



Common and idiosyncratic movements in Latin-American Exchange Rates

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Main findings

- We examine a simple factor model, that allows us to decompose theoretically and empirically the common and idiosyncratic movements in Latin-American exchange rates (Brazil, Chile, Colombia, Mexico, and Peru).
- We provide evidence that the regional common factor has a significant effect on the dynamics of the Latin-American exchange rates.
- In our estimations the association between exchange rates and the common factor is contemporaneous and stable during the studied period.
- This is the first paper we are aware of that documents the drivers of bilateral exchange rates in Latin America including the recent period of the Covid-19 pandemic using a simple theoretical and empirical approach that allows to differentiate between regional and idiosyncratic movements in exchange rates.
- This method permits us to examine if there is evidence of structural changes in the factors that explain comovements of exchange rates in the region.



Outline

STYLIZED FACTS

LITERATURE REVIEW

THEORETICAL APPROACH

METHODOLOGY AND DATA

RESULTS

FINAL REMARKS



Outline

STYLIZED FACTS

LITERATURE REVIEW

THEORETICAL APPROACH

METHODOLOGY AND DATA

RESULTS

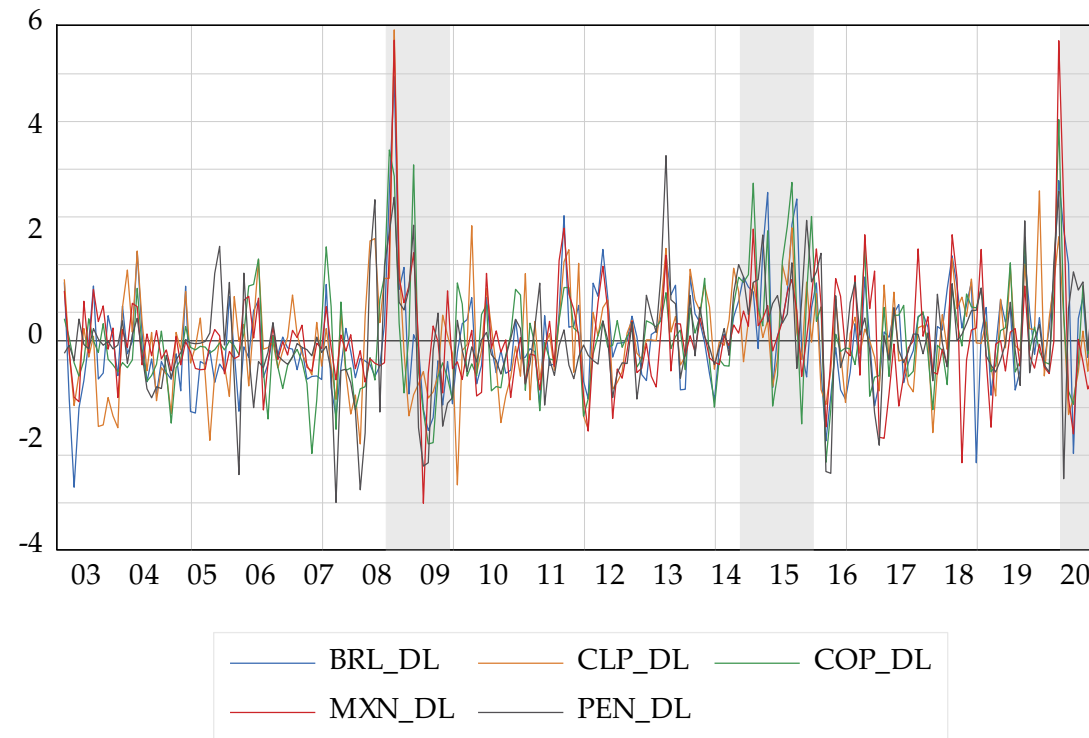
FINAL REMARKS

Stylized facts

- Throughout the nineties, central banks in LATAM economies switched to a system of flexible or managed exchange rates. After floating their currencies, central banks in these economies adopted an inflation targeting monetary framework. These characteristics make them very suitable for studying the pattern of their currencies and their main determinants.
- Exchange rates in Latin-America display an important level of comovement. This phenomenon may be related with investors' appetite and portfolio inflows to the region, despite the differences that may exist between these countries.

Stylized facts

- Despite practitioners and academics have pointed out that the comovement in Latin-American exchange rates may be related to the existence of underlying factors - *related to global financial cycles or commodity prices* - there is not a consensus regarding the drivers of this phenomenon.



LATIN-AMERICAN EXCHANGE RATE MONTHLY CHANGES (LOG-DIFFERENCES)

Notes: The series correspond to the log differences (monthly returns %) of the nominal exchange rate indices of the Brazilian Real (BRL_DL), the Chilean Peso (CLP_DL), the Colombian Peso (CLP_DL), the Mexican Peso (MXN_DL) and the Peruvian Sol (PEN_DL). The shaded areas correspond to the 2008/2009 Global Financial Crisis, the 2015/2016 Commodities price shock, and the Covid-19 Recession. Source: Bloomberg and authors' calculations.

Stylized facts

- Correlations between LATAM exchange rate returns are positive, and significantly different from zero.

Correlations between monthly changes of Latin-American Exchange Rates

Correlation [t-Statistic]	BRL_DL	CLP_DL	COP_DL	MXN_DL	PEN_DL
BRL_DL	1.00				

CLP_DL	0.60*** [11.20]	1.00			

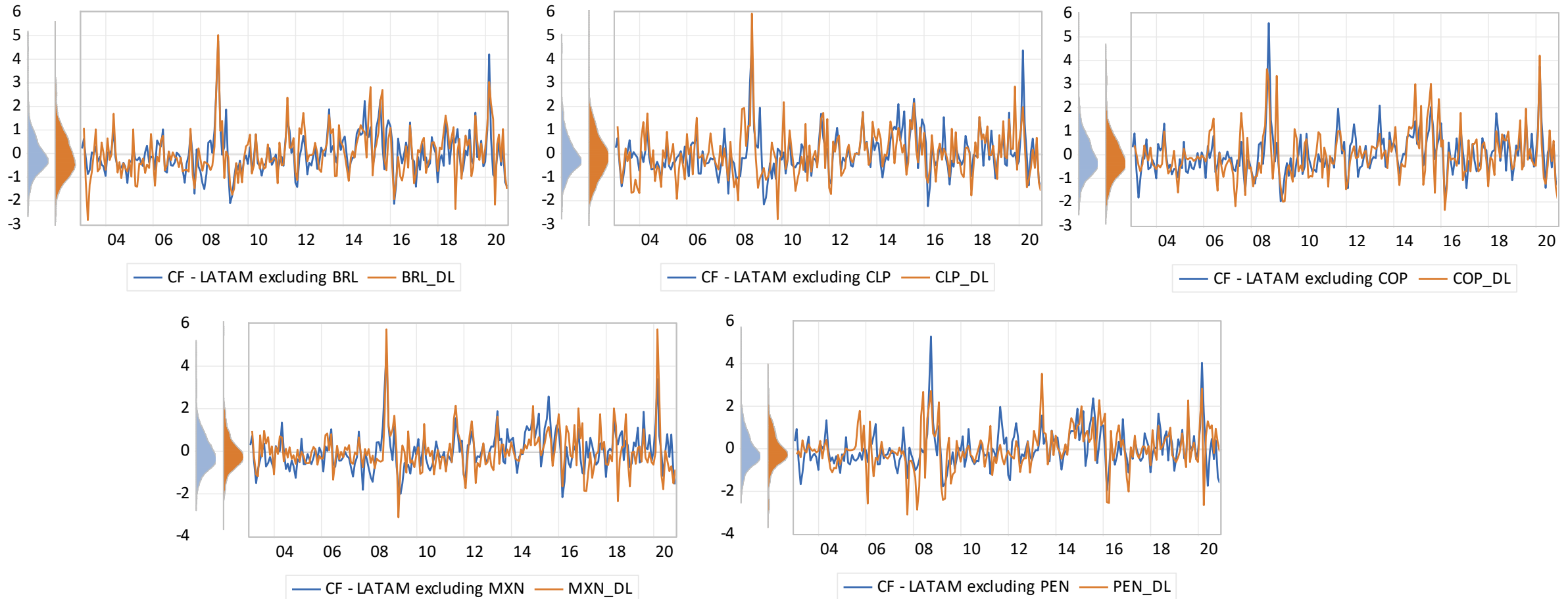
COP_DL	0.64*** [12.00]	0.51*** [8.69]	1.00		

MXN_DL	0.61*** [11.36]	0.48*** [8.19]	0.590*** [10.94]	1.00	

PEN_DL	0.42*** [6.84]	0.40*** [6.38]	0.52*** [8.87]	0.41*** [6.61]	1.00

Source: Authors' calculations. Notes: The series correspond to the log differences of the nominal exchange rate indices of the Brazilian Real (BRL_DL), the Chilean Peso (CLP_DL), the Colombian Peso (CLP_DL), the Mexican Peso (MXN_DL) and the Peruvian Sol (PEN_DL). [] t-stat. * p<0.1; ** p<0.05; *** p<0.01. These correlations are computed from 2003:01 to 2020:12.

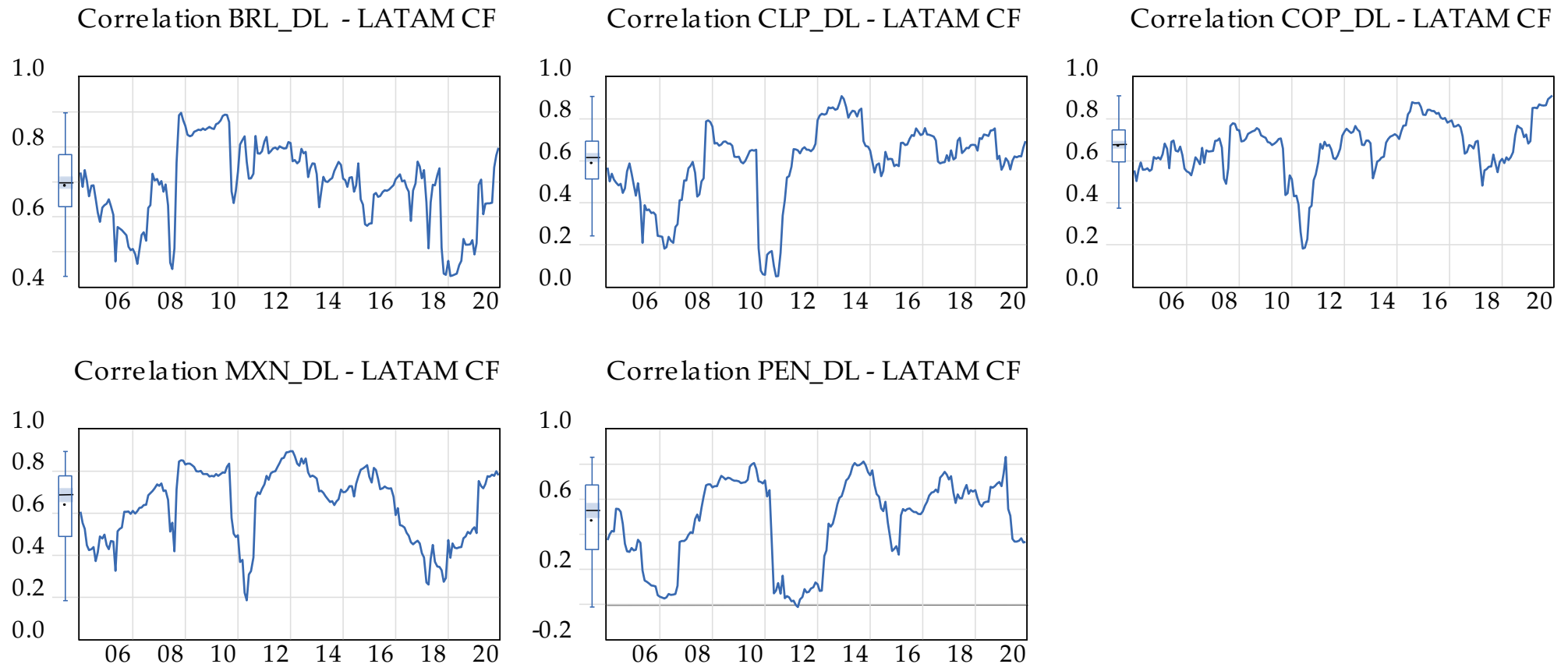
Stylized facts



Source: Bloomberg and Authors' calculations. Notes: For the graphs the common factor plotted against each exchange rate is computed as the dynamic factor of the other four currencies. The CF - LATAM represents monthly returns (%). Empirical (kernel) distributions are plotted in the RHA.

Stylized facts

- The correlation between exchange rate returns and the *common factor* is stronger and positive during turbulent periods such as the financial crisis of 2008-2009, the drop of commodity prices of 2014 and 2015, and the recent crisis caused by the COVID-19 pandemic.



CORRELATION BETWEEN LATIN-AMERICAN EXCHANGE RATE MONTHLY CHANGES AND THE REGIONAL COMMON FACTOR (LOG-DIFFERENCES)

Source: Authors' calculations. The correlations are computed using a 24-month rolling window. In the box-plot, the box portion represents the first and third quartiles (middle 50 percent of the data). The mean is represented by the black dots, the black line stands for the median and the blue shaded areas are the median 95% confidence interval.



Outline

STYLIZED FACTS

LITERATURE REVIEW

THEORETICAL APPROACH

METHODOLOGY AND DATA

RESULTS

FINAL REMARKS

Literature Review

- Different econometric methods have been extensively used to study the main determinants of bilateral exchange rate movements. Nevertheless, the literature on common currency factors driving exchange rates is not that abundant for emerging economies.
- The most influential research we are aware of are:
 - Cayen et al (2010) identify two common factors using a dynamic factor model for a panel of six developed economies over the 1980-2007 period.
 - Lustig, Roussanov, and Verdelhan (2011) find two factors that explain the variation in foreign currency excess returns: i) the dollar risk factor, and ii) the “slope factor” or carry trade risk factor.
 - Baku (2019) predicts currency returns for Brazil, Chile, Colombia, Mexico, and Peru between 2001 and 2016. The results indicate that the Global Exchange Rate Factor derived from a factor model approach is an important determinant of exchange rate movements.
 - Greenaway-McGrevy et al (2018) find that exchange rate returns from 1999 to 2015 in a sample of 27 currencies are driven by global factors (a dollar factor and a euro factor).
 - Aloosh and Bekaert (2019) explain which factors determine the comovements of exchange rates in G-10 countries. The authors find that a factor model including a commodity currency factor, and a world or market factor, explains much better currency variation than other models.



Outline

STYLIZED FACTS

LITERATURE REVIEW

THEORETICAL APPROACH

METHODOLOGY AND DATA

RESULTS

FINAL REMARKS

Theoretical approach

- We assume that the risk-neutral efficient market hypothesis does not hold due to risk aversion of market participants. Thus, the modified UIP is:

$$i_t - i_t^* = \Delta s_{t+1} + \rho_t$$

- Where Δs_{t+1} are the exchange rate returns, ρ_t is the time-varying risk premium, i_t and i_t^* are the domestic and the US interest rates, respectively.
- To derive the pattern of spot exchange rates, we specify the monetary policy rules followed by the domestic and foreign monetary authorities as follows:

$$i_t = \gamma_\pi \pi_t + \gamma_y y_t + \gamma_q q_t + \delta i_{t-1} + v_t$$

$$i_t^* = \gamma_\pi \pi_t^* + \gamma_y y_t^* + \delta i_{t-1}^* + v_t^*$$

- where π_t is the inflation rate, y_t is the output gap, and q_t is the real exchange rate. * denotes foreign variables. $\gamma_\pi, \gamma_y, \gamma_q$ are non-negative and $0 \leq \delta < 1$.

Theoretical approach

- Using the two Taylor rules above with the modified UIP we get the following equation:

$$\Delta s_{t+1} = \gamma_{\pi}(\pi_t - \pi_t^*) + \gamma_y(y_t - y_t^*) + \delta(i_{t-1} - i_{t-1}^*) + \gamma_q q_t + (v_t - v_t^* - \rho_t)$$

- If the output gap is a function of the terms of trade, and there is not any explicit reaction of the domestic economy to the real exchange rate ($q_t = 0$), we get the following expression for the spot exchange rate returns:

$$\Delta s_{t+1} = \gamma_{\pi}(\pi_t - \pi_t^*) + \gamma_y(T_t - T_t^*) + \delta(i_{t-1} - i_{t-1}^*) + \psi_t$$

- where T_t and T_t^* are measures of the terms of trade for the domestic and foreign economy, respectively, and ψ_t measures country risk and is approximately equal to $(v_t - v_t^* - \rho_t)$. As a proxy for ψ_t we use credit default swaps (CDS) for each country in our estimations.

Theoretical approach

- In this paper we describe the pattern of the spot exchange rate for each country as a function of the common exchange rate factor of LATAM region. Thus, if we have two countries A and B, we have the following set of equations:

$$\Delta s_{t+1}^A = \gamma_{\pi}(\pi_t^A - \pi_t^*) + \gamma_y(T_t^A - T_t^*) + \delta(i_{t-1}^A - i_{t-1}^*) + \psi_t^A$$

$$\Delta s_{t+1}^B = \gamma_{\pi}(\pi_t^B - \pi_t^*) + \gamma_y(T_t^B - T_t^*) + \delta(i_{t-1}^B - i_{t-1}^*) + \psi_t^B$$

- Therefore, in relative terms, variables for the US denoted by * cancel out in the equation as follows:

$$\Delta s_{t+1}^A = \Delta s_{t+1}^B + \gamma_{\pi}(\pi_t^A - \pi_t^B) + \gamma_y(T_t^A - T_t^B) + \delta(i_{t-1}^A - i_{t-1}^B) + (\psi_t^A - \psi_t^B)$$

Theoretical approach

- Then, the depreciation rate of country A relative to the average depreciation rate of the region must be equal to:

$$\Delta S_{t+1}^A = F_t + \gamma_\pi (\pi_t^A - \pi_t^{LATAM}) + \gamma_y (T_t^A - T_t^{LATAM}) \\ + \delta (i_{t-1}^A - i_{t-1}^{LATAM}) + (\psi_t^A - \psi_t^{LATAM})$$

- where F_t is the common exchange rate factor for LATAM economies excluding country A. π_t^{LATAM} , T_t^{LATAM} , i_{t-1}^{LATAM} , ψ_t^{LATAM} are the simple average of the corresponding variables for LATAM economies excluding country A.
- Finally, we transform the previous equation into this form:

$$\Delta S_{t+1}^A = F_t + \gamma_\pi \left(\frac{\pi_t^A}{\pi_t^{LATAM}} \right) + \gamma_y \left(\frac{T_t^A}{T_t^{LATAM}} \right) \\ + \delta (i_{t-1}^A - i_{t-1}^{LATAM}) + (\psi_t^A - \psi_t^{LATAM})$$



Outline

STYLIZED FACTS

LITERATURE REVIEW

THEORETICAL APPROACH

METHODOLOGY AND DATA

RESULTS

FINAL REMARKS

Methodology and data

- The previous Equation shows a simple way to divide exchange rate changes into a common factor and idiosyncratic elements.
- The common factor in LATAM currencies is estimated through a dynamic factor model following the methodology proposed by Solberger & Spånberg (2019). (See appendix).
- We test our model in both low (monthly) and high (daily) frequencies. For each model we computed a specific factor. For example, in the case of the Brazil Real, the estimated models use a LATAM common factor that includes Chilean peso, Colombian Peso, Mexican Peso and Peruvian Sol.
- We use both OLS and GMM models for monthly data while we use GARCH models for daily estimations.
- CUSUM and recursive OLS estimations are used to test parameter stability.
- We use local projections to assess the dynamic properties of our specifications.
- Standard in-sample forecast evaluation.

Methodology and data

- Source: To calculate exchange rate returns we use data from Bloomberg. Data on commodities (country specific and IMF's CToT), exchange rate returns, CDS, interest rates, and the emerging market currency index are obtained from the same source.
- Period: 2003:1 to 2020:12
- Frequency: Monthly and daily



Outline

STYLIZED FACTS

THEORETICAL APPROACH

METHODOLOGY AND DATA

RESULTS

FINAL REMARKS

Results – Monthly frequency

	BRAZIL (DLOG(BRL))		CHILE (DLOG(CLP))		COLOMBIA (DLOG(COP))	
	OLS	GMM	OLS	GMM	OLS	GMM
	DLOG(BRL)	DLOG(BRL)	DLOG(CLP)	DLOG(CLP)	DLOG(COP)	DLOG(COP)
DLOG(Latam common factor ex_)	1.247 *** [0.11]	1.643 *** [0.11]	0.699 *** [0.09]	1.001 *** [0.08]	0.979 *** [0.07]	1.089 *** [0.08]
D(Commodities ratio)	-0.006 [0.03]	-0.035 [0.04]	-0.033 *** [0.00]	-0.031 *** [0.00]	-0.060 ** [0.02]	-0.056 ** [0.03]
D(CDS spread ex_)	0.000 *** [0.00]	0.000 *** [0.00]	0.000 [0.00]	0.000 [0.00]	0.000 ** [0.00]	0.000 ** [0.00]
D(Interest rate spread ex_)	-0.009 ** [0.04]	-0.010 ** [0.00]	-0.006 [0.01]	-0.009 [0.01]	0.003 [0.01]	0.003 [0.01]
Intervention index	-0.002 * [0.00]	0.000 [0.00]	0.001 [0.00]	0.001 [0.00]	0.001 [0.00]	0.000 [0.00]
Adjusted R-squared	0.616	0.577	0.431	0.379	0.558	0.553
Durbin-Watson stat	1.727	1.700	1.634	1.580	1.731	1.737
BG Serial Correlation LM Test (up to lag 36):	0.923	-	1.568	-	1.422	-
BG No Serial Correlation P-value:	0.598	-	0.031	-	0.072	-

	MEXICO (DLOG(MXN))		PERU (DLOG(PEN))	
	OLS	GMM	OLS	GMM
	DLOG(MXN)	DLOG(MXN)	DLOG(PEN)	DLOG(PEN)
DLOG(Latam common factor ex_)	0.975 *** [0.10]	1.207 *** [0.09]	0.230 *** [0.03]	0.231 *** [0.03]
D(Commodities ratio)	-0.062 * [0.03]	-0.060 * [0.03]	0.000 [0.00]	0.000 [0.00]
D(CDS spread ex_)	0.001 *** [0.00]	0.001 *** [0.00]	0.000 ** [0.00]	0.000 ** [0.00]
D(Interest rate spread ex_)	0.010 ** [0.00]	0.010 ** [0.00]	-0.006 [0.00]	-0.006 [0.00]
Intervention index	-0.001 [0.00]	-0.001 [0.00]	-0.002 *** [0.00]	-0.002 *** [0.00]
Adjusted R-squared	0.530	0.501	0.372	0.372
Durbin-Watson stat	1.547	1.441	1.638	1.637
BG Serial Correlation LM Test (up to lag 36):	1.288	-	0.905	-
BG No Serial Correlation P-value:	0.145	-	0.626	-

[] Std. Error. HAC standard errors & covariance.

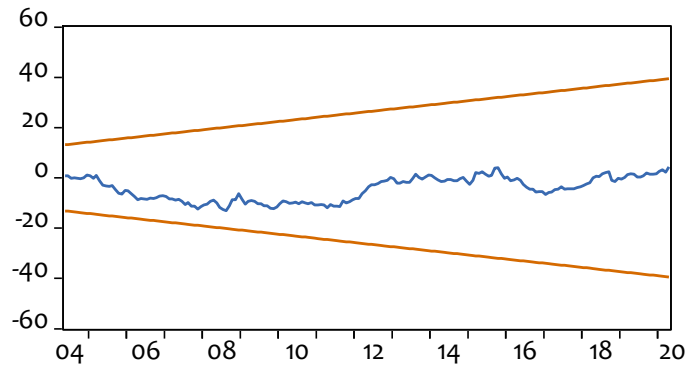
* p<0,10; ** p<0,05; *** p<0,01

Source: Authors' calculations.

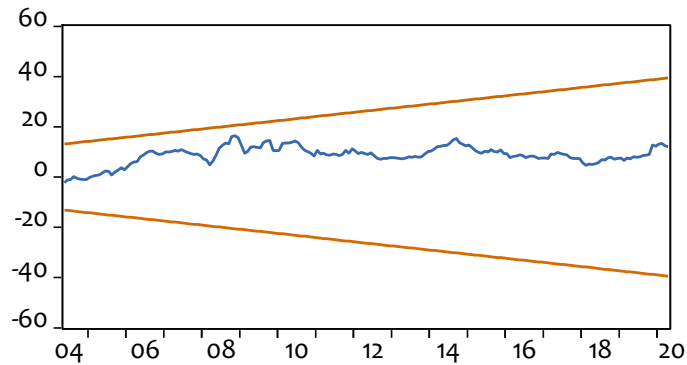
Sample: 2003M01-2020M12.

Results – Monthly frequency

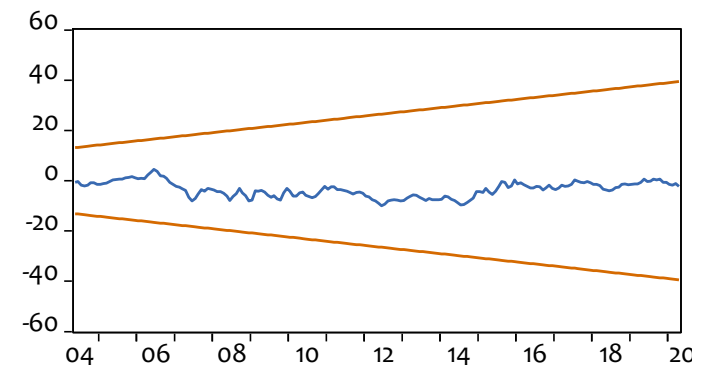
- In all specifications there is evidence of coefficients stability.



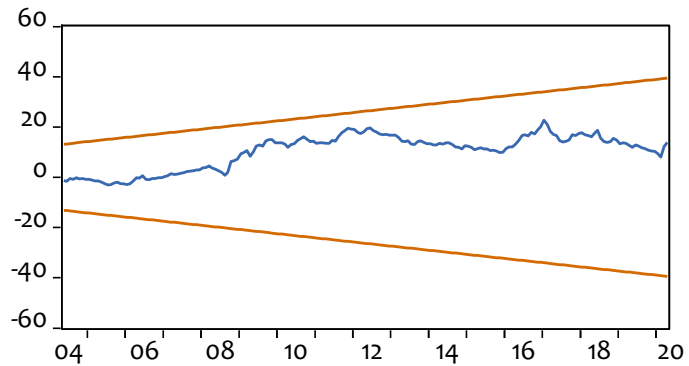
— BRL - CUSUM Test



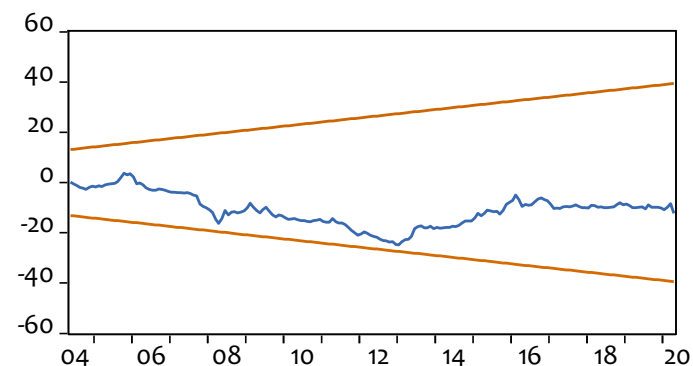
— CLP- CUSUM test



— COP - CUSUM Test



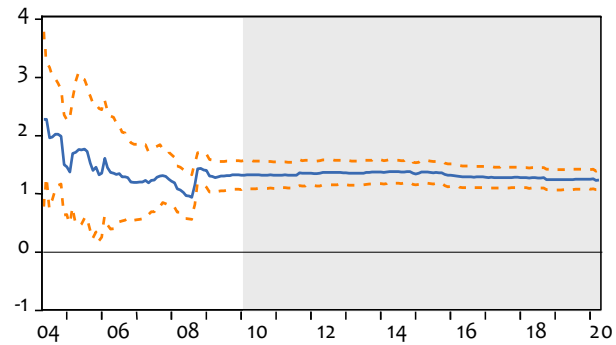
— MXN - CUSUM Test



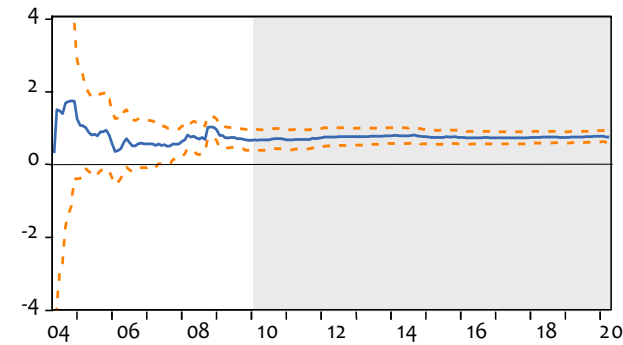
— PEN - CUSUM Test

Results – Monthly frequency

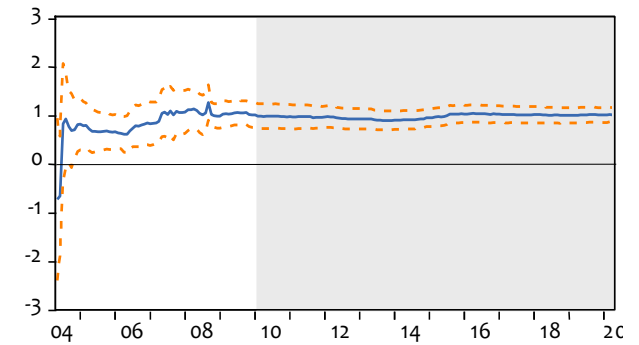
- The response of exchange rates to the common currency factor has been stable after de 2008-2009 financial crises.



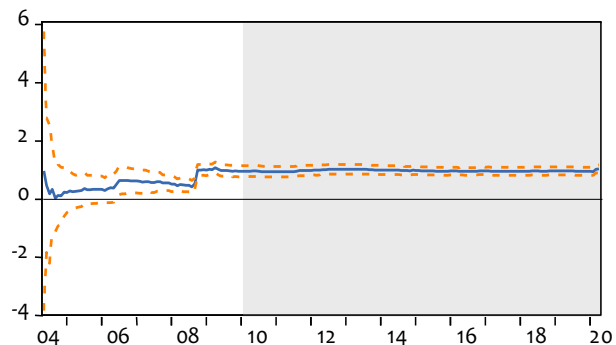
— Brazil - Recursive Estimates LatAm factor
- - ± 2 S.E.



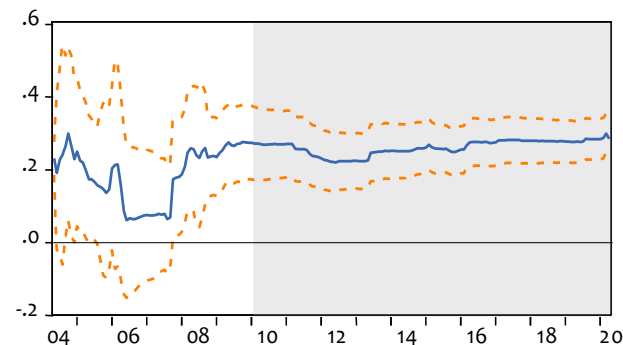
— Chile - Recursive Estimates LatAm factor
- - ± 2 S.E.



— Colombia - Recursive Estimates LatAm factor
- - ± 2 S.E.



— Chile - Recursive Estimates LatAm factor
- - ± 2 S.E.

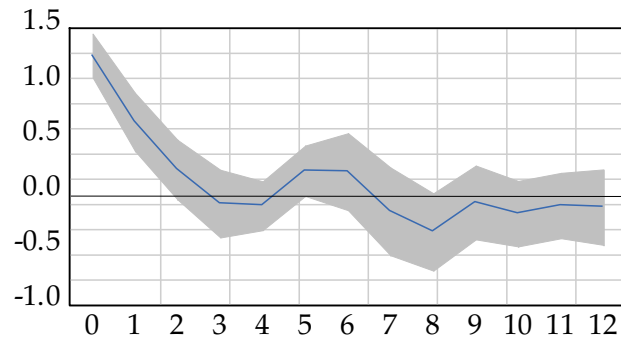


— Peru - Recursive Estimates LatAm factor
- - ± 2 S.E.

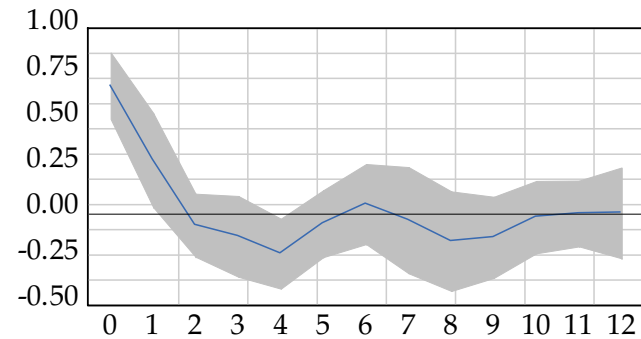
PARAMETER STABILITY. RECURSIVE OLS ESTIMATES. Notes: 5% confidence bands. Source: Authors calculations.

Results – Monthly frequency

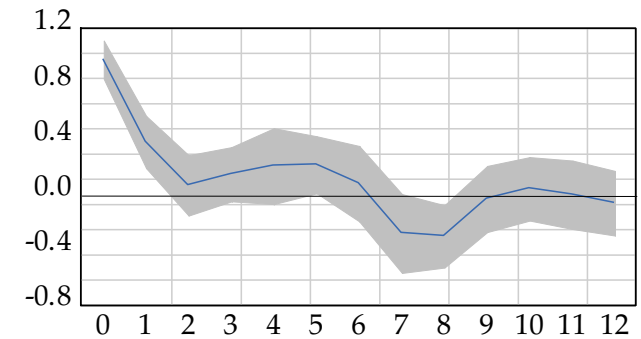
- According to the impulse response derived from local projections, the sensitivity of LATAM exchange rates to a shock in their common factor is positive and significant, with the total effect completed after two months.



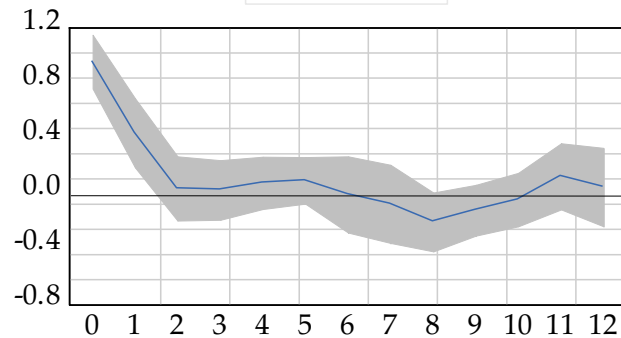
— BRAZIL



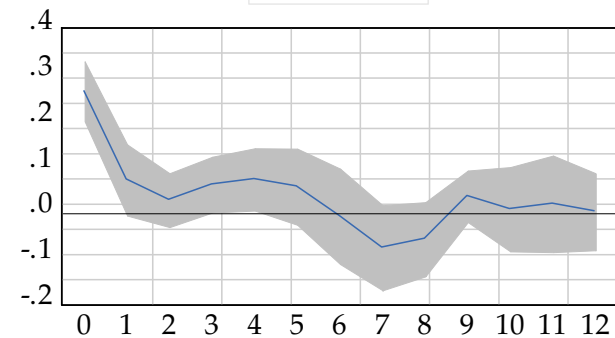
— CHILE



— COLOMBIA



— MEXICO



— PERU

LOCAL PROJECTIONS

SENSITIVITY OF LATAM EXCHANGE RATES TO THEIR RESPECTIVE COMMON FACTOR. Source: Authors calculations.

Source: Authors' calculations. Notes: Blue-shaded areas represents + 1 standard deviation.

Results – Monthly frequency

- We show the in-sample forecast evaluation of our proposed specification seems to perform better than a naïve random walk model (RW) in most cases.

Sample: 2003M01 2020M12

Included observations: 216

Forecast	RMSE	MAE	MAPE	U-Theil
Random Walk	0.043070	0.032529	301.3862	1.000000
OLS	0.023374	0.017869	203.4635	0.510226
GMM	0.024507	0.018853	239.1738	0.392982

FORECAST EVALUATION OF BRL ESTIMATIONS

Sample: 2003M01 2020M12

Included observations: 216

Forecast	RMSE	MAE	MAPE	U-Theil
Random Walk	0.036616	0.026181	252.2429	1.000000
OLS	0.021056	0.016589	178.3127	1.580276
GMM	0.021178	0.016808	192.5298	1.424707

FORECAST EVALUATION OF COP ESTIMATIONS

Sample: 2003M01 2020M12

Included observations: 216

Forecast	RMSE	MAE	MAPE	U-Theil
Random Walk	0.014316	0.009594	382.5134	1.000000
OLS	0.009362	0.006988	226.5614	0.981504
GMM	0.009362	0.006991	227.0610	0.983544

FORECAST EVALUATION OF PEN ESTIMATIONS

Sample: 2003M01 2020M12

Included observations: 216

Forecast	RMSE	MAE	MAPE	U-Theil
Random Walk	0.032379	0.025198	313.5806	1.000000
OLS	0.020156	0.014456	155.3592	0.725933
GMM	0.021059	0.015059	182.2111	0.700481

FORECAST EVALUATION OF CLP ESTIMATIONS

Sample: 2003M01 2020M12

Included observations: 216

Forecast	RMSE	MAE	MAPE	U-Theil
Random Walk	0.035204	0.025024	346.4360	1.000000
OLS	0.019804	0.015244	186.4113	0.805343
GMM	0.020414	0.016091	220.9546	0.874437

FORECAST EVALUATION OF MXN ESTIMATIONS

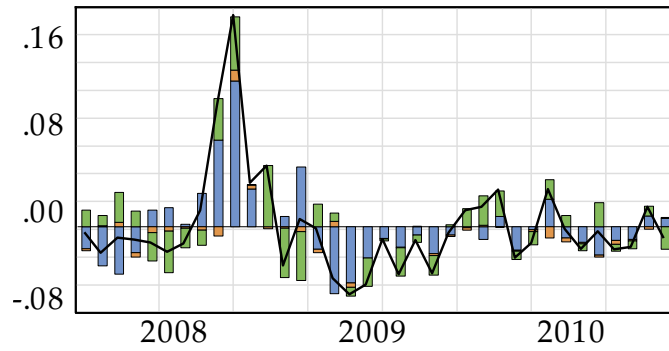
Source: Authors' calculations. Notes: For each model we compute the root mean square error (RMSE), the mean absolute error (MAE), the mean absolute prediction error (MAPE), and the U-Theil. The shaded cells represent the best model according to each forecast evaluation measure.

Results – Monthly frequency

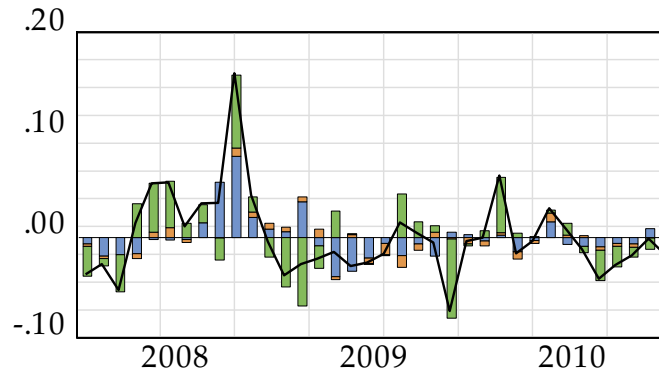
- *Using this approach, we find evidence that the common exchange rate factor is an important driver for LATAM exchange rates, especially during stressed episodes.*

Results – Monthly frequency –Global Financial Crisis

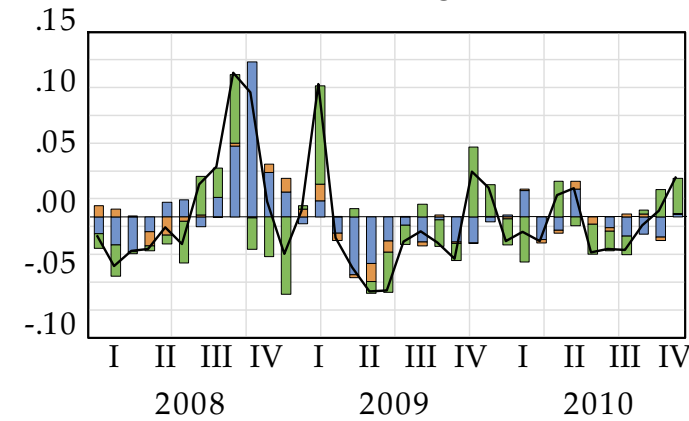
Brazil (dlog(BRL))



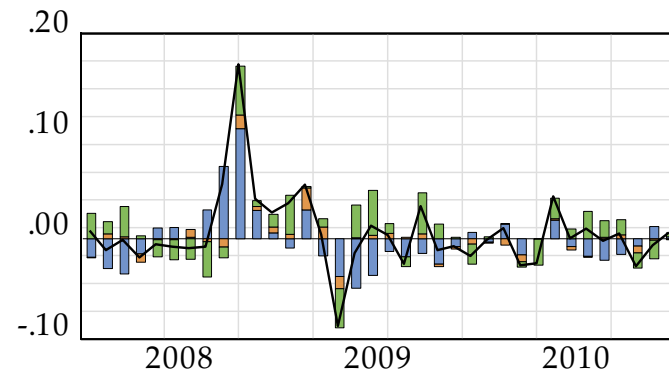
Chile (dlog(CLP))



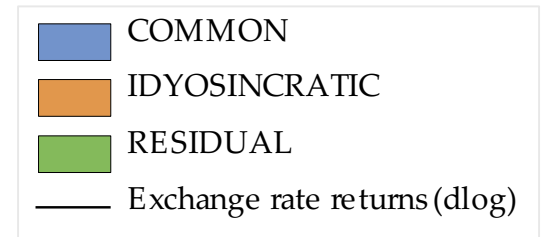
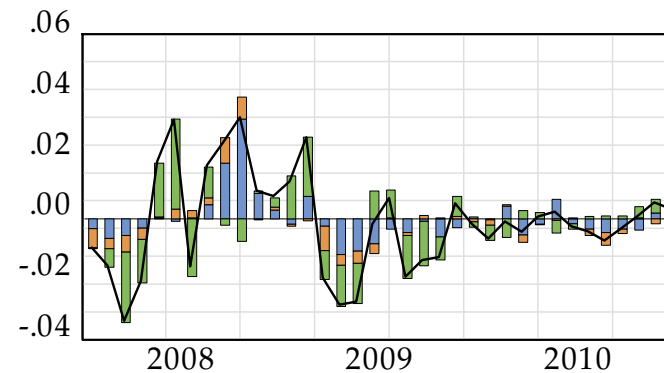
Colombia (dlog(CLP))



Mexico (dlog(MXN))



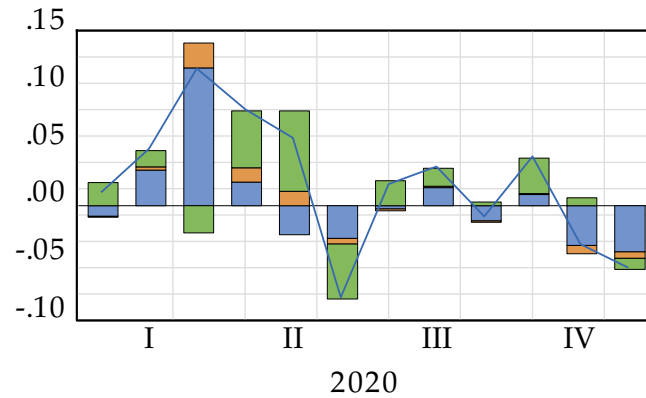
Peru (dlog(PEN))



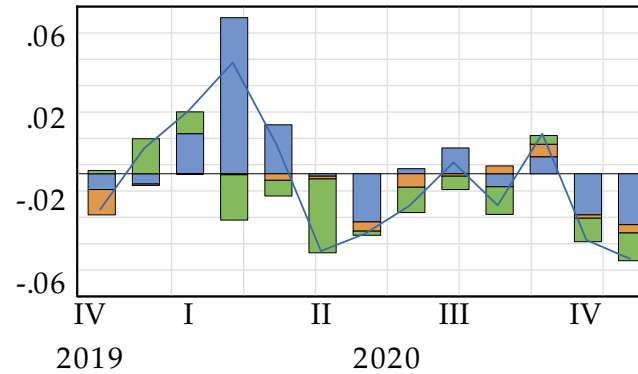
CONTRIBUTION OF COMMON AND IDYSINCRATIC FACTORS ON LATAM EXCHANGE RATES DURING THE GLOBAL FINANCIAL CRISIS. MONTHLY DEPRECIATION. *Source: Authors calculations.*

Results – Monthly frequency –COVID-19 Shock

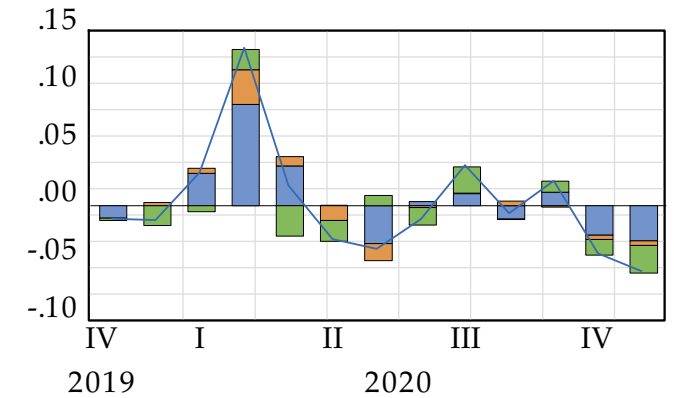
Brazil (dlog(BRL))



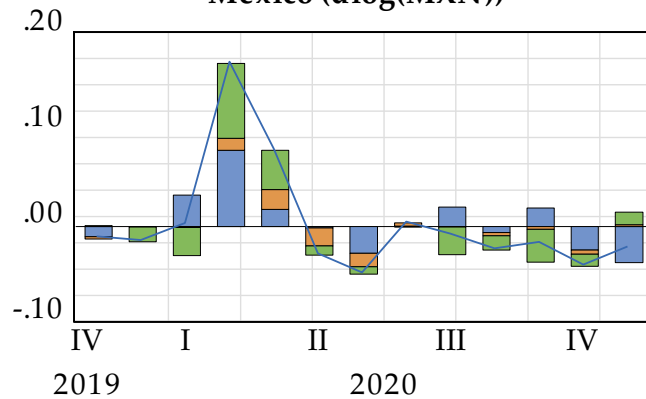
Chile (dlog(CLP))



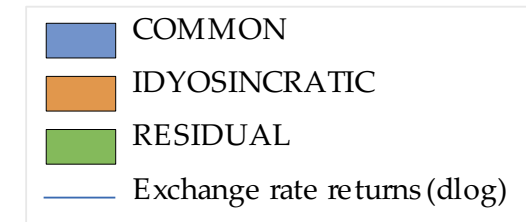
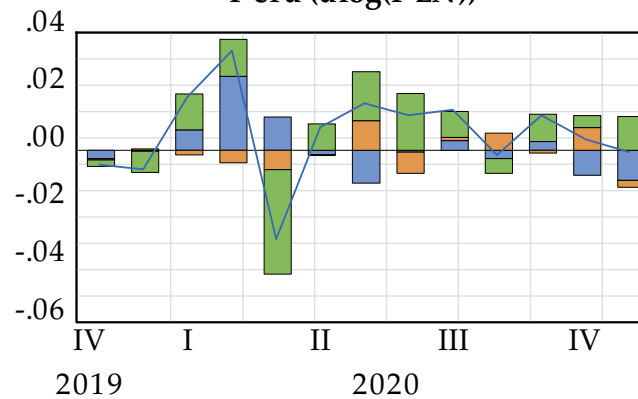
Colombia (dlog(CLP))



Mexico (dlog(MXN))



Peru (dlog(PEN))



Results – Daily frequency

- The previous results are consistent with the GARCH high frequency estimations as there is evidence that the common currency factor in LATAM economies is an important driver of daily exchange rates returns.

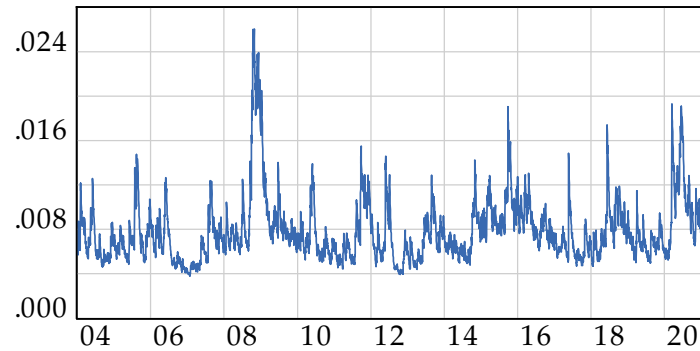
	DLOG(BRL)	DLOG(CLP)	DLOG(COP)	DLOG(MXN)	DLOG(PEN)
DLOG(Latam common factor ex_)	0.9645 *** [0.021]	0.5621 *** [0.013]	0.6348 *** [0.014]	0.6850 *** [0.012]	0.1280 *** [0.002]
DLOG(Commodities ratio)	-0.0030 [0.005]	-0.0123 *** [0.001]	-0.0142 *** [0.002]	0.0103 ** [0.004]	-0.0001 [0.000]
D(CDS spread ex_)	0.0004 *** [0.000]	-0.0001 *** [0.000]	0.0001 *** [0.000]	0.0001 *** [0.000]	0.0000 ** [0.000]
D(Interest rate spread ex_)	-0.0017 * [0.000]	-0.0013 * [0.000]	0.0001 [0.000]	0.0000 [0.000]	0.0004 [0.000]
Variance equation					
Constant	0.0000 *** [0.000]	0.0000 *** [0.000]	0.0000 *** [0.000]	0.0000 *** [0.000]	0.0000 *** [0.000]
α_1	0.0950 *** [0.006]	0.0551 *** [0.003]	0.1012 *** [0.005]	0.1003 *** [0.004]	0.2173 *** [0.006]
α_2	0.8891 *** [0.007]	0.9384 *** [0.002]	0.8914 *** [0.005]	0.8926 *** [0.005]	0.8079 *** [0.004]
Adjusted R-squared	0.346	0.251	0.301	0.300	0.138
Durbin-Watson stat	2.292	1.958	1.970	2.170	1.987
Log likelihood	15449.320	16955.940	16739.520	16826.660	21057.190

[] t-Statistic. HAC standard errors & covariance.

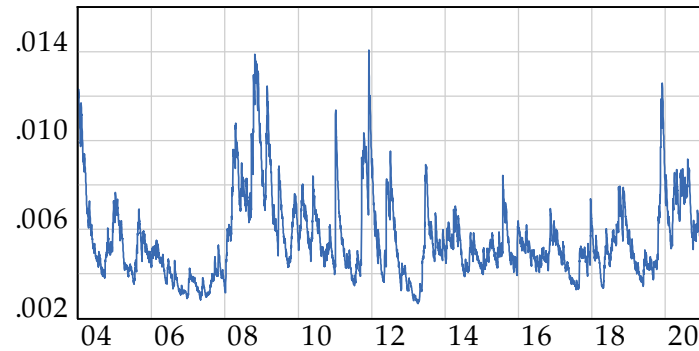
* p<0,10; ** p<0,05; *** p<0,01

Results – Daily frequency

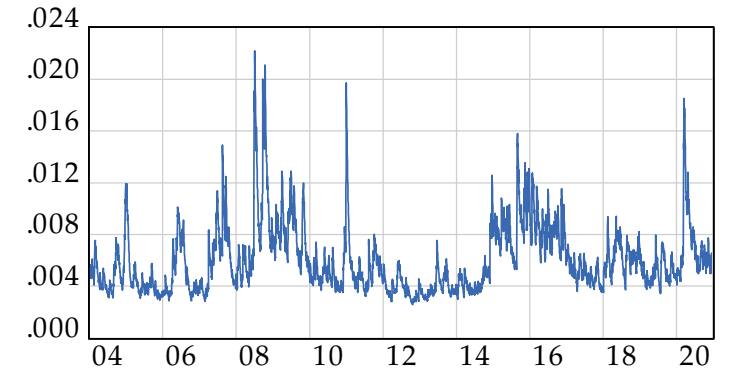
- The conditional volatility derived from GARCH models was higher during the financial crisis of 2008-2009 than during the recent Covid-19 crisis.



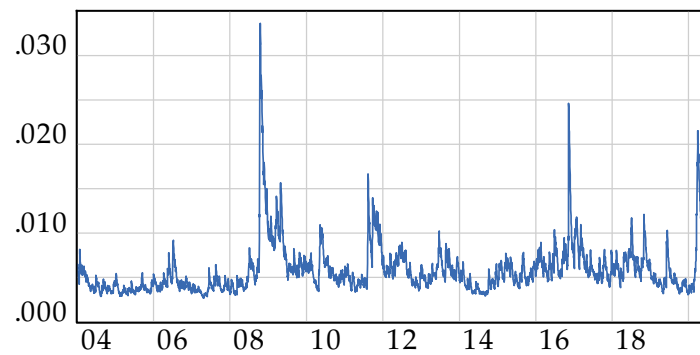
— Brazil - Conditional standard deviation



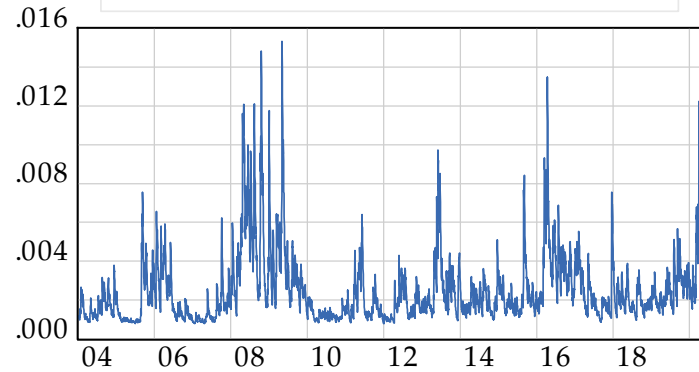
— Chile - Conditional standard deviation



— Colombia - Conditional standard deviation



— Mexico - Conditional standard deviation



— Peru - Conditional standard deviation



Outline

STYLIZED FACTS

LITERATURE REVIEW

THEORETICAL APPROACH

METHODOLOGY AND DATA

RESULTS

FINAL REMARKS

Final remarks...

- The regional factor seems to be a significant determinant of the Latin-American exchange rates, and we show evidence that its inclusion is suitable in both low and high frequency empirical settings.
- The regional common factor has been an important driver explaining exchange rates during stress episodes such as the Global Financial Crisis and the recent health crisis.
- Policy implications
 - The possibility to distinguish regional and idiosyncratic factors in a simple theoretical framework could be a useful tool that allows to explore the underlying drivers of FX movements, particularly during stressed episodes.
- Future research
 - Explore the drivers of the regional common factor, the influence of the global financial cycle on regional FX dynamics and the influence of the regional common factor on FX volatility.
 - Evaluating the forecasting capacity of the model using exchange rate expectations and forwards.

Thank you!



Appendix

Estimation of the common factor for LATAM currencies

Let $\mathbf{s}_t = (x_{1,t}, x_{2,t}, \dots, x_{N,t})'$ be a vector of N exchange rate log returns, each of which is a real-valued stochastic $\{s_{i,t}, t \in \mathbb{Z}\}$. Suppose we observe a finite realization of \mathbf{s}_t over some time points $t = 1, 2, \dots, T$, and let the empirical information available at time t be condensed into the information set $\mathcal{F}_t = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_t\}$. Thus, the dynamic factor model is specified such that each observable exchange rate $s_{i,t}$ ($i = 1, 2, \dots, N$) is the sum of two independent and unobservable components: a common component $\chi_{i,t}$, which is driven by a small number of factors that are common to all exchange rates, and a remaining idiosyncratic (individual-specific) component $\epsilon_{i,t}$. In panel notation, the model is:

$$s_{i,t} = \chi_{i,t} + \epsilon_{i,t}, \quad (i = 1, 2, \dots, N; t = 1, 2, \dots, T)$$

$$\chi_{i,t} = v_i(L)' \mathbf{z}_t$$

Where $v_i(L) = v_{i,0} + v_{i,1}L + \dots + v_{i,\ell}L^\ell$ ($\ell < \infty$) is a vector lag-polynomial of constants loading onto a vector of \mathcal{K} unobservable common factors, $\mathbf{z}_t = (z_{1,t}, z_{2,t}, \dots, z_{\mathcal{K},t})'$.

Estimation of the common factor for LATAM currencies

Thus, only the left-hand side of (1) is observed; the right-hand side is unobserved. If the dimension of \mathbf{z}_t is finite ($\mathcal{K} < \infty$), then there exists for every i an ($\mathcal{R} \leq \mathcal{K}$) of constants $\lambda_i = (\lambda_{i,1}, \lambda_{i,2}, \dots, \lambda_{i,\mathcal{R}})'$, such that $v_i(L)' = \lambda_i' \mathbf{C}(L)$, where $\mathbf{C}(L)$ is an $\mathcal{R} \times \mathcal{K}$ matrix lag-polynomial, $\mathbf{C}(L) = \sum_{m=0}^{\infty} \mathbf{C}_m L^m$, that is absolutely summable, $\sum_{m=0}^{\infty} \|\mathbf{C}_m\| < \infty$. Thus, letting $\mathbf{f}_t = (f_{1,t}, f_{2,t}, \dots, f_{\mathcal{R},t})' = \mathbf{C}(L)\mathbf{z}_t$, the dynamic factor model can be cast in the static representation:

$$s_{i,t} = c_{i,t} + \epsilon_{i,t},$$

$$c_{i,t} = \lambda_i' \mathbf{f}_t,$$

Estimation of the common factor for LATAM currencies

Which, equivalently, can be written in vector notation as

$$\mathbf{s}_t = \mathbf{c}_t + \boldsymbol{\epsilon}_t,$$

$$c_{i,t} = \boldsymbol{\Lambda} \mathbf{f}_t,$$

Where $\mathbf{c}_t = (c_{1,t}, c_{2,t}, \dots, c_{N,t})'$, $\boldsymbol{\epsilon}_t = (\epsilon_{1,t}, \epsilon_{2,t}, \dots, \epsilon_{N,t})'$ and $\boldsymbol{\Lambda} = (\lambda'_1, \lambda'_2, \dots, \lambda'_N)'$. The common factors in \mathbf{z}_t are often referred to as dynamic factors, while the common factors in \mathbf{f}_t are referred to as static factors. The number of static factors, \mathcal{R} , cannot be smaller than the number of dynamic factors, and is typically much smaller than the number of cross-sectional individuals, $\mathcal{K} \leq \mathcal{R} \ll N$. As with $\chi_{i,t}$, in the dynamic representation, the scalar process c_i or the multivariate process \mathbf{c}_t is the common component which we used to represent the LATAM common factor.