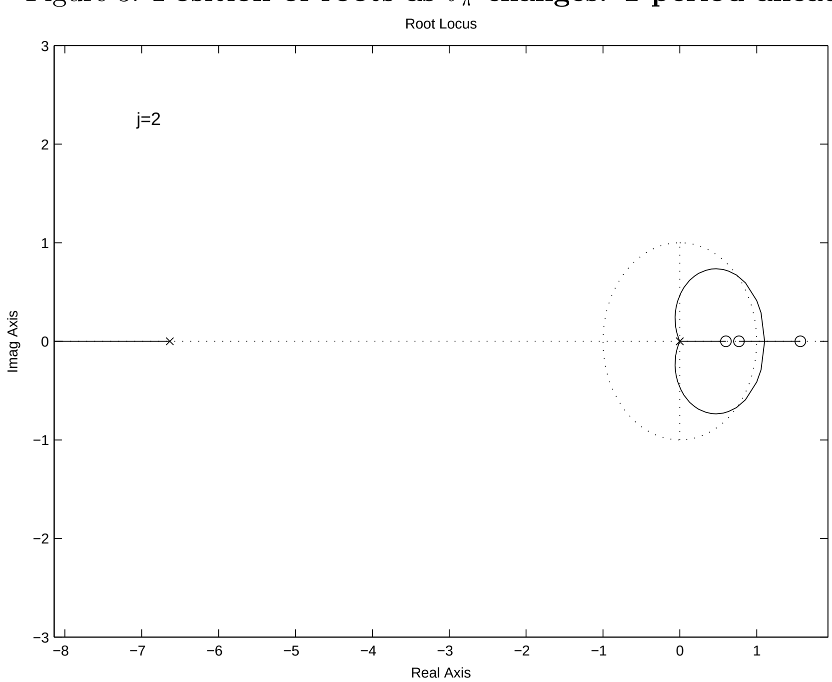

Figure 4: **Position of roots as θ_π changes: 2-period ahead expected inflation**

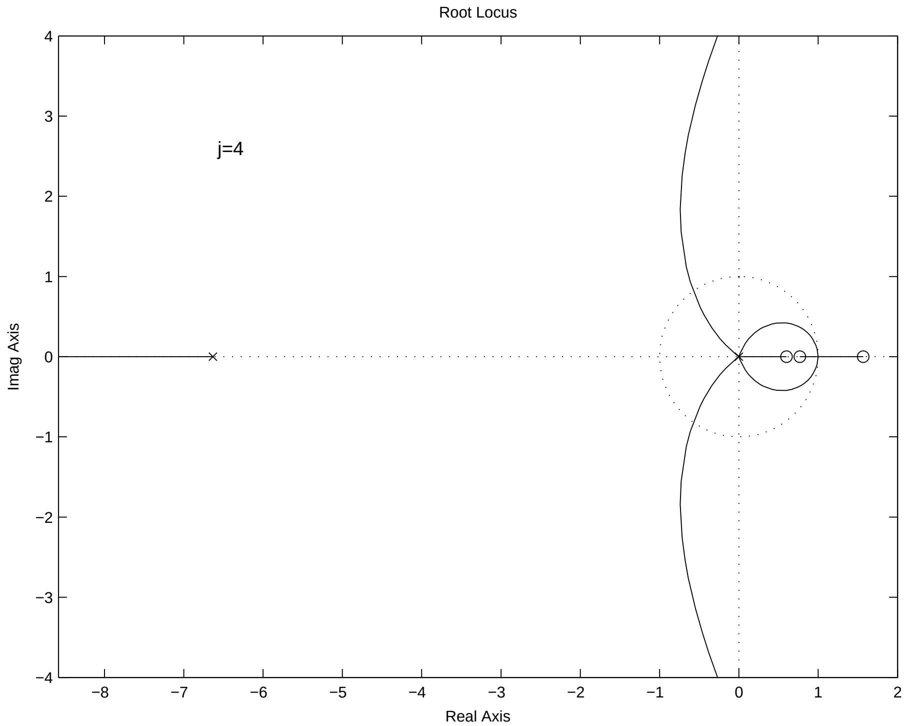

Figure 5: **Position of roots as θ_π changes: 4-period ahead expected inflation**

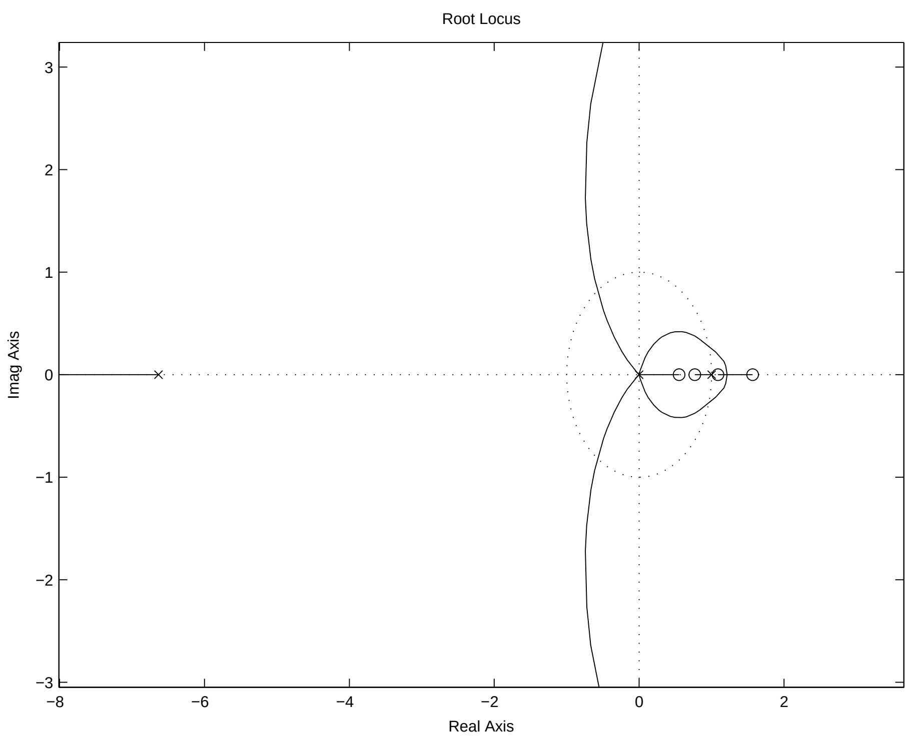

Figure 6: **Position of roots as θ_π changes: 4-period ahead expected inflation with managed exchange rate**

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