1 Model

1.1 Households

We adopt Hansen and Sargent (1986) straight time and overtime framework. In particular, we consider an economy populated by a family with a continuum of ex-ante identical infinitely live individuals along the unitary interval $i \in [0,1]$. The objective of the family is to maximize the social utility subject the budget constraint. Period utility is given by:

$$\frac{C_t^{1-\sigma}-1}{1-\sigma}-\chi\pi_{1t}\frac{h_1^{1+\zeta}}{1+\zeta}-\chi\pi_{2t}\frac{(h_1+h_2)^{1+\zeta}}{1+\zeta},$$

where C_t is consumption, π_{1t} is the probability of working h_1 hours, π_{2t} is the probability of working h_1+h_2 hours and $1-\pi_{1t}-\pi_{2t}$ is the probability of working zero hours. Expost π_{2t} is the number of people working (h_1+h_2) hours, which we denote N_{2t}. Similarly, $\pi_{1t}+\pi_{2t}$ is the fraction of people who work the first h_1 hours. We denote this fraction N_{1t}.

Period utility can be rewritten as:

$$\frac{C_t^{1-\sigma}-1}{1-\sigma}-\chi\left(N_{1t}-N_{2t}\right)\frac{h_1^{1+\zeta}}{1+\zeta}-\chi N_{2t}\frac{(h_1+h_2)^{1+\zeta}}{1+\zeta}.$$

Budget constraint of the family is:

$$w_{1t}N_{1t}+w_{2t}N_{2t}+r_tK_t+\int_0^1\pi_{jt}dj+T_t=C_t+K_{t+1}-(1-\delta)K_t.$$

where T_t represents a lump sum transfers of hiring and firing costs (defined below).

Recursive (family) problem:

$$V(\mathbf{s})=\max\frac{C(\mathbf{s})^{1-\sigma}-1}{1-\sigma}-\chi\left(N_1(\mathbf{s})-N_2(\mathbf{s})\right)\frac{h_1^{1+\zeta}}{1+\zeta}-\chi N_2(\mathbf{s})\frac{(h_1+h_2)^{1+\zeta}}{1+\zeta}+\beta E\left[V(\mathbf{s}')\,|\mathbf{s}\right]$$

subject to the constraints:

$$w_1(\mathbf{s})N_1(\mathbf{s})+w_2(\mathbf{s})N_2(\mathbf{s})+r(\mathbf{s})K+\int\pi(\varepsilon,n_1)\,d\mu(\varepsilon,n_1)+T(\mathbf{s})=C(\mathbf{s})+K'(\mathbf{s})-(1-\delta)K$$

where $\mu(\varepsilon,n_1)$ is the distribution of firms (defined below).

First order conditions

$$C(\mathbf{s}):C(\mathbf{s})^{-\sigma}=\lambda(\mathbf{s})$$

$$\begin{aligned}
N_1(\mathbf{s}) &: \chi\frac{h_1^{1+\zeta}}{1+\alpha}=\lambda(\mathbf{s})\,w_1(\mathbf{s})\\
N_2(\mathbf{s}) &: \chi\frac{(h_1+h_2)^{1+\zeta}}{1+\zeta}-\chi\frac{h_1^{1+\zeta}}{1+\zeta}=\lambda(\mathbf{s})\,w_2(\mathbf{s})\\
K'(\mathbf{s}) &: \lambda(\mathbf{s})=\beta E\left[\frac{\partial V(\mathbf{s}')}{\partial K'}|\mathbf{s}\right]
\end{aligned}$$

Envelope condition:

$$\frac{\partial V(\mathbf{s})}{\partial K}=\lambda(\mathbf{s})\left[r(\mathbf{s})+(1-\delta)\right]$$

Thus, the second foc can be rewritten as:

$$\lambda(\mathbf{s})=\beta E\left[\lambda'(\mathbf{s}')\left(r'(\mathbf{s}')+P'(\mathbf{s}')(1-\delta)\right)|\mathbf{s}\right]$$

Finally, notice that:

$$\frac{w_2(\mathbf{s})}{w_1(\mathbf{s})}=\frac{(h_1+h_2)^{1+\zeta}-h_1^{1+\zeta}}{h_1^{1+\zeta}}>1$$

which implies that the overtime wage presents a premium over the straight time wage.

1.2 Firms

We adapt Veracierto (2008) to a straight time and overtime framework which includes both hiring and firing costs. The economy is populated by a mass of firms $j \in [0,1]$ with idiosyncratic productivity. Output is produced using a decreasing returns to scale technology

$$y_{jt} = e^{z_t} e^{\varepsilon_{jt}} k_{jt}^{\alpha} \left[n_{1jt}'^{\nu} h_1 + n_{2jt}'^{\nu} h_2 \right], \;\; \alpha + \nu < 1$$

where z_t is an aggregate productivity, ε_{jt} is a productivity shock idiosyncratic to firm j, k_{jt} is the level of stock of capital rented by firm j, n_{1jt}' is the number of workers hired by firm j to work straight time, and n_{2jt}' is the number of workers hired by firm j to work the overtime shift. The production function expresses output at t as the sum of output obtained from the straight time shift, $e^{z_t} e^{\varepsilon_{jt}} k_{jt}^{\alpha} n_{1jt}'^{\nu} h_1$, and output obtained from the overtime shift, $e^{z_t} e^{\varepsilon_{jt}} k_{jt}^{\alpha} n_{2jt}'^{\nu} h_2$. Within each shift, we assume a team-production where every worker works the same amounts of hours.

The aggregate productivity shock follows an AR(1) process of the form:

$$z_{t+1} = \rho_z z_t + \sigma_z \omega_{t+1}^{z}, \text{ where } \omega_{t+1}^{z} \sim N(0,1)$$

The firm-specific productivity shock $\varepsilon_t \in \{\varepsilon_1, ..., \varepsilon_{n_\varepsilon}\}$ follows a Markov process with transition matrix Π. The number of idiosyncratic innovations is set at n_ε. This shock is independent across firms. Let $\pi(\varepsilon'|\varepsilon)$ be the conditional probability of $\varepsilon_{t+1} = \varepsilon'$ given that $\varepsilon_t = \varepsilon$.

Each period, the firm j observes the two productivity shocks, rents its capital stock at the rental rate r, hires workers for a straight time shift at a base wage rate w_1, retains workers for a overtime shift at a base wage rate w_2, produces and sells its output.

The firm faces hiring and firing costs. Similar to Hansen and Sargent (1986), we assume that it is costly to adjust straight time employment, but not to adjust overtime employment. This assumption is reasonable given that overtime workers are selected from among those working straight time, so that adjustment costs related to their employment have already been borne. Every period, a fraction q of straight time employees quit. That fraction is not subject to either hiring or firing costs.

We directly incorporate the implications of household optimization into firm's optimization problem:

$$\begin{aligned} \upsilon(\varepsilon, n_1; z, K, \mu) &= \lambda \max_{k, n_1', n_2'} \left[\begin{array}{c} e^{z} e^{\varepsilon} k^{\alpha} \left(n_1'^{\nu} h_1 + n_2'^{\nu} h_2\right) - rk - w_1 n_1' - w_2 n_2' \\ -\tau_h \max\left(0, n_1' - (1-q) n_1\right) - \tau_f \max\left(0, (1-q) n_1 - n_1'\right) \end{array} \right] \\ &\quad + \beta E\left[\upsilon(\varepsilon', n_1'; z', K', \mu') \,|\varepsilon, n_1; z, K, \mu\right] \end{aligned}$$

where K, is the aggregate stock of capital, μ is the distribution of firms over (ε, n_1) pairs and λ is the stochastic discount factor (defined above).

Period profits:

$$\begin{aligned} \pi(\varepsilon, n_1; \mathbf{s}) &\equiv e^{z} e^{\varepsilon} k^{\alpha} \left(n_1'^{\nu} h_1 + n_2'^{\nu} h_2\right) - rk - w_1 n_1' - w_2 n_2' \\ &\quad -\tau_h \max\left(0, n_1' - (1-q) n_1\right) - \tau_f \max\left(0, (1-q) n_1 - n_1'\right) \end{aligned}$$

Define $\mathbf{s} \equiv (z, K, \mu)$ as the aggregate state vector, then:

$$\begin{aligned} \hat{\upsilon}(\varepsilon, n_1; \mathbf{s}) &= \lambda(\mathbf{s}) \max_{k, n', h} \left[\begin{array}{c} e^{z} e^{\varepsilon} k^{\alpha} \left(n_1'^{\nu} h_1 + n_2'^{\nu} h_2\right) - rk - w_1 n_1' - w_2 n_2' \\ -\tau_h \max\left(0, n_1' - (1-q) n_1\right) - \tau_f \max\left(0, (1-q) n_1 - n_1'\right) \end{array} \right] \\ &\quad + \beta E\left[\hat{\upsilon}(\varepsilon', n_1'; \mathbf{s}') \,|\varepsilon, n_1; \mathbf{s}\right] \end{aligned}$$

Focs:

$$\begin{aligned} k &: \theta \frac{e^{z} e^{\varepsilon} k^{\alpha} \left[n_1'^{\nu} h_1 + n_2'^{\nu} h_2\right]}{k} = r(\mathbf{s}) \\ n_2' &: \nu \frac{e^{z} e^{\varepsilon} k^{\alpha} n_2'^{\nu} h_2}{n_2'} = w_2(\mathbf{s}) \end{aligned}$$

$$n_1' \quad : \quad \frac{\partial \hat{v}\left(\varepsilon, n_1; \mathbf{s}\right)}{\partial n_1'} = \lambda\left(\mathbf{s}\right) \left[\begin{array}{c} \nu \frac{e^z e^\varepsilon k^\alpha n_1'^\nu h_1}{n_1'} - w_1\left(\mathbf{s}\right) \\ -\tau_h \mathbb{I}\left(n_1' > \left(1-q\right) n_1\right) + \tau_f \mathbb{I}\left(n_1 > \left(1-q\right) n_1'\right) \end{array} \right] + \beta E\left[\frac{\hat{v}\left(\varepsilon', n_1'; \mathbf{s}'\right)}{\partial n_1'} | \varepsilon, n_1; \mathbf{s}\right] \leq 0$$

$$(= 0 \text{ if } n_1' > 0)$$

where $\mathbb{I}\left(\cdot\right)$ is one if the condition holds and zero otherwise.

Government

$$\begin{aligned} T\left(\mathbf{s}\right) &= \tau_f \int \max\left(0, \left(1-q\right) n_1 - n_1'\left(\varepsilon, n_1; \mathbf{s}\right)\right) d\mu\left(\varepsilon, n_1\right) \\ &+ \tau_h \int \max\left(0, n_1'\left(\varepsilon, n_1; \mathbf{s}\right) - \left(1-q\right) n_1\right) d\mu\left(\varepsilon, n_1\right) \end{aligned}$$

1.3 Equilibrium

A recursive competitive equilibrium for this model is a set $\hat{v}\left(\varepsilon, n_1; \mathbf{s}\right), n_1'\left(\varepsilon, n_1; \mathbf{s}\right), n_2'\left(\varepsilon, n_2; \mathbf{s}\right), k\left(\varepsilon, n; \mathbf{s}\right), \lambda\left(\mathbf{s}\right), w_1\left(\mathbf{s}\right), w_2\left(\mathbf{s}\right), r\left(\mathbf{s}\right), K'\left(\mathbf{s}\right), C\left(\mathbf{s}\right), N_1\left(\mathbf{s}\right), N_2\left(\mathbf{s}\right)$, and $\mathbf{s}' \equiv \left(z', K', \mu'; \mathbf{s}\right)$ such that:

1. Taking $w_1\left(\mathbf{s}\right), w_2\left(\mathbf{s}\right), r\left(\mathbf{s}\right), \lambda\left(\mathbf{s}\right)$ as given, $\hat{v}\left(\varepsilon, n_1; \mathbf{s}\right), k\left(\varepsilon, n_1; \mathbf{s}\right), n_1'\left(\varepsilon, n_1; \mathbf{s}\right)$ and $n_2'\left(\varepsilon, n_1; \mathbf{s}\right)$ solve the firm's optimization problem

2. Taking $w_1\left(\mathbf{s}\right), w_2\left(\mathbf{s}\right), r\left(\mathbf{s}\right), \lambda\left(\mathbf{s}\right)$ as given, $K'\left(\mathbf{s}\right), C\left(\mathbf{s}\right), N_1\left(\mathbf{s}\right)$ and $N_2\left(\mathbf{s}\right)$ solve the household's optimization problem

3. Markets clears:

$$\begin{aligned} \int n_1'\left(\varepsilon, n_1; \mathbf{s}\right) d\mu\left(\varepsilon, n_1\right) &= N_1\left(\mathbf{s}\right) \\ \int n_2'\left(\varepsilon, n_1; \mathbf{s}\right) d\mu\left(\varepsilon, n_1\right) &= N_2\left(\mathbf{s}\right) \\ \int k\left(\varepsilon, n_1; \mathbf{s}\right) d\mu\left(\varepsilon, n_1\right) &= K \\ \int y\left(\varepsilon, n_1; \mathbf{s}\right) d\mu\left(\varepsilon, n_1\right) &= C\left(\mathbf{s}\right) + K'\left(\mathbf{s}\right) - \left(1-\delta\right) K \end{aligned}$$

3. For all measurable set Δ_n, the law of motion for distribution $\mu\left(z, \mu\right)$ is given by

$$\mu'\left(z, \mu\right)\left(\varepsilon' \times \Delta_{n_1'}\right) = \sum_{\varepsilon} \pi\left(\varepsilon' | \varepsilon\right) \int \mathbb{I}\left(n_1'\left(\varepsilon, n_1; \mathbf{s}\right) \in \Delta_{n_1'}\right) d\mu\left(\varepsilon, n_1\right)$$

4. The law of motion for aggregate shocks $z' = \rho_z z + \sigma_z \omega_z'$ where $\omega_z' \sim N\left(0, 1\right)$

1.4 Calibration

Household preferences: Discount factor β equals 0.99

Household preferences: Relative risk aversion σ equals 1

Capital accumulation: Depreciation rate δ equals 0.025

Labor market: Quit rate q equals 0.06 - Veracierto (2008)

Firm's production function: Employment's exponent α equals 0.256 (Kanh and Thomas 2008)

Firm's production function: Employment's exponent ν equals 0.64 (Khand and Thomas 2008)

Aggregate TFP shock - Persistence AR(1) ρ_z equals 0.95

Aggregate TFP shock - Innovation's standard deviation AR(1) σ_z equals 0.01

Idiosyncratic productivity shock - Number of gridpoints n_ε equals 5

Idiosyncratic productivity shock - Markov transition matrix is calibrated to match a AR(1) process with persistence of 0.95 and innovation's standard deviation of 0.03. We use Tauchen's method. Maximum and minimum idiosyncratic shocks equal 1 and -1 standard deviations.

Household preference: Hours' exponent ζ to match the Frisch elasticity of the intensive margin in the representative agent model. We consider a Frisch elasticity of 0.5, which in the mid range of values, see Chan, Kim, Kwon and Rogerson (2012).

Straight time hours h_1

Overtime hours h_2

Household preference: scaling coefficient χ