An alternative view to the credit-to-GDP gap *

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Abstract

The credit-to-GDP gap, measured as the deviation of the credit-to-GDP ratio from a trend computed using the HP filter based on historical data, has gained a prominent role as a leading indicator of financial vulnerabilities to signal the activation of countercyclical capital buffers. In this paper we compare this gap to an alternative indicator that is derived from a cumulative sum statistic for parameter stability that can identify periods of unduly acceleration on the credit-to-GDP ratio. The CumSum indicator is easy to interpret and compute, and can be calibrated to replicate the workings of the HP gap.

JEL Classification	**:**	C50, G10, G14, G15
Keywords	**:**	Credit-to-GDP gap, HP filter, momentum and acceleration, Cumsum statistic.

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1 Introduction

The global financial crisis highlighted the need to mitigate systemic risk and led to the adoption of a macroprudential approach in the supervision and regulation of the financial systems around the globe. Within the intense debate that followed, the Basel Committee on Banking Supervision (BCBS henceforth) proposed, among other measures, the use of countercyclical capital buffers to protect the banking sector from periods of excess aggregate credit growth, often associated with the build-up of systemic risk and with subsequent periods of credit rationing (BCBS, 2010a).

The BCBS framework requires a variable (or group of variables) that can signal of the accumulation of financial vulnerabilities; the so-called credit-to-GDP gap has prominently played the role of such a signal (BCBS, 2010b). The gap is defined as the residual of the credit-to-GDP ratio minus a smooth trend estimated with a one-sided Hodrick and Prescott (1997, HP henceforth) filter, with a smoothing parameter (λ) of 400,000. The choice of this particular credit variable and detrending method over other measures of systemic risk responds mainly to its Early Warning Indicator (EWI) properties and its flexibility for setting policy thresholds, as thoroughly documented in Drehmann et al. (2010).

The main criticism on this reference measure focuses on: (i) the potential countercyclical behavior of the credit-to-GDP gap with GDP growth; (ii) its EWI abilities; (iii) measurement errors in the real-time estimation process; and (iv) difficulties in the calibration of λ and the interpretation of the associated gap. Points (i) to (iii) have been properly addressed in a series of studies, remarkably Drehmann and Tsatsaronis (2014) who provide a comprehensive evaluation of the pros and cons of the credit-to-GDP gap in these fields.

The discussion surrounding the choice of the HP filter, especially its calibration, is less clear cut. Contrary to the well-developed business cycle literature, studies that discuss the choice of a suitable smoothing parameter for characterizing *financial cycles* are still scarce (Borio, 2014, is a notable exception). Following Gomez (2001), if s denotes the number of times data is sampled over a year (1 if annual, 4 if quarterly, and so on), and T is the so-called *cutoff* period (in years), setting $\lambda = [2\sin(\pi/Ts)]^{-4}$ provides an HP trend that isolates the fluctuations in the data of T years or more, while the cycle series (i.e. the difference between the trend and the data) reflects the variation of less than T years. For quarterly data ($s = 4$), the popular choice $\lambda = 1,600$ implies a cutoff period (T) of about 10 years, well in agreement with the consensus in the business literature that fluctuations beyond 10 years correspond to trend developments (Maravall and del Rio, 2007).

On the other hand, the choice $\lambda = 400,000$, suggested by BCBS (2010b) to enhance the predictive power of the resulting gap, renders a cutoff period of near 40 years! This implies that the credit-to-GDP gap will still be influenced by the credit dynamics that were recorded, say, 30 years ago. Even though this result is consistent with the notion put forward in Borio (2014) of a longer financial cycle, we believe relevant discussion on this issue is still lacking.

In any case, it is important to acknowledge that the chosen λ for the buffer activation rule is implicitly linked with a set of assumptions about the period of the credit-to-GDP ratio cycle. In principle, this could be difficult to explain to a broad audience in a non-technical language. On account of this potential limitation, alternative readings of the signal obtained by observing the credit-to-GDP gap may come in handy for the design and implementation of macroprudential tools. As stated in Drehmann and Juselius (2014), "interpretability" is a central policy requirement of trigger variables and an easy-to-read indicator facilitates clear communication of policy decisions.

In this paper we develop an alternative measure of the credit-to-GDP gap that is much easier to interpret than the HP gap, and even to compute. We also show that, upon suitable calibration, our gap measure is able to replicate, almost exactly for all practical purposes, the workings of the HP gap. In this sense, we provide an alternative, clearer interpretation to the HP-based credit-to-GDP gap. Moreover, the proposed gap is potentially a better leading indicator of banking crises.

We begin with the simple idea, advanced in Franses (2016), that for a time series y_t, the evolution of the two-period difference $y_t - y_{t-2}$ is useful to identify periods of unstable growth in y_t (i.e., bubbles). Thus, in principle, this difference can serve the purpose of identifying dangerous patterns of credit growth.

The remainder of the paper is organized as follows. Section 2 reviews the literature on the credit-to-GDP gap as a trigger of the build-up phase of the countercyclical capital buffer, its main criticisms, and the subsequent replies. Section 3 describes analytically the properties, similarities and differences between the HP gap and the proposed indicator. Section 4 shows that, under suitable calibration, the proposed indicator can replicate the credit-to-GDP gap behavior and EWI properties. Section 5 concludes.

2 Literature review

The countercyclical capital buffer is designed to ensure that banking sector capital requirements internalize the risk of the macro-financial environment in which banks operate (BCBS, 2010b). Thus, when aggregate indicators (such as measures of credit growth) follow patterns associated with the accumulation of system-wide risk, it is expected that banks constitute sufficient capital to face future potential losses. This applies even if the distress situation is presumed to only impact directly a subset of institutions in the system (as network effects may be strong). Then, when the warning signal dissipates, any additional resources destined by banks to meet the countercyclical buffer are freed. With the goal of finding a suitable leading indicator of system-wide stressful conditions, Drehmann et al. (2010) evaluated a series of variables on the basis of their predictive power of banking crises and suitability criteria for setting an automatic buffer activation rule.[1]

Following Kaminsky and Reinhart (1999), the leading indicator properties of each candidate were assessed for different critical thresholds (i.e. the level over which a signal of an imminent banking crisis is sent) through the comparison of Type I and II error rates, the proportion of predicted crises, and the Noise-to-Signal ratio (NTSR).[2] On the side of suitability criteria, fulfillment of conditions such as that the indicator provided a wide range of informative critical thresholds was preferred. The latter was considered useful as it would allow the buffer to activate at certain level of the selected variable, and progressively require the constitution of a greater add-on of regulatory capital as subsequent (higher) critical thresholds are exceeded. Authors concluded that the credit-to-GDP gap (defined as in Section 1) was the best single-variable indicator for the build-up phase. However, as the identification of an unerring leading indicator is an impossible task, the implementation of a purely rule-based buffer may not be achievable. Therefore, they recommended the use of this trigger in combination with judgment as a starting point for designing the buffer in each jurisdiction.

As in the case of stabilization through monetary policy based on the output gap, criticism was drawn about the use of a one-sided HP trend for calculating the credit-to-GDP gap. Mainly, doubts centered around the potential costs of policy decisions that depended on a gap measure characterized by important end-of-sample errors when calculated in real-time. For example, see Orphanides and van Norden (2002), Marcellino and Musso (2011) and Ince and Papell (2013). In these studies, the lack of ex-post information for the end point estimate of the trend was identified as the main source of bias rather than revisions of the underlying data. Edge and Meisenzahl (2011) pointed out that this could lead to early/late deployments of the countercyclical capital buffer, when comparing to the signal sent by a two-sided version of the gap. Moreover, it was suggested that policy decisions would need to be revised as soon as the gap estimates were updated. Clearly, these problems would generate unnecessary costs to the banking system or reduce the regulator's credibility. Other objections stated that the credit-to-GDP gap may move countercyclically with GDP growth, thereby enhancing business cycle fluctuations (Repullo and Saurina, 2011), and that other single-variable EWIs could be used (Shin, 2013).

Drehmann and Tsatsaronis (2014) replied these lines of criticism. First, they explained that the well-known

[1] These included indicators of aggregate macroeconomic conditions (real GDP growth, aggregate real credit growth, credit-to-GDP gap, asset price growth), banking sector activity (banking sector growth/profits, aggregate losses) and cost of funding (banking sector credit spreads, cost of liquidity, corporate bond spreads).

[2] Type I error corresponds to the proportion of cases where no warning signals were sent by the variable but a posterior crisis took place; while Type II error, to the proportion of instances where warning signals were sent but no crisis followed (that is, false alarms). The proportion of predicted crises refers to the rate of crises in the sample anticipated by warning signals. The NTSR is defined as [Type II error/(1 − Type I error)].

end-of-sample bias of the HP filter does not invalidate the signaling abilities of the credit-to-GDP gap. As presented in BCBS (2010b), the buffer is meant to be constituted in anticipation of stressful conditions. On these grounds, as indicated also by van Norden (2011), there is no evidence that two-sided measures of the credit-to-GDP gap perform better in predicting banking crises (or other distress events). Second, despite the negative correlation between the gap and GDP growth registered in certain time intervals, the authors mentioned that this mainly occurred in periods without consequences to policy decisions. Particularly, the buffer would not have been activated in periods of economic downturn. However, they reminded that this macroprudential tool should be calibrated in consideration of the financial cycle rather than the business cycle. Third, statistical tests were presented to confirm the superiority of the credit-to-GDP gap over other indicators in several contexts.

On the side of calibration and interpretability concerns, literature about suitable values for the smoothing parameter (λ) and alternative readings of the credit-to-GDP gap has been limited. The former requires an in-depth exploration of the financial cycle in the frequency domain when distancing from the predictive exercise of Drehmann et al. (2010) and is not the intention of this paper. The latter, on the contrary, will be dealt with by developing an interpretation of the credit-to-GDP gap from a chartist's point of view. To the best of our knowledge, this is the first paper to develop a complementary understanding of this measure, which can be applied to simplify communication strategies of the countercyclical capital buffer.

3 Methodological discussion

3.1 The HP gap

The HP trend can be obtained as the optimal linear predictor of T_t in the signal-plus-noise model:

$$y_t = T_t + \varepsilon_t \qquad \text{and} \qquad \Delta^2 T_t = u_t \,, \tag{1}$$

where L is the lag operator ($L^k y_t = y_{t-k}$) and $\Delta = 1 - L$ so $\Delta^2 = (1-L)^2$. The noise ε_t satisfies $\text{var}(\varepsilon_t) = \lambda$ and is uncorrelated with the disturbance u_t, that satisfies $\text{var}(u_t) = 1$. The parameter λ is the smoothing constant: the predicted trend is smoother for large values of λ and gets closer to the original series y_t as λ decreases.

The predictor for the cycle $G_t = y_t - T_t$ can be expressed as two-sided weighted average of the data $G_t = W_G(L) y_t$. McElroy (2008) shows that this can be factorized as:

$$W_G(L) = G_F(L^{-1}) G_B(L) = \left[\frac{(1-L^{-1})^2}{\theta(L^{-1})} \right] \cdot \left[\frac{\theta_2 (1-L)^2}{\theta(L)} \right] , \tag{2}$$

where $\theta(z) = 1 + \theta_1 z + \theta_2 z^2$ is a quadratic polynomial whose coefficients $\theta_1 < 0$ and $\theta_2 > 0$ are known, but tedious, functions of λ.

From (2) it is apparent that we obtain the final predictor G_t in a two-stage process. First, we compute a predictor that uses exclusively past data, $g_t = G_B(L) y_t$; second, we smooth the g_t into the final predictor, $G_t = G_F(L^{-1}) g_t$. The first-stage predictor g_t corresponds to the gap of the one-sided HP filter.

From the definition of $g_t = G_B(L) y_t$ we can write $\theta(L) g_t = \theta_2 \Delta^2 y_t$. Thus, we can compute the gap g_t from a second-order recursion with a term proportional to the second differences of the original series:

$$g_t = -\theta_1 g_{t-1} - \theta_2 g_{t-2} + \theta_2 \Delta^2 y_t \,. \tag{3}$$

Thus, the HP gap is a weighted average of $\Delta^2 y_t$ and its history:

$$g_t = H(L) \Delta^2 y_t \,, \quad \text{where} \quad H(L) = \frac{\theta_2}{\theta(L)} = \sum_{k=0}^{\infty} H_k L^k \,. \tag{4}$$

The polynomial $H(L)$ is a transfer function that describes how the filter delivers g_t from the data $\Delta^2 y_t$ and, in particular, the timing between both series.

We can characterize the coefficients of H_k in terms of the roots of the polynominal $\theta(z) = 1 + \theta_1 z + \theta_2 z^2$, which are complex conjugates with modulus equal to ρ and argument equal to ω (expressions for both quantities, as a function of λ, are also provided by McElroy, 2008). In particular, the Appendix shows that:

$$H_k = \theta_2 \rho^k \left[\cos(\omega k) + \frac{\sin(\omega k)}{\tan(\omega)} \right] \quad \text{for } k = 1, 2, \ldots . \tag{5}$$

The weight profile $\{H_0, H_1, H_2, \ldots\}$ is hump-shaped. It starts with $H_0 = \theta_2 > 0$ and increases up to the point:

$$k^* = \frac{1}{\omega} \arctan\left(-\frac{\omega}{\ln(\rho)} \right) - 1 . \tag{6}$$

Then it decreases and converges to zero as a damped wave, crossing zero at $k_0 = \pi/\omega - 1$. Finally, the mean lag of this transfer function, which measures the delay in the transmission of shocks in $\Delta^2 y_t$ into g_t, is:

$$M_H = \frac{-(\theta_1 + 2\theta_2)}{1 + \theta_1 + \theta_2} . \tag{7}$$

It is simple to verify that the most important features of this transfer function, such as the sum of coeffcients $H(1)$, the peak in the weights profile k^*, the maximum weight H_{k^*} and the mean lag M_H, are all strictly increasing functions of λ. To illustrate, consider the values in Ravn and Uhlig (2002) for different frequencies, so that for $\lambda = 6.25$ (annual frequency), then $H(1) = 1.6$, $k^* = 0.7$, $H_{k^*} = 0.5$ and $M_H = 1.4$; for $\lambda = 1600$ (quarterly), then $H(1) = 35.8$, $k^* = 6.0$, $H_{k^*} = 2.6$ and $M_H = 8.0$; and for $\lambda = 129,600$ (monthly), then $H(1) = 346.8$, $k^* = 20.1$, $H_{k^*} = 8.3$ and $M_H = 25.8$. For the value $\lambda = 400,000$ proposed in BCBS (2010b), we obtain $H(1) = 614.9$, $k^* = 26.9$, $H_{k^*} = 11.1$ and $M_H = 34.6$.

3.2 Stability tests and the CumSum gap

Although most real economic and financial series behave as $I(0)$ or $I(1)$ processes, Franses (2016) argues that during a rapid escalation a series may behave temporarily as an $I(2)$ process. Based on this insight, he then proposes a simple method to unveil bubble-like behavior in a series.

Consider the population linear projection of Δy_t on $\Delta^2 y_t$:

$$\Delta y_t = \beta \Delta^2 y_t + e_t , \quad \text{where, by construction,} \quad \beta = \frac{\mathrm{cov}(\Delta y_t, \Delta^2 y_t)}{\mathrm{var}(\Delta^2 y_t)} \quad \text{and} \quad \mathrm{E}(e_t) = 0 . \tag{8}$$

If $y_t \sim I(0)$ or $y_t \sim I(1)$, then $\Delta y_t \sim I(0)$ and so $\mathrm{var}(\Delta y_t) = \mathrm{var}(\Delta y_{t-1})$ and:

$$\beta = \frac{\mathrm{var}(\Delta y_t) - \mathrm{cov}(\Delta y_t, \Delta y_{t-1})}{2\mathrm{var}(\Delta y_t) - 2\mathrm{cov}(\Delta y_t, \Delta y_{t-1})} = \frac{1}{2}. \tag{9}$$

In contrast, when $y_t \sim I(2)$ then β will be closer to one than to $\frac{1}{2}$.

Define:

$$x_t = \Delta y_t - \tfrac{1}{2}\Delta^2 y_t = \tfrac{1}{2}(y_t - y_{t-2}) , \tag{10}$$

which equals the residual in the population linear projection (8) when $\beta = \frac{1}{2}$. Thus, in normal times, when y_t behaves like a $I(0)$ or $I(1)$, then $x_t \simeq 0$. On the contrary, in turbulent times, when y_t behaves as an $I(2)$ process, then $x_t \neq 0$. For this reason, Franses (2016) proposes to perform standard stability tests on a regression model of x_t on a constant (i.e., a "location model"), as a formal procedure to detect episodes of unduly growth in y_t.

Following a long tradition started in Brown et al. (1975) and extended in Dufour (1982) and Chu et al. (1996), such tests are often based on the cumulated sum ("CumSum") of recursive residuals of the regression. In the case of a location model, the CumSum statistic is defined as:

$$\hat{c}_t = \sum_{\tau=0}^{t-1} \hat{r}_{t-\tau}\,, \quad \text{where} \quad \hat{r}_t = x_t - \hat{x}_t \quad \text{and} \quad \hat{x}_t = \frac{1}{t}\sum_{\tau=0}^{t-1} x_{t-\tau}\,. \tag{11}$$

where $\hat{c}_t$ cumulates residuals $\hat{r}_t$ obtained by deducting the average $\hat{x}_t$, computed with information up to period t, from the series. A sequence of values of x_t markedly different from $\hat{x}_t$ produces large values for $\hat{r}_t$ and a turning point in $\hat{c}_t$. Such points mark the beginning of accelerations.

More formally, the approach consists on comparing the discrepancy between x_t and its historical average, and to compute a confidence interval around it. This is equivalent to perform a t-test for the discrepancy, which can be done in a very standard fashion (see Dufour, 1982, section 4.3), as such tests are readily implemented in most commercial econometric packages. We return to this in our empirical exploration below.

We propose a "gap" indicator inspired by the $\hat{c}_t$ statistic. For mathematical convenience and to enhance the comparability with the HP gap, our CumSum gap is based on exponentially weighted, rather than simple, sums and averages. Thus, it depends on a decay parameter $a \in (0, 1)$, a "forgetting factor", that allocates exponentially decreasing weights to older data and residuals. We define the CumSum gap as:

$$c_t = \sum_{\tau=0}^{\infty} a^\tau r_{t-\tau}\,, \quad \text{where} \quad r_t = x_t - \bar{x}_t \quad \text{and} \quad \bar{x}_t = (1-a)\sum_{\tau=0}^{\infty} a^\tau x_{t-\tau}\,. \tag{12}$$

Note that $\bar{x}_t$ as an estimator of the mean of x_t is such that $\mathrm{E}(\bar{x}_t) = \mathrm{E}(x)$ if $\mathrm{E}(x_s) = \mathrm{E}(x)$ for all $s \leq t$, and $\mathrm{E}(\bar{x}_t) \neq \mathrm{E}(x)$ if $\mathrm{E}(x_s) \neq \mathrm{E}(x)$ for some $s \leq t$, which is a property that shares with the unweighed average $\hat{x}_t$. Thus, in analogy to (11), values of x_t markedly different from $\bar{x}_t$ produce a turning point in c_t. It is worth noting that as a approaches one, $\bar{x}_t$ approaches the sample average, and the results in (12) and (11) get closer.[3]

To describe the filter leading to the CumSum gap, and thus to deduce a recursion for its computation, first note that $(1 - aL)\bar{x}_t = (1 - a)x_t$ leads to $(1 - aL)r_t = a(1 - L)x_t$; then, note also that $(1 - aL)c_t = r_t$, making $(1 - aL)^2 c_t = a(1 - L)x_t$; finally, since $x_t = \frac{1}{2}(1 + L)(1 - L)y_t$ we conclude that $(1 - aL)^2 c_t = \frac{1}{2}a(1 + L)\Delta^2 y_t$. Therefore, we can compute the CumSum gap c_t from the second-order recursion:

$$c_t = 2a\,c_{t-1} - a^2 c_{t-2} + a\left(\frac{\Delta^2 y_t + \Delta^2 y_{t-1}}{2}\right). \tag{13}$$

Interestingly, the CumSum gap is also a moving average of the second differences of the series:

$$c_t = A(L)\Delta^2 y_t\,, \quad \text{where} \quad A(L) = \frac{a}{2}\frac{1+L}{(1-aL)^2} = \sum_{k=0}^{\infty} A_k L^k\,, \tag{14}$$

where:

$$A_k = \tfrac{1}{2}(a + (1+a)k)a^k \quad \text{for } k = 1, 2, \ldots\,. \tag{15}$$

The weight profile $\{A_0, A_1, A_2, \ldots\}$ is hump-shaped. It starts with $A_0 = a > 0$ and reaches a maximum at:

$$k^* = -\frac{1}{\ln(a)} - \frac{a}{1+a}\,, \tag{16}$$

[3] The original CumSum statistic (11) is non-stationary (see Brown et al., 1975; Chu et al., 1996) and once it crosses suitable confidence bands to signal instability, it rarely gets back within the bands. Thus, it design to detect instabilities once; after the alert, the process will be adjusted and reset (Box and Ramírez, 1992). In contrast, as long as $a < 1$ our CumSum gap is stationary and can be used to signal several instabilities: after the alert, it will mean-revert (towards zero) until the next episode of turbulence.

Figure 1. *Map from λ to a*

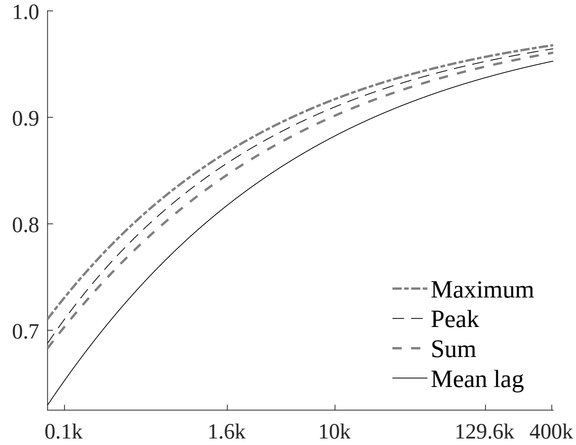
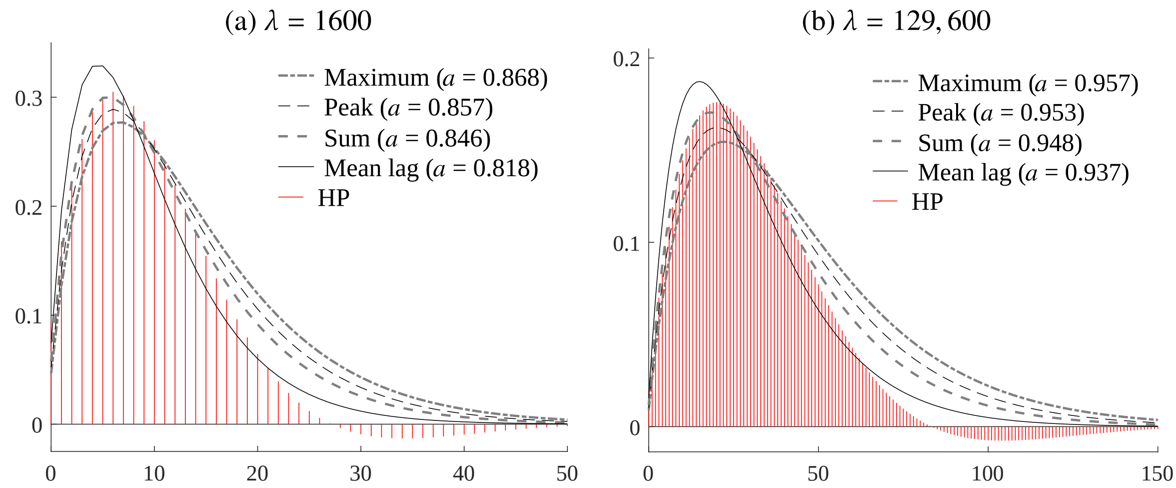


Figure 2. *Weight profiles*



Then it converges to zero monotonically. Finally, the mean lag of this transfer function is:

$$M_A = \frac{1}{2}\left(\frac{1+3a}{1-a}\right). \tag{17}$$

A useful convention among users of exponentially weighted moving averages is to use $a = (n-1)/(n+1)$ and interpret the filter as an n-period average. This is so because with such calibration of a, the weights will have the same "center of mass" as a simple n-period moving average. It is quite revealing that such a straightforward interpretation is adequate for the CumSum gap since the mean lag can be written as $M_A = n - \frac{1}{2}$.

The salient features of this transfer function, such as the sum of coefficients $A(1)$, the peak in the weights profile k^*, the maximum weight A_{k^*} and the mean lag M_A (or the period n), are strictly increasing functions of a. For instance, for $a = 0.80$, then $A(1) = 20.0$, $k^* = 4.0$, $A_{k^*} = 1.6$ and $M_A = 8.5$ (a 9-period filter); for $a = 0.85$, then $A(1) = 37.8$, $k^* = 5.7$, $A_{k^*} = 2.3$ and $M_A = 11.8$ (a 12-period filter); for for $a = 0.90$, then $A(1) = 90.0$, $k^* = 9.0$, $A_{k^*} = 3.5$ and $M_A = 18.5$ (a 19-period filter); finally, for $a = 0.95$, then $A(1) = 380.0$, $k^* = 19.0$, $A_{k^*} = 7.2$ and $M_A = 38.5$ (a 39-period filter).

3.3 Comparison between approaches

Values in Ravn and Uhlig (2002)

$$a = \frac{2M_H - 1}{2M_H + 3}$$

$A(1) = a/(1-a)^2$

$$a^2 - \left(2 + \frac{1}{H(1)}\right) a + 1 = 0$$

When applied in the context of the countercyclical capital buffer, instances where positive (negative) recursive residuals of the credit-to-GDP ratio are identified relate closely to positive (negative) HP gaps.[4] Figure **??** compares the evolution of the one- and two-sided versions of the HP gap to the recursive residuals of the credit-to-GDP ratio for the cases of United Kingdom, United States, Spain and Greece. A set of stylized facts can be established. First, the recursive residual contains more noise than the HP gaps as it depends directly on a measure of two-period variation. Second, despite the noise of the recursive residual, its movement resembles that of the credit-to-GDP gap. Nevertheless, the similarity is greater with the one-sided version of the gap as both measures only incorporate information available in real-time. For example, in the case of the United States, Spain and Greece both the recursive residuals and the one-sided gaps rose together during the early 1990s before the two-sided version began to adjust.[5] Third, when positive (negative) values are registered, both indicators reflect a higher (lower) growth rate of the credit-to-GDP ratio in comparison to a historical mean.

In addition, the EWI properties of each variable could be compared. In what follows, only the one-sided version of the HP filter will be considered when referring to the credit-to-GDP gap as it is the relevant measure for a real-time monitoring process. In the case of the credit-to-GDP gap, the commonly assessed variable in terms of its predictive power of banking crises is a dichotomous variable (i.e. a signal) triggered when the gap exceeds a critical threshold. In the case of the recursive residual, several approaches can be adopted for building the signal variable. On the one hand, signals can be constructed through a direct reading of the recursive residual. This implies activating warning signals when the indicator surpasses specific levels or only when it is positive and statistically significant, as suggested by Franses (2016). Despite the noise of the series, a signal of this kind may provide useful information. Figure **??** points out the periods when the recursive residuals registered positive and significant values at the 10% significance level. These can be interpreted as quarters with evidence of explosive behavior of the credit-to-GDP ratio, which include episodes such as the build-up phase of the banking crises of: 1991 (United Kingdom), 2007 (United Kingdom and United States) and 2008 (Spain and Greece)[6].

On the other hand, the noise in the series of the recursive residual could be taken into account before activating a warning signal. This could be dealt with through a range of options such as: (i) requiring the recursive residual of the credit-to-GDP ratio to exceed a critical threshold for a minimum number of periods in a time interval; (ii) asking that the recursive residuals of a major set of variables display a minimum number of alerts; or (iii) smoothing the recursive residual of the credit-to-GDP ratio with moving averages. The latter provides the advantage of controlling for outliers in the two-period variation series. Furthermore, requiring a moving average of the recursive residual to surpass a critical threshold already incorporates the smoothing strategy of point (i). On the grounds of these benefits, approach (iii) is explored in the next sections in order to compare the informative contents of the Basel III's credit-to-GDP gap and the recursive residual.

[4] Note that the recursive residuals refer to the differences between the current two-period variation x_t and its historical average, using information available up to $t-1$ (i.e. the previous period).

[5] A simple exercise can be done to give further evidence. The recursive residual and HP gaps (both the one- and two-sided versions with $\lambda = 400{,}000$) are calculated for the 39 countries of the BIS total credit to the private non-financial sector database (excluding the Euro area). Then, in each country, the Pearson correlation coefficient is calculated between the recursive residual and the one-sided HP gap, and between the former and the two-sided version. When averaging the correlation coefficients among countries, it is found that the recursive residual and the one-sided HP gap present an average correlation of 0.51, while the coefficient diminishes to 0.25 when measuring against the two-sided HP gap.

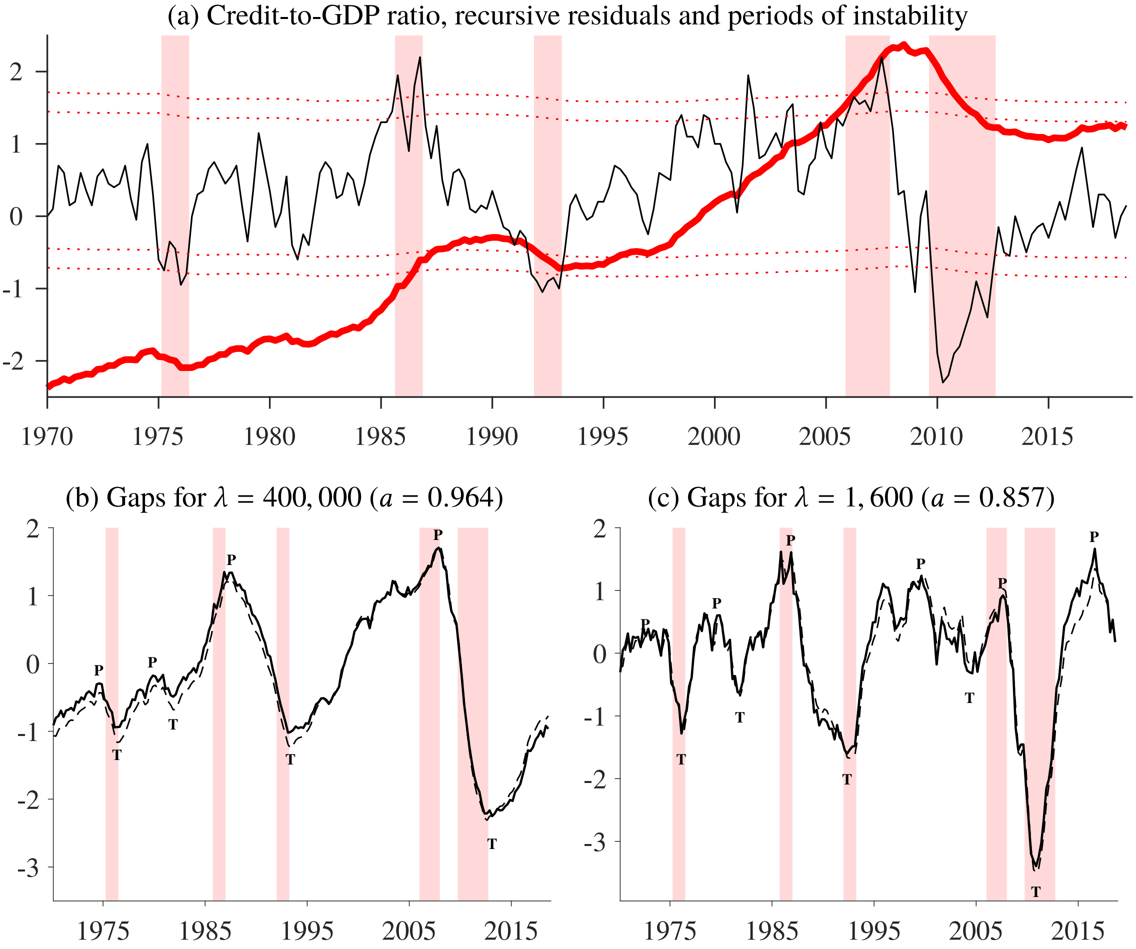
[6] The definition and date of outbreak of banking crises were obtained from Reinhart and Rogoff (2011).

4 Discussion

5 Conclusions

In the present paper, it was demonstrated that the credit-to-GDP gap, computed through a one-sided HP filter with λ set to 400,000, can be expressed in simpler terms as the difference between a short and a long moving average of the two-period variation of the credit-to-GDP ratio. This idea builds on Franses (2016) simple test for detecting bubble-like behavior, as well as the MACD literature widely applied in the financial industry. This alternative reading of the credit-to-GDP gap contributes to simplify the communication strategies of the countercyclical capital buffer (or other gap-based) policy decisions, and aids to meet the "interpretability" policy requirement of a trigger variable used for stabilization policy.

Figure 3. *Credit-to-GDP ratio and gaps for the United States*



Notes: Some notes

A Derivations

Transfer functions

The transfer functions in the text are proportional to the general function:

$$Q(L) = \frac{1 - q_0 L}{(1 - q_1 L)(1 - q_2 L)}, \tag{A1}$$

where the moduli of q_1 and q_2 are less than one. The mean lag of $Q(L)$ can be computed as $M_Q = m(1)$ for:

$$m(z) = \frac{d \ln Q(z)}{dz} = \frac{q_1}{1 - q_1 z} + \frac{q_2}{1 - q_2 z} - \frac{q_0}{1 - q_0 z}. \tag{A2}$$

Thus:

$$M_Q = \frac{(q_1 + q_2) - q_0(1 - q_1 q_2) - 2q_1 q_2}{(1 - q_0)(1 - q_1)(1 - q_2)}. \tag{A3}$$

On the other hand, to obtain the weight coefficients, use the partial fractions decomposition:

$$Q(L) = \frac{1}{q_1 - q_2}\left(\frac{q_1 - q_0}{1 - q_1 L} - \frac{q_2 - q_0}{1 - q_2 L}\right) = \sum_{k=0}^{\infty} v_k L^k, \tag{A4}$$

from whence it follows that:

$$v_k = \left(\frac{q_1 - q_0}{q_1 - q_2}\right)(q_1)^k - \left(\frac{q_2 - q_0}{q_1 - q_2}\right)(q_2)^k = \frac{(q_1)^{k+1} - (q_2)^{k+1}}{q_1 - q_2} - q_0\left(\frac{(q_1)^k - (q_2)^k}{q_1 - q_2}\right). \tag{A5}$$

Variances

Let η_t be a white noise. A textbook exercise is to note that the first three Yule-Walker equations of the ARMA(2,1) process:

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \eta_t + \varphi \eta_{t-1}.$$

are given by:

$$\begin{bmatrix} 1 & -\phi_1 & -\phi_2 \\ -\phi_1 & 1 - \phi_2 & 0 \\ -\phi_2 & -\phi_1 & 1 \end{bmatrix} \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \gamma_2 \end{bmatrix} = \sigma_\eta^2 \begin{bmatrix} 1 + \varphi(\varphi + \phi_1) \\ \varphi \\ 0 \end{bmatrix},$$

where γ_τ is the τ-th autocovariance of z_t and σ_η^2 is the variance of η_t. Upon solving for γ_0

$$\frac{\gamma_0}{\sigma_\eta^2} = \frac{(1 - \phi_2)(1 + \varphi^2) + 2\phi_1 \varphi}{(1 + \phi_2)(1 - \phi_2 - \phi_1)(1 - \phi_2 + \phi_1)} \tag{A6}$$

The one-sided HP filter

The coefficients in (5) are simply $H_k = \theta_2 v_k$ for the case $q_0 = 0$, and $q_1 = \rho e^{\mathrm{i}\,\omega}$ and $q_2 = \rho e^{-\mathrm{i}\,\omega}$ being complex conjugates with modulus ρ and argument ω (i is the imaginary unit). Thus, upon replacing $q_0 = 0$ and $(q_1)^p - (q_2)^p = 2\,\mathrm{i}\,\rho^p \sin(\omega p)$ for $p = k + 1$ and $p = 1$ in (A5):

$$v_k = \rho^k \frac{\sin(\omega(k + 1))}{\sin(\omega)}, \tag{A7}$$

where we use $\sin(\omega k + \omega) = \sin(\omega k)\cos(\omega) + \cos(\omega k)\sin(\omega)$ and $\cos(\omega)/\sin(\omega) = \cot(\omega)$ to simplify this expression into (5). Note that, as a function of k, v_k is a sine wave multiplied by a function that converges monotonically to zero, making it hump-shaped. Thus, v_k is initially positive and increasing up to close to $k_1 = \pi/(2\omega) - 1$, such that $\sin(\omega(k_1 + 1)) = 1$, and then remains positive but decreasing up to $k_2 = \pi/\omega - 1$, such that $\sin(\omega(k_2 + 1)) = 0$. The actual turning point is k^* as specified in the text, which also accounts for the effect of ρ^k in the profile v_k, but in practice it is close to k_1.

For the mean lag, replacing $q_0 = 0$, $(1 - q_1)(1 - q_2) = 1 + \theta_1 + \theta_2$, $q_1 q_2 = \theta_2$ and $q_1 + q_2 = -\theta_1$ into (A3) gives (7). Finally, to compute the variance of the gap relative to that of the data, consider $\phi_1 = -\theta_1$, $\phi_2 = -\theta_2$, $\varphi = 0$ and $\eta_t = \theta_2 \Delta^2 y_t$ so $\sigma_\eta^2 = (\theta_2)^2\sigma^2$. Replacing these into (A6) gives (**??**).

The CumSum filter

This case corresponds to $q_0 = -1$ and $q_1 = q_2 = a$. The weights can be derived by setting $q_1 = a$ and $q_2 = a - \epsilon$ and letting $\epsilon \to 0$. Thus, define $f(x) = x^p$ and note that:

$$\lim_{\epsilon \to 0} \frac{(q_1)^p - (q_2)^p}{q_1 - q_2} = \lim_{\epsilon \to 0} \frac{a^p - (a-\epsilon)^p}{\epsilon} = \lim_{\epsilon \to 0} \frac{f(a) - f(a-\epsilon)}{\epsilon} = f'(a) = pa^{p-1} .$$

Thus, upon replacing $q_0 = -1$ and the limit above for $p = k + 1$ and $p = k$ in (A5):

$$v_k = \left[1 + \left(\frac{1+a}{a}\right) k\right] a^k . \tag{A8}$$

The coefficients in (15) are simply $A_k = \frac{1}{2} a v_k$.

For the mean lag, we replace $q_0 = -1$ and $q_1 = q_2 = a$ into (A3) and obtain (17). Finally, we compute the variance of the gap relative to that of the data in (**??**), by plugging $\phi_1 = 2a$, $\phi_2 = -a^2$, $\varphi = 1$ and $\eta_t = \frac{1}{2} a^2 \Delta^2 y_t$, so $\sigma_\eta^2 = \frac{1}{4} a^2 \sigma^2$, into (A6).

B Hamilton Linear projection filter

$$z_t = \tfrac{1}{h}(y_t - y_{t-h}), \tag{A9}$$

The gap is the demeaned version of z_t. Using a decay parameter $0 < b < 1$:

$$c_t = z_t - \bar{z}_t \quad \text{and} \quad \bar{z}_t = (1-b) \sum_{\tau=0}^{\infty} b^\tau z_{t-\tau} . \tag{A10}$$

To obtain the filter leading to the gap c_t, first define $S_n(L) = 1 + L + L^2 + \cdots + L^n$ and note that $1 - L^h = (1 - L)S_{h-1}(L)$. Then note that $(1 - bL)\bar{z}_t = (1 - b)z_t$ leads to $(1 - bL)c_t = b(1 - L)z_t$; finally, since $z_t = \frac{1}{h}(1 - L)S_{h-1}(L)y_t$ we finally conclude that $(1 - bL)c_t = \frac{1}{h} b S_{h-1}(L)\Delta^2 y_t$. Therefore, the gap c_t can be obtained following the recursion:

$$c_t = b\, c_{t-1} + \frac{b}{h} \sum_{i=1}^{h} \Delta^2 y_{t-i} . \tag{A11}$$

Transfer function:

$$c_t = B(L)\Delta^2 y_t\,, \quad \text{where} \quad B(L) = \frac{b}{h}\left(\frac{1 + L + L^2 + \cdots + L^{h-1}}{1 - bL}\right) = \sum_{k=0}^{\infty} B_k L^k\,, \tag{A12}$$

Define a polynomial $W(L)$ such that the coefficients of $B(L)$ will be those of $W(L)$ multiplied by b/h; in other words, $(1 - bL) = S_{h-1}(L)W(L)$. This means that beggining with $w_0 = 1$ we have $w_k = 1 + bw_{k-1}$ for $k = 1, 2, \ldots, h-1$ and $w_k = bw_{k-1}$ for $k \geq h$:

$$w_k = \begin{cases} \dfrac{1 - b^{k+1}}{1 - b} & 0 \leq k < h \\ b^{k-h+1}\left(\dfrac{1 - b^h}{1 - b}\right) & k \geq h \end{cases}$$

The weight profile $\{B_0, B_1, B_2, \ldots\}$ is hump-shaped and reaches a maximum at $k^* = h - 1$.

The mean lag of $B(L)$ can be computed as $M_B = m(1)$ for:

$$m(z) = \frac{d \ln B(z)}{dz} = \frac{1 + 2z + 3z^2 + \cdots + (h-1)z^{h-2}}{1 + z + z^2 + \cdots + z^{h-1}} + \frac{b}{1 - bz}\,. \tag{A13}$$

Thus:

$$M_B = \frac{h-1}{2} + \frac{b}{1-b}\,. \tag{A14}$$

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