Optimal Monetary Policy and Endogenous Price Dollarization[*]

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Abstract

This paper studies the conditions under which it is optimal for an economy to have more than one unit of account (price dollarization) and how this equilibrium is related to monetary policy. To that purpose we build a simple stochastic general equilibrium model of a small open economy where firms have to set prices in advance and to choose endogenously between a domestic and a foreign currency to set prices. We find as in Mundell (1961), that the combination of sticky prices and sector specific productivity shocks make optimal for some firms to use a foreign currency as unit of account. The central bank sustains this equilibrium by actively using the nominal exchange rate as an instrument to partially offset the effect that sector specific productivity shocks generate on relative price distortions . Additionally, we find that an strict inflation target can reduce price dollarization, although this equilibrium is not optimal, and that "excessive fear of floating" induces inefficiently high degrees of price dollarization.

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1 Introduction

A practical issue for many central banks in emerging economies with a history of high inflation is whether or not they should accept the fact that a foreign currency shares the function of unit of account with the domestic one and if this is so, how monetary policy should be conducted in this type of environment. The use of a foreign currency, usually the US dollar, in countries like Bolivia, Egypt, Turkey and Russia, has emerged as an endogenous response of firms to a macroeconomic environment of high inflation. Using a more stable currency allow domestic agents to avoid the distortions that inflation generates, in particular it allows domestic firms to avoid inflation generated relative price distortions. However, the more recent experience of those economies seems to suggest that firms find optimal to set prices in foreign currency even when inflation is low and stable. In this paper we provide a rationale for that type of behavior and also we show the way monetary policy influence the degree of price dollarization.

The previous issue is, however, not only of practical interest for central banks in emerging economies, but one that it is at centre of modern monetary theory. Most of the recent models for analyzing the design of monetary policy in close and open economies[1],are general equilibrium microfounded models with sticky prices, where the fundamental, albeit the only, function that money plays is as unit of account. Thus understanding what factors determine the optimal election of one or more units of account and how this interact with the degree of price stickiness and monetary policy are essential issues in monetary theory. This paper provides some insight on this issue.

We use a simple fully microfounded general equilibrium model of an small open economy with three particular characteristics: a) firms face sector specific productivity shocks b) firms can choose freely between a domestic and foreign currency to set their prices, and c) a fraction of randomly selected firms set prices one period in advance. This set up contains the minimum ingredients to study the interaction between the election of a unit of account and optimal monetary policy. Firms select their unit of account by comparing their expected profits under domestic and foreign currency pricing, whereas the central bank implements its monetary policy aiming at maximizing the welfare of the representative agent of the economy. In equilibrium, the pricing decision of firms and the monetary policy implemented by the central bank are mutually consistent, thus an endogenous degree of PD is determined.

By using a second order approximation of the profits function of the representative firm we find the conditions that induce them to set prices in dollars. Firms have more incentives to set prices in foreign currency when domestic inflation is more volatile, when the covariance between

[1] Aoki (2001), Woodford (2003) for close economies, and Benigno (2004), De paoli(2005), and Gali and Monacelli (2005) for open economies.

the domestic and the nominal exchange rate is higher and when the volatility of nominal exchange rate is lower. The intuition of these relationships are easy to understand. Since firms set prices in advance, unexpected changes in domestic prices and the nominal exchange rate generate relative price misalignment that are costly for firms. When domestic inflation is more volatile, setting prices in a foreign currency isolate relative prices from domestic inflation volatility, similarly, when the nominal exchange rate and the real exchange are more stable, the benefits of using the foreign currency as unit of account are larger.

Likewise, by approximating up to second order the welfare function of the representative agent we derive the optimal lost function for the central bank. We show that the combination of sticky prices and sector specific productivity shocks makes zero domestic inflation not optimal even when the channel of terms of trade is not active. Only when firms face a common productivity shock, zero inflation becomes optimal in this economy, as in Clarida, Gali and Gertler (2001) and Monacelli (2005). In contrast, in our set up the central bank will have incentives to use actively the nominal exchange rate to partially offset the distortions in relative prices that sector specific productivity shocks and price stickiness generate. In turn, this incentive of the central bank to correlate fluctuations in the exchange rate with sector specific productivity shocks, generate the conditions for some firms to choose optimally set prices in dollars.

In equilibrium, we find that under optimal monetary policy a positive degree of PD, the use of two currency as unit of account, become optimal when sector specific productivity shocks are large enough relative to real exchange rate shocks. More precisely, only when the real exchange rate is stable enough, the foreign currency is attractive for both firms and the central bank as an instrument to stabilize relative prices. In particular, as the volatility of real exchange rate increases, the central bank faces a larger cost in terms of the volatility of output gap when stabilizing the nominal exchange rate, thus using it as an instrument to reduce relative price distortions generated by sector specific productivity shocks become less desirable. Our results also show that a central bank that aims at anchoring domestic inflation, as for instance in the case of adopting an explicit inflation targeting regime, induces a reduction on the degree of PD. On the contrary central banks that exhibit an excess of "fear of floating" would generate an excess of PD, which it would be suboptimal. In that sense the results of the paper suggest that there exist an optimal degree of "fear of floating" determined by the incentives that the central bank has to smooth exchange rate fluctuations in order to offset relative prices distortions.

Our setup is related to several recent papers that analyze optimal monetary policy in environments where there exist more than one sector. All these papers, including ours, share the same general conclusion that when there exist sector specific shocks the first best allocation is not attainable. Since, in this case, the central bank has only one instrument, but multiple objectives to achieve. For instance Aoki (2001), analyses optimal monetary policy for a two-sector close economy model, Benigno(2004) characterizes optimal monetary policy in a currency area, Erceg, et al (2000) consider the case of stickiness in wages and prices, and in Huang, X D Kevin y Zheng Liu (2005) analysis the case of price stickiness in the final and intermediate production sectors. Differently from the previous papers, we focus on the optimal election of unit of account besides optimal monetary policy. Our paper is more closely related to Loyo (2001) and Corsetti and Pensenti (2004), since in both papers firms have to decide optimally among different units of account. In Loyo (2001) firms have to decide between a real and an imaginary money, where the central bank can control directly the parity between these two types of currency, whereas in Corsetti and Penseti (2004), importing firms have to choose between domestic and foreign currency[2]. In contrast to Loyo (2001), in this setup, the central bank does not perfectly controls the parity between the domestic and foreign currency, since there exist shocks to the real exchange rate.

A number of simplifying assumptions have been made in the paper in order to gain in clarity. First, although our framework is of an small open economy where terms of trade surely play an important role, we have chosen preferences that shut off this channel, with the intention to highlight the effects of PD. Second, we have use a very simple structure of correlations among sector specific productivity shocks, which are enough to show qualitatively the implications of the model, however a more realistic assumption on this issue can be made as in Loyo (2001). We would like to explore these extensions in future research.

The remaining of this paper is organized as follows. Section 2 describes a simple general equilibrium model of an small open economy where firms faces sector specific productivity shocks and firms have to set prices in advance. Section 3 discuss the implication of price stickiness for the dynamic equilibrium of the economy when firms face sector specific productivity shocks, section 4 analyses the relevant lost function for the central bank and the design of optimal monetary policy under price dollarization. Section 5, discuss in detail the pricing decision of firms and the equilibrium under optimal monetary policy. The final section presents some concluding remarks.

2 The Model

We model an small open economy where price dollarization (PD) emerges as an endogenous choice of monopolistic competitive firms. Domestic goods producers can set their prices either in units of a foreign currency, "the dollar", or units of the domestic currency, "the peso". Prices

[2] A series of other papers study invoicing decisions in different contexts, for instance, Bacchetta, P and Wincoop (2001), Donnefeld, et al (1991), Giovanini (1988), Johnson et al (1997) and Klemperer and Meyer (1986).

are sticky in both the domestic and the foreign currency. Only a fraction of domestic firms can freely set prices each period, the remaining ones have to set prices one period in advance, relying only on information up to period $t-1$.

The small open economy model is derived as a limiting case of a two country world, where the size of the domestic economy, n, is taken to be arbitrary small. The word economy is populated by a continuum of households of mass one. A fraction n of agents lives in the domestic economy and the remaining one, $1-n$, live in the foreign economy. In each economy households receives utility from consuming a continuum of differentiated consumption goods and desutility from working. Agents in each economy can freely trade goods and assets with foreign agents, asset markets are complete, thus domestic and foreign households can share risks efficiently. Each variety of domestic differentiated goods is produced using labour in a constant returns to scale technology by firms that operate in a monopolistic competitive market with sector specific productivity shocks.

2.1 Preferences

We assume the following period utility function on consumption and labour

$$E_0\left[\sum \beta^t\left(\log C_t - \frac{N_t^{1+v}}{1+v}\right)\right] \tag{2.1}$$

Where $0<\beta<1$ represents the subjective discount factor, and v the inverse of the Frish labour supply elasticity, N_t represents hours of labour and C_t, the consumption basket index. We choose log preferences on consumption, which imply a unitary intertemporal elasticity of substitution, since this particular parametrization allows to eliminate the effects of terms of trade on the economy, making much easier to understand the interplay between PD and optimal monetary policy. The domestic consumption index is defined by:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}}\left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\left(C_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{1-\eta}} \tag{2.2}$$

where η represents the elasticity of substitution between domestic and foreign consumption goods, $C_{H,t}$ and $C_{F,t}$ and α is a preference parameter that, when $\eta=1$, measures the fraction of consumption allocated in foreign goods parameter . In turn, the domestic consumption basket is defined as a compositive of a continuum of differentiated consumption goods defined by a CES aggregator function, as follows:

$$C_{H,t} = \left(\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_0^n C_{s,t}(z)^{\frac{\varepsilon-1}{\varepsilon}}\, d(z)\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.3}$$

where $\varepsilon > 1$ represents the elasticity of substitution between differentiated domestic consumption goods. Associated to these set of preferences there exist consumption based price index, P_t and the corresponding domestic and foreign price indices $P_{H,t}$ and $P_{F,t}$, respectively. These price indexes are defined as follows:

$$P_t = \left((1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{2.4}$$

and

$$P_{H,t} = \left(\frac{1}{n} \int_0^n P_{H,t}^{1-\varepsilon}(z)\, dz \right)^{\frac{1}{1-\varepsilon}} \tag{2.5}$$

where, $P_{F,t} = e_t P_{F,t}^*$ represents the price index of foreign goods expressed in domestic currency, e_t nominal exchange rate, the price of foreign currency in terms of domestic currency, and $P_{F,t}^*$ the price index of foreign goods in foreign currency, defined by a similar aggregator as equation (2.5).

2.2 Asset Market Structure

A complete set of stage contingent bonds, denoted by, $B_t(\varpi)$, is available for households to smooth consumption. At each period of time agents can purchase a particular state contingent bond at price, $\xi_{t,t+1}(\varpi)$, that delivers one unit of domestic currency in the next period if the state, ϖ, occurs. Each household receives income flows from their wages $W_t N_t$ and from the profits that firms distribute Ξ_t. For simplicity, we assume that each household owns an equal proportion of all firms in the economy, thus the budget constraint of the household can be easily written as follows:

$$P_t C_t + E_t (Q_{t+1} B_{t+1}) = W_t N_t + B_t + \Xi_t \tag{2.6}$$

2.3 Optimality Household conditions

Each household in the domestic economy maximizes her utility function given by equation (2.1), subject to their flow budget constraint, equation (2.6). Her corresponding first order conditions are given by the following set of equations,

$$\xi_{t+1} = \beta \left(\frac{C_{t+1}^{-1}}{C_t^{-1}} \frac{P_t}{P_{t+1}} \right) \tag{2.7}$$

$$C_t N_t^v = \frac{W_t}{P_t} \tag{2.8}$$

$$
\begin{array}{ll}
C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t & C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \\
C_{H,t}(z) = \frac{1}{n}\left(\frac{P_{H,t}(z)}{P_{Ht}}\right)^{-\eta} C_{H,t} & C_{F,t}(z) = \frac{1}{1-n}\left(\frac{e_t P^*_{F,t}(z)}{P_{Ft}}\right)^{-\eta} C_{H,t}
\end{array}
\tag{2.9}
$$

Equation (2.7), is the standard Euler condition that defines the optimal path of consumption and savings. Under complete markets the free risk interest rate can be obtained by taking the conditional expectation of the state contingent bond prices across all states of nature. Thus, in terms of the free risk nominal interest rate, the Euler equation can be alternatively written as follows,

$$
E_t\left(\xi_{t+1}\right) = \frac{1}{1+i_t} = \beta E_t\left(\frac{C_{t+1}^{-1}}{C_t^{-1}}\frac{P_t}{P_{t+1}}\right) \tag{2.10}
$$

Furthermore, by using the first order condition that determines optimal consumption of foreign households, we can obtain the following risk sharing condition,

$$
\xi_{t+1} = \beta\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-1}\frac{P^*_t}{P^*_{t+1}}\frac{e_t}{e_{t+1}} = \beta E_t\left(\frac{C_{t+1}^{-1}}{C_t^{-1}}\frac{P_t}{P_{t+1}}\right) \tag{2.11}
$$

Denoting by Q_t the real exchange rate, the relative price of foreign goods in terms of domestic goods, $Q_t = \frac{P^*_t e_t}{P_t}$ we can rearrange the previous condition and obtain the following recursive equation,

$$
Q_{t+1} = \left(\frac{C^*_{t+1}}{C_{t+1}}\right)^{-1}\left(\frac{C_t}{C^*_t}\right)^{-1} Q_t \tag{2.12}
$$

Following Chari, Kehoe and McGratan (2002) we iterate backwards the previous equation to obtain the following risk sharing condition that relates the real exchange rate to the dynamics of domestic and foreign consumption,

$$
Q_t = \varsigma_0\left(\frac{C^*_t}{C_t}\right)^{-1} \tag{2.13}
$$

where ς_0 is a constant defined as follows, $\varsigma_0 = \frac{C_0^{-1}}{\left(C_0^*\right)^{-1}}Q_0$. Equation (2.12) summarizes the implication of complete markets for risk sharing. In an economy with complete markets agents can perfectly smooth consumption, therefore, the real exchange rate that measures the relative price of domestic and foreign consumption index becomes proportional to the ratio of consumption levels abroad and domestically. On the other hand, equation (2.8) determines labour supply. The household would supply labour up to the point where the marginal desutility of working equalizes its marginal benefit, given by the real wage expressed in units of utility. The disutility allocation of consumption across different types of consumption goods is determined by equation (2.9). The household allocates consumption among the different varieties of consumption goods by minimizing the total expenditure that the consumption of C_t involves. At the optimal the household demand for each type of consumption good is increasing in the level of total consumption, C_t and decreasing in their corresponding relative price. In the rest of the world households solve an identical problem to one detailed above. Therefore, a set of similar optimality conditions describes their behavior.

2.4 Firms

Consumption goods are produced by continuum of monopolistic competitive firms distributed between zero and one. Each firm produces using a constant returns to scale technology that transform labour services $N_t(z)$ into a particular variety of final consumption good:

$$A_t(z)Y_{m,t}(z) = N_t(z) \tag{2.14}$$

where $A_t(j)$ represents a negative technology shock, since, higher the value of $A_t(j)$ the higher the amount of labour required to produce the same amount of final output. We assume that $A_t(j)$ follows the following stochastic process,

$$\ln(A_t(z)) = \epsilon_t \tag{2.15}$$

with $\epsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$. The nominal marginal cost of a typical firm, z, is given by $nmc_t(z) = w_t A_t(z)$. Furthermore, we define the aggregate level of technology, A_t as an aggregate variable over the individual technology shocks,$A_t(z)$ using an aggregator function similar to one used for prices. Thus A_t is defined as follows,

$$A_t = \left(\frac{1}{n} \int_0^n A_t^{1-\varepsilon}(z)\, dz \right)^{\frac{1}{1-\varepsilon}}$$

For convenience we express the marginal cost in real terms, no respect to the consumer price index, but to respect the price that each firms set. This alternative representation of real marginal costs alleviates notation later when the Phillips curve is derived. Thus, the real marginal cost is given by,

$$mc_t(z) = \frac{mc_t A_t(z)}{Z_{j,t} Z_{H,t}} \tag{2.16}$$

where, $mc_t = \frac{W_t}{P_t}$ represents the real marginal cost in terms of the consumer price index, and $Z_{j,t} = \frac{P_{H,t}(j)}{P_{H,t}}$ and $Z_{H,t} = \frac{P_{H,t}}{P_t}$ represent the relative price of firm, j, respect to domestic prices and of domestic prices respect to the consumer prices, respectively. Each domestic producer faces a downward sloping demand function. We obtain those demands functions for a particular good z by aggregating the corresponding ones for both the domestic and foreign households ,

$$Y_{s,t}(z) = \int_0^n C^i_{H,t}(z)d(i) + \int_n^1 \left(C^*_{F,t}\right)^i (z)d(i) \tag{2.17}$$

Using equation(2.9) we can write the previous equation as follows:

$$Y_{H,t}(z) = \left(\frac{P_{H,t}\left(z\right)}{P_{h,t}}\right)^{-\varepsilon} Y_{H,t} \tag{2.18}$$

Where, $Y_{H,t}$ represents the total demand for domestic goods, which can be written as follows:

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left(\left(1-\alpha\right)C_t + \frac{\left(1-\alpha^*\right)\left(1-n\right)}{n} Q_t^\eta C_t^*\right) \tag{2.19}$$

Similarly, foreign firms face a downward demand function given by,

$$Y_{F,t}(j) = \left(\frac{P_{F,t}\left(j\right)}{P_{F,t}}\right)^{-\varepsilon} \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} \left(\frac{n}{1-n}\alpha C_t + \alpha^* Q_t^\eta C_t^*\right) \tag{2.20}$$

Where, α^* represents the participation of foreign goods in the consumption basket of foreign households. Notice that in this model, domestic consumption affects foreign demand through imports of foreign goods. However, since we want to focus on the case of a small open economy, SOE, here after, in the next subsection we derive this case by making the size of the domestic economy arbitrary small.

2.5 The Small Open Economy

Following Sutherland (2001) and De Paoli (2004) we set the parameter α that represents the participation of foreign goods in the consumption basket of domestic households, as a function of the size of the economy, n and its degree of openness, γ, such that $\alpha = (1-n)\gamma$. Similarly for the case of the foreign economy, $1-\alpha^* = n\gamma$. With this parametrization, domestic households consume more imported foreign goods when the economy is more open, this is when γ is larger, or when the size of domestic economy, n, is relatively small.

The SOE, is obtained as the particular case when the size of the domestic economy becomes arbitrary small, $n \to 0$. In this case, the participation of foreign goods in the domestic economy is given by γ, its maximum value under this parametrization, and foreign households consume only foreign produced final goods, $\alpha^* = 1$, consequently, changes of home aggregated demand have no effect on the foreign economy and the consumer price index in the foreign economy coincides with its producer price index $P^* = P_F^*$. When the economy is small, the demand functions for domestic and foreign final consumption goods, equations (2.19) and (2.20) become:

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left(\left(1-\gamma\right) C_t + \gamma Q_t^{\eta} C_t^*\right) \tag{2.21}$$

$$Y_{F,t}(j) = \left(\frac{P_{F,t}\left(j\right)}{P_{F,t}}\right)^{-\epsilon} \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} \left(Q_t^{\eta} C_t^*\right) \tag{2.22}$$

In what follows we restrict our analysis to the particular case of $\eta = 1$. We choose this parametrization because it allow us to eliminate the effects of terms of trade in the economy thus we can highlight the interaction between price dollarization and monetary policy. Under this parametrization equation (2.21) become,

$$P_{H,t} Y_{H,t} = P_t C_t \tag{2.23}$$

2.6 CPI inflation, Real exchange rate and terms of trade

An equation relating inflation of consumer price index, π_t, domestic inflation, $\pi_{H,t}$ and terms of trade, T_t, can be obtained from equation (2.4), as follows,

$$\left(\frac{\pi_t}{\pi_{H,t}}\right) = \left(\frac{T_t}{T_{t-1}}\right)^{\gamma} \tag{2.24}$$

Where, T_t, is defined as the price of foreign goods in terms of domestic goods:

$$T_t = \frac{P_{F,t}}{P_{H,t}} \tag{2.25}$$

since the domestic economy is small and the law of one price holds, the price of foreign goods is $P_{F,t} = e_t P_t^*$, therefore we have the following relationship between the terms of trade and the real exchange rate,Q_t ,

$$Q_t = T_t^{1-\gamma} \tag{2.26}$$

Whereas, the domestic relative price, $Z_{H,t}$ is related to terms of trade and the real exchange rate through the following condition,

$$Z_{H,t} = \frac{Q_t}{T_t} \tag{2.27}$$

Moreover, from the definition of terms of trade, we can link this latter variable to domestic inflation, foreign inflation and the depreciation of the nominal exchange rate,

$$T_t = T_{t-1}\frac{(1+\Delta e_t)\,(1+\Pi_t^*)}{(1+\Pi_{H,t})} \tag{2.28}$$

2.7 Price setting

A fraction θ of firms in the domestic economy can set prices observing the realization of all shocks, whereas the remaining fraction, $(1-\theta)$ set prices one period in advance. Among these latter subset of firms a smaller group of them, of mass s, choose to set their prices in foreign currency. Notice that the choice of unit of account for firms that can set prices observing the realizations of shocks is irrelevant since they can always choose an equivalent price in dollars for the corresponding optimal price in pesos by simple dividing the price in peso by the current nominal exchange rate. Pricing in foreign currency becomes relevant only when firms face uncertainty about the realization of shocks.

Thus in order to set prices, a typical firm, z, choose a price $P^o_{m,t}(z)$ to maximizes the expected discounted value of its flow of profits given by:

$$\Omega(z) = E_{t-1}\left[\left((P_{H,t}(z) - W_t A_t(z)\right)Y_t(z)\frac{1}{P_t C_t}\right] \tag{2.29}$$

and the demand for good z in period t given a fixed price, by the following condition,

$$Y_t(z) = \left(\frac{P_{H,t}\,(z)}{P_{H,t}}\right)^{-\epsilon} Y_{H,t} \tag{2.30}$$

Using the previous definition, equation (2.29), we can write the first order condition of a typical firm as follows,

$$E_{t-1}\left[\left((P_{H,t}(z) - \mu W_t A_t(z)\right)\left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\varepsilon}\frac{Y_{H,t}}{P_{H,t}}\frac{P_{H,t}}{P_t C_t}\right] = 0 \tag{2.31}$$

After solving for the optimal price of firm, z we have that,

$$P_{H,t}(z) = \mu\frac{E_{t-1}\left(W_t A_t(z)\,P_{H,t}^{\varepsilon-1}\right)}{E_{t-1}\left(P_{H,t}^{\varepsilon-1}\right)} \tag{2.32}$$

As we highlighted before, in this economy firms have the option to set prices using a different unit of account, the dollar. Thus every firm also solve their problem, by considering that prices are being set in foreign currency. Let's define the price of an individual domestic consumption good in dollars by $d_{H,t}(z)$ and the aggregate domestic price level expressed in dollars by, $d_{H,t}$, then, similarly to the firms problem when prices are set in pesos, the optimal dollar price for firm set will maximizes the following expected profit function,

$$\Psi(z) = E_{t-1}\left[\left(\left(d_{H,t}(z) - \frac{W_t}{e_t}A_t(z)\right)\right)Y_t(z)\frac{e_t}{P_tC_t}\right] \tag{2.33}$$

subject to the following demand constraint,

$$Y_{H,t}(z) = \left(\frac{d_{H,t}(z)}{d_{H,t}}\right)^{-\varepsilon} Y_{H,t} \tag{2.34}$$

Similarly to case of the pricing in pesos, the first order condition for pricing in dollars is given by the following condition,

$$d_{H,t}(z) = \mu\frac{E_{t-1}\left(\frac{W_t}{e_t}A_t(z)\right)d_{H,t}^{\varepsilon-1})}{E_{t-1}\left(d_{H,t}^{\varepsilon-1}\right)} \tag{2.35}$$

where $d_{H,t}$ represents the aggregate price index, $P_{H,t}$ expressed in foreign currency, thus it can be determined, which can be obtained using the following equation,

$$d_{H,t}e_t = P_{H,t}$$

3 The Dynamic Equilibrium

3.1 The steady state

In the steady state, the economy with PD behaves identically to an SOE economy without PD. In appendix A we provide a detailed derivation of the steady-state for this economy. In particular, since we assume a symmetric structure for both the domestic and the foreign economy, the level of domestic output is identical to that of the foreign one and depend only on the monopolistic competition distortion,Φ,

$$Y_H = Y^* = (1-\Phi)^{\frac{1}{\sigma+v}} \tag{3.1}$$

where, $1-\Phi = \frac{1+\tau}{\mu}$, and the real exchange rate and terms of trade are equal to 1. Also, since these relative prices are equal to 1 in steady-state, trade balance is nil and it holds that $C = Y_H$. However, notice that the level of output is distorted, since it is below its efficient level of 1. This distortion is generated by the degree of monopolistic competition existent in the economy. As in Woodford (2003), we assume that the government uses fiscal policy, more precisely, a subsidy, τ, to eliminate this distortion in output by properly choosing τ. With this extra assumption, the only distortion that would remain in the dynamic equilibrium with sticky prices is the distortion that inflation and idiosyncratic productivity shocks generate when prices do not fully adjust in response to shocks.

3.2 The flexible price Equilibrium

When prices are flexible firms can set prices every period observing the realizations of all shocks, therefore the pricing strategy of firms is irrelevant for the equilibrium allocation. In order to show this point, lets look at the optimal prices that a particular firm z would choose when setting prices in pesos and when setting prices in dollars. Under price setting in pesos and dollars the corresponding optimal prices for firm z will be given by,

$$P_{H,t}(z) = \mu W_t A_{z,t} \quad d_{H,t}(z) = \mu \tfrac{W_t}{e_t} A_{z,t} \tag{3.2}$$

From the previous equation is clear that since firms can perfectly observe their productivity shock, $A_{z,t}$ and the nominal exchange rate, it holds that,

$$P_{H,t}(z) = d_{H,t}(z) e_t \tag{3.3}$$

thus, the amount of good, z produced will be exactly the same in both cases, since the relative price of good z under pricing in dollars or pesos would be exactly the same. As Klemperer and Meyer (1986) have pointed out, the currency denomination of prices only affects the resources allocation of the firm when there is uncertainty. When prices are flexible firms do not face any kind of uncertainty since they know all the relevant variables for deciding how to allocate their resources. In contrast when prices are sticky, firms face uncertainty about future demand and cost conditions, making the pricing strategy a relevant one. Also, as it is show in appendix B, the flexible price level of output, up to a log linear approximation around the steady-state, the natural interest rate and the real exchange rate of the economy under PD can be characterized by the following set of equations,

$$y^n_{H,t} = -a_t \tag{3.4}$$

$$r^n_t = -E_t\left(a_{t+1} - a_t\right) \tag{3.5}$$

$$q_t = -\left(1-\gamma\right)\left(a_t - a^*_t\right) \tag{3.6}$$

From the previous equations notice that, even though our economy is open, both the natural level of output and the natural interest rate do not depend on foreign shocks. In that sense, these two equations are very alike the ones characterizing the flexible price allocation of a close economy. We obtain this result only because we have chosen a very special type of preferences, ones that exhibits both unitary intertemporal elasticity of substitution and unitary elasticity of substitution between domestic and foreign goods. Under this type of preferences, the substitution and income effect that movements in terms of trade generate cancel out each other eliminating any trade balance[3]. For a more general setup, both domestic output and real interest rate respond to foreign shocks even under perfectly flexible prices.

3.3 The Price Stickiness Distortion

As we discussed previously, PD does neither affect the steady-state nor the flexible price dynamic equilibrium of the economy. However, it does affect the sticky price equilibrium in a fundamental way. In an environment with sticky prices, idiosyncratic productivity shocks generate relative price distortions that reduce welfare. In order to illustrate this point, let's define the aggregate usage of labour in the economy by,

$$h_t = \int_0^1 h_t(z)d(z) \tag{3.7}$$

since,

$$h_t(z) = Y_t(z)A_t(z) = \left(\frac{P_{H,t}(z)}{P_t}\right)^{-\varepsilon} Y_{H,t} \tag{3.8}$$

Thus h_t can be written in terms of domestic output, $Y_{H,t}$,aggregate productivity, A_t, and relative price distortion term, Δ_t as follows,

$$h_t = Y_{H,t}A_t\Delta_t \tag{3.9}$$

where,

$$\Delta_t = \int_0^1 \left(\frac{P_{H,t}(z)}{P_t}\right)^{-\varepsilon} \frac{A_t(z)}{A_t} d(z) \tag{3.10}$$

Notice that when,$\Delta_t > 1$, there exist an extra cost of producing $Y_{H,t}$, given by the additional real resources required to produce this amount of output, $(\Delta_t - 1)$. This extra cost

[3] See Gali and Monacelli (2005) for a detailed discussion of a canonical representation of an small open economy, and De Paoli (2005) for its implications in optimal monetary policy.

of production is generated by price stickiness. When prices are sticky, some goods that are relatively expensive to produce are sold relatively cheap. In this case, the efficiency condition that equalizes the marginal rate of substitution to the marginal rate of transformation of any two goods does not hold. As we show in appendix, C, up to second order this cost can be expressed as follows,

$$\begin{aligned} \widehat{\Delta}_t \quad \simeq \quad & var_z a_t + \frac{1}{2}\varepsilon^2 \left(p_{h,t}(z) - p_{h,t}\right)^2 \\ & -\varepsilon \left(a_t(z) - a_t\right)\left(p_{h,t}(z) - p_{h,t}\right) \end{aligned} \tag{3.11}$$

From (3.11) it is clear that when $p_{h,t}(z) = p_{h,t}$ and $var_z a_t = 0$, the economy reaches its efficient allocation, one in which labour usage for production is not distorted by relative prices. This allocation can be achieved by a policy that fully stabilizes domestic inflation in an economy where there are no sector specific technology shocks. However, when there exist sector specific shocks, zero domestic inflation is not anymore the optimal policy, since the central bank can do better by inducing a positive correlation between relative prices and idiosyncratic productivity shocks.

In an open economy, the central bank can induce a correlation between the aggregate price level and sector specific technological shocks, if some firms set prices in dollars, by making the nominal exchange rate to react to these type of shocks. In turn, this potential behavior of the central bank generates the necessary conditions for some firms to choose to set prices in dollars.

We will discuss in detail in sections 4 and 5 the optimal policy of the central bank and the optimal pricing strategy of firms. However, for this section it is important to highlight that the interactions of sector specific shocks and sticky prices generates the incentives for the central bank to use more actively the exchange rate and for firms to set prices in a foreign currency. Next we derive the aggregate supply and demand of this economy, building blocks for analyzing the design of optimal policy.

3.4 Aggregate Supply

We derive the aggregate supply equation of this economy by aggregating over the continuum of firms the log linear approximation of their optimal pricing rule. In this economy, at each point in time there exist three type of firms differentiated by their pricing strategy. A first type, of mass θ, set prices flexibly observing the realization of all shocks in the economy. The remaining fraction, $1-\theta$, set prices one period in advance, thus they use only information up to period, $t-1$. From this second group of firms, a fraction $(1-\theta)\, s$ sets prices in foreign currency, whereas the remaining one, $(1-\theta)(1-s)$ does it in pesos. The fraction s will be endogenously determined as an equilibrium property of the equilibrium of the economy in section 5.

Since firms that set prices in advance are chosen randomly we define by Θ the set with mass θ of firms with flexible prices and by Σ the set with mass $(1-\theta)\,s$ of firms with sticky prices in dollars. Up to a first order approximation the aggregate domestic price index, defined in equation (2.5), is given by,

$$p_{h,t} = \int_0^1 p_{h,t}(z)d(z) \tag{3.12}$$

Notice, that the optimal pricing rules of the three types of firms given by equations (3.2), (2.32) and (2.35) up to a log linear approximation around the steady-state are given by,

$$p_{h,t}(z) - p_{h,t} = \begin{cases} w_t + a_t(z) \\ E_{t-1}\left(w_t + a_t(z)\right) - \left(p_{h,t} - E_{t-1}p_{h,t}\right) \\ E_{t-1}w_t + d_{h,t} + a_t(z)) - \left(p_{h,t} - E_{t-1}p_{h,t}\right) + e_t - E_{t-1}e_t \end{cases}$$

where, w_t represents real wages in terms of domestic prices. Notice that for firms that set prices flexible, up to a log linear approximation, the optimal relative prices is equal to the real marginal cost, first row in the previous table. For this group of firms neither unexpected inflation nor unexpected depreciation affects its relative price. In contrast, for firms with sticky prices, unexpected inflation reduces their relative prices, and for firms with prices in dollars, unexpected depreciation of the domestic currency increases their relative price. Aggregating over firms, we obtain the following condition for the aggregate supply,

$$\begin{aligned} p_{h,t} - E_{t-1}p_{h,t} &= \frac{\theta}{1-\theta}\left(w_t + a_t\right) + \left(E_{t-1}\left(w_t + a_t\right)\right) \\ &\quad + s\left(E_{t-1}\left(e_t\right) - e_t\right) \end{aligned} \tag{3.13}$$

Taking conditional expectations in period $t-1$ to this previous equation, we can easily show that, $E_{t-1}\left(w_t + a_t\right) = 0$, thus, we have,

$$p_{h,t} - E_{t-1}p_{h,t} = \frac{\theta}{1-\theta}\left(w_t + a_t\right) + s\left(E_{t-1}\left(e_t\right) - e_t\right) \tag{3.14}$$

Since, from the log linear approximation of equation (2.8) and equation (2.23) we obtain,

$$w_t = vh_t + c_t - z_{h,t} \tag{3.15}$$

$$c_t = y_{h,t} + z_{h,t} \tag{3.16}$$

By using equations (3.15), and (3.16) to eliminate real wages and the aggregate productivity shock, we obtain the following aggregate supply equation in terms of output gap, unexpected changes in prices and in the nominal exchange rate,

$$p_{h,t} = E_{t-1}\left(p_{h,t}\right) + \kappa x_t + s\left(e_t - E_{t-1}\left(e_t\right)\right) \tag{3.17}$$

Where, $x_t = y_t - y_t^n$, $y_t^n = -a_t$ and $\kappa = (1+\nu)\frac{\theta}{(1-\theta)}$. Notice that when, $s = 0$, the aggregate supply curve converges to the standard case of small open economy without price dollarization. Differently when a positive mass of firms sets prices in dollar, $s \neq 0$ unexpected changes in the nominal exchange rate, show up as a cost push shock in the aggregate supply curve. Therefore, to stabilize domestic prices, the central bank has to set both the output gap and the nominal exchange rate equal to zero. Using a simple transformation, equation (3.17), we obtain the Phillips curve in this economy,

$$\pi_{h,t} = E_{t-1}\pi_{h,t} + \kappa x_t + s\left(\Delta e_t - E_{t-1}\left(\Delta e_t\right)\right) \tag{3.18}$$

where, κ represents the slope of the Phillip Curve

3.5 Aggregate Demand

We derive the aggregate demand equation combining the following log linear approximation equations of the Euler condition, the demand for domestic goods, the risk sharing condition and consumer price index aggregator,

$$c_t = E_t c_{t+1} - \left(i_t - E_t \pi_{t+1}\right) \tag{3.19}$$

$$y_{H,t} = \frac{\gamma q_t}{1-\gamma} + c_t \tag{3.20}$$

$$q_t = c_t - y_t^* \tag{3.21}$$

$$\pi_t = \pi_{H,t} + \frac{\gamma}{1-\gamma}\Delta q_t \tag{3.22}$$

As we show in appendix, B, the aggregate demand equation in terms of output gap consistent with the previous four equations, is given by,

$$x_t = E_t x_{t+1} - \left(i_t - E_t \pi_{H,t+1} - r_t^n\right) \tag{3.23}$$

The other relevant condition that comes from the aggregate demand section is the one that determines the dynamics of the nominal exchange rate. The nominal exchange rate in this simple economy depends on domestic prices, the output gap and a compositive real exchange rate shock,

$$e_t = p_{H,t} + x_t + \eta_t \tag{3.24}$$

Where η_t, the composite real exchange rate shock is defined as follows,

$$\eta_t = -x_t^* + a_t^* - \pi_t^* - a_t \tag{3.25}$$

Thus our SOE with PD is fully determined by equations (3.18), (3.23) and (3.24). Notice that when, $s = 0$ our model economy collapses to a standard SOE, where there does not exist a trade-off between stabilizing domestic inflation and output gap.

4 Optimal Monetary Policy

In this section we follow closely Woodford (2003) and Benigno and Woodford (2004) in deriving a microfundated lost function for a central bank in an economy with PD. This lost function is obtained from a second order Taylor expansion of the utility function of the representative household around a deterministic steady-state. In order to simplify the analysis, we focus in an steady-state that is efficient, since in this case, a log linear solution of the rational expectations equilibrium of the model is enough to obtain an accurate measure of welfare. When the steady-state is not efficient, a second order solution of the rational expectations dynamic equilibrium is required. First, in order to fully characterize the efficient steady-state allocation we solve for the social planner problem. Then, we approximate the household welfare function using a second order Taylor expansion around this efficient steady-state. Finally, we analyze optimal monetary policy based on the microfundated welfare function obtained previously.

4.1 Optimal allocation

Since the economy is distorted by monopolistic competition and terms of trade, it is helpful to solve the problem of the social planner to fully characterize the optimal allocation in this economy. We assume that the social planner is a benevolent one, therefore it chooses an allocation that maximizes the welfare of the representative household, given by:

$$W = E_t \left[\sum_{t=0}^{t=\infty} \beta^t \left(\ln C_t - \frac{N_t^{1+v}}{1+v} \right) \right] \tag{4.1}$$

Subject to the resource constraint that, given the particular parametrization we have chosen, can be written as follows:

$$C_t = Y_t^{1-\gamma} Y_t^{*\gamma} \tag{4.2}$$

the production function

$$Y_t = \frac{h_t}{A_t \Delta_t} \tag{4.3}$$

where, Δ_t accounts for the distortion that inflation generates in the economy through relative price dispersion, defined in equation(3.10). Let's first solve for the conditions that characterize an steady-state equilibrium with zero inflation.

$$\Delta_t = 1 \tag{4.4}$$

Therefore, in steady- state, equation (4.2) becomes:

$$C = Y^{1-\gamma} Y^{*\gamma} \tag{4.5}$$

In this case, the first order condition for the efficient allocation of consumption and labour is given by:

$$v = (1-\gamma)\frac{C}{h} \tag{4.6}$$

Which can be also written in a more convenient way as follows:

$$Yv = C(1-\gamma) \tag{4.7}$$

Notice however that this condition for efficient allocation in steady-state, differs from its analog under the decentralized equilibrium. In this case, the marginal utility of consumption and the marginal disutility of labour differ by a factor,$(1-\Phi)$ that accounts for the distortion that monopolistic competition generates.

$$(1-\Phi)\,C = vY \tag{4.8}$$

where: $(1-\Phi) = \frac{1-\tau}{\mu}$, and τ represents a subsidy or tax, that can be used to eliminate the distortion that these two features of the economy generate. Thus, in order to make the steady-state of the decentralized equilibrium compatible with the efficient allocation that the social planner determines, we choose τ such that, $(1-\Phi) = (1-\gamma)$.

4.2 The Central Bank Welfare Lost Function under Price Dollarization

In order to obtain the lost function of the central bank, we follow most of the recent literature on optimal monetary policy by assuming that the central bank implement its policy aiming at maximizing the welfare of the representative agent in this economy. In particular we follow Woodford (2003)[4], and we approximate equation (4.1) by taking a second order Taylor expansion around the efficient steady-state equilibrium. Up to second order of accuracy, the welfare function of the representative agent in an economy with PD can be written as a quadratic function of output gap, $\widehat{x}_t$, unexpected changes in domestic prices, $\widetilde{p}_{h,t}$, unexpected changes in the nominal exchange rate, $\widetilde{e}_t$ the correlation between $\widetilde{p}_{h,t}$ and $\widetilde{e}_t$; and the correlation between $\widetilde{e}_t$ and $\widetilde{a}_{s,t}$. Where this last term represents, the average productivity dispersion of firms that have set prices in dollars. Thus, the lost function that the central bank aims at minimizing in an economy with PD, is given by the following equation,

$$-\frac{\Omega}{2}\sum_{t=0}^{t=\infty}\beta^t\left(\Lambda\widehat{x}_t^2+\widetilde{p}_{h,t}^2+\Lambda_e\widetilde{e}_t^2+2s\widetilde{p}_{h,t}\widetilde{e}_t-2\theta\widetilde{e}_t\widetilde{a}_{s,t}\right) \tag{4.9}$$

where, $\Omega=\overline{u}_c\overline{Y}\left(1-\gamma\right)\varepsilon^2\frac{(1-\theta)}{\theta}$, $\Lambda_e=s\theta\left(1+\frac{s(1-\theta)}{\theta}\right)$, $\Lambda=\left(1+v\right)\frac{\theta}{(1-\theta)\varepsilon^2}$ and
$\widetilde{a}_{s,t}=\int\limits_{\Sigma}\left(a_t(z)-a_t\right)d(z)$, $\widetilde{p}_{h,t}=\left(p_{h,t}-E_{t-1}p_{h,t}\right)$, and $\widetilde{e}_t=\left(e_t-E_{t-1}e_t\right)$

Several remarks are in order to qualify this welfare function. First, since we assume that fiscal policy is used to eliminate the distortion that monopolistic competition creates in production, the optimal target for the output gap is zero. Second, in an economy with PD exchange rate volatility generate welfare losses, thus the central bank has incentives to smooth exchange rate fluctuations. In that sense we can argue that it is optimal for the central bank to exhibit some degree of "fear of floating", in the terminology of Calvo and Reinhart (2001). How much fear of floating, it will depend on the degree of PD.

Third, the central bank can reduces welfare losses by generating a positive correlation between productivity shocks of firms with prices in dollars and the nominal exchange rate. The intuition of this effect is simple. Since firms with prices in dollars can not change their prices after the realization of shocks, a negative productivity shock, an increase in $a_t(z)$ deviates their relative price from its optimal level, generating more price distortions in the economy. Since price distortions generate efficiency losses, the central bank has the incentive to partially offset the effect of these shocks by increasing the nominal exchange rate. With a depreciation of the domestic currency, the relative price of firms affected by the negative productivity shock will increase, thus the gap of their relative price with its optimal value will diminish.

[4] The appendix D shows the details of this derivation

Similarly, in a economy with PD, according to equation (4.9), a positive correlation between the nominal exchange rate and domestic inflation generate welfare losses since this correlation increases the dispersion of relative prices between firms the fix prices in pesos and those that do it in dollars. It is important to notice that in an economy with PD the first best allocation is unattainable, since the central bank has just one instrument, either inflation, or the nominal exchange rate, but more than one objective, $p_{s,t} = e_{d,t} = \widehat{x}_t = 0$. Therefore, optimal monetary policy can achieve only a second best.

4.3 Optimal Monetary Policy Under Commitment

We use the lost function of the central bank, equation (4.9) to analyse the implementation of optimal monetary policy under commitment. To implement its policy, the central bank minimizes equation (4.9) subject to the Phillips curve, and the dynamics of the nominal exchange rate, equations (3.18) and (3.24) respectively[5]. The first order condition of this problem is given by the following equation,

$$(\Lambda_e + s)\,\widetilde{e}_t - \theta\widetilde{a}_{s,t} + (1+s)\,\widetilde{p}_{s,t} = 0 \tag{4.10}$$

Under optimal policy it is optimal for the central bank to generate unexpected movements in both the nominal exchange rate and the domestic prices that respond to productivity shocks of those firms that have set prices in pesos. Thus, the central is using the nominal exchange rate and domestic prices to try to minimize the relative price distortions that sector specific shocks generate when prices are sticky. Equations (4.10), (3.18) and (3.24) fully describe the rational expectations equilibrium of this economy. From the Phillips curve is easy to show that $E_{t-1}x_t = 0$, thus equation (3.24) can be written as follows,

$$\widetilde{e}_t = \widetilde{p}_{h,t} + x_t + \widetilde{\eta}_t \tag{4.11}$$

Using equations (4.11), (4.10) and (3.18) we can find the rational expectations equilibrium of this economy under optimal policy. Thus we have that under optimal policy the nominal exchange rate, the domestic prices and the output gap are given by,

$$\widetilde{e}_t = \varpi_1\widetilde{\eta}_t + \varpi_2\widetilde{a}_{s,t} \tag{4.12}$$

$$\widetilde{p}_{h,t} = -\varpi_3\widetilde{\eta}_t + \varpi_4\widetilde{a}_{s,t} \tag{4.13}$$

[5]The details of the Central Bank Problem under commitment are presented in appendix C

$$x_t = -\frac{(\varpi_3 + s\varpi_1)}{\kappa}\widetilde{\eta}_t + \frac{(\varpi_4 - s\varpi_2)}{\kappa}\widetilde{a}_{s,t} \tag{4.14}$$

Where the parameters, ϖ_1, ϖ_2,ϖ_3 and ϖ_4 are defined as follows,

$$\begin{array}{ll} \varpi_1 = \frac{\kappa(1+s)}{dd} & \varpi_2 = \frac{\theta(1+\kappa)}{dd} \\ \varpi_3 = \frac{\kappa(\Lambda_e+s)}{dd} & \varpi_4 = \frac{(\kappa+s)\theta}{dd} \end{array}$$

where,

$$dd = [(\kappa + s)(1 + s) + (1 + \kappa)(\Lambda_e + s)]$$

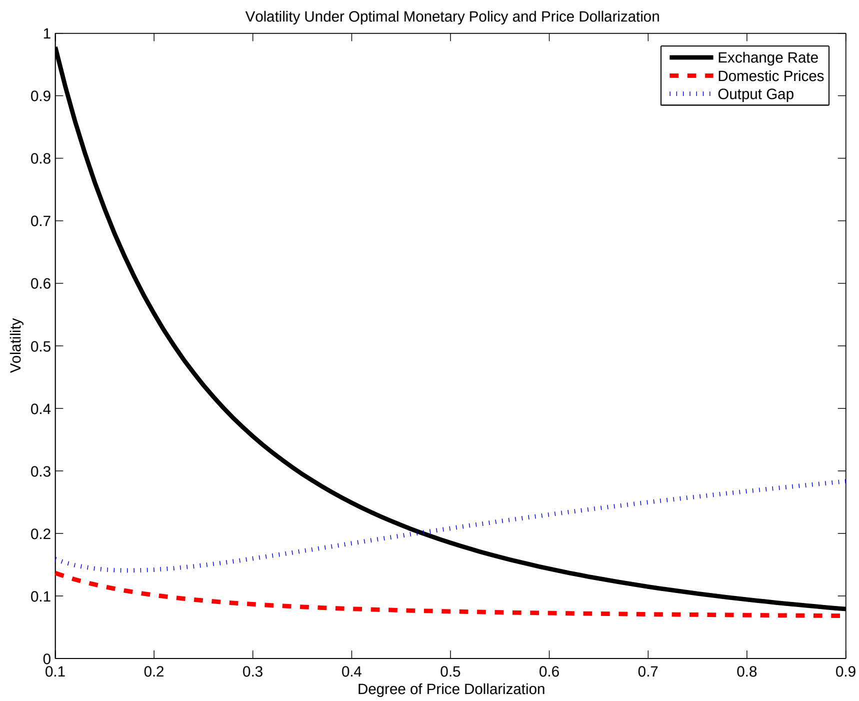
Notice that in the allocation under optimal monetary policy the nominal exchange rate is not fully flexible in the sense that respond not only to real exchange rate shocks, but also to domestic productivity shocks of a particular group of firms, those with prices in dollars. More precisely, when a negative productivity shocks hits firms with prices in dollars the central bank increases the nominal exchange rate. This reaction of the nominal exchange rate reduces relative price distortions, since with a higher nominal exchange rate, the relative price of dollar firms increases partially absorbing the impact of the negative productivity shock. When a positive real exchange rate shock hits the economy, the nominal exchange rate depreciates, as in economies without PD

Domestic prices also react to productivity shocks of firms with prices in dollars. As in the previous case, when $\widetilde{a}_{s,t}$ increases, goods with prices in dollars become relatively cheap respect to peso goods, an increase in domestic prices reduce this gap. Also, domestic prices fall when a negative real exchange shock hits the economy, since the nominal exchange do not fully adjust to real exchange rate shock, domestic prices and the output gap absorb part of the effect of this shock in the economy. Notice that a feature of the equilibrium of this economy under optimal monetary policy is that output gap falls when the real exchange rate increases. This effect of real exchange rates is explained by the fact that the central bank smooths the exchange rate, the economy can not reach its potential level of output, thus the output gap becomes negative.

It is interesting to notice that when there is no PD, this is when, $s = 0$ the rational expectations equilibrium under optimal policy for domestic inflation, the nominal exchange rate and the output gap are given by,

$$\begin{array}{ll} \widetilde{p}_{h,t} = 0 & \widetilde{e}_t = \widetilde{\eta}_t \\ \widetilde{x}_t = 0 & \end{array}$$

In this case, it is optimal for the central bank to make the price level constant, therefore, no unexpected inflation is generated in equilibrium, and to let the nominal exchange rate to float freely. In this equilibrium the nominal exchange rate fluctuates only responding to real exchange shocks. On the other hand, since without PD, the output gap is proportional to unexpected inflation, this latter variable will be also equal to zero in this equilibrium. In order to illustrate how the domestic price, the nominal exchange rate and the output gap are affected by the degree of PD when the central bank implements monetary policy optimally, we use a calibrated version of the model, where we set $\theta = 0.5$, $v = 1.5$ and $\varepsilon = 10$ and use conditions (4.12), (4.13) and (4.14) to show the implied variance of these three variables for different degrees of PD. The results are depicted in the next graph,



As the previous graph shows, when the degree of PD rises in the economy, the volatility of the nominal exchange rate and inflation falls, and the corresponding one for the output gap increases. This evolution in the volatilities of these three variable reflects the optimal response of the central bank. When the degree of PD increases, the weight that the nominal exchange rate has in the welfare lost of the central bank increases, consequently also the incentive of the central bank to smooth the exchange rate. Since the Central Bank has a limited number of instruments it can not simultaneously achieve all of its objectives, therefore, as exchange rate become more important, the central bank has to put less weight in output gap stabilization, thus, in equilibrium output gap volatility increases..

Up to this section we have derived results for optimal monetary policy but assuming that the degree of PD is given. However, this variable is not exogenous to monetary policy, by the contrary, it is determined by monetary policy. In the next section, we show how firms decide which currency to use for setting its prices and the interaction of this decision with optimal monetary policy.

5 Endogenous Price Dollarization

In this economy every firm decides in which currency to set prices by comparing the expected profits they obtain when setting prices in pesos with those obtained when prices are set in dollars. Let's denote by $\Omega(z)$ the level of profits in real terms that firm z will obtain under peso pricing, and by $\Psi(z)$ the level of real profits under dollar pricing. Firm z will set prices in pesos if and only if the expected profits of setting prices in pesos excceds the corresponding one of setting prices in dollars, thus for a firm to set prices in pesos it must be true that ,

$$\frac{E_{t-1}\Omega(z)}{E_{t-1}\Psi(z)} > 1 \tag{5.1}$$

Equations (2.32) and (2.35) gives us the optimal price levels under each pricing strategy. Plugging in these optimal pricing rules into their corresponding profit functions, equations (2.29), (2.33) allow us to rewrite the optimal profit function in a much simpler way,

$$\Omega(z) = \frac{(\mu-1)\,\mu^{-\epsilon}\left(E_{t-1}\left(w_t A_t(z) P_{H,t}^{\epsilon}\right)\right)^{1-\epsilon}}{E_{t-1}\left(P_{H,t}^{\epsilon-1)}\right)^{-\epsilon} P_t} \tag{5.2}$$

similarly for profits under dollar pricing we have,

$$\Psi(z) = \frac{(\mu-1)\,\mu^{-\epsilon}\left(E_{t-1}\left(w_t A_t(z) d_{H,t}^{\epsilon}\right)\right)^{1-\epsilon}}{E_{t-1}\left(d_{H,t}^{\epsilon-1}\right)^{-\epsilon} P_t e_t} \tag{5.3}$$

Using equations (5.2) and (5.3), the condition for setting prices in pesos, equation ( ) can be written as follows,

$$\frac{\left(E_{t-1}\left(w_t A_t(z) P_{H,t}^{\epsilon}\right)\right)^{1-\epsilon}}{\left(E_{t-1}\left(w_t A_t(z) d_{H,t}^{\epsilon}\right)\right)^{1-\epsilon}} \frac{\left(E_{t-1}\left(p_{H,t}^{\epsilon-1}\right)\right)^{\epsilon}}{\left(E_{t-1}\left(d_{H,t}^{\epsilon-1}\right)\right)^{\epsilon}} > 1 \tag{5.4}$$

In order to gain more intuition on this condition, we take a second order approximation of equation (5.4) around the steady-state. The details of the derivation are provided in appendix

E. We obtain the following condition,

$$
\begin{aligned}
0 \;>\; & \frac{1}{2}E_{t-1}\left(\widehat{p}_{h,t}^{2}-\widehat{d}_{h,t}^{2}\right)+E_{t-1}\left[e_t\left(a_t(z)-a_t\right)\right] \\
& +\frac{(1-\theta)}{\theta}E_{t-1}\left[e_t\left(p_{h,t}-E_{t-1}\left(p_{h,t}\right)\right)\right] \\
& -s\frac{(1-\theta)}{\theta}E_{t-1}\left[e_t\left(e_t-E_{t-1}\left(e_t\right)\right)\right]
\end{aligned}
\tag{5.5}
$$

Condition (5.5) has a very intuitive interpretation. First, when expected volatility of domestic prices is expected to be high, $E_{t-1}\widehat{p}_{h,t}^{2}$ setting prices in pesos exposes the firm to a higher relative price misalignment, consequently it expects to have lower profits. As its clear in the previous equation when $E_{t-1}\widehat{p}_{h,t}^{2}$ is relatively large, the likelihood that condition (5.5) holds, thus that a firm set prices in pesos, is lower. Similarly, when the volatility of prices in dollars is expected to be high, firms have a stronger incentive to set prices in pesos, since setting prices in dollars it would generate a lower expected profit.

A similar intuition applies for the case of the variance of the nominal exchange rate, as this variance is expected to be higher, firms have more incentives to set prices in pesos. However, the more interesting mechanism of equation (5.5) comes from the correlation of the nominal exchange rate with the productivity shock of firm z ,$a_t(z)$, since this correlation links monetary policy with the pricing decision of the firm. When $E_{t-1}\left[e_t\left(a_t(z)-a_t\right)\right]>0$, by setting prices in dollars firms will have a higher expected profits, since the nominal exchange rate will partially offset the negative effects of productivity shocks. Higher this correlation, higher the incentive the firms has to set prices in dollars, by generating high relative prices misalignments.

Finally, firms will have more incentive to set prices in dollars when the real exchange rate is more stable, this is precisely what a positive correlation between the nominal exchange rate and domestic prices will deliver and what it will make condition (5.5) less likely to hold .

With a bit of extra algebra, we transform equation (5.5) in the following equation that is more suitable to define the equilibrium level of price dollarization since uses unconditional moments.

$$
-cov((a_t(z)-a_t)\,,\widetilde{e}_t)+\frac{var(d_t)-var(p_{h,t})}{2\theta}+\left(s-\frac{1}{2}\right)\left(\frac{1-\theta}{\theta}\right)var\left(e_t\right)<0 \tag{5.6}
$$

Notice that condition (5.6) determine whether or not a particular firm set prices in dollars, not in pesos as in equation (5.5). Using the rational expectations equilibrium solution for the nominal exchange rate, the domestic price level, given by equations (4.12) and (4.13) we obtain the following condition,

$$cov\left[\left(a_t(z)-a_t\right), a_{s,t}\right] > \chi_1 var\left(a_{s,t}\right) + \chi_2 var\left(\eta_t\right) - \chi_3 cov\left(a_{s,t}, \widetilde{\eta}_t\right) \tag{5.7}$$

where the parameters χ_1, χ_2 and χ_3 are defined as follows,

$$\chi_1 = \frac{\left(\frac{\left[(\omega_4-\omega_2)^2-\omega_4^2\right]}{2\theta}+\left(s-\frac{1}{2}\right)\frac{1-\theta}{\theta}\omega_2^2\right)}{\omega_2} \qquad \chi_2 = \frac{\left(\frac{(\omega_3+\omega_1)^2-\omega_3^2}{2\theta}+\left(s-\frac{1}{2}\right)\frac{1-\theta}{\theta}\omega_1^2\right)}{\omega_2}$$

$$\chi_3 = \frac{\left(\frac{2[(\varpi_4-\varpi_2)(\varpi_3+\varpi_1)+\omega_3\omega_4]}{2\theta}-2\omega_1\omega_2\left(s-\frac{1}{2}\right)\frac{1-\theta}{\theta}\right)}{\omega_2}$$

Condition (5.7) allow us to define the equilibrium degree of PD in this economy as the size of set, Σ where all elements of this set satisfy condition (5.7), thus we have the following condition for the endogenous degree of PD.

$$\Sigma = \left\{z : cov\left[\left(a_t(z)-a_t\right), a_{s,t}\right] > \chi_1 var\left(a_{s,t}\right) + \chi_2 var\left(\eta_t\right) - \chi_3 cov\left(a_{s,t}, \widetilde{\eta}_t\right)\right\} \tag{5.8}$$

Notice that condition (5.8) defines a fixed point over the space of sets, since for evaluating condition (5.7) we need to know the set Σ, and to know the set Σ we need to know which mass of firms satisfy condition (5.8). Thus we can not tell much about the equilibrium degree of PD of this economy unless we specify some structure for the second moments of $a_t(z)$. Next we use a very simple case that is enough to allows to obtain qualitatively results about the equilibrium degree of PD and its relationship with optimal monetary policy.

5.1 The equilibrium Price Dollarization Under Optimal Monetary Policy

The simplest possible case that we can assume is one in which there exist only two sectors in the economy, thus we have only two sector specific shocks that we denote by $a_{1,t}$ and $a_{2,t}$, which have the same mean and variance but with a coefficient of correlation equal to -1,

$$\begin{array}{cc} E_{t-1}(a_{1,t}) = \mu & E_{t-1}(a_{2,t}) = \mu \\ E_{t-1}(a_{2,t} - E_{t-1}(a_{2,t}))^2 = \sigma^2 & E_{t-1}(a_{1,t} - E_{t-1}(a_{1,t}))^2 = \sigma^2 \end{array}$$

$$E_{t-1}\left(a_{1,t} - E_{t-1}(a_{1,t})\right)\left(a_{2,t} - E_{t-1}(a_{2,t})\right) = -\sigma^2$$

these assumptions provide a minimum set of conditions to have a well define equilibrium degree of PD. Furthermore, it simplify the analysis by making, $a_t = \int_0^1 (a_t(z) - a)dz = 0$ and

$$var\left(a_t\right)=var\left(\int_0^1 (a_t(z)-a)dz\right)=0.$$

In order to define the equilibrium let's start by defining $F\Sigma$ as the set of all possible elements that belong to Σ . Since there are only two sectors, we can have only the following possible cases, no one set prices in dollars, firms type 1 set prices in dollars, or firms type 2 set prices in dollars. Both firms can not set prices in dollars by assumption. We assume that when indifferent between setting prices in pesos and in dollars a firm sets prices in pesos. Since, $\theta = 1/2$ one firm has to set prices in pesos with probability 1. Thus all possible elements of the set that defines the firms that set prices in pesos is given by,

$$F\Gamma = \{\{\phi\}, z_1, z_2, \}$$

An equilibrium in this simple economy it will be given by the set of firms that satisfies equations (5.7). Since we are interested in analyzing an equilibrium where price dollarization exist, we are going to focus on the conditions that guarantee that $s = \frac{1}{2}$, which is equivalent to

$$\Gamma = \{z_1\}$$

or

$$\Gamma = \{z_2\}$$

Since under the assumptions we have made on the two sectorial shocks, $cov\left(a_{s,t}, \widetilde{\eta}_t\right) = 0$, we have that the conditions that guarantee $\Gamma = \{z_1\}$ as an equilibrium are,

$$cov\left[\left(a_{1t}-a_t\right),\left(a_{1t}-a_t\right)\right] > \chi_1 var\left(a_{1t}-a_t\right)+\chi_2 var\left(\eta_t\right) \tag{5.9}$$

$$cov\left[\left(a_{2t}-a_t\right),\left(a_{2t}-a_t\right)\right] < \chi_1 var\left(a_{1t}-a_t\right)+\chi_2 var\left(\eta_t\right) \tag{5.10}$$

rearranging the previous equations we have,

$$\left(1-\chi_1\right)\sigma_1^2 > \chi_2\sigma_\eta^2 \tag{5.11}$$

$$\sigma_{12}-\chi_1\sigma_1^2-\chi_2\sigma_\eta^2 < 0 \tag{5.12}$$

From equation (5.11) and (5.12) it is easy to see that a necessary condition to sustain an equilibrium with PD is that the size of domestic shocks relative to real exchange rate shocks reaches a critical value given by,

$$\frac{\sigma_1^2}{\sigma_\eta^2} > \frac{\chi_2}{(1-\chi_1)} \tag{5.13}$$

$$\rho_{12} < \chi_1 + \chi_2 \frac{\sigma_\eta^2}{\sigma_1^2} \tag{5.14}$$

where, χ_2 and χ_1 are evaluated at $s = \frac{1}{2}$ and $\theta = \frac{1}{2}$. In this simple example,for the calibration described in section 4.3 both conditions (5.13) and (5.14) are simultaneously satisfy when $\frac{\sigma_1^2}{\sigma_\eta^2} > 0.91$. Thus, for this calibration when domestic sectorial shocks are at least 90 percent as volatile as real exchange rate shocks, the equilibrium under optimal monetary policy implies a degree of PD of 50 percent. This example shows that the necessary condition to sustain an equilibrium with PD under optimal monetary policy, is to observe large enough domestic sector specific technology shocks. This result has a parallel with the intuition of Mundell (1961) on optimal currency areas. Mundell, defines an optimal currency area as geographical area that share common real shocks. In our model, within the domestic economy there exist two currency areas in the terminology of Mundell, when there exist enough volatility in the domestic sector specific shocks.

Also, this example provides some rationale for the high degree of persistence that PD has exhibited in emerging economies. Since having more than a currency might have some benefits for stable economies, as we previously show, agents in emerging economies with a history of high inflation, might not want fully switch back to setting prices in the domestic currency, to benefit of the advantage of having two currencies[6].

Next we explore the effects of deviations of monetary policy from its optimal rule on the degree of PD. We perform two simple exercises, in the first one we ask whether a central bank that is more adverse to inflation that what is optimal can achieve lower degrees of PD, whereas in the second one we look for the implications of excess of "fear of floating".

5.2 Price Dollarization and inflation aversion

In order to perform both exercises we parameterize deviations of the central bank from its optimal policy rule. For the first exercise, we use the following alternative central bank reaction function,

$$\left[(\Lambda_e + s)\,\widetilde{e}_t - \theta \widetilde{a}_{s,t}\right] \varrho + (1+s)\,\widetilde{p}_{s,t} = 0$$

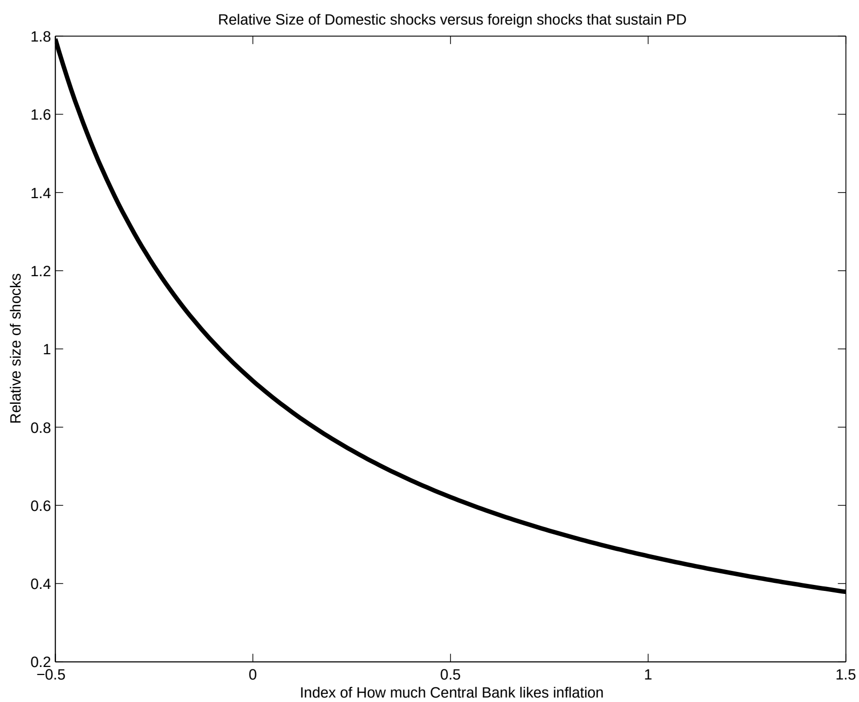
where we label by ϱ the index of how much the central bank dislike inflation. When, $\varrho = 0$

[6] One of the most important features of economies with history of dollarisation is their adaptation for the use of a foreign currency, at the level of being relatively cheap to use both currencies.

we have a central bank that is an inflation nutter, since, it this case, it would implement a policy where,

$$\widetilde{p}_{s,t} = 0$$

As ϱ increases we have a central bank who tolerates increasingly more inflation. The next graph shows in the vertical axis the size of relative shocks, $\frac{\sigma_1^2}{\sigma_\eta^2}$ that sustain an equilibrium with PD and in the horizontal one a measure of how much the central bank likes inflation, $(\varrho - 1)$. We have normalized this measure so that the case of optimal monetary policy coincides in this axis with zero. As this graph show, when the central banks deviates from this optimal behavior, in particular, as it tolerates higher levels of inflation, the size of relative shocks that sustain an equilibrium with PD falls, making more likely an equilibrium with PD.



When a central bank tolerates more volatility on inflation, firms that set prices in pesos are exposed to higher profit losses generated by deviations of their relative prices from their optimal levels. In this more volatility environment, it turns out optimal for some domestic firms to react by setting prices in dollars . By setting prices in dollars firms partially isolate their relative prices from inflation because movements on the nominal exchange rate, which are positively correlated in equilibrium with domestic inflation, tend to stabilize their relative prices. Thus in our example with just two sectors, when inflation is more volatile PD is sustain as an equilibrium for decreasing size of domestic productivity shocks.

The other interesting insight provided by the previous exercise is that a central bank who implements monetary policy by using an inflation target framework, with more weight on inflation stabilization than in other objectives, can effectively reduce PD. As the previous graph shows, as the central bank dislike inflation more, the relative size of domestic versus real exchange rate shocks that sustain an equilibrium with PD increases, thus making less likely this equilibrium.

5.3 Price Dollarization and Fear of Floating

In this second exercise we parameterize "fear of floating" by considering that the central bank deviates from the optimal weights that it puts on exchange rate volatility, Λ_e and on the cross terms between $\widetilde{e}_t$ and $a_{s,t}$, when implementing its policy. More precisely, we shift both parameters by a factor, ϱ_e. When $\varrho_e = 1$ the central banks is using the optimal weights, derived in section 4.2, for the exchange rate terms on the welfare function. Then, we calculate the critical relative size of domestic sector specific versus real exchange rate shocks, $\frac{\sigma_1^2}{\sigma_\eta^2}$, that sustain an equilibrium with PD, $s = \frac{1}{2}$.

The results are presented in the next graph. As in the previous case, we normalize the degree of fear of floating, by using $1 - \varrho$ on the horizontal axis. This normalization allow us to located the equilibrium under optimal monetary policy at point zero on this axis.

The equilibrium with excess of "fear of floating", can be located,in the previous graph, at the horizontal axis, on the left of the zero point. As degree of "fear of floating" increases, an equilibrium with PD can be sustained with relative smaller domestic sector specific shocks, making more likely an equilibrium with $s = \frac{1}{2}$. Notice that all points on the horizontal axis different from imply higher welfare losses than when the central bank implements monetary policy optimally. Thus, our model implies that excess of "fear of floating" induces an excess of price dollarization and consequently welfare losses. As the degree of fear of floating falls, as we move from left to right in the previous graph, the size of relative shocks that allow an equilibrium with PD increases, making less likely to observe an equilibrium where, $s = \frac{1}{2}$.

6 Conclusions

In this paper we have studied how monetary policy should be conducted in an small open economy where firms that faces sector specific shocks can set prices in two different currencies. The results of this paper suggest that in this type of economies optimal monetary policy involves some degree of exchange rate smoothing and an active reaction of the central bank to sector specific productivity shocks. When domestic sector specific productivity shocks are large enough, an equilibrium with positive degree of price dollarization is sustainable under optimal monetary policy. As in Mundell (1961), where it is optimal that two countries share a common currency when they face similar real shocks, in this paper, we show that it might be optimal for a particular economy to have more than one currency when there exist asymmetric productivity shocks within the economy.

The paper also explores the implications of deviations from optimal policy on the degree of price dollarization. In particular, we analyze two cases: when the central bank is an inflation nutter and it exhibits "excess of fear of floating". A central bank that is more adverse to inflation than society would generate, in equilibrium, a lower level of PD. In that sense, the model predicts that in countries where an explicit inflation targeting is successfully implemented it is less likely to observe price dollarization. However, in this model, an inflation nutter central bank would induces welfare losses by responding sub-optimally to sector specific productivity shocks.

On the contrary "excessive fear of floating" leads to an "excessive" degree of price dollarization, as firms try to take advantage the benefits that pricing in foreign currency offers in this case. However, excess degree of price dollarization induces welfare losses for the society, since by keeping the nominal exchange rate more stable, the central bank has to tolerate increasingly high levels of volatility in output gap and domestic inflation.

The results of the paper also provide some insight on the differences between the problem that a central bank faces in an small open economy and in a close one. Clarida, Gali and Gertler (2001) and Gali and Monacelli (2005) show, for a particular type of preferences, that the equilibrium dynamics of an small open economy model with sticky prices can be represented only by two equations, a dynamic aggregate demand equation and a Phillips Curve for domestic inflation, thus making the problem of the central isomorphic to the case of a close economy. By contrast, in this paper, we provide a model where this does not hold, even in the particular case analyzed by the aforementioned authors. In our model, even when the terms of trade channel is not present, the central bank problem in a small open economy differs from the corresponding one in a close economy. In particular, a fully flexible exchange rate is not optimal, but instead the central bank smooth exchange rate fluctuations.

Finally, the paper can be extended in many directions. First, we can use a more general assumption on preferences that allow to study simultaneously the interactions between the channel of terms of trade and sector specific productivity shocks on the design of monetary policy. Second, we can assume a more complex structure on the correlations among sector specific productivity shocks, as in Loyo (2001), which it will permit us to generate a continuous mapping between policy and the degree of price dollarization, finally we could at taxes to analyse the interaction between monetary and fiscal policy when a country faces sector specific productivity shocks.

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A The steady-state

From equation, (2.27) we define, the following implicit function of terms of trade,

$$Z_H = \frac{1}{h(T)} \tag{A.1}$$

therefore, we have that the real exchange rate can be written as follows,

$$Q = \frac{T}{h(T)} \tag{A.2}$$

Also, from the risk sharing condition, equation, (2.13), we obtain:

$$C = Y^* \left(\frac{T}{h(T)} \right) \tag{A.3}$$

Plugging in equations (A.2) and (A.3) into the steady-state version of equation (2.23) we obtain the following condition for the home output

$$Y_H = K(T)Y^* \tag{A.4}$$

where,

$$K(T) = T \tag{A.5}$$

Since at the steady-state, marginal costs of all domestic firms are the same, we have that,

$$\frac{1}{\mu} = mc_s = mc_d = \frac{Y_H^v C^\sigma}{Z_H} \tag{A.6}$$

and, consequently all relative prices, $\frac{P_H(z)}{P_H} = 1$. From equation (A.6), we have that,

$$\frac{1}{\mu} = Y_H^v \left(Y^*\right) \left(\frac{T}{h(T)} \right) h(T) \tag{A.7}$$

we also define, $\frac{1+\tau}{\mu} = 1 - \Phi$, as the distortion generated by monopolistic competition, therefore, equation, (A.7) can be written as:

$$Y_H = (1 - \Phi)^{\frac{1}{v}} \frac{1}{(Y^*)^{\frac{1}{v}} T^{\frac{1}{v}}} \tag{A.8}$$

Combining equations (A.4) and (A.8), we obtain the following condition to determine, T

$$K(T)T^{\frac{1}{v}} = \frac{(1-\Phi)^{\frac{1}{v}}}{(Y^*)^{\frac{v+1}{v}}} \tag{A.9}$$

In a symmetric equilibrium, where the foreign economy has an identical structure to the domestic one, it holds that,

$$Y^* = (1-\Phi)^{\frac{1}{1+v}} \tag{A.10}$$

therefore, we have the following condition to determine terms of trade a the steady-state.

$$K(T)T^{\frac{1}{v}} = 1 \tag{A.11}$$

From equation (A.11) we have that $T = 1$, using equation, (A.8), we obtain,

$$Y_H = Y^* = (1-\Phi)^{\frac{1}{\sigma+v}} \tag{A.12}$$

From equations (A.1) and (A.2),

$$Q = Z_H = 1 \tag{A.13}$$

and from equation (A.6), we obtain that,

$$C = Y_H = (1-\Phi)^{\frac{1}{\sigma+v}} \tag{A.14}$$

A.1 The flexible price equilibrium

We solve for the flexible price equilibrium of this economy by using the log linear approximation of the model equation around its steady-state. The set of equation characterizing this equilibrium is given by

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) \tag{A.15}$$

$$y_{H,t} = -\eta z_{H,t} + (1-\gamma)c_t + \gamma\eta q_t + \gamma y_t^* \tag{A.16}$$

$$q_t = \sigma(c_t - y_t^*) \tag{A.17}$$

$$t_t = \frac{q_t}{1-\gamma} \tag{A.18}$$

$$0 = v y_{H,t} + \sigma c_t - z_{H,t} + (1+v)a_t \tag{A.19}$$

$$z_{H,t} = q_t - t_t = -\gamma t_t \tag{A.20}$$

Using this system of six equations we can solve for the natural levels of the interest rate, output, the real exchange rate, terms of trade, consumption, and domestic relative price. Combining equations (A.17), (A.18) and (A.20), to eliminate, the relative domestic price, and terms of trade, equation (A.16) can be written as follows:

$$y_{H,t} = \frac{\gamma\omega}{\sigma(1-\gamma)} q_t + c_t \tag{A.21}$$

where,$\omega = \eta\sigma + (1-\gamma)(\eta\sigma - 1)$, the plugging in equation (A.17) into equation (A.21) we obtain:

$$y_{H,t} = \frac{1}{\sigma_y(1-\gamma)} q_t + y_t^* \tag{A.22}$$

where, $\sigma_y = \frac{\sigma}{\gamma\omega + (1-\gamma)}$. Using, equations (A.22) and (A.19) we obtain the natural level of output in terms of productivity shocks and the level of foreign output,

$$y_{H,t}^n = -\Gamma a_t + \gamma\Psi y_t^* \tag{A.23}$$

where, $\Gamma = \frac{1+v}{v+\sigma_y}$,$\Psi = -\frac{\sigma_y\Theta}{v+\sigma_y}$ and $\Theta = \omega - 1$. On the other hand, the natural interest rate can be obtained by rewriting equation (A.15) in terms of domestic and foreign output,

$$0 = E_t\left(\frac{\sigma_y}{\sigma}\Delta y_{t+1} + \frac{\gamma(\omega-1)\sigma_y}{\sigma}\Delta y_{t+1}^*\right) - \frac{1}{\sigma} r_t \tag{A.24}$$

Plugging in equation (A.23) into equation(A.24), we obtain the natural interest rate in terms of shocks, productivity and foreign output,

$$r_t^n = \sigma_y(1-\rho_a)\Gamma a_t + \gamma\sigma_y(\Theta + \Psi) E_t \Delta y_{t+1}^* \tag{A.25}$$

By setting $\sigma = \eta = 1$ we have $\Gamma = \sigma_y = 1, \Psi = \Theta = 0$ therefore, equations (A.23) and (A.25) become,

$$y_{H,t}^n = -a_t$$

$$r_t^n = (1-\rho_a) a_t$$

which correspond to equations (3.4) and (3.5) on the main text. Equation (3.6) of the main text is obtained from equation (A.22),

$$q_t = (1-\gamma)\left(y_{H,t}^n - y_t^*\right) = -(1-\gamma)(a_t - a_t^*)$$

B Sticky Price equilibrium

B.1 The aggregate supply curve

In order to obtain the domestic aggregate price level we approximate up to first order equation (2.5, thus we have,

$$p_{h,t} = \int_0^1 p_{h,t}(z)d(z) \tag{B.1}$$

however, after log linearize, $p_{h,t}(z)$ for the three relevant cases, we obtain the following condition,

$$p_{h,t}(z) = \begin{cases} w_t + p_{h,t} + a_t(z) \\ E_{t-1}\left(w_t + p_{h,t} + a_t(z)\right) \\ E_{t-1}\left(w_t + d_{h,t} + a_t(z) + e_t\right) \end{cases} \tag{B.2}$$

We use this previous condition to obtain equation (B.1) in terms of its determinants, thus we calculate,

$$\begin{aligned} p_{h,t} &= \int_\Theta \left[w_t + p_{h,t} + a_t(z)\right] d(z) + \\ &\quad \int_\Sigma \left[E_{t-1}\left(w_t + d_{h,t} + a_t(z) + e_t\right)\right] d\left(z\right) \\ &\quad + \int_{[0,1]\backslash\Sigma} \left[E_{t-1}\left(w_t + p_{h,t} + a_t(z)\right)\right] d\left(z\right) \end{aligned} \tag{B.3}$$

Using the fact that,

$$\int_\Theta a_t(z)d(z) = \theta a_t \tag{B.4}$$

$$\int_{[0,1]\backslash\Theta} a_t(z)d(z) = \left(1-\theta\right) a_t \tag{B.5}$$

We find that,

$$\begin{aligned} p_{h,t} &= \theta\left(w_t + p_{h,t} + a_t\right) + \\ &\quad \left(1-\theta\right) s\left(E_{t-1}\left(w_t + p_{h,t} + a_t + e_t\right) - e_t\right) \\ &\quad + \left(1-\theta\right)\left(1-s\right) E_{t-1}\left(w_t + p_{h,t} + a_t\right) \end{aligned} \tag{B.6}$$

Rearranging properly, we have that,

$$\begin{aligned} p_{h,t} &= \theta\left(w_t + a_t\right) + (1-\theta)\left(E_{t-1}\left(w_t + a_t\right)\right) \\ &\quad + (1-\theta)\, s\left(E_{t-1}\left(e_t\right) - e_t\right) + \theta p_{h,t} + (1-\theta) E_{t-1} p_{h,t} \end{aligned} \tag{B.7}$$

Using this equation, we can show that, $E_{t-1}\left(w_t + a_t\right) = 0$, thus, we have,

$$w_t = -a_t + \frac{(1-\theta)}{\theta}\left(p_{h,t} - E_{t-1}\left(p_{h,t}\right)\right) - \frac{(1-\theta)}{\theta} s\left(e_t - E_{t-1}\left(e_t\right)\right) \tag{B.8}$$

We know that real wages in terms of domestic prices, up to log linear approximation, are given by,

$$w_t = vh_t + c_t - z_{h,t} \tag{B.9}$$

Furthermore, given our assumption on consumer preferences, we have that the following relationship between consumption and domestic output holds,

$$c_t = y_{h,t} + z_{h,t} \tag{B.10}$$

also, we have that,

$$h_t = y_{h,t} + a_t \tag{B.11}$$

Plugging in equations (B.9), (B.10) and (B.11) into (B.8) we obtain the following aggregate supply equation in terms of output gap, inflation expectations errors, and exchange rate expectations errors,

$$p_{h,t} = E_{t-1}\left(p_{h,t}\right) + (1+\nu)\frac{\theta}{(1-\theta)} x_t + s\left(e_t - E_{t-1}\left(e_t\right)\right) \tag{B.12}$$

Where, $x_t = y_t - y_t^n$ and $y_t^n = a_t$. Using a simple transformation, equation (B.12), we obtain the Phillips curve in this economy,

$$\pi_{h,t} = E_{t-1}\pi_{h,t} + (1+\nu)\frac{\theta}{(1-\theta)} x_t + s\left(\Delta e_t - E_{t-1}\left(\Delta e_t\right)\right) \tag{B.13}$$

Aggregate Demand

The aggregate demand block is given by the following set of equations,

$$c_t = E_t c_{t+1} - \left(i_t - E_t \pi_{t+1}\right) \tag{B.14}$$

$$y_{H,t} = \frac{\gamma q_t}{1-\gamma} + c_t \tag{B.15}$$

$$q_t = c_t - y_t^* \tag{B.16}$$

$$\pi_t = \pi_{H,t} + \frac{\gamma}{1-\gamma}\Delta q_t \tag{B.17}$$

Plugging in equation (B.17) into equation (B.14), we obtain,

$$c_t = E_t c_{t+1} - \left(i_t - E_t \pi_{H,t+1} - \frac{\gamma}{1-\gamma} E_t \Delta q_{t+1} \right) \tag{B.18}$$

From equation, (B.15) we can relate consumption and output through the real exchange rate and eliminate consumption from equation(B.14), such that,

$$y_{H,t} = E_t y_{H,t+1} - \frac{\gamma}{1-\gamma} E_t \Delta q_{t+1} - \left(i_t - E_t \pi_{H,t+1} - \frac{\gamma}{1-\gamma} E_t \Delta q_{t+1} \right)$$

thus the aggregate demand equation for this small open economy will be given by,

$$y_{H,t} = E_t y_{H,t+1} - (i_t - E_t \pi_{H,t+1}) \tag{B.19}$$

Since a similar equation holds in the case of perfectly flexible prices, we can write equation (B.19) in terms of output gap, as follows,

$$x_t = E_t x_{t+1} - (i_t - E_t \pi_{H,t+1} - r_t^n) \tag{B.20}$$

Furthermore, we link the dynamics of the nominal exchange rate, to the real exchange rate as follows,

$$\Delta e_t = \pi_{H,t} + \frac{1}{1-\gamma}\Delta q_t - \pi_t^* \tag{B.21}$$

Whereas, the real exchange rate can be determined by the following condition,

$$q_t = q_t^n + (1-\gamma)(x_t - x_t^*) \tag{B.22}$$

where,

$$q_t^n = (1-\gamma)(a_t - a_t^*) \tag{B.23}$$

C Lost function of the Central Bank

In what follows we derive the microfundated lost function of the central bank by using a second order Taylor approximation of the utility function of the representative agent, equation (C.1) around the deterministic steady-state,

$$U = \ln C_t - \frac{h_t^{1+v}}{1+v} \tag{C.1}$$

We use a generic form of the previous utility function, in order to have a general result, thus we approximate,

$$U = U(C_t) - V(h_t)$$

Where total labor depends on output, productivity and relative price distortions,

$$h_t = Y_t \Delta_t A_t \tag{C.2}$$

and we know from section 5 that at the first best allocation it must hold that,

$$vY = C(1-\gamma) \tag{C.3}$$

The second order expansion of the utility generated by consumption is given by, :

$$u\left(C_t\right) = \overline{u} + \overline{u}_c \overline{C}\left(\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2\right) + \frac{1}{2}\overline{u}_{cc}\overline{C}^2 \widehat{C}_t^2 + o\left(\|\epsilon\|^3\right) \tag{C.4}$$

collecting terms we have that:

$$u\left(C_t\right) = \overline{u}_c \overline{C}\left(\widehat{C}_t + \frac{1}{2}\left(1-\sigma\right)\widehat{C}_t^2\right) + t.i.p + o\left(\|\epsilon\|^3\right) \tag{C.5}$$

For our particular case where, $\sigma = 1$. equation (C.5) becomes,

$$u\left(C_t\right) = \overline{u}_c \overline{C}\widehat{C}_t + t.i.p + o\left(\|\epsilon\|^3\right)$$

Next we take a second order expansion of $v(h_t)$, we use equation (C.2) to define the aggregate level of labour in terms of output,productivity shocks and price dispersion,thus a second order approximation for the dissutility of labor effort is given by,

$$\begin{aligned} v(h_t) \quad &= \quad \overline{v}_h \overline{Y}\left(\widehat{\Delta}_t + \widehat{Y}_t + \frac{1}{2}\left(1 + \frac{\overline{v}_{hh}\overline{Y}}{\overline{v}_h}\right)\widehat{Y}_t^2 + \left(1 + \frac{\overline{v}_{hh}\overline{Y}}{\overline{v}_h}\right)\widehat{Y}_t \widehat{A}_t\right) \\ &\quad\quad +t.i.p + o\left(\|\varepsilon\|^3\right) \end{aligned} \tag{C.6}$$

Notice that since $\widehat{\Delta}_t$ is of order $o\left(\|\varepsilon\|^2\right)$ all terms involving second order terms of $\widehat{\Delta}_t$ are dropped out from equation (C.6) , thus, we have,

$$\begin{aligned} u(C_t) - v(h_t) &= \overline{u}_c\overline{C}\widehat{C}_t - (1-\gamma)\overline{u}_c\overline{C}\left(\widehat{\Delta}_t + \widehat{Y}_t + \frac{1}{2}(1+v)\widehat{Y}_t^2 + (1+v)\widehat{Y}_t\widehat{A}_t\right) \\ &\quad + t.i.p + o\left(\|\epsilon\|^3\right) \end{aligned} \tag{C.7}$$

imposing the restriction on the coefficient of risk aversion, $\sigma = 1$, we have:

$$\begin{aligned} &= \overline{u}_c\overline{C}\left(\begin{array}{c} \widehat{C}_t - (1-\gamma)\widehat{Y}_t - \frac{1}{2}((1-\gamma)(1+v))\widehat{Y}_t^2 \\ -(1-\gamma)\widehat{\Delta}_t - (1-\gamma)(1+v)\widehat{Y}_t\widehat{A}_t \end{array}\right) \\ &\quad + t.i.p + o\left(\|\epsilon\|^3\right) \end{aligned} \tag{C.8}$$

Let´s define the following parameters:

$$u_{yy} = -(1-\gamma)(1+v) \tag{C.9}$$

$$u_{yA} = (1-\gamma)(1+v) \tag{C.10}$$

$$u_{\Delta} = (1-\gamma) \tag{C.11}$$

Moreover, since:

$$\widehat{C}_t = (1-\gamma)\widehat{Y}_t + \gamma\widehat{Y}_t^*$$

We can now write the utility function of the representative agent as follows:

$$\begin{aligned} u(C_t) - v(h_t) &= \overline{u}_c\overline{Y}\left(-\frac{1}{2}u_{yy}\widehat{Y}_t^2 - u_{\Delta}\widehat{\Delta}_t - u_{yA}\widehat{Y}_t\widehat{A}_t\right) \\ &\quad + t.i.p + o\left(\|\varepsilon\|^3\right) \end{aligned} \tag{C.12}$$

Rewriting appropriately the quadratic terms we have:

$$\frac{1}{2}u_{yy}\widehat{Y}_t^2 + u_{\Delta}\widehat{Y}_t\widehat{A}_t = \frac{1}{2}(1-\gamma)\left((1+v)\left(\widehat{Y}_t^2 - 2\widehat{Y}_t\widehat{A}_t + \widehat{A}_t^2\right)\right) \tag{C.13}$$

since we have eliminated all the distortions of the steady-state equilibrium, the quadratic terms of the approximated lost function of the central bank can be written as follows:

$$\frac{1}{2}u_{yy}\widehat{Y}_t^2 + u_{\Delta}\widehat{Y}_t\widehat{A}_t = \frac{1}{2}(1-\gamma)(1+v)\widehat{x}_t^2$$

$$u\left(C_t\right)-v(h_t)=\overline{u}_c\overline{Y}\left(-\frac{1}{2}\left(1-\gamma\right)\left(1+v\right)\widehat{x}_t^2-u_\Delta\widehat{\Delta}_t\right)+t.i.p+o\left(\|\varepsilon\|^3\right) \tag{C.14}$$

Now we have to find the second order approximation of $\widehat{\Delta}_t$,

$$\Delta_t=\int\left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\varepsilon}\frac{A_{z,t}}{A_t}dz \tag{C.15}$$

since, at each point in time, there exist three types of firms in this economy, we have that,

$$\begin{aligned}\Delta_t \;=\;& \int\limits_{\Theta}\left(\frac{P_{H,t}(z)}{P_{Ht}}\right)^{-\varepsilon}\frac{A_{z,t}}{A_t}dz \\ &+\int\limits_{\Sigma}\left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\varepsilon}\frac{A_{z,t}}{A_t}dz \\ &+\int\limits_{[0,1]\backslash\Sigma}\left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\varepsilon}\frac{A_{z,t}}{A_t}dz\end{aligned} \tag{C.16}$$

First let's find the second order Taylor approximation of,

$$\begin{aligned}\frac{\left(\frac{P_{H,t}(z)}{P_{Ht}}\right)^{-\varepsilon}\frac{A_{z,t}}{A_t}-\left(\frac{P_H(z)}{P_H}\right)^{-\varepsilon}\frac{A_z}{A}}{\left(\frac{P_H(z)}{P_H}\right)^{-\varepsilon}\frac{A_z}{A}} \;\simeq\;& 1-\varepsilon\left(p_{h,t}(z)+\frac{1}{2}p_{h,t}^2(z)-p_{ht}-\frac{1}{2}p_{h,t}^2\right)+ \\ & a_t(z)+\frac{1}{2}a_t^2(z)-a_t-\frac{1}{2}a_t^2+\frac{1}{2}\varepsilon\left(1+\varepsilon\right)p_{h,t}^2(z) \\ &+\frac{1}{2}\varepsilon\left(\varepsilon-1\right)p_{ht}^2-\varepsilon^2p_{h,t}(z)p_{ht} \\ &-\varepsilon\left(a_t(z)-a_t\right)\left(p_{h,t}(z)-p_{h,t}\right)\end{aligned} \tag{C.17}$$

Since,

$$\int\left(-\varepsilon\left(p_{h,t}(z)-p_{ht}\right)+a_t(z)-a_t\right)dz=0$$

Also, notice that,

$$\frac{1}{2}\varepsilon^2p_{h,t}^2(z)+\frac{1}{2}\varepsilon^2p_{ht}^2-\varepsilon^2p_{h,t}(z)p_{ht}=\frac{1}{2}\varepsilon^2\left(p_{h,t}(z)-p_{h,t}\right)^2 \tag{C.18}$$

up to second order we have that,

$$\Delta_t = \int_{\Theta} \left(a_t^2(z) - a_t^2 + \frac{1}{2}\varepsilon^2 \left(p_{h,t}(z) - p_{h,t} \right)^2 \right) dz \tag{C.19}$$
$$+ \int_{[0,1]\setminus\Theta U\Sigma} \left(a_t^2(z) - a_t^2 + \frac{1}{2}\varepsilon^2 \left(p_{h,t}(z) - p_{h,t} \right)^2 \right) d(z)$$
$$+ \int_{\Sigma} \left(a_t^2(z) - a_t^2 + \frac{1}{2}\varepsilon^2 \left(p_{h,t}(z) - p_{h,t} \right)^2 \right) dz$$

where we further define the cross product terms, as $\Delta_{c,t}$, such that,

$$\Delta_{c,t} = -\varepsilon \int_{\Theta} \left(a_t(z) - a_t \right) \left(p_{h,t}(z) - p_{h,t} \right) dz \tag{C.20}$$
$$+\varepsilon \int_{[0,1]\setminus\Theta U\Sigma} \left(a_t(z) - a_t \right) \left(p_{h,t}(z) - p_{h,t} \right) d(z)$$
$$+\varepsilon \int_{\Sigma} \left(\left(a_t(z) - a_t \right) \left(p_{h,t}(z) - p_{h,t} \right) \right) dz$$

noticing that, $\int \left(a_t^2(z) - a_t^2 \right) dz = var_z a_t$, we can write Δ_t up to second order as follows,

$$\Delta_t = var_z a_t + \frac{1}{2}\varepsilon^2 \int \left(p_{h,t}(z) - p_{h,t} \right)^2 dz + \Delta_{c,t} \tag{C.21}$$

Since, $p_{h,t}(z)$ and $p_{h,t}$ have second order effects on Δ_t we only need a first order approximation of $p_{h,t}(z)$, which from the previous section is given by,

$$p_{h,t}(z) - p_{h,t} = \begin{cases} p_{1,t}(z) \\ E_{t-1}p_{1,t}(z) - \left(p_{h,t} - E_{t-1}p_{h,t} \right) \\ E_{t-1}p_{1,t}(z) - \left(p_{h,t} - E_{t-1}p_{h,t} \right) + \left(e_t - E_{t-1}\left(e_t \right) \right) \end{cases}$$

where, we denote by $p_{1,t}$ the relative optimal price under flexible prices,

$$p_{1,t}(z) = w_t + a_t + a_t(z) - a_t$$

Since, $E_{t-1}(w_t + a_t) = 0$, we have that,

$$E_{t-1}p_{1,t}(z) = E_{t-1}\left(a_t(z) - a_t \right)$$

let's denote by, $\overline{p}_t = w_t + a_t$, thus we have that,

$$p_{1,t}^2(z) = \overline{p}_t^2 + (a_t(z) - a_t)^2 + 2\overline{p}_t\left(a_t(z) - a_t\right)$$

$$\begin{aligned} p_{2,t}^2(z) &= \left(E_{t-1}\left(a_t(z) - a_t\right)\right)^2 + \left(p_{h,t} - E_{t-1}p_{h,t}\right)^2 \\ &\quad -2E_{t-1}\left(a_t(z) - a_t\right)\left(p_{h,t} - E_{t-1}p_{h,t}\right) \end{aligned}$$

$$\begin{aligned} p_{3,t}^2(z) &= \left(E_{t-1}\left(a_t(z) - a_t\right)\right)^2 + \left(p_{h,t} - E_{t-1}p_{h,t}\right)^2 + \left(e_t - E_{t-1}\left(e_t\right)\right)^2 \\ &\quad -2E_{t-1}\left(a_t(z) - a_t\right)\left(p_{h,t} - E_{t-1}p_{h,t}\right) + \\ &\quad +2E_{t-1}\left(a_t(z) - a_t\right)\left(e_t - E_{t-1}\left(e_t\right)\right) + \\ &\quad -2E_{t-1}\left(e_t - E_{t-1}\left(e_t\right)\right)\left(p_{h,t} - E_{t-1}p_{h,t}\right) \end{aligned}$$

Furthermore, we have that,

$$\overline{p}_t = \frac{(1-\theta)}{\theta}\left(p_{h,t} - E_{t-1}\left(p_{h,t}\right)\right) - \frac{(1-\theta)}{\theta}s\left(e_t - E_{t-1}\left(e_t\right)\right)$$

therefore,

$$\begin{aligned} \overline{p}_t^2 &= \left(\frac{(1-\theta)}{\theta}\right)^2\left(p_{h,t} - E_{t-1}\left(p_{h,t}\right)\right)^2 + \\ &\quad \left(\frac{(1-\theta)}{\theta}\right)^2 s^2\left(e_t - E_{t-1}\left(e_t\right)\right)^2 \\ &\quad -2\left(\frac{(1-\theta)}{\theta}\right)^2 s\left(p_{h,t} - E_{t-1}\left(p_{h,t}\right)\right)\left(e_t - E_{t-1}\left(e_t\right)\right) \end{aligned}$$

and,

$$\begin{aligned} p_{1,t}^2(z) &= \left(\frac{(1-\theta)}{\theta}\right)^2\left(p_{h,t} - E_{t-1}\left(p_{h,t}\right)\right)^2 + \\ &\quad \left(\frac{(1-\theta)}{\theta}\right)^2 s^2\left(e_t - E_{t-1}\left(e_t\right)\right)^2 \\ &\quad -2\left(\frac{(1-\theta)}{\theta}\right)^2 s\left(p_{h,t} - E_{t-1}\left(p_{h,t}\right)\right)\left(e_t - E_{t-1}\left(e_t\right)\right) + \\ &\quad \left(a_t(z) - a_t\right)^2 + 2\overline{p}_t\left(a_t(z) - a_t\right) \end{aligned}$$

Thus we have that the aggregate price distortion is given,

$$
\begin{aligned}
\int \left(p_{h,t}(z) - p_{h,t}\right)^2 dz &= \int\limits_{\Theta} p_{1,t}(z)^2 dz + \int\limits_{[0,1]\backslash \Sigma U \Theta} p_{2,t}(z)^2 dz \\
&\quad + \int\limits_{\Sigma} p_{3,t}(z)^2 dz
\end{aligned}
$$

Aggregating we have,

$$
\int \left(p_{h,t}(z) - p_{h,t}\right)^2 dz = M_t + F_t + G_t + MF_t
$$

Where, M_t contains the quadratic terms that come from aggregate variables,

$$
\begin{aligned}
M_t &= \theta \left(\left(\frac{(1-\theta)}{\theta}\right)^2 \left(p_{h,t} - E_{t-1}\left(p_{h,t}\right)\right)^2 + \left(\frac{(1-\theta)}{\theta}\right)^2 s^2 \left(e_t - E_{t-1}\left(e_t\right)\right)^2 \right) \\
&\quad (1-\theta)(1-s)\left[\left(p_{h,t} - E_{t-1} p_{h,t}\right)^2\right] + (1-\theta) s \left[\left(p_{h,t} - E_{t-1} p_{h,t}\right)^2\right] + \\
&\quad (1-\theta) s \left(e_t - E_{t-1}\left(e_t\right)\right)^2
\end{aligned}
$$

Thus after simplifying the previous expression we have,

$$
\begin{aligned}
M_t &= \frac{(1-\theta)}{\theta} \left(p_{h,t} - E_{t-1}\left(p_{h,t}\right)\right)^2 + \\
&\quad \frac{(1-\theta)}{\theta} s\theta \left(1 + \frac{s(1-\theta)}{\theta}\right) \left(e_t - E_{t-1}\left(e_t\right)\right)^2
\end{aligned}
$$

Next we consider the quadratic terms specific to each group of firms,

$$
\begin{aligned}
F_t &= \int\limits_{\Theta} \left(a_t(z) - a_t\right)^2 d(z) + \int\limits_{[0,1]\backslash \Sigma U \Theta} \left(E_{t-1}\left(a_t(z) - a_t\right)\right)^2 d(z) \\
&\quad + \int\limits_{\Sigma} \left(E_{t-1}\left(a_t(z) - a_t\right)\right)^2 d(z)
\end{aligned}
$$

We can further simplify this expression as follows,

$$
F_t = \int\limits_{\Sigma} \left(a_t(z) - a_t\right)^2 d(z) + \int\limits_{[0,1]\backslash \Sigma} \left(E_{t-1}\left(a_t(z) - a_t\right)\right)^2 d(z)
$$

Next we consider the cross terms among aggregate variables,

$$\begin{aligned} G_t &= \theta \left(2 \left(\frac{(1-\theta)}{\theta} \right)^2 s \left(p_{h,t} - E_{t-1}\left(p_{h,t}\right)\right) \left(e_t - E_{t-1}\left(e_t\right)\right) \right) \\ &+ (1-\theta)(1-s)(0) + \\ &(1-\theta) s \left(2E_{t-1}\left(e_t - E_{t-1}\left(e_t\right)\right)\left(p_{h,t} - E_{t-1} p_{h,t}\right)\right) \end{aligned}$$

simplifying this expression we obtain,

$$G_t = -2 \left(\frac{(1-\theta)}{\theta} \right) s \left(p_{h,t} - E_{t-1}\left(p_{h,t}\right)\right) \left(e_t - E_{t-1}\left(e_t\right)\right)$$

Finally we have the cross terms between idiosyncratic shocks and aggregate variables,

$$\begin{aligned} MF_t &= 2\overline{p}_t \int\limits_{\Theta} \left(a_t(z) - a_t\right) d\left(z\right) \\ &\quad -2\left(p_{h,t} - E_{t-1} p_{h,t}\right) \int\limits_{[0,1]\backslash \Sigma U \Theta} E_{t-1}\left(a_t(z) - a_t\right) d(z) \\ &\quad -2\left(p_{h,t} - E_{t-1} p_{h,t}\right) \int\limits_{\Sigma} E_{t-1}\left(a_t(z) - a_t\right) d(z) \\ &\quad +2\left(e_t - E_{t-1}\left(e_t\right)\right) \int\limits_{\Sigma} E_{t-1}\left(a_t(z) - a_t\right) d(z) \end{aligned}$$

since, we know that,

$$\overline{p}_t = \frac{(1-\theta)}{\theta} \left(p_{h,t} - E_{t-1}\left(p_{h,t}\right)\right) - \frac{(1-\theta)}{\theta} s \left(e_t - E_{t-1}\left(e_t\right)\right)$$

MF_t can be further be written as,

$$\begin{aligned}
MF_t &= 2\frac{(1-\theta)}{\theta}\left(p_{h,t}-E_{t-1}\left(p_{h,t}\right)\right)\int\limits_{\Theta}\left(a_t(z)-a_t\right)d\left(z\right) \\
&\quad -2\frac{(1-\theta)}{\theta}s\left(e_t-E_{t-1}\left(e_t\right)\right)\int\limits_{\Theta}\left(a_t(z)-a_t\right)d\left(z\right) \\
&\quad -2\left(p_{h,t}-E_{t-1}p_{h,t}\right)\int\limits_{[0,1]\backslash\Sigma U\Theta}E_{t-1}\left(a_t(z)-a_t\right)d(z) \\
&\quad -2\left(p_{h,t}-E_{t-1}p_{h,t}\right)\int\limits_{\Sigma}E_{t-1}\left(a_t(z)-a_t\right)d(z) \\
&\quad +2\left(e_t-E_{t-1}\left(e_t\right)\right)\int\limits_{\Sigma}E_{t-1}\left(a_t(z)-a_t\right)d(z)
\end{aligned}$$

We can further simplify the previous expression as follows,

$$\begin{aligned}
MF_t &= 2\frac{(1-\theta)}{\theta}\left(p_{h,t}-E_{t-1}\left(p_{h,t}\right)\right)\int\limits_{A}\left(a_t(z)-a_t\right)d\left(z\right) \\
&\quad -2\frac{(1-\theta)}{\theta}s\left(e_t-E_{t-1}\left(e_t\right)\right)\int\limits_{A}\left(a_t(z)-a_t\right)d\left(z\right) \\
&\quad -2\left(p_{h,t}-E_{t-1}p_{h,t}\right)\int\limits_{[0,1]\backslash A}E_{t-1}\left(a_t(z)-a_t\right)d(z) \\
&\quad +2\left(e_t-E_{t-1}\left(e_t\right)\right)\int\limits_{\Gamma}E_{t-1}\left(a_t(z)-a_t\right)d(z)
\end{aligned}$$

Putting all the components of price distortion together we find,

$$
\begin{aligned}
\int (p_{h,t}(z) - p_{h,t})^2 \, dz &= \frac{(1-\theta)}{\theta} (p_{h,t} - E_{t-1}(p_{h,t}))^2 + \\
& \frac{(1-\theta)}{\theta} s\theta \left(1 + \frac{s(1-\theta)}{\theta}\right) (e_t - E_{t-1}(e_t))^2 + \\
& \int\limits_{\Theta} (a_t(z) - a_t)^2 \, d(z) + \int\limits_{[0,1]\setminus\Theta} (E_{t-1}(a_t(z) - a_t))^2 \, d(z) + \\
& -2 \left(\frac{(1-\theta)}{\theta}\right) s (p_{h,t} - E_{t-1}(p_{h,t})) (e_t - E_{t-1}(e_t)) + \\
& 2\frac{(1-\theta)}{\theta} (p_{h,t} - E_{t-1}(p_{h,t})) \int\limits_{\Theta} (a_t(z) - a_t) \, d(z) \\
& -2\frac{(1-\theta)}{\theta} s (e_t - E_{t-1}(e_t)) \int\limits_{\Theta} (a_t(z) - a_t) \, d(z) \\
& -2 (p_{h,t} - E_{t-1} p_{h,t}) \int\limits_{[0,1]\setminus\Theta} E_{t-1}(a_t(z) - a_t) \, d(z) \\
& +2 (e_t - E_{t-1}(e_t)) \int\limits_{\Sigma} E_{t-1}(a_t(z) - a_t) \, d(z)
\end{aligned}
$$

Using the following properties of large numbers,

$$\int\limits_{\Theta} a_t(z) d(z) = \theta a_t$$

$$\int\limits_{[0,1]\setminus\Theta} a_t(z) d(z) = (1-\theta) a_t$$

therefore, it follows that,

$$\int\limits_{\Theta} (a_t(z) - a_t) \, d(z) = \theta a_t - \theta a_t = 0$$

then, we can further simplify our lost function of price dispersion,

$$\begin{aligned}\int (p_{h,t}(z) - p_{h,t})^2 \, dz &= \frac{(1-\theta)}{\theta}\left(p_{h,t} - E_{t-1}\left(p_{h,t}\right)\right)^2 + \\ & \frac{(1-\theta)}{\theta} s\theta \left(1 + \frac{s\left(1-\theta\right)}{\theta}\right)\left(e_t - E_{t-1}\left(e_t\right)\right)^2 + \\ & \int\limits_{\Theta} \left(a_t(z) - a_t\right)^2 d\left(z\right) + \int\limits_{[0,1]\backslash\Theta} \left(E_{t-1}\left(a_t(z) - a_t\right)\right)^2 d(z) + \\ & -2\left(\frac{(1-\theta)}{\theta}\right) s\left(p_{h,t} - E_{t-1}\left(p_{h,t}\right)\right)\left(e_t - E_{t-1}\left(e_t\right)\right) + \\ & -2\left(p_{h,t} - E_{t-1}p_{h,t}\right) \int\limits_{[0,1]\backslash\Theta} E_{t-1}\left(a_t(z) - a_t\right) d(z) \\ & +2\left(e_t - E_{t-1}\left(e_t\right)\right) \int\limits_{\Sigma} E_{t-1}\left(a_t(z) - a_t\right) d(z)\end{aligned}$$

which after eliminate terms independent of policy, we obtain

$$\begin{aligned}\frac{1}{2}\int (p_{h,t}(z) - p_{h,t})^2 \, dz &= \frac{(1-\theta)}{\theta}\left(p_{h,t} - E_{t-1}\left(p_{h,t}\right)\right)^2 + \\ & \frac{(1-\theta)}{\theta} s\theta \left(1 + \frac{s\left(1-\theta\right)}{\theta}\right)\left(e_t - E_{t-1}\left(e_t\right)\right)^2 + \\ & -2\left(\frac{(1-\theta)}{\theta}\right) s\left(p_{h,t} - E_{t-1}\left(p_{h,t}\right)\right)\left(e_t - E_{t-1}\left(e_t\right)\right) + \\ & -2\left(p_{h,t} - E_{t-1}p_{h,t}\right) \int\limits_{[0,1]\backslash\Theta} E_{t-1}\left(a_t(z) - a_t\right) d(z) \\ & +2\left(e_t - E_{t-1}\left(e_t\right)\right) \int\limits_{\Sigma} E_{t-1}\left(a_t(z) - a_t\right) d(z)\end{aligned}$$

in order to save notation, we define the following gaps,

$$\widetilde{a}_t(z) = a_t(z) - a_t$$

$$\widetilde{p}_{h,t} = \left(p_{h,t} - E_{t-1}p_{h,t}\right)$$

$$\widetilde{e_t} = \left(e - E_{t-1}e_t\right)$$

therefore, we have,

$$\begin{aligned}
\frac{1}{2}\int \left(p_{h,t}(z)-p_{h,t}\right)^2 dz &= \frac{(1-\theta)}{2\theta}\widetilde{p}_{h,t}^2 + \\
&\quad \frac{(1-\theta)}{2\theta}s\theta\left(1+\frac{s\,(1-\theta)}{\theta}\right)\widetilde{e}_t^2-\left(\frac{(1-\theta)}{\theta}\right)s\widetilde{p}_{h,t}\widetilde{e}_t \\
&\quad +\widetilde{e}_t\int\limits_{\Sigma} E_{t-1}\widetilde{a}_t(z)d(z)
\end{aligned}$$

Now we need to calculate,

$$\begin{aligned}
\Delta_{c,t} &= -\varepsilon\int\limits_{\Theta}\widetilde{a}_t(z)p_{1,t}(z)dz \\
&\quad +\varepsilon\int\limits_{[0,1]\setminus\Sigma U\Theta}\widetilde{a}_t(z)p_{2,t}(z)d(z) \\
&\quad +\varepsilon\int\limits_{\Sigma}\widetilde{a}_t(z)p_{3,t}(z)dz
\end{aligned}$$

where, as we defined previously,

$$p_{1,t}(z)=\overline{p}_t+\widetilde{a}_t(z)$$

$$p_{2,t}(z)=E_{t-1}\widetilde{a}_t(z)-\widetilde{p}_{h,t}$$

$$p_{3,t}(z)=E_{t-1}\widetilde{a}_t(z)-\widetilde{p}_{h,t}+\widetilde{e}_t$$

and,

$$\overline{p}_t=\frac{1-\theta}{\theta}\widetilde{p}_{h,t}-\frac{1-\theta}{\theta}s\widetilde{e}_t$$

thus we have,

$$\begin{aligned}
\Delta_{c,t} &= -\varepsilon\int\limits_{\Theta}\widetilde{a}_t(z)\left(\widetilde{a}_t(z)+\frac{1-\theta}{\theta}\widetilde{p}_{h,t}-\frac{1-\theta}{\theta}s\widetilde{e}_t\right)dz \\
&\quad -\varepsilon\int\limits_{[0,1]\setminus\Sigma U\Theta}\widetilde{a}_t(z)\left(E_{t-1}\widetilde{a}_t(z)-\widetilde{p}_{h,t}\right)d(z) \\
&\quad -\varepsilon\int\limits_{\Sigma}\widetilde{a}_t(z)\left(E_{t-1}\widetilde{a}_t(z)-\widetilde{p}_{h,t}+\widetilde{e}_t\right)dz
\end{aligned}$$

Simplifying the previous expression we obtain,

$$
\begin{aligned}
\Delta_{c,t} &= -\varepsilon \int_{\Theta} \widetilde{a}_t^2(z)dz - \int_{[0,1]\backslash\Theta} E_{t-1}\widetilde{a}_t(z)\widetilde{a}_t(z)d(z) \\
&\quad -\varepsilon \frac{1-\theta}{\theta}\widetilde{p}_{h,t}\int_{\Theta} \widetilde{a}_t(z)dz + \varepsilon\frac{1-\theta}{\theta}s\widetilde{e}_t\int_{\Theta}\widetilde{a}_t(z)dz \\
&\quad +\varepsilon\widetilde{p}_{h,t}\int_{[0,1]\backslash\Theta} \widetilde{a}_t(z)d(z) - \varepsilon\widetilde{e}_t\int_{\Sigma}\widetilde{a}_t(z)dz
\end{aligned}
$$

since, as we shown previously, $\int_{[0,1]\backslash\Theta} \widetilde{a}_t(z)d(z) = 0$, we have that,

$$
\Delta_{c,t} = -\varepsilon\widetilde{e}_t\int_{\Sigma}\widetilde{a}_t(z)dz + t.i.p + o\left(\|\varepsilon\|^3\right)
$$

Plugging in the relative price dispersion equation into the welfare function derived previously, we obtain the following quadratic welfare function for this economy with price dollarization,

$$
u\left(C_t\right) - v(h_t) = \overline{u}_c\overline{Y}\left(-\frac{1}{2}\left(1-\gamma\right)\left(1+v\right)\widehat{x}_t^2 - u_{\Delta}\widehat{\Delta}_t\right) + t.i.p + o\left(\|\varepsilon\|^3\right) \tag{C.22}
$$

$$
\Delta_t = var_z a_t + \frac{1}{2}\varepsilon^2\int\left(p_{h,t}(z) - p_{h,t}\right)^2 dz + \Delta_{c,t}
$$

Thus the lost function of the central bank, presented in the main text as equation (4.9) can be written as,

$$
W = -\Omega E_0 \sum_{t=0}^{\infty}\beta^t L_t
$$

$$
L_t = \Lambda\frac{1}{2}\widehat{x}_t^2 + \frac{1}{2}\widetilde{p}_{h,t}^2 + \frac{\Lambda_e}{2}\widetilde{e}_t{}^2 - s\widetilde{p}_{h,t}\widetilde{e}_t - \Lambda_{ea}\widetilde{e}_t a_{s,t} + t.i.p + o\left(\|\varepsilon\|^3\right)
$$

where,

$$
\begin{aligned}
\Omega &= \overline{u}_c\overline{Y}\left(1-\gamma\right)\varepsilon^2\tfrac{(1-\theta)}{\theta} & \Lambda_{ea} &= \tfrac{\theta}{1-\theta} \\
\Lambda &= \left(1+v\right)\tfrac{\theta}{(1-\theta)\varepsilon^2} & \overline{a}_{se,t} &= \int_{\Sigma} E_{t-1}\left(a_t(z) - a_t\right)d(z) \\
\overline{a}_{ss,t} &= \int_{\Sigma}\left(a_t(z) - a_t\right)d(z) & a_{s,t} &= \left(\overline{a}_{ss,t} + \tfrac{1}{\varepsilon}a_{se,t}\right) \\
\widetilde{p}_{h,t} &= \left(p_{h,t} - E_{t-1}p_{h,t}\right) & \widetilde{e}_t &= \left(e - E_{t-1}e_t\right)
\end{aligned}
$$

D Monetary Policy Under Commitment

The central bank has to choose the domestic prices, the output gap and the exchange rate to minimize the following lost function

min

$$L_t = \Lambda \frac{1}{2} \widehat{x}_t^2 + \frac{1}{2} \widetilde{p}_{h,t}^2 + \frac{\Lambda_e}{2} \widetilde{e}_t{}^2 + s \widetilde{p}_{h,t} \widetilde{e}_t - \Lambda_{ea} \widetilde{e}_t a_{s,t}$$

subject to the constraint of the Phillips curve and the dynamics of the nominal exchange rate, equations (D.1) and (D.2), presented next,

$$\widetilde{p}_{h,t} = \kappa x_t + s \widetilde{e}_t \tag{D.1}$$

$$e_t = p_{h,t} + x_t + \eta_t \tag{D.2}$$

where, $\kappa = (1+\nu) \frac{\theta}{(1-\theta)}$, denotes the slope of the Phillips curve, and η_t represents a shock to the real exchange rate that summarizes the effect of the following structural shocks on the nominal exchange rate,

$$\eta_t = -x_t^* - (a_t - a_t^*) - \pi_t^*$$

We solve for the optimal monetary policy under commitment by maximizing the following Lagrangian function, which after applying the law of iterative expectations, can be written as follows,

$$\begin{gathered} E_0 \{ \sum_{t=0}^{\infty} \{ \Lambda \tfrac{1}{2} \beta^t \widehat{x}_t^2 + \tfrac{1}{2} \beta^t \widetilde{p}_{h,t}^2 + \beta^t \tfrac{\Lambda_e}{2} \widetilde{e}_t{}^2 + \\ \beta^t s \widetilde{p}_{h,t} \widetilde{e}_t - \Lambda_{ea} \beta^t (e_t - E_{t-1} e_t) \widetilde{e}_t a_{s,t} \\ + \iota_{1,t} \beta^t (p_{h,t} - p_{h,t} - \kappa x_t - s (e_t - e_t)) + \\ + \iota_{2,t} \beta^t (e_t - p_{h,t} - x_t - \eta_t) + \} \} \end{gathered}$$

where, $\iota_{1,t}$ and $\iota_{2,t}$ are the Lagrange multipliers of the Phillips curve and of the equation that constraints the dynamics of the nominal exchange rate. The first order conditions are given by the following three equations,

$$\Lambda \widehat{x}_t - \kappa \iota_{1,t} - \iota_{2,t} = 0 \tag{D.3}$$

$$\widetilde{p}_{h,t} + s \widetilde{e}_t - \iota_{2,t} = 0 \tag{D.4}$$

$$\Lambda_e \widetilde{e}_t + s \widetilde{p}_{h,t} - \Lambda_{ea} a_{s,t} + \iota_{2,t} = 0 \tag{D.5}$$

These conditions hold at each t, with $t \succeq 1$. They also hold at time 0, given the initial conditions,

$$\iota_{1,-1} = \iota_{2,-1}$$

The optimal plan is a set of policy functions for $\widehat{x}_t$, $p_{h,t}$, e_t, $\iota_{1,t}$,and $\iota_{2,t}$, that satisfy conditions,(D.1),(D.2), (D.3),(D.4),and (D.5), given the initial conditions and the processes for the exogenous variables, $\overline{\overline{a}}_{tss}$, and η_t^*. To find the allocating under optimal policy we can combine equations (D.4) and (D.5) to eliminate, $\iota_{2,t}$ as follows,

$$(\Lambda_e + s)\,\widetilde{e}_t - \Lambda_{ea} a_{s,t} + (1+s)\,\widetilde{p}_{h,t} = 0 \tag{D.6}$$

the remaining equations that define the economy are given by,

$$\Lambda \widehat{x}_t - \kappa \iota_{1,t} - \widetilde{p}_{h,t} + s\widetilde{e}_t = 0 \tag{D.7}$$

$$e_t = p_{h,t} + x_t + \eta_t \tag{D.8}$$

$$\widetilde{p}_{h,t} = \kappa x_t + s\widetilde{e}_t \tag{D.9}$$

Since from the Phillips curve we have that, $E_{t-1}x_t = 0$, thus we can write equation (D.8) as follows,

$$\widetilde{e}_t = \widetilde{p}_{h,t} + x_t + \widetilde{\eta}_t \tag{D.10}$$

using the previous equation and the Phillips curve we can eliminate the output gap,thus we have, a second condition that relates exchange rate and domestic prices,

$$(\kappa + s)\,\widetilde{e}_t = (1+\kappa)\,\widetilde{p}_{h,t} + \kappa \widetilde{\eta}_t \tag{D.11}$$

combining equations (D.6) and (D.11) we solve for the exchange rate and level of domestic prices,

$$\widetilde{e}_t = \varpi_1 \widetilde{\eta}_t + \varpi_2 a_{s,t} \tag{D.12}$$

where,

$$\varpi_1 = \frac{\kappa}{\left[(\kappa+s) + \frac{(1+\kappa)}{(1+s)}(\Lambda_e + s)\right]} \quad \varpi_2 = \frac{\Lambda_{ea}}{\left[\frac{(1+s)}{(1+\kappa)}(\kappa+s) + (\Lambda_e+s)\right]}$$

using, equation (D.6) and equation (D.12) we can find the rational equilibrium of prices,

$$\widetilde{p}_{h,t} = -\varpi_3 \widetilde{\eta}_t + \varpi_4 a_{s,t} \tag{D.13}$$

where the parameters, ϖ_3 and ϖ_4 are defined as follows,

$$\varpi_3 = \frac{\kappa(\Lambda_e+s)}{[(\kappa+s)(1+s) + (1+\kappa)(\Lambda_e+s)]} \quad \varpi_4 = \frac{(\kappa+s)\Lambda_{ea}}{[(1+s)(\kappa+s) + (1+\kappa)(\Lambda_e+s)]}$$

E Endogenous Price Dollarization

E.1 Deriving profit function,

In this appendix we show how to obtain equations (5.3) and (5.4) of the main text. The profit function of a particular firm, z is given by,

$$\Omega\left(z\right) = E_{t-1}\left[\left(P_{H,t}(z) - W_t A_t(z)\right) P_{H,t}^{-\varepsilon}(z) P_{H,t}^{\varepsilon-1}\right] \tag{E.1}$$

From the first order condition of optimal pricing, we have that,

$$E_{t-1}\left[\left(P_{H,t}(z) - \mu W_t A_t(z)\right) P_{H,t}^{\varepsilon-1}\right] = 0 \tag{E.2}$$

rearranging equation (E.1), we can use equation (2.32) to eliminate, the $P_{H,t}(z)$,

$$\Omega\left(z\right) = E_{t-1}\left[\left(P_{H,t}(z) - \mu W_t A_t(z) + \left(\mu - 1\right) W_t A_t(z)\right) P_{H,t}^{-\varepsilon}(z) P_{H,t}^{\varepsilon-1}\right] \tag{E.3}$$

which can be written also as,

$$\begin{aligned} \Omega\left(z\right) &= E_{t-1}\left[\left(P_{H,t}(z) - \mu W_t A_t(z)\right) P_{H,t}^{-\varepsilon}(z) P_{H,t}^{\varepsilon-1}\right] \\ &\quad + E_{t-1}\left[\left(\mu - 1\right) W_t A_t(z) P_{H,t}^{-\varepsilon}(z) P_{H,t}^{\varepsilon-1}\right] \end{aligned} \tag{E.4}$$

By equation (E.2), the first part of the previous equation is zero, we have that the optimal level of profits are given by,

$$\Omega\left(z\right) = \left(\mu - 1\right) E_{t-1}\left[W_t A_t(z) P_{H,t}^{\varepsilon-1}\right] P_{H,t}^{-\varepsilon}(z) \tag{E.5}$$

thus, plugging the optimal price, we have,

$$P_{H,t}^{-\varepsilon}(z) = \mu^{-\varepsilon} \frac{\left(E_{t-1}\left(W_t A_t(z)\, P_{H,t}^{\varepsilon-1}\right)\right)^{-\varepsilon}}{\left(E_{t-1}\left(P_{H,t}^{\varepsilon-1}\right)\right)^{-\varepsilon}} \tag{E.6}$$

$$\Omega\left(z\right) = \left(\mu - 1\right) \mu^{-\varepsilon} \frac{\left(E_{t-1}\left(W_t A_t(z)\, P_{H,t}^{\varepsilon-1}\right)\right)^{1-\varepsilon}}{\left(E_{t-1}\left(P_{H,t}^{\varepsilon-1}\right)\right)^{-\varepsilon}} \tag{E.7}$$

In terms of real wages, it can be written as follows,

$$\Omega\left(z\right)=\left(\mu-1\right)\mu^{-\varepsilon}\frac{\left(E_{t-1}\left(w_{t}A_{t}(z)\,P_{H,t}^{\varepsilon}\right)\right)^{1-\varepsilon}}{\left(E_{t-1}\left(P_{H,t}^{\varepsilon-1}\right)\right)^{-\varepsilon}} \tag{E.8}$$

similarly for the case of pricing in foreign currency,

$$\Psi\left(z\right)=\left(\mu-1\right)\mu^{-\varepsilon}\frac{\left(E_{t-1}\left(w_{t}A_{t}(z)\,d_{H,t}^{\varepsilon}\right)\right)^{1-\varepsilon}}{\left(E_{t-1}\left(d_{H,t}^{\varepsilon-1}\right)\right)^{-\varepsilon}} \tag{E.9}$$

Equation (E.8) correspond to equation (5.3) , whereas equation (E.9) to equation (5.4), in the main text.

E.2 Equilibrium Condition for PD

After taking a log linear approximation of equation (5.4) we obtain the following condition for pricing in pesos,

$$\begin{aligned} 0 \quad < \quad & -(\epsilon-1)\left(E_{t-1}\left(\widehat{w_{t}A_{t}(z)}P_{H,t}^{\epsilon}\right)-E_{t-1}\left(\widehat{w_{t}A_{t}(z)}d_{H,t}^{\epsilon)}\right)\right) \\ & +\epsilon\left(\widehat{E_{t-1}\left(p_{H,t}^{\epsilon-1}\right)}-\widehat{E_{t-1}\left(d_{H,t}^{(\epsilon-1)}\right)}\right) \end{aligned}$$

Now we take a second order approximation of, each component in the previous expression

$$\frac{\left(d_{H,t}^{(\epsilon-1)}-d_{H}^{(\epsilon-1)}\right)}{d_{H}^{(\epsilon-1)}}\cong\left(1+(\epsilon-1)\widehat{d}_{h,t}+\frac{1}{2}(\epsilon-1)^{2}\widehat{d}_{h,t}^{2}\right)$$

similarly for the case of the price in pesos,

$$\frac{\left(p_{H,t}^{(\epsilon-1)}-p_{H}^{(\epsilon-1)}\right)}{p_{H}^{(\epsilon-1)}}\cong\left(1+(\epsilon-1)\widehat{p}_{h,t}+\frac{1}{2}(\epsilon-1)^{2}\widehat{p}_{h,t}^{2}\right)$$

let's now look at the case of,

$$
\begin{aligned}
\frac{w_t A_t(z) P_{H,t}^{\epsilon} - wAP_H^{\epsilon}}{wAP_H^{\epsilon}} &\cong 1 + \widehat{w}_t + a_t(z) + \epsilon\left(\widehat{p}_{h,t} + \frac{1}{2}\widehat{p}_{h,t}^2\right) \\
&+\frac{1}{2}\widehat{W}_t^2 + \frac{1}{2}a_t^2(z) + \epsilon(\epsilon-1)\frac{1}{2}\widehat{p}_{h,t}^2 \\
&+\epsilon\left(\widehat{p}_{h,t}\widehat{W}_t + \widehat{p}_{h,t}a_t(z)\right)
\end{aligned}
$$

similarly for the case of pricing in foreign currency,

$$
\begin{aligned}
\frac{w_t A_t(z) d_{H,t}^{\epsilon} - wAd_H^{\epsilon}}{w_t A d_H^{\epsilon}} &\cong 1 + \widehat{w}_t + a_t(z) + \epsilon(\widehat{d}_{h,t} + \frac{1}{2}\widehat{d}_{h,t}^2) \\
&\frac{1}{2}\widehat{w}_t^2 + \frac{1}{2}a_t^2(z) + \epsilon(\epsilon-1)\frac{1}{2}\widehat{d}_{h,t}^2 \\
&+\epsilon\left(\widehat{d}_{h,t}\widehat{w}_t + \widehat{d}_{h,t}a_t(z)\right)
\end{aligned}
$$

Thus, using the previous expression we can calculate the following,

$$
H \cong E_{t-1}\left((\epsilon-1)\left(\widehat{p}_{h,t} - \widehat{d}_{h,t}\right) + \frac{1}{2}(\epsilon-1)^2\left(\widehat{p}_{h,t}^2 - \widehat{d}_{h,t}^2\right)\right)
$$

similarly for

$$
J = \left(E_{t-1}\left(\widehat{w_t A_t(z)} P_{H,t}^{\epsilon}\right) - E_{t-1}\left(\widehat{wA_t(z)} d_{H,t}^{\epsilon}\right)\right)
$$

we have that,

$$
\begin{aligned}
J \;\cong\;& 1 + E_{t-1}\widehat{w}_t + E_{t-1}a_t(z) + \epsilon E_{t-1}\widehat{p}_{h,t} + \\
& \frac{1}{2}E_{t-1}\widehat{w}_t^2 + \frac{1}{2}E_{t-1}a_t^2(z) + \epsilon^2\frac{1}{2}E_{t-1}\widehat{p}_{h,t}^2 + \\
& \epsilon\left(E_{t-1}\widehat{p}_{h,t}\widehat{w}_t + E_{t-1}\widehat{p}_{h,t}a_t(z)\right) - 1 - E_{t-1}\widehat{w}_t - E_{t-1}a_t(z) \\
& -\epsilon E_{t-1}\widehat{d}_{h,t} - \frac{1}{2}E_{t-1}\widehat{w}_t^2 - \frac{1}{2}E_{t-1}a_t^2(z) \\
& -\epsilon^2\frac{1}{2}E_{t-1}\widehat{d}_{h,t}^2 - \epsilon\left(E_{t-1}\widehat{d}_{h,t}\widehat{w}_t + E_{t-1}\widehat{d}_{h,t}a_t(z)\right)
\end{aligned}
$$

simplifying the previous expression, we obtain,

$$
\begin{aligned}
J \;\cong\;& \epsilon E_{t-1}\left(\widehat{p}_{h,t} - \widehat{d}_{h,t}\right) + \epsilon^2\frac{1}{2}E_{t-1}\left(\widehat{p}_{h,t}^2 - \widehat{d}_{h,t}^2\right) \\
& +\epsilon E_{t-1}\left(\widehat{p}_{h,t} - \widehat{d}_{h,t}\right)\left(\widehat{w}_t + a_t(z)\right)
\end{aligned}
$$

Since we are looking for

$$K = \epsilon H - (\epsilon - 1)J$$

we get,

$$\begin{aligned}
K &= \epsilon(\epsilon-1)E_{t-1}\left(\widehat{p}_{h,t} - \widehat{d}_{h,t}\right) + \frac{1}{2}\epsilon(\epsilon-1)^2 E_{t-1}\left(\widehat{p}^2_{h,t} - \widehat{d}^2_{h,t}\right) \\
&\quad -\epsilon(\epsilon-1)E_{t-1}\left(\widehat{p}_{h,t} - \widehat{d}_{h,t}\right) - (\epsilon-1)\epsilon^2\frac{1}{2}E_{t-1}\left(\widehat{p}^2_{h,t} - \widehat{d}^2_{h,t}\right) - \\
&\quad (\epsilon-1)\epsilon E_{t-1}\left(\widehat{p}_{h,t} - \widehat{d}_{h,t}\right)\left(\widehat{w}_t + a_t(z)\right)
\end{aligned}$$

simplifying the previous expression we obtain the following condition,

$$\begin{aligned}
K &= -\frac{1}{2}\epsilon(\epsilon-1)E_{t-1}\left(\widehat{p}^2_{h,t} - \widehat{d}^2_{h,t}\right) \\
&\quad (\epsilon-1)\epsilon E_{t-1}\left(\widehat{p}_{h,t} - \widehat{d}_{h,t}\right)\left(\widehat{w}_t + a_t(z)\right)
\end{aligned}$$

Notice that from the definition of $\widehat{d}_{h,t}$ we have that, $\widehat{d}_{h,t} - \widehat{p}_{h,t} = e_t$, thus the previous expression becomes,

$$\begin{aligned}
K &= -\frac{1}{2}E_{t-1}\left(\widehat{p}^2_{h,t} - \widehat{d}^2_{h,t}\right) \\
&\quad -E_{t-1}\left[e_t\left(\widehat{w}_t + a_t(z)\right)\right]
\end{aligned}$$

Thus a firms will set prices in pesos, when, $K > 0$, which holds, when,

$$E_{t-1}\left(\widehat{p}^2_{h,t} - \widehat{d}^2_{h,t}\right) + 2E_{t-1}\left[e_t\left(\widehat{w}_t + a_t(z)\right)\right] < 0$$

since we know that,

$$w_t = -a_t + \frac{(1-\theta)}{\theta}\left(p_{h,t} - E_{t-1}\left(p_{h,t}\right)\right) - \frac{(1-\theta)}{\theta}s\left(e_t - E_{t-1}\left(e_t\right)\right)$$

$$\begin{aligned}
E_{t-1}\left[e_t\left(\widehat{w}_t + a_t(z)\right)\right] &= E_{t-1}\left[e_t\left(a_t(z) - a_t\right)\right] \\
&\quad + \frac{(1-\theta)}{\theta}E_{t-1}\left[e_t\left(p_{h,t} - E_{t-1}\left(p_{h,t}\right)\right)\right] \\
&\quad - s\frac{(1-\theta)}{\theta}E_{t-1}\left[e_t\left(e_t - E_{t-1}\left(e_t\right)\right)\right]
\end{aligned}$$

we can thus write the condition for setting prices in pesos as follows,

$$\begin{aligned}
0 &> \frac{1}{2}E_{t-1}\left(\widehat{p}_{h,t}^{2}-\widehat{d}_{h,t}^{2}\right)+E_{t-1}\left[e_t\left(a_t(z)-a_t\right)\right] \\
&+\frac{(1-\theta)}{\theta}E_{t-1}\left[e_t\left(p_{h,t}-E_{t-1}\left(p_{h,t}\right)\right)\right] \\
&-s\frac{(1-\theta)}{\theta}E_{t-1}\left[e_t\left(e_t-E_{t-1}\left(e_t\right)\right)\right]
\end{aligned}$$

This last equation corresponds to equation (5.5) in the main text,